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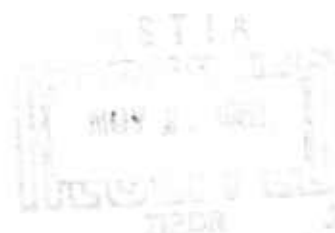
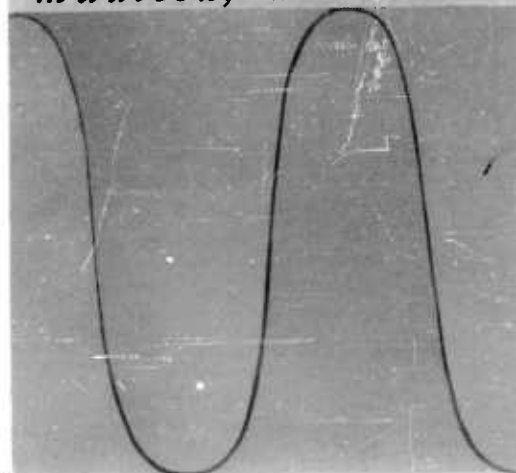
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COALITION BARGAINING IN N-PERSON  
GAMES

Evar D. Nering

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## COALITION BARGAINING IN N-PERSON GAMES

Evar D. Nering

1. Introduction: Since von Neumann and Morgenstern first advanced a theory of n-person games there have been suggestions that their theory is not satisfactory in all respects, and many alternatives have been proposed. [4], [2] It is evident that no one theory is going to meet all demands. The theory we wish to advance here also makes no claim to universal applicability. It deals with a restricted type of coalition formation in n-person games for relatively small values of n. It is a normative, rather than a descriptive, theory in that it provides a rational estimate of the bargaining position of each player in terms of his position in the structure of the game.

We shall consider n-person zero-sum games in which the sole strategic maneuvers are the formations of coalitions. When a coalition S is formed, the rules shall specify a payment  $v(S)$  to the coalition as a whole. Nothing is specified about the payments to individuals within a coalition. That is left for the bargaining process. How strong is each person's bargaining position? What demands can he reasonably make?

2. The bargaining process and the contract: It is not reasonable to try to answer questions about the relative strength of a player's bargaining position until some assumptions about the nature of the bargaining process itself are made. We shall assume the following bargaining and coalition formation structure.

A coalition already formed can carry on negotiations with another disjoint coalition to form a larger coalition. The coalition functions as a unit in such negotiations and an individual within a coalition may not negotiate separately. Since an individual cannot engage in further bargaining once he has joined a coalition he must negotiate at the time he enters any coalition for his share of any future gains of the coalition he joins. Thus at each stage of coalition formation the potential gains at all subsequent steps must be anticipated and the division of these net gains provided for.

Let A and B be two disjoint coalitions that are negotiating with each other. If they form a larger coalition (A, B) a binding contract must be signed at that time. This contract will specify the shares going to A as a whole and to B as a whole as a result of future gains of the coalition (A, B). This contract need not specify how these gains are distributed within each of the two coalitions since this was decided in the earlier contracts as the coalitions A and B were built up.

It is not unreasonable to demand that contracts for the coalitions be drawn up in this manner. In many economic situations there are devices to take care of this feature. When corporations merge, the new corporation issues stock in exchange for the stock of the merging corporations. If the corporations are A and B the exchange value of the stocks determines the distribution of the gains of the new corporation to A as a whole and to B as a whole. In turn, the holdings of each individual stockholder determines his share in the gains of his old corporation and, thereby, his

share in the gains of the new corporation.

In many situations where the gains are relatively unimportant compared to the efforts of negotiation, many persons resort to the "split-the-difference" principle to resolve conflict. The stock method for corporations is only slightly more sophisticated. We propose to show by example that the power positions of the bargainers are considerably more subtle. Of course, we shall assume that every possible gain, no matter how small, will be sought and bargained for.

We shall make additional assumptions about the nature of the bargaining. We shall assume that the process of building up the coalitions proceeds in steps. At each step two coalitions can unite into a larger coalition. Thus the number of coalitions is reduced by exactly one at each step. Three or more coalitions cannot unite into a larger coalition at the same time. These conditions are not unduly unrealistic because negotiations among three or more parties are difficult to carry out. However, other assumptions are clearly possible.

3. The inequalities: Let  $N = \{ 1, 2, \dots, n \}$  be the set of players. Let  $\pi_k = \{ A_1, A_2, \dots, A_k \}$  be a partition of  $N$  into  $k$  disjoint coalitions. If this is the set of coalitions that has been achieved by bargaining up to a certain point,  $A_i$  and  $A_j$  can consider the possibility of uniting to form a larger coalition,  $(A_i, A_j)$ . If they unite they can get the amount  $v(A_i, A_j)$  even if they do not extend their coalition any further. At any rate they will find themselves in a coalition game involving  $k - 1$  coalitions. We shall assume for the moment that we can resolve the shares for the

players in a coalition game with fewer than  $k$  players and that the share that  $(A_i, A_j)$  can expect for extending its coalition in any conceivable way can be evaluated. Let  $u(A_i, A_j)$  be the share of  $(A_i, A_j)$  at the termination of a particular sequence of coalition formations. The contract between  $A_i$  and  $A_j$  will have to specify how this amount will be split between them, if it should ever be realized.

Let  $x_{ij}$  be  $A_i$ 's share of  $u(A_i, A_j)$  and let  $x_{ji}$  be  $A_j$ 's share. Then

$$x_{ij} + x_{ji} = u(A_i, A_j). \quad (1)$$

Each of  $A_i$  and  $A_j$  can consider other coalition possibilities. Suppose that  $A_i$  considers forming a coalition with each of the other coalitions in  $\pi_k$ , and suppose that  $u(A_i, A_1), u(A_i, A_2), \dots$  are the payoffs to other coalitions including  $A_i$  at the end of a sequence of coalition formations in some sense comparable to each other. We shall go into detail later on what we mean by comparable sequences of coalition formations. Let

$\{x_{i1}, x_{i2}, \dots, x_{ik}\}$  be  $A_i$ 's shares of the payoffs to all these other possible coalitions. Let

$$x_i = \max \{x_{i1}, x_{i2}, \dots, x_{ik}\}. \quad (2)$$

We shall assume that  $A_i$  will consider only those coalitions in which its share is  $x_i$ . It may well be argued that  $A_i$  might be willing to join in a coalition in which its share is less than  $x_i$  in anticipation of a larger potential gain. This consideration has merit, but for this discussion we shall take the simpler assumption.

Combining (1) and (2) we obtain the system of inequalities



$$x_i + x_j \geq u(A_i, A_j) \text{ for all } i, j. \quad (3)$$

This is the fundamental system of inequalities with which we have to deal.

Certainly, all the inequalities would be satisfied if the  $x_i$  are taken large enough. However, the parties  $A_i$  and  $A_j$  can realize the amounts  $x_i$  and  $x_j$  by joining in a coalition if and only if

$$x_i + x_j \leq u(A_i, A_j). \quad (4)$$

This additional condition means that at least some of the inequalities in the system (3) must be satisfied as equalities. Let  $X_0 = \{x_1, x_2, \dots, x_k\}$  be a solution of the system (3) of inequalities. Those inequalities that are satisfied as equalities by  $X_0$  are said to be effective for the solution  $X_0$ , and the coalitions involved in the effective inequalities are said to be effective for that solution.

Let  $E$  be the set of effective inequalities for a particular solution  $X_0$ , and let  $r$  be the rank of  $E$ . The rank of all the right-hand sides of the inequalities in (3) is  $k$  so that  $r \leq k$ . Suppose  $r < k$ . In this case  $E$  has a family of solutions of which  $X_0$  is only one. The other inequalities in (3) are satisfied as strict inequalities so that there is an open range of values of the parameters of the solutions of  $E$  which includes  $X_0$  and still satisfies all the inequalities of the system (3). Thus the shares of the players could be adjusted so that another inequality becomes effective while the inequalities which are already effective remain so. We interpret this as meaning that the full bargaining possibilities of the situation are not being realized. We therefore define an effective solution of the system (3) as a solution in which the set of effective inequalities has rank  $k$ . We shall consider only effective solutions.

4. The 3-person coalition game: Let  $N = \{1, 2, 3\}$ . In the first round of coalition negotiation we have the following system of inequalities:

$$\begin{aligned} x_1 + x_2 &\geq v(1, 2) \\ x_1 + x_3 &\geq v(1, 3) \\ x_2 + x_3 &\geq v(2, 3). \end{aligned} \tag{5}$$

The additional condition that the system of effective inequalities be of rank 3 implies that all three inequalities are equalities. The solution is then

$$\begin{aligned} x_1 &= \frac{1}{2} \{v(1, 2) + v(1, 3) - v(2, 3)\} \\ x_2 &= \frac{1}{2} \{v(1, 2) - v(1, 3) + v(2, 3)\} \\ x_3 &= \frac{1}{2} \{-v(1, 2) + v(1, 3) + v(2, 3)\} \end{aligned} \tag{6}$$

No further coalitions are profitable because of the assumption that we are dealing with zero-sum games. Since these are precisely the conditions derived by von Neumann and Morgenstern in their analysis of the 3-person game it is no surprise that the results obtained here are the same. The three possible contracts are the three imputations of the finite solution which they obtained.

5. The 4-person coalition game: Let  $N = \{1, 2, 3, 4\}$ . In the first round of coalition negotiation we have the following system of inequalities:

$$\begin{aligned}
x_1 + x_2 &\geq v(1, 2) \\
x_1 + x_3 &\geq v(1, 3) \\
x_1 + x_4 &\geq v(1, 4) \\
x_2 + x_3 &\geq v(2, 3) \\
x_2 + x_4 &\geq v(2, 4) \\
x_3 + x_4 &\geq v(3, 4).
\end{aligned} \tag{7}$$

In this case the condition that the solution be effective implies that all inequalities are effective because of the zero-sum condition. Thus the solution is

$$\begin{aligned}
x_1 &= \frac{1}{2} \{v(1, 2) + v(1, 3) + v(1, 4)\} \\
x_2 &= \frac{1}{2} \{v(1, 2) + v(2, 3) + v(2, 4)\} \\
x_3 &= \frac{1}{2} \{v(1, 3) + v(2, 3) + v(3, 4)\} \\
x_4 &= \frac{1}{2} \{v(1, 4) + v(2, 4) + v(3, 4)\}.
\end{aligned} \tag{8}$$

After a coalition of two persons has been formed the three parties find themselves in a 3-person game. By an application of the previous arguments we can find the reasonable shares for each of these parties. But one of these parties is a coalition and their share must be divided between them.

Suppose, to be particular, that  $A = (1, 2)$  is the coalition formed in the first round. If they do not succeed in extending their coalition their joint share is  $v(1, 2)$ . If they do succeed in extending their coalition, their joint share is

$$\frac{1}{2} \{v(1, 2, 3) + v(1, 2, 4) - v(3, 4)\} = u(1, 2). \tag{9}$$

We have denoted this joint share by  $u(1, 2)$ . There are five other first-round coalitions that could have been formed. If one of these other coalitions had been formed and extended, it would also have earned a second-round share similar to (9). Since the first-round shares for each player do not depend upon the identity of the other partner in the coalition, the second-round shares will also have to compete or else the coalition that proposes too low a second-round share will not be formed. Thus, if  $\{z_1, z_2, z_3, z_4\}$  represent the maximum share to each player in the second step of coalition formation, we have

$$\begin{aligned}
 z_1 + z_2 &\geq u(1, 2) \\
 z_1 + z_3 &\geq u(1, 3) \\
 z_1 + z_4 &\geq u(1, 4) \\
 z_2 + z_3 &\geq u(2, 3) \\
 z_2 + z_4 &\geq u(2, 4) \\
 z_3 + z_4 &\geq u(3, 4).
 \end{aligned} \tag{10}$$

The system of inequalities (10) resembles the system of inequalities (7), but there is a slight difference. The function  $u(A)$  does not satisfy the zero-sum condition. However,

$$u(1, 2) + u(3, 4) = \frac{1}{2} \{v(1, 2, 3) + v(1, 2, 4) + v(1, 3, 4) + v(2, 3, 4)\}. \tag{11}$$

Since the expression on the right is fully symmetric in the four players, we have  $u(1, 2) + u(3, 4) = u(1, 3) + u(2, 4) = u(1, 4) + u(2, 3)$ . This is enough to guarantee that if a subsystem of rank 4 is effective, then all six inequalities are effective. Thus

$$\begin{aligned}
z_1 &= \frac{1}{2} \{u(1, 2) + u(1, 3) - u(2, 3)\} \\
&= \frac{1}{4} \{v(1, 2, 3) + v(1, 2, 4) - v(3, 4) + v(1, 2, 3) + v(1, 3, 4) \\
&\quad - v(2, 4) - v(1, 2, 3) - v(2, 3, 4) + v(1, 4)\} \\
&= \frac{1}{4} \{v(1, 2, 3) + v(1, 2, 4) + v(1, 3, 4) + v(1, 2) + v(1, 3) + v(1, 4) + v(1)\}.
\end{aligned} \tag{12}$$

The fact that this expression for  $z_1$  is fully symmetric in the players  $\{2, 3, 4\}$  shows that 1's second-round share is the same for any first-round coalition he should choose to join. Furthermore, we can write down immediately corresponding expressions for the second-round share of each of the other three players.

6. A numerical example: Consider the 4-person game with characteristic function

$$\begin{array}{llll}
v(1) = -20 & v(1, 2) = 30 & v(3, 4) = -30 & v(2, 3, 4) = 20 \\
v(2) = -40 & v(1, 3) = 0 & v(2, 4) = 0 & v(1, 3, 4) = 40 \\
v(3) = -40 & v(1, 4) = -10 & v(2, 3) = 10 & v(1, 2, 4) = 40 \\
v(4) = -20 & & & v(1, 2, 3) = 20.
\end{array}$$

According to formula (8) we have

$$x_1 = 10, x_2 = 20, x_3 = -10, x_4 = -20.$$

According to formula (12) we have

$$z_1 = 25, z_2 = 20, z_3 = 5, z_4 = 10.$$

In addition, there remains the calculations of the shares for those players that do not succeed in joining a coalition on the first round but do succeed on the second round. All the calculations will be summarized in the following table. In this table we shall use a vinculum to denote the first coalition and parenthesis to denote the second. Thus  $\overline{12}(34)$  means that a co-

alition of 1 and 2 was formed in the first round and in the subsequent 3-person game 3 and 4 formed a coalition.

Share for player

| Coalition structure | 1   | 2   | 3   | 4   |
|---------------------|-----|-----|-----|-----|
| $\overline{12}(34)$ | 10  | 20  | -25 | -5  |
| $(\overline{123})4$ | 25  | 20  | -25 | -20 |
| $(\overline{124})3$ | 25  | 20  | -40 | -5  |
| $\overline{13}(24)$ | 10  | -10 | -10 | 10  |
| $(\overline{132})4$ | 25  | -10 | 5   | -20 |
| $(\overline{134})2$ | 25  | -40 | 5   | 10  |
| $\overline{14}(23)$ | 10  | 5   | 5   | -20 |
| $(\overline{142})3$ | 25  | 5   | -40 | 10  |
| $(\overline{143})2$ | 25  | -40 | 5   | 10  |
| $\overline{23}(14)$ | -5  | 20  | -10 | -5  |
| $(\overline{231})4$ | -5  | 20  | 5   | -20 |
| $(\overline{234})1$ | -20 | 20  | 5   | -5  |
| $\overline{24}(13)$ | 10  | 20  | -10 | -20 |
| $(\overline{241})3$ | 10  | 20  | -40 | 10  |
| $(\overline{243})1$ | -20 | 20  | -10 | 10  |
| $\overline{34}(12)$ | 25  | 5   | -10 | -20 |
| $(\overline{341})2$ | 25  | -40 | 5   | 10  |
| $(\overline{342})1$ | -20 | 5   | 5   | 10  |

Several interesting and important observations can be made from an examination of the table. Player 2 finds himself in a strong position for the first round. He gains 60 units by joining any coalition. But he cannot share in the future gains of any of these coalitions. On the other hand player 4 gains nothing from the first round in joining a coalition. All his gains would come if the coalition is extended. For either player to demand more would have the effect of making him an undesirable member of a proposed coalition.

This game was used in an experiment performed by Kalisch, Milnor, Nash, and Nering. [1] The subjects used in the experiment were intelligent but untutored in any aspects of game theory. Furthermore, the stakes in the outcome were so small that motivation for a careful analysis was slight. Under the circumstances it is not surprising that the outcomes did not resemble the outcomes proposed here. All the players looked to the 30 units awarded to the coalition (1, 2) as dominating the game. Players 3 and 4 considered their positions so hopeless that they did not try to engage in a coalition with 1 or 2. Players 1 and 2 split not their net gains but the absolute reward evenly, i.e., each took 15 units. When the first-round coalitions were formed there were no agreements about the division of future gains. But all future gains were divided evenly according to a tacit understanding of fair play.

Our analysis shows that players 3 and 4 grossly underestimated their bargaining positions. As a result player 1 was assured a coalition with player 2 and at a favorable price. Actually, although the various players differ in bargaining strengths and the demands they can make, each makes



just as good a coalition partner as anyone else.

There are six 3-person games that result after the first coalition is formed, and they are quite different from each other. Player 4 finds himself in a better position if players 1 and 3 form a coalition than he does if 1 and 2 or 2 and 3 form a coalition. This raises an interesting point that will not be pursued at this moment. Player 4 cannot force two other players to form a coalition. But in this case he might find it advisable to start negotiations with player 2. If he succeeds in forming a coalition it doesn't really matter who it is with. But perhaps by negotiating with player 2 he can at the same time impede formation of the coalitions (1, 2) and (2, 3). Exactly the same kind of reasoning would lead player 2 to initiate negotiation with player 4, player 1 with player 2, and player 3 with player 2.

7. General considerations: Let  $\pi_k$  be a particular partition of  $N$  into  $k$  coalitions. We say that  $\pi_{k-1}$  follows  $\pi_k$  immediately if  $\pi_{k-1}$  is obtained from  $\pi_k$  by uniting two coalitions in  $\pi_k$ , provided that there is at least one effective solution of the corresponding system of inequalities for which the united coalition is effective. The coalitions which unite are said to be active. The sequence  $\pi = \{\pi_n, \pi_{n-1}, \dots, \pi_2\}$  is a play of the coalition game if each  $\pi_{k-1}$  follows  $\pi_k$  immediately.

For each player  $i$  and each play  $\pi$ , let  $(a_n, a_{n-1}, \dots, a_3)$  be a sequence of 0's and 1's where  $a_k$  is a "1" if  $i$  is in one of the two active coalitions in  $\pi_k$ , and  $a_k$  is a "0" if  $i$  is in one of the inactive coalitions in  $\pi_k$ . We call this sequence  $i$ 's play index. As far as  $i$  is concerned, two plays represent comparable strategic maneuvers if they have the same play index. Therefore,  $i$  will expect a comparable share in two plays with



the same play index regardless of which partner he chooses, and this is the basis for comparing the payoffs.

We have seen that the coalition games for 3- and 4- person games can be analyzed and fair shares for each play determined. Assume that coalition games with fewer than  $n$  players can be analyzed and fair shares determined. After the first coalition is formed, the players find themselves in a coalition game with  $n - 1$  players. If a player is inactive in the first round he becomes an individual in the resulting  $n - 1$  person game. By assumption his share as a result of future coalition formation can be determined. Our problem concerns the shares due to a player who is active in the first round.

First we must consider all plays with play index  $(1, 0, \dots, 0)$ . Since the coalition active in such a play is not extended, each coalition  $(i, j)$  must divide the amount  $v(i, j)$ . This leads to a system of inequalities like (3) in which the right sides are the  $v(i, j)$ . There may be several different effective solutions to this system of inequalities, and several different sets of effective coalitions. Let one effective solution  $X_1$  be chosen.

Then, consider all plays with play index  $(1, 1, 0, \dots, 0)$  for which the coalitions active in the first step are effective for  $X_1$ . If the coalition  $(i, j)$  is active in the first step it finds itself in a coalition game with  $n - 1$  persons and it extends its coalition one more step. Let  $u(i, j)$  be the share to  $(i, j)$  in this shorter game. We have another system of inequalities like (3) in which the right sides are these  $u(i, j)$ , but only those inequalities

can be considered which were effective for the solution  $X_1$ . All other coalitions are ignored in the first step of coalition formation and have no future shares to consider. The remaining system of inequalities is still of rank  $n$  and they have an effective solution. Let  $X_2$  be the effective solution chosen at this step.

We next consider all plays with play index  $(1, 1, 1, 0, \dots, 0)$ , etc. At each stage only those inequalities which are effective for all previously chosen solutions will be considered. A coalition which eventually becomes ineffective in this way will not be considered as a feasible coalition in the first stage of coalition formation. If we drop these coalitions and the corresponding inequalities from consideration from the first, the solutions will not be changed because the remaining inequalities are of rank  $n$ . The coalitions which remain will depend upon the entire sequence of solutions  $\mathcal{X} = \{X_1, X_2, \dots\}$ , and these coalitions will be called effective for  $\mathcal{X}$ .

In the manner described we can determine the fair share for each player in all plays in which he is active in the first step, remains active for a while, and then is inactive for the rest of the play. Consideration by each player of these plays determines which coalitions will be considered and which plays will occur. But there is always the possibility that after a player's chain of active participation is interrupted he may become active again. In such a case there is no difficulty deciding upon the fair share for the 2-person coalition formed in the first step; that is part of our induction assumption. But there is a problem in deciding how these two will

split their gains.

To see the problem in this case, consider the example given in Paragraph 6 as representing the terminal play of a 5-person game in which the player designated as "4" is the coalition (45) which was active in the first step. For players 4 and 5 there are three plays with play index (1, 0, 1). They are  $\overline{12}(3(45))$ ,  $\overline{13}(2(45))$ , and  $\overline{23}(1(45))$ . The shares going to (45) in these cases are -5, 10, and -5, respectively. There is no question of what they would prefer, but they cannot decide which coalition will form at a step where they are inactive. Thus all three possibilities must be considered.

Let  $E$  be the set of coalitions effective for  $\mathcal{X}$ , and let  $(i, j)$  be a coalition in  $E$ . For a given broken play index, e.g., (1, 1, 0, 1, ...), they can order the plays with that play index for both of them according to their preference. Other coalitions in  $E$  can do the same for all plays with that same play index for them. If  $u(i, j)$  is the value of the first preference for  $(i, j)$ , etc., then we must find an effective solution to the system

$$x_i + x_j \geq u(i, j) \text{ for } (i, j) \in E. \quad (13)$$

We then solve the corresponding system for the second preferences, etc.

8. Summary and conclusions: The theory of coalition negotiation advanced here is normative. As such it has an ad hoc flavor, which is to be expected. It is not intended to describe how people do bargain. Nor is there any guarantee that a person who bargains this way will necessarily be successful in bargaining with people who do not. If the game involves a large number of players the contracts would necessarily be so involved

that a sophisticated player could not induce an unsophisticated player to consider it. However, in games where the contracts would not have to be complicated it should be expected that a person who can evaluate his bargaining position accurately will fare better, on the average.

The spirit of the analysis is a direct extension of the analysis given the zero-sum 3-person game by von Neumann and Morgenstern. The extension is obtained by emphasizing the balance of bargaining forces rather than the stability of the payoff. In this way we are concerned more with the dynamics of coalition formation than with the static stability of the payoffs. This difference is clear in the numerical example of Section 6. The fair shares obtained there are not contained in any solution in the sense of von Neumann and Morgenstern. However, the Shapley quota point is one of the distributions of shares. [3]

The theory advanced here differs from the von Neumann - Morgenstern theory, and most alternatives proposed, in another important respect. In solving the system of inequalities (3) there is nothing to guarantee that a player's share will be at least as much as he could get alone, i.e., it might happen that  $x_i < v(i)$ . The assumption that  $x_i \geq v(i)$  has been one of the least challenged assumptions of n-person game theory.

Actually, the condition  $x_i \geq v(i)$  can be included without difficulty. Just add these inequalities to the system (3) and proceed with the analysis. If it has any effect at all, it will be to exclude some players from any coalition. We can call these additional conditions conservative conditions and the lack of such conditions as we have been assuming can be called risky

conditions. A position in between is also possible with some players playing conservatively and some playing with risk. In any case the same principles apply and only the actual calculations are affected.

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