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THE DESIGN AND DEVELOPMENT OF A HELE-SHAW APPARATUS FOR FLOW VISUALIZATION

THESIS

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Preface

An apparatus demonstrating the two-dimensional potential flow pattern about the cross-section of any arbitrary shape was designed by the author. The method first was demonstrated nearly 65 years ago by Professor Henry Selby Hele-Shaw. Since practically no technical drawings were available, the design of this study is original, except for certain physical arrangements of various components. The apparatus, known generally as a Hele-Shaw apparatus, was constructed in the Air Force Institute of Technology machine shop. Most of this investigation was spent building and perfecting the apparatus along with the other components necessary for its operation. After dependable operation was established, the cross-sections of several body shapes were tested under various conditions of flow. The most satisfactory operation was achieved with a Reynolds Number of about 1.

The graphical results of this investigation are shown in the appendices. I feel the most important contribution of a device of this type is toward the basic understanding of ideal fluid theory by the student of aerodynamics. This method presents to the student a vivid graphic solution of textbook theory and to the researcher the solution to an infinite array of unsolved problems.

I wish to acknowledge my indebtedness to the following for their contributions to the preparation of this report: The Staff and Faculty of the Institute of Technology, and especially to Professor H. C. Larson, Head of the Department of Aeronautical Engineering, for his aid and enthusiasm, Capt. R. R. Maestri and Capt. E. M. Romer for their help and encouragement, Mr. J. W. Haines and Mr. M. Wolfe of the Institute of

Technology technical staff for their assistance in the construction and operation of this apparatus, and finally, to my wife for her help in the final preparation of this study.

Henry J. Mehserle

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A	area, sq ft
a	radius of circular cylinder, ft
b	laminar boundary layer thickness
h	height of two-dimensional test section, ft
p	pressure, lb/sq ft
t	temperature, F
t	time, sec
u	velocity in x direction, ft/sec
V	velocity, ft/sec
V_u	uncorrected velocity, ft/sec
V_∞	freestream velocity, ft/sec
v	velocity in y direction, ft/sec
w	velocity in z direction, ft/sec
Δ	finite change
θ	angle, radians
μ	viscosity, lb_f-sec/ft^2
ρ	density, lb/ft^3
φ	isotherm
ψ	stream function

Abstract

A fluid flow mapper showing the exact solution of the two-dimensional potential flow streamline geometry about any arbitrary shape was constructed after the method of Hele-Shaw. Photographs were made of classical aerodynamic flow fields and an analysis to predict two-dimensional wind-tunnel boundary effects on streamline curvature was made. The application of potential flow theory to two-dimensional steady heat transfer was considered.

THE DESIGN AND DEVELOPMENT OF A HELE-SHAW APPARATUS FOR FLOW VISUALIZATION

I. Introduction

Background

The Hele-Shaw analogy is a very powerful demonstration of the two-dimensional steady-potential flow of an incompressible fluid around an obstacle. The apparatus consists of two closely spaced plates of glass, a device for inserting dye lines, and a liquid source. The demonstration of the flow is achieved by inserting the dye into the liquid sheet passing between the plates of glass (Ref 7:575). Figure 1 shows the general arrangement of the apparatus.

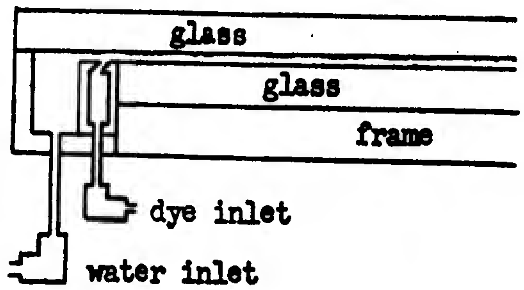

Fig. 1

Schematic Cross-section of a Hele-Shaw Apparatus

(From Ref 7:576)

During the year 1897, Professor Henry Selby Hele-Shaw demonstrated the device described above to the Institution of Naval Architects in Britain. The interest of that time was concerned with the surface resistance of a fluid to a body in motion. It was not until after the turn of the century that Prandtl presented his boundary layer theory which was one explanation of the region of surface resistance.

Interest was immediately gained by Hele-Shaw because the pictures made during his experiments were quite revolutionary. He compared his photographic plates to analytical plots of known stream functions and found very good agreement between the streamline geometries (Ref 4:22). This, then, was the beginning of what is now called the Hele-Shaw analogy. It is an analogy because a real viscous fluid is used to demonstrate the flow of a nonviscous frictionless fluid about a body. Hele-Shaw's apparatus did not help greatly with the surface resistance problem, but it did open a practical, easy way to analyze an infinite array of models which could not be reduced simply to a mathematical model.

Objective

The objective of this investigation was to build and operate a working model of the described apparatus. The report will discuss the history and theory, the design and fabrication, the actual operation of the apparatus, and the areas of investigation with the completed apparatus. The Hele-Shaw analogy probably provides the student of aerodynamics with the most vivid demonstration of streamline flow that can be devised. The choice of a working fluid in this investigation was water and the dye used was potassium permanganate. This dark purple dye, photographed over a lighted frosted-glass background, provides a fascinating picture of the flow geometry.

II. History and Theory of the Hele-Shaw Apparatus

History

During the last years of the nineteenth century, the investigation of the fluid region near the skin of sea going vessels was receiving much attention. The greatest minds of the time were beginning to shape what now is known as the boundary layer theory. Numerous investigations of this region were in progress and the apparatus described by this report was an outgrowth of one of these endeavors. The first apparatus designed by Professor Hele-Shaw used air bubbles for study of the character of the flow. The object of his first experiments appeared to be the examination of the layer of fluid adjacent to the shapes he had placed between the glass plates. Later, when dye lines were used in place of the air bubbles, the layer of water adjacent to the models was noticeably clean and without the dye. Thus, the existence of a layer which now is called the boundary layer was noticed during the experiments with this apparatus. However, no practical use could be made of this information. It could only be observed and measured (Ref 3:28).

Professor Hele-Shaw performed experiments with many differently shaped models. Pictures of these experiments can be seen in all of the early periodicals listed in the bibliography; however, one set of experiments should be mentioned because it is significant in demonstrating the power of the analogy. Early experiments showing the flow about a circular cylinder were conducted with water as the working fluid. No correction for streamline curvature was made at the boundary of the apparatus. A photographic plate was made of the flow with this setup and a comparison with theory begun. A corrected formula for the streamline position with straight boundaries was derived in order to obtain the best correlation possible (Ref 4:28). After careful plotting of the theoretical values, the two streamline pictures were almost identical (Ref 4:29). The next step involved a correction for streamline curvature at the boundary of the channel, thus enabling the photographing of the circular cylinder in an infinite field. A change was made in the working fluid. A more viscous substance, glycerine, was used. Dye lines again were forced into the flow and a photographic plate made of the results. The well known formula for the stream function of a flow field about a circular cylinder was used to plot the same field from theory. The result was a dramatic success -- complete agreement of the experimental with the theoretical. This, then, is really the value of Hele-Shaw's early experiments. With a relatively simple device, the theoretical flow about any shape could be demonstrated. G. C. Stokes verified the analogy with the Navier-Stokes equations and stated that the apparatus yielded "a complete graphical solution" (Ref 3:28).

Theory

The theoretical basis for the Hele-Shaw analogy was carefully developed by Stokes. The article appeared as "Mathematical Proof of the Identity of the Streamlines Obtained by Means of a Viscous Film with Those of a Perfect Fluid Moving in Two Dimensions" (Ref 4:43). The model for this analysis is an incompressible inviscid fluid flowing around a two-dimensional body. Since the flow pattern must satisfy the equation of continuity, the stream function ψ exists for the two-dimensional incompressible field. The stream function can be used to express the condition of irrotationality, as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \qquad (1)$$

If the velocity components of the flow are u and v in the x and y direction, then the stream function defines these velocities by the equations:

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad (2)$$

Thus Eq (1) becomes
$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad (3)$$

The flow through the apparatus is considered to be developed and down the channel as shown in Figure 2. The distance between the plates is of the order of twice the boundary layer thickness and in this case defined as 2b. The flow velocity down the channel is restricted to very slow or "creeping" velocity in the order of .1 fps. At this point, it is necessary to realize that the incompressible inviscid fluid of the two-dimensional steady-potential flow does not

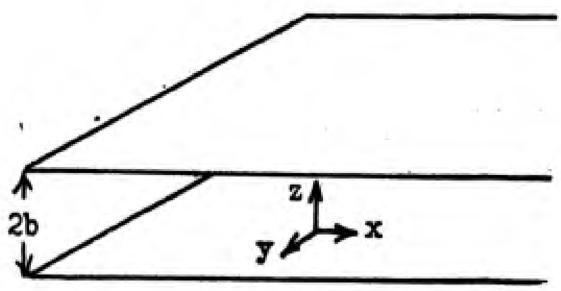

Fig. 2
Sketch of Co-ordinate System Orientation

exist and that the geometry of the streamlines is to be demonstrated by an incompressible viscous fluid. The dynamical equations of motion of a viscous incompressible flow are given by the Navier-Stokes equations, as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{\rho}\left\{-\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]\right\} \quad (4)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \frac{1}{\rho}\left\{-\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right]\right\} \quad (5)$$

and

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \frac{1}{\rho}\left\{-\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right]\right\} \quad (6)$$

These equations are simplified greatly by assuming steady flow, predominance of the viscous stresses over the inertia stresses, and no flow in the z direction. Thus, the first assumption causes $\partial u/\partial t$, $\partial v/\partial t$, and $\partial w/\partial t$ to vanish. Since the flow is slow, the viscous stresses assume great magnitude compared to the inertia stresses.

Therefore, the left-hand sides of the Navier-Stokes equations become zero. With w equal to zero Eq (6) becomes

$$\frac{\partial p}{\partial z} = 0 \tag{7}$$

Another reduction of terms can be made on the right hand side of the equations if the distance between the plates be kept small in comparison with the object around which the flow is to be observed. Since the velocity must be zero at the surface of the plates, it can be concluded that $\partial^2 u/\partial x^2$, $\partial^2 u/\partial y^2$, $\partial^2 v/\partial x^2$, and $\partial^2 v/\partial y^2$ are small enough to be neglected when compared with $\partial^2 u/\partial z^2$ and $\partial^2 v/\partial z^2$. The Navier-Stokes equations now take the form of

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} \quad \text{and} \quad \frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} \tag{8}$$

With Eq (8) it is possible to develop the solution of the velocity profile between the two parallel plates. Differentiating Eq (8) gives

$$\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}\right) = \mu \frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial z^2}\right)$$

Because the fluid is a continuum the order of differentiation can be changed so that

$$\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}\right) = \mu \frac{\partial}{\partial z^2}\left(\frac{\partial u}{\partial x}\right)$$

But $\frac{\partial u}{\partial x} = 0$ since the channel is uniform in area.

Therefore, $\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}\right) = 0$ and it can be concluded that $\frac{\partial p}{\partial x}$ is a constant.

The integration of Eq (8) is now a simple problem, and the resultant velocity profile is the exact solution. The boundary conditions are u = o when $y = \pm b$. The solution of Eq (8) now yields

$$u = -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[b^2 - y^2\right] \tag{9}$$

Dividing by b^2 Eq (9) becomes

$$u = -\frac{b^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[1 - \frac{y^2}{b^2}\right] \tag{10}$$

This is the velocity profile in a rectangular channel far upstream from the body where the coefficient of the bracketed term is U_∞. Hence, Eq (10) can be written

$$u = U_\infty\left[1 - \frac{y^2}{b^2}\right] \tag{11}$$

If u_o (x, y), v_o (x, y), and p_o (x, y) are the velocity and pressure distribution of the two-dimensional potential flow past the given body, the solution to Eq (8) is

$$u = u_o(x,y)\left[1 - \frac{y^2}{b^2}\right] \tag{12}$$

$$V = V_0(x,y)\left[1 - \frac{z^2}{b^2}\right] \tag{13}$$

and
$$p = -\frac{2\mu}{b^2}\int_{x_0}^{x} u_0(x,y)\,dx = -\frac{2\mu}{b^2}\int_{y_0}^{y} v_0(x,y)\,dy \tag{14}$$

Thus, u_0, v_0, and p_0 satisfy continuity and Eqs (4) and (5) for steady, incompressible nonviscous flow. Differentiating Eqs (12) and (13) twice and substituting into Eq (8), it follows that

$$\frac{\partial p}{\partial x} = -\frac{2\mu}{b^2}u_0 \quad \text{and} \quad \frac{\partial p}{\partial y} = -\frac{2\mu}{b^2}v_0 \tag{15}$$

By differentiating crosswise, the terms of the last result can be made equivalent.

Thus $\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}\right) = -\frac{2\mu}{b^2}\frac{\partial u_0}{\partial y}$ and $\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial y}\right) = -\frac{2\mu}{b^2}\frac{\partial v_0}{\partial x}$

are equated to show

$$\frac{\partial v_0}{\partial x} - \frac{\partial u_0}{\partial y} = 0 \qquad \text{(Ref 10:105)} \tag{16}$$

A summary of the above analysis can be made by listing the principle operations. The first step was a reduction of the terms in the Navier-Stokes equations. This was possible because it was assumed that there is steady flow, no flow in the z direction, and control of the flow by the viscous stresses. Other terms were dropped in the right hand side of the equations. Then, it was established that the pressure gradient in the x direction is a constant. Knowing this, it was possible to integrate Eq (8) for the velocity profile. All the required quantities are known and the solution to the problem follows as outlined above. Since Eq (16) is the same form as Eq (3), the condition of irrotationality is satisfied by the Hele-Shaw flow. The development of the theory in this section is given generally in reference 7, pages 578-580.

The interesting point of the Hele-Shaw flow is that the viscous forces of the parallel flow help demonstrate the potential flow. This is what Hele-Shaw found in his experiments conducted in 1897-1899.

III. The Design of the Apparatus

Factors

During the design of this apparatus two important factors were recognized. First, accuracy in all measurements was required. Therefore, to assist the machinist, the apparatus design was kept as simple as possible. All close tolerance work was held to the flat surfaces with .001 of an inch being the maximum deviation. The distance between the plates, 2b, was designed to .01 inch. Thus, the maximum allowable error was hoped to be 10 percent or less. The second factor was the criteria for the design, this being that all material be standard available stock found in any machine shop with the exception of the glass plates. Fortunately, the glass plate was already available in surplus photo windows. The machine shop supply provided brass stock, plexiglass, bolts, and all the other necessary fittings.

Principal Components

The principal components of the device are the test section composed of a brass frame and the two glass plates, the water manifold or inlet reservoir, the dye line manifold, and the dye reservoir. The detail drawings of Appendix A show the size, shape, measurements, and method of attachment of the principal components of the device; however, a few general comments about each component are necessary.

Glass Plates. The glass plates used in a Hele-Shaw apparatus should be of near optical quality. This insures the surface smoothness of the glass far more than standard plate. Standard plate works very well, but its accuracy cannot be assured. The bottom glass plate butts up against the combined water and dye manifold set.

This plate must have a carefully ground face that is flat with a sharp edge at the top. Where the ground face and the manifold set meet, a small amount of sealer will stop all leaks when the apparatus is under operating pressure. If this glass face is not ground true, heavy duty sealers can stop the leaks but the flow may be disturbed. During the experimentation with the apparatus as designed with the .357 inch glass plate, it was observed by the investigator that the water pressure on the flat plate surface caused a slight heaving or increase in the .01 inch distance. This troublesome effect can be overcome by using at least a .75 inch plate glass. It is believed that this thickness will prevent noticeable heaving at even the highest operating pressures.

Water Manifold. The water manifold was constructed of brass stock. The brass was milled to the specifications with careful attention to the tolerances of the top face. This face requires extreme accuracy since it seats against the upper glass plate. The water manifold is mounted to the frame with 1/4 inch set screws. The holes through the manifold are slotted so that adjustments can be made when it is secured to the apparatus frame.

Dye Manifold. The dye manifold was fabricated from plexiglass. This material is very easy to work with and permits the operator of the apparatus to watch the progress of the dye as it is bled into the flow. The dye outlet holes were drilled with a .013 inch drill to allow very fine bands of color. The dye manifold outlets were spaced every 1/2 inch in the central portion of the test section, and at the extremes 1 inch spacing was used. The outlets were drilled at a 45° angle to allow a forward component with the flow. Before the dye manifold was positioned in the water manifold, its contact surfaces were carefully coated with sealer to prevent leakage when the apparatus was under operating pressure. The effect of the sealer was enhanced by the fact that eight screws were used to hold the dye manifold tightly in place. A metal ring around the dye manifold inlet fitting prevents the brass male fitting from cracking the dye manifold inlet (see Appendix A, Figure 5).

Dye Reservoir. The dye reservoir is really the main control of the apparatus. The device is extremely simple to operate and has two moving parts, these being the two needle valves. Figure 3 of Appendix A shows the relation of the dye reservoir with the entire Hele-Shaw apparatus. The main control valve admits the water to the system. The water manifold inlet valve is open to permit water to pass through the apparatus. As this valve is adjusted to reduce the flow, a differential pressure is developed between the water and dye sides of the reservoir. Then the dye is forced to pass into the dye manifold and ultimately into the test section. The pressure gauge was included to help in the dye reservoir filling process and to register operating pressure at all times.

IV. Fabrication and Initial Operation

Cost

This apparatus was constructed in the Air Force Institute of Technology machine shop at Wright Field, Dayton, Ohio. The cost of materials was $5.50, not including the glass plate, and the shop time was approximately 124 man-hours. The criteria of exacting measurement was met with no more than .001 of an inch deviation in any critical measurement, that being the distance between the plates.

Initial Operation

The assembled apparatus did not operate perfectly the first time. Several problems had to be solved before the flow was satisfactory. These were air bubbles in the entire system, leakage, and turbulence in the water manifold.

Air Blockage. The problem of air in the system was solved by use of an air purging tank located between the dye reservoir water outlet and the water manifold inlet. Small amounts of air did manage to enter the flow through the dye inlet system, but were of no consequence. When model changes were made, air was admitted in great quantities. Again, this proved to be of no consequence since the flow forced all the air downstream to the exit. At no time was it necessary to make elaborate preparations to assemble the apparatus under water. Perfect operation was attained after assembly of all components in air.

Leakage. The leakage problem was never completely solved, but it was stemmed to a small drop or two. The use of nondrying commercial waterproof sealers helped tremendously at most places. Two types were used. For surfaces that have close tolerances a light to medium duty sealer was used because of its ability to be squeezed down. Where pressure in the apparatus would force out this light weight sealer, the heavy duty sealer was used with a high degree of success. Even after experience with the apparatus and sealers, a leak would appear now and then.

Turbulence. The dye manifold as shown in Figure 5 is a slightly modified version of the original. The original had a sloped entry to the test section. Turbulence in the water manifold, even at very low flows, caused disturbances which affected the bands of color emanating from the dye outlets. For this reason the squared-off dye manifold design was adopted. This permitted the dye lines to enter in a developed flow.

After these problems were treated, a relatively true flow was possible. However, the surface distortions of the glass and the spreading of the plates under pressure caused a slight distortion of the flow. It was possible to make some adjustment for these conditions. With carefully selected glass plates and very precise assembly, it is possible to get exact streamline analysis around any arbitrary shape.

V. Operation of the Apparatus

Working Fluid

The Hele-Shaw apparatus uses the water system in three ways. The water itself is used as a working fluid and the water pressure is used to provide a pressure gradient and a differential pressure. Only two needle valves and two hose clamps are needed to yield complete and efficient operation of this apparatus. These operations are the filling of the dye reservoir, the valve adjustment for normal Hele-Shaw flow, and the clearing of air bubbles from the water and dye manifolds.

Dye Filling Process

The dye filling process of the dye reservoir was recognized as being the most unpleasant task of the entire operation. Therefore, the design eliminated the necessity of touching and pouring the dye. This was accomplished by the following method. The schematic drawing, Figure 3 of Appendix A, shows the general setup of the operating apparatus. The main control valve is closed at the beginning to prevent accidental rupture of the dye reservoir diaphragm. Then the water manifold inlet valve is closed as is also the dye inlet clamp. After the dye reservoir filling tube is placed in the solution of mixed dye, the tube clamp is opened. The main control valve is now opened very carefully to allow the water system to pressurize the reservoir. The diaphragm is forced into the dye reservoir and purging begins. A pressure of 50 psi maximum is sufficient for this process and is read directly from the pressure gauge on the reservoir. As the desired pressure is reached, the main control valve is shut off tightly. To fill the reservoir, the water manifold inlet valve is opened. The water pressure immediately drops and the dye solution is drawn in through the filling tube. When the reservoir is full, the filling tube clamp is closed and the dye inlet clamp is opened. The apparatus is now ready for operation.

Normal Hele-Shaw Flow

The Hele-Shaw flow is begun by opening the main control valve. Very little pressure is necessary. During the experiments by the investigator, the pressure was set at approximately 1 to 2 psi. The water reservoir inlet valve can now be adjusted to cause the dye to flow into the test section. The width of the dye lines can be controlled by the water inlet valve or the dye line inlet clamp. The best operation was achieved when the flow was very slow. The Reynolds Number was usually held near 1. The characteristic length used was .005 inch or b (1/2 the plate separation). Velocities of the fluid were in the vicinity of .1 fps.

Air Bubbles

During most dye reservoir refills, a few air bubbles managed to get forced into the dye inlet system. This proved to be no problem since the mass flow of normal operation caused the bubbles to be carried along the channel. The greatest problem with air in the apparatus came during the changing of a model in the test section. The model change required that the top plate be removed. The air trapped in the two manifolds is expelled with high pressure operation (10-20 psi) after the top plate is secured in place again. However, great difficulty was encountered in attempting to remove air bubbles from the stagnation points of the models. After many methods had failed, a piece of fine wire was used to pull them into the main stream where they could be carried off by the flow.

Plate Separating Material

The models used in the experimental investigation of this report were all constructed of plastic electrical tape or celluloid. The same material used for the model was used for the gasket around the three sides of the apparatus. The exit end of the apparatus has no gasket since it must be open for the flow. All the material used for plate separation was .01 inch in thickness. The plastic tape was by far the easiest material to use since it adhered to the glass surface. The celluloid must be seated with sealing material.

Changing the model in the test section of the apparatus is accomplished by removing the top plate. Care must be exercised in securing the top plate for operation. If the wing-nuts are torqued too much, the glass can be broken.

VI. Discussion of Results and Recommendations

Results

The array of models that can be investigated in the Hele-Shaw apparatus is unlimited. The models chosen were considered of typical interest to the student of classical aerodynamics. The resulting photographs of the circular cylinder, the NACA 0012 airfoil, the flat plate normal to the flow, and the nozzle are mounted in Appendix B. Specially mounted photographic transparencies showing the wall effects on streamline curvature are mounted in Appendix C. All the photographic material of the Appendices was made during a steady-flow situation. However, the nature of the dynamics of the flow field can be observed while the pattern of the streamlines is being established. As the dye lines enter the flow and create the geometry of the field, it is possible to observe the division of the flow at the model's stagnation point, the acceleration of the particles, and in some cases, the appearance of a thin boundary layer on the model's surface. This boundary layer zone is characterized by the absence of the dye in the vicinity of the surface and can be seen most clearly along the top of the NACA 0012 airfoil at 30° angle of attack in Figure 8, Appendix B. Hele-Shaw commented about the appearance of this same zone of clear water many years ago (Ref 2:149). A color motion picture has been made by this investigator depicting the formation of the flow field about all of the models tested. Copies of this film are available through the School of Engineering of the Air Force Institute of Technology.

Discussion

The photographs of Appendix B show many of the important conclusions reached in the ideal flow of classical aerodynamics. Figure 8 shows the streamline geometry for the airfoil at high angle of attack. The steep gradient over the leading edge demonstrates clearly continuity and a decrease in pressure. The $\psi = 0$ streamline connected to the top surface is displaced from the trailing edge. This, of course, is the condition of zero circulation. Figure 7 shows the NACA 0012 airfoil at 10° angle of attack.

Figure 9 has the interesting feature of showing clearly the $\psi = 0$ streamline downstream of the flat plate. Mechanical arrangements made it difficult to align a dye line with the upstream $\psi = 0$ streamline in some cases. The presence of very great shearing stress is demonstrated in the blurring effect of the dye line behind the flat plate.

The nozzle of Figure 13 depicts the low velocity venturi effect (Ref 8). The narrow spacing of the dye lines at the throat show the higher velocity and the decreased pressure that is expected. However, the photograph makes it clear that the one dimensional treatment of venturi theory is somewhat in error.

Applications

The Hele-Shaw analogy can be useful in the calculation of wind tunnel corrections. The transparencies of Appendix C were analyzed for the effects of boundaries on the free stream velocity. Two-dimensional solid blocking by a circular cylinder is corrected by the formula

$$\frac{\Delta V}{V_u} = \frac{\pi^2 a^2}{3 h^2} \qquad \text{(Ref 9:278)} \qquad (17)$$

Eq (17) yields $\Delta V / V_u$ as .101 and .155 for the bottom transparencies of Figures 13 and 14 respectively. If the equation of continuity is used in conjunction with the stream function for the circular cylinder, relationships can be derived to yield a qualitative comparison with the theoretical values.

The stream function is given as

$$\psi = V_\infty \left(r - \frac{a^2}{r} \right) \sin\theta \tag{18}$$

Differentiating with respect to r yields V_θ. Solving for V_θ at r = a and $\theta = \frac{\pi}{2}$ where θ is measured from either stagnation point gives $V_\theta = 2V_\infty$. Thus, the ideal velocity across the top of the model is known. With a comparator, the distances between the center streamline and the first dye line were measured. To get the average value both distances on each side of the cylinder were calculated and averaged.

Thus

$$\frac{\Delta V}{V_u} = \frac{V_{actual} - V_{ideal}}{V_{actual}} \tag{19}$$

where V actual is V_θ at the top of the cylinder.

V_∞ was assumed to be the average velocity at the entrance to the test section. By continuity

$$V_\theta = \frac{A_\infty}{A_\theta} V_\infty \tag{20}$$

But theory predicts $V_\theta = 2V_\infty$. Hence, the ratio can be formed

$$\frac{\Delta V}{V_u} = \frac{\frac{A_\infty}{A_\theta}V_\infty - 2V_\infty}{\frac{A_\infty}{A_\theta}V_\infty} \tag{21}$$

or in the final form $$\frac{\Delta V}{V_u} = 1 - 2\frac{A_\theta}{A_\infty} \tag{22}$$

The values calculated for the transparencies of Figures 13 and 14 of Appendix C were .123 and .150 respectively. These results show definite agreement with theory. More important is the fact that the graphical method of finding the two-dimensional boundary correction has been shown. It could be extended to find the solid blocking factor about any arbitrary shape. Better agreement in the derived values with the theoretical values given by Eq (17) would be possible if the dye lines were closer to the cylinder surface.

Another application of the Hele-Shaw analogy can be made in the field of heat transfer. For two dimensions the heat conduction equation for an isotropic, homogeneous material in which the temperature distribution is invariant with time and in which there is an absence of heat sources can be written as

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0 \tag{23}$$

A proof has been given in reference 1, page 64-69 which shows that Eq (23) can be written in the form

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \qquad (24)$$

or

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \quad \text{where} \qquad (25)$$

φ = t, the temperature, and ψ is the path of constant heat flow. Thus ψ here is equivalent to the ψ of Eq (1). The lines of constant temperature given by a solution of Eq (24) would form an orthogonal set to the streamlines. Therefore, the Hele-Shaw analogy can be used quite easily to show the case of steady two-dimensional heat transfer. The same mathematical approach has been used to extend the analogy to the field of electromagnetism (Ref 6).

Recommendations

This apparatus can be extremely useful in analyzing the effect of the wind-tunnel boundaries on streamline curvature. Useful data could be derived with this apparatus in the calculation of difficult boundary problems which arise with the advent of new aircraft and re-entry vehicle shapes.

With slight modification, the present apparatus could be useful in demonstrating a source or sink in a uniform flow. The photographing of many basic flow fields would be possible. The requirement for demonstration of this type of flow is a small hole drilled in the center of the present apparatus test section where a small amount of fluid could be added or subtracted at will.

For very exacting work the recommendation of this investigator is that a new apparatus be constructed with at least .75 inch plate glass. The reasons for this recommendation have been stated. Also, more than one water inlet to the water manifold would be desireable, and a reduction of the distance between the plates is deemed feasible. Of course, the requirement for exact measurements becomes even more important as the distance decreases.

All of the ideal flow fields depicted by Appendix B have zero circulation. It is believed that a small amount of circulation can be induced with the proper technique. Possible schemes that can be tried for inducing circulation about the circular cylinder are rotation of the circular cylinder cross-section in a uniform flow, variation of the distance between the plates above and below the cross-section, and insertion of fake solid streamlines.

Another interesting problem that can be analyzed with this apparatus is the problem of turbulent flow. With a sizeable increase in the distance between the plates, the inertia terms of the Navier-Stokes equations are no longer negligible and with larger velocities the "creeping flow" analysis does not apply. The resulting flow will tend to be turbulent beyond a disturbing body in the flow field. The characteristic dye lines will blur suddenly at this point. Since no work was done in this area, the actual appearance of the turbulent zone could be the topic of an entire new investigation.

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Appendix A

Detail Drawings of Hele-Shaw Apparatus

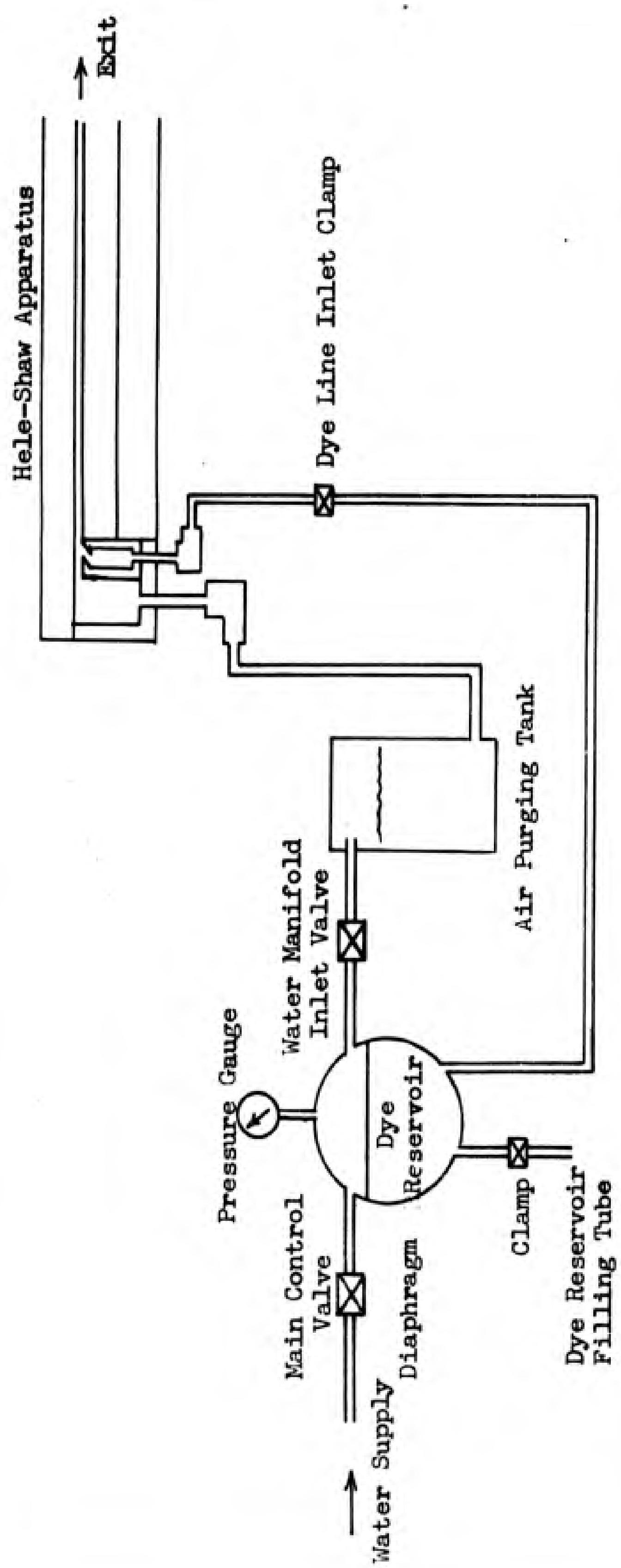


Fig. 3

Schematic Cross-section Showing the Arrangement of Dye Reservoir, Control Valves, and Air Purging Tank

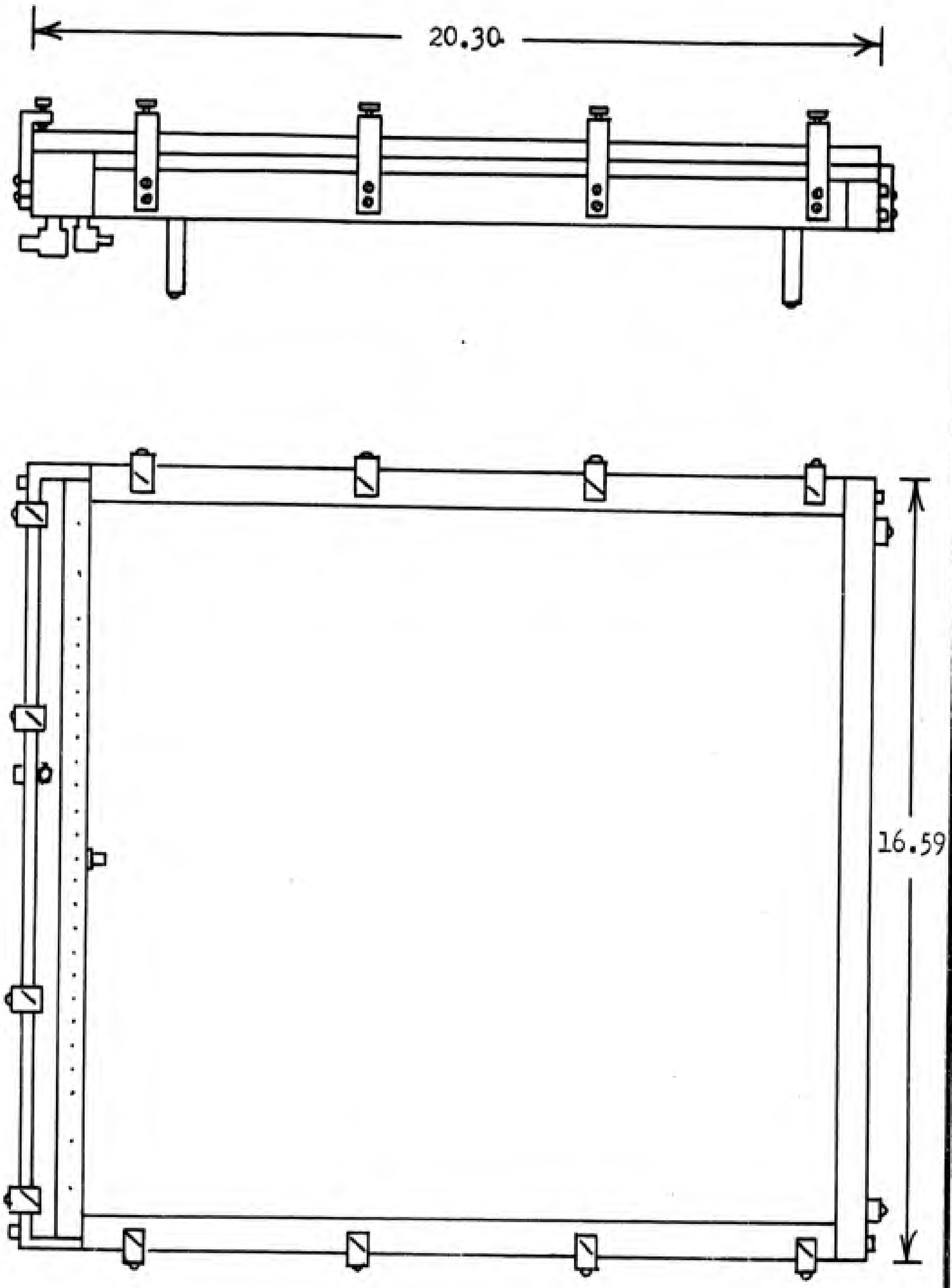


Fig. 4

Detail Drawing of Hele-Shaw Apparatus

Scale: 1 inch = 4 inches

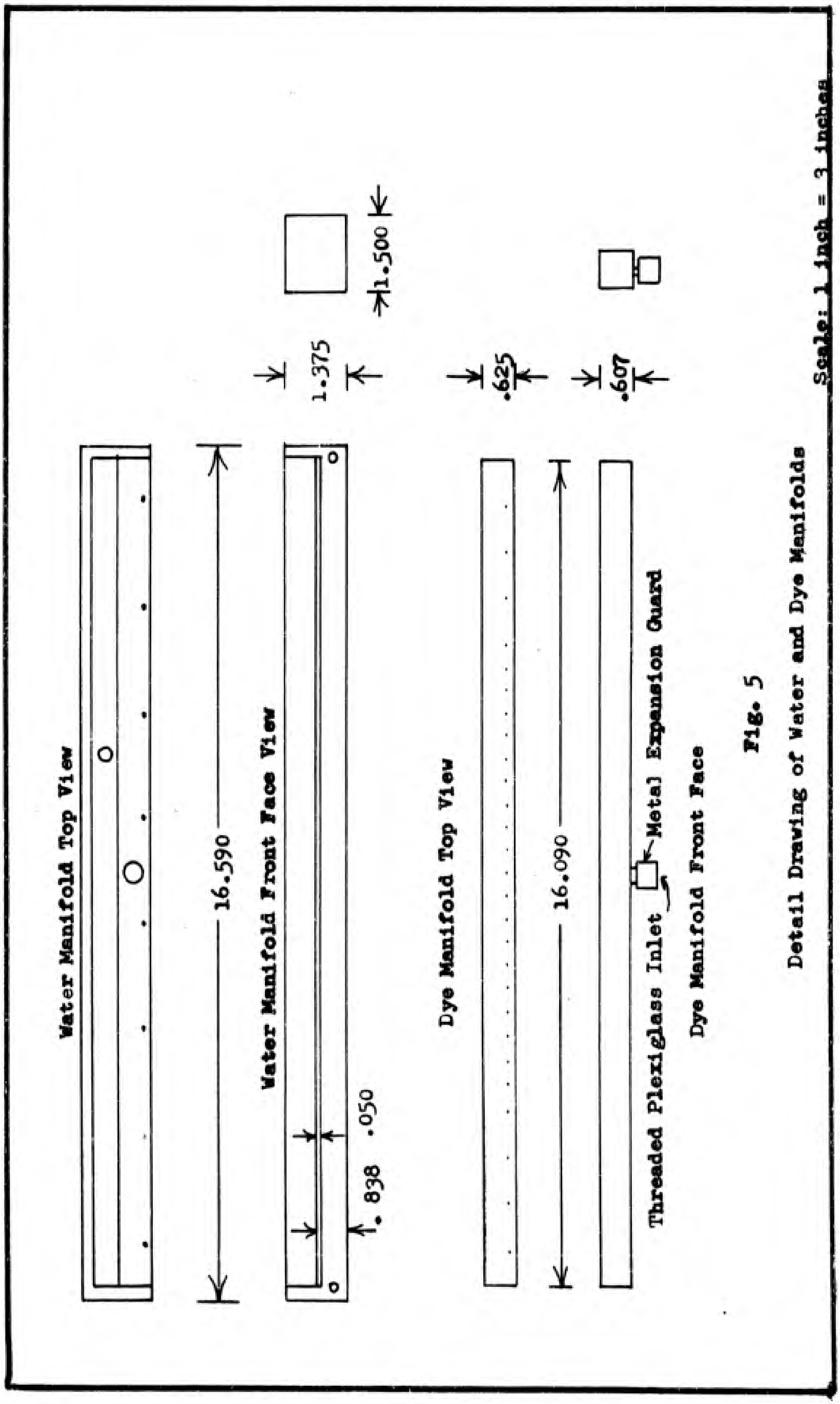


Fig. 5

Detail Drawing of Water and Dye Manifolds

Scale: 1 inch = 3 inches

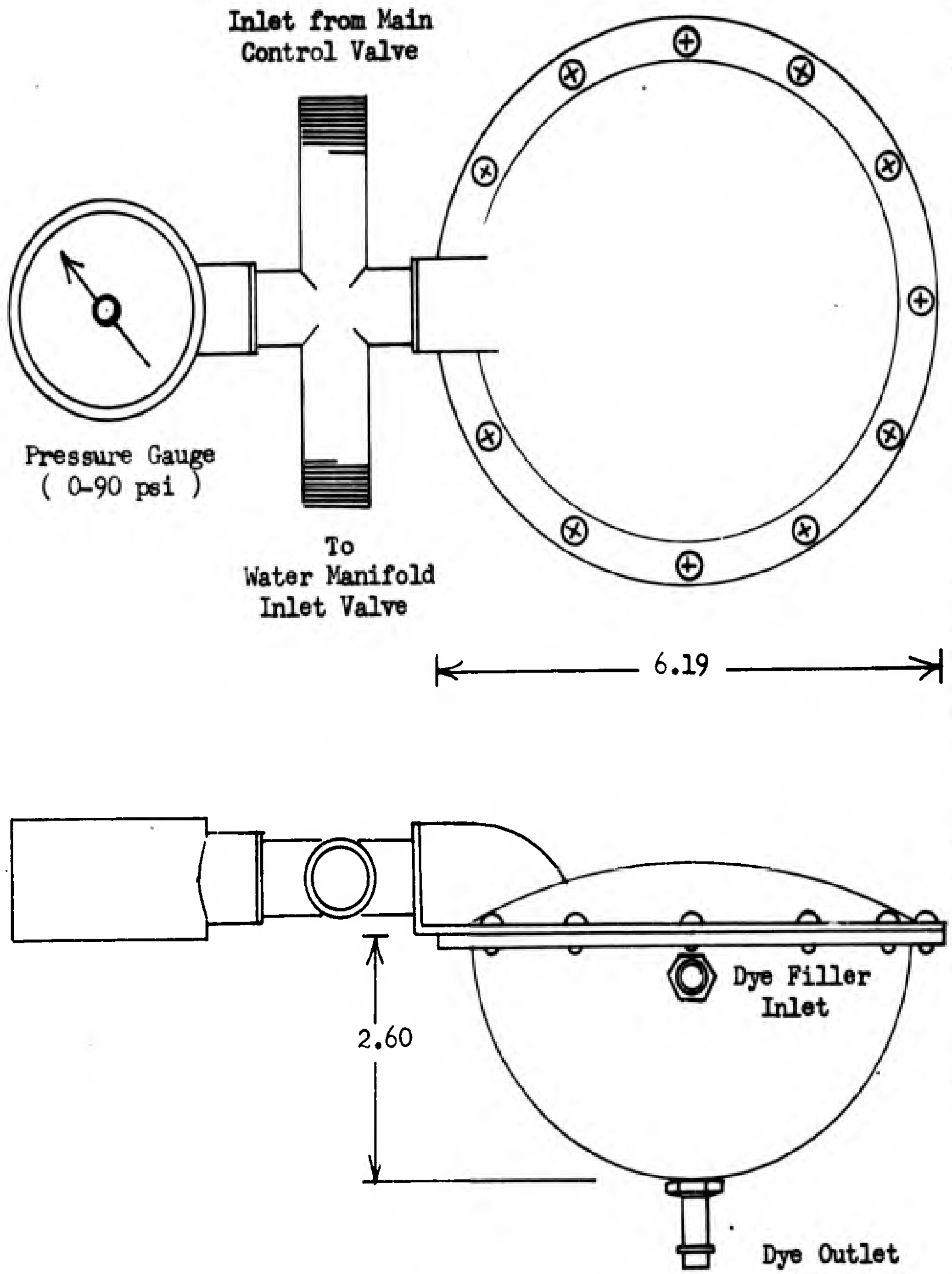


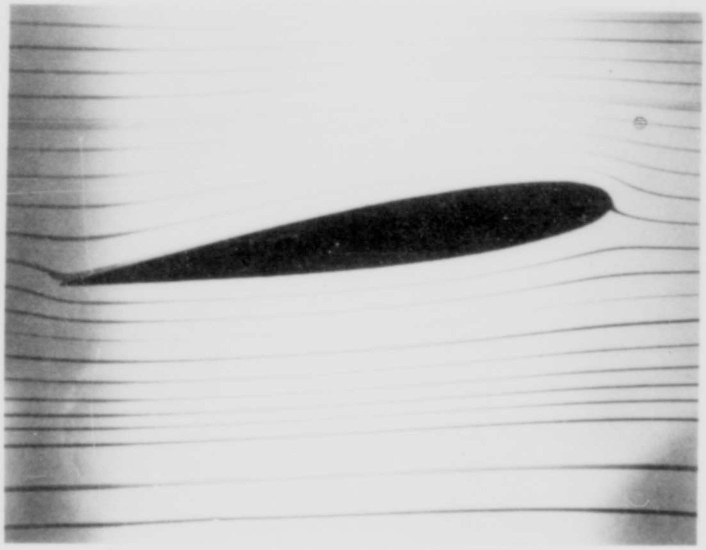
Fig. 6

Detail Drawing of Dye Reservoir

Scale: 1 inch = 2 inches

Appendix B

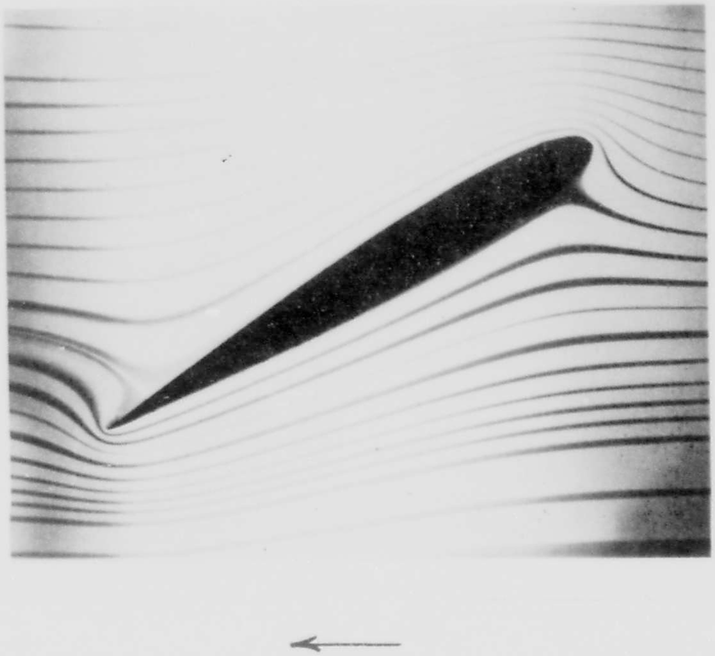
Typical Two-Dimensional Potential Flow Fields



Flow

Fig. 7

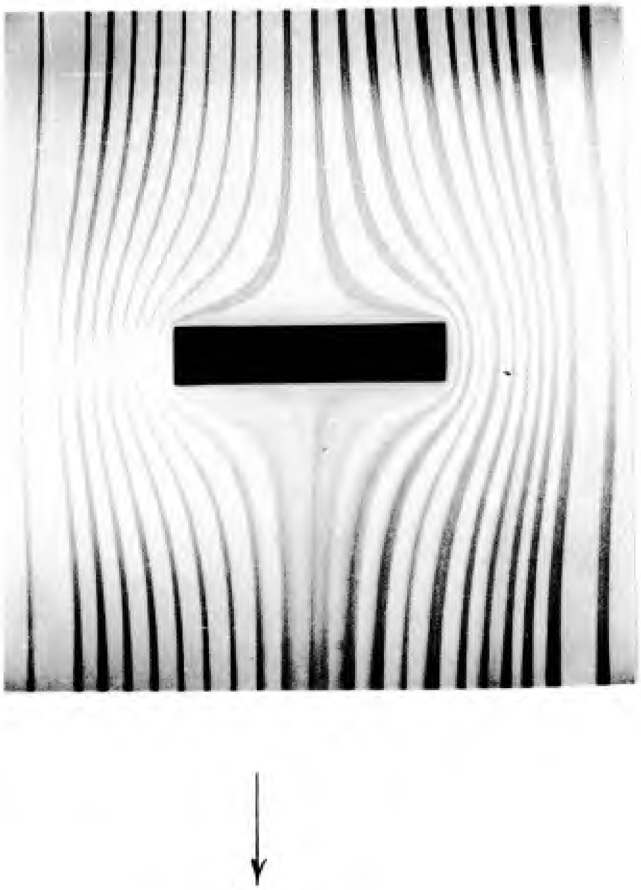
Two-Dimensional Potential Flow Field about
NACA 0012 Airfoil at 10° Angle of Attack



Flow

Fig. 8

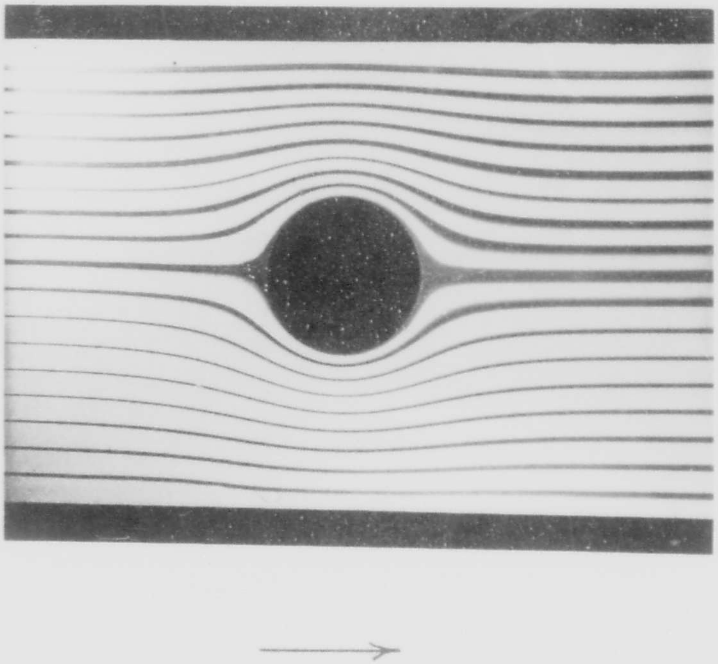
Two-Dimensional Potential Flow Field about NACA 0012 Airfoil at 30° Angle of Attack



Flow

Fig. 9

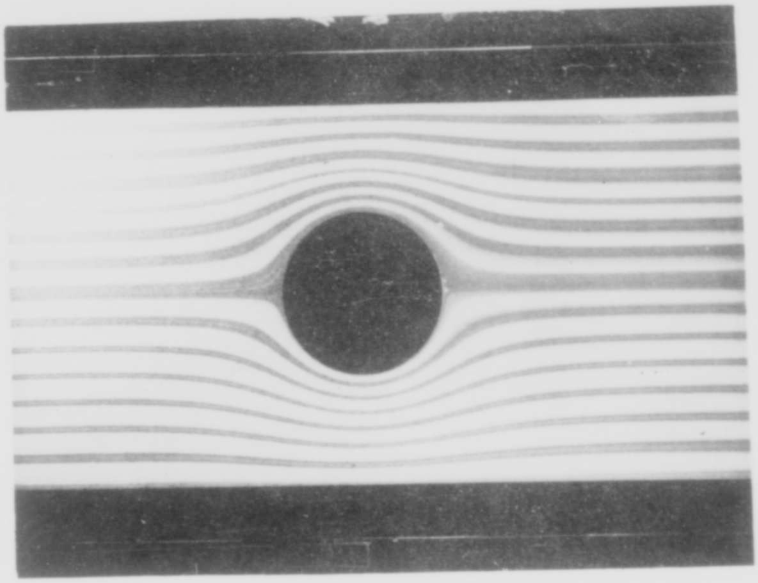
Two-Dimensional Potential Flow Field
about Flat Plate Normal to Flow



Flow

Fig. 10

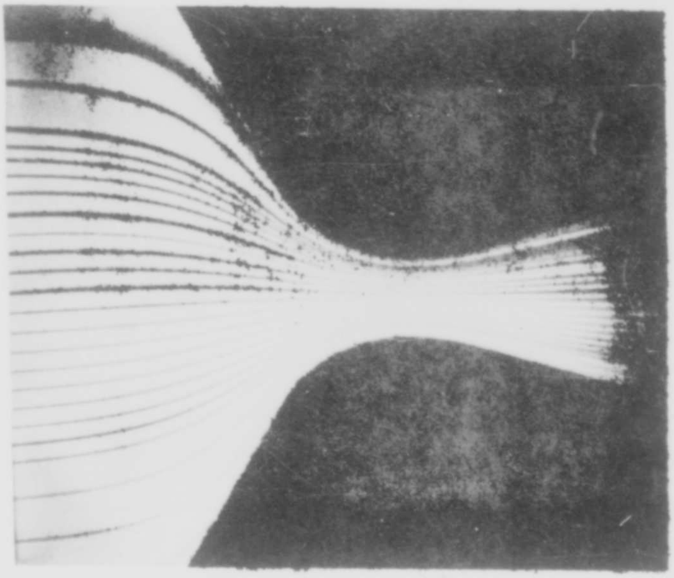
Two-Dimensional Potential Flow Field
about Circular Cylinder (a/h = .175)



Flow

Fig. 11

Two-Dimensional Potential Flow Field
about Circular Cylinder (a/h = .217)



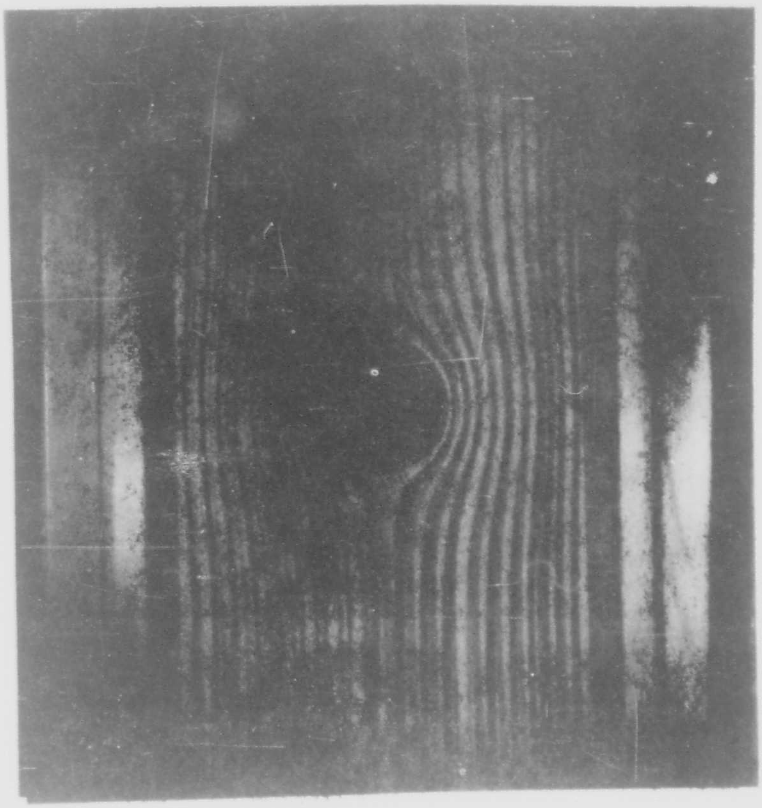
Flow

Fig. 12

Two-Dimensional Potential Flow Field
in Expanded Nozzle

Appendix C

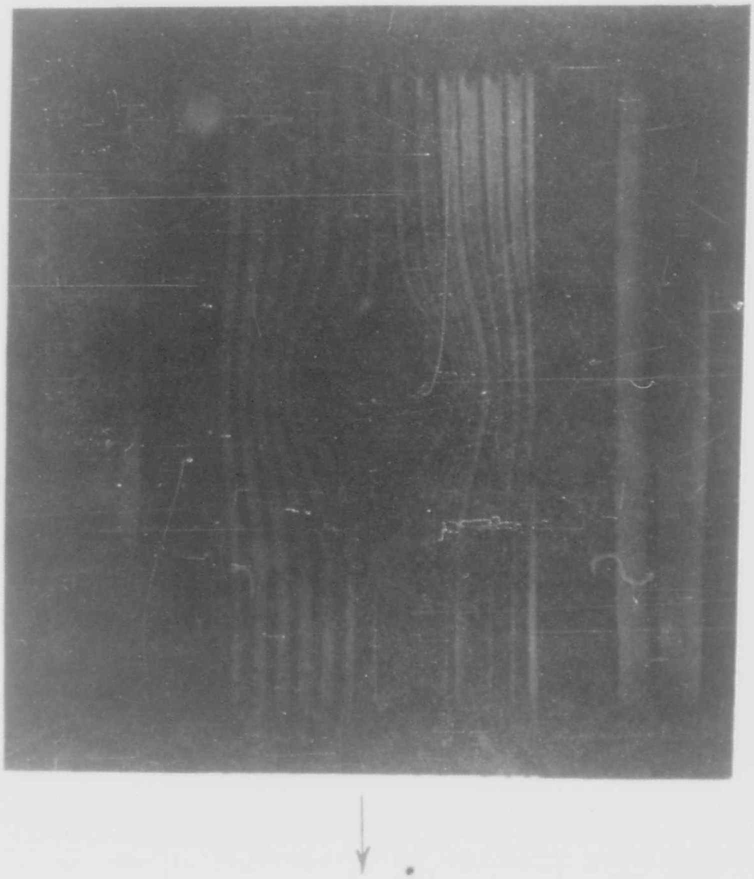
Boundary Effects on Streamline Curvature



Flow

Fig. 13

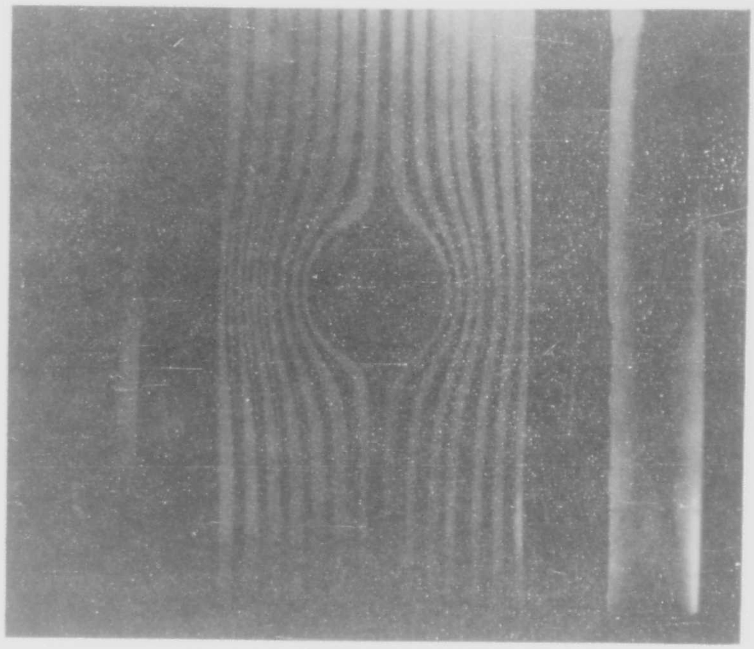
Transparency of Streamline Geometry about Circular Cylinder with Boundaries Corrected for Streamline Curvature and Circular Cylinder Overlay (a/h = .175)



Flow

Fig. 14

Transparency of Streamline Geometry about Circular Cylinder with Boundaries Corrected for Streamline Curvature and Circular Cylinder Overlay (a/h = .217)



Flow

Fig. 15

Transparency of Circular Cylinder with a/h = .175
Compared to Circular Cylinder with a/h =.217

Vita

Henry James Mehserle was born on [redacted] the son of Henry Joseph Mehserle and Malinda [redacted]erle. After graduation from the [redacted] School, [redacted] in 1950, he entered the United States Military Academy at West Point, N. Y. Upon graduation, he was awarded the degree of Bachelor of Science in Engineering and was commissioned as a Second Lieutenant in the United States Air Force. He entered pilot training in 1954 and graduated in August 1955. His military assignments prior to his coming to the Air Force Institute of Technology were as fighter-interceptor pilot in the 58th and 44th Fighter-Interceptor Squadrons and as Aide-de-Camp to the Commander of the 11th Air Division (DEF) in Alaska.

Permanent address: [redacted]

This thesis was typed by Nancy Mehserle