MEMORANDUM
RM-2735-PR
DECEMBER 1961

OPERATIONAL CRITERIA FOR THE DESIGN OF MISSILE READINESS TESTING PROGRAMS AND EQUIPMENT

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PREPARED FOR:

UNITED STATES AIR FORCE PROJECT RAND

The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

This is one of a series of memoranda produced by Project ACE (automatic checkout equipment) on the broad subject of automatic test and checkout equipment. This memorandum is intended to serve two basic purposes:

- to demonstrate how missile readiness is influenced by equipment, weapon systems, and operational factors

and

- to develop aids toward determining an effective readiness testing program for each component, hence for the entire weapon system.

The work is intended to be useful to designers of checkout equipment and ground support equipment, and to persons concerned with over-all integration of weapon systems.

Appendix C, 962 pages of machine-computed inputs for readiness testing programs, is not included in this volume. Requests for it should be addressed to The RAND Corporation.

SUMMARY

The increasing complexity of modern weapons and the growing importance of their near-instantaneous readiness for action have forced development of very rapid readiness testing procedures, which of necessity, had to be at least partially automated for speed. Because much of the automatic checkout equipment (ACE) was developed to meet immediate needs, without careful integration of its design with that of the prime equipment and ground support equipment, technical misfits and inefficiency have resulted.

This memorandum is intended to demonstrate how missile readiness (the probability that the missile is operative and ready to launch at any future time) is influenced by equipment, weapon systems, and operational factors. It also aims at developing aids toward determining an effective readiness testing program for each component, and hence for the entire missile or weapon system.

The report develops a mathematical model that relates missile readiness to relevant physical and operational factors. The first part of the model assumes no physical constraints inhibiting the design of the ground system. For this, the designer need only develop a readiness testing program compatible with the system concept and missile design, assuring adequate safety and yielding the best potential missile readiness (Sec. III and Appendix B). A numerical demonstration is included (Sec. IV). The second part of the model assumes a constraint on the design problem, such as limited silo space for test equipment (Appendix A).

In the model, the missile is considered a combination of several independent components, but to broaden the usefulness of this work, extensions to a system concept are included (Sec. V).

This memorandum is one of a family of related studies, devoted to aspects of automatic checkout equipment, that have been produced by Project ACE. These are cited as appropriate throughout the text.

Since production of the numerical illustration required use of a high-speed computer, a data volume was prepared for users of the model who might not have access to computing service. These data (Appendix C) permit a user to choose his own set of input parameters and to read missile readiness values directly from the appendix. However, because of its bulk (962 pages), Appendix C is bound separately. Requests for it should be addressed to The RAND Corporation.

CONTENTS

Appendix C, Machine Computations, because of its bulk (962 pages), is not included in this volume. Requests for Appendix C should be addressed to The RAND Corporation.

I. INTRODUCTION

The increasing complexity of modern weapon systems, as exemplified by the ICBM, has made imperative a new concept of weapon readiness testing. Any system as complex as an ICBM requires a significant manual effort to check out thoroughly the parts and subsystems that, by a malfunction, would likely abort the missile's mission. Thus, engineers studied the systems and subsystems, and, using computers and modern techniques, began to develop automatic readiness testing equipment and procedures. This memorandum will discuss readiness testing, with two primary objectives:

- to demonstrate how missile readiness is influenced by equipment, weapon system, and operational factors

and

- to develop aids toward determining a readiness testing program for the entire missile

To these ends, the memorandum will develop a mathematical model that relates missile readiness to relevant physical and operational factors.

The following definitions and discussions are offered as an introduction to the problem addressed and as an aid to understanding the language used in this memorandum.

MISSILE READINESS is the probability that each function (on a higher level, the entire missile) is operative and ready for launch at any future time.

READINESS TESTING* is that part of the test operations performed while the missile is on the ground.

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*Testing and checking are synonymous in this memorandum. However, there are some persons in the field of checkout equipment who differentiate between these terms. For example, they consider maintenance checking as a more detailed examination of hardware than would be done in confidence testing of the same hardware.

Parts of a missile (missile functions) perform individual roles, e.g., they propel the missile, or they guide it, etc. In addition, these functions must operate together to accomplish a mission. Because missile hardware demonstrates failure characteristics, this hardware may require readiness testing.

Readiness testing is a set of tests performed during the interval from the time the missile is in place to the time a launch order is given. Because of present rapid reaction weapon system requirements, the testing is usually done with automatic checkout equipment (ACE). This testing has two purposes: (1) to detect failures that have occurred, and (2) to inform the commander of missile force status. As already noted, failures can be caused by missile hardware unreliability. They also can be caused by a testing procedure. Since all failures act to degrade the missile's readiness, the possibility of test-caused failures must also be considered. In any event, prior to a launch attempt, failures can be detected only by testing. Three kinds of readiness testing methods are considered--continuous monitor, periodic check, and leave alone.

A READINESS TESTING PROGRAM is the combination of each testing method applied to each function of the missile.

Factors that enter the development of a readiness testing program are the potential capabilities of the test equipment to detect and to isolate faults, and the expected propensities of the test equipment to indicate falsely a defective condition. Missile failures that are undetected will potentially degrade the effectiveness of the force. Good parts that are called bad by the test equipment will result in unnecessary down time and expense for repair.

Time, such as the time required to test a missile and to repair malfunctioning parts, is another factor that must be considered. It is possible that the system maintenance concept includes a planned delay in repairing a known malfunction. For example, it may be more effective or more economical to spend money for more missiles than to spend money for a massive maintenance system capable of responding immediately to a known malfunction. The reasons for requirements to account for test, repair, and maintenance times are obvious. A missile must be ready to launch in order to be launched. If a missile is undergoing test, maintenance, or awaiting maintenance, it is certainly not launch ready.

It can be seen, then, that more than simply the details of such physical factors as the test signal requirements and measurement techniques enter into the determination of a readiness testing concept and program. Readiness testing plays an operational role. It affects, and in turn is affected, by other operations. Therefore, these operational factors should be accounted for when determining the concepts and the programs. An operational analysis should be performed to specify what tests are best done, with what type of equipment, in what manner, and with what frequency. This memorandum is an aid to such an operational analysis. The analysis should be integrated with hardware analyses, as estimates based on physically oriented studies are necessary inputs to an operational study. The operational study would then define the design goals that tend to give the best operational capability, and these goals must then be translated into physical items.

PHYSICAL FACTORS are specifications of material aspects. Areas, volumes, and weights are examples.

OPERATIONAL FACTORS are statements specifying what the equipment is to do, how well it must do these tasks, and how often.

A statement that a ballistic missile is to travel 1200 n mi, carrying a payload of 1000 lb, describes some operational factors.

CONSTRAINTS limit design freedom.

For example, the military may impose a safety requirement that certain functions on a ballistic missile warhead be monitored continuously. On the other hand, the model may show that for maximum missile readiness, these same functions should be checked only periodically, or even left alone. In such a situation, maximum missile readiness is reduced by the imposition of the constraint.

A MATHEMATICAL MODEL is a mechanism to describe a physical entity or operation in a manner that lends itself to logical interpretation or analysis.

Mathematically, the present analysis views the ground life of a missile as a compound Markov process. For application, a detailed understanding of the mathematical methods underlying the design aid developed is not necessary. The body of the memorandum is devoted to a discussion of readiness testing considerations, an application-oriented description of the design aid developed, and an example of the use of the method. The mathematics employed is described and developed in an appendix.

In constructing such models, certain simplifications and assumptions must be employed. Situations arise in which it is either impractical or beyond the analyst's ability to quantify properly certain phenomena. Consequently, a model should be used with caution. The degree of caution, in general, will be dictated by the crudeness of the model and the uncertainty surrounding the input data. In addition, a system under consideration may have different characteristics than those postulated in the model development, and it is quite likely that the results of the model will not be appropriate. Finally, the problem is aggravated by the fact that this analysis, and others like it, deal with idealized conditions. It is implicitly assumed that all missiles of a type will be identical, as will all units that use the missile. Both assumptions require qualification. Missiles of a type will vary in their detailed characteristics. Units within a missile force could have different operating methods or rules. Because of these considerations, general solutions may require model modification to adapt to varied situations.

The model described in this memorandum has two adverse characteristics. First, the model uses a "lumped-parameter" description of the physical system. A missile function (possibly the result of the operation of many components and even several "black boxes") is assumed to be described completely by a single set of parameters. This grossness of system description is justified in part by the nature of the equipment design and operation, and further, is in keeping with the nature of the descriptive data available.

The second adverse characteristic of the model is that its estimates of reliability must be based on anticipated standing and operating failure rates and checkout-caused failures. Checkout equipment capability estimates are also anticipated. Some of these estimates must be considered as purely speculative, especially during the early stages of a weapon system program. Unfortunately, it is at this time that preliminary decisions as to ground system design must be made. However, this model will provide an objective means for making the best use of reliability estimates. As the reliability and equipment performance estimates improve, early decisions can be reviewed and this method is useful for the re-evaluation.

This memorandum was prepared within the context of a broad study of automatic test and checkout equipment. It addresses just one problem in a spectrum of ground system design problems; other memoranda address closely related problems. These companion works are cited as appropriate throughout this text.

II. READINESS TESTING CONSIDERATIONS

PURPOSES OF READINESS TESTING

Readiness testing of ballistic missiles during peacetime is performed to detect malfunctions that have occurred, and to inform the commander of the status of his missiles. Although any sharp differentiation between these two purposes could be called arbitrary, the purposes describe different aspects of missile testing.

The detection of malfunctions is necessary to keep missiles on alert, even at the expense of causing malfunctions by testing. Subsequent actions repair the malfunctioned missile.

The commander needs to know which missiles in his force are available for use, i.e., mission ready. Perhaps some time estimate of the missile's return to availability, which is a purely military requirement, can be included. Hence, readiness testing is involved in the two roles, missile support and command.

Different methods of readiness testing supply information to the missile support and command systems in different quantities and in different degrees of accuracy and timeliness. (As an illustration, continuous monitoring of a missile function yields more data than would a 30-day periodic check.) Because of these differences, the requirements for readiness testing for each of the two roles should be examined separately. Hopefully, the readiness testing concept that is preferred for one role will also be effective for the other. It is not clear that either role can be made subordinate to the other. A "down" missile that the commander thinks is "up" could severely degrade a military mission, and lack of status knowledge could handicap military planning. On the other hand, perfect command knowledge is not desirable if this knowledge comes at the expense of test-induced failures. Perhaps information needed for maintenance can be obtained with a test frequency that is appropriate for requirements of command information.

Reference 1, a companion report, is a study of the data-processing implications of automatic equipment. It addresses, in part, the problem of obtaining support and command data from automated ground systems, the type of data required, the desirable frequency of data generation, and the interaction of command-data requirements with support-data requirements.

TEST LEVELS

Basically, there are two levels for testing, a system level and a functional level. Consider the system level first. A missile system, by the nature of its design and anticipated employment, will tend to dictate an overall concept for its readiness testing. (For example, a missile that is to have an in-silo repair capability probably could be checked periodically with little or no additional cost for checkout equipment, as maintenance testing equipment could also serve as readiness testing equipment.) Within the broad context dictated by the missile's operational and physical characteristics, other considerations would then be brought to bear to determine the system readiness testing concept. System cost is the primary consideration here.

Missile readiness inspections require equipment and people, tend to cause or to induce failures, and tend to wear out missiles and ground equipment. When determining the system concept, it appears that a reasonable criterion for the concept's effectiveness (neglecting command data) is the maximization of the total number of missiles ready for a given system cost. This is equivalent to maximizing missile up-time per dollar spent on testing and repair. This criterion is used in Ref. 2, a closely related report that can be used to determine the best time between inspections for a static, alert missile.

Another important consideration is that at the system level, system characteristics must be used, e.g., the determination of a system periodic inspection policy must be based on the over-all system mean-time-to-failure (MTF). Average repair times and average repair costs are elements of an investigation on the system level, but these averages aggregate the effects of the individual components--as planning factors are aggregates in a military war game. This aggregation results in reduced sensitivity to the contribution of the individual components.

This report addresses the second level of testing, that of the functions. For this, we assume three initial inputs:

- a system time-between-periodic-tests, e.g., T = 30 days
- a system time-between-periodic-inspections for those functions monitored continuously, e.g., A = 60 days
- a system replacement period for functions never tested, e.g., R = 6 months

Given this system description, then the individual idiosyncrasies of each missile function are related to testing methods to find the best method for maximum readiness of each function. If a particular component, because of a high failure rate, requires continuous monitoring for maximum readiness, the analysis either will indicate this fact or show the consequences of adopting a periodic check for this function. If some other function, because of a low failure rate, is best left alone, the analysis shows that. By maximizing component readiness, the readiness of the missile as an entity is maximized.

Admittedly, the two levels can be profitably combined. The system method[2] depends, in essence, on a missile MTF that is in turn affected by the choice of testing methods using the functional method. (For example, if the test method is continuous monitor, the missile function(s) must be operating. If the test method is leave alone, the missile function is standing. Operating

MTF is usually much less than standing MTF.) Thus, an iterative design process could be needed. First a system test concept would be chosen and used in the functional method. Then the system concept should be re-examined in the light of the missile system failure characteristics that result from the functional method.

TEST METHODS

Many ballistic missile weapon systems incorporate a prelaunch checkout. The technical desirability of a prelaunch checkout will certainly be dependent on the method of readiness testing employed and on the frequency of such actions. At a hypothetical extreme, if all functions on a missile are continuously monitored with perfect test efficiency, then a prelaunch checkout would be operationally unnecessary. For any readiness testing procedure other than this, the operational desirability of a prelaunch checkout could be examined using the method developed in another related report.[3] Even so, regardless of any relation between prelaunch checkout and these other readiness testing procedures, there could be overriding military or technical reasons either to check or not check in the prelaunch situation. In one instance potential damage to friendly areas could be an overriding consideration, while in another instance, the missile's operational concept may not allow time to perform other than the most cursory prelaunch checkout.

Independent of whether a prelaunch checkout is to be performed, readiness testing of an entire missile or a black box on the missile can be performed by continuous monitor or by periodic check, or the item can be left alone. (Although leave alone is not a testing procedure, it may be the preferred course of action for some situations.) For a given set of missile or function characteristics, each method affords a different operational capability, and induces certain costs for test equipment, manpower, and spares. Some missile functions require a particular mode of test, although design freedom exists for most. The operational capability afforded by each method is described in Sec. III in terms of physical system parameters. We shall next describe the three test methods.

Continuous Monitor

In this method of readiness testing, key indicators or parameters in the missile or black box are monitored at all times. Measurements are made continuously to show the status (go/no-go), or quantitative level of a parameter or missile function. Typically, the information obtained in this manner about physical parameters (gyro fluid temperature, line continuity) is complete and accurate. On the other hand, this method of testing tends to yield incomplete information about more complex functions (the dynamics of the flight control system). For this discussion, a function or missile will be considered to be monitored continuously if the check is at least each hour.

Periodic Check

This readiness testing method involves "looking" at the missile or black box periodically, say every week or month. This test method, the most common to date, usually incorporates a mix of quantitative and qualitative tests,* and is performed using either mobile test equipment or test equipment that is organic to the silo.

Leave Alone

This involves simply leaving the missile or black box alone for extended periods of time, say six months to three years, after which the item would be replaced with one that is known to be in a "design" condition.

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*Reference 4 discusses these and other test methods.

EFFECTIVENESS CRITERION

The designing of appropriate readiness testing equipment can be done only after selecting the type of equipment and its method of use. For such preliminary design decisions, one reasonable criterion of readiness testing effectiveness is:

> the probability that each function, P_{ijk}, (or on a higher level, the entire missile, P) is operative and ready for launch at any future time.

One testing method (continuous monitor, periodic check, or leave alone) will tend to maximize this measure of effectiveness, P_{ijk} , for each function of the missile. A combination of these methods will be best for the missile system as a whole. This combination is called a readiness testing program. (Appendixes A and B present the underlying mathematics.) As an aid to using the method, characteristic numerical ranges for each of the input factors or parameters (Sec. III) were assumed. Then a high-speed computer determined P_{ijk} as a function of the numerical ranges (App. C). With these data, the readiness testing concept can be determined by finding the testing method that yields the highest P_{ijk} for each function of the missile. The readiness probability for the entire missile, P, can be computed (Sec. III) once the P_{ijk}s are known. It should be remembered, however, that the system time-between-periodic-tests, time-between-periodic-inspections for functions monitored continuously, and a replacement period for functions never tested, are all given values. Therefore, whenever the words maximum P_{ijk} or highest P_{ijk} are used, the phrase within the given system context is implied. In Sec. V, design exercises are discussed in which these system parameters are open for choice.

CONSTRAINTS

Choice of the testing method yielding the highest P_{ijk} may be barred by design constraints. These could result from cost considerations, operational restrictions, physical limitations (space, weight), or from the design of the missile itself. We shall now identify and clarify some common design constraints and discuss the apparent applicability of each constraint to the readiness testing design process. No single constraint seems to pervade all design applications, and it is quite reasonable to expect that some system exists for which none of these constraints apply. Yet the problem of constraints cannot be ignored because the effectiveness of the design must be maximized within the constraints on the system.

Physical Restrictions

Physical restrictions are perhaps the simplest area of design constraints. When matching a support concept to a missile design and operation, the physical limitations imposed by the missile can be recognized and the ground system designed to accommodate these limitations. This constraint applies during both the system-concept-design phase and the equipment-design and detailed-support-design phases.

Examples of physical attributes of the missile that impose restrictions on ground system design and operation are:

1. Electronic equipment insulated from other parts of the missile. Such equipment cannot be operated continuously because it is so packaged that the heat generated cannot be dissipated. This particular condition may be changed, however, if it is shown that the function should be monitored continuously for maximum P_{ijk}. Otherwise, such packaging precludes continuous monitoring of the unit.

2. Solid-propellant engines that are part of a missile in a silo. To check for deterioration or cracks in the grain requires both bore-scope and X-ray equipment. This sort of equipment is certainly not operated continuously. When mobile versions of the equipment are developed, perhaps a periodic test could be performed at the operational missile site. Until then, a leave alone philosophy is required (with infrequent higher echelon inspection or replacement).

3. Gyro fluid temperature. This is a physical restriction of a different sort, one that "requires," rather than limits, testing. For military reasons, the entire guidance system of some missiles and the gyroscopes of other missiles must be operated continuously. In this case, power is applied continuously to the gyro package and thermocouples monitor the temperatures. Because the thermocouple line is there, and because the measurement made does not require any unique actions of the ground system, designers are prone to include continuous temperature monitoring, with an appropriate output display. This "restriction" makes some additional testing possible at almost no added cost. Numerous other missile and ground support equipment could have these necessary attributes and thus could also be monitored continuously at essentially no cost.

Physical restrictions have implications to both the system concept designs and specific equipment designs. If some function must be tested in a particular manner, e.g., periodically rather than continuously, then it could make sense to harness the testing of other functions to the one that is constrained. For example, if the evaporation rate of a liquid fuel or oxidizer is such that the missile fuel supply must be refilled every 60 days, it could prove to be an overriding consideration, from the manpower and equipment cost view, for specifying a checkout period that is either 60 days or some fraction or multiple thereof.

On the equipment design level, if some function being continuously monitored has the capability of monitoring other functions, then it could prove expedient to enlarge the scope of the monitored test to include the other functions. For example, if a guidance computer is continuously monitoring certain elements in the guidance and control system, and other missile or ground elements have normal states that could be monitored without adding stress to functions, then the additional tests could be done continuously to improve the over-all P.

If some of the checkout equipment is located aboard the missile, other physical restrictions could be important. Heat generated by the checkout equipment would have to be dissipated and this could limit both the quantity of equipment and the permissible length or frequency of testing operation. The use of a coolant would add a premium to locating the checkout equipment near other electronic equipment that also uses a coolant. Such location could also prove beneficial for ease of maintenance. Parts that are operated will have a tendency to fail more frequently than parts that remain dormant; therefore, for ease of maintenance the checkout equipment should be located near other equipments that are operated during the ground life of the missile.

There are other missile-centered design constraints that are concerned with the weight added to the missile. Signal measurement and conversion equipment add weight to a missile and their value should be questioned in the light of other uses for the weight. Wires that connect missile-borne checkout equipment with the ground equipment add pounds to the missile and should be short or non-existent, if possible. Lastly, in a multi-stage missile, equipment used only for ground operations (like checkout) should be located in the first stage, if possible, so that second-stage engines would not be burdened by this non-productive (in flight) weight.

Safety Restrictions

Functions on a missile that could cause catastrophic damage to the missile, missile carrier, silo, surrounding equipment, or surrounding area are often monitored continuously--and rightfully so. Examples include pressures in tanks used for structural integrity, and warheads. Gyro temperatures are monitored because of potential damage to the missile. Safety alone could preclude using completely remote, unattended, inert missile basing with testing that consists simply of an infrequent look at the missile. However, continuous monitoring for safety is not all negative because once equipment and cables are installed, other functions requiring monitoring can be accommodated easily.

Dollar Cost Restrictions

The commonest constraint is dollar cost. In the system sense, costs do play an important part. However, this constraint is sometimes used when it should not be. Costs may not, in fact, operate as a design constraint, particularly at a functional level. In addition, the relevant cost factors may not be available in either appropriate accuracy or detail. Therefore, cost constraints should be employed with caution. First we present some of the basic considerations of a cost context. Following is a hypothetical missile system to illustrate the points presented.

A cost constraint is both useful and necessary during certain phases of system design. When determining the system operational and design concepts, one should be highly concerned with the system cost. Each element of the system design and operation should be evaluated against the dollar sign, and the system readiness testing method--continuous monitor, periodic check, or leave alone--should be chosen using the dollar sign as a critical decision factor. Ideally at least, support dollars can be exchanged for missile dollars when making a system proposal or evaluation. Missile systems are often compared on a cost-effectiveness basis, and in this type of evaluation, one important effectiveness determinant is the number of missiles launched for a given system cost. Missiles launched per system dollar are determined by how many are procured, how they are based, and how they are supported. At this stage, it is relatively straightforward and certainly meaningful to evaluate alternative support concepts on how they contribute to the over-all system effectiveness when their costs are compared to alternative uses for the same dollars.

As an example of the system sense, costs should be considered when deciding whether to check an entire missile periodically or to monitor most functions continuously and check the rest infrequently. If one decides to check the missile periodically (realizing that some functions must be monitored continuously for reasons described later), one would use cost considerations to determine whether mobile test equipment or on-site test equipment should be used. A specific instance of cost influence is in the frequency of checks by the van equipment, because at some test frequency the number of vans needed for the missile force will take a discrete, non-linear jump. At one (low) level of testing the number of vans required will be determined largely by the geography of the missile force. Above some critical test frequency, the number of vans would increase and because of geography, this increase could be a factor of two.

Once the system support concept has been determined, along with the system operating and basing concepts, the cost constraint can change. Now it is more important to match the physical designs of the missiles, facilities, and ground equipment. In our example, but from the functional standpoint, cost should not be the prime criterion for deciding what functions are to be monitored continuously (even though the system concept is to check periodically) and which are to be checked periodically. Cost should be considered only secondarily in deciding how frequently the missile should be checked. For this decision, one should be concerned with finding faults that are there, not causing faults by testing, wearing out the parts--in short, keeping the missile "up" at all times.

To help clarify the difference between a "system" and a "functional" approach to cost considerations, suppose we generate part of a hypothetical missile system. The military (for safety reasons) insists that the warhead be monitored continuously. Therefore our system must have men and equipment to do the monitoring. Our system cost effectiveness study (using system MTF and average repair costs) shows that all other missile functions are best checked every 30 days, using silo-located equipment. Our system concept is now specified. At this point, we look at the individial missile functions. With present-day design knowledge and military launch requirements, it is quite likely that the guidance package will be running at all times. Further, using the functional analysis of this paper, it turns out that an operating guidance package requires continuous monitoring for maximum readiness. Since we already made allowances for men and equipment to monitor the warhead, the cost of adding monitoring of the guidance package could be small. If so, we would decide to monitor the guidance package, not on the basis of cost, but of missile readiness. As a second example, our functional approach would probably discover a missile function that exhibits long MTFs but is susceptible to testing stresses. The functional analysis may show a 60-day check to be best. The "added" cost of checking this function every 60 days instead of the system's 30 is trivial if it exists at all. So once again we would decide to check every 60 days based upon maximum readiness, noting that in either example our decision is imbedded in a cost framework. At an extreme, one can envision situations wherein missile readiness is to be maximized independently of cost. This paper presents aids for such design analyses. By computing this extreme, one can see how non-optimum testing procedures affect missile readiness.

Manpower Restrictions

A manpower restriction is exceedingly difficult to quantify in any analysis that is made prior to specific design. Yet, because it is one of the most stringent constraints facing the Air Force today, it deserves treatment in system planning analyses. Since the cost of people in a system can be determined, manpower is often treated as a cost factor in an analysis. This is not strictly correct. Although people can be exchanged for dollars in system planning, dollars cannot always be exchanged for people in system implementation.

Shortages of both skill types and numbers of people could potentially limit the operational capability of the Air Force. This manpower restriction provides a guideline for test equipment design. If a replacement of manual readiness testing methods with automatic methods means a decrease in manpower requirements, the Air Force probably would welcome the change, even if the dollar cost of the system rose somewhat. On the other hand, if the reverse is true, more people are required,* the automatic system probably would not be wanted unless automation significantly improves the system's operational capability or gives other necessary and otherwise unobtainable capability.

This manpower limitation could manifest itself in unusual ways when planning a readiness testing concept. For example, when determining the preferred frequency of periodic tests using mobile test equipment, a cost constraint, per se, may have less meaning than the number of people required to operate the test equipment. Similarly, when deciding between periodic check and continuous monitor, the true determinant could be the manpower required--regardless of small (or perhaps not so small) differences in P or dollar cost.

Missile-to-checkout-control Communication

To monitor continuously a missile function means that some information must be continuously (or nearly so) flowing between the missile and the checkout control equipment. If this checkout control equipment is separate from the missile, then some communication channel is needed. This could be hardened cable, radio transmission equipment, earth current equipment, or other means. It is reasonable to ask if there are any important constraints on the amount of information that can be transmitted. By time sharing on information channels, additional lines in hardened cables, and additional amplifying equipment, the physical problem of transmitting data or signals should, in general, not be a prime determinant of the feasibility of continuously monitoring missile functions. The space required for the test and communication equipment could, on the other hand, become a design constraint.

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*Reference 5, a companion work, shows that this is often true.

Space and Weight Requirements

Test, checkout, and communication or data-transmission equipment could be restricted in space or weight. It is conceivable, for example, to envision a silo design that is fixed with a certain space allocated to test equipment. Mobile test units, whether truck or helicopter mounted, could have both space and weight limitations. The number of wires, or transducers, or signal conversion equipment carried aboard the missile itself may have practical bounds. The total quantity and types of testing are affected by these kinds of constraints. However, with proper design, even small test sets can be made to control most or all of the tests to be performed. As a last resort, tests can be designed to encompass large parts of the system at the expense of a less accurate indication of system condition.

Summary

This discussion of design constraints indicates that no physical constraint other than those imposed by safety and missile design and operation is pervasive through all applications of ground system design. Indeed, an examination of several systems in being and discussions with persons from the developing organizations have indicated that other than those above, there may be no true constraints on ground system design, and that manpower is considered more as a design boundary condition than as a contraint. (Design for minimum manpower is a framework within which a testing system design is accomplished.) Actually, it is not clear that system manning can operate as a quantitative constraint on a design. Efforts are being expanded by several research organizations to quantify maintainability and a priori maintenance requirements (as these are reflected in manpower requirements).

Thus far, these efforts have yielded only subjective guides that cannot be employed as quantitative constraints. Because of this inability to quantify this most important consideration, manpower requirements must be employed, as they have been, as a design framework. Perhaps future research will yield a basis for a priori quantification.

III. SPECIFICATION OF READINESS TESTING PROGRAMS AND EQUIPMENT

DISCUSSION

If no physical constraints inhibit design of the ground system, the designer is faced with a relatively straightforward problem--that of developing a readiness testing program compatible with the system concept and missile design, assuring adequate safety, and yielding the best potential missile readiness. The compatible problem is considered in this section. A more complicated, constrained, design problem is discussed in Appendix A.

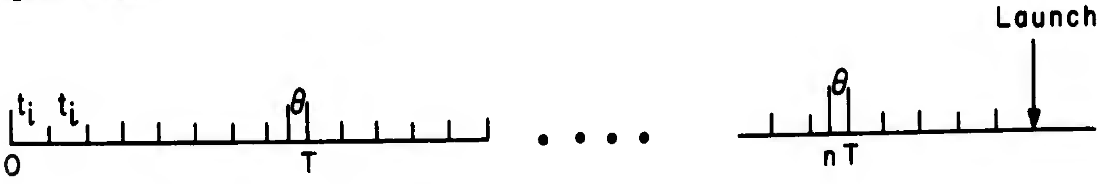
Two points bear repeating for emphasis here. Within the system context:

- The intent of a readiness testing program (other than supplying command data) is to detect faults that are present in the missile
- The criterion is the maximization of the missile's readiness (or P), which is defined as the probability that all functions on the missile are operative and ready for launch at any future time. (The missile could be needed at the beginning of hostilities, which could start at any time)

Numerous design and operational factors affect the determination of the best program of readiness testing for functions of a missile. These factors must be quantified and integrated into a model that describes the missile system's operation. To avoid sidetracking the reader with complicated mathematical formulas, most of the details of the analysis and all methods of estimating the parameters have been relegated to appendixes. Formulas are used within the body of the text only where necessary to show dependencies and relevancies.

PERIODIC CHECK

The life span of a missile function that is checked periodically can be depicted as:



Every T days* the function is given a periodic test. The test requires θ days for the entire missile. The term θ is usually less than 1. It is assumed that, because of the potential use of automatic checkout equipment, $\theta \ll 1$, say, $\theta = \frac{1}{12}$ or $\frac{1}{6}$. The time between the periodic tests has been divided into n_i increments of t_i days. For convenience of analysis, t_i is taken as the time required to repair the i^{th} function. The parameter t_i includes whatever time is used to travel to the missile, disassemble it, and after repair, assemble it. Therefore, $T = n_i t_i + \theta$.**

Since testing is concerned with detecting faults that have occurred since the last test, a function's ground reliability is the characteristic of interest for test planning. Assuming that an exponential failure relationship is appropriate, the measure of ground reliability for the i^{th} function is taken as α_i, the function's failure rate measured in failures per hour. During the periodic test, a different failure rate could obtain. The term δ_i is defined as the probability that the i^{th} function survives the periodic

---

*Recall from p. 7, T is one of the given system parameters.

**The equation is not an equality because: since T and θ are specified for the entire missile and t_i relates to a particular function, n_i may not be an integer. Because of the way n_i enters into the development of the mathematical method, the error introduced by rounding n_i to the nearest integer is not great enough to warrant model modification. In other words, T is taken to be equal to $n_i t_i + \theta$.

test. Turn-on and operating stresses determine δ_i and are, in turn, influenced by the method of test used.

Three more terms are required before a missile function's P_{ijk} can be defined. They are M, p_i, and q_i. The term M describes part of the system's maintenance concept. Once a missile function is known to be in an inoperative (failed) state, the maintenance system must perform a repair operation. This repair could be done immediately or it could be delayed. To account for the possibility of such delays, and moreover, to help evaluate the desirability of delays, M is included. It is defined as the average delay from the time a function's failed state is known to the time repairs are started.

The terms p_i and q_i are measures of the periodic test equipment's capability to detect faults. The term p_i is defined as the probability that the tester indicates that a good function is good, and q_i is defined as the probability that the tester detects a malfunction present in the i^{th} function. Both are difficult terms to estimate. The value of q_i depends on the degree of access the tester is given to the function, the accuracy of the measurements, and the extent to which the test simulates the requirements placed on the function during flight. The value of p_i is completely dependent on test equipment design.

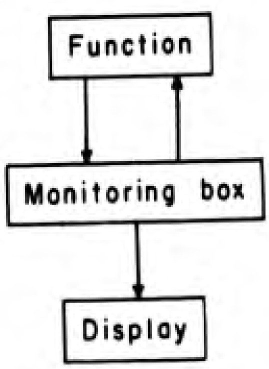
The dependence of P_{ijk} on the parameters (T, t_i, α_i, δ_i, M, p_i, q_i) is derived in Appendix B.

As the next step, we assumed characteristic ranges of numerical values for each of the parameters. Then we used a high-speed computer to determine P_{ijk} by varying one parameter throughout its assumed range of values while holding the other parameters fixed. Each parameter, in turn, was varied as described. Such numerical runs yield great quantities of data, which, because of sheer bulk, have been placed in Appendix C. With these data, any user can select his own set of input parameters and quickly find a corresponding value for P_{ijk}. Or the user may study how P_{ijk} varies with changes in hardware or operational design as reflected in the input parameters.

One bonus of the numerical runs is a kind of sensitivity study. For example, as would be expected, a variation in the function's failure rate, α_i, from 10^{-1} to 10^{-6} produces a range of P_{ijk} from almost zero to almost one. Time, whether related to operational concept (T, t) or delays (M), has a lesser effect. The terms q_i, ranging from 0.7 to 0.99, and δ_i, ranging from 0.9 to 1.0, cause a change in P_{ijk} of roughly 10 per cent. The term p_i is the least important. A variation of p_i from 0.9 to 0.99 produced less than a 5 per cent change in P_{ijk}.

CONTINUOUS MONITOR

Here is an idealized sketch of a function that is monitored continuously:



This indicates that a monitoring box is associated with each function. This box could be nothing more than a power source that constantly supplies an input voltage. The display shows the status of the function as indicated by the box; this indication could be erroneous.

For this analysis it is assumed that a failure in the box itself will always be known, but that a failure in the function might not be detected by the box. The detection weakness arises if the test applied continuously is incomplete; the failure is in a portion of the function or mode of operation that is not tested.

As before, it is assumed that the life span of the function can be depicted as:

where A is the number of days between inspections of the function using test equipment other than that used for continuous monitoring (similar to the periodic tests). The value of A probably would be some multiple of the T, p. 7. The test equipment used for the inspection has characteristics p_i and q_i, as before, and the function's failure characteristics are assumed to be described by α_i and δ_i. The incorporation of the monitoring box into the test system requires additional parameters to specify the part played by the box. The term p_i' is defined as the chance that the box calls a good function good, and q_i' is the chance that the box detects a failure. In addition, the monitoring box could fail during operation or during the inspection. The appropriate failure characteristics are given by β_i and γ_i.

Using these parameters, P_{ijk} terms have been derived for a function that is monitored continuously. The derivation is presented in Appendix B. Once again, a wide range of numerical values for the parameters were assumed and high-speed computer runs made. These latter data are given in Appendix C.

The numerical runs show the same kind of sensitivity as discussed in the previous section. The box parameters have the following effects. The term p_i' has the same degree of sensitivity as p_i, on the order of 5 per cent. On the other hand, q_i', ranging from 0.5 to 0.99, varies P_{ijk} by approximately a factor of 20. This is understandable, for if the monitoring box misses a fault, the missile is not ready to go, even though a display would say it is. This condition could last for long periods. The chance that the box survives the inspection test is not too important. When γ is varied from 0.9 to 1.0, it has about a 5 per cent effect on P_{ijk}.

LEAVE ALONE

Some missile functions are best left alone for extended periods of time and then replaced. If this replacement takes place every R days and if α_i again designates the function's failure rate (under the stress level appropriate to being left alone), then the P_{ijk} for the function is as derived in Appendix B. Data from numerical runs are in Appendix C. Observe, if it is known with certainty that a particular function will not fail over R days, then the failure rate α_i equals zero and P_{ijk} equals one. This is probably the case for many mechanical systems and could be the case for other systems if the shelf life can be determined.

The factors that must be considered in an evaluation of readiness testing concept and equipment have now been delineated. The stage is set for an examination of a method for determining the best readiness testing program and operational design criteria.

COMPATIBLE READINESS TESTING PROGRAM

We repeat here the definition of a readiness testing program: that combination of each testing method applied to each function of the missile that tends to maximize P.

The first readiness testing program considered is designed to be compatible with safety and missile design factors previously discussed, but is not otherwise constrained, i.e., there are no weight, volume, or other limitations on the checkout equipment. Recall that each missile function can, in general, be tested by periodic check, continuous monitor, or leave alone. Further, the test equipment can, in general, be located in a van or helicopter, in the silo, or aboard the missile itself. The goal of this analysis is to choose that readiness testing method-equipment location combination for each function yielding the highest P_{ijk}, considering (1) the stress levels that the equipment would be subjected to because of the method and (2) the capabilities for fault detection demonstrated by each kind of test equipment using each method in each location. This goal can be stated as, choose which $x_{ijk} = 1$ to maximize

$$P = \prod_{ijk} (P_{ijk})^{x_{ijk}} \quad (1)$$

*

where P ≡ probability that the entire missile is operative

P_{ijk} ≡ probability that function is in ready condition if readiness testing method j is used with an equipment location k

and

$$x_{ijk} \equiv \begin{cases} 1, & \text{if readiness testing method j using equipment in location k is used for function i} \\ 0, & \text{otherwise} \end{cases}$$

*See Appendix B for a further discussion of this equality.

The generalized formula for P given above must be modified to account for compatibilities mentioned earlier.

As a second program, consider the case in which certain functions are restricted, e.g., there is a military requirement to monitor the warhead continuously for safety reasons. Now there is no freedom to choose j and possibly k for the warhead (a particular i^{th} function). So P becomes

$$P = \left\{ \prod_{i=i_c} (P_{ijk})^{x_{ijk}} \right\} \left\{ \prod_{i \neq i_c} (P_{ijk})^{x_{ijk}} \right\} \qquad (2)$$

where $i = i_c$ denotes restricted functions.

As an aid to understanding the mechanics of maximizing Eq. 2, consider the following sequence:

1. Set aside the restricted functions ($i = i_c$). From the data appendix, draw out and list the appropriate P_{ijk}s for them. The product of the listed P_{ijk}s is the meaning of the first bracket in Eq. 2.

2. Form a table as illustrated below for the remaining unrestricted functions.

i = 1	i = 2		i = n
P_{111}	P_{211}	. . . .	P_{n11}
P_{121}	P_{221}	. . . .	P_{n21}
P_{131}	P_{231}	. . . .	P_{n31}
⋮	⋮	⋮	⋮
P_{112}	P_{212}	. . . .	P_{n12}
⋮	⋮	⋮	⋮

The numerical entries for the table are found in Appendix C.

Each missile function is considered in turn. For example, the guidance package may be function number one; the engines may be function number two; and so on. Suppose periodic check is test method number one or $j = 1$. Suppose a silo is location number one or $k = 1$. Then, a periodic check of the guidance package using silo equipment (read from Appendix C) is entry P_{111}. With all the P_{ijk}s displayed, it is a simple matter to choose the largest. This selection is illustrated by the circled entries, one per column. The product of the circled numbers is the meaning of the second bracket in Eq. 2. (There is a numerical example of such a table on p. 54.) It is a maximum product, and hence, the theoretically best readiness testing program.

Although the purpose of the table is to maximize P by circling the largest entries, there are other uses for a full display of the P_{ijk}s. There can be cases where it is best to monitor one function continuously but to check a second function periodically. The development of a piece of equipment to do each testing job may have serious cost or operational concept implications. It may be that there are truly significant savings if, for example, both functions are checked periodically. Attracted by the possibility of obtaining these savings, a system designer would want to know the detrimental effects of adopting a compromised testing program. The table displays the reduction in P_{ijk} resulting from checking the first function periodically rather than monitoring continuously--the preferred way. If the reduction is small, the designer may be justified in grouping the two functions into one and then checking the combined function on a periodic basis.

One must be careful when grouping tests and finding a grouped P_{ijk}. As a first example, suppose the grouping is merely a physical union. Each tester part would have its own probability that the tester indicates a good function good, p, and its own probability of detecting a malfunction, q.

The grouping would be as shown in Fig. 1.

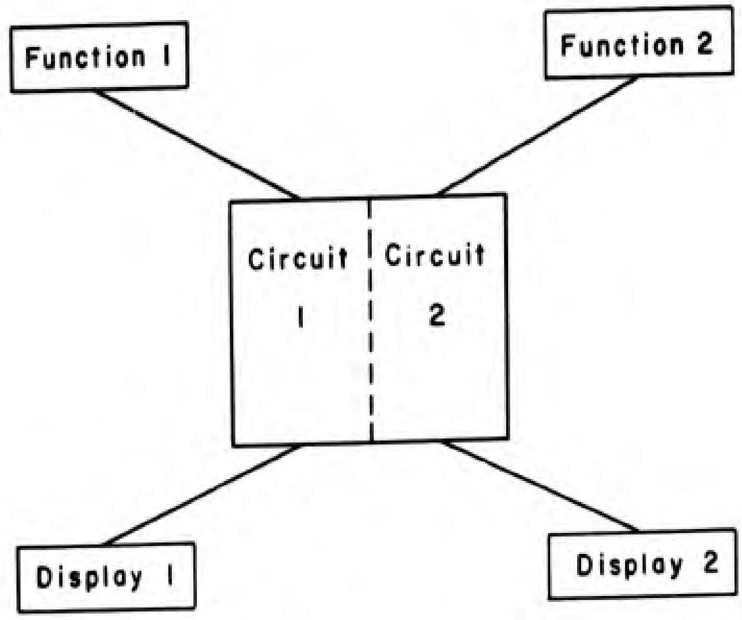


Fig. 1—Combined P_{ijk} test

The P_{ijk} table, in part, might have the following entries.

Test	P_{ijk}		
	1	2	Combined 1 and 2
Continuous monitor	.97	.94	.91
Periodic check	.84	.95	.80

Maximum P_1P_2 is .97 times .95 or approximately .92. Physically grouped P_{ijk} (i = 1 and 2) for periodic check is .84 times .95 or about .80. The loss in P_{ijk} in going to periodic check for both functions is .12.

Now suppose that the test unit is of different circuitry with a single p and q. The test grouping appears as in Fig. 2.

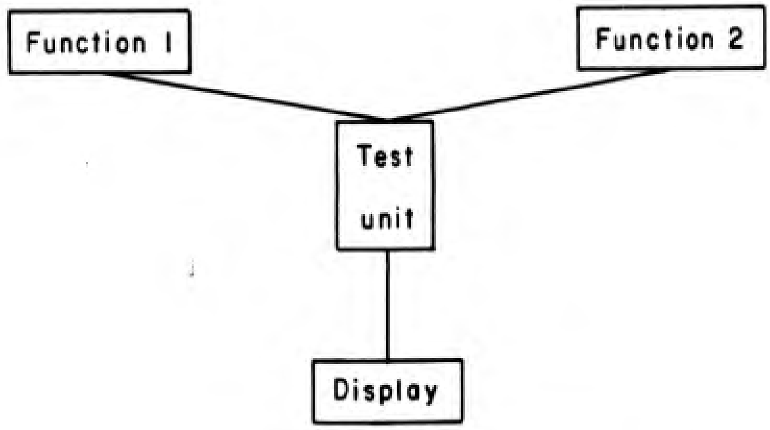


Fig. 2 — Combined P_{ijk} tester

This partial P_{ijk} table could be as follows:

Test	P_{ijk}		
	1	2	Combined 1 and 2
Continuous monitor	.97	.94	.94
Periodic check	.84	.95	.88

Once again, if two testers are used, one for continuous monitor of the first function and another for periodic check of the second function, maximum P_1P_2 is (as given in the first grouping example) .92. But now, if both functions are checked periodically with a redesigned tester, then P_{ijk} (i = 1 and 2) is .88. The P_{ijk} loss is only .04, instead of .12 as in the first example.

It can be seen that grouped P_{ijk} entries in a full display table could have different values depending on how the test grouping is done.

The discussion following Eq. 2 has been brought together in a single design aid, the design tableau of Fig. 3. In the tableau the functions 1, 2, 3, ..., represent single functions or groups of functions that are always best tested as a group. The functions m and n show the possibilities for different grouping depending on the test method and test equipment location. To use such a tableau to aid in the design decision process, one would:

1. Indicate by circling those tests that must be done for reasons of safety or physical requirements.

2. Fill in the balance of the readiness terms for each test method and equipment location combination.

3. Select the largest readiness probability for each unrestricted function and indicate it with a V.

4. Form the product of those P_{ijk} terms (one from each column) that are marked with a V or are circled; this is the "best" over-all P.

5. To examine other convenient test groupings, determine what tests can be grouped with other tests and enter these in the row "Tests to Group."

6. Use the "Grouping Comparison" row to compare the products of the grouped terms with the products of the "best" P terms and determine the worth of non-best patterns.

Equipment location	Test method	1	2	3	•••	m_1	m_2	m_3	•••	n_1	n_2	n_3	•••
Van	CP*	p_1	p_2	(p_3)		(p_{m_1})	p_{m_2}	p_{m_3}					
							p_m						
Silo	CP												
	CM									p_{n_1}	p_{n_2}	p_{n_3}	
											p_n		
Missile	CP												
	CM												
None	LA	p ✓											
Check if done													
P_{ijk} chosen		✓		✓							✓		
Tests to group	CP CM												
Grouping comparison													

CP* — Periodic Check
CM — Continuous Monitor
LA — Leave Alone
✓ — Best choice of P_{ijk}
( ) — Constrained by safety or physical considerations

Fig. 3 — Design tableau

IV. APPLICATION OF THE METHOD

This section illustrates the application of the readiness testing model to a missile system.

THE MISSILE

A complete description of all missile parts, of the grouping of parts into functions, and of the interrelations among the functions would be as complicated as describing the mathematical analysis in this text. Inclusion of a complete description would defeat the purpose of this section. Instead, we shall describe a ballistic missile only in sufficient detail to illustrate how the model may be applied to any actual missile.*

Our exemplary missile does not exist. It represents an advanced, storable-propellant ICBM. In some cases, the choice of a major subassembly is based upon proposed advanced designs. In other cases, availability of input data determines the choice. The missile has two stages and a re-entry vehicle.

The first stage consists of a dual-thrust-chamber rocket engine, a fuel tank, an oxidizer tank, and a tank pressurization system. It is controlled by a timed programmer operating through an autopilot, both located in the second stage. Autopilot signals go to the Stage 1 accessory power supply which, by means of a hydraulic actuator system, controls engine movement. The engine runs for three minutes and propels the whole missile through the lower dense atmosphere.

The second stage contains a single-thrust-chamber engine along with fuel and oxidizer tanks and a tank pressurization system. Although the engine is swiveled, four solid-propellant jets are included for fine adjustments of speed, direction, and orientation. This stage is controlled by an internal inertial guidance package operating through the autopilot to the accessory

*And to avoid the need for classification.

power supply. As in Stage 1, a hydraulic actuator system modulates engine and vernier jet movement. Other ancillary equipments, such as range safety equipment, may rightfully be part of a real missile but they are not included in this example. Stage 2 provides the re-entry vehicle with proper trajectory conditions.

The re-entry vehicle consists of a structure assembly, arming and fusing circuits, and a warhead.

For illustration, the missile is described by eight major subassemblies:

1. re-entry vehicle
2. airframe
3. control system
4. first-stage accessory power supply
5. second-stage accessory power supply
6. first-stage engine
7. second-stage engine
8. inertial guidance package

THE RE-ENTRY VEHICLE

The re-entry vehicle is shown in Fig. 4, with its principal parts, nose cone, fusing mechanism, and warhead, emphasized.

It is useful to convert such a design drawing (Fig. 4) into a functional block diagram (Fig. 5) in which each box contains those functions (or grouping of parts) that have dependent failure characteristics, that is, dependent in the sense that a failure in any one function causes a failure in other functions. Consequently, each box has independent failure characteristics relative to each other. As implied earlier, the model analyzes functions with independent failure qualities.

The analyst's task, perhaps aided by a missile design engineer, is to group missile functions into boxes characterized by independent failures and then to estimate the numerical failure rates for each box. This aggregation for the re-entry vehicle is presented in Fig. 5 and the failure rates are given in Table 1.

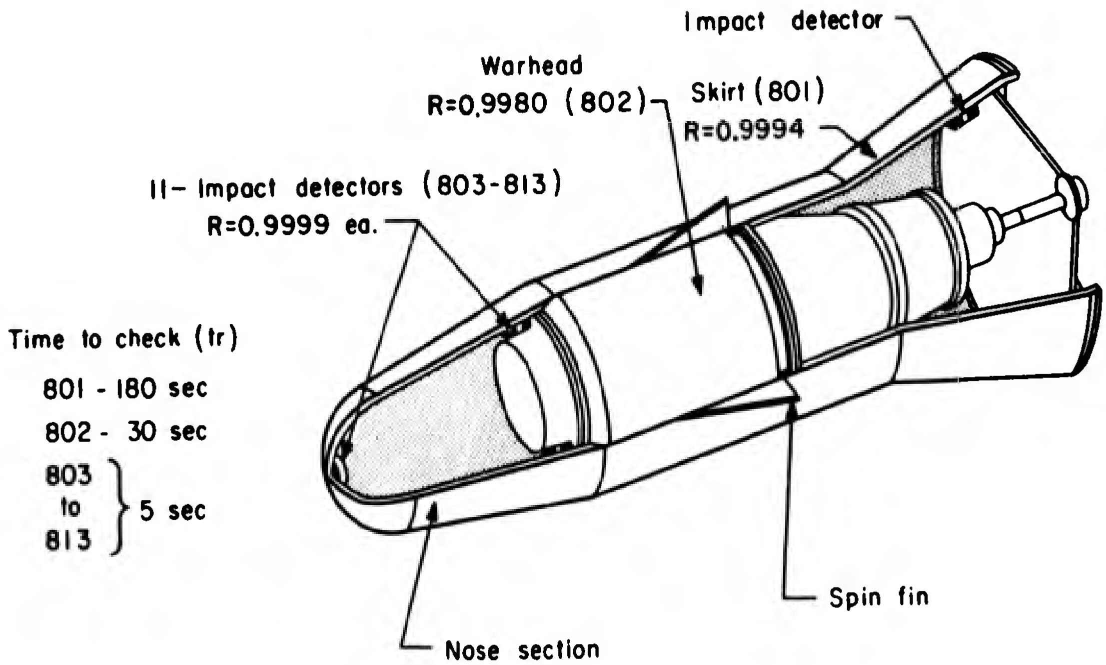


Fig. 4 — Re-entry body

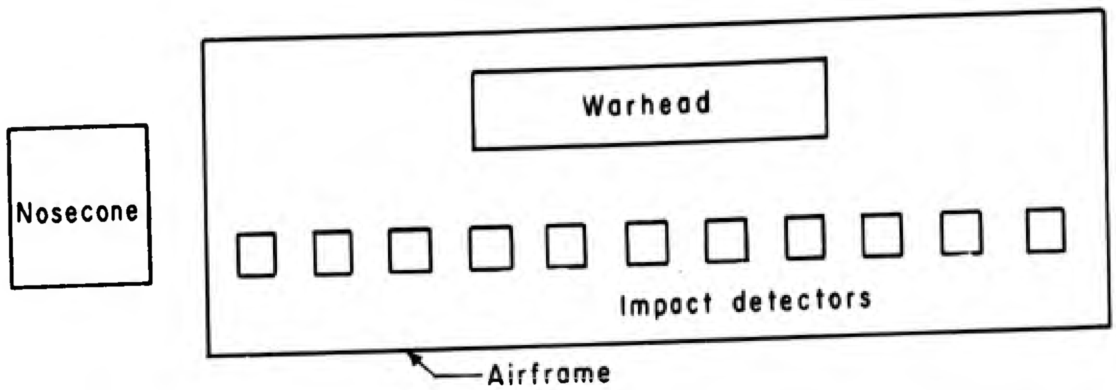


Fig. 5 — "Function" block diagram of re-entry vehicle

Table 1

RE-ENTRY BODY FAILURE RATES

Item	In-flight Reliability	Countdown Failure Rate
Nosecone	.9320	.00908
Warhead	.9800	.00261
Airframe	.9994	.00008
Impact detectors (each of eleven)	.9999	.00001

The method for computing failure rates under various stress levels is based upon that in Ref. 6. The method requires in-flight reliability as an input and determines the countdown failure rate. These two quantities for the exemplary missile are given in Tables 1 through 10. Other stress level failure rates are in later tables.

There are, of course, other sources of failure rate data and methods for estimating reliability.[7,8,9]

The Airframe

The missile airframe consists of fuel tanks connected by suitable structures. Starting at the top of the missile, the re-entry vehicle is joined to the Stage 2 airframe by a transition structure that houses electronic equipment. The Stage 2 airframe has fuel and oxidizer tanks separated by an inter-tank structure. Below the tanks is the Stage 2 engine. Stages 2 and 1 are connected by a second transition structure. The Stage 1 airframe is similar to the Stage 2 airframe.

Missile airframes are highly reliable and normally are not tested by automatic checkout equipment. However, for illustration, the airframe components are included as described in Table 2.

Control System

The flight control system uses a timed programmer that sends signals through gyros and electronic circuits to hydraulic actuators. There are two sets of gyros. The first set comprises three displacement gyros, for the pitch, yaw, and roll axes. These gyros, operating in conjunction with the programmer, control missile alignment during flight. The second set has three rate gyros, pitch, roll, and yaw, used to stabilize missile flight.

Table 2

AIRFRAME AND FUEL TANKS FAILURE RATES

Item	In-flight Reliability	Countdown Failure Rate
Tail skirt assembly	.9999	.000001
Tail fairing assembly	.9999	.000001
Transition structure (each of two)	.9999	.000001
Inter-tank structure (each of two)	.9999	.000001
Fuel tanks (each of two)	.9999	.000001
Oxidizer tanks (each of two)	.9999	.000001

The hydraulic actuators swivel the engine. The functional block diagram, including information flow, is in Fig. 6. Failure rates are in Table 3.

Table 3

FLIGHT CONTROL SYSTEM FAILURE RATES

Item	In-flight Reliability	Countdown Failure Rate
Timer	.9978	.00038
Programmer	.9990	.00017
Displacement gyros (each of three)	.9988	.00021
Rate gyros (each of three)	.9959	.00072
Amplifier channel (each of six)	.9992	.00001
Mixer (each of two)	.9950	.00087
Servo amplifier and hydraulic actuator (each of two)	.9990	.00017

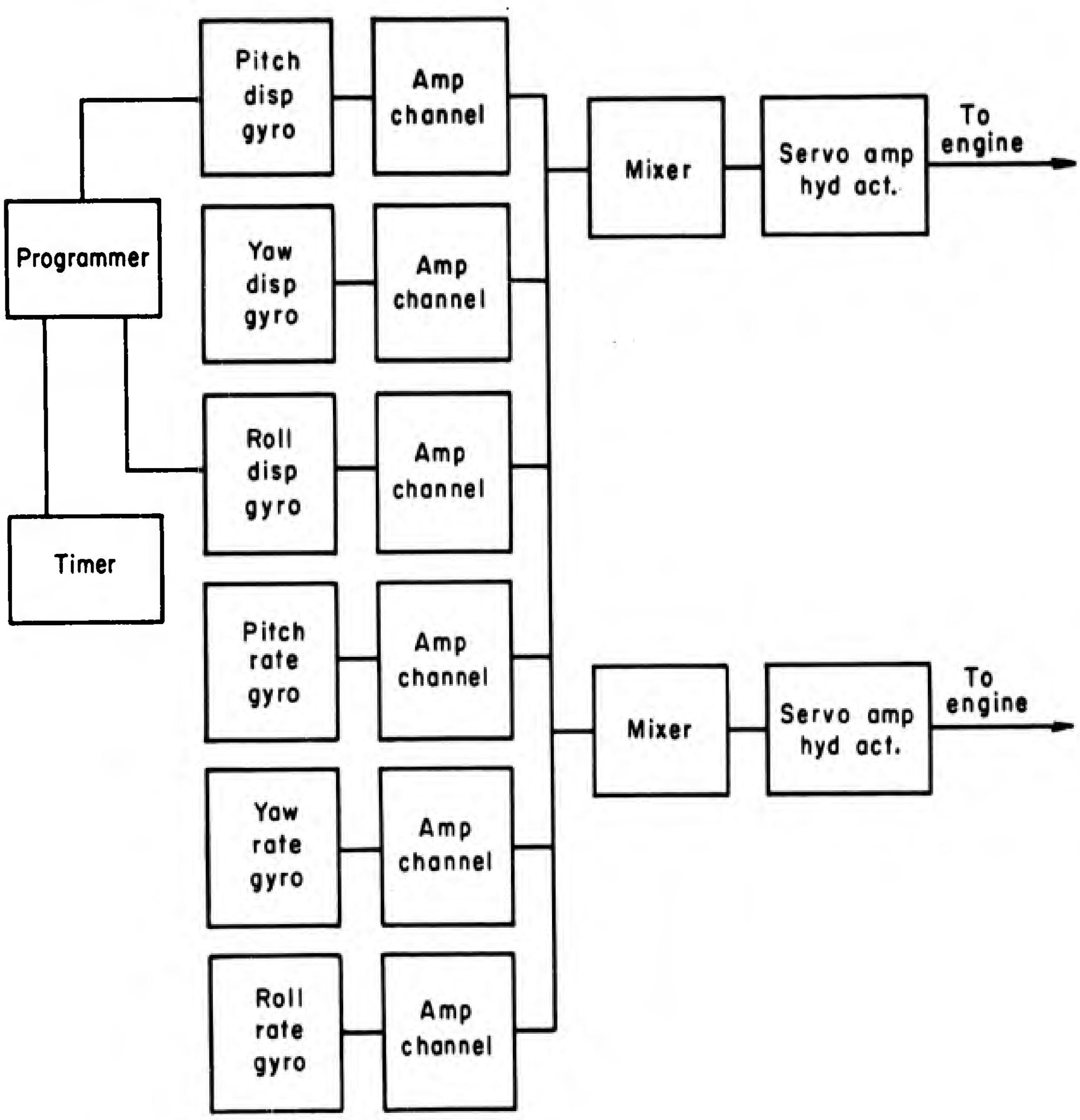


Fig. 6 — Flight control system

Stage 1 and Stage 2 Accessory Power Supplies

Stage 1 and Stage 2 have identical, yet independent, accessory power supplies. They consist of an electrical supply and a hydraulic system. The electrical supply has one battery bank for dc current and a second battery bank plus inverter for ac current. The hydraulic system has such components as motors, pumps, and valves. The electrical power supply is presented in Table 4. The hydraulic system appears in Fig. 7 and Table 5.

Table 4

STAGE I POWER SUPPLY FAILURE RATES

Item	In-flight Reliability	Countdown Failure Rate
Battery bank (dc)	.9996	.00007
Battery bank plus inverter (ac)	.9996	.00007

Table 5

STAGE I HYDRAULIC SYSTEM FAILURE RATES

Item	In-flight Reliability	Countdown Failure Rate
Electric motor	.9999	.00002
Hydraulic pump	.9959	.00072
Reservoir	.9990	.00017
Quick disconnect (each of two)	.9995	.00009
Relief valve	.9992	.00014
Accumulator	.9969	.00054

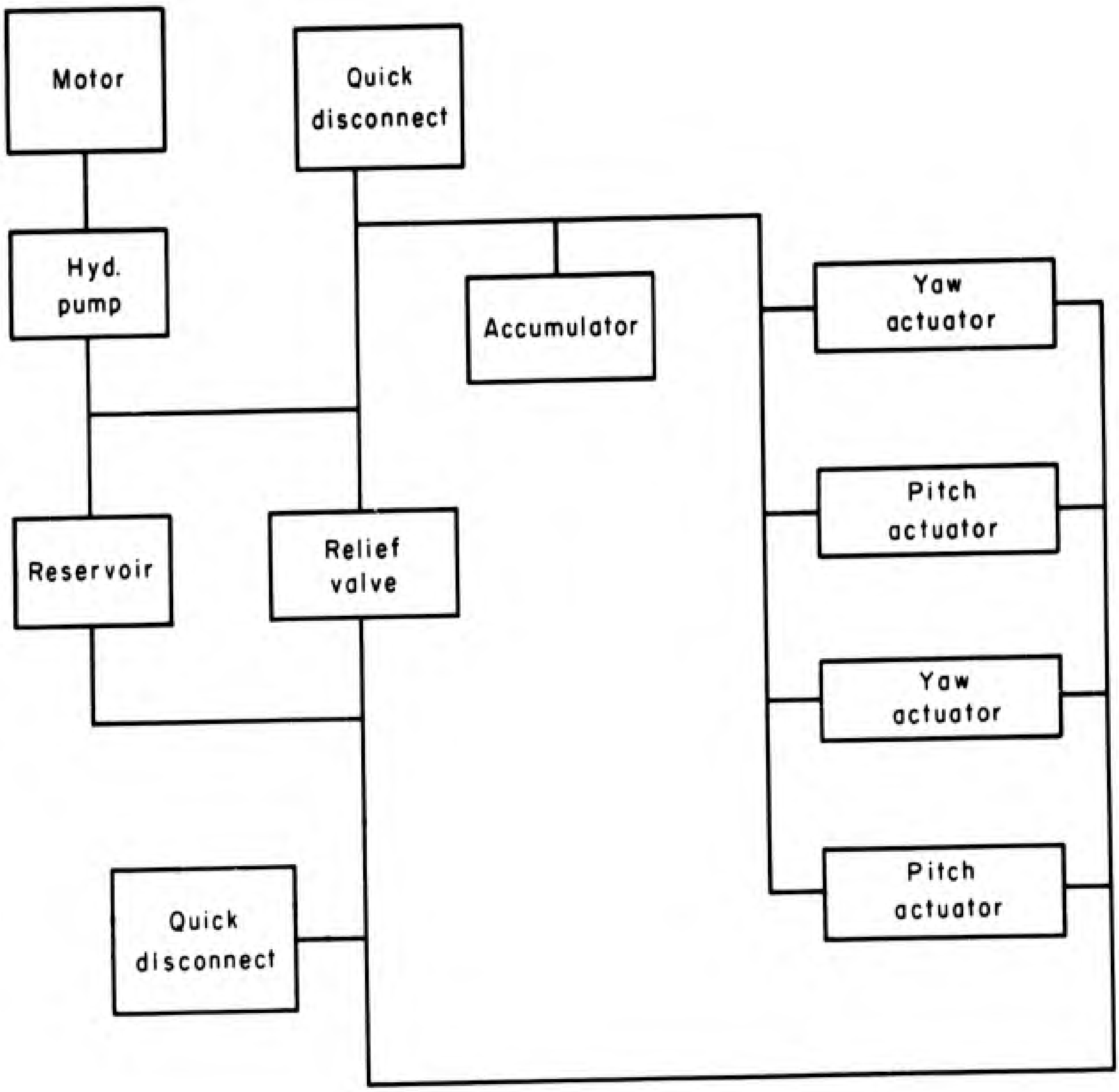


Fig. 7 — Stage I hydraulic system

Engines

The engines use storable liquid propellants of a nitrogen tetroxide/hydrazine and unsymmetrical dimethylhydrazine combination. The Stage 1 engine has two thrust chamber systems, one of which is shown in Fig. 8. The Stage 2 engine is a single-thrust chamber design as shown in Fig. 8. These depicted components are combined with suitable mounting structures, connecting lines and fittings. Once properly timed signals are sent to the solid-propellant starters, the engines automatically develop rated thrust. No throttling device, based upon individual chamber thrust, is used.

The second stage also has a vernier engine system to control roll and velocity vector during a short period following Stage 2 engine cut-off. This vernier system uses solid propellants. Because of their high reliability and the inability to check them without consumption, the vernier engine system is not included in this example. Vernier engine control is accomplished by exhaust vanes positioned by the second-stage hydraulic control system.

Table 6 listing component failure rates completes the engine description.

Two ancillary systems, not part of the engines, are presented here for convenience. The first, a fuel flow system, consists of a series of valves controlling fuel flow from tanks to engines. The valves are actuated by timed signals from the electronic equipment. Figure 9 and Table 7 present this system. The second is the tank pressurization system, in which gases from the generator of Fig. 8 are bled through various regulators and valves to pressurize the fuel tanks. Figure 10 and Table 8 describe this system.

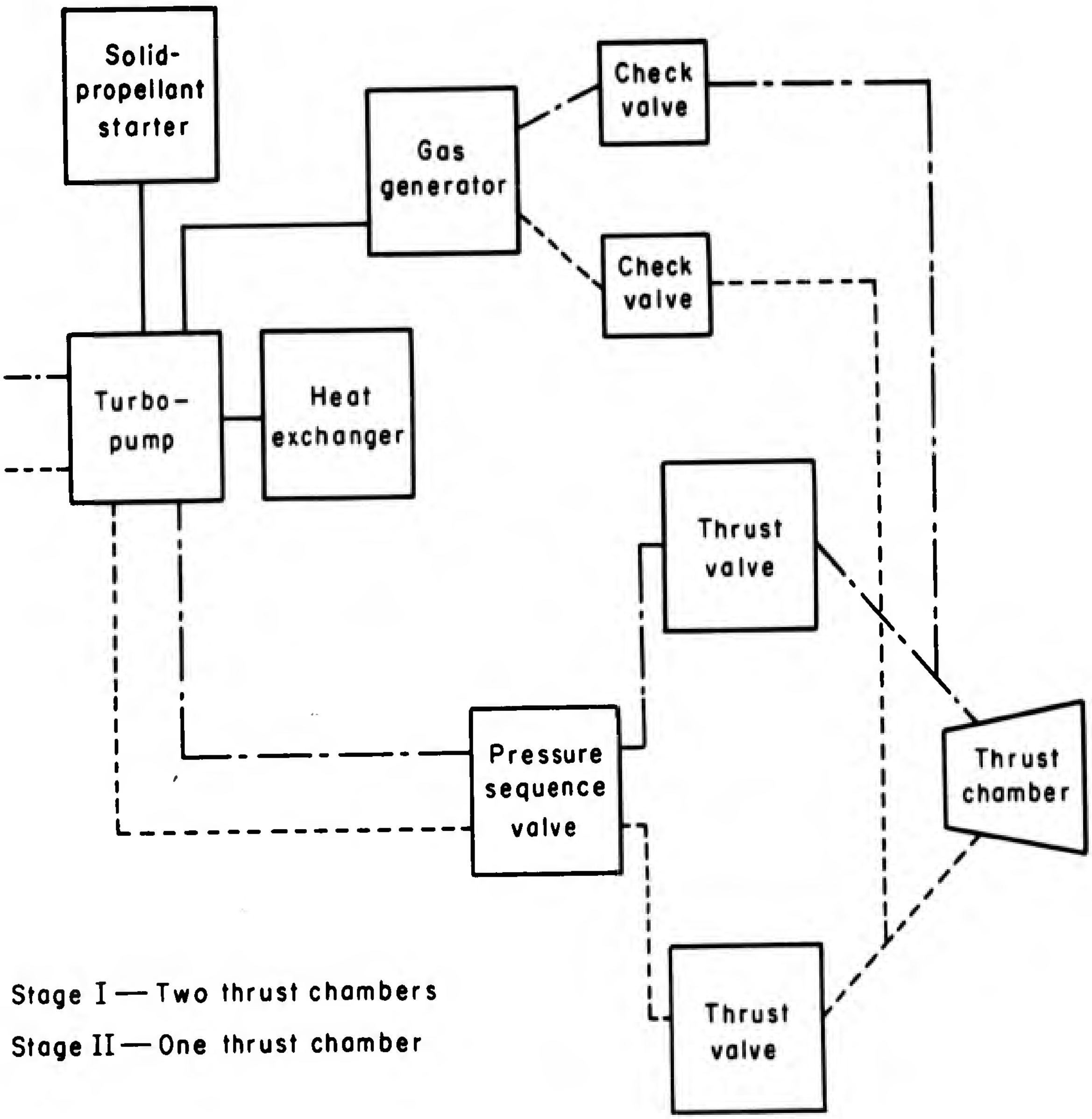


Fig. 8 — Thrust chamber

Table 6

THRUST CHAMBER FAILURE RATES

Item	In-flight Reliability	Countdown Failure Rate
Solid propellant starter	.9975	.00044
Turbopump	.9980	.00035
Heat exchanger	.9980	.00035
Gas generator	.9975	.00044
Check valve (each of two)	.9985	.00026
Pressure sequence valve	.9955	.00078
Thrust valve (each of two)	.9970	.00052
Thrust chamber	.9925	.00131

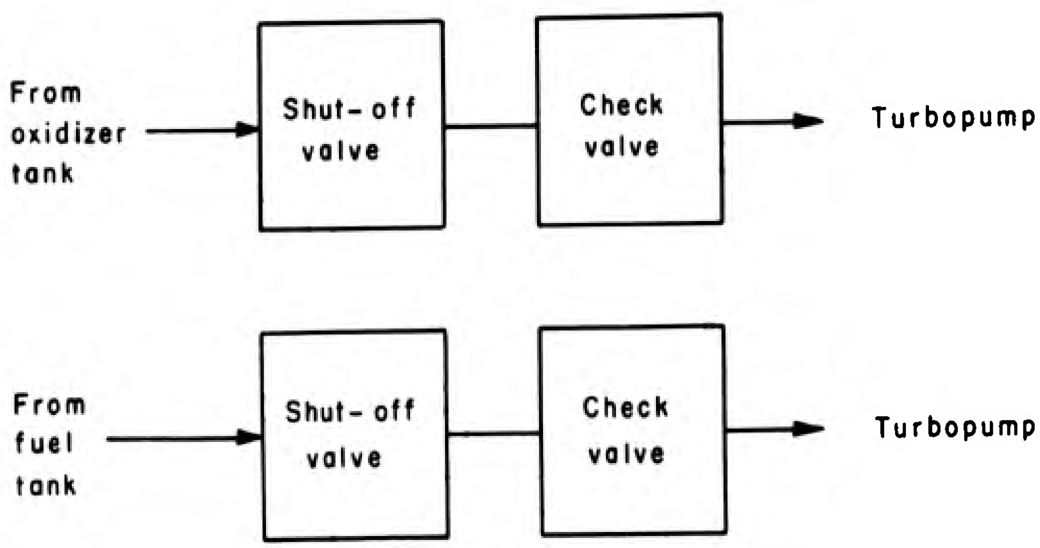


Fig. 9 — Stage I fuel flow

Table 7

STAGE I FUEL FLOW

Item	In-flight Reliability	Countdown Failure Rate
Shut-off valves (each of two)	.9988	.00021
Check valves (each of two)	.9988	.00021

Table 8

STAGE I TANK PRESSURIZATION FAILURE RATES

Item	In-flight Reliability	Countdown Failure Rate
Primary pressure regulator	.9997	.00005
Relief valves (each of three)	.9974	.00045
Secondary pressure regulator (each of two)	.9985	.00026
Accumulator tank	.9999	.00002
Burst diaphragm (each of two)	.9988	.00021

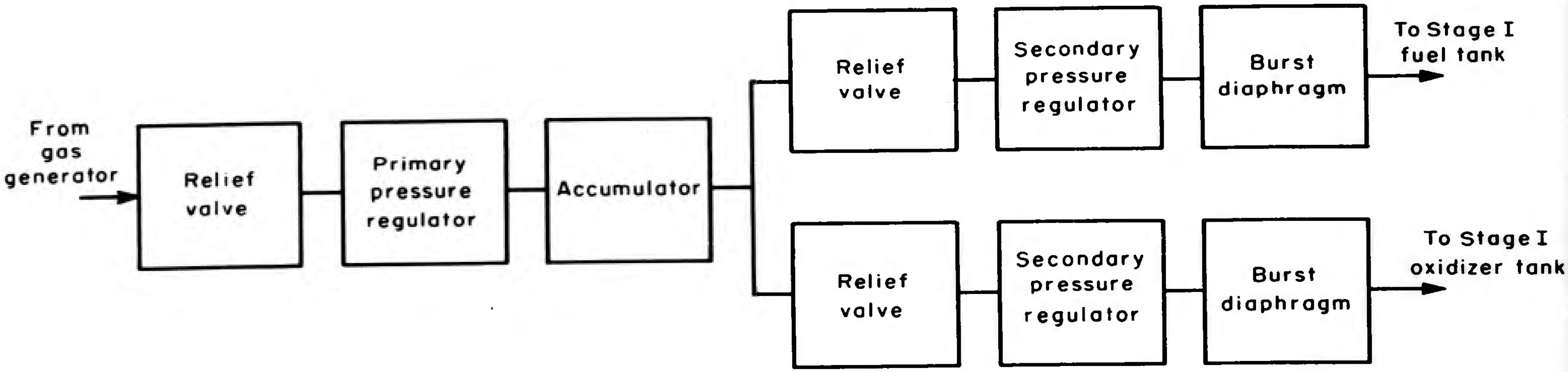


Fig. 10 — Stage I tank pressurization

The Inertial Guidance Package

As discussed earlier, Stage 1 flight operation is controlled by a timed programmer working through the flight control system. This procedure guides the missile along a predetermined flight path. During this initial period, the inertial guidance system (IGS) is inert in a guidance sense but does compute position and velocity data. Upon start of the Stage 2 engine, the IGS assumes active missile guidance. Noting the position and velocity history of Stage 1 flight, the IGS steers the missile to a point in space and a proper velocity such that the free ballistic trajectory of the re-entry vehicle terminates at the desired ground zero.

The stable platform, computer, and electronic control form the main components of the IGS. The stable platform provides acceleration outputs and pitch and roll error signals. Principal parts are three mutually orthogonal accelerometers and a pair of two-degree-of-freedom gyros plus two pendulums. The gyros and pendulums serve to stabilize the platform. Subsidiary parts include heaters, resolvers, and servo drives. Prior to flight, the platform is aligned by a ground-based optical system.

The computer is digital. It accepts platform acceleration outputs and integrates them to give position and velocity data. Additional computer functions involve gravity vector corrections, engine cut-off signals, and destruct signals.

The electronic control has mode switches, power supplies, and amplifiers needed by the stable platform and digital computer.

Figure 11 and Table 9 present the IGS.

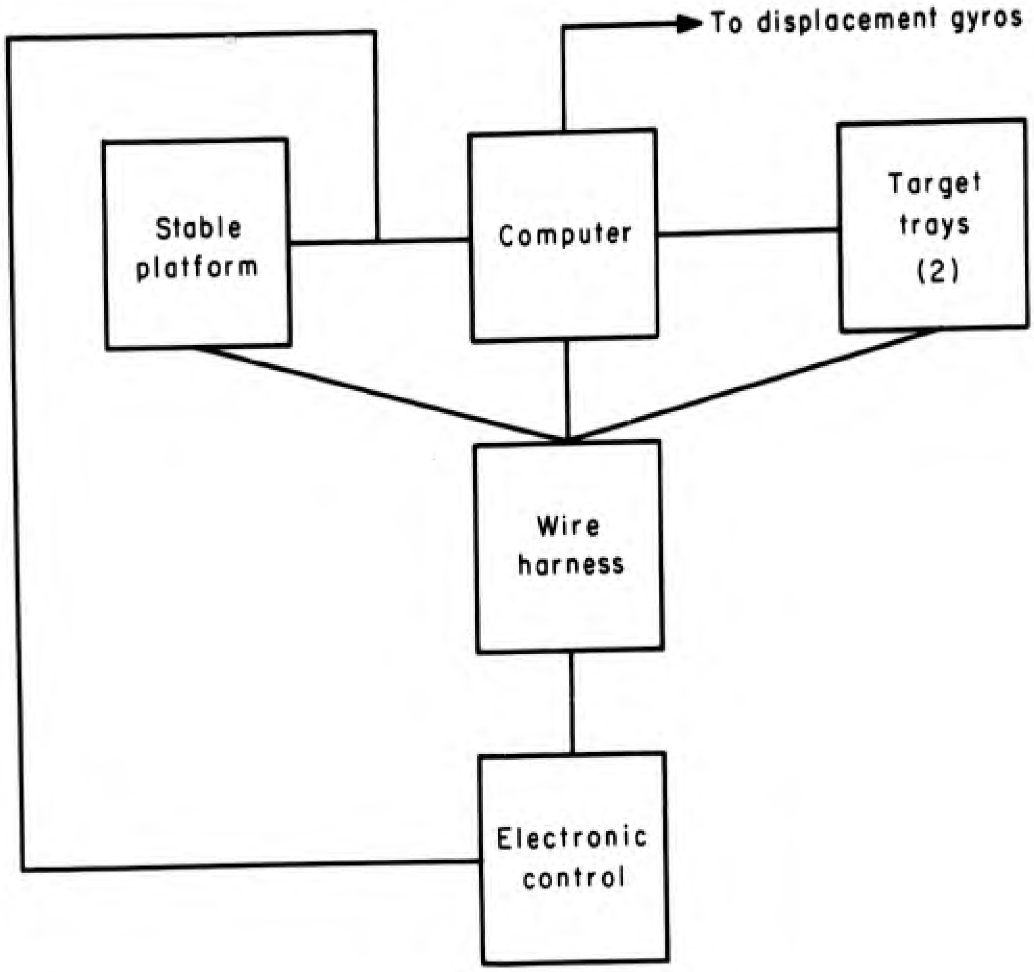


Fig.11—Inertial guidance

Table 9

INERTIAL GUIDANCE FAILURE RATES

Item	In-flight Reliability	Countdown Failure Rate
Stable platform	.9743	.00454
Computer	.9515	.00865
Target trays (each of two)	.9886	.00199
Wire harness	.9970	.00052
Electronic control	.9629	.00658

Failure Rate Conversion Table

The previous tables of missile functions contain countdown failure rates. To convert to other stress levels, Table 10 was assumed. The numerical entries are based on previous RAND studies.

Table 10

FAILURE RATE CONVERSION

Components	$\frac{\text{Operating Failure Rate}}{\text{Countdown Failure Rate}}$ (K_p)	$\frac{\text{Standing Failure Rate}}{\text{Countdown Failure Rate}}$ (K_s)
Electronic parts	.2	.002
Electrical parts	.065	.00065
Electrical-mechanical parts	.15	.002
Mechanical moving parts	.065	.0013
Mechanical non-moving parts	.05	.00025

NUMERICAL EXAMPLE

For this example, we plotted P_{ijk} versus function failure rate to avoid leafing through many pages in Appendix C. This requires that the input parameters of Sec. III be specified, and so the following inputs are assumed.

$T = 30$ days	$^*p_i' = 0.9$
$t_i = 24$ hours	$q_i' = 0.75$
$\delta_i = 0.9$	$\alpha_i = \beta_i$
$M = 5$ days	$^*\gamma_i = 0.9$
$^*p_i = 0.99$	$R = 180$ days
$q_i = 0.85$	k = silo location for all tests

Using these inputs, values of P_{ijk} versus function failure rate were drawn from Appendix C and plotted as shown in Fig. 12. The information in Fig. 12 and in Tables 1 through 10 were combined to form Table 11.

It is most important to understand that Fig. 12 and Table 11 pertain to a specific set of input parameters. Different sets produce different results. For example, Fig. 12 shows that if the function failure rate is between .00002 and .000001, then leave alone is the best testing method--when the curve is based upon a replacement period (R) of 180 days. As R goes up, P_{ijk} goes down. Make R high enough and the leave-alone curve will drop beneath both the continuous-monitor and periodic-check curves. In that event, the character of Fig. 12 is quite different. Table 11 is quite different, too, since Fig. 12 is reflected in the table. Consequently, one should not draw general conclusions about missile testing from these two presentations alone. The example is intended to be illustrative.

With results like those in Table 11, a system designer can begin to say something about missile readiness testing relative to a particular application. Consider the amplifier channel entry in the flight control section.

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*Denotes parameters that have a small effect on P_{ijk}.

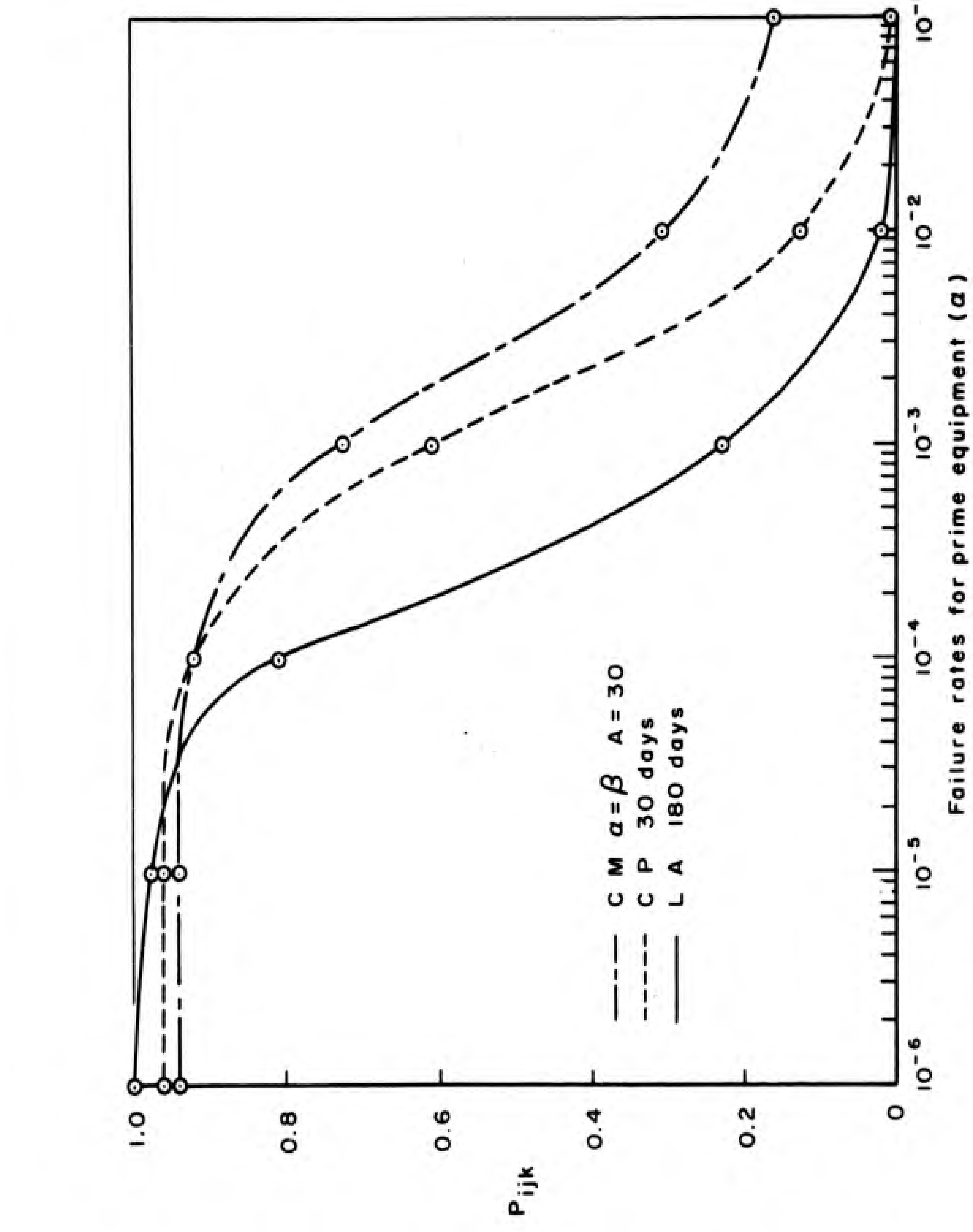


Fig. 12 — P_{ijk} versus function failure rate

Table 11

DISPLAY OF P_{ijk} VALUES

Item	Continuous Monitoring		Check Periodic		Leave Alone	
	Failure Rate	P_{ijk}	Failure Rate	P_{ijk}	Failure Rate	P_{ijk}
Re-entry body						
Nosecone	.00000227	.940	.00000227	.962	.00000227	.997
Warhead	.00000522	.940	.00000522	.960	.00000522	.990
Airframe	$< 10^{-6}$ [a]	.940	$< 10^{-6}$ [a]	.964	$< 10^{-6}$ [a]	1.000
Impact detectors (11)	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Airframe plus fuel tanks						
Tail skirt assembly	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Tail fairing assembly	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Transition structure (2)	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Inter-tank structure (2)	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Fuel tanks (2)	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Oxidizer tanks (2)	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Fuel flow						
Shut-off valves (4)	.000032	.938	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Check valves (4)	.000014	.939	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Tank pressurization						
Primary pressure valve (2)	.0000032	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Relief valve (6)	.000029	.938	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Secondary pressure valve (4)	.000017	.939	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Accumulator tank (2)	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Burst diaphragm (4)	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Flight control						
Timer	.000057	.935	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Programmer	.000026	.939	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Displacement gyros (3)	.000032	.938	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Rate gyros (3)	.00011	.921	.00000144	.963	.00000144	.998
Amplifier channel (6)	.000002	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Mixer (2)	.000174	.910	.00000174	.962	.00000174	.997
Servo amplifier and hydraulic actuator (2)	.000026	.938	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Accessory power supply						
Battery bank (DC)	.000038[b]	.938	.000038	.950	.000038	.912
Battery bank (AC) (2)	.000038	.938	.000038	.950	.000038	.912
Electric motor	.000003	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Hydraulic pump	.000047	.937	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Reservoir	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Quick disconnect (2)	.000006	.940	.000006	.960	.000006	.984
Relief valve	.000009	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Accumulator	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Engine						
Solid-propellant starter (3)	$< 10^{-6}$	.940	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Turbopump (3)	.000023	.938	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Heat exchanger (3)	.000018	.939	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Gas generator (3)	.000022	.938	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Check valve (6)	.000017	.939	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Pressure sequence valve (3)	.000051	.936	.000001	.964	.000001	1.000
Thrust valve (6)	.000034	.938	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Thrust chamber (3)	.000066	.935	$< 10^{-6}$	.964	$< 10^{-6}$	1.000
Inertial guidance						
Stable platform	.000681	.788	.000681	.698	.000681	.300
Computer	.00173	.638	.00173	.495	.00173	.140
Target trays (2)	.00013	.918	.00013	.910	.00013	.755
Wire harness	.000026	.938	.000026	.958	.000026	.939
Electronic control	.00131	.718	.00131	.590	.00131	.200

[a] $< 10^{-6}$ indicates function is not critical.

[b] An assumed 3-year life.

P_{ijk} for continuous monitor is 0.940; for check periodic, 0.964; and for leave alone, 1.0. The leave-alone testing method is best--best in the sense that it affords the highest ready probability. The numerical differences are due mainly to the assumed test inaccuracies and test-caused failures. There is a distinction in ranking, but the differences are small, especially in light of the uncertainties and inaccuracies surrounding present-day input data. Therefore, the important point is not that leave alone is best, but rather that all three testing methods produce similar results. This similarity permits the system designer to choose among the three on different grounds than ready probabilities, namely, cost. Certainly it must be cheaper to leave the amplifier alone than to develop, purchase, and operate checkout equipment.

The situation in the inertial guidance package is quite different. By assumption, the package is running at all times to meet a military launch requirement. Operating failure rates are higher than would be the case if the unit were in the standing state of leave alone. The stable platform P_{ijk} for continuous monitor is more than twice the value for leave alone. A factor of two is meaningful in its own right, but it also points to a second consideration for units that are in operation. In the leave-alone case, the functions within the guidance package are not checked, in the other two testing cases, they are. It is conceivable that missile designers might plan to turn one function on to check a second function, but not plan to check the first function itself. Such "planning" has occurred. The difference in P_{ijk} illustrates the potential penalties involved in such design practices. If something is to be turned on, it should be checked.

In this numerical application (recall the 6-month replacement assumption), all 8 major missile functions came out best when left alone, except for the inertial guidance package. There, continuous monitor is a good choice. A complete testing program for the entire missile would follow the outline just given, i.e., leave everything alone but the guidance package.

Even with this best program, the ready probability for the entire missile, P, as determined by the product of the best ready probabilities for the individual functions, P_{ijk}s, is only .25. This low number is caused principally by the P_{ijk}s for the inertial guidance package. Further, this indicates that for this exemplary missile, one might gain substantially by developing a guidance package that can be brought from rest to launch-ready in a short time, rather than developing continuous-monitor checking equipment. If this redesign can be done, the inertial guidance package could be kept in the standing stage. The failure rates would be lower, resulting in higher P_{ijk}s for the inertial guidance package and a higher P for the entire missile.

V. EXTENSIONS TO A SYSTEM CONCEPT

An application of the method described so far requires that certain weapon system parameters be specified. For instance, in the numerical example of the previous section, the time between periodic checks, T, is 30 days. It is necessary to specify T before the P_{ijk} display table can be generated. Three extensions to a system concept are presented in this section. The first two use the parameter T as a means for presenting the arguments. The first assumes that there is freedom to choose T so as to yield the best over-all P. The second assumes that the choice of T is constrained. A dollar-cost constraint is selected because it is a common one and can be understood intuitively. The third extension correlates certain readiness testing and prelaunch checkout parameters.

If there is design freedom to choose the system readiness testing concept and if there is no significant direct dollar cost associated with a test of the missile, the method of this memorandum can be used directly to determine some elements of the system concept. This situation can arise, for example, if the readiness testing equipment is contained in the silo, the launch site is manned continuously, and an inspection of the missile can be initiated and remotely controlled from some console. Here, the only measurable costs associated with an inspection are missile down-time during the inspection and the time and resource costs resulting from a test-caused malfunction or wear-out.

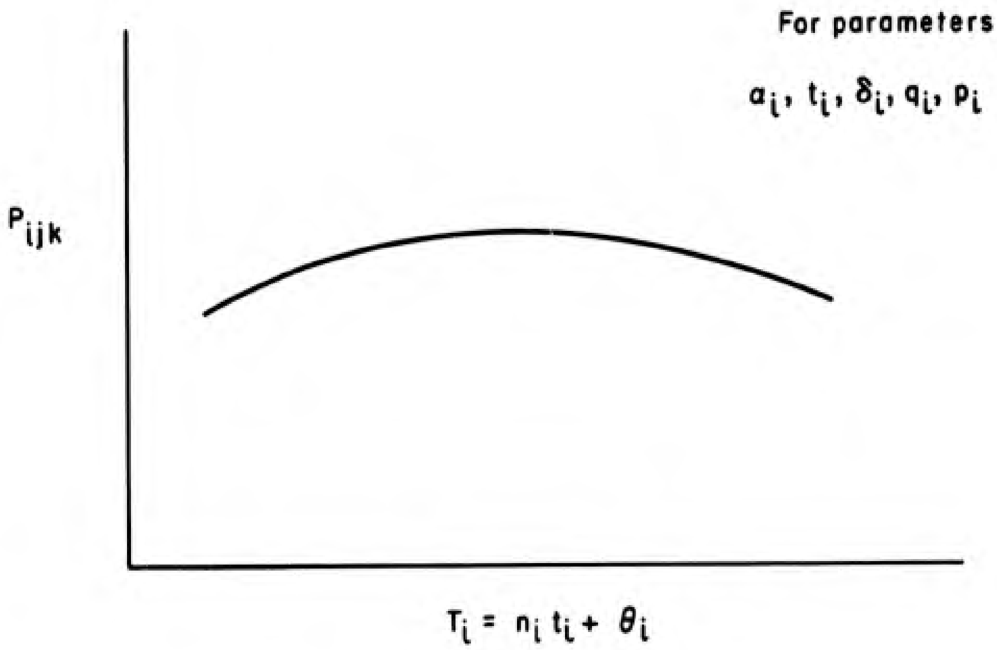
On the other hand, system cost can be strongly dependent upon the time interval between periodics (p. 15). The results of this memorandum can be combined with cost data to determine the best time between periodics.

Failures caused by the inspection can result in costs that are significant. Two characteristics of tests, their severity and their frequency, give rise to these failure-associated costs. Test severity is beyond the scope of this memorandum, except that account is kept of test-caused malfunctions. The missile is penalized for these malfunctions in terms of non-operative time. Methods that choose a test program tending to maximize a missile's up-time (P) will also tend to minimize the time a missile is undergoing maintenance for all reasons, including test-caused malfunctions. This implicit down-time minimization will also tend to minimize the repair-associated costs for a given test severity.

Test frequency has implications pertinent to the problem of wear-out, but in general, the problem of wear-out can only be alleviated, not eliminated. Obviously a component should not be exercised so much that it is worn out in an unacceptably short time. If a function is subject to both random and wear-out failures, one must be willing to accept some cost of wear-out for assurance of missile readiness. The appropriate trade-off involves system economics and estimates of wear-out phenomena. This analysis assumes that the missile is to be rejuvenated every N years, so that the problem is that of not causing a wear-out failure within the N years. For mechanical systems this means that N/T (in years) must be less than some critical value. For electronic systems the same general criterion holds, but is more difficult to apply.

FIRST EXTENSION

Using the results of the analyses, it is possible to prepare a plot for each function checked with period T_i.



where

n_i = 1, 2, 3, ... n

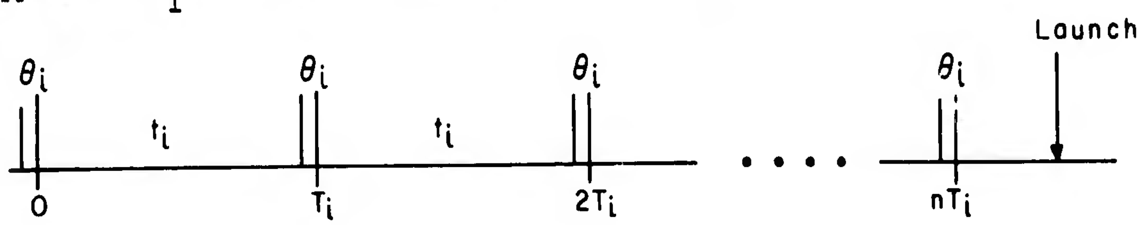
t_i = average repair time for the i^{th} function

θ_i = average test time for the i^{th} function

T_i = average test interval for the i^{th} function.

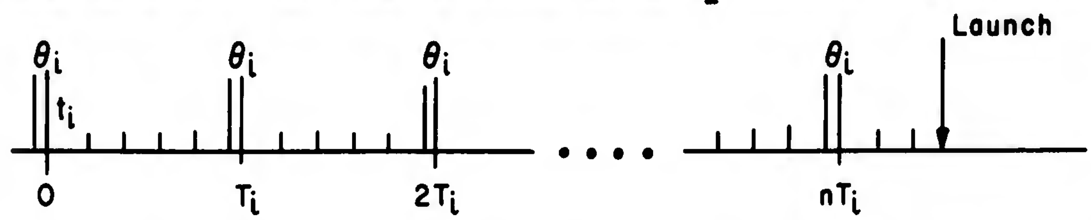
It is logical to expect a maximum value of P_{ijk} at some value of T_i as illustrated above. The arguments follow.

Consider the life span of a function when n_i = 1. As shown, t_i approaches T_i. By definition the function (and thus the missile) is not



operative during θ_i but can be operative only, if at all, during the interval $(T_i - \theta_i)$. If the periodic test shows a missile malfunction, then the missile would spend the entire interval $(T_i - \theta_i)$ in maintenance.

Now suppose n_i is 5. The life span is as follows. With $n_i = 5$, a repaired missile, say repaired in the second t_i interval, has the chance



of remaining operative in the last three t_i intervals. Therefore, it is reasonable to expect P_{ijk} to increase as n_i increases above 1.

However, at large values of n_i, the function failure rate acts to reduce P_{ijk}. For example, the function's MTF is exceeded at very large values of n_i. With a large n_i, a failed function may have a long downtime until checked again.

From the above reasoning comes the a priori statement that there is a maximum for P_{ijk} as a function of T_i.

At this point, a system designer, working with a missile of more than 100 functions, may be faced with more than 100 test time-intervals. Intuitively, even though missile readiness is improved, it does not seem economically or operationally desirable to conduct tests of separate parts of a missile at all these different times. In practice, <u>one</u> system time-between-periodic-tests is generally specified. Consequently, the system designer would probably be forced to give a compromised solution specifying one system T.

As a means of computing a system T based on missile function considerations, the following method is offered. First, choose several plausible periods, like T = 7, 10, 30, 60, 180 days. Second, work the design problem for each T using the appropriate method of either Sec. III (unconstrained) or Appendix A (constrained). Solutions give the best mix of readiness testing methods and thereby the best over-all P for the particular T and constraints. Third, plot the results as indicated in Fig. 13.

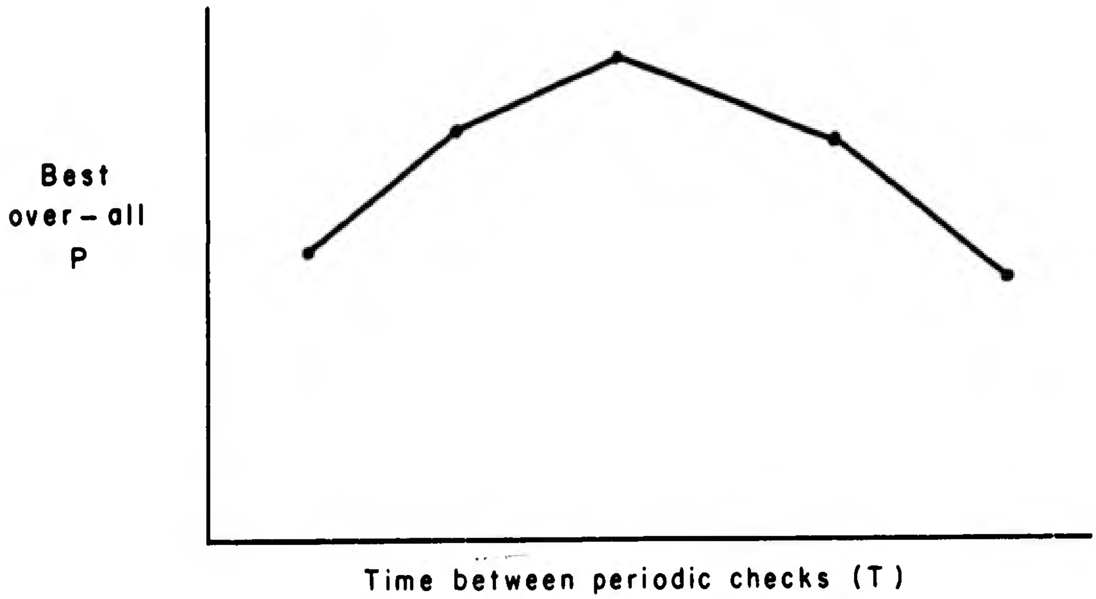


Fig. 13 — Method of finding best T

The best T can be seen readily. The plot will also afford the necessary data to estimate the loss in P from using a non-best T; say, one established by some other, more desirable, operational trait.

If the level of some other possible design constraint (such as silo volume in Appendix A) is undetermined, then presumably the preceding analysis could be used for that. In such an analysis, either the best level of the constraint (or design parameter) would be found, or the change in P caused by a change in the parameter could be estimated.

SECOND EXTENSION

For those system design cases where system dollar costs strongly depend on the schedule of readiness testing, the data from a plot like Fig. 13 can be used along with system cost data to find the system best-time-between-periodics. Suppose, as described in companion Refs. 2 and 10, the number of missiles procured, based, and supported for a given system cost could be estimated relative to T, with all other system design parameters held fixed. Then the numerical estimate can be combined with a P estimate to give the total number of missiles ready to go for the fixed condition.

The best T could be obtained from a plot of the data as in Fig. 14. The best system T is that which gives the largest expected number of missiles ready to go. For each level of T, the expected number is the product of two curves:

1. P versus T from Fig. 13, and
2. Total number of missiles for given cost versus T from Fig. 14.

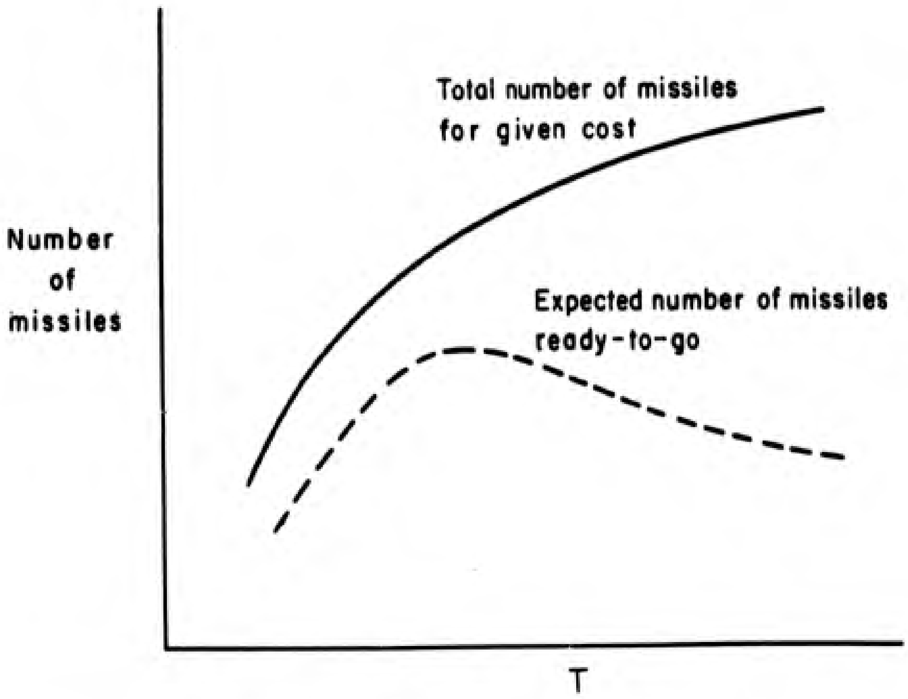


Fig. 14 — Best T for fixed cost

An effectiveness table could be useful if more than one T is to be considered. For example, if the system description specifies periodic check with period T, most functions would be checked every T. However, there may be a few functions with low failure rates for which checks at some multiple of T would produce greater P_{ijk}. As a second example, the system may prescribe periodic check with period T, but allow for continuous monitor or leave alone for some functions. Here, the system description could include T and multiples of T which become either A or R. The extension of Fig. 14 to an effectiveness table is relatively straightforward.

In Fig. 14, the problem was to choose that T that gives the best system capability. But as discussed in a companion work, Ref. 11, the definition of best system capability can have more than one meaning. For instance, if one is concerned with a strike-second capability, the greatest number of missiles after an attack could result from spending a fixed budget less on checking (producing lower readiness) but more on missiles (hence more targets for the enemy to cover in his initial strike). Alternatively, if one is concerned with a first-strike capability, the greatest number of ready missiles for the fixed system cost could be the criterion. Table 12 could supply data for both types of decisions in a system framework.

Table 12

EFFECTIVENESS TABLE

Test Schedule	Over-all P	Test-associated Costs	Number of Missiles Procured and Based	Expected Number of Ready Missiles
T				
T, mT				
T, A				
T, R				

The first column contains the various test schedules that are to be considered. The second lists the corresponding Ps computed as described. The third and fourth columns result from cost analyses similar to those presented in the companion works, Refs. 10 and 11. The last column is simply the product of columns two and four.

PRELAUNCH CHECKOUT

Prelaunch checkout and its relation to readiness testing is briefly described on p. 8. A companion work on prelaunch checkout[3] uses a parameter p_r, defined as the probability that the r^{th} defect does not exist prior to checkout, i.e., has not occurred since the last checkout. This parameter, p_r, depends on the time since the last accurate checkout, but the authors of Ref. 3 indicate that it could be more appropriate to base the prelaunch checkout on the expected values of p_r. The P_{ijk} terms in this memorandum are exactly those expected values and hence can be used as inputs to the prelaunch checkout analysis in Ref. 3.

Appendix A

CONSTRAINED READINESS TESTING PROGRAMS

Ideally, one might think of a design exercise where complete freedom exists to choose missile and checkout equipment design to maximize P_{ijk}. However, such complete freedom rarely exists. Often design constraints are imposed by safety or physical design reasons. One example of a safety constraint is a requirement for monitoring continuously the warhead for safety reasons. To understand the meaning of a physical design constraint, consider an electric motor that is heavily insulated to protect it from the heat of the propulsion system. Such a motor cannot be run continuously, even if analysis showed continuous monitor to be the best testing method, because it own heat would soon destroy the motor. In Sec. III, this first general area of design constraint is handled by showing that missile readiness is given by

$$P = \prod_{i=i_c} (P_{ijk})^{x_{ijk}} \cdot \prod_{i \neq i_c} (P_{ijk})^{x_{ijk}} \qquad (3)$$

where

$P \equiv$ probability that the entire missile is operative

$P_{ijk} \equiv$ probability that function i is in ready condition if readiness testing method j is used with an equipment location k

$$x_{ijk} = \begin{cases} 1, & \text{if readiness testing method } j \text{ using equipment in location } k \text{ is used for function } i \\ 0, & \text{otherwise} \end{cases}$$

$i_c =$ restricted functions.

The problem can be stated as one of choosing a set of $x_{ijk} = 1$ so that P given by Eq. 3 above is a maximum and

$$\sum_{ijk} x_{ijk} = 1 \text{ for each } i \tag{4}$$

A design aid for the mathematics above is part of Sec. III (see Fig. 3, p. 33).

Other design constraints are discussed in Sec. II. For example, silo space for test equipment may be limited. Although there may not be a single "other" constraint that pervades all design applications, it does not follow that there will never be such a constraint. Therefore, this appendix presents a treatment of these other design constraints.

Equation 3, by use of natural logarithms, can be rewritten as

$$\ln P = \sum_{\substack{i=i_c \\ jk}} x_{ijk} \ln P_{ijk} + \sum_{\substack{i \neq i_c \\ jk}} x_{ijk} \ln P_{ijk} \tag{5}$$

By definition,

$$Z = \ln P - \sum_{i=i_c} x_{ijk} \ln P_{ijk} = \sum_{\substack{i \neq i_c \\ jk}} x_{ijk}\, p_{ijk} \tag{6}$$

where

$$p_{ijk} = \ln P_{ijk}$$

The constrained program problem is now one of maximizing a function Z, subject to constraints.

The general form of the (other) constraint equation is

$$\sum_{ijk} x_{ijk} c_{ijk} \leq C \tag{7}$$

where

x_{ijk} = 1 or 0 as in Eq. 3

c_{ijk} = that part of the constraint required for test of i^{th} function using method j from location k

C = total constraint.

For the example mentioned earlier, i.e., constrained silo volume, Eq. 7 becomes

$$\sum_{ij} x_{ij2}\, sv_{ij2} \leq SV, \text{ the silo volume allowance}$$

where

sv_{ij2} = silo volume of equipment required for test of the i^{th} function using method j from silo location (arbitrarily taken as location 2).

It can be seen that both the effectiveness equation, Eq. 6, and the general constraint equation, Eq. 7, are linear. At first, a solution to the constrained program problem was attempted using integer linear programming, but no programming could be found. The difficulty is that the grouping of functions is not constant for all test methods. For example, suppose the warhead is to be monitored continuously as a safety constraint. If some other function can be tested using the same equipment, then there would be no c_{ijk} penalty in Eq. 7 for this second test. On the other hand, if periodic check is chosen for the second function, there may or may not be a penalty depending on whether extra periodic check equipment is needed. Therefore, the method of solution presented next is offered as a stopgap measure in case the problem arises before an efficient solution method is developed. It is intended to yield good, feasible solutions as a point of departure. Its tabular form also facilitates the examination of alternative groupings.

It is assumed that functions of a missile can be tested by two methods, continuous monitor and periodic check, using equipment either in a van or in a silo. An analysis of the whole system, perhaps using the methods of Ref. 2, has indicated that for maximum missile readiness per system dollar, the system concept should be periodic check with a period of T_0 days. In addition, some functions must be monitored continuously for safety reasons, and a portion of the hardened silo volume has been allocated to this checkout equipment. The van has a limited volume, and as a final constraint, the weight of the test equipment to be placed aboard the missile is limited. Therefore, the problem is that of choosing which tests to do periodically using the constrained van volume, which tests to do periodically and which to do continuously using the constrained silo volume, and which functions to leave alone.

These constraints can be written as

$$\sum_{i} x_{i11} v_{i11} \leq V, \text{ the van volume allowance}$$

$$\sum_{ij} x_{ij2} s_{ij2} \leq S, \text{ the silo volume allowance}$$

$$\sum_{ijk} x_{ijk} w_{ijk} \leq W, \text{ the missile weight allowance}$$

where

$j = 1$ for check periodically

$j = 2$ for continuous monitor

$j = 3$ for leave alone

$k = 1$ for van

$k = 2$ for silo

$k = 3$ for missile

and

w_{ijk} = weight of missile-carried equipment required for test of the i^{th} function using method j from location k

v_{i11} = volume of van equipment required for test of the i^{th} function using method 1 from location 1

s_{ij2} = silo volume of equipment required for test of the i^{th} function using method j from location 2.

No constraints apply for leave alone.

Note that if the same test equipment can be used for the test of more than one function, then these functional groupings should be accounted for in the constraint relationships. In general, these sorts of groupings give different indications of function condition. Some functions can, for example, be monitored continuously, but because of the extra testing stress and the less effective testing that would result, the function would not demonstrate as high a ready probability.

These, then, are the sorts of complicating factors that must be accounted for and overcome in the design analysis. The tool or method of analysis must allow for arbitrary and non-constant grouping of tests (i.e., the grouping may be different within each constraint) and should afford a means of incorporating an examination of design trade-offs. A design tableau, such as Table 13, is suggested. The tableau provides a layout for examining the division of equipment among van, silo, and missile. (The theoretical basis of the problem and some of the steps of the procedure are in Appendix B.)

To use this tableau to help determine the best readiness testing design and to examine whatever alternative design mixes are efficient, the following steps are appropriate:

1. Enter the appropriate data in each row (i.e., p_{ijk}, v_{ijk}, ...).

2. In the rows labelled "check," indicate by a symbol ⊗ the tests that are constrained by safety or physical reasons.

3. For each other test, find the method of test that yields the largest p_{ijk} and indicate it with an x. For these tests that are possibly grouped, the p_{ijk} appropriate for the group is the sum* of the individual p_{ijk}s. If there are alternative groupings, they can be entered and compared in the bottom row.

4. Construct the sum of the requirements for the tests, i.e., add the weight and volume requirements for the tests marked by x. Enter the sums in the appropriated rows of the column marked "Sum of Requirements."

5. Compare the sum of the requirements with the appropriate constraint.

6. If all the constraints have been met, then the assignment of tests is acceptable, and depending on the groupings, near optimum.

7. If one or more of the constraints has been exceeded, then pick one of the constraints and for that constraint, say the allowable volume of the van, rank the tests according to p_{ijk}/v_{ijk}. The tests that have the lowest ratios should be changed to their next best test method, i.e., next largest

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*Recall that $p_{ijk} = \ln P_{ijk}$.

Table 13

CONSTRAINED DESIGN TABLEAU

Equipment Location	Test Method	Quantity	Function: Single 1	2	3	...	Grouped m_1m_2	$n_1n_2n_3$	...	Sum of Requirements
VAN	C.P.	p_{i11}								
		v_{i11}								Σv
		w_{i11}								Σw
		p_{i11}/v_{i11}								
		p_{i11}/w_{i11}								
		CHECK								
SILO	C.P.	p_{i12}								
		s_{i12}								Σs
		w_{i12}								Σw
		p_{i12}/s_{i12}								
		p_{i12}/w_{i12}								
		CHECK								
	C.M.	p_{i22}								
		s_{i22}								Σs
		w_{i22}								Σw
		p_{i22}/s_{i22}								
		p_{i22}/w_{i22}								
		CHECK		$\otimes$						
MISSILE	C.P.	p_{i1k}								
		w_{i1k}								Σw
		p_{i1k}/w_{i1k}								
		CHECK								
	C.M.	p_{i2k}								
		w_{i2k}								Σw
		p_{i2k}/w_{i2k}								
		CHECK								
NONE	L.A.	p_{i34}								
		CHECK								
GROUPING COMPARISONS										

p_{ijk}, until this constraint is met. The logic of the ranking is that the tests with the greatest contribution per volume are chosen, in this example, for use in the limited van volume.

7a. Two alternatives in Step 7 exist. First, it may prove desirable, in light of the alternative test assignments, to obtain a constraint deviation, i.e., permission to exceed one of the constraints. In the example above, it may be possible to support arguments for the use of two vans or a larger van. Second, it may prove desirable, again in the light of alternative test assignments, to regroup the tests, meet the constraint(s), and still obtain an acceptable ready probability. As suggested earlier, the space labelled "Grouping Comparisons" is for this iteration and examination purpose.

8. If Step 7 causes another constraint to be exceeded or if more had been exceeded initially, then Step 7 should be repeated for that constraint until all constraints are met.

The above process will yield the best solution in some cases and, depending on the compromises made, it will yield just a "good" solution in other cases. The tableau allows examination of alternative ways to organize, and readily displays the consequences of these alternatives in terms of both capability and resources. It shows what constraints could well be stretched, and what would be gained in terms of p_{ijk}, and what it would cost in W, V, and S. It is felt that in light of the possible crudeness of the input data, this method of analysis, affording in some cases just a "good" solution, is appropriate for use until an exact solution method becomes available.

If the input data are crude, one might ask, what is the advantage of the method in this appendix over the current methods of evaluation, experience, design judgment, and rules of thumb? This method, like others developed during the course of this project, is intended to bring objectivity into design decisions. If the data are crude, this method at least affords the means of making best use of these data. Admittedly, it is an aid to design intuition. However, as design and experimental testing continue, the decisions made using this method could be re-evaluated. Afterward, when the ground system design must be made firm, the available data estimates could be much improved and therefore the final design decisions could also be better.

Appendix B

MODEL DEVELOPMENT

For a strategic missile to be effective in a retaliatory mode, it must be ready to go as much of the time as is economically and technically feasible. It is the role of readiness testing and maintenance operations to keep this weapon in a ready condition. In the system planning phase, when in theory at least, dollars for maintenance, missile-site active defense, basing, and offensive subsystem design and procurement can be balanced to give the greatest total missile system effectiveness, an analysis is needed to provide the best system readiness testing concept per total system dollar.

No one has been able, as yet, to solve this over-all system optimization problem, but several studies of this operational-economic balance for readiness testing suboptimization on the system level have determined such necessary system parameters as best time between periodic inspections and overhauls, and system economic characteristics that favor either periodic check or continuous monitor testing methods.[12-15]

The remaining task is to determine, within the system concept and safety, or physical, constraints, the readiness testing method for each part or function of a missile so that the entire missile system demonstrates the best readiness characteristics. The mathematical model for this is developed in this appendix.

A missile and ground operating equipment (GOE) necessary for launch are assumed to be composed of N statistically independent parts or functions. It is further assumed that the state--go/no go--of each of these functions can be determined in terms of function performance necessary to insure proper system performance. (Because it forces limits for a decision of good or bad to be made in an a priori sense, this assumption neglects the chance that while a function parameter may have drifted beyond an a priori set limit, other system changes may have occurred to compensate for this drift. This undesirable feature can be alleviated by aggregating missile parts into larger test units.)

A missile can be called upon at any time. In order for a missile to be ready to go at any instant, each function must also be ready to go at that time. This characteristic for the entire missile and the launch GOE is called P, i.e., the probability that the missile is ready to go at any time. Missile readiness, P, is dependent on the readiness probability of each missile function, P_{ijk}.

Missile functions can be checked periodically, monitored continuously, or left alone. Test equipment to do readiness testing can, in general, be located in a van, helicopter, or other mobile device, in the silo, or possibly in the missile itself. Each combination of test method-equipment location ordinarily produces different readiness probabilities for each function. Thus, one needs to determine what test actions should be taken to maximize missile readiness within the constraints imposed by missile design, system operation, and safety. In other words, within the confines of the system concept, how does one choose among periodical check, continuous monitor, or leave alone, and among all possible test equipment locations, for each function in a missile system in order to make the best use of physically constrained tests and obtain the greatest missile readiness?

PROBLEM STATEMENT

It follows from the assumption of statistical independence* that missile readiness, noted simply by P, is given by

$$P = \Pi (P_{ijk})^{x_{ijk}} \tag{8}$$

where

P_{ijk} = probability that function i is in ready condition if readiness testing method j is used with an equipment location k

$$x_{ijk} = \begin{cases} 1, & \text{if readiness testing method } j \text{ using equipment in location } k \text{ is used for function } i \\ 0, & \text{otherwise} \end{cases}$$

*The assumption of statistical independence does not hold strictly for all test methods. For example, in continuous monitor, a common tester may monitor several functions. Suppose a unit of test equipment is monitoring three individual functions, a, b, and c. There are three separate P_{ijk}s, for a, b, and c. Each of the three P_{ijk}s reflects the test equipment's failure probability. Now suppose that the functions are grouped into one unit. There is one P_{ijk}. It does not follow that

$$(P_{ijk} \text{ for a}) (P_{ijk} \text{ for b}) (P_{ijk} \text{ for c}) = P_{ijk} \text{ for abc.}$$

When this independent assumption fails, the effects on P could be significant but the decision process to be described will be largely unaffected. This is because each value of P_{ijk} is compared to each alternative value of P_{ijk} on an individual basis. In this context, each value is separate, in essence, from the other values (except of course, that all terms are based on the same system concept) because at that time just one is being examined, with all other factors held constant.

To separate those functions that are constrained by safety or physical reasons from those for which design freedom exists, let the constrained tests be indicated by i_c. Then

$$P = \prod_{\substack{i=i_c \\ jk}} (P_{ijk})^{x_{ijk}} \cdot \prod_{\substack{i \neq i_c \\ jk}} (P_{ijk})^{x_{ijk}} \qquad (9)$$

where the symbols are defined as in Eq. 8.

The problem to be solved is: choose a set of $x_{ijk} = 1$ so that P given by Eq. 9 is a maximum and

$$\sum_{jk} x_{ijk} = 1 \text{ for each } i \qquad (10)$$

DERIVATION OF P_{ijk} TERMS

Under the assumption that a missile function is either good or not good at any time, the portion of the life span of a function that is spent in the silo can be divided into five states:

1. Operative--there are no failures (malfunctions)
2. Inoperative-unknown--a disabling failure has occurred in the function so that it is inoperative, but the failure is undiscovered. (This state will also be called "failed-unknown")
3. Inoperative-awaiting maintenance--a disabling failure has occurred in the function, has been discovered, and the missile is down awaiting maintenance. (This state will also be called "failed-known")
4. Being maintained--the inoperative function is being repaired or replaced and the missile is down
5. Undergoing periodic inspection or preventive maintenance --the missile is down and the function (or all functions) are being inspected or replaced

Thus, all functions on a missile must be in State 1, in order for the missile to be ready to go. (It is assumed that preventive maintenance is performed after a missile is removed from the silo and replaced with another. The down time is counted against the maintenance operation. This time should be considered when developing the system maintenance concept, but is not germane to this analysis of readiness testing.)

Looking now at each function separately, it is seen that its life is spent in a series of states, with defined paths of possible transitions from state to state. If the missile is rejuvenated every several years (this period could be determined by the shelf life of storable propellants or composition seals), it appears that wear-out failures can be safely neglected in an examination of readiness testing, except that proper attention must be given to wear-out constraints imposed on some types of tests for some functions. For example, mechanical parts with definite wear-out characteristics probably should not be exercised continuously. Wear-out phenomena should be considered when determining the preventive maintenance concept --especially for functions that are operated continuously. For the readiness testing concept, however, the concern should be centered about the randomly occurring failures. And, for this analysis, one should also take account of the failures that are caused by the readiness testing; different methods of testing impose different stresses on the missile functions.

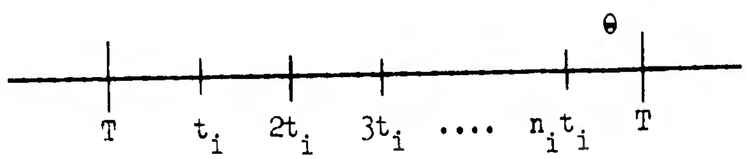
One further assumption is needed before the analysis can continue; that failures are exponentially distributed. The justification for this assumption and the conditions under which it is reasonable have been discussed in numerous reports,[2] and will not be discussed here. This appears to be a reasonably good assumption for many physical items, other than electronic devices, and there is much experimental evidence to justify its (cautious) use.

Because of the preceding assumptions on the statistics of the failure distribution, the probability of a function failing in a given period is dependent only on the length of the period and the operational stresses, and is independent of the number of periods of previous use. All other state-to-state transitions also depend only on the existing state and the state to which the movement is to be made. Therefore, the progress of a particular function from state to state through its operational lifetime can be viewed as a Markov Process, which will be described later.

PERIODIC CHECK

The process of checking periodically is the most tractable of the two active readiness testing methods and will be treated first. The length of time between periodic tests that gives the best system performance is an element of the system concept, and can be determined by using results in Ref. 2 or by using the method described in Sec. V. The time between periodic tests will be called T. Using the periodic check, each missile function is tested every T days, and by assumptions, defects that have occurred in the function can be detected only at that time.

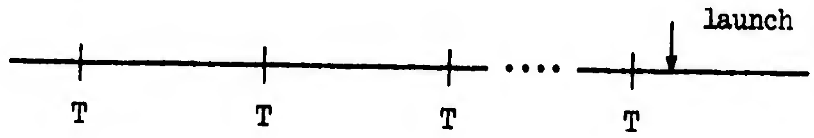
It will prove useful to break the T days down into n_i smaller time periods of t_i days each and into the time required for the missile's periodic test, θ, so that a periodic interval appears as



To further simplify the analysis, the quantity of time, t_i, is defined to be the average length of time required to repair or replace the i^{th}

function. This includes all time required to travel to the missile, unbutton the silo, remove and repair or replace, etc.

Projecting the function's operations into the future, they appear as a series of intervals of length T, that end at the time of launch.



The movements of a missile function through this pattern must of necessity demonstrate repetitive properties. The chance of a transition from one of the five states to another will depend on whether the movement, if it occurs, takes place during a periodic test or between tests, but not upon the function's previous transition history; this gives a Markovian property to the process. It will be convenient, therefore, to define three transition matrices: Matrix A defined over t_i, B defined over θ, and C defined over T. These three describe the Markov Process.[16]

For States 1 through 4:

A $\equiv$ matrix of single-step transition probabilities in time t_i, for the interval between periodic tests.

$$A = \{a_{rs}\};\ \text{a matrix of terms } a_{rs} \qquad (11)$$

where

$a_{rs} \equiv P$ (transition from State r to State s in one interval of length t_i)

Similarly, for the periodic test

B $\equiv$ matrix of single-step transition probabilities in time θ, for the periodic test.

$$B = \{b_{rs}\} \qquad (12)$$

where

$b_{rs} \equiv$ P (transition from State r to State s during the periodic test)

The process to be described moves through n_i steps of length t_i described by A, and one step of length θ described by B, for each large step, or period T, in its life. A matrix is needed to describe transitions over the entire period T.

C $\equiv$ matrix of transition probabilities in time $T \cong n_i t_i + \theta$.*

If

$c_{rs} \equiv$ P (transition from State r to State s in one interval of length T)

and if it is observed that each term c_{rs} is a term compounded from a_{rs} and b_{rs} terms,** then

$$c_{rs} = p_{rs}(n_i t_i + \theta) \tag{13}$$

$$= \sum_v p_{rv}(n_i t_i) p_{vs}(\theta) \tag{14}$$

By induction, it can be shown that

$$A^{n_i} = \left\{ p_{rv}(n_i t_i) \right\} \tag{15}$$

and

$$B = \left\{ p_{vs}(\theta) \right\} \tag{16}$$

---

*See p. 22.

**That is (changing notation slightly for clarity), $a_{rv} = p_{rv}(t_i)$, the probability of moving from State r to State v in time t_i, and $b_{vs} = p_{vs}(\theta)$, the probability of moving from State v to State s in time θ.

so that

$$C = \{c_{rs}\} \tag{17}$$

is given by

$$C = A^{n_1} B \tag{18}$$

The next step in finding the long-run probability of being in State 1 is to define the m-step absolute probability vector $x^{(m)}$.

$$x^{(m)} \equiv \left(x_1^{(m)}, x_2^{(m)}, x_3^{(m)}, x_4^{(m)}\right) \tag{19}$$

where

$$x_k^{(m)} \equiv P \text{ (function being in State k after m periodic intervals)}$$

and

$$x^{(m+1)} = x^{(m)} C \tag{20}$$

If matrix C is either regular or ergodic, this vector has the property that if the process is allowed to contimue long enough that the effects of the initial distribution of states have been dissipated, then for some large m, a steady state, or fixed, probability vector is given by

$$x = xC \tag{21}$$

That is, the long-run distribution of states following a periodic test should be unaffected by an additional period.

Solving for the vector x (for this problem, this involves the solution of 5 simultaneous linear equations; one for each x_k and one wherein the sum of the x_ks equals unity) gives the probability (of the function's being in each state following a periodic test) that is conveniently far removed from the time of initial installation. In particular, this could be the test that precedes the beginning of hostilities.

Hostilities can begin at any randomly chosen time (with respect to the readiness testing schedule) and, therefore, a measure is needed for the probability of each function being operative at any randomly chosen future time. If the vector describing the state probabilities in the time following a (steady-state) periodic test is given by x, as before, then for r time periods of length t_i later

$$x^{(r)} = xA^r \qquad (22)$$

where

$$x^{(r)} = \left(x_1^{(r)}, x_2^{(r)}, x_3^{(r)}, x_4^{(r)}\right)$$

Neglecting the θ time interval ($\theta < t_i$, and in turn $t_i \ll T$, therefore only a small error will result*), the probability of the function being operative at time of launch is

$$E\left[x_1^{(r)}\right] = P_{ijk} = \frac{1}{n_i} \sum_{r=1}^{n_i} x_1^{(r)}, \text{ for each j and k.} \qquad (23)$$

This form of the expectation of $x_1^{(r)}$ over the n_i time interval is used because the launch attempt could occur at any time with equal likelihood.

The necessary matrices must now be developed using terms describing the missile function's physical characteristics, the test equipment's capabilities and error propensities, and the system operation and maintenance concepts.

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*Another way of stating this effect is the assumption that

$$\frac{T - \theta}{T} = 1 .$$

Matrix A

Matrix A is of the form

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & a_{34} \\ a_{41} & 0 & 0 & 0 \end{bmatrix}$$

The terms are given as follows:

$a_{11} \equiv$ P (function does not fail in t_i/operative at beginning of t_i)

and

$a_{12} \equiv$ P (function fails in t_i/operative at beginning of t_i)

If $\lambda_i \equiv$ basic failure rate of the i^{th} function, failures/hour. (This is the failure rate, under some normal operating conditions, of the i^{th} function.)

and $k_{i\ell} \equiv$ stress factor for the i^{th} function in ℓ^{th} mode of use, dimensionless. (This factor, obtained either experimentally or from some source such as Ref. 17, accounts for the difference in function failure rate from that considered as normal.)

(NOTE: $\ell = 1$ indicates mode and stress appropriate to i^{th} function in operations between periodic tests.)

Then, by the exponential failure assumption*

$$a_{11} = \exp(-k_{i\ell}\lambda_i t_i)$$

$$= \exp(-\alpha_i t_i) \qquad (24)$$

---

*If the function basic failure rate λ_i is obtained from a consideration of the component failure rates by a relationship such as and

$$a_{12} = 1 - \exp(-\alpha_i t_i) \tag{25}$$

The term a_{13} is zero because failures can be detected only during the periodic test, and a_{14} is zero because a failure must be detected before it can be repaired.

Once a function has failed, it will remain in a failed-unknown state until at least the next periodic test. Therefore,

$$a_{22} = 1$$

and all other transitions from State 2 have probability zero.

Once a function has a known failure, it can make one of two possible moves during each time t_i; it will remain in its present state, or move into the state of being repaired. Maintenance policy will decide whether a malfunctioned missile will be repaired immediately, or whether it will be placed into a "missiles-down, awaiting-maintenance" list that is planned for the sake of support economies. It could prove advantageous from a system planning point of view to allow some missiles to remain with failures, until others in the area have also failed, then bring a repair crew into the area

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$$\lambda_i = \sum_r \lambda_{i_r}$$

where

λ_{i_r} = failure rate of r^{th} component in the i^{th} function, then the term

$$k_{i\ell}\lambda_i$$

should be replaced by

$$\sum_r k_{i_{r\ell}} \lambda_{i_r}$$

to account for the individual component reactions to the stresses imposed. (17).

and fix them all. As a consequence, the transition probability a_{34} will be handled as a parameter. This expedites an examination of the impact of different maintenance policies on the system ground effectiveness.

Three levels of a_{34} will be considered in the data presented in Appendix C, $a_{34} = 0.9, 0.2$, and 0.1. As a_{34} is defined as the probability of a transition from the state "failed-known" to the state "being repaired," it can be paraphrased as

$$a_{34} \equiv P\,(\text{function goes to repair in time } t_i/\text{no repair prior})$$

Then, from any arbitrary time when the failure is known, if the probability that the function will have gone to repair by the end of m time periods of length t_i is defined as P_m, it follows that

$$P_1 = a_{34}$$

$$P_2 = 1 - (1 - a_{34})^2 \qquad (26)$$

$$\vdots$$

$$P_m = 1 - (1 - a_{34})^m \qquad (27)$$

This probability is plotted on Fig. 15. These curves allow one to choose among a rapid ($a_{34} = 0.9$), moderate ($a_{34} = 0.2$) or slow ($a_{34} = 0.1$) maintenance policy.

If $p_m \equiv P$ [function goes to repair in m^{th} interval of length t_i], then

$$p_m = a_{34}(1 - a_{34})^{m-1} \qquad (28)$$

The average delay time for each policy is given by

$$E(m) = \sum_{m=1}^{n_i} m p_m$$

$$\cong \frac{1}{a_{34}} \qquad (29)$$

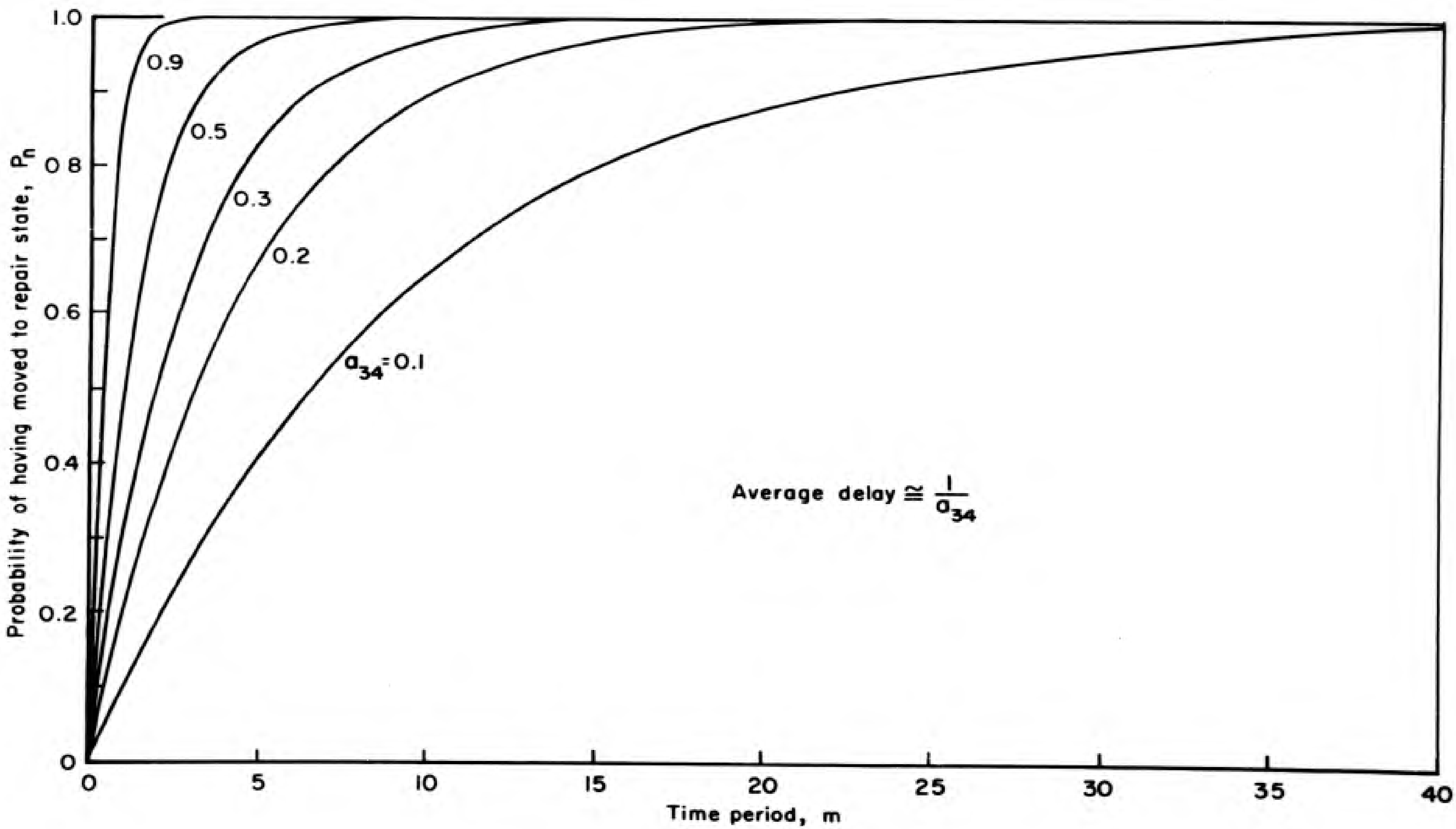


Fig. 15 — Effects of maintenance delay

Policy	a_{34}	Average Delay
rapid	0.9	1.1 t_i
moderate	0.2	5 t_i
slow	0.1	10 t_i

The term t_i was defined to be the average time to repair the i^{th} function. Nevertheless, it will be assumed that if a missile is in maintenance at the beginning of one period t_i, it will be operative at the beginning of the next t_i. Hence:

$$a_{41} = 1$$

This assumption could be readily changed to reflect errors in maintenance, need to do again, extreme times, etc. However, at most $2t_i$ would be reasonably required for the repair operation, and the results will be shown to be insensitive to such changes for most levels of the other parameters. Therefore, within context this simplifying assumption appears justified.

For use in determining C, the matrix A must be raised to the n^{th} power. For $n \geq 3$,

$$A^n = \begin{bmatrix} a_{11}^n & 1 - a_{11}^n & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - \left[a_{32}^{(n)} + a_{33}^{(n)} + a_{34}^{(n)}\right]^* & a_{34} \sum_{i=0}^{n-3} a_{33}^i \left(1 - a_{11}^{n-2-i}\right) & a_{33}^n & a_{34} a_{33}^{n-1} \\ a_{11}^{n-1} & 1 - a_{11}^{n-1} & 0 & 0 \end{bmatrix}$$

* $a_{ij}^{(n)}$ ≡ n-step transition probability from State i to State j.

Matrix B

Matrix B that describes the probabilities of movements between states during the periodic test is of the form:

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 \\ 0 & b_{22} & b_{23} & 0 \\ 0 & 0 & 0 & b_{34} \\ 0 & 0 & 0 & b_{44} \end{bmatrix}$$

Test Equipment Capabilities

Two definitions are needed to describe the transition probabilities of matrix B; these terms describe the periodic test equipment's capability of detecting faults, and propensity toward errors.

p_i = P (test equipment calls a good i^{th} function good)

q_i = P (test equipment calls a defective i^{th} function defective)

These two test equipment qualities are quite different in their natures, and would be estimated in different ways. The first term, p_i, is a measure of test equipment accuracy and consistency; it depends on the nature of the test and the design of the test equipment. To call a good function bad, a tester must sense a measurement that is within allowable limits, and present an incorrect response. It appears that the only way to truly estimate the p_i terms is by an experimental program. Fortunately, as discussed in Sec. III, the readiness probability is only weakly dependent on p_i.

The choice of the best P_{ijk} is more heavily dependent upon the term q_i. This probability (that the periodic inspection detects a failure that exists in the i^{th} function) is another extremely difficult parameter to estimate. Depending on the conditions of its use in the model, there may be two problems concerning this parameter: given an existing testing device and a procedure, what detection probability does it afford, and given a test situation for which equipment is to be designed, what is the lowest detection probability that would be acceptable?

Addressing the second question first, it is seen that q_i can operate as a design goal for the test equipment. Reference 3 derives a straightforward expression for the minimum acceptable test efficiency, but it is not useful here because of the large number of interacting parameters. However, within the context of the other parameters, a certain level of test efficiency could cause a decision between one test method or another for a particular function. The data appendix is useful for this.

The first question—that of estimating the test efficiency for equipment in being—is addressed in Ref. 3. There it is shown that this first meaning of q_i depends on

1. the extent to which all components are exercised in the checkout, and
2. the extent to which the ground test simulates or duplicates the in-flight performance requirements.

The referenced work contains means for making this estimation. Quantification of the latter extent requires previous test experience. Lacking this experience, then other experimental or subjective means must be used.

For test equipment that is known to be operative the operating conditions described and assumptions employed in defining the transition probabilities are as follows.

In order for a function to be operative following a periodic test, it must truly operate, and the tester must indicate that it is operative. If the tester erroneously indicates that the function has failed, then that function moves into State 3 (failed-known). It follows then that:

$$b_{11} = p_i \exp(-k_{i3}\lambda_i\theta) \tag{30}$$

where $k_{i3}\lambda_i\theta$ could be nothing more than a pseudo-factor to help approximate the chance of the function surviving the test. (Note: $\ell = 3$ is the mode of operation associated with the periodic test.) It could prove true, for example, that for the short time θ, the turn-on stress of testing is the most severe element, or the only element of significance. To more easily account for these alternatives, b_{11} shall be written and treated as

$$b_{11} = p_i\delta_i \tag{31}$$

where

$$\delta_i = P(i^{th} \text{ function survives the periodic test})$$

A function will move from State 1 (operative) to State 2 (failed-unknown) if it fails during the test and the tester does not detect the failure. Consequently,

$$b_{12} = \left(1 - \exp(-k_{i3}\lambda_i\theta)\right) \quad (1 - q_i)$$

$$= (1 - \delta_i)(1 - q_i) \tag{32}$$

A function could move from State 1 to State 3 in one of two ways; it could be erroneously called inoperative, or it could fail and be detected during the test. Therefore,

$$b_{13} = (1 - p_i)\exp(-k_{13}\lambda_i\theta) + q_i\left(1 - \exp(-k_{13}\lambda_i\theta)\right)$$

$$= (1 - p_i)\,\delta_i + (1 - \delta_i)\,q_i \qquad (33)$$

To allow for such operations as failure reporting, maintenance scheduling, etc., a minimum of one period (of length t_i) of delay in going into State 4 (being repaired) from States 1 and 2 will be built into the model. This will be done by setting

$$b_{14} = b_{24} = 0$$

This causes the function to move through an intermediate state, State 3 (failed-known), for at least one period of t_i.

Because the function is assumed to be non-regenerative, and a test decision is firm, once made,

$$b_{21} = b_{31} = 0$$

If a function is in State 2 (failed-unknown), it can go to only one other state, failed-known. It will make this transition if the tester detects the fault; otherwise it will remain in the state failed-unknown. Therefore,

$$b_{22} = 1 - q_i \qquad (34)$$

$$b_{23} = q_i \qquad (35)$$

A function can move into the state failed-known only during a periodic test. It would then be expected to move into the state being repaired shortly thereafter. It is unlikely, therefore, that the function will still be in the failed-known state when the next periodic test is due. It is also unlikely that it would be scheduled for maintenance (where the malfunction was discovered in a previous periodic test) on the day of the periodic. However, if either event should transpire, it is assumed that the periodic test would be conducted as scheduled and the function repaired immediately thereafter. Therefore,

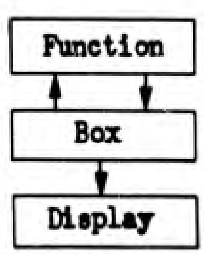
$$b_{34} = b_{44} = 1$$

For example, if the van comes to test, it will also repair; if the van comes to repair, it will also test--if the test is due. The possibility of unscheduled periodic tests is not accounted for in this model. If such tests are contemplated, the error in using these results will decrease with increasing T and increasing a_{34}.

The matrix B is now completely defined and, therefore, so are all terms necessary to the computation.

CONTINUOUS MONITOR

Derivation of the P_{ijk} terms for the continuous monitor mode of readiness testing is somewhat more difficult than for the check periodic mode, because of the necessity to account for the "box" that does the monitoring. An idealization of the relationship between the box and the function is as follows.



The display is assumed to read go and no go. Readings would result as shown in Table 14.

Table 14

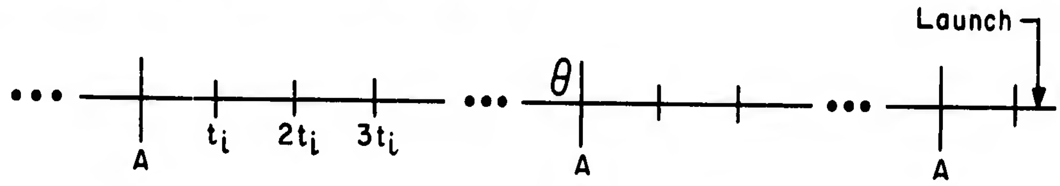
POSSIBLE DISPLAY INDICATIONS, CONDITIONS, AND STATES FOR CONTINUOUS MONITOR

Display	Condition	State
Go	function operative box operative	operative
	function failed box operative, but makes error	failed-unknown
	function failed box operative, senses fault	failed-known
No go	function failed box failed	failed-known
	function operative box failed	failed-known

As Table 14 shows, even with the continuous monitoring box, it is possible to have the failed-unknown state. Therefore, continuous monitor should include a thorough periodic inspection similar to that conducted during the θ interval of the check periodic testing method.

Notice that a failure in the box is assumed to be always known, and furthermore, in order for a function to be truly operative, the box must so indicate. A failure in the box will disable the function--because the operator must then assume that the function has failed, because he cannot, at that stage, distinguish between a function failure and a box failure.

As before, the four states will be employed for the analysis, and two matrices will be used to describe the process. Matrix A will again be defined over times of length t_1, and matrix B will be defined over an overhaul or complete inspection of all missile and ground functions that is assumed to take place after a large number, called n_i, of t_is have elapsed. The time diagram for this process is



where $A = n_i t_i + \theta$.

Matrix A

Matrix A, defined over t_i, has the form

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \\ 0 & a_{22} & a_{23} & 0 \\ 0 & 0 & a_{33} & a_{34} \\ a_{41} & 0 & 0 & 0 \end{bmatrix}$$

For an operative function to remain operative over a period t_i, neither it nor the box can fail. Assuming the applicability of the exponential failure distribution, this transition probability is given by

$$a_{11} = \exp\left(-\sum_{j=i,bi} k_{j2}\lambda_j t_i\right)$$

$$= \exp\left(-(\alpha_i + \beta_{bi})t_i\right) \qquad (36)$$

where: bi designates the box for the i^{th} function, and

k_{j2} = stress factor for second mode of employment--continous operation for j^{th} item

α_i = failure rate under continuous operation mode for i^{th} function

β_{bi} = failure rate under continuous operation mode for box of i^{th} function.

The α_i and β_{bi} notation will be used hereafter.

A function will make the transition from operative to failed-unknown only if it fails and the box, still operating, does not detect the failure. The probability of this transition is given by

$$a_{12} = \left(1 - \exp(-\alpha_i t_i)\right)\left(\exp(-\beta_{bi} t_i)\right)\left(1 - q_i'\right) \tag{37}$$

where q_i' is continuous monitor term corresponding to q_i.

The transition from operative to failed-known can be made in three mutually exclusive ways: (a) the function fails, the box, still operating, detects the failure, (b) the function and the box both fail, or (c) the box fails while the function remains operative. The probability of this transition is given by

$$\begin{aligned} a_{13} &= \left(1 - \exp(-\alpha_i t_i)\right)\left[q_i' \exp(-\beta_{bi} t_i) + \left(1 - \exp(-\beta_{bi} t_i)\right)\right] \\ &\quad + \left(\exp(-\alpha_i t_i)\right)\left(1 - \exp(-\beta_{bi} t_i)\right) \\ &= 1 - \exp(-\beta_{bi} t_i) + q_i' \exp(-\beta_{bi} t_i)\left[1 - \exp(-\alpha_i t_i)\right] \end{aligned} \tag{38}$$

As before, it is assumed that a function cannot move from an operative state or from the state failed-unknown to the state of being repaired in one step. Therefore,

$$a_{14} = a_{24} = 0$$

From the state failed-unknown, the function can move only to the state failed-known, or remain in its present state. If the function is in the state failed-unknown, the failure will become known only if the box fails. Therefore,

$$a_{22} = \exp(-\beta_{bi} t_i) \tag{39}$$

$$a_{23} = 1 - \exp(-\beta_{bi} t_i) \tag{40}$$

The balance of the matrix is the same as Matrix A of the check periodic method.

Matrix B

The matrix B is defined for the periodic inspection. This inspection is assumed to be a complete inspection of both the missile functions and the monitoring boxes. Test equipment is brought to the missile site, and it is known that the test equipment is in good operating condition. This does not preclude tester errors, however, of both kinds previously described. As before, the length of the inspection is noted as θ.

Matrix B is of the form

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 \\ 0 & b_{22} & b_{23} & 0 \\ 0 & 0 & 0 & b_{34} \\ 0 & 0 & 0 & b_{44} \end{bmatrix}$$

The test equipment will test both the function and the box. The function will be operative following the inspection, given that it was operative going into the inspection, only if both it and the box survive the test and the testers call both operative. This transition probability is given by

$$b_{11} = \left(\exp(-\sum_{j=i,bi} k_{j3}\lambda_j\theta_i)\right) p_i p_{bi}$$

$$= \delta_i \eta_{bi} p_i p_{bi} \qquad (41)$$

where

η_{bi} = P (box for i^{th} function survives the inspection), and

p_{bi} = P (tester calls a good box for i^{th} function good)

By assumption, any failure in the box will be detected with probability one. Therefore, a function that is operative before the inspection will be in state failed-unknown after the inspection only if it fails, its failure is undetected, and the box is called operative. In symbols

$$b_{12} = (1 - \delta_i)\eta_{bi}(1 - q_i)p_{bi} \qquad (42)$$

The transition from an operative condition to the state failed-known can occur in several ways: (a) the function can fail, the box not fail, and either the failure is detected, or it is not detected and the box is erroneously called bad, (b) both the function and the box can fail (as $q_{bi} = 1$, this will lead to detection with certainty), (c) the box can fail while the function remains operative, or (d) both function and box can remain good, but one or both are called bad. Therefore,

$$b_{13} = (1 - \delta_i)\eta_{bi}\left[q_i + (1 - q_i)(1 - p_{bi})\right] + (1 - \delta_i)(1 - \eta_{bi}) + \delta_i(1 - \eta_{bi}) + \delta_i\eta_{bi}\left[1 - p_i p_{bi}\right]$$

$$= 1 - \delta_i \eta_{bi} p_i p_{bi} - (1 - \delta_i)\eta_{bi} p_{bi}(1 - q_i) \qquad (43)$$

For a function that is in the state failed-unknown to move into the state failed-known, either its box must fail during the inspection, or if its box survives the inspection, the function's failure must be detected, or if it is not detected the box must be erroneously called inoperative. The function will remain in the state failed-unknown if the box survives the test and is called operative and the function is erroneously called operative. Symbolically

$$b_{22} = \eta_{bi} p_{bi}(1 - q_i)$$

$$b_{23} = (1 - \eta_{bi}) + \eta_{bi}\left[q_i + (1 - q_i)(1 - p_{bi})\right]$$

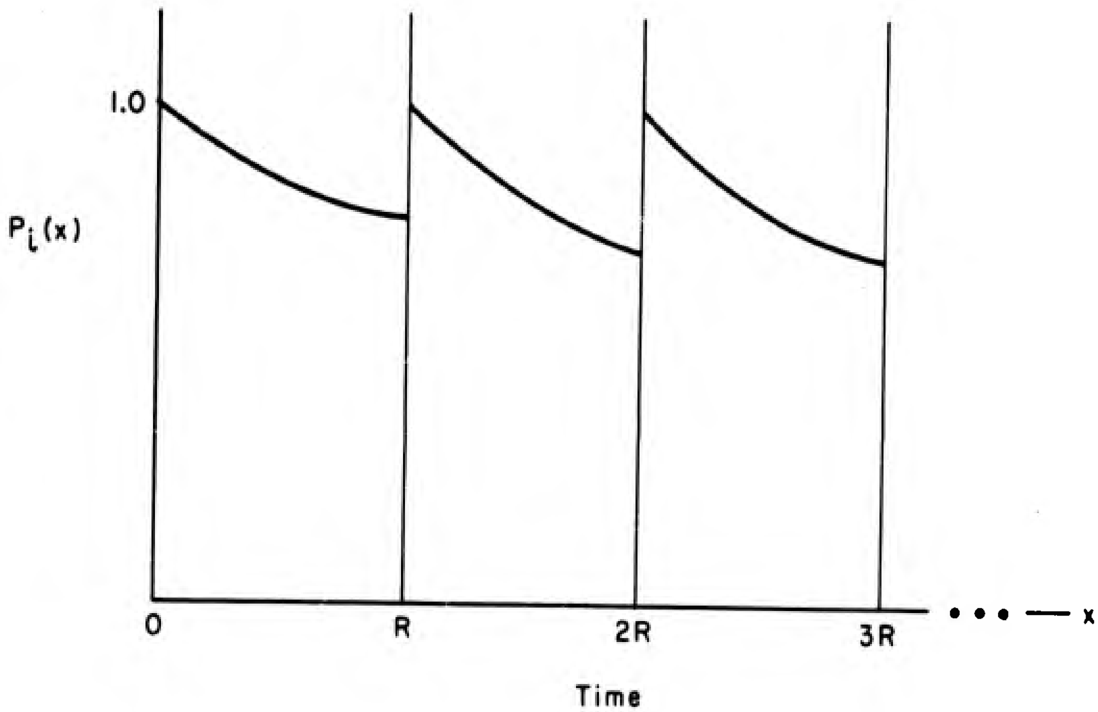
$$= 1 - \eta_{bi} p_{bi}(1 - q_i) \qquad (44)$$

The balance of this matrix is the same as matrix B for the periodic check method. Matrix C and x vector can now be obtained as before, and, therefore, the development of the continuous monitor model is complete.

LEAVE ALONE

This third method of readiness testing consists of leaving the function alone for extended periods of time and then replacing it with one that is known to be operative. For some mechanical items, the life length under the anticipated silo conditions is known to be much greater than the replacement period; consequently, $P_i \cong 1$. (Note that the usual P_{ijk} has been shortened to P_i for the leave alone testing method.) For other functions, failures are possible during the replacement interval.

For these other functions, the probability that the function is operative looks, in general, as shown below:



where

x is a simple time axis

R is the replacement period

$P_i(x)$ is the probability that the i^{th} function is operative at some time x.

It is assumed that the time needed to replace the functions is negligible with respect to R.

The leave alone testing problem to be solved is simply to find an average value for $P_i(x)$.

As introduced on p. 85,

$$P_i(x) = \exp(-k_{i\ell}\lambda_i x)$$

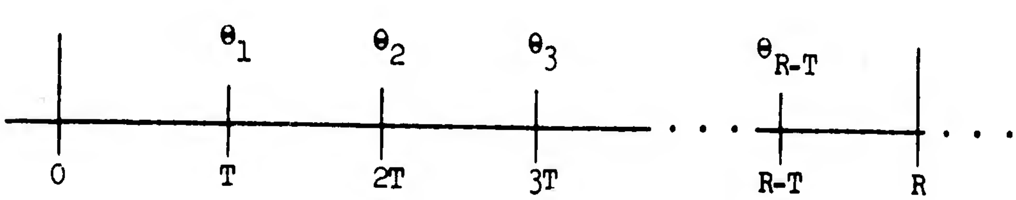
$$= \exp(-\alpha_i x) \qquad (45)$$

The average value is

$$\overline{P_i(x)} = \frac{1}{R}\int_0^R \exp(-\alpha_i x)dx$$

$$\frac{1}{R\alpha_i}\left[1 - \exp(-\alpha_i R)\right] \qquad (46)$$

There may be testing programs wherein some functions are left alone and others are checked periodically with interval T. Further, within this testing program, some of the left alone functions may be energized but not tested directly during the periodic checks. If these periodic stresses are significantly different from the stresses imposed on the functions during the remainder of the time between replacements, then the additional stresses should be accounted for in determing P. Now the time pattern changes to the following.



As before, the function is replaced every R periods. R is assumed to be a multiple of T. During the time from 0 to (R - T), the function would be subjected to a series of test-induced stresses. During the time interval from (R - T) to R, the function would be in the leave alone or quiescent condition.

Using the same notation as for the unstressed leave alone example

$$P_i(x) = \exp\left\{-\lambda_i \left[(T-\theta)\left[\frac{x}{T}\right]\right] + \left(x - \left[\frac{x}{T}\right]T - \theta\right)\right\} \cdot \exp\left\{-k\lambda_i\left(\left[\frac{x}{T}\right]\theta + \theta\right)\right\} \qquad (47)$$

where

$$\left[\frac{x}{T}\right] \equiv \text{highest integer of the ratio } \frac{x}{T}$$

The probability that the function is ready over the interval R becomes

$$\overline{P_i(x)} = \frac{1}{R}\int_0^R \exp\left\{-\lambda\left(x - \left\{\left[\frac{x}{T}\right] + 1\right\}\theta\right)\right\} \cdot \delta^{\left[\frac{x}{T}\right]+1} dx \qquad (48)$$

Therefore,

$$P_i = \frac{e^{\lambda T} - 1}{\lambda R} \sum_{y=1}^{\frac{R}{T}} \left(\delta e^{\lambda\theta - \lambda T}\right)^y \qquad (49)$$

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