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MEASUREMENT OF DIELECTRIC CONSTANT

BY USING A RADIAL WAVEGUIDE

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THESIS

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Preface

This paper is the result of an attempt to find a method for measuring the relative dielectric constant, ϵ_r', and the loss tangent, tan δ, of dielectric materials at high temperatures when in the presence of a high frequency signal. The method sought will be evaluated by a measurement of the dielectric samples of known values at room temperatures and X band frequencies where the values obtained may be compared to accepted values.

The method to be explored was suggested by G. Weinstein as a part of his recommendations in his similar thesis (Ref 9).

With the guidance of Dr. C. M. Zieman as thesis advisor, I evaluate this method, a radial wave guide slotted line system, as a means to measure these values and then measure several samples to evaluate the accuracy of the method.

The theory supporting this method is developed starting with Maxwell's equations and is so presented that a reader who is unfamiliar with the method of manipulation of field equations may follow the development readily.

No attempt has been made to measure materials at the high temperatures and frequency because of the limited time, facilities, and the lack of standard values with which to compare the results of such measurements. The problem of temperature and frequency and how it affects the errors in measurements is discussed, however, in the body of the report.

I would like to express my appreciation to Dr. C. M. Zieman for his guidance throughout this study and to Mr. W. Bahrett for his help in procuring materials for the actual measurements and also to Mr. Hamilton for his splendid machining and assembly of the radial wave guide slotted line.

And finally my deepest affections to my wife, Joan, who ran smoothly our happy home throughout this ordeal and in addition presented me with the newest addition to our family, Linda.

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ABSTRACT

A method for measuring permittivity, ϵ , is derived from the equations of the fields in a shorted radial wave guide obtained from the Maxwell equations. The method is checked by measuring ϵ_r' , the relative dielectric constant, and tan δ , the loss tangent, for three samples by measuring in a radial slot in the guide, the wave length and the VSWR at one minimum of known distance from the short at room temperature and at a frequency of 9375 mc/s. The method shows promise of measuring ϵ_r' at frequencies as high as k band and at high temperatures limited only by the metal used in the guide itself.

MEASUREMENT OF DIELECTRIC CONSTANT BY USING A RADIAL WAVEGUIDE

I. Introduction

The subject of this paper is the measurement of the permittivity of dielectric solids. The electromagnetic properties of a substance may be fully described when its permittivity, ϵ^*, permeability, μ^*, and conductivity, σ, are known. A dielectric is a material having the property of low conductivity, 10^{-4} mhos per meter and the permeability is essentially that of free space, $4\pi \times 10^{-7}$ henries per meter. The permittivity, generally called the dielectric constant, varies considerably for different dielectrics and is in general a complex number,

$$\epsilon^* = \epsilon_o \epsilon_r' \left[1 - j \frac{\sigma}{\omega \epsilon_o \epsilon_r'} \right] \qquad (1)$$

where ϵ_o is the permittivity of air, 8.854×10^{-12} farads per meter; ϵ_r' is the relative dielectric constant of the material; σ is the conductivity of the material; and ω is the angular frequency of the electromagnetic field.

The purpose of this study is to examine the radial wave guide and to adapt it as a tool useful for measuring dielectric constant. This report is used then to present the method for measuring ϵ^*, both its real part and its imaginary part, using the radial wave guide. Also the results of the author's measurements of five samples will be given and compared with accepted standard values.

The complex relative dielectric constant, ϵ_r^* , may be defined as

$$\epsilon_r^* = \frac{\epsilon^*}{\epsilon_0} = \epsilon_r' \left[1 - j \, \frac{\epsilon_r''}{\epsilon_r'} \right] \qquad (2)$$

where ϵ_r''/ϵ_r' is called the loss tangent, tan δ , and is the ratio of energy loss per cycle in the dielectric to the energy stored per cycle by the dielectric. The real part, ϵ_r' , is a figure of merit of the ability of the material to store the energy of the field. A "low loss" dielectric is characterized by a tan δ much less than one.

Variations in temperature, frequency, and humidity change the value of ϵ_r^* for most materials. The imaginary part, tan δ , will vary greatly with these quantities, while the real part, ϵ_r' , varies to a lesser extent. Errors associated with measuring these quantities increase as temperature and frequency increase (1000° C and 100,000 mc/s). Here the source of heating may distort the wave guide, or even the material itself while the limited resolution of the scales limits the accuracy of physical measurements in a slotted line at these short wave lengths. The presence of a slot in the smaller sized wave guide increases the perturbations in the electrical fields. Many methods are known for measuring the dielectric constant, but not all are suited for measuring ϵ_r' and tan δ at these high temperatures and frequencies (Ref 5:633-673).

This report is divided into seven sections: The Introduction, The Theory of Radial Wave Guides, Derivation of the Complex Relative Dielectric Constant, Laboratory Equipment and Procedures, Sources of Error in Measuring Dielectric Constant, Results of Measurements for Dielectric Samples, and Conclusions and Recommendations.

The theory and the derivation of the field equations for radially traveling waves are presented in Section II, Theory of Radial Wave Guides. The development of the fields in radial wave guides is presented; the field modes of operation are described; and, the impedance of radial lines is found.

The equations describing ϵ_r' , and tan δ are extracted from the theory of Section II and presented in Section III, Derivation of the Complex Relative Dielectric Constant.

A special piece of equipment, a parallel-plate, radial wave guide slotted line, not normally found in a microwave laboratory, is required for these measurements. This equipment, as well as the method for using it, is described in Section IV, Laboratory Equipment and Procedures.

The sources of error in measuring ϵ_r' and tan δ are qualitatively discussed in Section V, Sources of Error in Measuring Dielectric Constant. The effect of high temperatures and frequencies on measurements is also presented.

The results of calibrating the radial wave guide by measuring ϵ_r' and tan δ for five samples of known values of ϵ_r' and tan δ at a nominal frequency of 9330 mc/s and at room temperature are tabulated and compared to the known values in Section VI, Results of Measurements for Dielectric Samples.

Conclusions drawn from the results of this study and the advantages and disadvantages of the method presented for measuring ϵ_r' and tan δ are given in Section VII, Conclusions and Recommendations. Ideas to improve the accuracy of measurements are also presented.

II. Theory of Radial Wave Guides

The equations describing the electric and magnetic fields in radial wave guide must satisfy the Maxwell equations and are best expressed in the cylindrical coordinate system. Solutions for the field components derived from the Maxwell equations must simultaneously satisfy the Helmholtz wave equation, and the boundary conditions of the guiding structure.

In the remainder of this section the field components E_r , E_ϕ , H_r , and H_ϕ , are obtained implicitly from the Maxwell equations in terms of E_z and H_z ; the Helmholtz wave equation is solved for the E_z and H_z field components; the field components E_z and H_z are substituted in the implicit equations of E_ϕ , E_r , H_ϕ , and H_r with assumptions made that fit the radial wave guide; the modes of transmission are described and the cut-off frequency expressions are derived; then the impedance of the line is obtained.

Maxwell's Equations

The Maxwell's equations for homogeneous isotropic media are:

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \qquad (3)$$

$$\nabla \times \bar{H} = \sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t} \qquad (4)$$

When the time varying factor is expressed as $\epsilon^{j\omega t}$ and σ is assumed zero (σ may be re-entered by letting

$$\epsilon = \epsilon_c = \epsilon\left[1 + \frac{\sigma}{j\omega\epsilon}\right] \tag{4a}$$

(Ref 6:306), equations (3) and (4) become

$$\nabla \times E = -j\omega\mu\bar{H} \tag{5}$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E} \tag{6}$$

Expressing these equations in cylindrical components of $\bar{E}$ and $\bar{H}$, they become

$$\left[\frac{1}{r}\left(\frac{\partial E_z}{\partial \phi}\right) - \frac{\partial E_\phi}{\partial z}\right] = -j\omega\mu H_r \tag{7}$$

$$\left[\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r}\right] = -j\omega\mu H_\phi \tag{8}$$

$$\left[\frac{1}{r}\left(\frac{\partial (rE_\phi)}{\partial r}\right) - \frac{1}{r}\left(\frac{\partial E_r}{\partial \phi}\right)\right] = -j\omega\mu H_z \tag{9}$$

$$\left[\frac{1}{r}\left(\frac{\partial H_z}{\partial \phi}\right) - \frac{\partial H_\phi}{\partial z}\right] = j\omega\epsilon E_r \tag{10}$$

$$\left[\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r}\right] = j\omega\epsilon E_\phi \tag{11}$$

$$\left[\frac{1}{r}\left(\frac{\partial (r H_\phi)}{\partial r}\right) - \frac{1}{r}\left(\frac{\partial H_r}{\partial \phi}\right)\right] = j\omega\epsilon E_z \tag{12}$$

Assuming propagation in the z direction so that,

$$E = E_n \epsilon^{-\gamma z} \quad \text{, then} \quad \frac{\partial E}{\partial z} = -\gamma E_n \epsilon^{-\gamma z} = -\gamma E$$

the components of E_r may be found from equation (10) by differentiating H_ϕ with respect to z in equation (8) and substituting $\partial H_\phi/\partial z$ in equation (10) resulting in

$$E_r = \frac{1}{(\gamma^2 + k^2)}\left[\gamma\frac{\partial E_z}{\partial r} + j\frac{\omega\mu}{r}\frac{\partial H_z}{\partial \phi}\right] \tag{13}$$

H_r is found by differentiating equation (11) with respect to z and substituting $\partial H_r/\partial z$ in equation (7) resulting in

$$H_r = \frac{1}{(\gamma^2 + k^2)}\left[j\frac{\omega\epsilon}{r}\frac{\partial E_z}{\partial \phi} - \gamma\frac{\partial H_z}{\partial r}\right] \tag{14}$$

In a similar fashion, H_ϕ can be found by differentiating equation (13) and substituting $\partial E_r/\partial z$ in equation (8) as

$$H_\phi = \frac{1}{(\gamma^2 + k^2)}\left[j\omega\epsilon\frac{\partial E_z}{\partial r} + \frac{\gamma}{r}\frac{\partial H_z}{\partial \phi}\right] \tag{15}$$

By differentiating equation (14) and substituting $\partial H_r/\partial z$ in equation (11), E_ϕ is found as

$$E_\phi = \frac{1}{(\gamma^2 + k^2)} \left[j\omega\mu \frac{\partial H_z}{\partial r} - \frac{\gamma}{r}\frac{\partial E_z}{\partial \phi} \right] \tag{16}$$

Wave Equation

Maxwell's equations may be solved explicitly for $\bar{E}$ or $\bar{H}$ when a charge-free region is assumed; i.e., $\rho = 0$. If the curls of Eqs (5) and (6) are taken then,

$$\nabla \times \nabla \times \bar{E} = -j\omega\mu \nabla \times \bar{H} \tag{17}$$

$$\nabla \times \nabla \times \bar{H} = j\omega\epsilon \nabla \times \bar{E} \tag{18}$$

and remembering that $\nabla \cdot D = \rho = 0$ and that the vector identity,

$$\nabla \times \nabla \times \bar{E} = \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} \tag{19}$$

Eq (17) becomes

$$\nabla^2 \bar{E} = -\omega^2 \mu\epsilon \bar{E} \tag{20}$$

and since $\nabla \cdot \bar{B} = 0$ Eq (18) gives

$$\nabla^2 \bar{H} = -\omega^2 \mu \epsilon \bar{H} \tag{21}$$

Writing Eq (20) for E_z in the cylindrical coordinate system gives,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \epsilon E_z \tag{22}$$

To solve this equation, a solution of the product form will be tried. That is, let

$$E_z = R\Phi Z \tag{23}$$

where R is a function of r alone, Φ of ϕ alone, and Z of z alone. Preforming the differentiation required on Eq (23) and substituting into Eq (22) gives

$$\frac{1}{rR}\cdot\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{1}{r^2\Phi}\frac{d^2\Phi}{d\phi} + \frac{1}{Z}\frac{d^2 Z}{dz^2} + k^2 = 0 \tag{24}$$

where $k^2 = \omega^2 \mu \epsilon$.

Here the third term must be a constant if the sum of terms is zero and the third term depends on z alone. Therefore, let the third term

$$\frac{1}{Z}\frac{d^2 Z}{dz^2} = -\gamma^2 \tag{25}$$

Substituting Eq (25) in Eq (24) then multiplying Eq (24) by r^2 shows that we can let

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\phi} = -n^2 \tag{26}$$

for the same reasons as for Eq (25).

Substituting Eq (26) in Eq (24) then gives

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) - n^2 + (k^2-\gamma^2)r^2 = 0 \tag{27}$$

Now defining

$$k_r^{\,2} = k^2 - \gamma^2 \tag{28}$$

and substituting $k_r^{\,2}$ in Eq (27)

$$r\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \left[(k_r r)^2 - n^2\right]R = 0 \tag{29}$$

To summarize, E_z is defined by equation (23), where the variables R, Φ , and Z are the solutions of Eqs (25), (26), and (29). Solutions of R, Φ , and Z may be as follows:

$$R = B_n(k_r r) \tag{30}$$

where $B_n(k_r r)$ is representative of the Bessel functions that may be used.

$$\Phi = \mathcal{H}(n\phi) \tag{31}$$

where $\mathcal{H}(n\phi)$ may be solutions such as sin(nϕ) or cos(nϕ), for the harmonic differential equation, Eq (23).

$$Z = \mathcal{H}(\gamma z) \tag{32}$$

where $\mathcal{H}(\gamma z)$ is also the solution of a harmonic differential equation. Combining the solutions of R, Φ , and Z then show that

$$E_z = B_n(k_r r)\ \mathcal{H}(n\phi)\ \mathcal{H}(\gamma z) \tag{33}$$

Any linear combination of these functions is also a solution for E_z for differing values of n and γ , or n and k_r. Solutions where n, γ , and k_r are independent variables will not be correct as k_r and γ are not independent variables but are related by Eq (28).

The values of n and m, with m related to γ and k_r by Eqs (28) and (42), describe the "mode" of propagation in the z direction and are used as subscripts; i.e., TM_{mn} or TE_{mn}, when referring to a particular mode.

TM_{mn} Field Equations

Solutions TE or TM to the z direction are possible only when $E_z = 0$ or $H_z = 0$ respectively (Ref 4:202). The TM_{mn} wave components will be evaluated here.

Substituting E_z from Eq (33) into Eqs (13), (14), (15), and (16) with $H_z = 0$ gives,

$$E_r = -\frac{1}{(\gamma^2+k^2)}\left[k_r\gamma\, B_n'(k_r r)\, \mathcal{H}(n\phi)\, \mathcal{H}(\gamma z)\right] \tag{34}$$

$$E_\phi = -\frac{1}{(\gamma^2+k^2)}\left[\frac{n\gamma}{r}\, B_n(k_r r)\, \mathcal{H}'(n\phi)\, \mathcal{H}(\gamma z)\right] \tag{35}$$

$$H_r = \frac{1}{(\gamma^2+k^2)}\left[j\frac{n\omega\epsilon}{r}\, B_n(k_r r)\, \mathcal{H}'(n\phi)\, \mathcal{H}(\gamma z)\right] \tag{36}$$

$$H_\phi = -\frac{1}{(\gamma^2+k^2)}\left[j\omega\epsilon k_r\, B_n'(k_r r)\, \mathcal{H}(n\phi)\, \mathcal{H}(\gamma z)\right] \tag{37}$$

$$H_z = 0 \tag{38}$$

Radial Wave Guide Equations

Applying boundary conditions to the TM_{mn} wave now will give the particular solution for the radial wave guide system as shown in Figure 1.

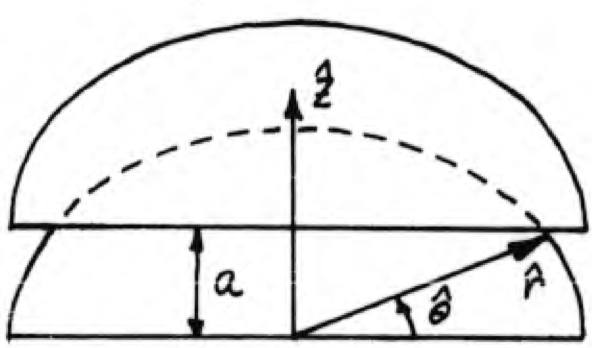


FIGURE 1
Cylindrical Coordinate System
For Radial Wave Guide

The boundary conditions for this case, assuming perfectly conducting plates, are: E_ϕ and $E_r = 0$ at $z = 0$ and a. Thus

$$\mathcal{H}(\gamma z) = 0 \tag{39}$$

for z = 0 and z = a and

$$\mathcal{H}(0) = 0 ; \qquad \mathcal{H}(\gamma a) = 0 \tag{40}$$

Letting

$$\mathcal{H}(\gamma z) = A\cos(\gamma z) + B\sin(\gamma z) \tag{41}$$

and imposing the above conditions we see A = 0 and

$$\gamma a = m\pi \tag{42}$$

or

$$\gamma = \frac{m\pi}{a}, \qquad m = 0, 1, 2, \cdots \tag{43}$$

Since the fields are assumed single valued

$$\mathcal{H}(n\phi) = \mathcal{H}(n\phi + 2\pi) \tag{44}$$

and if the excitation has no variations with the angle, ϕ , n may be chosen equal to zero by this excitation of the wave guide and the ϕ variation for all field components will be zero. That is, $\mathcal{H}(n\phi)$ is a constant and $\mathcal{H}'(n\phi) = 0$ and the field components E_ϕ and H_r will equal zero.

Looking toward propagation in the radial direction we choose $B_n(k_r r)$ as a sum of two Hankel functions,

$$B_n(k_r r) = C H_n^{(2)}(k_r r) + D H_n^{(1)}(k_r r) \tag{45}$$

where the origin r = 0 is excluded since here $H_n^{(2)}(k_r r)$ approaches an infinite value and is not physically realizable. Then the field components become,

$$E_z = B \sin\left(\frac{m\pi z}{a}\right)\left[C H_0^{(2)}(k_r r) + D H_0^{(1)}(k_r r)\right] \tag{46}$$

where $$k_r^{\,2} = k^2 - \gamma^2 = k^2 - \left(\frac{m\pi}{a}\right)^2 \tag{47}$$

$$E_r = -\frac{1}{(\gamma^2 + k^2)}\left[\gamma k_r\left(C H_0^{(2)\prime}(k_r r) + D H_0^{(1)\prime}(k_r r)\right)\right] \times \left[B \sin\left(\frac{m\pi}{a} z\right)\right] \tag{48}$$

$$H_\phi = -\frac{1}{(\gamma^2 + k^2)}\left[j\omega\epsilon k_r \; C H_0^{(2)\prime}(k_r r) + D H_0^{(1)\prime}(k_r r)\right] \times \left[B \sin\left(\frac{m\pi}{a} z\right)\right] \tag{49}$$

$$H_r = 0 \tag{50}$$

$$E_\phi = 0 \tag{51}$$

From the asymptotic expressions for $H_0^{(2)}(k_r r)$ and $H_0^{(1)}(k_r r)$ it is seen that these field equations are of the form of a traveling wave in the radial direction,

$$\underset{x \to \infty}{H_n^{(2)}(x)} = \underset{x \to \infty}{H_n^{(1)*}(x)} = \sqrt{\frac{2}{\pi x}}\, \epsilon^{-j\left[x - \frac{\pi}{4} - n\frac{\pi}{2}\right]} \tag{52}$$

(Ref 6:165), where the magnitude is not constant but decreases as the argument x increases.

If propagation in the z direction is not assumed, as it was in Eq (25), then the derivative must be set equal to $+\gamma^2$ for that equation. When the boundary conditions, Eq (35), are imposed on E_r and E_ϕ the following solution,

$$Z = A\epsilon^{\gamma z} + B\epsilon^{-\gamma z} \tag{53}$$

of the modified Bessel equation is used and gives that, where z = 0 and Z = 0, A = -B, and where z = a and Z = 0,

$$\epsilon^{\gamma a} - \epsilon^{-\gamma a} = 0 \tag{54}$$

or

$$\sinh \gamma a = 0 \tag{55}$$

Thus $\gamma a = 0$ and since a must not equal zero

$$\gamma = 0 \tag{56}$$

Setting $\gamma = 0$ is then a special case where m = 0 in the total solution described in Eqs (46), (47), (48), (49), (50), and (51). But setting m = 0 in these equations gives the trivial solution where all variables are zero as $\sin \frac{m\pi z}{a} = 0$ for this case. It is therefore necessary to eliminate γ in the step before these equations; that is, let $\gamma = 0$ in Eqs (33), (34), (35), (36), and (37). Then if $H'(n\phi)$ is assumed zero again, the field equations where m = 0 and $\gamma = 0$ are

$$E_z = C H_0^{(2)}(kr) + D H_0^{(1)}(kr) \tag{57}$$

$$H_\phi = \frac{j\omega\epsilon}{k}\left[C H_1^{(2)}(kr) + D H_1^{(1)}(kr)\right] \tag{58}$$

$$\left.\begin{aligned} E_r &= 0 \\ E_\phi &= 0 \\ H_r &= 0 \\ H_z &= 0 \end{aligned}\right\} \tag{59}$$

and

$$k_r = k \tag{60}$$

Now ϵ must be replaced by ϵ_c from Eq (4a) to represent the lossy case. Combining ϵ_c then in the factor k shows that

$$k = \alpha + j\beta = \omega^2\mu\epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right]^{1/2} \tag{61}$$

where

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\left[1 + \frac{\sigma^2}{\omega^2\epsilon^2}\right]^{1/2} - 1\right)} \tag{62}$$

and

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\left[1 + \frac{\sigma^2}{\omega^2\epsilon^2}\right]^{1/2} + 1\right)} \tag{63}$$

(Ref 6:306), which when $\frac{\sigma^2}{\omega^2\epsilon^2} << 1$ reduce to

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \frac{k\,\epsilon_r''}{2\,\epsilon_r'} \tag{64}$$

(Ref 6:310), or

$$\alpha = \frac{\pi}{\lambda_g}\tan\delta \tag{65}$$

(Ref 9:84), and

$$\beta = k\left[1 + \frac{1}{8}\left(\frac{\epsilon_r''}{\epsilon_r'}\right)^2\right] \tag{66}$$

or

$$\beta = k\left[1 + \frac{1}{8}(\tan\delta)^2\right] \tag{67}$$

(Ref 6:310).

Now considering the outward traveling wave and assuming $\alpha << \beta$,

$$H_0^{(2)}[(\beta - j\alpha)r] = |H_0^{(2)}(\beta r)| \, \epsilon^{-\alpha r} \underline{/ \arctan \alpha/\beta} \qquad (68)$$

(Ref 1:336)

Here the $H_0^{(2)}(\beta r)$ term gives the propagating wave in the radial direction while the $\epsilon^{-\alpha r}$ term attenuates the wave as it moves outward radially.

<u>Impedance of Radial Lines</u>

The radially directed wave impedance of the outward traveling TM_{mo} mode with no ϕ variation is found from Eqs (46) and (49) as

$$Z_r^{+TM} = -\frac{E_z^+}{H_\phi^+} = \frac{k_r}{j\omega\epsilon} \frac{H_0^{(2)}(k_r r)}{H_1^{(2)}(k_r r)} \qquad (69)$$

while for inward traveling waves

$$Z_r^{-TM} = \frac{E_z^-}{H_\phi^-} = -\frac{k_r}{j\omega\epsilon} \frac{H_0^{(1)}(k_r r)}{H_1^{(1)}(k_r r)} \qquad (70)$$

When $\frac{m\pi}{a} > k$ it is seen from Eq (28) that k_r is imaginary and letting $k_r = -j\alpha$,

$$Z_r^{+TM} = j \frac{j\alpha}{\omega\epsilon} \frac{H_0^{(2)}(-j\alpha r)}{H_1^{(2)}(-j\alpha r)} = \frac{j\alpha}{\omega\epsilon} \frac{K_0(\alpha r)}{K_1(\alpha r)} \qquad (71)$$

Since $K_0(\alpha r)$ is positive and $K_1(\alpha r)$ is negative for all αr, Z^+_r is always a capacitive reactance (Ref 4:210). Therefore if $\frac{m\pi}{a} > k$ the TM_{mo} mode will not propagate in the radially direction.

Examining the equation of impedance when m = 0

$$Z_r^{+TM} = -j\eta \frac{H_n^{(2)}(kr)}{H_n^{(2)'}(kr)}$$

$$= \frac{\eta}{|H_n^{(2)'}(kr)|^2} \left\{ \frac{2}{\pi kr} - j\left[J_n(kr) J_n'(kr) + N_n(kr) N_n'(kr)\right]\right\} \qquad (72)$$

when kr becomes small it is seen that the imaginary part of Eq (72) becomes quite large compared to the real part (Ref 4: 211). Therefore the TM_{oo} wave would propagate energy more effectively than the higher TM_{on} waves when the radius at the input terminals is small. For this reason a TM_{oo}, z wave, would be easier to propagate in the radial direction. If the m = 0 variation were attained then the TM_{oo} mode would be equivalent to a TEM radially directed mode.

III. Derivation of the Complex Relative Dielectric Constant

The relative dielectric constant may be found from the field equations of the TEM radial mode; this involves measuring the distance between successive field minima in the radial direction. Placing a slot in the radial line then is all that is necessary to measure the voltage minimum positions. The attenuation may be measured also assuming that the Hankel functions at large radii approximate a plane wave.

Real Part, ϵ'_r

The dielectric constant may be obtained from the field equations of E_z, Eq (57), if the position of successive minima in the shorted radial wave guide are known. By imposing boundary conditions on E_z the position of successive minima will be obtained. Letting $E_z = 0$ at the short circuit of radius r_0 then the following equation is found:

$$E_z = A\left[H_0^{(2)}(kr) - \frac{H_0^{(2)}(kr_0)}{H_0^{(1)}(kr_0)}\, H_0^{(1)}(kr)\right] \tag{73}$$

Using the large-argument form of the Hankel functions, Eq (52).

$$E_z = A\sqrt{\frac{2}{\pi kr}}\left[\epsilon^{-j\left[kr-\frac{\pi}{4}\right]} - \epsilon^{-j2\left[kr_0-\frac{\pi}{4}\right]}\epsilon^{j\left[kr-\frac{\pi}{4}\right]}\right] \tag{74}$$

Setting $E_z = 0$ or E_z^2 in this case;

$$|E_z|^2 = E_z \cdot E_z^* = 0$$

$$= \frac{2A^2}{\pi kr}\left[\epsilon^{-j[kr-\frac{\pi}{4}]} - \epsilon^{-j2[kr_0-\frac{\pi}{4}]}\epsilon^{j[kr-\frac{\pi}{4}]}\right]$$

$$\times \left[\epsilon^{j[kr-\frac{\pi}{4}]} - \epsilon^{j2[kr_0-\frac{\pi}{4}]}\epsilon^{-j[kr-\frac{\pi}{4}]}\right] \tag{75}$$

and

$$\left[1 - \epsilon^{-j2[kr_0-\frac{\pi}{4}]}\epsilon^{j2[kr-\frac{\pi}{4}]} - \epsilon^{j2[kr_0-\frac{\pi}{4}]}\epsilon^{-j2[kr-\frac{\pi}{4}]}\right] = 0 \tag{76}$$

thus

$$2 - \epsilon^{j2(kr-kr_0)} - \epsilon^{-j2(kr-kr_0)} = 0 \tag{77}$$

or

$$\cos 2k(r_0-r) = 1 \tag{78}$$

therefore

$$2k(r_0-r) = 2n\pi, \quad n=0,1,2,\cdots \tag{79}$$

and

$$r_0 - r = \frac{n\pi}{k}, \quad n=0,1,2,\cdots \tag{80}$$

By taking two successive values of n, n_1 and n_2, and two values of r, r_1 and r_2, and then subtracting the larger r from the other,

$$r_2 - r_1 = (n_1 - n_2)\frac{\pi}{k} \tag{81}$$

Rearranging Eq (81) and defining Δ as the distance between successive minima,

$$\Delta = r_2 - r_1 = \frac{\pi}{k} = \frac{\pi}{\omega\sqrt{\mu\epsilon}} \tag{82}$$

then

$$\epsilon = \frac{\pi^2}{(2\pi f)^2 \mu \Delta^2} \tag{83}$$

As Δ is defined by any two specific values of r_1 and r_2 an average of two or more successive minima may be used for determining Δ . Eq (83) may be simplified to facilitate future calculations by defining

$$K = \frac{1}{4\mu\epsilon_0} = .2246 \times 10^{17} \tag{84}$$

then

$$\epsilon_r' = \frac{K}{f^2 \Delta^2} \tag{85}$$

As ϵ is a real number equal $\epsilon_0 \epsilon_r'$ then Eq (85) defines the relative dielectric constant.

The positions of maximum field strength, E_z max, in the radial direction may be obtained by setting the derivative of E_z, $\partial E_z / \partial r$, equal to zero and solving for r. The results show that the maxima are positioned equally between the minima along the radial direction as would be expected as the Hankel functions in their large argument form are cosine waves of decreasing amplitude. Thus the position between adjacent maxima is also defined as Δ, but because the position of the nulls can be more accurately measured, and because the probe used to sample the fields disturbs the fields less in the null, Δ is best determined by the distance between nulls.

Imaginary Part, Tan δ

Let E^+ be the magnitude of the incident field in the direction of the short circuit and E^- be that of the reflected field. Then the field equations present a plane traveling wave front since $kr >> 1$ and the assumption

$$E^- = E^+ \epsilon^{-2\alpha \ell} \tag{86}$$

will be correct. Here

$$\alpha = \frac{\pi}{\lambda_g} \tan \delta \tag{87}$$

(Ref 9:43) and ℓ is the distance from the short to the point where E^+ and E^- are measured. The use of Eq (86) is based on the Eq (68) which is essentially the form for the E^+ wave used in Eq (86). Then as the voltage standing wave ratio is defined

$$VSWR = \frac{E^+ + E^-}{E^+ - E^-} \tag{88}$$

substituting Eq (86) for E^- and solving for α gives

$$\alpha = \frac{1}{2\ell} \ln\left(\frac{VSWR+1}{VSWR-1}\right) \tag{89}$$

(Ref 9:43). Thus from Eqs (87) and (89)

$$\tan\delta = \frac{1}{2\ell}\frac{\lambda_g}{\pi} \ln\left(\frac{VSWR+1}{VSWR-1}\right) \tag{90}$$

Now using Eqs (85) and (90), ϵ_r' and tan δ may be found with only the following data known: the voltage standing wave ratio (VSWR), at a known distance ℓ from the shorting cylinder, the distance between successive minima, and the frequency of the propagating wave.

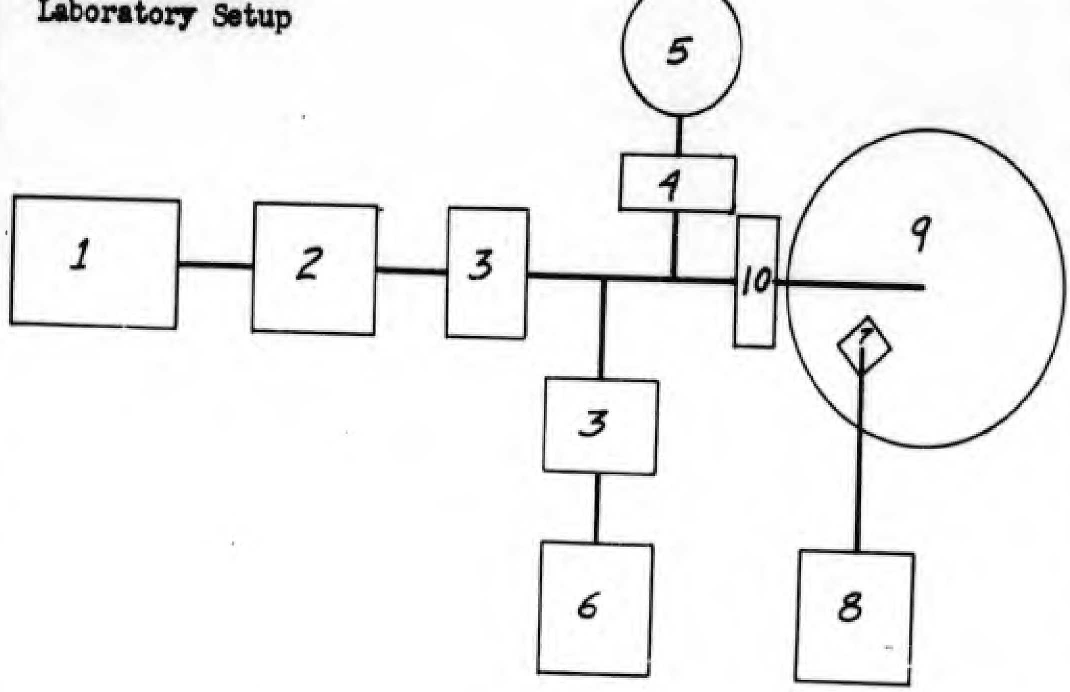
IV. Laboratory Equipment and Procedures

The laboratory equipment and procedures described in this section were used by the author to find, for five samples, the values of ϵ_r' and tan δ as defined by Eqs (85) and (90). The equipment used, with the exception of the radial wave guide slotted line, was standard laboratory equipment. The procedures used entailed finding the position of voltage minima along the slotted line and measuring the VSWR at one minimum position. In the following subsections a block diagram of the laboratory setup is given; the radial wave guide slotted line is described; the dielectric samples used are listed; and procedures for measuring ϵ_r' and tan δ are given. Only the manufacturer and model number of the standard laboratory equipment are given as the reader may refer to the manufacturer's literature for complete descriptions of each device. A detailed description of the radial wave guide slotted line and dielectric samples is given.

Block Diagram

Figure 2 shows the laboratory setup in block diagram form. Below the diagram the name, manufacturer, and model number of each item are recorded. Figure 3 shows a photograph of the laboratory setup with the teflon dielectric sample leaning against the radial wave guide slotted line.

FIGURE 2
Block Diagram of
Laboratory Setup



1. Universal Klystron Power Supply: Polytechnic Research and Development Company, Type 801.
2. Reflex Klystron: Varian Associates, X-13.
3. Ferrite Isolator: Raytheon, Model 1XH7.
4. Detector Mount: Hewlett Packard, X485B, Crystal Diode: Sylvania, 1N23B.
5. Cathode Ray Oscillograph: Dumont, Type 304-A.
6. Echo Box: U. S. Air Force, Fairchild Camera and Instrument Corporation, TS 488/U.
7. Probe and Crystal Detector Mount: Unknown, (Ref 5:492-494).
8. Standing Wave Indicator: Hewlett Packard, Model 415B.
9. Radial Wave Guide Slotted Line: See Subsection.
10. Precision RF Attenuator: Polytechnic Research and Development Company, Model 195B, Serial 1290.

Figure 3. Laboratory Setup

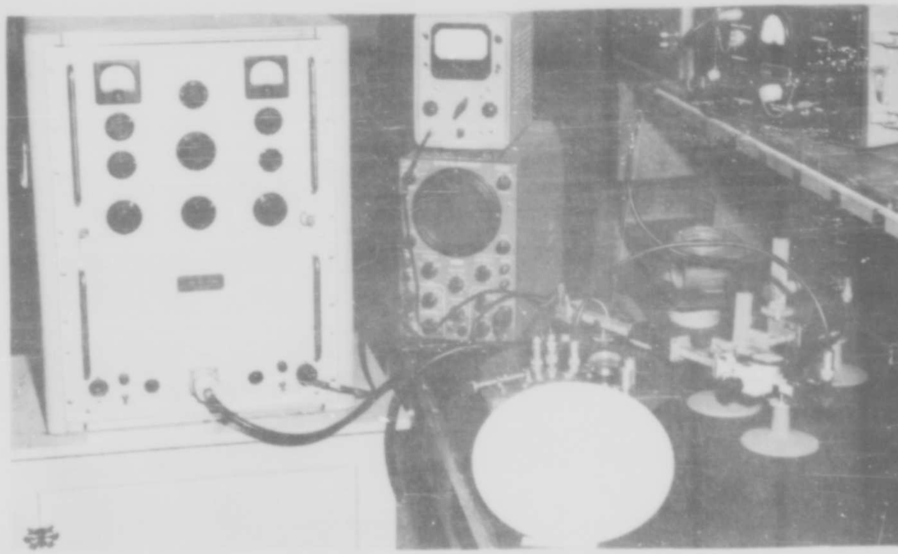
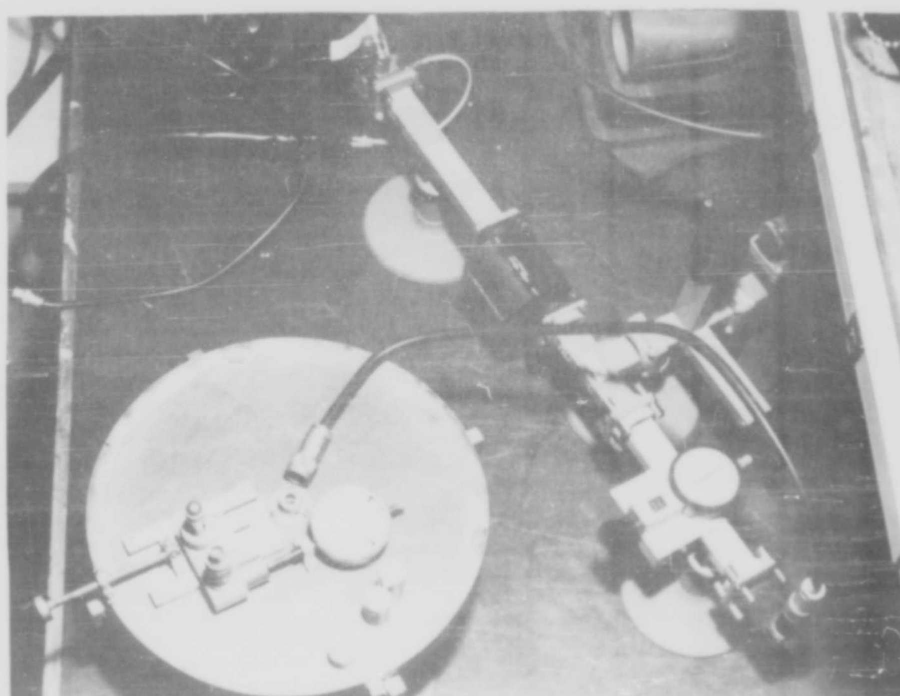


Figure 4. Radial Wave Guide Slotted Line



Radial Wave Guide Slotted Line

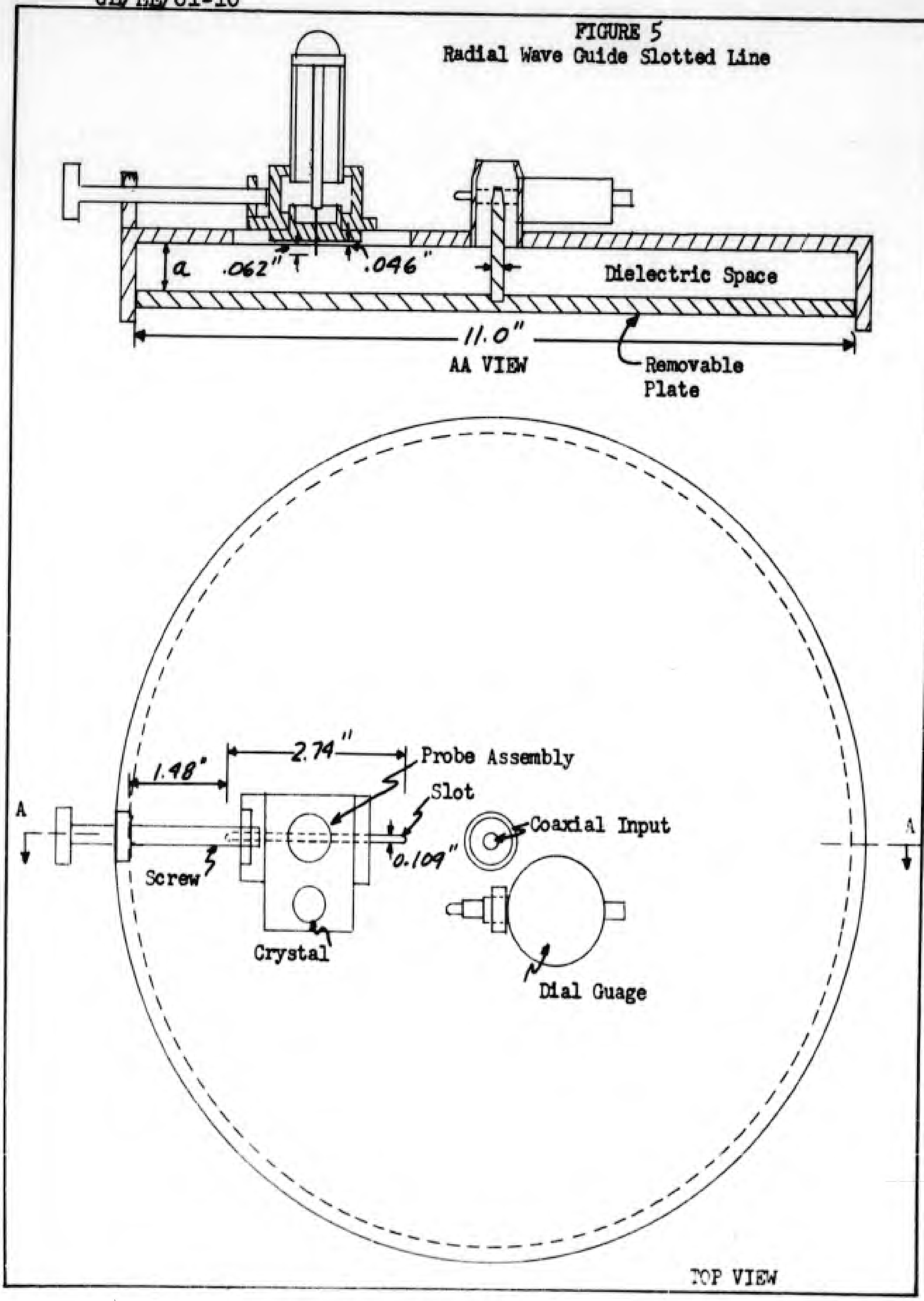
The radial wave guide slotted line consists of two flat circular plates surrounded by a cylindrical shorting bar. The line is fed from the center by a cylindrical metal bar that is the antenna. The coaxial line from the generator connects to the antenna with a standard type-N connector.

The plates are held apart by the dielectric between them. One plate is attached permanently to the cylindrical shorting bar while the other may be lifted from the shorting bar so that dielectric samples can be interchanged. Two clamps hold the removable plate against the dielectric sample and in case no sample is used they may be used to adjust the spacing between the plates.

Figure 5 shows a top and a cut-away side view of the radial line used. The drawing does not show the two clamps described above and also was not made to scale so that smaller details could be expanded. The dimensions given, however, are taken directly from the model used by the author which is shown in Figures 3 and 4. An accurate drawing of the probe and crystal detector mount is given by Montgomery (Ref 5:492).

The slot is cut radially in the top plate starting 1.48 in. from the shorting cylinder so that the standing wave from the short circuit will contain two or three minima before the opening can disturb the fields. With only E_z and

FIGURE 5
Radial Wave Guide Slotted Line



H_ϕ components present, surface current will flow radially; therefore, the slot will not greatly disturb the fields. The total travel of the probe in the slot covers at least three minima so that an averaging process can be used to measure Δ.

The position of the probe radially in the slot is measured with a millimeter dial gage that follows the probe travel. Four bars of known length were used to extend the range of the gage, which measured approximately 1.0 cm in increments of 1/100th of a millimeter.

Dielectric Samples

The dielectric samples and the accepted values of ϵ_r' and tan δ for each are shown in Table I. The thickness of the sample and the computed safety factor, F, using the values of ϵ_r' measured, are also given.

The solid samples (teflon and plexiglas) were machined in the form of flat circular disks that fit snugly inside the shorting cylinder. It was necessary to place a small slot in the sample so that the probe could be extended into the guide. Figure 3 shows a typical sample. A hole in the center of the sample allows the antenna to extend through the guide.

The air dielectric was obtained by positioning the parallel circular plates the required distance apart. The liquid dielectric was merely poured in the guide at the antenna connector after the removable lower plate was soldered the required distance from the upper plate. The thickness of the dielectric samples must be such that the TM_{00} mode only will propagate in the radial line. That is the sample thickness, a, is limited to the range

$$a \le \frac{\pi}{k} \tag{91}$$

where

$$k = \omega\sqrt{\mu \epsilon} = \frac{2\pi}{\lambda_0}\sqrt{\epsilon_r'} \tag{92}$$

Then a may be expressed as

$$a \le \frac{\lambda_0}{2\sqrt{\epsilon_r'}} \tag{93}$$

Thus the dimension a depends on the frequency and the dielectric constant.

To insure that higher modes, if excited, will be evanescent in nature the dimension a is generally less than the maximum value. This may be expressed as

$$a = F\frac{\lambda_0}{2\sqrt{\epsilon_r'}} \tag{94}$$

where F is the safety factor and is limited to the range

$$0 \le F \le 1 \tag{95}$$

Measuring ϵ_r'

The procedure to determine ϵ_r' entails measuring the frequency of the generator and then measuring the quantity Δ , the distance between successive minima. The frequency was measured with an echo box, which is essentially a tuned cavity wavemeter. Rotating the calibration dial changes the resonant frequency of the cavity. The frequency is then read from the dials when resonance is detected by observation of the maximum rectified current from the output of the cavity.

The position of voltage minima are determined in conjunction with the twice minimum method for measuring high VSWR (Ref 9:50). The positions on either side of the null 3db above the null power level are recorded and the average of the two readings is used as the position of the minima. The average distance between minima is obtained by measuring the first and last minima positions, subtracting the two values, and then dividing by the number of maxima between these two minima. The values f and Δ are then entered in Eqs (85) to determine ϵ_r'.

Measuring Tan δ

The procedure to determine tan δ entails measuring the VSWR at a known distance from the shorting cylinder. The VSWR is measured by the twice minimum method as high

VSWR may be expected in low loss materials. The data necessary to calculate tan δ was taken while finding ϵ_r', only a different interpretation of the data is needed.

First, the VSWR is computed from the equation

$$VSWR = \frac{2\Delta}{\pi d} \tag{96}$$

where Δ is the distance between successive minima and D is the difference between the points that are 3db above the null power level. Then knowing the position, ℓ, of the first minimum from the shorting cylinder, tan δ is computed from Eq (90) where again $\lambda_g = 2\Delta$.

V. Sources of Error in Measuring Dielectric Constant

The ability to measure ϵ_r' and tan δ accurately with use of the radial line method will depend on four items:

1. The accuracy of the assumptions made
2. The errors in constructing the radial wave guide
3. The operational limitations of the equipment used
4. The operator errors in measurement

The assumption that a pure TEM mode was launched in the radial wave guide led to the expression for ϵ_r' as the distance between successive minima. If the TEM wave were not pure, the value of Δ would change depending on the radius. Also it was assumed that the Hankel functions could be approximated by their large argument form, Eq (52), as kr was much larger than one. This led to the equation for finding tan δ by use of the plane wave theory to measure VSWR and calculate α. Also the crystal diode was assumed a square law device when relative power points for finding VSWR were measured.

The construction of the radial wave guide will only approximate the boundary conditions imposed on the field equations as the metal walls and the shorting cylinder have a finite conductivity. In addition, the slot in the wave guide, the slot in the dielectric sample, the penetration of the probe into the guide, and the air space between the guide walls and dielectric sample alter the fields at these discontinuities from the strictly mathematical solutions; thus the values of VSWR and Δ may be in error. The penetration uniformity of the probe along the slot directly affects the VSWR reading also.

As ϵ_r' and tan δ were based on measuring f, Δ, D, VSWR, and the accuracy in measuring these quantities greatly affects the accuracy of ϵ_r' and tan δ. The stability of the klystron power source, the accuracy of the wave meter, the accuracy of the VSWR meter, and the accuracy of the distance-measuring equipment then directly affect the final values to be measured and are considered operational sources of error.

No matter how accurately a piece of equipment may be used for measuring a quantity, the operator is the final determining factor in the total error of the measurement. That is, the operator error in measuring frequency and reading VSWR and probe positions must be considered. In addition, the operator must calculate, using the data obtained, the final answers from the corresponding equations--another possible source of error.

As all measurements of Δ and VSWR were made where $kr >> 1$, the Hankel functions approximation will be accurate and there will be no error in measuring ϵ_r' from this point as long as the safety factor F is less than one. The operator errors may also be neglected if the averaging of several minimum positions is used to determine Δ.

The errors in ϵ_r' then will be the result of the measuring limitations, and of the construction of the radial wave guide slotted line. The accuracy in ϵ_r' will be limited by the resolution of the dial gage used to measure Δ and the accuracy will be decreased by the construction errors. In Appendix A the expected error for values of ϵ_r' from 1 to 10 is calculated by taking the total derivative of ϵ_r', Eq (85). The results are tabulated in Table II for expected deviations in dD = 0.002 cm and df = 0.5 mc/s.

The assumption that the radial wave approximates the plane wave as $kr >> 1$ will be good over small distances. Therefore, the VSWR measurement technique should be correct as long as ℓ is small. However, construction errors also affect VSWR. The slot discontinuity will cause reflections and thus the VSWR as measured by the twice minimum method will be affected.

Observations from actual measurements showed that the distance ℓ, supposedly an integral number of Δ's from the short circuit, was, in fact, sometimes greater and sometimes less than an integral amount. Also the VSWR varied considerably from one minimum to the next as the value D varied greatly in each case. The VSWR should decrease as the distance, ℓ, increases but this was not observed for all samples. Variations in dl and dD were observed as high as 0.17 cm. and 0.05 cm. These discrepancies may be attributed to the conductivity of the metal and to the reflections from the slot ends. Additional errors in ℓ come from the air gap between the shorting cylinder and the dielectric sample.

As tan δ was computed from only one voltage minimum no averaging process was used to minimize operator error. The value of tan δ could be more accurate if two of three minima were used to calculate VSWR's and tan δ 's and then take the average as the results.

Stability of the klystron source, level of power output, and sensitivity and noise level of VSWR meter are related factors in determining VSWR and the location of minima positions. The twice minimum method for measuring VSWR requires a highly stabilized klystron with a power output capability at the pickup probe in the slot large enough to register above the noise level of the VSWR meter as a nonvarying signal. If these requirements are not met either because the material is very "lossy" or because it is lossless, i.e. high VSWR, then for these two cases the VSWR cannot be calculated as the power level of the minimum is too low to register above the noise level of the meter.

Typical errors to expect for tan δ based on measurement errors only, i.e., $d\Delta$, dl, d(VSWR), have been calculated for a few points around the values measured in this paper. These calculations are shown for Appendix A and the results tabulated in Table III for variations of $d\Delta = 0.002$ cm. and $d\ell = 0.002$ cm. for several values of d(VSWR) based on the values of VSWR as measured by the twice minimum method. It is seen that the error d(tan δ) alone is greater than the actual value quoted as the standard for teflon in Table I. Thus it appears that there can be calculated by this method a limiting minimum tan δ which would be greater than 0.0005 if 100% error or less is to be expected.

Errors as a result of high temperature may come from three sources:

1. The mechanical movement necessary to position the probe
2. Buckling of the metallic radial wave guide
3. Expansion of the metal increasing the air gap between the dielectric and the shorting cylinder

The mechanical movement necessary to position the probe in the slot will require a means of observing the scale accurately. The result of this movement would be a high value for dD, $d\Delta$, and $d\ell$, which would affect ϵ_r' and tan δ as previously mentioned.

Alternately heating and cooling the dielectric material may cause the wave guide walls to buckle. This condition would alter the boundary conditions and moding would take place. As the moding would be irregular no corrections could be determined for ϵ_r' or tan δ , so a proper choice of metals must be used to prevent this from happening.

The expansion of the metal would also increase the air gap between the solid dielectric and the walls and the shorting cylinder. This would not affect ϵ_r' as the clamp may be turned to tighten the metal plates to the dielectric. However, tan δ would be affected as the distance ℓ is changed by the air gap between the short and the material. It would then be necessary to adjust the value of ℓ for this air gap.

As the frequency is increased, the accuracy of results is changed by these factors:

1. Accuracy of measuring frequency
2. Wave length decreases
3. The slot size becomes a significant part of a wave length
4. The spacing of the plates becomes small

The accuracy of measuring frequency does not become a significant factor in measuring ϵ_r' as the error in measuring Δ , dΔ , will be the larger part of the total error. However, the problem of frequency stability is present when measuring tan δ .

As the wave length decreases, the error dΔ becomes a significant factor in measuring ϵ_r'. By taking the average of many minima, this effect may be decreased. This is not so when measuring tan δ as dD/D becomes a larger value and will limit the accuracy of VSWR calculated, as no averaging of the measurement D is taken.

As a rectangular wave guide decreases in size, the slot appears as a larger portion of the guide and soon the guide is not a guide but a three-sided trough with no top to direct the wave. Thus the fields in the slot cease to represent the true fields in the covered guide. With the radial guide, the effect should be smaller as the slot is not a significant part of the guide surface. However, the slot depth in the solid dielectric samples will be a larger part of the material thickness and may alter the fields in that way. Thus the spacing which must be less than the critical value that allows higher mode propagation will be the limiting factor in the frequency range and in the values of ϵ_r' .

VI. Results of Measurements for Dielectric Samples

The results of measuring ϵ_r' and tan δ for five samples are compared to accepted standard values in Table I. Included in the table are the sample thickness and the computed safety factor of propagation of the TEM mode. The percentage of error in the measured values from the standards is also given.

TABLE I

Results of Measurements of ϵ_r' and Tan δ
Compared to Standard Values

Sample	Thickness (in.)	ϵ_r' Meas.	ϵ_r' Stand.	% Error	Tan δ Meas.	Tan δ Stand.	% Error	F
a-air	.250	1.00	1.00	0.0	.0063	--x	--x	.40
b-air	.375	1.00	1.00	0.0	.0062	--x	--x	.60
c-teflon	.250	1.92	2.08	7.7	.0041	.00037	975.	.55
d-plexiglas	.250	2.56	2.59	1.2	.0073	.0067	12.	.63
e-plexiglas	.361	2.60	2.59	0.4	.0087	.0067	30.	.92
f-methyl alcohol	.177	--y	8.90	--y	--y	.8100	--y	.85

x--No standard given
y--Power level low, no measurements taken
Note: Standards for samples a and b (Ref 3:2524), for samples c, d, and e (Ref 8:332,334,362)
Frequency - 9330 mc/s.

It is seen that no correspondence between safety factor, F, and accuracy could be determined as all values of F were with in the acceptable range. Attempts to measure ϵ_r' of air when F was equal to 1.25 showed that moding was taking place as only one minimum occurred in the length of the slot.

The accuracy of ϵ_r' values were good except for the teflon sample "c". No explanation for this departure is known other than that the sample is not of the same composition as that used for the standard. The error in tan δ is large for all cases; however, as the losses increase the accuracy becomes better. Again the teflon sample departed considerably from the standard. This fact caused the author to have an independent measurement of these values taken. The values obtained for a frequency of 8.5 kmc/s were $\epsilon_r' = 2.05$ and tan $\delta = 0.0068$. Higher frequency measuring equipment was not available but these values would still indicate a more accurate value of tan δ was actually attained by the author's measurements.

The values for the air samples "a" and "b" of tan δ are also quite high. The value of tan δ was not given in the references but is generally assumed quite small--less than the minimum variation of tan δ as found in Appendix A, Table III.

Values for methyl alcohol, sample "f", were not found because the power output from the klystron was too small to produce an indication on the VSWR meter above the noise level.

VII. Conclusions and Recommendations

Measurements of samples by the author show that this method for measuring ϵ_r' may be used with an expected accuracy around 2% when ϵ_r' is 2.6 or less. The expected errors for materials with higher dielectric constants will remain in this same range.

Measurements of tan δ show that, at best, an error of 30% to 50% may be expected using an unstabilized klystron. The errors in measuring tan δ of low loss material were shown to be larger than the tan δ itself as it approached a value of 0.0005. This would limit the range of tan δ that can be measured to this limiting value if 100% error is tolerable. A higher limit would actually exist as this error takes into account only the accuracy of measuring frequency, distance between minimum, and VSWR and does not allow for any assumptions made as to the accuracy of the plane wave approximation.

An attempt to measure ϵ_r' and tan δ of methyl alcohol, a lossy material, was fruitless as the power output from the klystron source was too small to register above the noise level of the standing wave meter.

Further study is needed to resolve the problems in measuring tan δ . To this end the following ideas may be studied:

1. Provide a frequency-stabilizing network to improve measuring VSWR and to increase the consistency in measuring the distances D, ℓ , and Δ .

2. Decrease the radius of the radial line and increase the power to the line so higher loss materials may be measured.

3. Either extend the slot to the shorting cylinder or use a coupling hole that will take the place of the slot and measure the distance ℓ at the first minimum from the shorting cylinder. This should give a more accurate value when calculating tan δ as the Hankel function will approximate a plane wave better at the larger radius.

4. Correct the distance ℓ for the air gap at the shorting cylinder.

The upper limit on the frequency of measurement is a function of the value of ϵ_r' to be measured and the spacing of the parallel plates. Decreasing the spacing, i.e. increasing frequency, will increase the errors as the slot becomes a larger fraction of a wave length. Higher ϵ_r' values just increase this fraction and then the fields in the area of the slot will no longer represent the true fields in the guiding structure. The temperature of the specimen would not limit measurements.

The advantages in measuring the dielectric constant with the radial guide are:

1. The measurements are quick as the samples are not taken out of the guide during the measuring. Only the first and last minima positions and the distance between 3db points on each side of the last minima need be measured.

2. The computation of tan δ would include only one correction term, that for ℓ .

3. A stabilized frequency source is not necessary to measure ϵ_r' accurately.

4. Liquids and gases may be measured with the same basic equipment.

5. Measurements at high temperatures will not present a problem.

The disadvantages to this method are:

1. The radial wave guide is not a standard laboratory item and must be specially made.

2. The measurements depend on knowing the range of ϵ_r' of the material so that the sample may be properly dimensioned.

3. A stabilized frequency source to measure tan δ accurately is required.

4. Tan δ measurements are of limited accuracy.

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Appendix A

Accuracy in ϵ_r' and Tan δ Measurements

The accuracy of ϵ_r' and tan δ will be limited by the errors in the physical measurements made in the laboratory. By taking the total derivative of Eqs (85) and (90) the error in ϵ_r' and tan δ may be found by substituting the values of f, Δ, VSWR, ℓ, and the expected changes in each into the resulting equation.

The total derivative of ϵ_r' is defined as

$$d\epsilon_r' = \frac{\partial \epsilon_r'}{\partial f} df + \frac{\partial \epsilon_r'}{\partial \Delta} d\Delta \tag{97}$$

and for the case of ϵ_r' defined by Eq (85)

$$d\epsilon_r' = -\frac{2K}{f^2\Delta^2}\left[\frac{df}{f} + \frac{d\Delta}{\Delta}\right] \tag{98}$$

Evaluating $d\epsilon_r'$ will depend on the values of f and Δ as df and $d\Delta$ are assumed constants. For the measurement described in this report the values of df and $d\Delta$ as a result of instrument errors are df = 0.5 mc/s, $d\Delta$ = 0.002 cm. These variations do not include those from frequency instability and slot reflections. Variations in Δ as measured along the slot were as large as 0.04 cm.

The variation, $d\,\epsilon_r'$, in the relative dielectric constant and the percentage change in the relative dielectric constant are tabulated in Table II for a nominal frequency of 9375 mc/s and a $\Delta = \lambda_g/2$ for values of ϵ_r' from 1 to 10.

TABLE II

Per Cent Error in ϵ_r' for Frequency Error of 0.5 mc/s and Δ Error of 0.002 cm.

ϵ_r'	Δ (cm)	$d(\epsilon_r')$	% of ϵ_r'
1.0	1.600	0.00261	0.26
2.0	1.311	0.00729	0.36
3.0	0.924	0.0133	0.44
4.0	0.800	0.0204	0.51
5.0	0.716	0.0285	0.57
6.0	0.653	0.0374	0.62
7.0	0.605	0.0471	0.67
8.0	0.566	0.0575	0.71
9.0	0.533	0.0686	0.76
10.0	0.506	0.0802	0.80

ℓ = 4.75 cm
f = 9375 mc/s

The accuracy of tan δ is greatly dependent on the error in measuring ℓ and Δ. The expected accuracy of VSWR based on the error of the physical measurements D and Δ can be evaluated by the same method as that used for ϵ_r'. That is

$$dS = \frac{\partial S}{\partial \Delta}\, d\Delta + \frac{\partial S}{\partial D}\, dD \qquad (99)$$

and for VSWR defined by Eq (96) where VSWR is equal S,

$$dS = \frac{2\Delta}{\pi D}\left[\frac{d\Delta}{\Delta} + \frac{dD}{D}\right] \tag{100}$$

Evaluating dS then depends on Δ and D and on the variations in each. The variations $d\Delta$ and dD are assumed constant and have the values $d\Delta = 0.002$ cm. and dD = 0.002 cm. based on the accuracy of the scales in measuring these quantities. The variations in D noted in the measurement made were sometimes as high as 0.03 cm. Therefore the values given in Table III are the minimum variations in VSWR and the error in the actual values of VSWR would not be limited by the scales but by the power level and how well the $\sqrt{2}\,E_{min}$ positions could be found.

The variation in tan δ can now be calculated, knowing the variation in VSWR, Δ, and ℓ by the total derivative method. Thus

$$d(\tan\delta) = \frac{\Delta}{\pi \ell}\left[\frac{d\ell}{\ell}\ln\left(\frac{S+1}{S-1}\right) + \frac{d\Delta}{\Delta}\ln\left(\frac{S+1}{S-1}\right) + \frac{2dS}{S^2-1}\right] \tag{101}$$

The value of d(tan δ) for values of $d\ell = 0.002$ cm. and $d\Delta = 0.002$ cm. have been calculated for differing values of ϵ_r', VSWR, and d(VSWR) and are tabulated in Table III. The value of ℓ was held constant at 4.75 cm. as an average value obtained from the actual measurements in this paper.

TABLE III

Error in Tan δ vs VSWR in Materials with Dielectric Constants of 1.0, 2.0, and 3.0

ϵ_r' / VSWR	1.0				2.0			
VSWR	D (cm)	d(VSWR)	Tan δ	d(Tan δ)	D (cm)	d(VSWR)	Tan δ	d(Tan δ)
10	0.1020	0.21	0.02152	0.000488	0.0720	0.29	0.01521	0.000486
20	0.0509	0.81	0.01073	0.000453	0.0360	1.14	0.00759	0.000452
30	0.0339	1.81	0.00715	0.000442	0.0240	2.55	0.00506	0.000442
40	0.0254	3.19	0.00536	0.000437	0.0180	4.51	0.00379	0.000436
50	0.0204	4.97	0.00429	0.000434	0.0144	7.03	0.00303	0.000433
60	0.0169	7.14	0.00357	0.000431	0.0120	10.10	*	
70	0.0145	9.70	0.00306	0.000430	0.0103	13.73		
80	0.0127	12.66	0.00268	0.000429	0.0090	17.91		
90	0.0113	16.02	0.00238	0.000428	0.0080	22.65		
100	0.0102	19.76	0.00214	0.000427	0.0072	27.94		

ϵ_r' / VSWR	3.0			
VSWR	D (cm)	d(VSWR)	Tan δ	d(Tan δ)
10	0.0588	0.36	0.01242	0.000485
20	0.0294	1.40	0.00620	0.000452
30	0.0196	3.12	0.00413	0.000441
40	0.0147	5.52	0.00310	0.000436
50	0.0117	8.61	0.00248	0.000433
60	0.0098	12.37		
70	0.0084	16.81		
80	0.0073	21.93		
90	0.0065	27.70		
100	0.0059	34.20		

$d(\ell) = 0.002$ cm
$d\Delta = 0.002$ cm
$\ell = 4.75$ cm
$f = 9375$ mc/s

*-Values were not calculated that are left blank

Appendix B

Laboratory Work Sheets for Calculating ϵ_r' and Tan δ

The data taken and the results calculated for ϵ_r' and tan δ are given in the following pages. The formulas used to calculate ϵ_r' and tan δ are:

$$\epsilon_r' = \frac{0.2246}{f^2 \Delta^2} \tag{85}$$

f -- Frequency in mc/s

Δ -- Average distance between minimum in meters

$$VSWR = \frac{2\Delta}{\pi D} \tag{96}$$

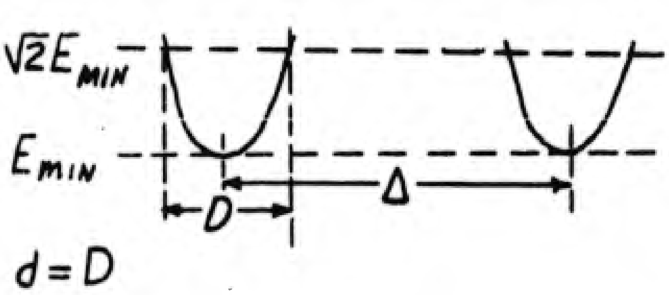
D -- Distance between 3db position of null in meters

$$Tan\ \delta = \frac{1}{2\ell} \frac{\lambda_g}{\pi} \ln\left(\frac{VSWR+1}{VSWR-1}\right) \tag{90}$$

ℓ -- Distance from null to short circuit

λ_g = 2Δ in meters

Voltage Standing Wave



Appendix B

Work Sheet for Calculating ϵ_r' and Tan δ

Sample		a-air		
Thickness	(in.)	0.250		
Run		1		
Frequency	(mc/s)	91.065	9330.4	9558.0
n minima		3	3	3
1st minimum	(mm)	46.344	47.506	32.740
nth minimum	(mm)	13.291	15.581	1.891
Δ	(mm)	16.526	15.962	15.924
ϵ_r'		0.99	1.01	0.97
Average ϵ_r'				0.99
Short position	(mm)	95.656		
"ℓ"	(mm)	49.312	48.150	62.916
3 db	(mm)	4-8429	4-9551	3-4271
3 db	(mm)	4-8060	4-9361	3-4060
"d"	(mm)	00.369	0.190	0.211
VSWR		28.5	53.5	48.0
Tan δ		0.0075	0.0039	0.0034
Average tan δ				0.0049
Run		2		
Frequency	(mc/s)	9099.8	9332.7	9557.3
n minima		3	3	3
1st minimum	(mm)	46.271	47.463	32.787
nth minimum	(mm)	0.551	15.592	1.888
Δ	(mm)	16.510	15.935	15.449
ϵ_r'		0.99	1.01	1.03
Average ϵ_r'				1.01
Short position	(mm)	95.656		
"ℓ"	(mm)	49.385	48.193	62.869
3 db	(mm)	4-8389	4-9480	3-4339
3 db	(mm)	4-7954	4-9246	3-4085
"d"	(mm)	0.435	0.517	0.254
VSWR		24.2	19.6	38.7
Tan δ		0.0087	0.0107	0.0040
Average tan δ				0.0078
Average Run 1 and 2				
ϵ_r'				1.00
Tan δ				0.0063

Work Sheet for Calculating ϵ_r' and Tan δ

Sample		b--air		
Thickness	(in.)	0.375		
Run		1		
Frequency	(mc/s)	9118.5	9321.2	9562.8
n minima		3	3	3
1st minimum	(mm)	46.462	47.536	32.625
nth minimum	(mm)	13.438	15.521	1.907
Δ	(mm)	16.512	15.980	15.859
ϵ_r'	(mm)	0.99	1.01	0.98
Average ϵ_r'				0.99
Short position	(mm)	95.656		
"ℓ"	(mm)	49.194	48.160	63.031
3 db	(mm)	4-8557	*	3-4322
3 db	(mm)	4-8167	*	3-3779
"d"	(mm)	00.390	0.247**	0.543
VSWR		27	41.2	18.6
Tan δ		0.0079	0.0078	0.0087
Average tan δ				0.0081
Run		2		
Frequency	(mc/s)	9118.3	9319.6	9558.0
n minima		3	3	3
1st minimum	(mm)	48.448	47.595	32.840
nth minimum	(mm)	13.449	16.020	1.818
Δ	(mm)	16.499	16.020	15.511
ϵ_r'		0.99	1.01	1.02
Average ϵ_r'				1.01
Short position	(mm)	95.656		
"ℓ"	(mm)	49.208	48.160	62.816
3 db	(mm)	4-8491	4-9495	3-4370
3 db	(mm)	4-8206	4-9298	3-4161
"d"	(mm)	0.285	0.197	0.209
VSWR		40.5	57.0	51.9
Tan δ		0.0058	0.0041	0.0033
Average tan δ				0.0044
Average Run 1 and 2				
ϵ_r'				1.00
Tan δ				0.0062

*Beyond slot
**Average value from preceding nulls

Work Sheet for Calculating ϵ_r' and Tan δ

Sample	c--teflon		
Thickness (in.)	0.250		
Run	1		
Frequency (mc/s)	9090.2	9333.3	9553.3
n minima	4	4	4
1st minimum (mm)	37.306	37.149	40.318
nth minimum (mm)	1.614	2.753	6.261
Δ (mm)	11.897	11.383	11.352
ϵ_r'	1.92	1.99	1.91
Average ϵ_r'			1.94
Short position (mm)	95.656		
"ℓ" (mm)	58.350	58.507	55.348
3 db (mm)	3-8812	3-8711	4-2479
3 db (mm)	3-8651	3-8438	4-1958
"d" (mm)	0.161	0.273	0.321
VSWR	47.0	26.5	21.5
Tan δ	0.0030	0.0047	0.0058
Average tan δ			0.0045
Run	2		
Frequency (mc/s)	9099.2	9334.3	9563.6
n minima	4	4	4
1st minimum (mm)	37.413	38.529	40.287
nth minimum (mm)	2.005	3.352	6.011
Δ (mm)	11.802	11.725	11.425
ϵ_r'	1.95	1.88	1.88
Average ϵ_r'			1.90
Short position (mm)	95.656		
"ℓ" (mm)	58.243	57.127	55.369
3 db (mm)	3-8972	4-0487	4-2310
3 db (mm)	3-8704	4-0372	4-2065
"d" (mm)	0.268	0.115	0.245
VSWR	28.0	65.0	30.0
Tan δ	0.0046	0.0020	0.0044
Average tan δ			0.0037
Average Run 1 and 2			
ϵ_r'			1.92
Tan δ			0.0041

Work Sheet for Calculating ϵ_r' and Tan δ

Sample	d--plexiglas		
Thickness (in.)	0.250		
Run	1		
Frequency (mc/s)	9090.0	9330.6	9577.6
n minima	5	5	5
1st minimum (mm)	44.777	45.500	46.856
nth minimum (mm)	3.035	5.299	7.629
Δ (mm)	10.435	10.050	9.806
ϵ_r'	2.50	2.55	2.55
Average ϵ_r'			2.53
Short position (mm)	95.656		
"ℓ" (mm)	50.879	50.156	48.800
3 db (mm)	4-6984	4-7660	4-8812
3 db (mm)	4-6370	4-7232	4-8701
"d" (mm)	0.614	0.519	0.111
VSWR	10.8	12.3	56.0
Tan δ	0.0121	0.0104	0.0023
Average tan δ			0.0083
Run	2		
Frequency (mc/s)	9061.0	9326.8	9567.2
n minima	5	5	5
1st minimum (mm)	43.979	45.476	46.895
nth minimum (mm)	2.828	5.576	7.582
Δ (mm)	10.287	9.925	9.828
ϵ_r'	2.59	2.62	2.56
Average ϵ_r'			2.59
Short position (mm)	95.656		
"ℓ" (mm)	51.677	50.180	48.761
3 db (mm)	4-5961	4-7664	4-8854
3 db (mm)	4-5797	4-7084	4-8637
"d" (mm)	0.164	0.575	0.217
VSWR	40.0	11.0	28.8
Tan δ	0.0032	0.0115	0.0044
Average tan δ			0.0064
Average Run 1 and 2			
ϵ_r'			2.56
Tan δ			0.0073

Work Sheet for Calculating ϵ_r' and Tan δ

Sample		e--plexiglas		
Thickness	(in.)	0.361		
Run		1		
Frequency	(mc/s)	9152.2	9335.4	9556.3
n minima		5	5	5
1st minimum	(mm)	44.872	45.833	47.136
nth minimum	(mm)	4.124	6.009	7.940
Δ	(mm)	10.186	9.956	9.769
ϵ_r'		2.58	2.63	2.57
Average ϵ_r'				2.59
Short position	(mm)	95.656		
"ℓ"	(mm)	50.784	49.823	48.520
3 db	(mm)	4-6995	4-7906	4-8685
3 db	(mm)	4-6550	4-7559	4-8685
"d"	(mm)	0.445	0.347	0.565
VSWR		14.6	18.3	11.0
Tan δ		0.0087	0.0070	0.0117
Average tan δ				0.0091
Run		2		
Frequency	(mc/s)	9106.3	9325.9	9567.2
n minima		5	5	5
1st minimum	(mm)	44.621	45.776	47.136
nth minimum	(mm)	3.602	5.906	8.032
Δ	(mm)	10.254	9.969	9.776
ϵ_r'		2.58	2.60	2.57
Average ϵ_r'				2.58
Short position	(mm)	95.656		
"ℓ"	(mm)	51.035	49.880	48.520
3 db,	(mm)	4-6735	4-7859	4-9272
3 db	(mm)	4-6308	4-7493	4-8800
"d"	(mm)	0.427	0.366	0.472
VSWR		15.3	17.9	13.2
Tan δ		0.0084	0.0073	0.0097
Average tan δ				0.0084
Average Run 1 and 2				
ϵ_r'				2.59
Tan δ				0.0087

Vita

James Gerald Klaus, Jr., was born on [redacted], he entered the Case Institute of Technology, Cleveland, Ohio, and was graduated with a Bachelor of Science degree in Electrical Engineering on 9 June 1955. Receiving a reserve commission as a Second Lieutenant at Case, he entered the United States Air Force after an eight-month stay working for the Martin Company, Baltimore, Maryland, as an Associate Engineer in the Radio Frequency Section of the Electronics Group. His first assignment, after completing Primary, Basic, and Advanced Pilot training, was to the 436th Tactical Fighter Squadron, George Air Force Base, California. He left George Air Force Base in January 1960 to enter the graduate electrical engineering class, GE-61, at the Institute of Technology (Air University), Wright-Patterson Air Force Base, Ohio.

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