AD 270 270

INSTITUTE OF TECHNOLOGY

AIR UNIVERSITY

UNITED STATES AIR FORCE

SCHOOL OF ENGINEERING

THESIS

WRIGHT-PATTERSON AIR FORCE BASE, OHIO

AF-WP-B-SEP 60 5M

A UNIFIED APPROACH TO THE IMPEDANCE STABILIZATION OF VACUUM TUBE AND TRANSISTOR L-C FEEDBACK OSCILLATORS

THESIS

Presented to the Faculty of the School of Engineering
The Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the
Master of Science Degree
in Electrical Engineering

By

John Daniel Kelly, B.S.
1st Lt. USAF

GE-61

August 1961

AF-WP-O-DEC 61 19

Preface

The linear analysis of oscillator circuits by the $\Delta = 0$ method is a relatively straightforward process. However, as the oscillator circuit progresses from the simple to the complex, the process becomes quite tedious and time-consuming. I thought there must be an easier way of calculating expressions for frequency of oscillation and various stabilizing impedances; so I set out to find one. But an investigation of textbooks and articles revealed that, although many texts explain the $\Delta = 0$ method of analysis, and illustrate its application for one or two common configurations (usually Colpitts or Hartley), the analyzer (myself) was left no choice but to carry out the process for configurations other than those used as examples in texts. This report, then, is my attempt to develope an approach to L-C feedback oscillator impedance stabilization which will apply to both transistor and vacuum tube oscillators, and which will eliminate the necessity of performing the tedious calculations required by complete linear analysis.

I wish to acknowledge the invaluable assistance given to me by Prof. Thaddeus L. Regulinski, EE Dept., AFIT, without whose advice and guidance my task would have been a far more difficult one. I also wish to acknowledge the help given to me by my wife, Violet, who, although she cannot type, performed the equally important functions of supplying moral support and managing three small children, without whose silence my task would also have been a far more difficult one.

John D. Kelly

Contents

Contents

List of Figures

List of Figures

List of Tables

Abstract

A general oscillator circuit, into which either a vacuum tube or a transistor can theoretically be inserted, is developed for use as a vehicle for linear analysis. Six different oscillator configurations are then analysed; and, from specific expressions calculated, general expressions for stabilized frequency and stabilizing impedances are synthesized. After rules for interpreting the general expressions are set forth, the expressions are tested for three oscillator configurations not used in their synthesis. The results of the tests prove the validity of the expressions. It is concluded that these expressions hold for all vacuum-tube and transistor L-C feedback oscillators, and therefore the desired unified approach to impedance stabilization is established.

A UNIFIED APPROACH TO THE IMPEDANCE STABILIZATION OF VACUUM TUBE AND TRANSISTOR L-C FEEDBACK OSCILLATORS

I. Introduction

One of the points of interest in oscillator analysis is the frequency of oscillation. Undesired frequency changes in both transistor and vacuum tube oscillators may result from three causes: changes in the mechanical arrangement of the oscillating circuit elements; changes in the values of the circuit parameters; and changes in the tube or transistor parameters themselves because of aging and climatic conditions as well as power supply variations. The first two changes can be minimized or overcome by careful mechanical and electrical design, by temperature control, and by the use of thermally-compensated inductances and temperature-controlled compensating capacitors.

The general method of eliminating frequency instability caused by changes in tube or transistor parameters is not to attempt to prevent the parameters from varying, but to design the oscillator in such a way that the frequency is independent of these parameters.

Purpose

Llewellyn has shown that impedance (capacitance or inductance) stabilization results in the independence of the oscillating frequency from variable device parameters, and hence from battery voltages (Ref 14:2064). The purpose of this report is to develope a unified approach to impedance stabilization that will work for both vacuum tube and transistor L-C feedback oscillators. This entails designing a single circuit into which either a vacuum tube or a transistor may be inserted to form an oscillator. From this circuit, which serves as a tool for linear analysis, two expressions are derived: first, an expression for the stabilized frequency, i.e., the oscillating frequency independent of the device parameters; second, an expression for the stabilizing impedance, either inductive or capacitive, needed to produce this stabilized frequency.

Report Organization

This report is divided into four chapters. Chapter II is a brief presentation of the basic theory of linear analysis by the $\Delta=0$ method. A complete linear analysis of one oscillator configuration is presented for both the transistor and vacuum tube circuits. Expressions for the stabilized frequency and the various stabilizing impedances are derived in detail.

In the third chapter a general oscillator circuit is designed. Various oscillator configurations (tuned-plate, tuned grid, Hartley, etc.), using both transistors and vacuum tubes, are analyzed by the $\Delta=0$ method outlined in the first chapter. These analyses produce sufficient data to form a basis for the synthesis of expressions for the stabilized frequency and the various stabilizing impedances. Tables are compiled showing the stabilized frequencies and the stabilizing impedances for the configurations analyzed. The data in these tables is compared to determine what each category (stabilized frequency, plate-stabilizing impedance, grid-stabilizing impedance, etc.) has in common. Then inductive reasoning is used to derive general expressions for each category. Rules for interpreting the general expressions are set forth, and examples of applying these rules are given.

The fourth chapter is the "test" chapter. In it the general stabilizing impedance expressions are tested for several oscillator configurations other than those used in their synthesis. The report is brought to a close with a final chapter in which results are summarized and conclusions are made.

Scope

The L-C oscillators considered in this report fall under Martin's classification of feedback oscillators (Ref 16:359). This classification excludes negative resistance oscillators such as dynatron or transitron types. Crystal-controlled oscillators and oscillators using either a mechanical oscillating system or selective filters are not considered. Since the frequency of oscillation is stabilized by the inclusion of a crystal or a filter in the circuit, such oscillators do not require stabilizing impedances.

Because this investigation is based primarily on Llewellyn's analysis of oscillator impedance stabilization, his assumptions are still applicable: the effect of grid current is taken into account, but distortion resulting from the non-linearity of tube or transistor parameters is neglected. It is further assumed that losses in the oscillating circuits are so small as to be negligible, and that the load is applied to the oscillator through a buffer stage which draws no power from the oscillator.

To achieve some standardization in regard to the approach to the circuit-design problem, all the oscillator configurations analyzed were either common-cathode (vacuum tube) or common-emitter (transistor); and the resulting circuit design and the unified approach to impedance stabilization are based on these particular device configurations.

II. Linear Analysis of Oscillator Circuits

A mathematical linear analysis of an oscillator circuit may be performed by writing a series of loop equations from which any of the circuit currents may be calculated by means of determinants. However, in an oscillating circuit, an alternating current flows for an indefinite period without any apparent source of emf. Therefore, in writing a set of loop equations of the form

$$Z_{11}I_1 + Z_{12}I_2 + --- + Z_{1n}I_n = E \tag{1}$$

E would be equal to zero in all cases. Solving for any desired I would therefore lead to an equation involving two determinants:

$$I = Dn/Dd \tag{2}$$

In (2) Dn is the determinant of the numerator and Dd is the determinant of the denominator. Dn will always equal zero because all loop voltage sources will total zero. Since a finite I does exist, Dn/Dd must be indeterminate to avoid a trivial solution. Hence Dd must also equal zero. The elements of this determinant are the circuit impedances; hence the linear criterion for any oscillator circuit is that the determinant of the loop equations (Δ) be equal to zero.

When expanded, the Δ determinant will contain a real and an imaginary part. In general, by setting the real part equal to zero, the conditions for oscillation can be calculated; by setting the imaginary part equal to zero, the frequency of oscillation can be calculated. By including the stabilizing impedances in the oscillator circuit, and hence in the Δ determinant, these impedances can also be calculated.

To demonstrate the Δ = 0 method, a complete analysis of a Colpitts oscillator is now presented for both the vacuum tube and transistor cases. Because this investigation is concerned with the frequency of oscillation and the values of the stabilizing impedances, it is assumed that the condition for oscillation is met. Setting the real part of the expanded Δ determinant equal to zero and solving for the condition for oscillation is therefore omitted.

Figures 1 and 2 on the next page show a Colpitts vacuum tube oscillator and its linear equivalent circuit respectively. Writing loop equations for the circuit in Fig. 2 yields:

$$I_1(r_p + jX_p - jX_1) + I_2(jX_1) + I_3(\mu r_g) = 0 \qquad (3)$$

$$I_1(jX_1) + I_2(jX_T) + I_3(jX_2) = 0 \qquad (4)$$

$$I_1(0) + I_2(jX_2) + I_3(r_g - jX_2 + jX_g) = 0 \qquad (5)$$

From these three equations the Δ determinant can be formed by taking the coefficients of each I term and arranging them as follows:

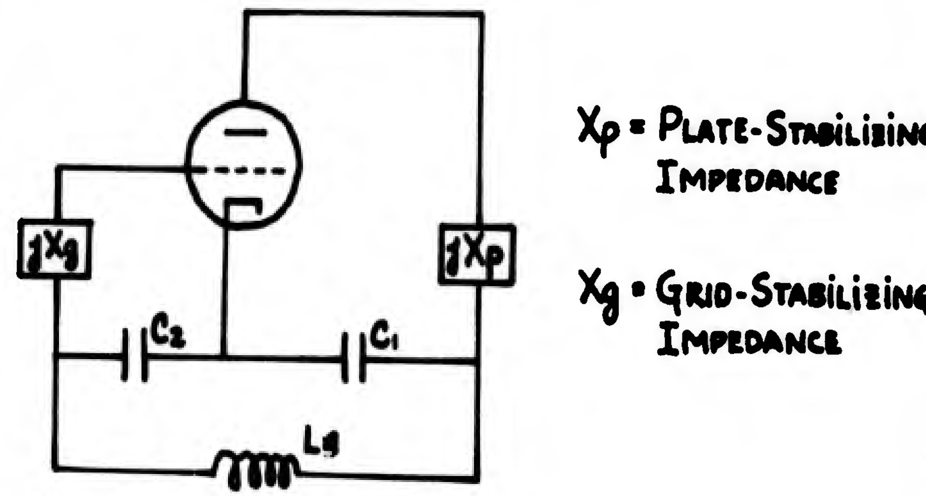

Figure 1

Colpitts Vacuum Tube Oscillator

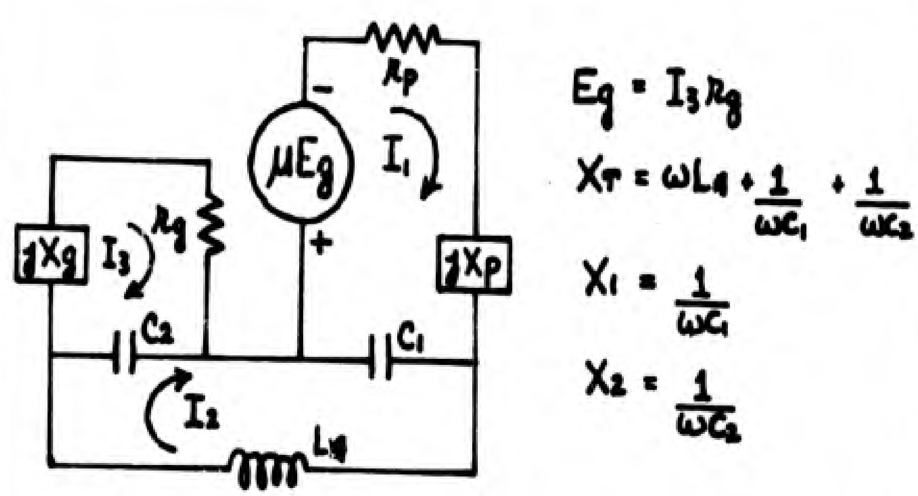

Figure 2

Colpitts Vacuum Tube Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} r_p + jX_p - jX_1 & jX_1 & \mu r_g \\ jX_1 & jX_T & jX_2 \\ 0 & jX_2 & r_g - jX_2 + jX_g \end{vmatrix} = 0 \qquad (6)$$

Expanding the Δ and summing the imaginary terms yields:

$$X_T(r_p r_g + X_p X_2 - X_p X_g - X_1 X_2 + X_1 X_g) + X_1(X_1 X_2 - X_1 X_g) + X_2(-X_p X_2 - X_1 X_2) = 0 \qquad (7)$$

Equation (7) can be broken down into two equations, namely:

$$X_T(r_p r_g + X_p X_2 - X_p X_g - X_1 X_2 + X_1 X_g) = 0 \qquad (8)$$

$$X_1(X_1 X_2 - X_1 X_g) + X_2(-X_p X_2 - X_1 X_2) = 0 \qquad (9)$$

where (9) includes all terms in (7) which do not contain X_T as a factor. It follows that if Xg and Xp have such values as to satisfy (9), then the frequency of oscillation is that which will force X_T to zero, thereby satisfying (8); and the frequency will remain constant regardless of variations of r_p, r_g and μ.

Therefore, solving for ω by setting X_T equal to zero yields:

$$\omega = \sqrt{\frac{C_1 + C_2}{L_4(C_1 C_2)}} \qquad (10)$$

To solve for Xp, setting Xg = 0 in (9) yields:

$$X_p = \frac{X_1}{X_2}(X_1 + X_2) = \frac{C_2}{\omega C_1}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \tag{11}$$

$$X_p = \omega L_p \tag{12}$$

$$L_p = \frac{C_2}{\omega^2 C_1}\left(\frac{C_1 + C_2}{C_1 C_2}\right) = \frac{C_2}{\frac{(C_1 + C_2)C_1}{L_4 C_1 C_2}}\left(\frac{C_1 + C_2}{C_1 C_2}\right) \tag{13}$$

$$L_p = \frac{L_4 C_2}{C_1} \tag{14}$$

To solve for Xg, setting Xp = 0 in (9) yields:

$$X_g = \frac{X_2}{X_1}(X_1 + X_2) = \frac{C_1}{\omega C_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \tag{15}$$

$$X_g = \omega L_g \tag{16}$$

$$L_g = \frac{C_1}{\omega^2 C_2}\left(\frac{C_1 + C_2}{C_1 C_2}\right) = \frac{C_1}{\frac{(C_1 + C_2)C_2}{L_4 C_1 C_2}}\left(\frac{C_1 + C_2}{C_1 C_2}\right) \tag{17}$$

$$L_g = \frac{L_4 C_1}{C_2} \tag{18}$$

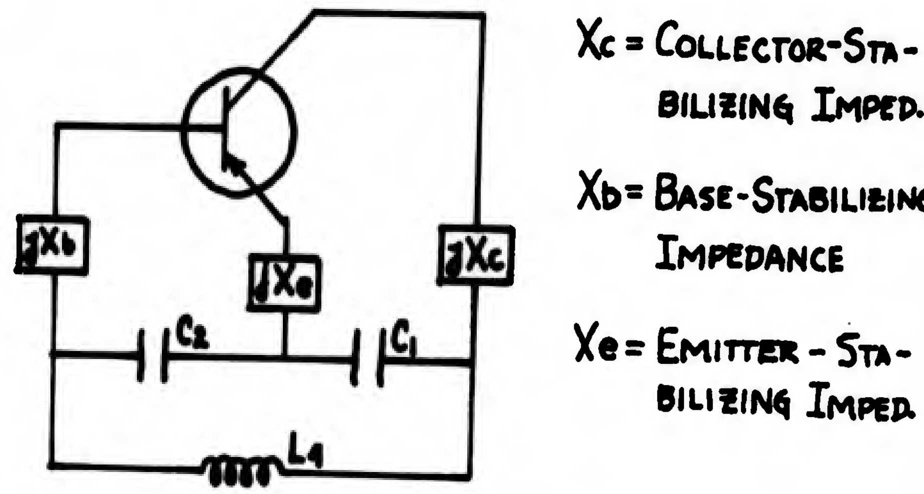

Figure 3

Colpitts Transistor Oscillator

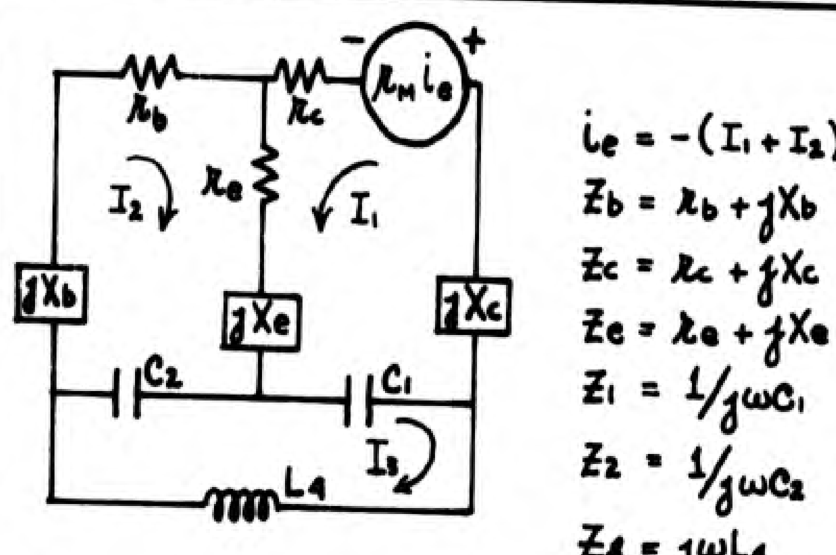

Figure 4

Colpitts Transistor Oscillator
Linear Equivalent Circuit

The analysis of the Colpitts transistor oscillator, shown in Fig. 3, follows the same procedure, with the exception that there is an additional stabilizing impedance in the emitter lead.

Writing loop equations for the linear equivalent circuit of Fig. 4 yields:

$$I_1(Z_c + Z_e + Z_1 - r_M) + I_2(Z_e - r_M) + I_3(Z_1) = 0 \quad (19)$$

$$I_1(Z_e) + I_2(Z_e + Z_b + Z_2) + I_3(-Z_2) = 0 \quad (20)$$

$$I_1(Z_1) + I_2(-Z_2) + I_3(Z_1 + Z_2 + Z_3) = 0 \quad (21)$$

The Δ for these three equations is therefore

$$\Delta = \begin{vmatrix} Z_c + Z_e + Z_1 - r_M & Z_e - r_M & Z_1 \\ Z_e & Z_e + Z_b + Z_2 & -Z_2 \\ Z_1 & -Z_2 & Z_1 + Z_2 + Z_3 \end{vmatrix} = 0 \quad (22)$$

Expanding the Δ and summing the imaginary terms yields:

$$\left[X_1 + X_2 + X_3\right]\left[(r_b r_c + r_b r_e + r_e r_c - r_b r_m) - (X_2 + X_b)(X_c + X_e + X_1) - X_e(X_1 + X_c)\right] + 2X_e X_1 X_2 + X_2^2(X_c + X_e + X_1) + X_1^2(X_b + X_e + X_2) = 0 \quad (23)$$

Following the same procedure as in the vacuum tube analysis,

(23) can be broken down into two equations:

$$\left[X_1 + X_2 + X_3\right]\left[\left(r_b r_c + r_b r_e + r_e r_c - r_b r_m\right) - \left(X_2 + X_b\right)\left(X_c + X_e + X_1\right) - X_e\left(X_1 + X_c\right)\right] = 0 \qquad (24)$$

$$2X_e X_1 X_2 + X_2^2\left(X_c + X_e + X_1\right) + X_1^2\left(X_b + X_e + X_2\right) = 0 \qquad (25)$$

To satisfy (24), $(X_1 + X_2 + X_3)$ must equal zero, thus yielding the frequency of oscillation which is independent of variations of the transistor parameters:

$$\omega = \sqrt{\frac{C_1 + C_2}{L_4 (C_1 C_2)}} \qquad (26)$$

To solve for Xc, setting $X_e = X_b = 0$ in (25) yields:

$$X_c = -\frac{X_1 X_2^2 + X_2 X_1^2}{X_2^2} = -\frac{X_1}{X_2}\left(X_1 + X_2\right) \qquad (27)$$

$$X_c = \omega L_c \qquad (28)$$

$$L_c = \frac{C_2}{\omega^2 C_1}\left(\frac{C_1 + C_2}{C_1 C_2}\right) = \frac{C_2}{\frac{(C_1 + C_2) C_1}{L_4 C_1 C_2}}\left(\frac{C_1 + C_2}{C_1 C_2}\right) \qquad (29)$$

$$L_c = \frac{L_4 C_2}{C_1} \qquad (30)$$

To solve for Xb, setting $X_c = X_e = 0$ in (25) yields:

$$X_b = -\frac{X_2}{X_1}(X_1 + X_2) = \frac{C_1}{\omega C_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \quad (31)$$

$$X_b = \omega L_b \quad (32)$$

$$L_b = \frac{C_1}{\omega^2 C_2}\left(\frac{C_1 + C_2}{C_1 C_2}\right) = \frac{C_1}{\frac{(C_1+C_2)C_2}{L_4 C_1 C_2}}\left(\frac{C_1 + C_2}{C_1 C_2}\right) \quad (33)$$

$$L_b = \frac{L_4 C_1}{C_2} \quad (34)$$

And, finally, to solve for Xe, setting Xc = Xb = 0 in (25) yields:

$$X_e = \frac{-X_2 X_1^2 - X_1 X_2^2}{(X_1 + X_2)^2} = \frac{-X_1 X_2}{(X_1 + X_2)} \quad (35)$$

$$X_e = \omega L_e \quad (36)$$

$$L_e = \frac{\frac{1}{C_1 C_2}}{\omega^2\left(\frac{C_1 + C_2}{C_1 C_2}\right)} = \frac{\frac{1}{C_1 C_2}}{\frac{C_1 + C_2}{L_4 C_1 C_2}\left(\frac{C_1 + C_2}{C_1 C_2}\right)} \quad (37)$$

$$L_e = \frac{L_4 C_1 C_2}{(C_1 + C_2)^2} \quad (38)$$

This completes the linear analysis of the Colpitts oscillator. Summarizing briefly, the $\Delta = 0$ method of analysis provides a straightforward approach for solving for the stabilized frequency of oscillation and for the various stabilizing impedances in both the vacuum tube and the transistor oscillators. Also worthy of note is the fact that, for the Colpitts oscillator, the plate-stabilizing impedance in the vacuum tube oscillator and the collector-stabilizing impedance in the transistor oscillator are identical, as are the grid-stabilizing and base-stabilizing impedances. In addition, the stabilized frequency takes the form

$$\omega = \sqrt{\frac{1}{L_{TOTAL}\, C_{TOTAL}}} \qquad (39)$$

where L_{total} and C_{total} are the sum of the inductances and the capacitances, series connected, in the tank loop. The significance of these observations will be shown in the next chapter.

III. Circuit and Expression Synthesis

Using the $\Delta=0$ method of linear analysis detailed in the previous chapter, a general oscillator circuit and expressions for the stabilized frequency and the various stabilizing impedances will now be derived.

Circuit Synthesis

The purpose of inserting a stabilizing impedance in an oscillating circuit is to make the frequency of oscillation independent of the tube or transistor parameters. Since the derivation of expressions for the stabilized frequency and the stabilizing impedances is of primary interest, it seems logical to adopt the "black box" concept and replace the vacuum tube or transistor in the circuit with a box labeled "device". It must be further stipulated that all the tube or transistor parameters are included within this box, and, since they are eliminated in the final analysis, need not be considered further. Adopting the "black box" concept not only eliminates the difficulty of accounting for the different parameters of the vacuum tube and transistor, but also provides an expedient means of designing a circuit which, used as a vehicle for linear analysis, is theoretically capable of containing either a vacuum tube or a transistor.

Boxes can also be included to represent the required stabilizing impedances. And, finally, a box can be included to represent the frequency-determining part of the oscillator, or that part of the circuit which distinguishes one oscillator configuration from the other.

Basically, then, the general oscillator circuit will take the form shown in Fig. 5.

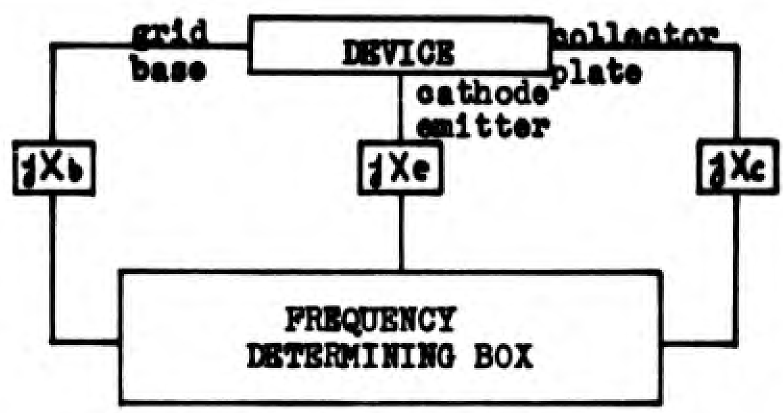

Figure 5

General Oscillator Circuit

The expressions derived later in this chapter will determine what is included in the stabilizing impedance (X) boxes. At this point, the remaining step is to determine what elements must be included in the frequency determining box.

This can be accomplished by comparing the circuits of six different oscillator configurations--tuned-plate, tuned-grid, Hartley, Colpitts, Clapp and Meissner--hereafter referred to as the six synthesizing configurations. The frequency-determining box must contain a network such that each of these six configurations can be made to "fit" by including some network elements and excluding others. This requirement is fulfilled by the network shown in Fig. 6.

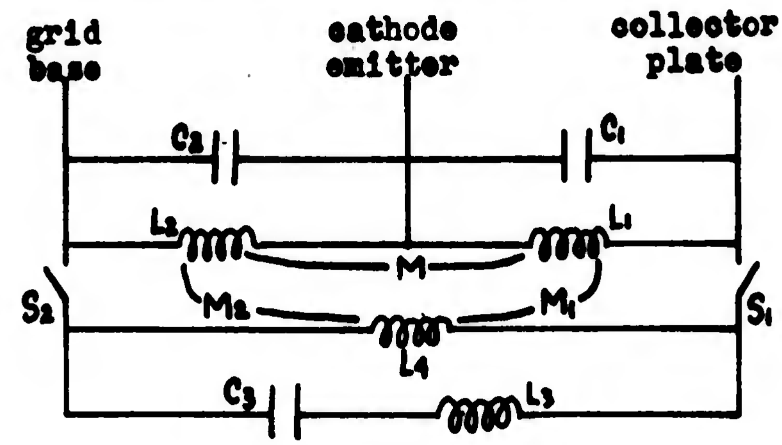

Figure 6

Frequency Determining Network

The elements in the above figure are numbered such that the subscript "1" signifies elements in the plate (collector) circuit, the subscript "2" elements in the grid (base) circuit, and the subscripts "3" and "4" elements common to both. This subscript system will be followed throughout, and was also followed in the linear analysis of the Colpitts oscillator in the previous chapter.

The two switches, S_1 and S_2, are included to cover the Meissner oscillator. These two switches are closed at all times except when considering the Meissner oscillator.

The final form of the general oscillator circuit can be derived by replacing the frequency-determining box in

Fig. 5 by the network shown in Fig. 6. This results in the circuit shown in Fig. 7.

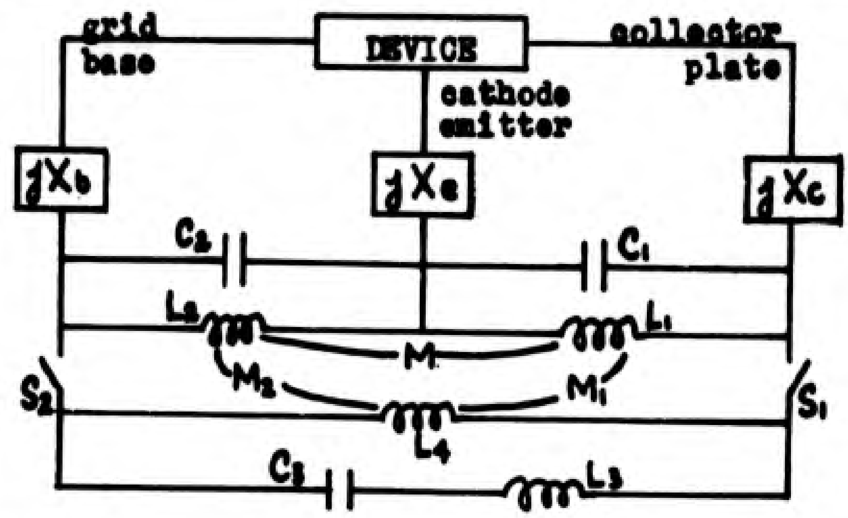

Figure 7

General Oscillator Circuit
With Frequency-Determining Network

The Hartley oscillator, as an example, would be derived from Fig. 7 by including L_1, L_2, M and C_3 and excluding all other elements. The Meissner oscillator would include L_1, L_2, M_1, M_2, C_3 and L_4, switches S_1 and S_2 would be open, and all other elements would be excluded.

Derivation of the Expression for Stabilized Frequency

Using the general oscillator circuit of Fig. 7 and the Δ=0 method of analysis, expressions for the stabilized frequency for each of the six synthesizing configurations were found and are tabulated in Table I on the next page.

TABLE I

Stabilized Frequencies (ω_{STAB}) for Six Oscillator Configurations

OSCILLATOR	STABILIZED FREQUENCY
Tuned-Plate	$\sqrt{\frac{1}{L_1C_1}}$
Tuned-Grid	$\sqrt{\frac{1}{L_2C_2}}$
Hartley	$\sqrt{\frac{1}{C_3(L_1+L_2+2M)}}$
Meissner	$\sqrt{\frac{1}{L_4C_3}}$
Colpitts	$\sqrt{\frac{C_1+C_2}{L_4(C_1C_2)}}$
Clapp	$\sqrt{\frac{1}{L_5(1/C_1+1/C_2+1/C_3)}}$

The linear equivalent circuits and the Δ determinants used in the analysis of the six vacuum tube configurations and the six transistor configurations are found in Appendices A and B respectively.

Comparing the frequency expressions in Table I, it can be seen that, in each case, ω assumes the general form

$$\omega_{STAB} = \sqrt{\frac{1}{L_{TOTAL}\,C_{TOTAL}}} \tag{40}$$

where L_{total} and C_{total} are the sums of the inductances and capacitances, series connected, in the loop of the frequency determining tank. For example, in the tuned-plate oscillator, considering the plate tank loop as the frequency-determining loop, L_{total} is simply L_1 and C_{total} is C_1.

Thus the following four-step approach provides a means of deriving the expression for the stabilized frequency without actually performing the complete $\Delta = 0$ analysis:

1. Starting with the general oscillator circuit of Fig. 7, eliminate those elements not needed in the desired configuration.

2. Determine the frequency-determining tank loop. This loop will be the one which does not contain a stabilizing impedance element.

3. Take the sum of the inductances and capacitances in this loop.

4. Substitute the expressions for L_{total} and C_{total} calculated in Step 3 in equation (40) to obtain ω_{stab}.

As was seen in the detailed analysis of the Colpitts oscillator in Chapter II, ω_{stab} must be known, or calculated before any expression for a stabilizing impedance can be calculated. Therefore, the detailed analysis of an oscillator can be expedited by the application of the above approach.

Derivation of Expressions for Stabilizing Impedances

The expressions for the various stabilizing impedances for the six synthesizing configurations, found by the $\Delta = 0$ analysis, are tabulated in Tables II and III on the following page.

As stated in the introduction, the method of deriving

TABLE II

Plate- and Grid-Stabilizing Impedances for Six Vacuum-Tube Oscillator Configurations

OSCILLATOR	STABILIZING IMPEDANCE	
	PLATE	GRID
Tuned-Plate	$C_c = \frac{C_1 M^2}{L_1 L_2 - M^2}$	$C_b = \frac{C_1 L_1^2}{L_1 L_2 - M^2}$
Tuned-Grid	$C_c = \frac{C_2 L_2^2}{L_1 L_2 - M^2}$	$C_b = \frac{C_2 M^2}{L_1 L_2 - M^2}$
Hartley	$C_c = \frac{C_3 (L_2 + M)^2}{L_1 L_2 - M^2}$	$C_b = \frac{C_3 (L_1 + M)^2}{L_1 L_2 - M^2}$
Meissner	$C_c = \frac{L_3 C_3 M_2^2}{L_2 M_1^2 + L_1 M_2^2}$	$C_b = \frac{L_3 C_3 M_1^2}{L_2 M_1^2 + L_1 M_2^2}$
Colpitts	$L_c = \frac{L_4 C_2}{C_1}$	$L_b = \frac{L_4 C_1}{C_2}$
Clapp	$L_c = \frac{L_3 C_2}{C_1}\left[\frac{1}{1 + \frac{C_1 C_2}{C_3 (C_1 + C_2)}}\right]$	$L_b = \frac{L_3 C_1}{C_2}\left[\frac{1}{1 + \frac{C_1 C_2}{C_3 (C_1 + C_2)}}\right]$

TABLE III

Collector-, Base- and Emitter-Stabilizing Impedances for Six Transistor-Oscillator Configurations

OSCILLATOR	STABILIZING IMPEDANCE		
	COLLECTOR	BASE	EMITTER
Tuned-Collector	$C_c = \frac{C_1 M^2}{L_1 L_2 - M^2}$	$C_b = \frac{C_1 L_1^2}{L_1 L_2 - M^2}$	$C_e = \frac{C_1 (L_1 - M)^2}{L_1 L_2 - M^2}$
Tuned-Base	$C_c = \frac{C_2 L_2^2}{L_1 L_2 - M^2}$	$C_b = \frac{C_2 M^2}{L_1 L_2 - M^2}$	$C_e = \frac{C_2 (L_2 - M)^2}{L_1 L_2 - M^2}$
Hartley	$C_c = \frac{C_3 (L_2 + M)^2}{L_1 L_2 - M^2}$	$C_b = \frac{C_3 (L_1 + M)^2}{L_1 L_2 - M^2}$	$C_e = \frac{C_3 (L_1 + L_2 + 2M)^2}{L_1 L_2 - M^2}$
Meissner	$C_c = \frac{L_3 C_3 M_2^2}{L_2 M_1^2 + L_1 M_2^2}$	$C_b = \frac{L_3 C_3 M_1^2}{L_2 M_1^2 + L_1 M_2^2}$	$C_e = \frac{L_3 C_3 (M_2 - M_1)^2}{L_2 M_1^2 + L_1 M_2^2}$
Colpitts	$L_c = \frac{L_4 C_2}{C_1}$	$L_b = \frac{L_4 C_1}{C_2}$	$L_e = \frac{L_4 C_1 C_2}{(C_1 + C_2)^2}$
Clapp	$L_c = \frac{L_3 C_2}{C_1\left[1 + \frac{C_1 C_2}{C_3 (C_1 + C_2)}\right]}$	$L_b = \frac{L_3 C_1}{C_2\left[1 + \frac{C_1 C_2}{C_3 (C_1 + C_2)}\right]}$	$L_e = \frac{L_3 C_1 C_2}{(C_1 + C_2)^2\left[1 + \frac{C_1 C_2}{C_3 (C_1 + C_2)}\right]}$

a single expression for each stabilizing impedance--plate, grid, emitter, etc.-- is by inductive reasoning. This process entails comparing the individual impedance expressions to find some common factor, or group of factors, which could be grouped together to form a composite expression. Upon comparing the expressions in Tables II and III, three facts become apparent:

1. The expressions for the plate- and grid-stabilizing impedances for the six vacuum tube circuits and the collector- and base-stabilizing impedances for the six transistor circuits are, respectively, identical. This similarity comes as no surprise because the complete analysis of the Colpitts oscillator in Chapter II showed these impedances to be identical for that particular configuration.

2. Three of the stabilizing impedances in each group share a common factor in the denominator, namely $L_1L_2-M^2$.

3. With the exception of the Clapp and Colpitts oscillators, all other stabilizing impedances are capacitive.

The conclusions which can be drawn from these facts are as follows: first, because of the similarity of the plate-collector and grid-base expressions, only three composite expressions will be required--plate-collector, grid-base and emitter; second, each composite expression, in order to fit all six oscillator configurations, must be composed of an inductive part and a capacitive part; and third, the three expressions containing the common factor can be grouped together to form a single term.

Based on the first conclusion, then, let the three composite expressions be defined arbitrarily as Xc, Xb and Xe, representing the plate-collector, grid-base and emitter-stabilizing impedances respectively. Each of these, in turn, must further be broken down into capacitive and inductive elements, i.e.

$$X_c = C_c + L_c \tag{41}$$

$$X_b = C_b + L_b \tag{42}$$

$$X_e = C_e + L_e \tag{43}$$

where C and L represent the capacitive and inductive parts of each expression.

As an example of the expression derivation, consider Xc, the plate-collector impedance. Grouping together the three expressions with the common factor and solving for Cc and Lc yields:

$$C_c = \frac{C_1 M^2 + C_2 L_2^2 + C_3 (L_1 + M)^2}{L_1 L_2 - M^2} + \frac{L_4 C_3 M_2^2}{L_2 M_1^2 + L_1 M_2^2} \tag{44}$$

$$L_c = \frac{L_4 C_2}{C_1} + \frac{L_3 C_2}{C_1}\left[\frac{1}{1 + \frac{C_1 C_2}{C_3 (C_1 + C_2)}}\right] \tag{45}$$

Modifying the expression for Cc by substituting $K = M/\sqrt{L_1 L_2}$

yields:

$$C_c = \frac{\frac{L_2}{L_1}\left[\frac{C_1K^2L_1}{L_2} + C_2 + C_3\left(1+K\sqrt{\frac{L_1}{L_2}}\right)^2\right]}{1-K^2} + \frac{L_4C_3M_2^2}{L_2M_1^2+L_1M_2^2} \tag{46}$$

Substituting (45) and (46) in (41) yields:

$$X_c = \frac{\frac{L_2}{L_1}\left[\frac{C_1K^2L_1}{L_2} + C_2 + C_3\left(1+K\sqrt{\frac{L_1}{L_2}}\right)^2\right]}{1-K^2} + \frac{L_4C_3M_2^2}{L_2M_1^2+L_1M_2^2} + \frac{L_4C_2}{C_1} + \frac{L_3C_2}{C_1}\left[\frac{1}{1+\frac{C_1C_2}{C_3(C_1+C_2)}}\right] \tag{47}$$

Equation (47) would seem to be a general expression for the plate-collector stabilizing impedance. The problem which now arises is one of interpretation. How is an expression for the plate (collector) stabilizing impedance for one particular oscillator configuration extracted from the general expression in (47)?

The rules for the interpretation of the general expression are propounded as follows:

1. Starting with the general oscillator circuit in Fig. 7, derive the circuit for the oscillator configuration desired by including those elements necessary to complete the circuit and excluding all others.

2. Referring to the expression for Xc in (47), consider

each term involved. If the elements in a term appear in the oscillator circuit, the term is retained. However, if <u>any</u> element in a term is <u>not</u> contained in the oscillator configuration, the term is omitted. The same can be said for individual factors comprising a single term. The sum of the terms retained is the expression for Xc.

3. Xc is capacitive or inductive depending upon whether the terms retained are part of Cc or Lc.

As an example, consider the Colpitts oscillator already analyzed in Chapter II. Following the three rules of interpretation yields:

1. The Colpitts configuration would contain only C_1, C_2 and L_4. All other elements would be eliminated.

2. Referring to the general expression for Xc, because the first term contains L_2/L_1 as a factor, it is eliminated. Because the second term contains $C_3M_2^2$ as a factor, it is eliminated. Because the fourth term contains L_3 as a factor, it too is eliminated. The only remaining term is L_4C_2/C_1.

3. Because the remaining term is part of Lc, the expression for the plate(collector) stabilizing impedance for a Colpitts oscillator is

$$Lc = L_4C_2/C_1 \tag{48}$$

which agrees with the calculated value in (14) and (30).

As another example, consider the Meissner oscillator. Again following the rules of interpretation yields:

1. The Meissner oscillator would contain elements L_1, L_2, M_1, M_2, L_4 and C_3. All other elements would be eliminated.

2. Referring to the expression for Xc, repeating the factor elimination process, and summing the retained terms yields:

$$X_c \doteq C_c = \frac{L_2 C_3}{L_1} + \frac{L_4 C_3 M_2^2}{L_1 M_2^2 + L_2 M_1^2} \qquad (49)$$

However, comparing this expression with the ones contained in Tables II and III indicated that (49) is not the expression for the plate(collector) stabilizing impedance for a Meissner oscillator. Therefore the expression for Xc, as stated in (47), is incorrect. But it was impossible to foresee that (47) was incorrect until some rules for interpreting this equation were set forth. With the exception of the Meissner case, the expression for Xc as stated in (47) holds for the other five synthesizing configurations. To include the Meissner oscillator, then, it is necessary to revise the second term of (47), $(L_4C_3M_2^2)/(L_1M_2^2 + L_2M_1^2)$, so that, when the revised term is added to C_3L_2/L_1, the resulting term will be the correct expression for the plate (collector) stabilizing impedance.

Performing this revision changes the expression for Cc from the one expressed in (46) to the one expressed in (50). The numerator and denominator of (50) have been left in fractional form to facilitate addition.

$$C_c = \frac{\frac{L_2}{L_1}\left[\frac{C_1K^2L_1}{L_2} + C_2 + C_3\left(1 + K\sqrt{\frac{L_1}{L_2}}\right)^2\right]}{1-K^2} + \frac{\left[\frac{L_4C_3M_2^2}{L_1} - \frac{L_2^2C_3M_1^2}{L_1^2} - \frac{L_2C_3M_2^2}{L_1}\right]}{\frac{M_1^2L_2}{L_1} + M_2^2} \tag{50}$$

Now, substituting (45) and (50) into (41) yields the correct expression for X_c, i.e.,

$$X_c = C_c + L_c = \frac{\frac{L_2}{L_1}\left[\frac{C_1K^2L_1}{L_2} + C_2 + C_3\left(1 + K\sqrt{\frac{L_1}{L_2}}\right)^2\right]}{1-K^2} + \frac{\left[\frac{L_4C_3M_2^2}{L_1} - \frac{L_2^2C_3M_1^2}{L_1^2} - \frac{L_2C_3M_2^2}{L_1}\right]}{\frac{M_1^2L_2}{L_1} + M_2^2} + \frac{L_4C_2}{C_1} + \frac{L_3C_2}{C_1}\left[\frac{1}{1 + \frac{C_1C_2}{C_3(C_1+C_2)}}\right] \tag{51}$$

Expressions for X_b and X_e, similarly derived, are:

$$X_b = C_b + L_b = \frac{\frac{L_1}{L_2}\left[\frac{C_2K^2L_2}{L_1} + C_1 + C_3\left(1 + K\sqrt{\frac{L_2}{L_1}}\right)^2\right]}{1-K^2} + \frac{\left[\frac{L_4C_3M_1^2}{L_2} - \frac{L_1^2C_3M_2^2}{L_2^2} - \frac{L_1C_3M_1^2}{L_2}\right]}{\frac{M_2^2L_1}{L_2} + M_1^2} + \frac{L_4C_1}{C_2} + \frac{L_3C_1}{C_2}\left[\frac{1}{1 + \frac{C_1C_2}{C_3(C_1+C_2)}}\right] \tag{52}$$

$$X_e = C_e + L_e = \frac{\left[\frac{L_2C_2}{L_1}\left(1-K\sqrt{\frac{L_1}{L_2}}\right)^2 + \frac{L_1C_1}{L_2}\left(1-K\sqrt{\frac{L_2}{L_1}}\right)^2 + \frac{C_3}{L_1L_2}\left(L_1+L_2+2M\right)^2\right]}{1-K^2} +$$

$$\frac{\left[\frac{L_4C_3(M_2-M_1)^2}{L_1L_2} - \frac{C_3M_2^2(L_1+L_2)^2}{L_1L_2^2} - \frac{C_3M_1^2(L_1+L_2)^2}{L_2L_1^2}\right]}{\frac{M_2^2}{L_2} + \frac{M_1^2}{L_1}} +$$

$$\frac{L_4C_1C_2}{(C_1+C_2)^2} + \frac{L_3C_1C_2}{(C_1+C_2)^2}\left[\frac{1}{1+\frac{C_1C_2}{C_3(C_1+C_2)}}\right] \qquad (53)$$

As a guide in applying the rules of interpretation, the following table gives the elements included in the six configurations used in synthesizing (51) through (53).

TABLE IV

Elements Included In Six Oscillator Configurations

OSCILLATOR	ELEMENTS
Tuned-Plate	L_1 L_2 C_1 M
Tuned-Grid	L_1 L_2 C_2 M
Hartley	L_1 L_2 C_3 M
Meissner	L_1 L_2 L_4 C_3 M_1 M_2
Colpitts	C_1 C_2 L_4
Clapp	C_1 C_2 C_3 L_3

In summary, the general oscillator circuit of Fig. 7 is synthesized by including a device box, which contains all device parameters, three stabilizing impedance boxes and a frequency-determining, or configuration-determining, network. Then an approach is set forth for determining the stabilized frequency of oscillation without performing the complete $\Delta=0$ analysis. Finally, the inductive process used in deriving the three general expressions for stabilizing impedances is explained, the expressions are derived, rules of interpretation for extracting any desired stabilizing impedance for any desired oscillator configuration are set forth, and two examples of applying these rules are given.

IV. Test Cases

The question which now arises is "Do the general expressions for the three stabilizing impedances hold for oscillator configurations other than those used in their synthesis?" The purpose of this chapter is to test the expressions. The unity-coupled, electron-coupled and tuned-plate tuned-grid oscillators are the three configurations which will be used as test cases.

Unity-coupled Oscillator

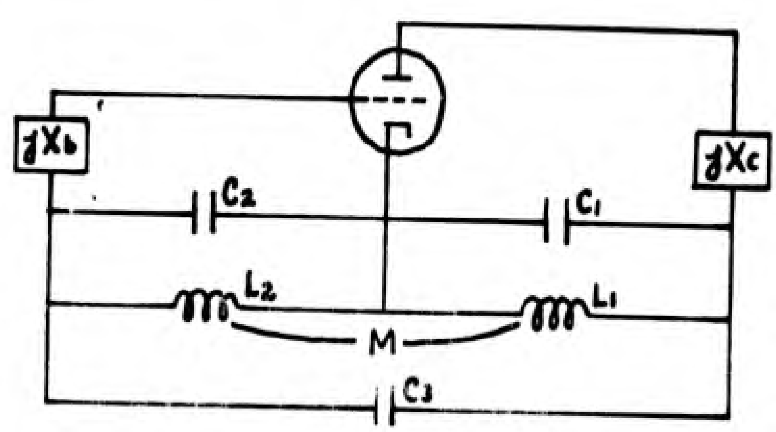

Figure 8

Unity-Coupled Vacuum Tube Oscillator

Fig. 8 shows the circuit for the unity-coupled vacuum tube oscillator. As was the case for the six synthesizing configurations used in the previous chapter, the linear equivalent circuits and the Δ determinants for the three oscillator configurations tested in this chapter appear in the

Appendix (Appendix C).

Both Llewellyn (Ref 14:2079-2082) and Terman (Ref 29:487) have analyzed this particular configuration. They have derived expressions for the plate- and grid-stabilizing impedances. They are:

$$X_c = C_c = \frac{\frac{L_2}{L_1}\left[\frac{C_1K^2L_1}{L_2} + C_2 + C_3\left(1+K\sqrt{\frac{L_1}{L_2}}\right)^2\right]}{1-K^2} \qquad (54)$$

$$X_b = C_b = \frac{\frac{L_1}{L_2}\left[\frac{C_2K^2L_2}{L_1} + C_1 + C_3\left(1+K\sqrt{\frac{L_2}{L_1}}\right)^2\right]}{1-K^2} \qquad (55)$$

By referring to equations (51) and (52), these two impedance expressions for the unity-coupled oscillator can be extracted exactly by retaining the entire first term of each of the general expressions and omitting the last three terms of each by virtue of the fact that M_1, M_2, L_3 and L_4 are not contained in the circuit. Thus the general expressions hold for this configuration.

Electron-coupled Oscillator

This oscillator, shown in Fig. 9 on the next page, may at first seem unsuited for adaptation in the general oscillator circuit of Fig. 7, and hence invalidate the general expressions in (51) through (53), because a multi-grid tube is used in place of the conventional triode. However, both Terman's (Ref 29:481) and Edson's (Ref 8:178) dis-

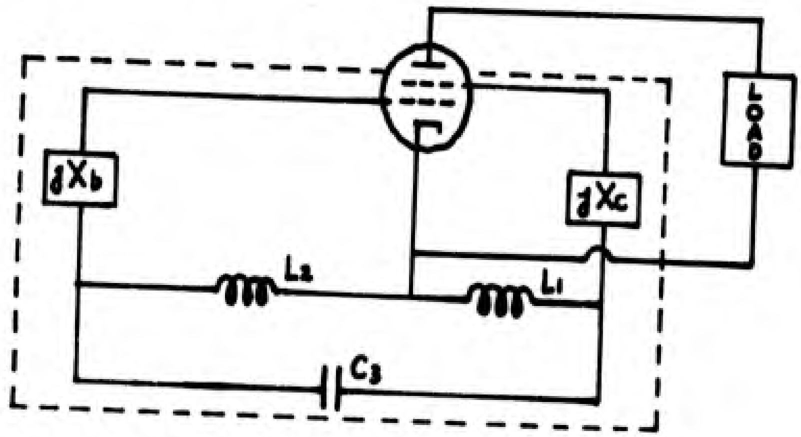

Figure 9

Electron-Coupled Vacuum Tube Oscillator

cussions of this particular configuration point out the fact that only part of the tube, and circuit, is used as the oscillator:

"In the electron-coupled circuit, the cathode, grid, and screen grid of a screen grid tube are operated as a triode oscillator, with the screen serving as the anode. Only a small fraction of the space current is intercepted by the screen, but the oscillator circuits are so designed that this will maintain the oscillations properly. The remaining electrons, which represent most of the space current, go on to the plate and produce power output by flowing through the load impedance that is connected in series with the plate electrode." (Ref 29:481)

Thus, in Fig. 9, the portion of the complete circuit enclosed by the dotted lines represents the oscillator part of the electron-coupled oscillator. As can be seen by referring to Fig. C-2 in Appendix C, this oscillator is quite similar to the Hartley oscillator with M set equal to zero. However, it differs from the conventional triode oscillator in that r_c replaces r_p, and an equivalent μ and Eg are used. By performing the $\Delta = 0$ analysis, the following relationships are found:

$$\omega_{STAB} = \sqrt{\frac{1}{C_3(L_2 + L_1)}} \tag{56}$$

$$X_c = C_c = \frac{L_2 C_3}{L_1} \tag{57}$$

$$X_b = C_b = \frac{L_1 C_3}{L_2} \tag{58}$$

The L_{total} and C_{total} of the stabilized frequency expression are indeed $(L_2 + L_1)$ and C_3, respectively. Applying the rules of interpretation to the general expressions of (51) and (52), with C_1, C_2, M, M_1, M_2, L_3 and L_4 excluded, the only terms retained yield:

$$X_c = C_c = \frac{L_2 C_3}{L_1} \tag{59}$$

$$X_b = C_b = \frac{L_1 C_3}{L_2} \tag{60}$$

These agree exactly with the calculated values expressed in equations (57) and (58).

Tuned-Plate Tuned-Grid Oscillator

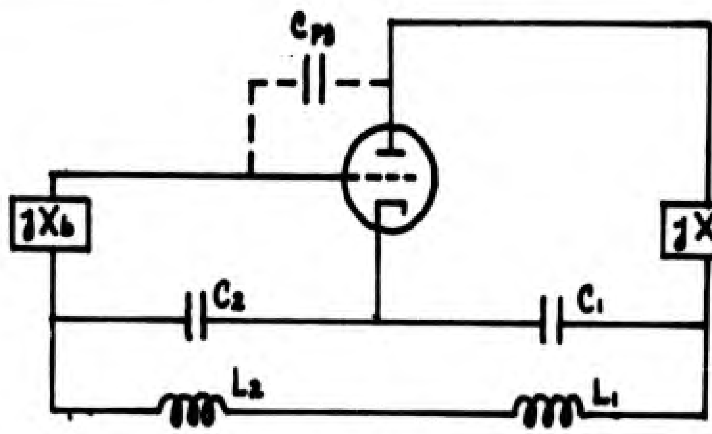

Figure 10

Tuned-Plate Tuned-Grid
Vacuum Tube Oscillator

The actual circuit of a tuned-plate tuned-grid oscillator is shown in Fig. 10. However, for oscillation to occur, both the plate and grid tanks are adjusted so as to offer inductive reactance at the frequency to be generated (Ref. 29:481). Therefore, L_1 and C_1 become an equivalent inductance L_{1eq}, and L_2 and C_2 become L_{2eq}. The linear equivalent circuit, Fig. C-3, takes on the form of a modified Hartley oscillator, with L_{1eq} replacing L_1, L_{2eq} replacing L_2, M=0, and C_3 representing Cpg. The analysis of this equivalent circuit, then, parallels that of the Hartley oscillator, or, for that matter, that of the electron-coupled oscillator. The results of this analysis are:

$$\omega_{STAB} = \sqrt{\frac{1}{C_3(L_{1eq} + L_{2eq})}} \tag{61}$$

$$X_c = C_c = \frac{C_3 L_{2eq}}{L_{1eq}} \tag{62}$$

$$X_b = C_b = \frac{C_3 L_{1eq}}{L_{2eq}} \tag{63}$$

It would seem that the general impedance expressions would not hold for this oscillator because of the absence of both L_{1eq} and L_{2eq}, per se, from any of the terms comprising the expressions. However, a reexamination of the $\Delta = 0$ method of linear analysis in general, and its application to this oscillator in particular, results in the following conclusions:

1. All expressions for the stabilizing impedances are calculated from the $\Delta = 0$ analysis of a linear equivalent circuit. Hence the general expressions for these impedances are derived from equivalent circuit analysis.

2. Of all the oscillators considered thus far, the tuned-plate tuned-grid is the only one in which the actual elements contained in the oscillator circuit do not appear in their specific form in the linear equivalent circuit, e.g., L_1 and C_1 do not appear specifically in the linear equivalent circuit, but are incorporated in a "fictitious"

element labeled L_{1eq}; and it is this element which is found in the linear equivalent circuit.

3. Heretofore, all elements appearing in the general expressions for the stabilizing impedances have represented actual elements appearing in a particular oscillator configuration. However, should an element labeled, for example, L_1 appear in an equivalent circuit, it would be treated the same in the analysis of that circuit whether it represent an actual inductance or an equivalent inductance.

Based on these conclusions, the tuned-plate tuned-grid oscillator must be analyzed in a slightly different manner, i.e., instead of applying the rules of interpretation to the actual circuit model, they must be applied to the linear equivalent circuit. Actually, all other circuits could have had the rules applied equally well to their equivalent circuit. The reason the actual circuits were chosen was to facilitate the task of the analyzer in establishing a desired configuration by eliminating the necessity of converting the actual circuit to an equivalent circuit.

Applying, then, the rules of interpretation to the linear equivalent circuit of the tuned-plate tuned-grid oscillator (Fig. C-3, Appendix C), it is seen that the only elements now retained are L_1, representing L_{1eq}, L_2, representing L_{2eq}, and C_3 (C_{pg}). As in the case of the electron-coupled oscillator, C_1, C_2, M, M_1, M_2, L_3 and L_4 are excluded. Therefore the only remaining terms yield:

$$X_c = C_c = \frac{C_3 L_2}{L_1} \tag{64}$$

$$X_b = C_b = \frac{C_3 L_1}{L_2} \tag{65}$$

where

$$L_1 = L_{1eq} = \frac{L_1 (C_1 + C_3)}{C_3} \tag{66}$$

$$L_2 = L_{2eq} = \frac{L_2 (C_2 + C_3)}{C_3} \tag{67}$$

The derivation of the expressions for L_{1eq} and L_{2eq} is included in Appendix C.

By comparing (62) and (63) with (64) and (65), it can be seen that, by applying the rules of interpretation to the linear equivalent circuit of the tuned-plate tuned-grid oscillator, the correct expressions for the plate- and grid-stabilizing impedances can be extracted from the general expressions in (51) and (52) respectively.

Summarizing, the general expressions for the stabilizing impedances were tested on three oscillator configurations not used in their synthesis. In each case the correct expressions for the stabilizing impedances were obtained, even though, in two of the three cases, the oscillator configuration varied slightly from the conventional configuration.

V. Summary and Conclusions

First, a general oscillator circuit was developed to serve as a tool for linear analysis. By using a combination of the "black box" concept and a frequency-determining network, the general circuit, shown in final form in Fig. 7, was made compatible to either a vacuum tube or transistor oscillator. It follows that expressions derived from this circuit would also hold for either type oscillator.

Deriving general expressions for the stabilized frequency of oscillation and for the plate-collector, grid-base and emitter-stabilizing impedances was the next step. Specific expressions calculated from individual analyses of six different oscillator configurations were tabulated and compared. General expressions were derived by inductive reasoning, modified as a result of applying rules of interpretation, and expressed in final form in equations (51) through (53).

Then the general expressions were checked for validity by applying them to three "test" oscillators. In each case the stabilizing impedances extracted from the general expressions were compared to values calculated by detailed analysis and found to be identical, thereby proving the validity of both the rules of interpretation and the general expressions themselves.

As far as is known, the known vacuum tube or transistor L-C feedback oscillators are identical to, or can be derived from, the nine oscillator configurations analyzed in this report. Therefore it is concluded that the general expressions for Xc, Xb and Xe derived in Chapter III are applicable to all L-C feedback vacuum tube and transistor oscillators of common-cathode and common-emitter configuration respectively. Thus a valid unified approach to impedance stabilization has been established.

It is felt that, by eliminating the arithmetic manipulations associated with detailed, individual linear analysis, the unified approach is a superior method of determining values for stabilizing impedances. This superiority is not based on the criterion of accuracy, for the results obtained by either method are identical. However, the unified approach is superior when the oscillator circuit becomes rather complex, and the excessive time element involved in obtaining results by linear analysis is substantially reduced.

Bibliography

1. Arguimbau, L.B. Vacuum Tube Circuits. New York: John Wiley and Sons, Inc., 1948.

2. Boone, E.M. Circuit Theory of Electron Devices. New York: John Wiley and Sons, Inc., 1953.

3. Carroll, J.M. Modern Transistor Circuits. New York: McGraw-Hill Book Co. Inc., 1959.

4. Clifford, H.E. and A.H. Wing (Ed.) Electronic Circuits and Tubes. New York: McGraw-Hill Book Co. Inc., 1947.

5. Coblenz, A. Transistors: Theory and Application. New York: McGraw-Hill Book Co. Inc., 1955.

6. DeWitt, D. Transistor Electronics. New York: McGraw-Hill Book Co. Inc., 1957.

7. Eastman, A.V. Fundamentals of Vacuum Tubes. New York: McGraw-Hill Book Co. Inc., 1941.

8. Edson, W.A. Vacuum Tube Oscillators. New York: John Wiley and Sons, Inc., 1953.

9. Eltgroth, G.V. "Frequency Stability of Tuned Circuits." Electronics, 17:118 (Feb. 1944).

10. Fitchen, F.C. Transistor Circuit Analysis and Design. Princeton, New Jersey: Van Nostrand, 1960.

11. Garmash, E.N. "Analysis of Transistor Circuits." Radio Engineer, 13: No. 7, C2-C71 (1958).

12. Gibson, J.W., et al. A Study of Oscillating Circuits for Telemetering Systems. WADC Technical Report 54-408. Springfield, Ohio: Carpenter Litho and Prtg. Co., 1955.

13. Keonjian, E. "Stable Transistor Oscillator." Institute of Radio Engineers Transactions on Circuit Theory, CT-3: 38 (1956)

14. Llewellyn, F.B. "Constant Frequency Oscillators." Proceedings of the Institute of Radio Engineers, 20: 2063-2094 (1931).

15. Lurch, E.N. Fundamentals of Electronics. New York: John Wiley and Sons, Inc., 1960.

16. Martin, T.L. Electronic Circuits. New York: Prentice-Hall, Inc., 1955.

17. McSpadden, W.R. "Graphical Designing of Transistor Oscillators." Electronics, 31: 90-93 (Dec. 1958).

18. Milnes, A.G. "Transistor Circuits and Applications." Proceedings of the Institution of Electrical Engineers, 104 pt. B: 565-580 (Nov. 1957).

19. Oakes, J.B. "Transistor Oscillators." Proceedings of the Institute of Radio Engineers, 42: 1235 (1954).

20. Oser, J.E., et al. "Transistor Oscillators." RCA Review, 13-3: 369-385 (Sept. 1952).

21. Reich, H.J. Theory and Applications of Electron Tubes. New York: McGraw-Hill Book Co. Inc., 1944.

22. Richter, W. Fundamentals of Industrial Electronic Circuits. New York: McGraw-Hill Book Co. Inc., 1947.

23. Riddle, R.L. Transistor Physics and Circuits. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1958.

24. Rowley, C.S. A Standardized Procedure for the Linear Analysis of Oscillators by Matrix Methods. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, August, 1959.

25. Ryder, J.D. Electronic Fundamentals and Applications. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1959.

26. Shea, R.F. (Ed.) Transistor Circuit Engineering. New York: John Wiley and Sons, Inc., 1957.

27. Strauss, L. Wave Generation and Shaping. New York: McGraw-Hill Book Co. Inc., 1960.

28. Terman, F.E. Electronic and Radio Engineering. New York: McGraw-Hill Book Co. Inc., 1955.

29. Terman, F.E. Radio Engineers' Handbook. New York: McGraw-Hill Book Co. Inc., 1943.

30. Thomas, D.E. "Low-drain Transistor Audio Oscillators." Proceedings of the Institute of Radio Engineers, 40: 1385-1395 (Nov. 1952).

31. Turner, R.P. Transistors: Theory and Practice. New York: Gernsback Library, 1958.

32. Witt, S.N. Jr. "Designing Oscillators for Greater Stability." Electronics, 30: 180-182 (Nov. 1957).

APPENDIX A

Linear Equivalent Circuits and Δ Determinants for Vacuum Tube Oscillators Used in Expression Synthesis

Definition of Symbols

X_c = Plate-stabilizing Impedance

$X_g = X_b$ Grid-stabilizing Impedance

$X_1 = \omega L_1$ $X_{c1} = 1/\omega C_1$

$X_2 = \omega L_2$ $X_{c2} = 1/\omega C_2$

$X_3 = \omega L_3$ $X_{c3} = 1/\omega C_3$

$X_4 = \omega L_4$ $X_{M1} = \omega M_1$

$X_M = \omega M$ $X_{M2} = \omega M_2$

r_p = Plate Resistance

r_g = Grid Resistance

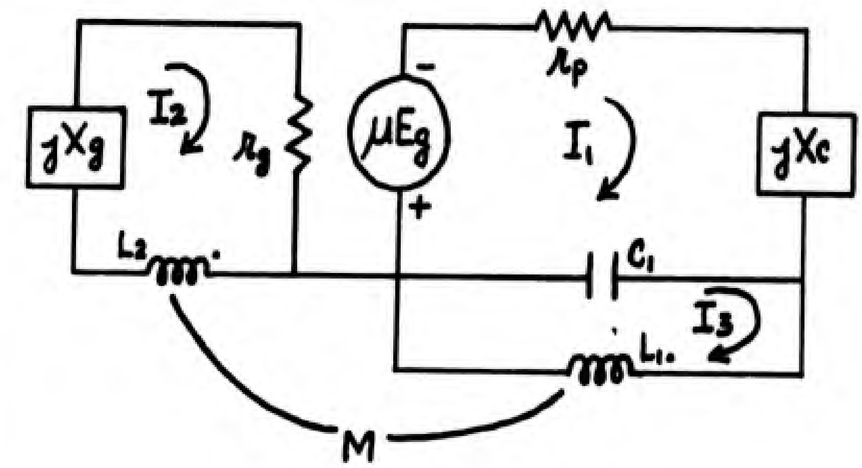

Figure A-1
Tuned-Plate Vacuum Tube Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} r_p + jX_c - jX_{c1} & \mu r_g & jX_{c1} \\ 0 & r_g + jX_2 + jX_g & jX_M \\ jX_{c1} & jX_M & jX_1 - jX_{c1} \end{vmatrix} = 0$$

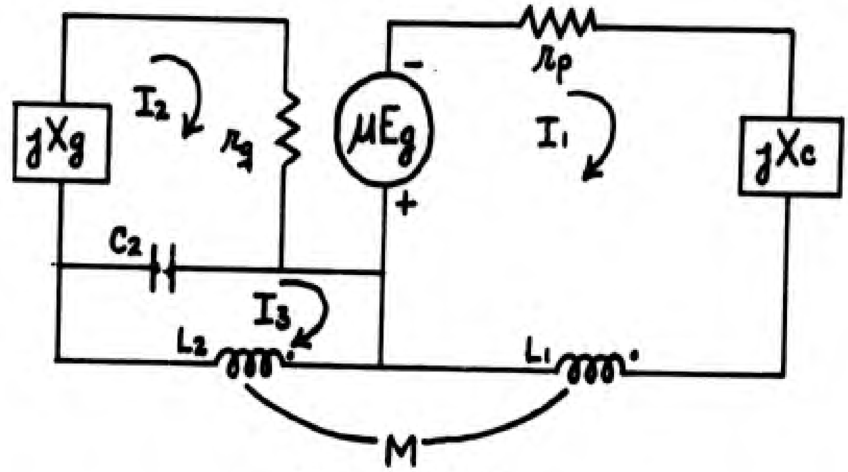

Figure A-2

Tuned-Grid Vacuum Tube Oscillator Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} r_p + jX_c + jX_1 & \mu r_g & jX_M \\ 0 & r_g + jX_g - jX_{c2} & jX_{c2} \\ jX_M & jX_{c2} & jX_2 - jX_{c2} \end{vmatrix} = 0$$

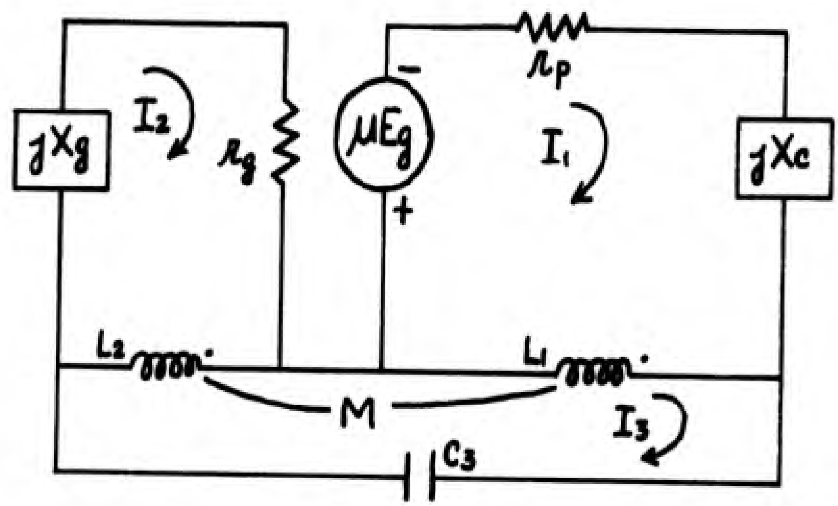

Figure A-3

Hartley Vacuum Tube Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} r_p + jX_c + jX_1 & \mu r_g + jX_M & -jX_1 - jX_M \\ jX_M & r_g + jX_2 + jX_g & -jX_2 - jX_M \\ -jX_1 - jX_M & -jX_2 - jX_M & jX_1 + jX_2 + jX_3 + j2X_M \end{vmatrix} = 0$$

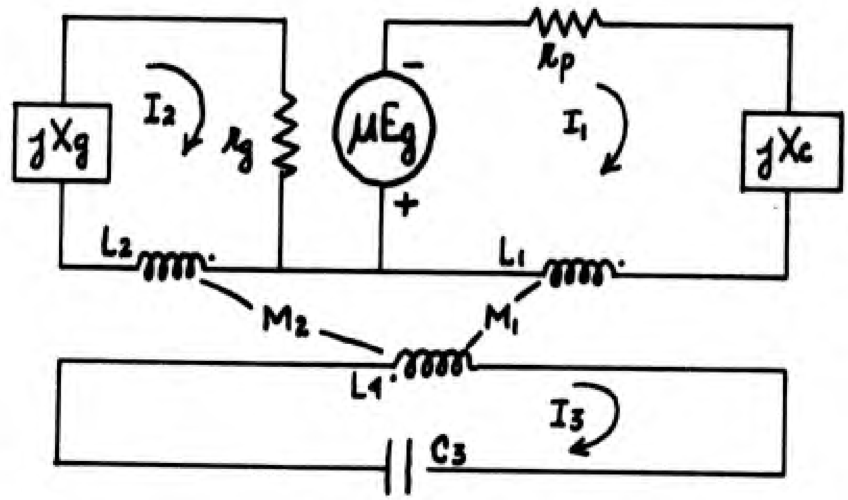

Figure A-4

Meissner Vacuum Tube Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} r_p + jX_1 + jX_c & +\mu r_g & jX_{M_1} \\ 0 & r_g + jX_g + jX_2 & jX_{M_2} \\ jX_{M_1} & jX_{M_2} & jX_4 - jX_{C3} \end{vmatrix} = 0$$

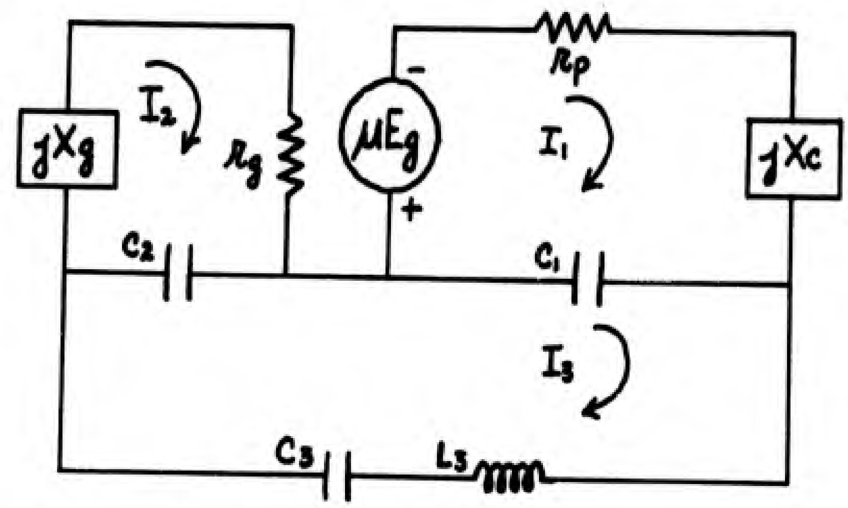

Figure A-5

Clapp Vacuum Tube Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} r_p + jX_c - jX_{c1} & \mu r_g & jX_{c1} \\ 0 & r_g - jX_{c2} + jX_g & jX_{c2} \\ jX_{c1} & jX_{c2} & jX_3 - jX_{c1} - jX_{c2} - jX_{c3} \end{vmatrix} = 0$$

APPENDIX B

Linear Equivalent Circuits and Δ Determinants for Transistor Oscillators Used in Expression Synthesis

Definition of Symbols

Xc = Collector-stabilizing Impedance

Xb = Base-stabilizing Impedance

Xe = Emitter-stabilizing Impedance

r_c = Collector Resistance

r_b = Base Resistance

r_e = Emitter Resistance

$Zc = r_c + jXc$	$X_4 = \omega L_4$
$Zb = r_b + jXb$	$X_{M_1} = \omega M_1$
$Ze = r_e + jXe$	$X_{M_2} = \omega M_2$
$Zm = jXm$	$X_{c1} = 1/\omega C_1$
$X_1 = \omega L_1$	$X_{c2} = 1/\omega C_2$
$X_2 = \omega L_2$	$X_{c3} = 1/\omega C_3$
$X_3 = \omega L_3$	

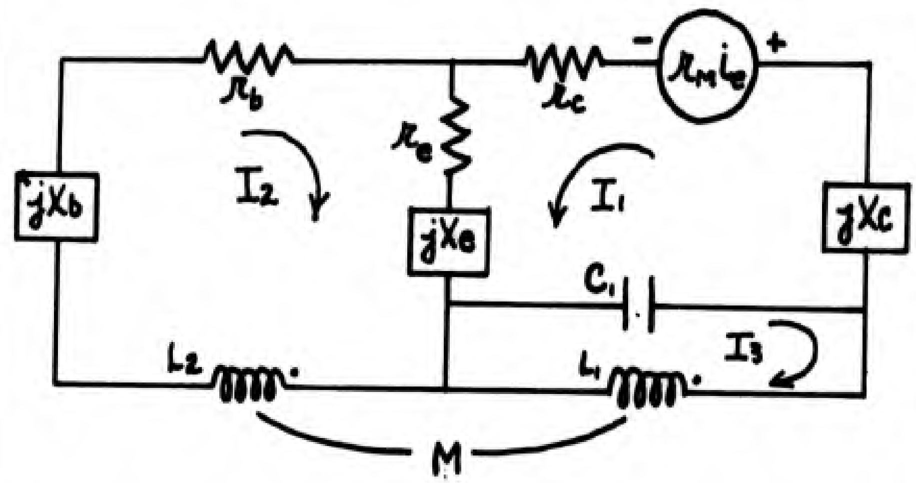

Figure B-1

Tuned-Collector Transistor Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} Z_c + Z_e - jX_{C1} - r_M & Z_e - r_M & -jX_{C1} \\ Z_e & Z_e + Z_b + jX_2 & Z_M \\ -jX_{C1} & Z_M & jX_1 - jX_{C1} \end{vmatrix} = 0$$

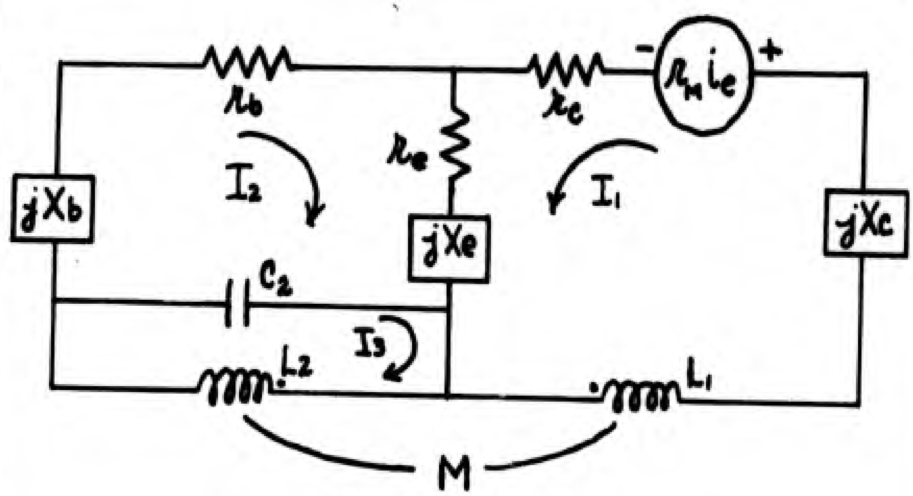

Figure B-2

Tuned-Base Transistor Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} Z_c + Z_e + jX_1 - r_M & Z_e - r_M & Z_M \\ Z_e & Z_e + Z_b - jX_{c2} & jX_{c2} \\ Z_M & jX_{c2} & jX_2 - jX_{c2} \end{vmatrix} = 0$$

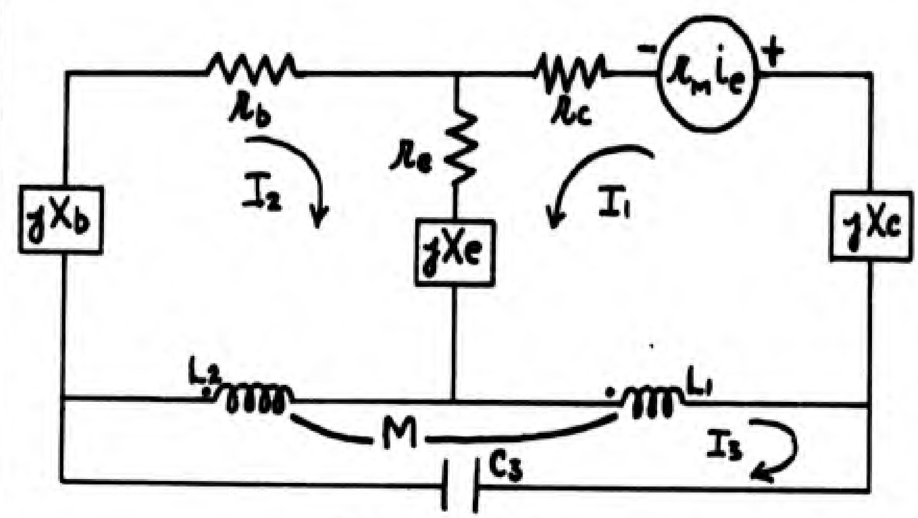

Figure B-3

Hartley Transistor Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} Z_c + Z_e + jX_1 - r_M & Z_e - Z_M - r_M & jX_1 + Z_M \\ Z_e - Z_M & Z_e + Z_b + jX_2 & -jX_2 - Z_M \\ jX_1 + Z_M & -jX_2 - Z_M & jX_1 + jX_2 - jX_{c3} + 2Z_M \end{vmatrix} = 0$$

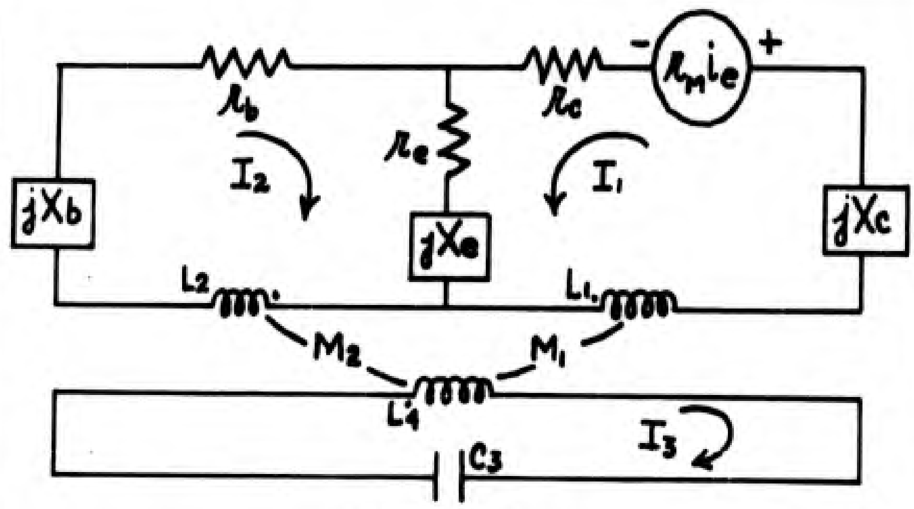

Figure B-4

Meissner Transistor Oscillator Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} Z_c + Z_e + jX_1 - r_M & Z_e - r_M & jX_{M_1} \\ Z_e & Z_e + Z_b + jX_2 & jX_{M_2} \\ jX_{M_1} & jX_{M_2} & jX_4 - jX_{C3} \end{vmatrix} = 0$$

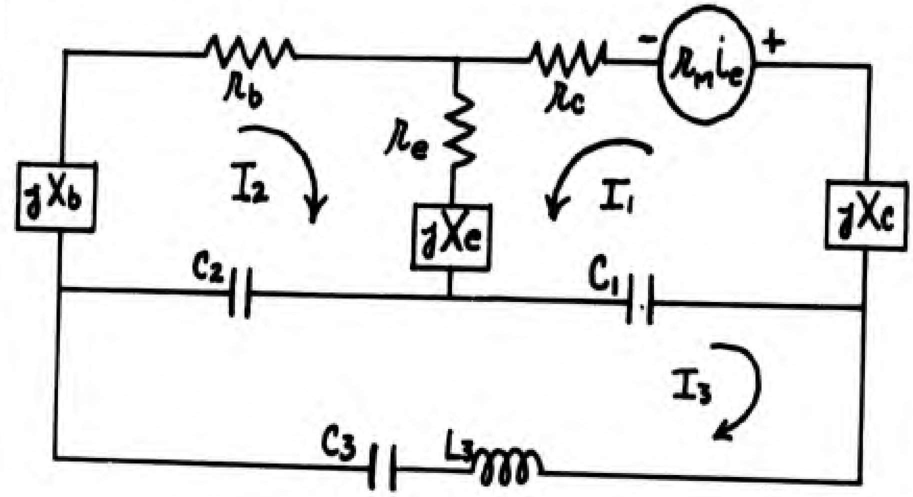

Figure B-5

Clapp Transistor Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} Z_c + Z_e - jX_{c1} - r_m & Z_e - r_m & -jX_{c1} \\ Z_e & Z_e + Z_b - jX_{c2} & jX_{c2} \\ -jX_{c1} & jX_{c2} & jX_3 - jX_{c1} - jX_{c2} - jX_{c3} \end{vmatrix} = 0$$

APPENDIX C

Linear Equivalent Circuits and Δ Determinants for Oscillators Used as Test Cases

Definition of Symbols

X_c = Plate-stabilizing Impedance

X_b = Grid-stabilizing Impedance

r_p = Plate Resistance

r_g = Grid Resistance

$X_1 = \omega L_1$

$X_2 = \omega L_2$

$X_{2_{eq}} = \omega L_{2eq}$

$X_{1_{eq}} = \omega L_{1eq}$

$X_{c_3} = 1/\omega C_3$

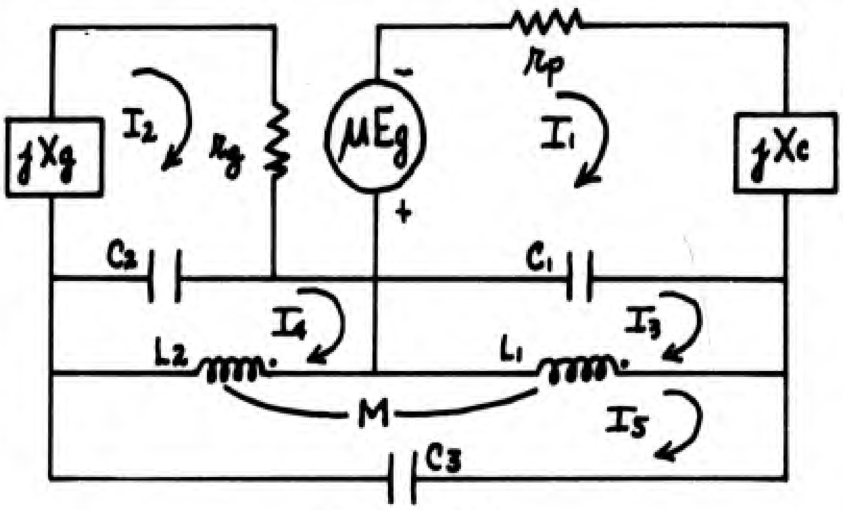

Figure C-1

Unity-Coupled Vacuum Tube Oscillator
Linear Equivalent Circuit

Δ = | A complete analysis of the Unity-coupled vacuum tube oscillator can be found in Llewellyn (Ref 14: 2079-2082). | = 0

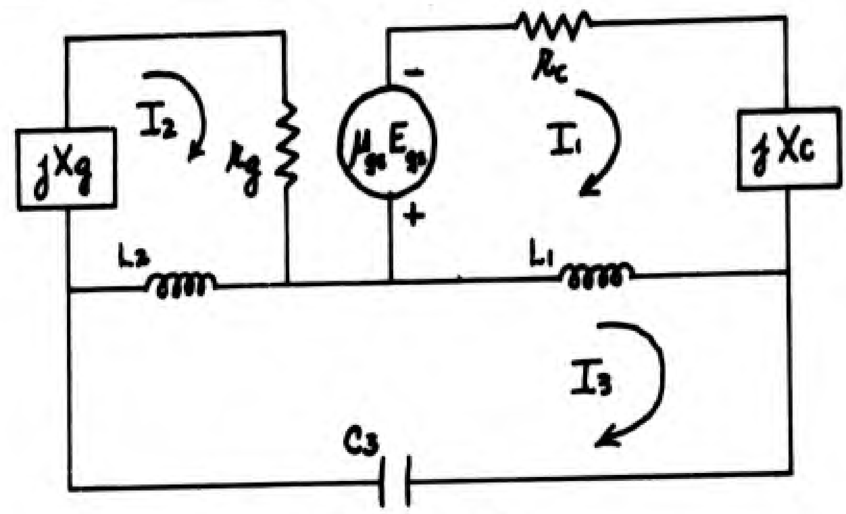

Figure C-2

Electron-Coupled Vacuum Tube Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} R_c + jX_c + jX_1 & \mu_{gs} R_g & -jX_1 \\ 0 & R_g + jX_2 + jX_g & -jX_2 \\ -jX_1 & -jX_2 & jX_1 + jX_2 - jX_{c3} \end{vmatrix} = 0$$

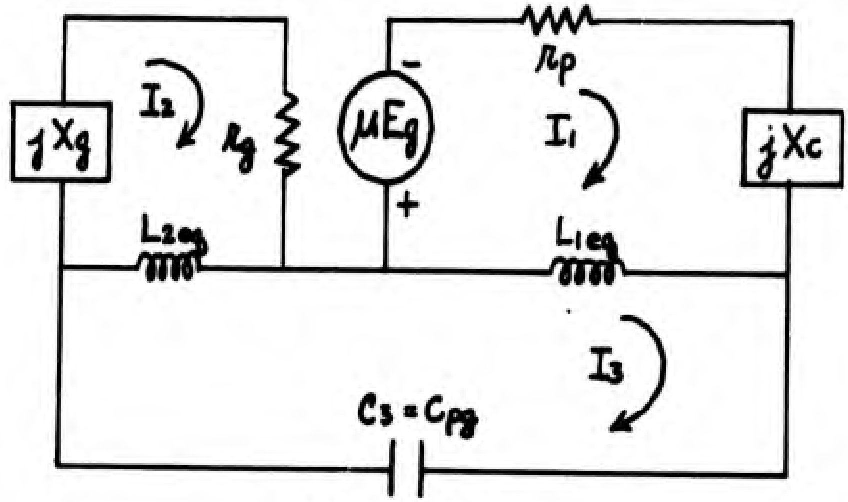

Figure C-3

Tuned-Plate Tuned-Grid Vacuum Tube Oscillator
Linear Equivalent Circuit

$$\Delta = \begin{vmatrix} r_p + jX_c + jX_{1_{eq}} & \mu r_g & -jX_{1_{eq}} \\ 0 & r_g + jX_g + jX_{2_{eq}} & -jX_{2_{eq}} \\ -jX_{1_{eq}} & -jX_{2_{eq}} & jX_{1_{eq}} + jX_{2_{eq}} - jX_{C3} \end{vmatrix} = 0$$

DERIVATION OF EXPRESSIONS FOR L_{1eq} AND L_{2eq}

$$j\omega L_{1eq} = \frac{1}{j\omega C_1 + \frac{1}{j\omega L_1}} = \frac{j\omega L_1}{1 - \omega^2 L_1 C_1} \tag{C-1}$$

$$L_{1eq} = \frac{L_1}{1 - \omega^2 L_1 C_1} \tag{C-2}$$

SUBSTITUTING $\omega^2 = \frac{1}{C_3(L_{1eq} + L_{2eq})}$ YIELDS:

$$L_{1eq} = \frac{L_1 C_3 (L_{1eq} + L_{2eq})}{C_3 (L_{1eq} + L_{2eq}) - L_1 C_1} \tag{C-3}$$

$$L_{1eq}^2 + L_{1eq}\left(L_{2eq} - \frac{L_1 C_1}{C_3} - L_1\right) - L_1 L_{2eq} = 0 \tag{C-4}$$

SOLVING Eq. (C-4) BY QUADRATIC FORMULA YIELDS:

$$L_{1eq} = \frac{-\left(L_{2eq} - \frac{L_1 C_1}{C_3} - L_1\right) + \sqrt{\left(L_{2eq} - \frac{L_1 C_1}{C_3} - L_1\right)^2 + 4 L_1 L_{2eq}}}{2} \tag{C-5}$$

$$L_{1eq} = \frac{-\left(L_{2eq} - \frac{L_1 C_1}{C_3} - L_1\right) + \sqrt{\left(L_{2eq} + \frac{L_1 C_1}{C_3} + L_1\right)^2}}{2} \tag{C-6}$$

$$L_{1eq} = \frac{-\left(L_{2eq} - \frac{L_1C_1}{C_3} - L_1\right) + \left(L_{2eq} + \frac{L_1C_1}{C_3} + L_1\right)}{2} \quad \text{(C-7)}$$

$$L_{1eq} = \frac{L_1C_1}{C_3} + L_1 \quad \text{(C-8)}$$

$$L_{1eq} = \frac{L_1(C_1 + C_3)}{C_3} \quad \text{(C-9)}$$

SINCE THE EQUATION FOR L_{2eq} HAS THE SAME FORM, I.E.,

$$L_{2eq}^2 + L_{2eq}\left(L_{1eq} - \frac{L_2C_2}{C_3} - L_2\right) - L_2L_{1eq} = 0 \quad \text{(C-10)}$$

IT FOLLOWS THAT

$$L_{2eq} = \frac{L_2(C_2 + C_3)}{C_3} \quad \text{(C-11)}$$

Vita

John D[redacted] Kelly was born on [redacted], the son of John Francis Kelly and Margaret Clarke Kelly. After graduating from [redacted], [redacted], in 1951, he enrolled in the School of Engineering at Villanova University, Villanova, Pennsylvania. In May of 1952 he was appointed a Midshipman at the United States Naval Academy, Annapolis, Maryland. He graduated from USNA in 1956 and was commissioned a Second Lieutenant in the Air Force. After completing pilot training at Goodfellow Air Force Base, San Angelo, Texas, he was assigned to the 1850th AACS Squadron, Hamilton Air Force Base, California, as a limited flight-check pilot. Two years later he was selected to attend Squadron Officer School at Maxwell Air Force Base, Montgomery, Alabama. This was his last assignment prior to entering the Air Force Institute of Technology in February of 1960.

Permanent address: [redacted]

This thesis was typed by Mrs. Frances L. Athey