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PRELIMINARY DESIGN OF A SUSTAINED-FLIGHT VEHICLE IN THE ATMOSPHERE OF MARS

THESIS

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By
John Ross Stetson B. S.
First Lieutenant USAF
Graduate Astronautics
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Preface

This thesis is the result of an attempt to synthesize a logical aerodynamic vehicle for use by future expeditions on the planet Mars. It represents a reasonable extrapolation of the elements of Earth aircraft and engine design to the Martian environment, based on current technology. It is not my intent to imply that the numerical values presented are absolutely certain, but rather to show their magnitude relative to the design and performance of Earth aircraft.

I wish to thank Capt. E. M. Romer of the Aeronautical Engineering Department, Institute of Technology, for his suggestions and guidance as my adviser. Useful suggestions for the engines were also made by Capts. J. P. Thomas and L. A. Hamilton of the Mechanical Engineering Department. Many other suggestions were obtained from discussions with Capt. W. L. Haygood, who was doing a companion thesis on aerodynamic flight in the atmosphere of Venus. Many thanks also go to my wife, Carol, for her encouragement and invaluable assistance in preparation of the manuscript.

One final point I should like to make, in deference to Congressman Daniel Flood of Pennsylvania, who claims that military pilots should do nothing but fly. I feel that my pilot experience greatly increased my knowledge and insight of the problems of aircraft design.

John R. Stetson

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Abstract

This thesis presents the preliminary design of a long range, powered aerodynamic vehicle for subsonic flight in the Martian atmosphere. From a survey of literature concerning Mars, a logical atmospheric structure is defined. Performance calculations and specific weight estimates of several powerplant types show an atmosphere-breathing turboprop engine is the most efficient. Calculations of range variation with wing loading and aspect ratio show that a vehicle with a Martian wing loading of 15 and aspect ratio of 16 has a range of 8100 miles with a 0.038 payload fraction. Structural weight, layout, performance, and static stability are discussed for a 26,300 Earth pound vehicle.

PRELIMINARY DESIGN OF A SUSTAINED-FLIGHT VEHICLE IN THE ATMOSPHERE OF MARS

I. Introduction

A manned expedition to Mars must have the means for mobility at the destination. The evidence that Mars possesses an atmosphere suggests an aircraft as one likely method of transportation about the planet. It is the purpose of this paper to propose a powered aerodynamic vehicle that will operate efficiently in the atmosphere of Mars.

Although Mars has only one fourth the surface area of Earth, no measurable amount of surface water has been discovered. Thus the Martian land mass to be explored is nearly ninety per cent of the Earth's land mass, about thirty million square miles. The circumnavigation distance is 13,300 miles. An aircraft may be a most useful vehicle for reconnaissance and for transportation of men and supplies, since travel by air avoids many unforeseen problems of surface mobility.

Initial design of a Martian aircraft should be directed toward a vehicle capable of long range and short field operation. Thus the vehicle to be considered will be of low mass and subsonic design. The structure, powerplant, and fixed equipment will be fabricated on Earth and transported to Mars by rocket. For extensive operations on Mars, it will be necessary to manufacture fuels locally. Thus it is further assumed that suitable raw materials for fuel production exist on the Planet.

The most distinctive features of Mars that affect powered aerodynamic flight are the low density and inert nature of the atmosphere, and the reduced gravitational force. From a survey of the literature concerning Mars, a model atmosphere is defined. The thermodynamic performance of several types of propulsion systems is then investigated. Consideration is given to both self-contained propellant engines and atmosphere-breathing engines using chemical heat sources. Engine selection is based on minimum mass of powerplant and fuel per horsepower hour.

The airframe is then designed by estimating the ranges of a family of vehicles as a function of wing loading and aspect ratio for a payload of 1000 Earth pounds. From selection of a wing loading and aspect ratio, the preliminary layout of a vehicle is then synthesized. Consideration is given to methods for short field operation and to structural loads.

From the preliminary layout, a more accurate weight estimate is obtained and the variation of range and landing field length with payload is then calculated. Finally aerodynamic performance, single-engine operation, and static stability are investigated. Systems design is not considered.

Preliminary studies of the feasibility and uses for a Martian aircraft have also been done by Cartaino (Ref 4) and Godwin (Ref 11: 125-129).

II. The Atmosphere of Mars

Positive evidence of the existence of an atmosphere on Mars is indicated from observations of these planetary features:

(1) Seasonal variation of the size of polar caps.

(2) Increase in brightness of the planet toward the edge of the disc.

(3) Existence of a twilight zone on the disc.

(4) Occasional appearance of haze and clouds.

Detailed investigation of the properties of the Martian atmosphere has been continuing since as early as 1923 (Ref 36).

Composition

Carbon dioxide is the only atmospheric gas that has been positively identified. The amount of CO_2 is estimated to be two per cent by volume, which is 50 times the concentration of CO_2 on Earth (Ref 28). Water vapor and free oxygen, if present, exist in quantities too small to be detected by observation through the Earth's atmosphere. Infrared spectra indicate that the polar caps are a thin layer of ice or hoarfrost, which implies that water vapor is present (Ref 28). The kinetic theory of gases predicts that lighter gases (hydrogen, helium, methane) will easily escape the weak gravitational attraction of the planet. Thus the remainder of the atmosphere is presumed to be nitrogen plus a small amount (1-2%) of argon from radioactive decay of potassium 40 (Ref 22).

Structure and Thermodynamic Properties

Surface temperatures measured by infrared emission have been recorded as high as 33 degrees C at subsolar points, with a diurnal variation of 50 degrees C or more in the equatorial regions. Night time temperatures in the winter hemisphere are as low as -70 degrees. The maximum daily temperature occurs about one-half hour after local noon, which indicates a surface thermal diffusivity higher than that of the moon, but not as high as loam on Earth (Ref 30). Because of the large diurnal variation of surface temperature, it is estimated that the temperature of the base of the atmosphere is 20 to 30 degrees below daytime surface measurements. Therefore an average ambient surface temperature of 0 degrees C is generally assumed. It is expected that this will result in strong vertical convection currents during mid-day (Ref 36:292).

Because of the lower surface gravity of Mars, the structure of the atmosphere is expected to be much "flatter" than that of Earth. A temperature lapse rate of about -3.7 degrees C per kilometer is estimated by measurement of cloud temperatures (Ref 18). A stratosphere in radiative equilibrium is assumed to exist, but estimates of the height of a definite tropopause vary from 5 to 45 Km (Ref 36). The temperature of the stratosphere is estimated to be between -90 and -140 degrees C. (Ref 35).

Surface pressure is calculated from measurements of cloud heights and temperatures and from the polarization of reflected light. Agreement exists among several observers for an average surface pressure of 85 millibars (1.234 psi). This gives a surface atmospheric density of 1.06×10^{-4} gms/cm^3 (0.000206 slugs/ft^3). The variation of pressure and density with altitude is assumed to be adiabatic. Deviations from adiabatic will probably be minor in the lower atmosphere, but will not be in the region of a stratosphere (Ref 13:5). The surface pressure corresponds roughly to Earth atmospheric pressure at 57,000 feet; but pressure decreases much less rapidly with height on Mars, so that atmospheric pressure is the same on both planets at an altitude of about 92,000 feet. Table I gives a comparison of data on Mars from several investigators. Reasonable agreement is obtained for all values except for the height of the tropopause.

Table I

A Comparison of Data on Mars' Atmosphere

Investigator	(Ref)	$\%CO_2$ [a]	Surface Pressure (mb)	Max Surface Temp (deg C)	Tropopause Height (Km)	Lapse Rate (deg/Km)	g ($\frac{cm}{sec^2}$)
Hess	(13	-	80	30	45	3.7	380
Kopal	(20)	0.66	85	30	$8\frac{1}{2}$-25	-	372
Kuiper	(22)	0.3	-	-	-	3.9	380
Urey	(35)	2.0	89	30	30	3.75	391
Vaucouleurs	(37)	2.2	85	30	5-45	3.7	-

(a) Early measurements of CO_2 were found to be low and were revised upwards (Ref 28:3)

<u>Other Atmospheric Features</u>

From extensive temperature and cloud movement data gathered over certain periods, attempts have been made to construct surface weather charts for Mars. From plots of surface temperature gradients, it is estimated that vertical wind shear and wind velocities in the free atmosphere are somewhat less than on Earth. Observations of the terminator, the dividing line between day and night, indicate that the surface is smooth and free of sharp mountains and depressions greater than 2500 feet. The absence of sharp mountain ranges and oceans, the lesser water vapor content than on Earth, and the longer Martian year (695 Earth days) are all factors that would give a more pronounced regularity of weather on Mars (Ref 13:1-11).

The density and size of Mars suggest that no more than a small, heavy (iron) core exists. Thus it is probable that a magnetic field, if one exists, is very weak and there would be no strong Van Allen type belts around the planet. It is suggested, however, that a Martian ionospheric "E" layer exists near an altitude of 220 Km (Ref 20).

A Model Atmosphere

Based on the assumptions of perfect gas law behavior and an adiabatic troposphere, a model atmosphere for Mars is defined as shown in Table II. Additional data on the atmosphere is shown in Appendix A. Ibele (Ref 15:384) has tabulated values of Martian atmospheric viscosity, thermal conductivity, and Prandtl number as a function of temperature for more detailed thermodynamic calculations.

Table II

A Model Martian Atmosphere

Composition	
Nitrogen	97 mol%
Carbon Dioxide	2
Argon	1
Molecular Weight	28.3
Mean Surface Air Temperature	0 deg C
Surface Pressure	85 mb (1.234 psi)
Surface Density	1.06×10^{-4} gm/cc (0.000206 slugs/ft^3)
Temperature Lapse Rate	-3.75 deg C/Km
Tropopause Height	30 Km
Ratio of Specific Heats	1.405
Surface Sonic Velocity	1100 feet/sec
Surface Gravity	372 cm/sec^2
Relative Gravity (Earth = 1)	0.38

III. Selection of a Propulsion Unit

The two most important factors that influence selection of a power plant are specific weight and specific propellant consumption. As the flight time increases, however, the specific propellant consumption, hereafter referred to as SPC, becomes the dominant factor. Thus in order to select the best propulsion system for a long range subsonic aircraft it is necessary to compare the SPC on Mars of several types of engines.

Atmosphere-Breathing Engines

Although the Martian atmosphere is presumed to be chemically inert, the performance of gas turbine engines will be very similar to performance in the Earth's atmosphere, except that both fuel and oxidizer must be supplied to the thermal cycle. Specific thrust, specific horsepower, and SPC have been calculated for several theoretical turboprop, turbojet, and turbofan engines on Mars at subsonic flight velocities and different cycle pressure ratios (Appendix B). Figure 1 shows the variation of horsepower SPC (HPSPC) with compressor pressure ratio for different engine types and using two different fuel/oxidizer heat sources. Stoichiometric mixture ratios were assumed for simplicity with a combustion efficiency of 0.95. Machinery efficiency was taken to be 0.9. A turbine inlet temperature of 2000° F was assumed, since the inertness of the Martian atmosphere should permit the use of higher temperature resistant metals such as columbium or molybdenum for turbine blades (Ref 21:9; ref 2:26).

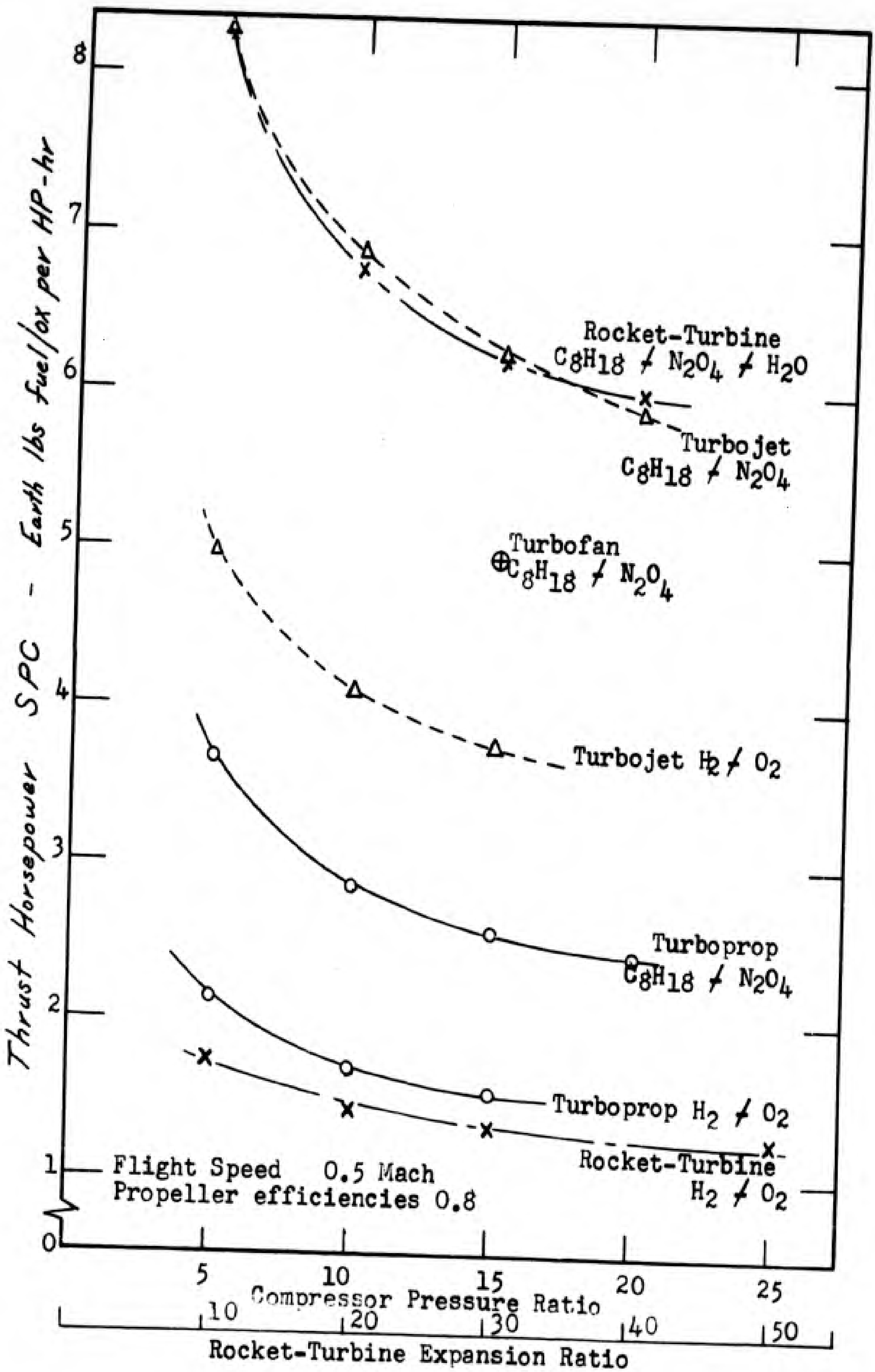

Figure 1

Horsepower Specific Propellant Consumption of Powerplants in a Martian Atmosphere

Rocket-Turbine Engines

The use of a rocket-turbine, or "turborocket", is suggested by Krase (Ref 21) as an efficient propulsive unit in very thin atmospheres. In such a unit the high kinetic energy of the exhaust gases of a conventional chemical rocket is converted into shaft horsepower by interposition of a multistage turbine between the rocket thrust chamber and expansion nozzle. The advantage of such an engine is that no compressor power is required, as in a turboprop, and thus the weight of a compressor is eliminated. Such an engine will be very light, but all of the working fluid must be carried by the vehicle. Because of the presence of a turbine, the temperature of the combustion chamber must be lowered to turbine-blade limits by dilution of the stoichiometric mixture ratio.

The SPC of a rocket-turbine has been calculated for two different fuel/oxidizer combinations (Appendix B) and is shown in Figure 1. The SPC of a high temperature rocket, for comparison, is more than 5 times higher at 0.7 Mach (Table III). Although the rocket-turbine will have a low specific weight, it is evident that for long flight times the rocket-turbine will be superior only if hydrogen is one of the propellants.

Other Powerplant Types

Because of the similarity of the Martian atmosphere to very high altitude conditions on Earth, certain qualitative conclusions may be made regarding other chemical propulsion units. A ramjet cycle, using bipropellants as the heat source in an inert atmosphere, has been found to give poor performance relative to pure rockets (Ref 12). The pulsejet, while limited to the subsonic speeds under consideration, is undesirable for piloted aircraft due to its severe vibration and high intensity of noise (Ref 39:103). Krase (Ref 21:16) suggests that a reciprocating (Otto cycle) engine may have a lower SPC than a rocket turbine but will have a higher specific weight.

Nuclear Propulsion

Although the SPC for an atmosphere-breathing nuclear propulsion unit will be nearly zero, the specific mass of such a system is high even on Earth. Because of the very low atmospheric density on Mars, the power-plant mass for a given power would increase by a factor of 4 to 5 over that for direct cycle nuclear-powered aircraft (ANP) on Earth. Thus the minimum mass for ANP systems on Mars may be so large that they will never be competitive with systems in which chemical propellants are produced from a stationary nuclear power source (Ref 21:5-7).

Propellant Selection

For extensive airborne operations on Mars, it will then be necessary to produce propellants from native materials. It is premature to speculate which substances exist on the crust of Mars that could most easily be converted into energetic fuels. Only carbon dioxide has been positively detected (Ref 35). Vaucouleurs (Ref 37) suggests that the infrared spectrum of the dark areas gives evidence of C-H heavy organic molecule bonds. Thus it is possible that hydrocarbons and hence petroleum may be present. Spilhaus (Ref 29) suggests that the seasonal color changes on Mars are similar to the color changes of nitrogen tetroxide with temperature.

Possible reactions with atmospheric nitrogen were considered by Krase (Ref 21:23). Only reaction with boron to produce non-crystalline boron nitride shows any promise of practical application, however the temperature of reaction is about 2200° F which is rather high for turbomachinery. Most other exothermic reactions with nitrogen yield crystalline products which are also unsuitable for turbomachinery. The same problems occur with the reaction of high energy borane, silane, and lithium fuels with oxygen (Ref 34).

An estimate of HPSPC of a theoretical turboprop and turbojet for 15 different fuel-oxidizer combinations has been calculated (Appendix B) and is shown in Table III. Although the liquid hydrogen-liquid oxygen system has by far the lowest SPC (highest thermal efficiency), the low bulk density and the cryogenic nature of these chemicals would require a massive fuel system and external storage system to reduce boil-off to acceptable levels. The production of liquid hydrogen and liquid oxygen from water vapor on Mars would also require the development of lightweight, efficient dissociation equipment.

From Table III it is seen that a fuel-oxidizer combination, both of which are non-cryogenic, that has the lowest SPC is a stoichiometric mixture of n-octane and nitrogen tetroxide (N_2O_4). Such a mixture is non-hypergolic, which is a desirable safety feature of a manned vehicle, and one or both of these substances may exist in its native state on Mars.

Engine Selection

From Figure 1 it is seen that, at subsonic speeds, the turboprop engine operates at the lowest SPC for n-octane - N_2O_4 fuel. Table III shows that choice of propellants does not alter the optimum engine unless hydrogen is used as a propellant for a rocket-turbine.

Table III

Horsepower Specific Propellant Consumption of Turboprop and Turbojet Engines on Mars for Selected Fuel/Oxidizer Combinations

Flight Speed: 770 ft/sec (0.7 Mach)
Compressor Pressure Ratio: 15
Turbine Inlet Temperature: 2000 deg F
Machinery Efficiency: 0.9
Combustion Efficiency: 0.95
Turboprop Propellor Efficiency: 0.8
Stoichiometric Mixture Ratio
SPC: Earth lbs fuel per thrust HP hr.

Symbol	Chemical
NH_3	Ammonia
C_2H_2	Acetylene
$n\text{-}C_8H_{18}$	Normal octane
N_2O_4	Nitrogen tetroxide
N_2H_2	Hydrazine
C_2H_5OH	Ethyl alcohol

Fuel/Oxidizer	Bulk Specific Gravity [a]	Turboprop SPC	Turbojet SPC	Remarks
$H_2 + O_2$	0.25	1.46	2.72	(b)
$C_2H_2 + O_2$	0.33	1.72	3.07	
$n\text{-}C_8H_{18} + O_2$	0.96	1.99	3.61	
$C_2H_2 + N_2O_4$	1.09	2.19	3.97	
$N_2H_2 + O_2$	1.05	2.25	4.25	
$C_2H_5OH + O_2$	0.98	2.28	4.18	
$NH_3 + O_2$	0.88	2.38	4.51	
$n\text{-}C_8H_{18} + N_2O_4$	1.19	2.43	4.55	(c)
$C_2H_2 + H_2O_2$	-	2.46	4.62	
$N_2H_4 + N_2O_4$	1.19	2.64	5.00	
$C_2H_5OH + N_2O_4$	-	2.80	5.19	
$N_2H_4 + H_2O_2$	1.24	2.82	5.50	
$n\text{-}C_8H_{18} + H_2O_2$	1.20	2.83	5.40	
$NH_3 + N_2O_4$	1.16	2.86	5.50	
$C_2H_5OH + H_2O_2$	1.24	2.95	5.70	

(a) Approximate (Ref 19)
(b) Turborocket SPC 1.34; Rocket HPSPC 7.0
(c) Turbofan SPC 3.60

Source: Appendix B

The performance of an n-octane - N_2O_4 turboprop is discussed more fully in Appendix C. From the performance charts in Appendix C it is seen that for a flight speed of 0.5 Mach the maximum specific equivalent horsepower occurs at a compressor pressure ratio of 13, while the equivalent SPC decreases asymptotically as pressure ratio increases. Thus an operating pressure ratio of 15 is selected for a turbine temperature of 2000° F. Assuming an arbitrary 500 lbs. thrust required at 0.5 Mach, the specific weights of a turboprop, turbojet, turbofan, and rocket-turbine are estimated (Appendix D). Figure 2 shows the total weight of powerplant and fuel as a function of time of flight. Evidently a turboprop is the optimum powerplant for flight times in excess of 2¼ hours.

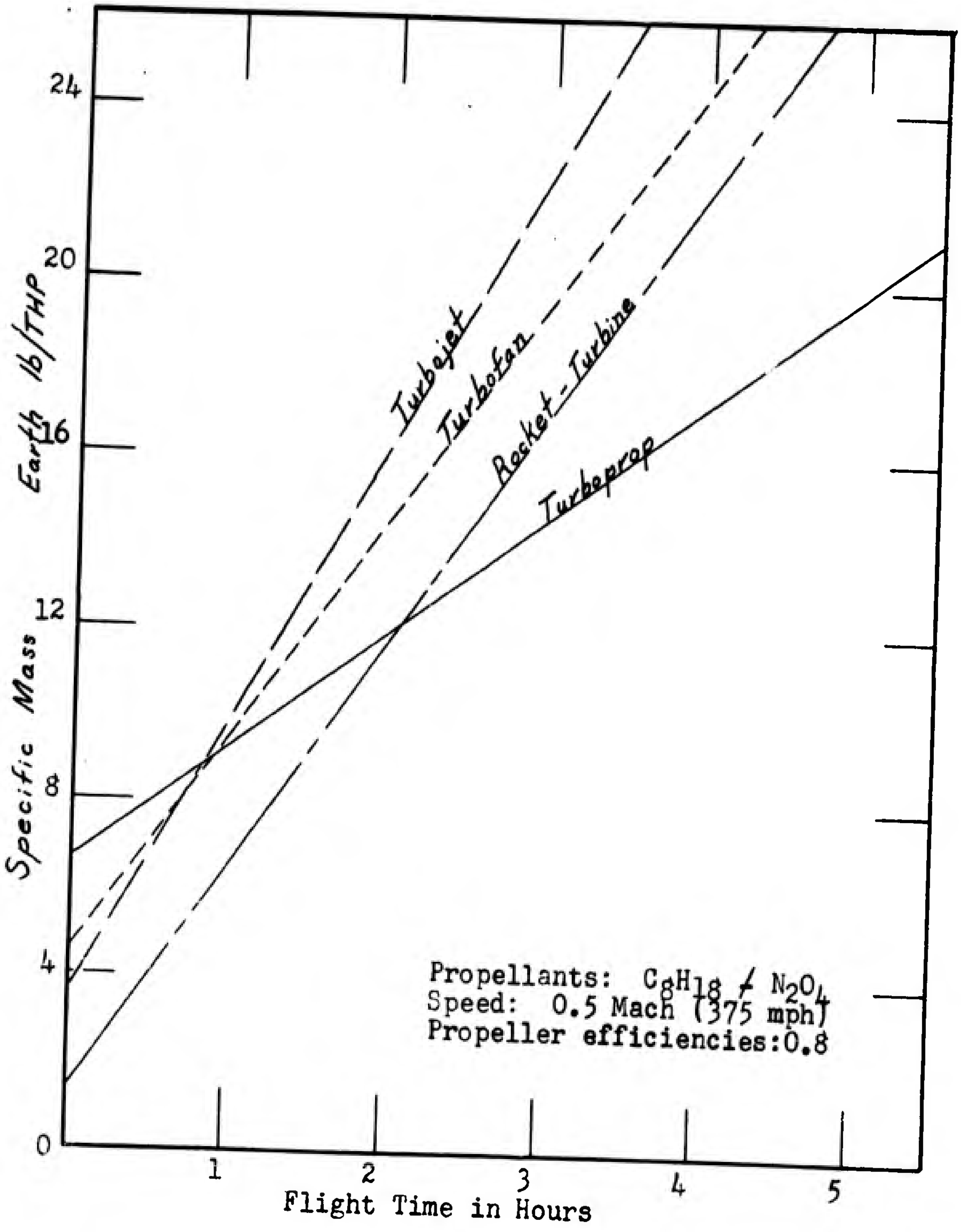

Figure 2

Specific Mass of Powerplant and Fuel as a Function of Flight Time

IV. Aerodynamic Design

Selection of an airframe must be based upon assumption of a mission profile. For a utility vehicle capable of both reconnaissance and transport, a compromise must be made between long range and endurance and short field capability. These requirements suggest a "conventional" subsonic design. The empty mass of the vehicle should be of such size and Earth weight that it can be manufactured and packaged on Earth and transported to Mars by rocket projects of the foreseeable future (Ref 5). The vehicle should also have safety features that give it a reliability approaching that for Earth flying, since escape by parachute will not be desirable in the hostile Martian environment.

Thus the following criteria are specified as a design basis:

Range	Maximum possible but not over 13,300 miles (circumference of Mars).
Speed	550 ft per sec (0.5 Mach at surface).
Payload	1000 Earth pounds.
Field length	As short as possible.
Gross weight	26,300 Earth lbs (10,000 lb wt. on Mars).
Engines	Two turboprop.
Crew	Two

For ease of calculation, some data is presented in terms of Martian pounds weight, since it is the vehicle weight that must be aerodynamically supported. Data is presented in Earth pounds only when comparison with Earth values is desired:

$$W(\text{lbs wt. on Mars}) = g'M$$

where $g' = 0.38$ is Martian gravity relative to that on Earth, and M is Earth weight in pounds.

<u>Wing Thickness Ratio, Sweepback, and Taper Ratio</u>

Wing thickness ratio and sweepback are determined directly from the design cruise Mach number. The wing critical Mach number is set at cruise Mach number plus 50 mph. From the generalized data of Corning (Ref 6: Chap. 2, p. 2-13), a thickness ratio of 0.153 is selected for zero sweepback of the wing quarter-chord line. Wing taper is selected so as to give an elliptical spanwise load distribution. Such a load distribution may be approximated by a taper ratio of 0.4 (Ref 6: Chap. 3, p. 2).

<u>Wing Loading and Aspect Ratio</u>

For the design of aircraft that have a specified range requirement, the optimum wing loading is the highest that will permit a given landing field length. The optimum aspect ratio is then that which gives the lowest operating cost per mile. Given the above design basis for a Martian craft, a compromise must be made between a high wing loading for maximum range and a low wing loading for short field length. However, as is shown in Figure 3, the range for a given wing loading is greatly affected by choice of aspect ratio.

Nominal ranges of a family of vehicles with four different wing loadings, and aspect ratios from four to 24, were calculated (Appendix E). The family of vehicles considered was based on the design criteria listed above plus the additional assumption that the fuselage length be three-fourths of the wing span. The following variables were considered in estimating the ranges of the vehicles:

(1) Wetted area of airframe.

(2) Changes of Reynolds number with dimensions of major components.

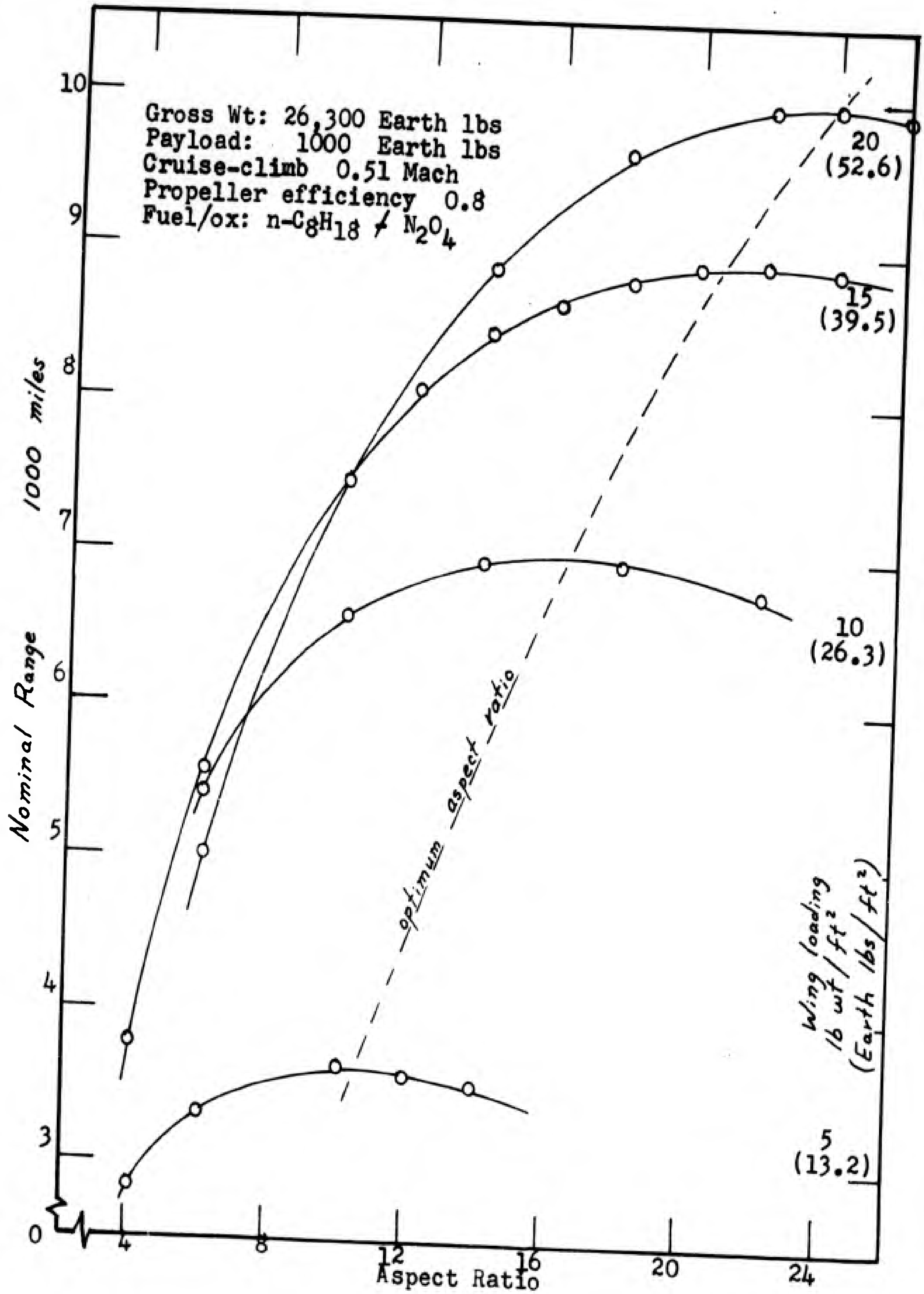

Figure 3

Range of Martian Turboprop Vehicles
as a Function of Wing Loading and Aspect Ratio

(3) Change of wing thickness ratio due to varying wing loading and aspect ratio.

(4) Variation of structural weight with wing loading, thickness ratio, and aspect ratio.

(5) Variation of powerplant weight due to differences in the power required at cruise.

Weights of structure and fixed equipment were estimated from Corning (Ref 7: Chap 2, p. 27-35) with the modification that structural mass, rather than weight, is a function of gross weight on Mars. An effective span efficiency factor (e) of 0.85 was used for estimation of induced drag. Although in general the span efficiency will decrease with increased aspect ratio, thereby reducing the optimum aspect ratio at a given wing loading, it is general practice for preliminary design studies to assume a constant value for (e) (Ref 7: Chap. 2, p. 37; Chap. 7, p. 14).

From the calculations of lift-drag ratio and fuel weight, range was then estimated from the Brequet range equation assuming flight at constant L/D (cruise-climb). A mean value for HPSPC of a turboprop engine at 0.5 M and 6 Km altitude (20,000 ft) was assumed since this method is only approximate for a turboprop (Ref 26:193). Figure 3 shows the nominal range as a function of wing loading and aspect ratio, and Figure 4 shows the relation between range and landing field length.

Figure 4 suggests a wing loading of 15 lb/ft^2 on Mars as a compromise for long range and short field length. This is equivalent to an Earth wing loading of 39.5. Although Figure 3 indicates an optimum aspect ratio of 21 with this wing loading, a lower span efficiency factor and possible aeroelastic and control problems may be realized from such a high aspect ratio. Therefore, a wing aspect ratio of 16 is selected for further study.

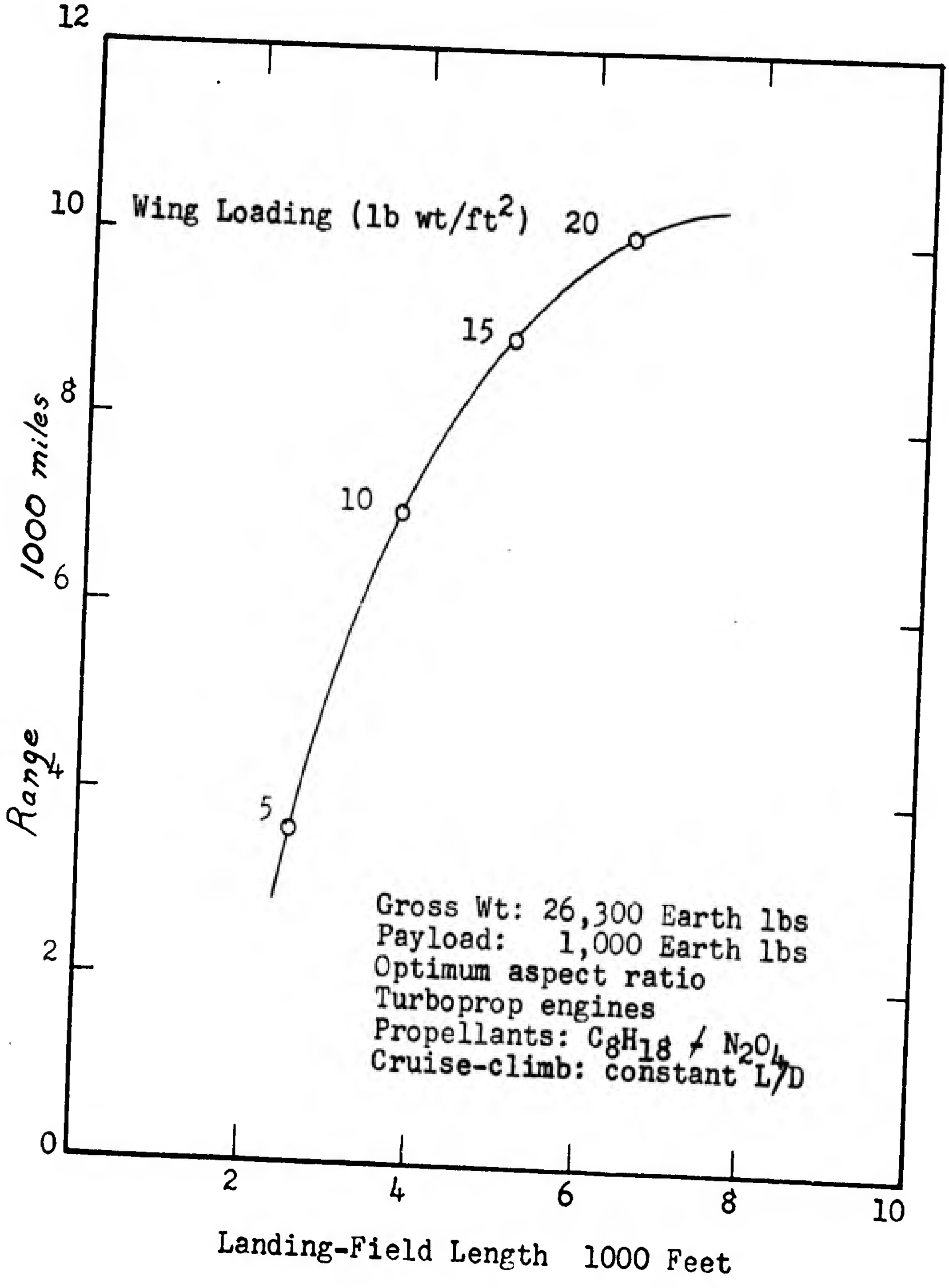

Figure 4

Variation of Range with Landing-Field Length for a Family of Martian Aircraft

Airfoil Selection

For the selected wing loading of 15, the desired lift coefficient will be 0.534 or greater and a thickness ratio of 0.153 has already been established (Appendix E). The cruise Reynolds number based on mean geometric chord is about 2×10^6. From a study of several NACA airfoils (Ref 1), the airfoil NACA 63_2-615 was selected as suitable for the design conditions. The 63_2-615 has the following two-dimensional characteristics at $Re_n = 3 \times 10^6$:

Aerodynamic Center	0.266 chord
Pitching Moment Coefficient about Aerodynamic Center	-0.11 (constant)
Angle of Attack for Zero Lift	-4 deg
Lift Curve Slope	0.11 deg^{-1}
Maximum Lift Coefficient	1.45
Minimum Drag Coefficient	.006

High Lift Devices

For take-off and landing a maximum lift coefficient of 3.0 was assumed. From data on high lift devices (Ref 26:80-81), it is estimated that a flap area/wing area ratio of 0.30 will give a $C_{L_{max}}$ of 3.0 for 0.40c Fowler flaps deflected 40 degrees and a Handley-Page automatic leading edge slot. For the wing planform selected, a flap area/wing area ratio of 0.30 corresponds to 65 per cent span flaps. After correcting for drag due to landing gear, flaps, slots, and ground effect on induced drag, the L/D for take-off and landing configuration is estimated to be 9.85 (Appendix F).

Power Loading

In general the power loading is determined by the take-off field length requirement. However, a brief calculation shows that because of the high specific weight of turboprop powerplants on Mars, the power required for a take-off field length equal to landing field length would require an engine whose weight approaches the gross weight of the aircraft. Since powerplant weight is critical, the power required for cruise at 550 fps at 3 Km (10,000 feet), plus a margin of 10%, is used as basis for engine selection. Thus for a propeller efficiency of 0.80, the total power required for two engines is 612 ESHP at cruise. This is a power loading of 17.5 corrected to Mars' static surface conditions.

Propeller Selection

Given the power per engine required for cruise at 550 fps, two propeller diameters were selected from a general propeller chart (Ref 7: Chap. 8, p. 7; Ref 26:149) and the variation of propeller efficiency with speed was calculated (Appendix F). A four bladed propeller with a diameter of 14.4 feet and a total activity factor of 600 (wide blade area) will have efficiency of 0.84 from 0.3 to 0.5M at sea level. A constant speed propeller should have a maximum rpm of 1060 at 550 fps. It must be full-feathering and reversible in pitch for maximum reverse power during approach and landing. Assuming a reduction gear efficiency of 0.95, the propulsion efficiency of the turboprop engine will be 0.80.

Assist Take-Off System

The static thrust of the two turboprop engines may be estimated at 2.5 times the static ESHP, and a 10% thrust augmentation may be obtained by water-alcohol injection (Ref 23:223-225). Thus the static thrust of the turboprops is 1570 lbs. Using the approximations for take-off distance suggested by Perkins and Hage (Ref 26:194-196), the unassisted take-off ground run would be 21,000 feet for a coefficient of rolling friction of 0.05 (Ref 17: Chap. 14, p. 62). Take-off distances may be reduced to the order of the estimated landing field lengths (about 5000 feet) by use of rocket thrust on the underside of the fuselage. For example, a total engine thrust of 5470 lbs acting for 36 seconds will give a take-off ground run of 3900 feet. A brief investigation shows that an auxiliary rocket engine using the same propellants as the turboprop engines produces 3900 lbs thrust with a propellant mass flow rate of 0.47 slugs per second and an expansion pressure ratio of 21. The nozzle exit diameter would be 6.1 inches and the engine hardware would have a weight of about 15 lbs wt. (Ref 31:223). The total weight of propellant burned on take-off is 208 lbs wt. This weight may be added to the design gross weight of the vehicle since it is never aerodynamically supported.

Structural Load Factor

From the discussion of the Martian atmosphere by Hess (Ref 13:9-11), it is expected that wind velocities on Mars will in general be less than those encountered on Earth with the exception of strong vertical currents near the surface during daytime. Table IV shows gust load factors at Martian ground level as a function of flight speed and gust velocity. It is seen that a load factor of 2.5 would provide structural protection against gust velocities equal to those of heavy thunderstorms on Earth (Ref 32:55). Since 2.5 is generally accepted as the lower limit for maneuvering load factor (Ref 36:57), this value is selected as the limit load factor. Using a safety factor of 1.5, this gives a structural load factor of 3.75.

Table IV

Gust Loads at Martian Surface as a Function of Flight Speed Expressed in Martian "g"

Level Flight

Gust Velocity (feet per second)	Flight Speed (feet per second)		
	330	440	550
30	1.33	1.44	1.55
60	1.66	1.88	2.10
90	1.99	2.32	2.64

Source: Appendix F

Weight Estimation

Empty weight of the vehicle may now be more accurately estimated. Structural weight is estimated from empirical relations used in the aircraft design course of the Institute of Technology, modified to express structural mass as a function of vehicle weight on Mars. The estimated structural weight is in reasonable agreement with the values obtained previously from Corning (Ref 7: Chap. 2, p. 27-29) that were used to select optimum aspect ratio. Table V compares component weight fractions on Mars with those for Earth aircraft of the same gross mass.

The weight values obtained by the above method may be checked for validity by comparing the estimated wing weight with the theoretical weight of material needed to support the maximum bending moment on the wing. Assuming a single aluminum spar of rectangular cross section and a parabolic mass distribution, the minimum structural mass needed to support the wing is 30.8 slugs (Appendix F). This compares favorably with the estimated wing mass of 54.6 slugs (670 lbs wt.), which includes mass of Fowler flaps, control surfaces, slats, and other equipment.

Vehicle Layout

The center of gravity of each major component is estimated from Corning (Ref 7: Chap. 4, p. 41). Fuel, fuel system, payload, landing gear, and fixed equipment are assumed to be centered about the aircraft c.g. The wing is then positioned longitudinally on the fuselage such that the aircraft c.g. is at the one-quarter mean aerodynamic chord position. The empty weight total moment of the vehicle is 119,200 ft-lb (Table VI gives dimensions and positions of major components referred to the nose of the aircraft).

Layout calculations are presented in Appendix F. From the data of Corning (Ref 7: Chap. 2, p. 58), the fuel capacity of a wet wing is 9330 lbs wt. of propellant. Thus all fuel can easily be stored in the wings about the aircraft c.g. A tricycle landing gear is selected. The main gear are suspended from the underslung nacelles and retract into the inboard wing section.

The optimum angle for the thrust vector on take-off roll is three degrees above horizontal for a smooth runway. The wing angle of incidence with the fuselage is also three degrees. Figure 5 shows a preliminary three-view of the aircraft.

Table V

Weight Estimation of Martian Aircraft
Gross wt. 10,000 lbs on Mars (26,300 Earth lbs.)

Component	Weight on Mars[a] (lbs)	Weight / Gross Weight	Weight Fraction for[b] Similar Earth Aircraft
Wing (including flaps and slats)	670	.067	.130
Fuselage	427	.043	.076
Tail Surfaces	178	.018	.036
Landing Gear	235	.023	.065
Nacelles (2)	100	.010	.034
Structure	1610	.161	.323
Engines (2)	1100	.110	
Propellers (2)	220	.022	
ATO Rocket	15	.002	
Powerplant	1335	.134	.167
Fixed Equipment (Systems)	620	.062	.068
Crew (2)	175	.018	.018
Fuel System	180	.018	
Total Empty Weight	3920	.392	.667
Weight Remaining for Payload and Fuel	6080	.608	.333
Total Weight	10,000		

(a) Calculations in Appendix F (b) Ref 7: Chap. 8, p. 12

Table VI

Layout Dimensions of a Martian Aircraft

Gross Weight 10,000 lbs. (26,300 Earth lbs.)		
Fuselage Length	75	Feet
Fuselage Maximum Diameter	7	
Wing Span	103	
Root Chord	9.23	
Tip Chord	3.69	
Mean Aerodynamic Chord	6.87	
Quarter-chord Sweepback	0	
Horizontal Tail Span	16.4	
Horizontal Tail Root Chord	7.80	
Horizontal Tail Tip Chord	3.12	
Vertical Tail Height	14.1	
Vertical Tail Root Chord	10.1	
Vertical Tail Tip Chord	4.04	
Nacelle Length	11.0	
Nacelle Diameter	3.4	
Propeller Diameter	14.4	
A.T.O. Rocket Nozzle Diameter	0.52	

Stations (Referred to nose of aircraft):

Crew	5	Feet
Plane of Propeller	25.7	
Leading Edge of Wing Root Chord	28.12	
Leading Edge of Wing M.A.C.	28.70	
Aircraft c.g.	30.42	
Stick-fixed Neutral Point	31.88	(Appendix G)
A.T.O. Rocket Nozzle	60	
Leading Edge, Vertical Tail Root Chord	64.9	
Leading Edge, Horizontal Tail Root Chord	67.2	
One-quarter Vertical Tail M.A.C.	69.37	
One-quarter Horizontal Tail M.A.C.	69.65	

Lateral Positions (Ref. Fuselage Center-Line):

Nacelle Center-line	11.0	Feet
Half-wing M.A.C.	22.1	

Angle Between Wing M.A.C. and Fuselage	3.0	Degrees (up)
Angle Between Tail M.A.C. and Fuselage	4.6	Degrees (Down)
Angle of Incidence of A.T.O. Nozzle	3	Degrees

Source: Appendix F

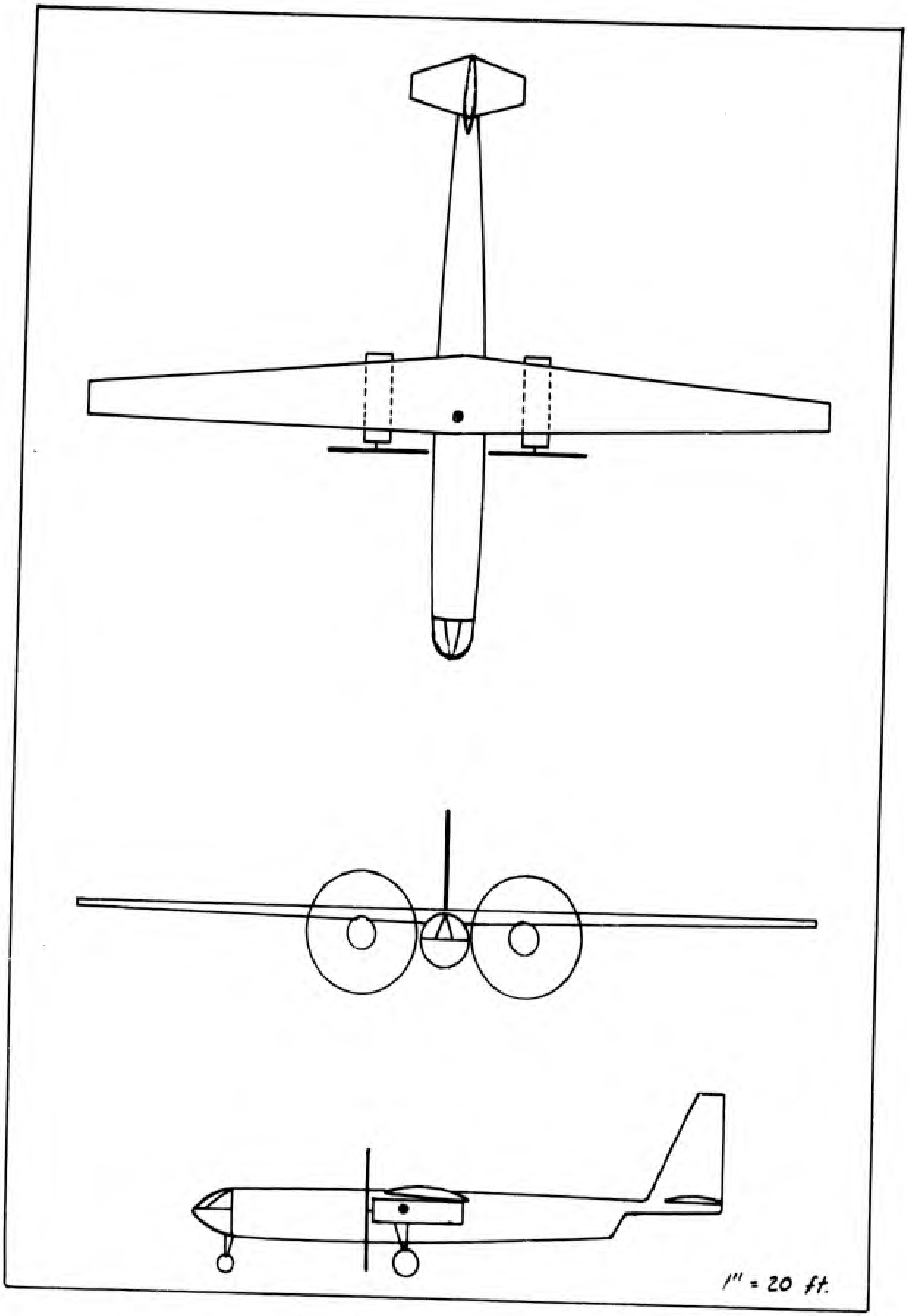

Figure 5

Three-View of a Martian Aircraft

V. Flight Performance

Range

Based on the design of Chapter IV, the variation of range with payload is determined graphically. The variation of theoretical equivalent parasite drag coefficient with speed and altitude was calculated, and drag rise due to compressibility was estimated. Curves of power required as a function of weight, speed, and altitude were then drawn. Variation of equivalent specific propellant consumption with speed and altitude for rated power is obtained from Appendix C. Equivalent SPC is assumed to be nearly independent of power output for a given flight condition. Thus the variation of miles per pound weight of fuel with gross weight, speed, and altitude is determined. From a graph of miles per pound fuel vs. aircraft weight for a given flight plan, a graphical integration gives range as a function of fuel weight or payload (Appendix G).

Figure 6 shows all-out range as a function of payload for a vehicle of 26,300 Earth pounds (818 slugs) at take-off. For 1000 Earth lbs. payload, the all-out range is 8100 miles for a cruise climb from 6 Km (20,000 feet) to more than 18 Km (60,000 feet). The speed for maximum range in cruise-climb averages 0.45 Mach, which is an average speed of 308 miles per hour. The average lift-drag ratio for maximum range is 23.5.

Landing Field Length

Figure 6 also shows landing field length as a function of payload for landing with 81 pounds weight of fuel reserve. The field length is based on the distance from clearing a 50 foot obstacle at a speed of 1.3 times power-off stall speed, to touchdown at 1.2 times power-off stall

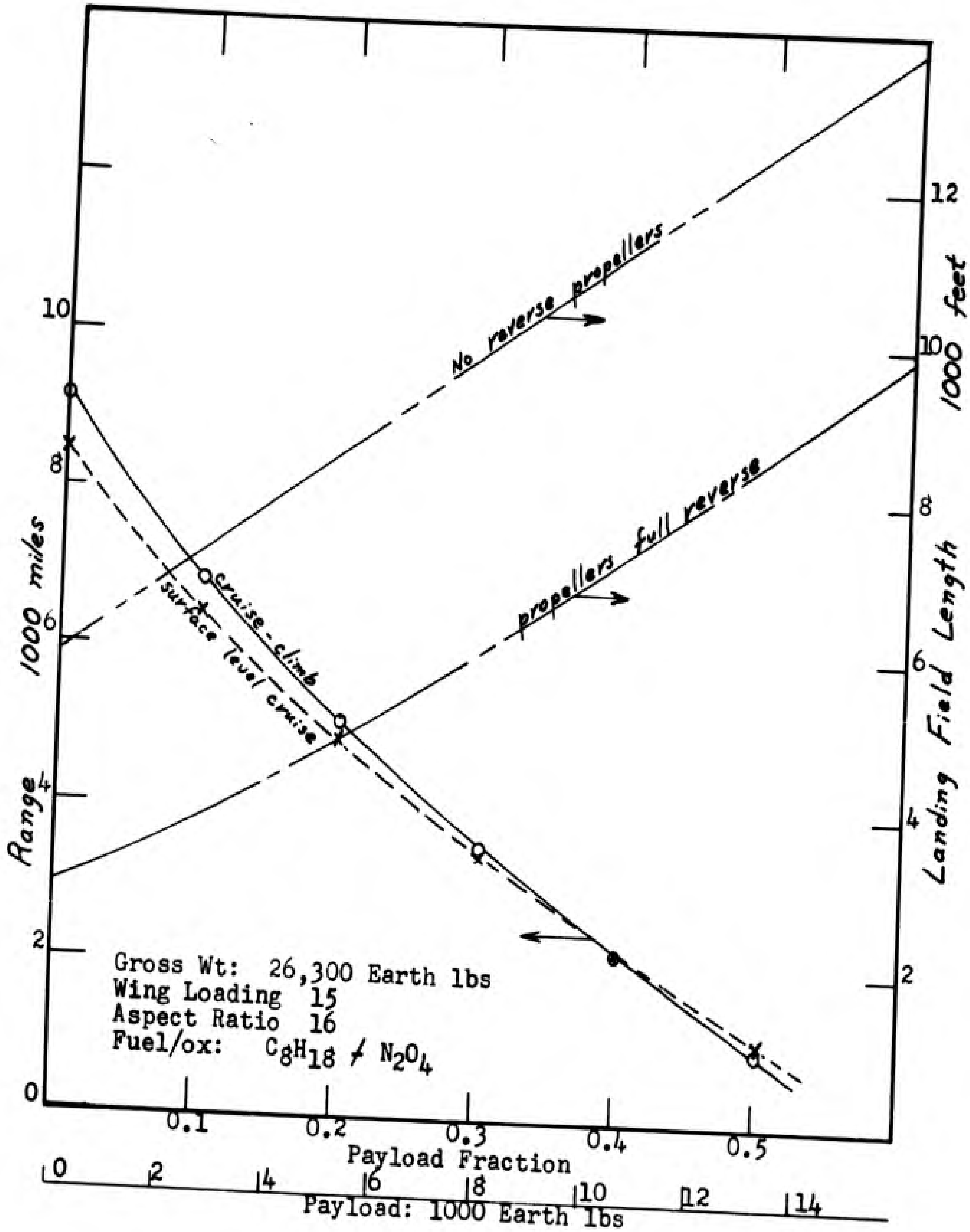

Figure 6

Range and Landing-Field Length as Function of Payload Ratio for a Martian Turboprop speed, to a full stop. The total distance is then multiplied by a safety factor of 1/0.7. The high touchdown speed is assumed due to possibility of surface turbulence during the daytime. For a C_LMax of 3.0, the lift-drag ratio during landing approach is estimated to be 9.65. A ground deceleration rate of 8 ft/sec^2 is assumed for the ground run. Touchdown speeds vary from 167 fps (114 mph) with minimum fuel to 264 fps (180 mph) for landing immediately after take-off.

The field length can be reduced considerably by decreasing the lift-drag ratio during approach. Because of the thin atmosphere, the use of air brakes would not be efficient. A brief calculation shows that to decrease L/D by a factor of one-half would require a flat plate surface area of 160 ft^2. By use of reverse propeller pitch during approach, L/D can be effectively decreased to 2.63 for landing at minimum weight (4,000 lbs wt.). This gives a glide angle of 20.8 degrees. Use of reverse propeller during ground run will also increase the deceleration rate. A value of 10 ft/sec^2 is assumed for calculation of ground run with reverse power. For short field operations, the field length may be reduced further by use of devices such as landing skids or arresting gear.

<u>Rate of Climb and Gliding Distance</u>

Figure 7 shows the power available and required at ground level. Additional power charts are shown in Appendix G. From the charts of power available and required, the maximum rate of climb at ground level, fully loaded, is 710 feet per minute without use of rocket assist. The time to climb to 6 Km (20,000 feet) is 34 minutes. The service ceiling fully loaded is about 10 Km (33,000 feet), but increases rapidly with decreasing weight to over 20 Km (67,000 feet).

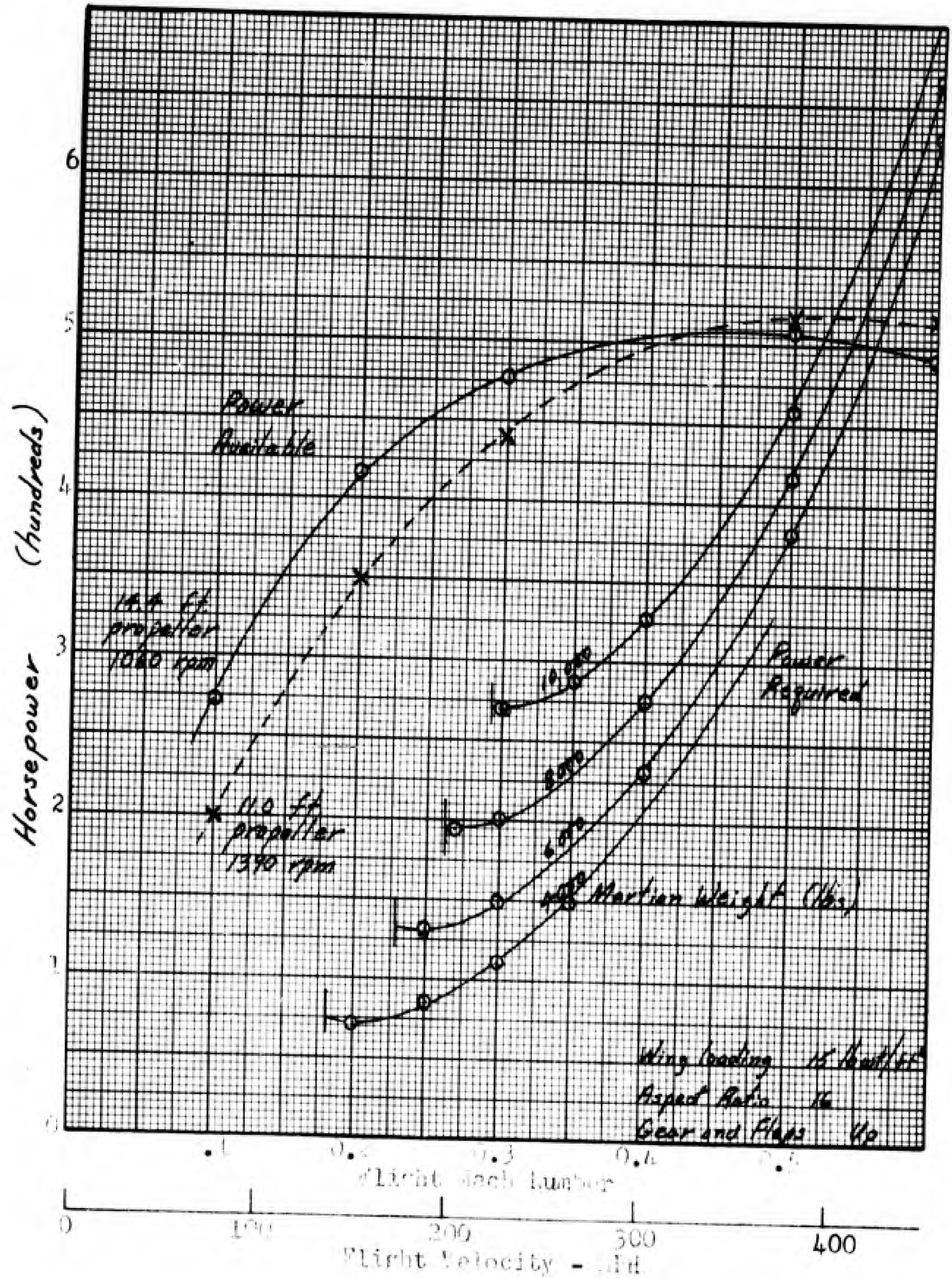

Figure 7

Power Curves for a Martian Turboprop at Surface Level

From the power charts, the minimum rate of sink at ground level varies from 890 feet per minute fully loaded to 540 feet per minute at 4000 lbs wt. The maximum aircraft lift-drag ratio with no propeller drag varies from 24.8 fully loaded at ground level, to 23 at high altitude and low weight. This gives a nominal glide ratio of 15 miles per kilometer altitude.

Single Engine Performance

Power required in take-off configuration (gear and flaps down), fully loaded, is 406 HP at take-off speed. From the power charts, it is seen that level flight can be sustained with two engines operating without rocket-assist. For one engine failure at take-off, flight can easily be maintained by use of rocket-assist, which will rapidly reduce fuel weight for landing immediately after take-off. Without rocket-assist the single-engine ceiling is ground level at 9000 lbs wt. The rudder is designed so that minimum control speed for single-engine operation at full power is always less than stall speed. Using the approximations of Perkins and Hage (Ref 26:324-329), a rudder area of 0.27 vertical tail area and a rudder deflection of 5.6 degrees will give full single-engine directional control.

Static Stability

Longitudinal stick-fixed stability was estimated using data from Etkin (Ref 10:446-479) and Perkins and Hage (Ref 26:88). The airfoil data for the NACA 63_2-615 was corrected for the effects of aspect ratio, body and nacelles, and downwash at the tail. For a horizontal tail surface of 89.1 ft^2, the stick-fixed neutral point of the aircraft is at

0.463 M.A.C. Thus the stick-fixed static margin is 0.213 M.A.C. or 1.46 feet. The pitching moment coefficient about the aerodynamic center is 0.170 for a symmetrical tail section at an angle of incidence of 0.6 degrees to the fuselage center-line. The aircraft is trimmed to the lift coefficient for maximum range ($C_L = 0.8$), and has a pitching moment coefficient derivative with respect to angle of attack of 0.207 per degree. These values are in reasonable agreement with those for Earth aircraft (Ref 26:216).

Directional stick-fixed stability was estimated from data of Perkins and Hage (Ref 26:318-326). For a vertical tail surface equal to 15 per cent of wing area, high wing configuration, and power-off flight, the yawing moment coefficient derivative with respect to yaw is 0.0011 per degree. This value is acceptable for Earth aircraft.

VI. Summary and Conclusions

The results of this investigation show that powered aerodynamic flight appears to be an efficient means of transportation on Mars. The design that has been considered is capable of reaching any point on the surface of Mars from any other point with a payload of 2700 Earth pounds. It has a cruise speed of 0.45 Mach and a maximum speed of 410 miles per hour at ground level. The two engines use relatively low-energy, storable fuels, and the vehicle size and mass is comparable to that of an Earth DC-3.

There appear to be three major differences in the environment of Mars that alter the results of aircraft design from conditions on Earth:

(1) The atmosphere is inert.

(2) Atmospheric surface density is only one-eleventh of Earth sea-level density.

(3) The force of gravity is only two-fifths of Earth's gravity.

The result of the first difference is that both fuel and oxidizer must be supplied to any thermodynamic engine cycle. This increases the mass specific fuel consumption of an engine nearly five-fold over equivalent Earth values. The effect of the second difference is an eleven-fold increase in the specific mass of propellers and atmosphere-breathing engines on Mars. The low density also increases stall speeds and take-off and landing distances. The third effect is a reduction in the weight of a given mass that must be aerodynamically supported. This not only results in a significant reduction of vehicle structural mass ratio, but it also partially offsets the mass penalties imposed by (1) and (2). Thus the net result of all three factors is a considerable increase in range and payload capabilities over that for corresponding Earth aircraft.

Despite the high specific mass of propellers and atmosphere-breathing turbomachinery, for long ranges at subsonic speeds a turboprop engine has the highest overall efficiency using storable fuels and oxidizers. A light-weight rocket-turbine engine offers improved efficiency for flight times less than two hours of if hydrogen is used as a working fluid.

Range is highly dependent upon choice of wing loading and aspect ratio. In addition, because of the high powerplant specific mass, power loading should be as high as cruise drag permits. A Martian wing-loading of 15, a power loading of 17.5, and an aspect ratio of 16 combine to give an aircraft range of 8100 miles with a 1000 Earth lb. payload. However, this results in a take-off ground run of 21,000 feet. Use of a 40 lb. liquid rocket-assist take-off unit will reduce take-off distance to 3900 feet with a range penalty of only 36 miles. Landing field length can be reduced to 3000 feet by use of reverse power during approach and ground roll.

Considerations of fuel storage, engine-out performance, balance, and static stability indicate that no special problems exist for this type of design. Preliminary studies of the Martian atmosphere indicate that strong thermal currents may exist near the surface during the daytime, but otherwise the air may be less disturbed than is Earth's atmosphere. The structure of this aircraft is designed to withstand severe gust loads, but the effect of gusts on the stability of the vehicle is unknown.

Recommendations for further study include investigation of the nature of Martian surface terrain, and an estimation of vehicle operating cost as a function of design variables.

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Appendix A

Structure of the Martian Atmosphere

Given the composition of the atmosphere (Table II), the molecular weight is obtained from

$$MW = X_{N_2}(MW)_{N_2} + X_{CO_2}(MW)_{CO_2} + X_A(MW)_A \tag{A 1}$$

where X is the mole fraction of each component. Thus

$$MW = (0.97)(28) + (0.02)(44) + (0.01)(40) = 28.3$$

Similarly, the molal heat capacity of the atmosphere at 0 deg. C. is given by

$$C_p = X_{N_2}(C_p)_{N_2} + X_{CO_2}(C_p)_{CO_2} + X_A(C_p)_A \tag{A 2}$$

Using values for C_p at 0 deg. C. from Perry (Ref 27),

$$C_p = (0.97)(6.96) + (0.02)(8.61) + (0.01)(4.97) =$$
$$6.97 \text{ Btu/mole-deg F}$$

Then from

$$\gamma = \frac{C_p}{C_p - R_o} \tag{A 3}$$

the ratio of specific heats is

$$\gamma = 1.405$$

Surface sonic velocity is then given by

$$a_{sL} = \sqrt{\frac{\gamma R_o T g_c}{(MW)}} \tag{A 4}$$

Thus

$$a_{sl} = \sqrt{\frac{(1.405)(1545)(492)(32.2)}{(28.3)}} = 1100 \text{ ft/sec}$$

The variation of temperature, pressure, and density with altitude is defined by assuming an adiabatic troposphere (Ref 13:3) and a temperature lapse rate of -3.75 deg C/Km. Then

$$T = T_{sl} - 3.75\ h\ (\text{Km}) \qquad \text{(A 5)}$$

which becomes

$$\frac{T}{T_{sl}} = 1 - 0.0137\ h \qquad \text{(A 6)}$$

Thus

$$\left(\frac{P}{P_{SL}}\right) = \left(\frac{T}{T_{SL}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T}{T_{SL}}\right)^{3.5} \qquad \text{(A 7)}$$

$$\left(\frac{\rho}{\rho_{SL}}\right) = \left(\frac{T}{T_{SL}}\right)^{\frac{1}{\gamma-1}} = \left(\frac{T}{T_{SL}}\right)^{2.5} \qquad \text{(A 8)}$$

and

$$\left(\frac{a}{a_{sl}}\right) = \left(\frac{T}{T_{sl}}\right)^{\frac{1}{2}} \qquad \text{(A 9)}$$

Table VII shows these properties of the Martian atmosphere to a height of 21 kilometers.

Table VII

Properties of the Martian Atmosphere

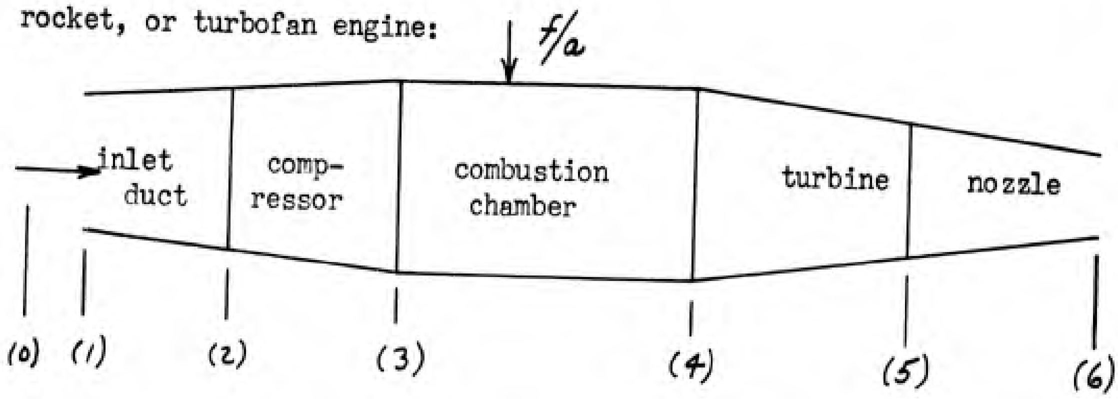
to 21 Km (69,000 feet)

h Km	Ft	$\frac{T}{T_{sl}}$	$\frac{p}{p_{sl}}$	$\frac{\rho}{\rho_{sl}}$	$\frac{a}{a_{sl}}$
0	0	1.0	1.0	1.0	1.0
3	9,850	.959	.864	.900	.970
6	19,700	.918	.742	.808	.959
9	29,550	.876	.630	.719	.937
12	39,400	.835	.532	.638	.915
15	49,200	.794	.446	.562	.892
18	59,050	.753	.371	.492	.870
21	68,900	.712	.305	.427	.845

Appendix B

Performance of Gas Turbine and Reaction Powerplants on Mars

General equations may be developed for a thermal jet cycle, and then be modified for application to a turbojet, turboprop, rocket-turbine, rocket, or turbofan engine:



Station
- (0) ambient conditions
- (1) air intake
- (2) compressor face
- (3) compressor discharge
- (4) turbine inlet
- (5) turbine discharge
- (6) nozzle exit

Note: The unit of mass used in engine performance calculations is that mass which weighs one pound on Earth

The work done by the compressor per pound of entering air (W_c), for an adiabatic compression with an efficiency η_c, is given by

$$W_c = \frac{J}{\eta_c}(h_3 - h_2) \qquad \text{(B 1)}$$

where h is specific enthalpy and J is mechanical equivalent of work. Now for constant specific heat

$$(h_3 - h_2) = C_p\,(T_3 - T_2) \qquad \text{(B 2)}$$

Then from the isentropic relation

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \equiv R^{\frac{\gamma-1}{\gamma}} \qquad \text{(B 3)}$$

thus for C_p constant

$$W_c = \frac{J C_p T_2}{\eta_c}\left[R^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad \text{(B 4)}$$

where T_2 is stagnation temperature for a given flight Mach number:

$$\frac{T_2}{T_0} = 1 + \frac{\gamma-1}{2} M_0^2 \qquad \text{(B 5)}$$

Similarly the work done by the not gases expanding through the turbine per pound of entering air (W_t), is expressed by

$$W_t = J\eta_t\,(1 + f/a)(h_4 - h_5) \qquad \text{(B 6)}$$

where f/a is lbs of fuel/oxidizer added per pound of entering air and η_t is turbine efficiency.

The kinetic energy of exhaust gases leaving the nozzle with velocity Ve may be expressed by

$$\frac{V_e^2}{2g_c} = J\eta_N (h_5 - h_6) \qquad \text{(B 7)}$$

for a nozzle efficiency η_N. For the special case of $\eta_t = \eta_N \equiv \eta_{tN}$, (B 6) and (B 7) may be combined to give the total work done by the exhaust gases during expansion through turbine and nozzle, W_{TN} .

$$W_{TN} = W_T + (1 + f/a)\frac{Ve^2}{2g_c} = J \quad {}_{TN}(1+f/a)(h_4-h_6) \qquad \text{(B 8)}$$

Assuming a mean specific heat for the hot gases in expansion, and using the relation

$$\frac{T_6}{T_4} = \left(\frac{P_6}{P_4}\right)^{\frac{\gamma'-1}{\gamma'}} \qquad \text{(B 9)}$$

equation (B 8) becomes

$$W_{tN} = J\eta_{tN} C_p' T_4 (1 + f/a)\left[1 - \left(\frac{P_6}{P_4}\right)^{\frac{\gamma'-1}{\gamma'}}\right] \qquad \text{(B 10)}$$

For optimum expansion of the exhaust gases $P_6 = P_o$, and for burner pressure efficiency ϵ

$$P_4 = \epsilon P_3 = \epsilon P_2 \left(\frac{P_3}{P_2}\right) = \epsilon \mathcal{R} P_2 \qquad \text{(B 11)}$$

For a ram efficiency η_R

$$P_2 = \eta_R \left(\frac{T_2}{T_0}\right)^{\frac{\gamma}{\gamma-1}} P_0 \qquad \text{(B 12)}$$

Substituting (B 11) and (B 12) into (B 10) gives

$$W_{tN} = \eta_{tN} J C_p' T_4 (1 + f/a) \left[1 - \left(\frac{1}{\epsilon \mathcal{R} \eta_R}\right)^{\frac{\gamma'-1}{\gamma'}} \left(\frac{T_0}{T_2}\right)^{\frac{\gamma}{\gamma'}\left(\frac{\gamma'-1}{\gamma-1}\right)}\right] \qquad \text{(B 13)}$$

where primed quantities refer to values at mean expansion temperatures of exhaust gases.

Since both fuel and oxidizer are being added to the air cycle, the quantity f/a may not in general be negligible. The value of f/a may be estimated by

$$f/a = \frac{C_{pa}(T_4 - T_3)}{-\eta_b \, \Delta H_c^{T_4}} \qquad \text{(B 14)}$$

for a combustion efficiency η_b. C_{pa} is mean C_p of the air alone in combustion chamber and $\Delta H_c^{T_4}$ is the heat of combustion of the fuel/ oxidizer combination per pound of propellant from reactants at ambient temperature to products of combustion at temperature T_4. Assuming a stoichiometric reaction:

$$aA + bB \longrightarrow mM + nN$$

then

$$\Delta H_c^{T_4} = \frac{m\,\Delta H_{Form,\,M}^{T_4} + n\,\Delta H_{Form,\,N}^{T_4} - a\,\Delta H_{Form,\,A}^{T_2} - b\,\Delta H_{Form,\,B}^{T_2}}{m\,(MW)_M + n\,(MW)_N} \quad \text{(B 15)}$$

where $\Delta\, H_{form}$ is heat of formation of component in Btu/mole. $\Delta\, H_{form}$ data is tabulated at different temperatures by Edse (Ref 9).

For a compressor efficiency η_c, the temperature T_3 is given by

$$T_3 = T_2 + \frac{T_2}{\eta_c}\left[\mathcal{R}^{\frac{\gamma-1}{\gamma}} - 1\right] \quad \text{(B 16)}$$

where T_2 is defined by (B 5).

Thus for a given T_0, M_0, $\mathcal{R}$, T_4 (turbine inlet temperature), and fuel/oxidizer combination, equations B 4, B 5, B 13, and B 14 define the specific power output and specific fuel consumption of any thermal-jet cycle.

Turbojet Performance

For the turbojet cycle, the work done on the turbine wheel equals the work done by the compressor, neglecting accessory loads

$$W_t = W_c \qquad \text{(B 17)}$$

Substituting into equation (B 8)

$$W_{tn} = W_c + (1 + f/a)\frac{Ve^2}{2g_c} \qquad \text{(B 18)}$$

or

$$Ve^2 = \frac{2g_c}{(1+f/a)} \quad (W_{tn} - W_c) \qquad \text{(B 19)}$$

From the momentum theorem, the specific thrust of the turbojet is

$$F_s = \frac{(1 + f/a)Ve-Vo}{g_c} \qquad \text{(B 20)}$$

Then the thrust specific propellant consumption is

$$TSPC = \frac{3600\ (f/a)}{Fs} \quad \frac{\text{lb fuel/ox}}{\text{lb thrust hr}} \qquad \text{(B 21)}$$

For comparison with propeller-powered units, it is desirable to express (B 20) and (B 21) in horsepower units. Since thrust specific horsepower is

$$(THP)_{sp} = \frac{FsVo}{550} \qquad \text{(B 22)}$$

the thrust horsepower specific propellant consumption becomes

$$\text{HPSPC} = \frac{3600\ (f/a)}{(\text{THP})_{sp}} \quad \frac{\text{lb fuel/ox}}{\text{HP hr}} \qquad \text{(B 23)}$$

Turboprop Performance

For a turboprop engine with propulsive efficiency η_p , there are two contributions to the thrust horsepower. The thrust horsepower of the propeller is given by

$$(\text{THP})_{prop} = \eta_p\ (W_t - W_c)/550 \qquad \text{(B 24)}$$

neglecting accessory loads. The thrust horsepower of the nozzle exhaust gases is then

$$(\text{THP})_n = \frac{(1 + f/a)\ VeVo - Vo^2}{550\ g_c} \qquad \text{(B 25)}$$

Thus the total specific thrust horsepower of the turboprop is

$$(\text{THP})_{sp} = (\text{THP})_{prop} + (\text{THP})_n$$

$$= \frac{\eta_p}{550}\ (W_t - W_c) + \frac{(1 + f/a)\ V_eV_o - V_o^2}{550\ g_c} \qquad \text{(B 26)}$$

It is convenient to reduce the dependence of horsepower on η_p by defining an equivalent horsepower for the turboprop by

$$(\text{EHP})_{sp} = \frac{(\text{THP})_{sp}}{\eta_p} \qquad \text{(B 27)}$$

Then from (B 26)

$$(EHP)_{sp} = \frac{W_t - W_c}{550} + \frac{(1 + f/a)\ V_e V_o^2}{550 \qquad g_c} \tag{B 28}$$

But from (B 8)

$$W_t = W_{tn} - (1 + f/a)\ \frac{V_e^2}{2g_c} \tag{B 29}$$

Then (B 28) becomes

$$(EHP)_{sp} = \frac{W_{tn} - W_c}{550} + \frac{(1+f/a)V_e V_o^2}{550\ \eta_p\ g_c} - \frac{(1+f/a)V_e^2}{1100\ g_c} \tag{B 30}$$

Since W_{tn} and W_c are independent of V_e, it can be seen by differentiation of (B 30) with respect to V_e, that $(EHP)_{sp}$ is a maximum (Ref 14:379) for

$$V_e = \frac{V_o}{\eta_p} \tag{B 31}$$

Substituting (B 31) into (B 30) and simplifying them gives

$$(EHP)_{sp} = \frac{W_{tn} - W_c}{550} + \frac{V_o^2}{1100 g_c\ \eta_p^2}\left[1+f/a - 2\eta_p\right] \tag{B 32}$$

which defines the specific power output of a turboprop. The equivalent specific propellant consumption is then given by

$$ESPC = \frac{3600(f/a)}{(EHP)_{sp}}\ \frac{\text{lbs fuel/ox}}{\text{HP} \quad \text{hr}} \tag{B 33}$$

and from (B 27) the thrust horsepower specific propellant consumption will be

$$HPSPC = \frac{3600(f/a)}{(EHP)_{sp}\,\eta_p} \qquad \frac{\text{lbs fuel/ox}}{\text{HP} \quad \text{hr}} \tag{B 34}$$

Rocket-Turbine Performance

For the rocket-turbine, the compressor work term vanishes, $W_c = 0$, and W_t becomes the turbine work done per pound of exhaust gases. Then equation (B 8) is modified to

$$W_{tn} = W_t + \frac{V_e^2}{2\,g_c} \tag{B 35}$$

and (B 10) becomes

$$W_{tN} = J\,\eta_{tN}\,C_p'\,T_4\left[1-\left(\frac{P_6}{P_4}\right)^{\frac{\gamma'-1}{\gamma'}}\right] \tag{B 36}$$

For optimum expansion $P_6 = P_o$, and P_4 will be combustion chamber pressure. As for the turboprop, the specific thrust horsepower consists of two terms

$$(THP)_{sp} = (THP)_{prop} + (THP)_n = \frac{\eta_p W_t}{550} + \frac{V_e V_o}{g_c 550} \tag{B 37}$$

Then from (B 27) and (B 35)

$$(EHP)_{sp} = \frac{W_{tn}}{550} + \frac{V_e V_o}{550 g_c \eta_p} - \frac{V_e^2}{1100 g_c} \tag{B 38}$$

and, as for the turboprop, EHP can be shown to be a maximum when

$$V_e = \frac{V_o}{\eta_p} \qquad \text{(B 31)}$$

Substituting into (B 38) gives

$$(EHP)_{sp} = \frac{W_{tn}}{550} + \frac{V_o^2}{1100 g_c \eta_p^2} \quad \frac{HP}{\text{lb fuel/sec}} \qquad \text{(B 39)}$$

The thrust horsepower specific propellant consumption is then

$$HPSPC = \frac{3600}{\eta_p (EHP)_{sp}} \quad \frac{\text{lb propellant}}{\text{HP} \quad \text{hr}} \qquad \text{(B 40)}$$

In general stoichiometric mixture ratios cannot be used for rocket-turbines designed to operate at a given turbine temperature. The combustion temperature must be lowered by dilution of the propellants with fuel or an inert substance such as water. The HPSPC can be calculated much more easily if specific impulse data is available for a given propellant mixture at a given combustion temperature and chamber pressure (Ref 21:11).

Since ideal exhaust velocity, V_e, is related to specific impulse by

$$V_e = Isp \, g_c \qquad \text{(B 41)}$$

the kinetic energy of the exhaust is

$$\frac{V_e^2}{2 g_c} = \frac{g_c Isp^2}{2} \qquad \text{(B 42)}$$

If this kinetic energy is substantially all converted into shaft horsepower by a turbine with efficiency η_t, then the thrust HPSPC is given by

$$\text{HPSPC} = \frac{(3600)(550)}{g_c \frac{Isp^2}{2} \eta_t \eta_p} \quad \frac{\text{lb propellant}}{\text{HP} \quad \text{hr}} \qquad \text{(B 43)}$$

Rocket Performance

The thrust HPSPC of a pure rocket can be obtained directly from specific impulse data by

$$\text{HPSPC} = \frac{(3600)(550)}{\eta_N \, \text{IspVo}} \quad \frac{\text{lb propellant}}{\text{HP} \quad \text{hr}} \qquad \text{(B 44)}$$

Turbofan Performance

Consider an auxiliary compressor (or fan) placed forward of the primary compressor in a turbojet cycle (station 2 on the diagram of page 43). From the discharge of the auxiliary compressor, let X lbs of air per lb of combustion (primary compressor) air be bypassed around the combustion and turbine section and discharged through a nozzle. Both compressors are driven by expansion of primary exhaust gases through the turbine. The derivation of performance equations is lengthy; in the interests of brevity, only the results are presented:

The exhaust velocity of the bypass air, Ve_2, is given by

$$\frac{Ve_2^2}{2 g_c J} = \eta_N C_p T_2 \left[1 + \frac{1}{\eta_{c_2}} \left(\pi_2^{\frac{\gamma-1}{\gamma}} - 1 \right) \right] \left[1 - \left(\frac{1}{\eta_R \pi_2} \right)^{\frac{\gamma-1}{\gamma}} \left(\frac{T_0}{T_2} \right) \right] \qquad \text{(B 45)}$$

where η_N is nozzle efficiency of bypass duct,

T_2 is total temperature of ambient air (as before)

π_2 is pressure ratio of auxiliary compressor

η_{c_2} is auxiliary compressor efficiency

The exhaust velocity of the combustion products and primary (Ve_1) is given by

$$\frac{Ve_1^2}{2 g_c J} = \eta_{tw} C_p' T_4 \left[1 - \left(\frac{1}{\epsilon \eta_R \mathcal{R}_1 \mathcal{R}_2} \right)^{\frac{\gamma'-1}{\gamma'}} \left(\frac{T_0}{T_2} \right)^{\frac{\gamma}{\gamma'}\left(\frac{\gamma'-1}{\gamma-1}\right)} \right] - \frac{W_c}{(1 + f/a)} \qquad \text{(B 46)}$$

where $\mathcal{R}_1$ is pressure ratio of primary compressor, and W_c is total compressor work:

$$W_c = W_{c_1} + W_{c_2}$$

$$= \frac{C_p T_2}{\eta_c} \left[\left(\mathcal{R}_1^{\frac{\gamma-1}{\gamma}} - 1 \right) + \frac{1}{\eta_c} \left(\mathcal{R}_2^{\frac{\gamma-1}{\gamma}} - 1 \right) \left(\mathcal{R}_1^{\frac{\gamma-1}{\gamma}} - 1 \right) + (1 + X) \left(\mathcal{R}_2^{\frac{\gamma-1}{\gamma}} - 1 \right) \right] \qquad \text{(B 47)}$$

for $\eta_{c_1} = \eta_{c_2} \equiv \eta_c$

Then the specific thrust is given by

$$F_s = \frac{\left(\frac{1 + f/a}{1 + X} \right) V_{e_1} + \left(\frac{X}{1 + X} \right) V_{e_2} - V_0}{g_c} \quad \frac{\text{lb thrust}}{\text{lb total air/sec}} \qquad \text{(B 48)}$$

and the thrust specific fuel/ox consumption is

$$TSFC = \frac{3600 \ (f/a)}{F_s \ (1 + x)} \quad \frac{\text{lb fuel/ox}}{\text{lb thrust-hr}} \qquad \text{(B 49)}$$

From a graphical plot of TSPC vs. X with $\mathcal{R}_2$ as a parameter (Figure 8) it is seen that the optimum SPC occurs at $Ve_1 = Ve_2$.

Calculations

Values of thrust HPSPC have been calculated from the preceding equations for the following systems:

(1) Turboprop and turbojet engines using 15 different combinations of fuel and oxidizer at a speed of 0.7 Mach at ground level. A compressor pressure ratio of 15 and turbine-inlet temperature of 2000 deg F was assumed. The results are presented in Table III of the test.

(2) Turboprop and turbojet engines using a liquid H_2 - liquid O_2 heat source. A flight speed of 0.5 Mach was selected for comparison, and compressor pressure ratio was varied from 5 to 15.

(3) Turboprop and turbojet engines using an n-octane -N_2O_4 heat source at a flight speed of 0.5 M. Compressor pressure ratio was varied from 5 to 20.

(4) Rocket-turbine engines using an H_2 - O_2 heat source and an n-octane - N_2O_4 - H_2O propellant combination. Expansion ratio was varied from 10 to 40.

(5) A high-temperature rocket engine using H_2 - O_2 propellant, at a speed of 0.7 M.

(6) A turbofan engine family using an n-octane - N_2O_4 heat source at a speed of 0.7 M. For a total compressor pressure ratio of 15, the optimum bypass ratio and ausiliary compressor pressure ratio was determined graphically from Figure 8.

The results of (2) through (4) are shown in Figure 1 of the text. The minimum SPC of the turbofan was corrected to a speed of 0.5 Mach for comparison with other engines in Figure 1.

Mechanical efficiency of compressor and turbine, nozzle and ram efficiencies, were taken to be 0.9 for all calculations. Burner pressure efficiency and combustion efficiency was assumed to be 0.95. Propeller efficiency, when used, was taken to be 0.8 and includes the efficiency of reduction gearing. Temperature of reactants was taken as 25 deg C., which is near the stagnation temperature of the ambient air. Heat of formation values for reactants were obtained from Refs. 19, 25, and 27. Heat capacity data was obtained from Ref. 31:98-99. Mean heat capacity values were used for the compressor, combustion, and expansion sections of the cycle. The ratio of specific heats for compressor and expansion section was obtained from the relation

$$\gamma = \frac{C_p}{C_p - R_o} \tag{B 50}$$

where R_0 is the universal gas constant.

<u>Sample Calculation</u>

Turboprop and turbojet at 0.5 Mach

$\mathcal{R} = 15$
$T_4 = 2000$ deg F
$T_0 = 32$ deg F
Fuel/ox: $C_8H_{18} + N_2O_4$

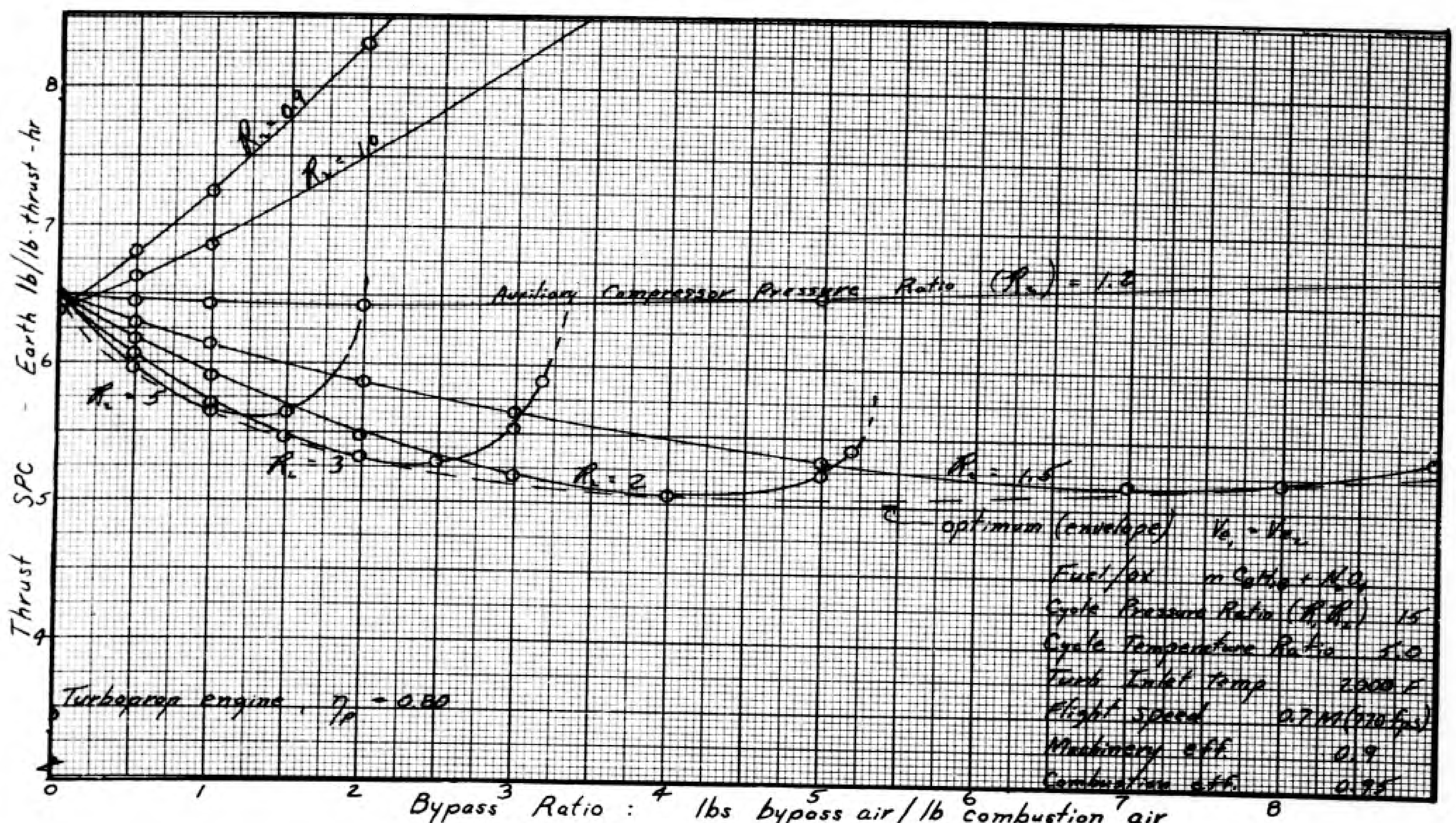


Figure 8

Thrust Specific Fuel/ox Consumption of a Turbofan Bypass Engine

Assuming a stoichiometric reaction

$$C_8H_{18} + 6\tfrac{1}{4}\, N_2O_4 \longrightarrow 6\tfrac{1}{4}\, N_2 + 8CO_2 + 9H_2O$$

From Ref (27)

n-C_8H_{18}(l)	ΔH^{o}_{Form}	= -107,500	Btu/mole
N_2O_4 (g)	ΔH^{o}_{Form}	= + 4010	Btu/mole

From Ref (9)

N_2	ΔH^{2000F}_{Form}	= + 14,550	Btu/mole
CO_2	ΔH^{2000F}_{Form}	= - 146,000	Btu/mole
H_2O	ΔH^{2000F}_{Form}	= - 85,900	Btu/mole

Then from equation (B 15)

$$\Delta H_c^{T_4} = \frac{-9(85,900)\ -8(146,000)\ +\ 6\tfrac{1}{4}(14,550)\ -6\tfrac{1}{4}(4010)\ +\ 107,500}{514\ +\ 6\tfrac{1}{4}\ (28)}$$

$$= -2550 \text{ Btu/lb fuel-ox}$$

at 0.5 M, from equ (B 5)

$$T_2 = 492 \left[1 + \frac{0.4}{2}(0.5)^2 \right] = 517\ ^\circ R$$

From (B 16)

$$T_3 = 517 + \frac{517}{0.9} \left[(15)^{0.286} - 1 \right] = 1190\ ^\circ R$$

From Ref 31:98-99, at mean temperatures of air during compression and in the combustion chamber, the specific heats are

$$C_p = \frac{7.2}{28} = 0.255 \text{ Btu/lb } ^oF$$

$$C_{pa} = \frac{7.8}{28} = 0.28 \text{ Btu/lb } ^oF$$

Then from (B 14)

$$f/a = \frac{(0.28)(2460 - 1190)}{(0.95)(2550)} = 0.147 \frac{\text{lb prop.}}{\text{lb air}}$$

Assuming a mean expansion temperature of 2000 ^oR, from Ref 31:99 the heat capacity of combustion products is

$$C_{p\,\substack{comb\\prod}} = \frac{9(10.1) + 8(13.3) + 6\frac{1}{4}(7.9)}{514 + 6\frac{1}{4}\ (28)} = 0.360 \text{ Btu/lb } ^oF$$

while the heat capacity of expanding hot air is

$$C_{pa}' = \frac{7.9}{28} = 0.282 \text{ Btu/ lb } ^oF$$

Thus the mean heat capacity of the exhaust gases is (approximately)

$$C_p' = \frac{0.282 + (0.147)(0.360)}{1.147} = 0.290 \text{ Btu/lb } ^oF$$

The molecular weight of combustion products is given by

$$MW_{\substack{comb\\prod}} = \frac{9\ (18) + 8(44) + 6\frac{1}{4}(28)}{9 + 8 + 6\frac{1}{4}} = 29.6$$

Thus the molecular weight of the exhaust is

$$MW_{exhaust} = \frac{28.3 + (0.147)\ 29.6}{1.147} = 28.5$$

Then from equation (B 50)

$$\gamma' = \frac{0.290}{0.290 - \frac{1.99}{28.5}} = 1.32$$

From equation (B 13)

$$W_{tn} = (0.9)(778)(0.290)(2460)(1.147)\left[1-\left(\frac{1}{(0.95)(0.9)15}\right)^{0.242}\left(\frac{492}{517}\right)^{\frac{.242}{.286}}\right]$$

$$= 276{,}000 \ \frac{\text{ft lb}}{\text{lb/sec}}$$

while from equation (B 4)

$$W_c = (778)(0.255)\frac{(517)}{0.9}\left[15^{0.286}-1\right] = 133.000 \ \frac{\text{ft lb}}{\text{lb/sec}}$$

Then from equation (B 32) for a turboprop

$$(EHP)_{sp} = \frac{143.000}{550} + \frac{(550)^2(1.147 - 1.60)}{(1100)(32.2)(0.8)^2} = 255 \ \frac{\text{HP}}{\text{lb/sec}}$$

and from equation (B 34)

$$HPSPC = \frac{(3600)(0.147)}{(0.8)\ (255)} = 2.59 \ \frac{\text{lb fuel}}{\text{HP hr}}$$

For a turbojet, from (B 19)

$$Ve = \sqrt{\frac{(2)(32.2)(143{,}000)}{(1.147)}} = 2840 \quad \text{fps}$$

From (B 20)

$$Fs = \frac{(1.147)(2840) - 550}{32.2} = 84.5 \quad \frac{\text{lb thrust}}{\text{lb/sec}}$$

From (B 22) and (B 23)

$$HPSPC = \frac{(3600)(0.147)(550)}{(84.5)(550)} = 6.25 \quad \frac{\text{lb fuel}}{\text{HP} \quad \text{hr}}$$

Appendix C

Performance of an n-Octane/N_2O_4 Turboprop on Mars

From the general equation for turboprop equivalent horsepower, equation (B 32), the variation of equivalent horsepower with compressor pressure ratio is calculated by assuming different values of $\mathcal{R}$. Figure 9 shows the variation with $\mathcal{R}$ at a speed of 0.5 Mach. The calculation is identical to the sample of Appendix B. For comparison, curves for different turbine-inlet temperatures and for turbojet TSPC are shown.

Figure 9 shows a maximum EHP for a compressor pressure ratio of $12\frac{1}{2}$ for a 2000 deg F turbine-inlet temperature. From a comparison of the corresponding SPC curve of Figure 1, a pressure ratio of 15 is selected for a speed of 0.5 Mach.

The variation of specific equivalent horsepower with speed and altitude is then determined by the assumption of constant compressor blade-tip speed (suggested by E. M. Romer). The condition of constant blade-tip speed gives

$$T_2 \left[\mathcal{R}^{\frac{\gamma-1}{\gamma}} - 1 \right] = constant \qquad \text{(C 1)}$$

for any M. From equation (B 16), then

$$T_3 - T_2 = \frac{T_2}{\eta_c} \left[\mathcal{R}^{\frac{\gamma-1}{\gamma}} - 1 \right] = Const. \qquad \text{(C 2)}$$

and hence the compressor temperature rise is constant.

Further, from equation (B 4)

$$\frac{W_c}{J C_p} = \frac{T_2}{\eta_c}\left[R^{\frac{\gamma-1}{\gamma}} - 1\right] = \text{Const} \tag{C 3}$$

Then, neglecting small changes in mean C_p of the compressor gases, the compressor power is constant for all M and altitudes.

Variation of T_0 and sonic velocity with altitude is obtained from Appendix A. Variation of T_2 with M is obtained from equation (B 16). Then from (B 13), (B 14), (B 28), and (B 33), the variation of specific EHP and ESPC is determined as shown by the sample calculation of Appendix B. Variation of ESPC with Mach number and altitude up to 12 Km is shown in Figure 10. This data is used in future range calculations.

The variation of mass rate of flow through a given engine is then determined by assuming constant engine rpm. For constant rpm the volume flow rate of air through the compressor face is essentially constant (Ref 14:387). Then the mass rate of flow is given by

$$\dot{m} = \rho_2 A_2 V_2 = (\text{const})\,\rho_2 \tag{C 4}$$

From the perfect gas law

$$\frac{\rho_2}{\rho_0} = \left(\frac{P_2}{P_0}\right)\left(\frac{T_0}{T_2}\right) \tag{C 5}$$

But from equation (B 12)

$$\frac{P_2}{P_0} = \eta_R \left(\frac{T_2}{T_0} \right)^{\frac{\gamma}{\gamma-1}} \qquad \text{(C 6)}$$

and for an adiaba tic atmosphere

$$\frac{\rho_0}{\rho_{SL}} = \left(\frac{T_0}{T_{SL}} \right)^{\frac{1}{\gamma-1}} \qquad \text{(C 7)}$$

Substituting (C 7) and (C 6) into (C 5)

$$\frac{\rho_2}{\rho_{SL}} = \eta_R \left(\frac{T_2}{T_{SL}} \right)^{\frac{1}{\gamma-1}} \qquad \text{(C 8)}$$

where T_2 is related to speed and T_0 by (B 5) and T_0 is a function of altitude (Appendix A).

Thus for an arbitrary engine developing 1000 EHP at 0.5 Mach at surface level

$$\dot{m} = \frac{EHP}{(EHP)_{sp}} = \frac{1000}{254} = 3.94 \text{ lb/sec}$$

$T_2 = 517^\circ$ R (Appendix B)

$$\rho_{SL} = (0.000206)(32.2) = 0.00662 \text{ lb/ft}^3 \text{ (Appendix A)}$$

So

$$\rho_2 = (0.9)\left(\frac{517}{492}\right)^{2.5} = 0.00673$$

Thus

$$\text{Vol. flow rate} = A_2V_2 = \frac{3.94}{0.00673} = 585 \text{ ft}^3\text{/sec}$$

$$= \text{constant}$$

Then for any speed and altitude

$$\dot{m} = 585\,\rho_2 = (585)(0.9)\left(\frac{T_2}{492}\right)^{2.5}$$

or

$$\dot{m} = 526\left(\frac{T_2}{492}\right)^{2.5} \tag{C 9}$$

From equation (C 9) the variation of $\dot{m}$ with speed and altitude is determined, and from

$$EHP = \dot{m}\,(EHP)_{sp} \tag{C 10}$$

the equivalent horsepower as a function of speed and altitude is calculated and shown in Figure 11.

The static surface EHP of this engine is 887 EHP and from equation (C 1) the static compressor pressure ratio is 16.5.

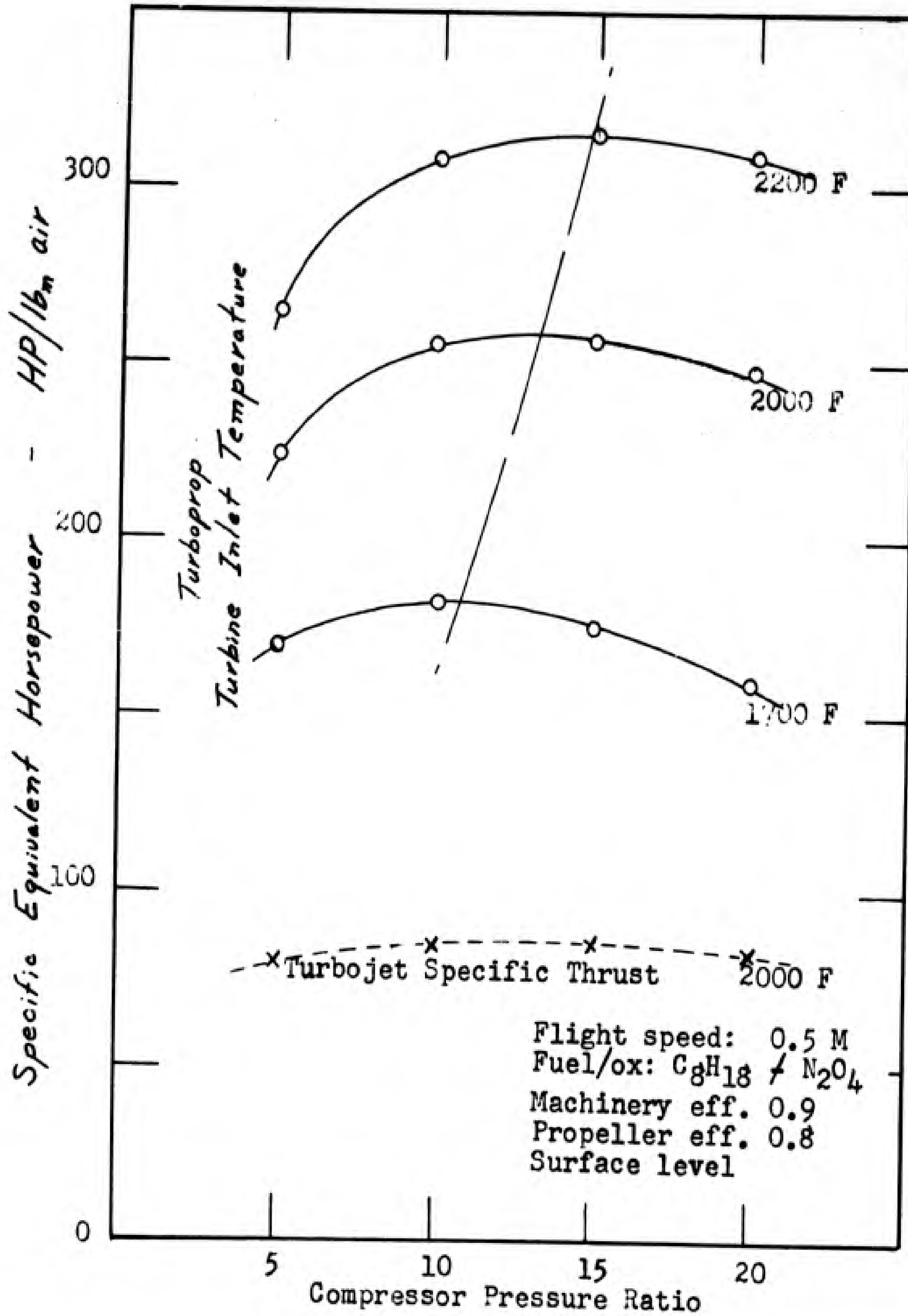


Figure 9

Specific Horsepower of
Jet Engines on Mars

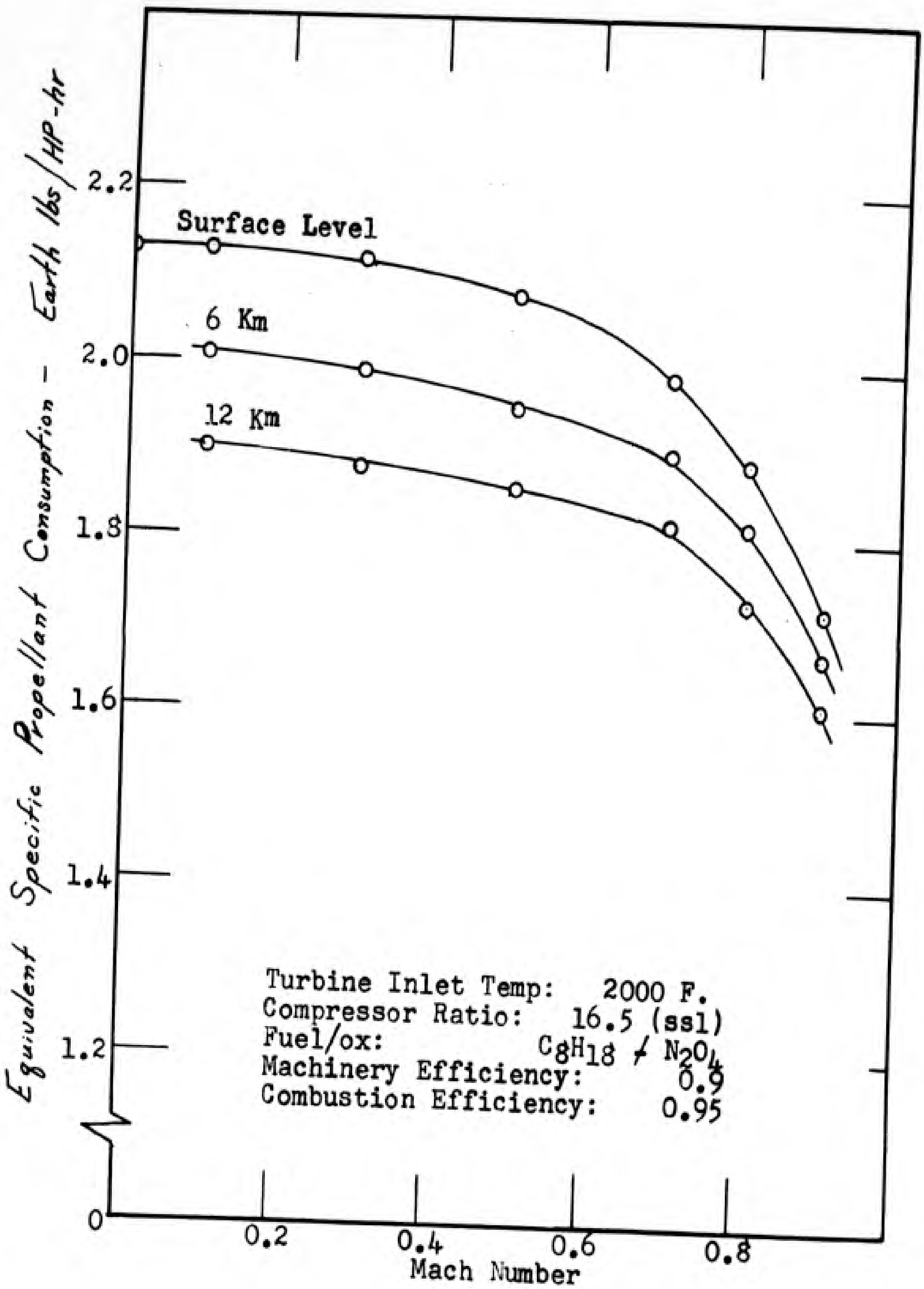


Figure 10

Equivalent Specific Propellant Consumption of a Turboprop on Mars

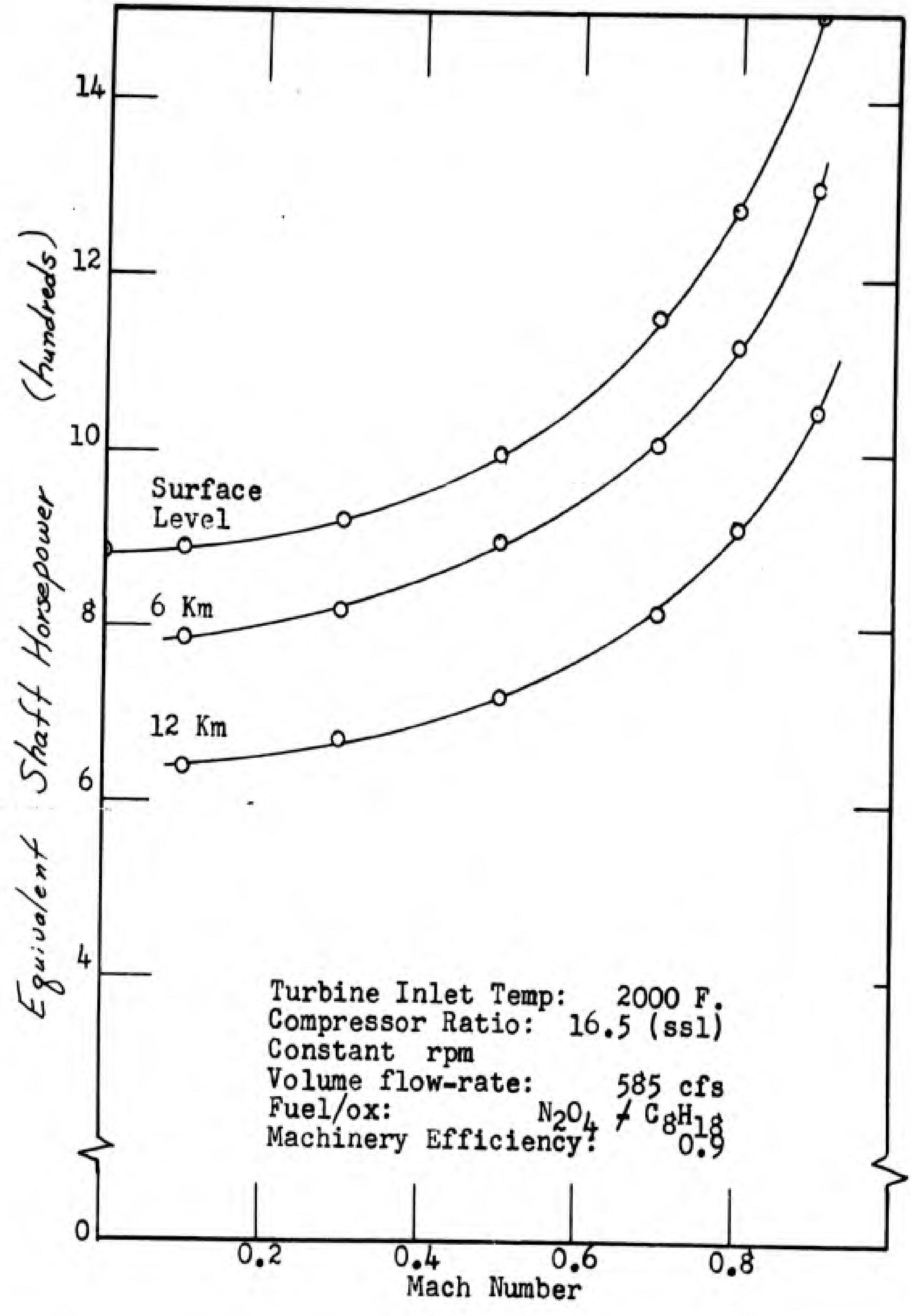


Figure 11

Equivalent Horsepower of a Turboprop Engine on Mars

Appendix D

Estimation of Powerplant Weights

Turboprop

The specific mass of a Martian turboprop engine can be estimated by comparing the horsepower that the engine would develop at Earth sea-level with similar Earth engines. For a given cycle pressure ratio and temperature ratio, the specific horsepower of a Mars engine will be the same on Earth, if the effect of different blade-Reynolds numbers on machinery efficiency may be neglected. Then for a given engine size and rpm (constant volume flow rate)

$$\frac{(EHP)_{♂}}{(EHP)_{\oplus}} = \frac{\rho_{♂}}{\rho_{\oplus}} = \frac{\rho_{o♂}}{\rho_{o\oplus}} \qquad \text{(D 1)}$$

where the subscripts refer to Mars and Earth conditions respectively.

Specific Earth weights for 41 turboprop engines have been tabulated by Lancaster (Ref 23:261-262), for 18 engines by Wilkinson (Ref 38:33), and for 13 engines by Judge (Ref 16:424-425). A graphical plot of specific weight vs. EHP does not reveal any clear-cut empirical relation between weight and power. Specific weights range from 0.38 lb/EHP to 0.83 for 5000 HP class engines, with the lower values being indicated for axial-flow high pressure ratio engines. Thus, assuming the mass penalty of a more complicated fuel/ox metering system is offset by high turbine temperature operation, the value suggested by Hesse (Ref 14:375) of 0.44 lb/EHP is assumed to be reasonable. Then from (D 1) the specific mass of a static turboprop on Mars is given by

$$(0.44)\left(\frac{\rho_{o\oplus}}{\rho_{o♂}}\right) = (0.44)\ \frac{0.002378}{0.000206} = 5.1\ \frac{\text{Earth lbs}}{\text{EHP}}$$

weight of reduction gearing is included in this figure.

Propeller

In general, propeller specific mass is a function of shaft horsepower rating. Specific Earth weights for propellers are tabulated by Teichmann (Ref 32: 261-262). From a general propeller chart (Ref 26:149), it is seen that for a given propeller efficiency, a Martian propeller may be related to an Earth propeller by the approximate relation

$$\left[\frac{J}{C_p^{1/3}}\right]_{\oplus} = \left[\frac{J}{C_p^{1/3}}\right]_{♂} \qquad \text{(D 2)}$$

where J is the advance ratio and C_p is the power coefficient, given by

$$J = \frac{60V}{ND} \qquad \text{(D 3)}$$

$$C_p = \frac{HP}{\rho N^3 D^5} \qquad \text{(D 4}$$

where N is rpm and D is propeller diameter.

Substituting into (D 2) gives

$$\left[\frac{\rho^{1/3} V D^{2/3}}{(HP)^{1/3}}\right]_{\oplus} = \left[\frac{\rho^{1/3} V D^{2/3}}{(HP)^{1/3}}\right]_{♂} \qquad \text{(D 5)}$$

Then comparing Earth and Mars propellers that absorb the same power at the same velocity gives

$$\left[\rho^{1/3} D^{2/3}\right]_{\oplus} = \left[\rho^{1/3} D^{2/3}\right]_{♂} \qquad \text{(D 6)}$$

from which the diameter of a Mars propeller must be related to an Earth propeller by

$$\frac{D_{♂}}{D_{\oplus}} = \sqrt{\frac{\rho_{\oplus}}{\rho_{♂}}} \qquad \text{(D 7)}$$

Substituting the respective densities gives

$$\frac{D_{♂}}{D_{\oplus}} = \sqrt{\frac{2378}{206}} = 3.4$$

Thus the diameter of a Mars propeller must be about 3.4 times the diameter of an Earth propeller in order to have the same efficiency at the same power loading and speed. Assuming propeller mass is proportional to diameter, then the specific mass of a Mars propeller is 3.4 times the specific mass of an Earth propeller for the same power. From Ref 32:261-262, the specific mass of 4-blade Earth propellers at low powers varies from 0.25 to 0.30 lb/HP. Thus a reasonable value for specific mass of propeller on Mars is 1 Earth lb/HP.

The specific mass of a turboprop propulsion unit including propeller is then 6.1 at static conditions. Assuming a cruise speed of 0.5 Mach (550 fps) and a propeller efficiency of 0.8, from the power chart for a turboprop (Figure 11), the cruise specific mass is

$$\frac{(6.1)(887)}{(0.8)(1000)} = 6.75 \ \frac{\text{Earth lb}}{\text{THP}}$$

Turbojet

Several empirical formulas for the specific Earth weight of turbojet engines have been developed:

From Corning (Ref 6:chap 2, p.31)

$$\text{Weight (lbs)} = 2.8(T_{ssl})^{.75} \qquad \text{(D 8)}$$

where T_{ssl} is static sea-level thrust.

From Corning (Ref 7:chap 2, p. 31)

$$\text{Weight (lbs)} = 1.95\ (10^{-3})\ T_{ssl}^{\ 1.55} \qquad \text{(D 9)}$$

From the aircraft design course of the Institute of Technology

$$\text{Specific wt.} = 0.15 + 5 \times 10^{-6} T_{ssl} \qquad \text{(D 10)}$$

From Figure 9 for a turbojet at 0.5 M and $\pi_c = 15$

$$F_s = 84.6$$

then using relation (C 1) and the turbojet equations of Appendix B

$$F_{s_{static}} = 107$$

Then for an engine developing 500 lb thrust at 0.5 M, the static thrust at Earth sea-level may be approximated by

$$T_{ssl} = (500) \frac{F_{s_{static}}}{F_s}$$

$$= (500) \frac{(107)(.0765)}{(84.6)(.00673)}$$

$$= 7200 \text{ lbs thrust}$$

The weights from equations (D 8 - D 10) are then

(D 8)	2200 lbs on Earth
(D 9)	1890 lbs on Earth
(D 10)	1360 lbs on Earth

Using a mean value from the above of 1820 lbs the cruise specific mass of the turbojet on Mars is 3.63 Earth lb/lb thrust = Earth lb/HP.

<u>Others</u>

Mass of a turbofan is estimated as 125% of turbojet mass for the same thrust level or 4.55 Earth lb/HP at 0.5 Mach.

A specific mass of 0.15 Earth lb/EHP for rocket-turbine engines on Mars is suggested by Krase for large units (Ref 21:13). To this must be added the same propeller as for a turboprop, thereby giving the rocket-turbine unit a specific mass of 1.44 Earth lb/THP at a propeller efficiency of 0.8.

<u>Total Specific Mass of Powerplant plus Fuel</u>

The total specific mass per horsepower is then given by

$$(\text{sp mass})_{\text{pp + fuel}} = (\text{sp mass})_{\text{pp}} + (\text{THPSFC})(t) \tag{D 11}$$

From the SPC values at 0.5 Mach (Figure 1), the engine and fuel mass required per HP is shown as a function of flight time in Figure 2 of the text.

The unit of mass used in these calculations is that mass which weighs one pound on Earth.

Appendix E

Range and Field Length of Martian Aircraft As a Function of Wing Loading and Aspect Ratio

Range

The range of an aircraft flying in the Martian atmosphere can be estimated by a modification of the Breguet range equation (Ref 4:26). For a vehicle of mass m

$$\frac{dm}{dt} = -(THPSPC)(THP) \qquad (E\ 1)$$

where THPSPC is thrust horsepower SPC in terms of Earth lbs. of propellant per HP-hr.

$$THPSPC = \frac{ESPC}{\eta_p} \qquad (E\ 2)$$

But

$$THP = \frac{DV}{375} \qquad (E\ 3)$$

for V in miles per hour, and

$$Range = \int V\,dt \qquad (E\ 4)$$

Combining these four equations gives

$$Range = -\int_{m_0}^{m} \frac{375\,\eta_p\,dm}{D\,(ESPC)} \qquad (E\ 5)$$

For cruising flight

$$L = W \qquad (E\ 6)$$

where W is vehicle weight on Mars. But

$$W = mg' \tag{E 7}$$

where m is the unit of mass that weighs one pound on Earth and g' is the relative gravity on Mars with respect to Earth. Combining with equation (E 5) then gives

$$Range = 375 \int_{m}^{m_0} \frac{\eta_p}{ESPC} \left(\frac{L}{D}\right)\left(\frac{1}{g'}\right) \frac{dm}{m} \tag{E 8}$$

In general ESPC varies with both speed and altitude for a turboprop engine (Fig 10). As a first approximation, however, some mean value for (ESPC) may be selected. Then for flight at constant (L/D) and η_p

$$\text{Range} = \frac{375}{ESPC}\left(\frac{\eta_p}{g'}\right)\left(\frac{L}{D}\right) \log \frac{m_0}{m} \tag{E 9}$$

Now

$$\frac{m_0}{m} = \frac{W_0}{W} \tag{E 10}$$

and for a fuel weight W_f

$$W = W_0 - W_f \tag{E 11}$$

Substituting into (E 9)

$$\text{Range in miles} = \frac{375}{ESPC}\left(\frac{\eta_p}{g'}\right)\left(\frac{L}{D}\right) \log \frac{1}{1 - \frac{W_f}{W_0}} \tag{E 12}$$

Now by definition of lift and drag coefficients

$$\frac{L}{D} = \frac{C_L}{C_D} \qquad \text{(E 13)}$$

where, for cruising flight, from equation (E 6)

$$C_L = \frac{W/S}{\frac{1}{2}\rho V^2} \qquad \text{(E 14)}$$

and

$$C_D = C_{D_P} + \Delta C_{D_{comp}} + C_{D_i} \qquad \text{(E 15)}$$

where

$$C_{D_i} = \frac{C_L^2}{\pi e AR} \qquad \text{(E 16)}$$

Let span efficiency factor (e) be constant at 0.85 (Ref 7: chap 7, p. 13). Drag rise due to compressibility, $\Delta C_{D_{comp}}$, is determined by flight Mach number, t/c, AR, and C_L. Parasite drag coefficient, C_{DP}, is a function of wetted area of the airframe and Reynolds number of flight.

Fuel weight can be found from

$$W_f = W_o - W_{struct.} - W_{pp} - W_{payload} - W_{fixed\ equip.} \qquad \text{(E 17)}$$

From Corning (Ref 7: chap 2, p.29), for a given sweepback and taper ratio, structural weight can be expressed as a function of W_o, W/S, t/c, and AR. Powerplant weight, W_{pp}, is a function of powerplant specific weight, already determined (Appendix D), and power loading. Let power loading be specified by the power required at cruise. Since

$$HP_{reqd} = \frac{DV}{550} = \frac{\frac{1}{2}\rho S V^3 C_D}{550} \qquad \text{(E 18)}$$

then powerplant weight is a function of speed, W/S and C_D. Fixed equipment can be assumed to be a function of gross weight (W_o). Then for a given payload, fuel weight is a function of W_o, W/S, t/c, AR, C_D, ρ, and speed.

Consider a family of vehicles with W/S and AR as the independent variables:

Cruise-climb	constant (L/D)
initial cruise	550 fps@ 3 Km (0.51M)
sweepback ($\frac{1}{4}$ chord)	0
taper ratio	0.4
payload	380 lbs wt (1000 Earth lbs)
gross weight	10,000 lbs wt on Mars
engines	two turboprop
crew	two
fuselage length	3/4 of wing span
average fuselage diameter	5 ft
$M_{CR_o} = M_{cruise} + \Delta M$ equal to 50 mph	

For a given W/S and AR, wing-thickness ratio (t/c) can be determined from the general charts of Corning (Ref 6: chap 2, pp 2-13). $\Delta C_{D_{comp}}$ is then obtained from p. 39 of the same reference. With W/S, t/c, and AR specified, wetted area of wing and fuselage is known for the above conditions. From a study of turboprop engine sizes (Refs 16; 23; 38), a nacelle length of 11 ft. and cross-sectional area of 9 ft^2 are reasonable values for 5000 HP class engines on Earth. From Corning (Ref 7: chap 4, pp. 38-39), reasonable values for tail-surface area may be given by

$$S_{H.T.} = \frac{2}{AR} S_{wing} \qquad \text{(E 19)}$$

and

$$S_{V.T.} = 0.1\, S_{wing} \qquad \text{(E 20)}$$

Thus the wetted area of the aircraft is known as a function of wing loading and aspect ratio.

The skin friction coefficient, C_f, for each major component is a function of Reynolds number based on component length. Assuming turbulent flow because of propeller slipstream, C_f is given by the equation of Prandtl (Ref 3:70)

$$C_f = \frac{0.455}{(\log_{10} Re_n)^{2.58}} \qquad \text{(E 21)}$$

For the wing, the characteristic length is the mean geometric chord, $\bar{c}$. But

$$\bar{c} = \sqrt{\frac{S}{AR}} \qquad \text{(E 22)}$$

For the fuselage

$$\ell_{fuse} = 3/4 b = 3/4 \sqrt{S(AR)} \qquad \text{(E 23)}$$

For the nacelles, l_{nac} = 11 ft. For both tail surfaces at low tail aspect ratios, characteristic length for each will be nearly the same. Then assume

$$\ell_{tail} = \bar{c}_{V.T.} = \bar{c}_{H.T.} = \sqrt{0.1\, S_{wing}} \qquad \text{(E 24)}$$

Structural weight may be estimated by an empirical equation of Corning (Ref 7: chap 2, p. 29). In general, structural mass will be a function of gross weight (not mass). Thus the equation is modified for use on Mars to

$$\text{weight of structure in lbs. wt.} = g'\left[0.16\,W_0 + \frac{9.8}{\left(\frac{100}{W/S}\right)^{.63}}\left(\frac{W_0}{W/S}\right)\right](K_{t/c})(K_{AR}) \qquad \text{(E 25)}$$

where K terms are correction factors for wing-thickness ratio and aspect ratio. Weight of fixed equipment is taken as 5% of gross weight plus two crew at 230 Earth lbs each.

Sample Calculation

Let Martian wing loading be 15 lbs wt/ft^2 and aspect ratio equal 16. From equation (E 14) and using ρ at 3 Km

$$C_L = \frac{15}{\frac{1}{2}(.90)(2.06)(10^{-4})(550)^2} = 0.534$$

then

$$C_D = \frac{(0.534)^2}{\pi(.85)(16)} = 0.0067$$

$$\Delta M(50\text{mph}) = (50)\left(\frac{88}{60}\right)\left(\frac{1}{1100}\right)\left(\frac{1}{0.97}\right) = 0.07$$

From Fig 2:23 (Ref 6)

$$\Delta C_{D_{comp}} = 0.0005$$

Mach number of initial cruise is

$$M = \frac{550}{1100}\left(\frac{1}{0.97}\right) = 0.51$$

Then from Figs. 2:7, 2:8, 2:9 (Ref 6)

$$t/c = 0.153$$

From Ibele (Ref 15:385), the viscosity of the Martian atmosphere at 3 Km is 3.44 (10^{-7}) slugs/ft-sec. Then

$$Re_n = \frac{(2.06)(10^{-4})(.90)(550)L}{(3.44)(10^{-7})} = 297{,}000\ L \qquad \text{(E 26)}$$

where (L) is component length.

Now

$$C_{D_p} = C_{f_{wing}} \frac{A_{wing}}{S} + C_{f_{fuse}} \frac{A_{fuse}}{S} + C_{f_{nac}} \frac{A_{nac}}{S} + C_{f_{tail}} \frac{A_{tail}}{S} \qquad \text{(E 27)}$$

where C_f is given by equation (E 21). Wetted area of wing is corrected for blanketing effect of fuselage and nacelles and for thickness ratio (K_{th}). Then

$$\frac{C_{D_p}}{0.455} = \frac{\left[1 - \frac{5}{\sqrt{S\,AR}} - 4\sqrt{\frac{9}{\pi S\,AR}}\right] K_{th}}{\left(\log_{10} 297{,}000 \sqrt{\frac{S}{AR}}\right)^{2.58}} + \qquad \text{(E 28)}$$

$$\frac{\frac{15\pi}{4}\sqrt{\frac{AR}{S}}}{\left[\log_{10} 297{,}000\left(\frac{3}{4}\right)\sqrt{S\,AR}\right]^{2.58}} +$$

$$\frac{(2)(2)(11)\ \frac{\sqrt{9\pi}}{S}}{\left[\log_{10}(297{,}000)(11)\right]^{2.58}} + \frac{\left(\frac{2}{AR} + 0.1\right) K_{th}}{\left(\log_{10} 297{,}000\sqrt{0.1\,S}\right)^{2.58}}$$

Now

$$S = \frac{10,000}{15} = 667 \text{ ft}^2$$

From Fig. 6:3 (Ref 7)

$$K_{th} = 2.06$$

for wing and tail thickness. Substituting into (E 28) with AR = 16 gives

$$\frac{C_{Dp}}{0.455} = 0.0302$$

C_{Dp} should be increased by a factor of 10% to account for miscellaneous interferences (Ref 7: chap 6, p. 5). Thus

$$C_{Dp} = (0.0302)(0.455)(1.10) = 0.0166$$

and

$$C_D = 0.0166 + 0.0005 + 0.0067 = 0.0238$$

So

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{0.534}{0.0238} = 22.4$$

From (E 18)

$$HP_{reqd} = \frac{1}{2}(2.06)(10^{-4})(.90)\frac{(550)^3}{550}(667)(.0238)$$

$$= 445 \quad HP$$

From Figure 11, for the same propeller efficiency, the power required at 550 fps at surface level is then

$$THP = (448)\ \frac{(1000)}{(950)} = 470\ HP$$

Let engine power available be 110% of power required. From Appendix D, the specific weight on Mars of a turboprop at 550 fps is

$$(0.38)\ 6.75\ \frac{\text{Earth lb}}{THP} = 2.56\ \frac{\text{lb. wt}}{THP}$$

Thus powerplant weight is

$$(1.10)(469)(2.56) = 1,320 \text{ lbs. wt.}$$

Structural weight is given by (E 25). $K_{t/c}$ and K_{AR} are given by Figs 2:19 and 2:20 (Ref 6):

$$W_{struct} = (0.38)\left[1600 + \frac{9.8}{\left(\frac{100}{15}\right)^{.63}}\ \frac{10,000}{15}\right](.92)(1.04)$$

$$= 1300 \text{ lbs wt}$$

Weight of fixed equipment is

$$(0.05)(10,000) + (2)\ (230)(0.38) = 675 \text{ lbs wt.}$$

Payload weight is 380 lbs wt. Thus weight of fuel plus fuel system is

$$10,000 - 1320 - 1300 - 675 - 380 = 6325 \text{ lbs wt}$$

For this large fuel fraction, let fuel system weight be 0.03 times fuel weight. Then

$$W_f = \frac{6325}{1.03} = 6140 \text{ lbs wt.}$$

For cruise at constant (L/D), assume a mean value for ESPC at 0.5 Mach and

6 Km altitude. From Figure 10, ESPC = 1.95 Earth lbs/HP-hr. Then for a constant propeller efficiency of 0.8, from equation (E 12)

$$Range = \frac{(375)(0.8)(22.4)}{(1.95)(0.38)} \log \frac{1}{1 - 0.614}$$

$$= 8650 \quad miles$$

Figure 3 shows calculated range for W/S of 5, 10, 15, and 20, and at aspect ratios from 4 to 24.

Landing Field Length

Landing field length is based upon the horizontal distance from clearing a 50 ft. obstacle to touchdown (S_{50}), plus the ground roll distance to a full stop (S_G). The total distance is then multiplied by a safety factor of 1/0.7 to allow for variations in pilot technique. The loss of kinetic and potential energy of the vehicle during approach is equal to the work done by retarding forces:

$$\frac{W}{2 g_{\male}} (V_{50}^2 - V_{TD}^2) + 50\,W = \frac{F S_{50}}{\cos \gamma} \qquad \text{(E 29)}$$

where V_{50} is speed clearing a 50 ft. obstacle, V_{TD} is touchdown speed, F is retarding force (drag), and γ is the angle of approach given by

$$\gamma = \arctan \frac{1}{L/F} \qquad \text{(E 30)}$$

The aircraft load factor during approach is

$$n = \frac{L}{W} \quad (E\ 31)$$

but

$$L = W \cos \gamma \quad (E\ 32)$$

thus

$$\cos \gamma = n \quad (E\ 33)$$

Because of the possibility of vertical currents near the Martian surface, it is desirable to touchdown at a speed well above stall speed, Vso. Therefore, let

$$V_{50} = 1.3\ V_{SO} \quad (E\ 34)$$

and

$$V_{TD} = 1.2\ V_{SO} \quad (E\ 35)$$

Combining (E 31) through E 35) with (E 29) gives

$$S_{50} = \frac{L}{F}\left[0.0102\ V_{50}^{2} + 50\right] \quad (E\ 36)$$

The ground run is given by

$$S_G = \frac{V_{TD}^{2}}{-2a} \quad (E\ 37)$$

where a is ground deceleration rate. For the retarding force all drag, then combining (E 36) and (E 37)

$$S_F = \frac{L}{D}\left[0.0102\, V_{so}^2 + 50\right] + \frac{0.72\, V_{so}^2}{-a} \qquad \text{(E 38)}$$

where

$$V_{so}^2 = \frac{\left(\frac{mW}{S}\right)_{land}}{\frac{1}{2}\rho\, C_{L_{Max}}} \qquad \text{(E 39)}$$

and for landing, wing loading is

$$\left(\frac{W}{S}\right)_{land} = \left(\frac{W}{S}\right)_{T.O.}\left[1 - \frac{W_f}{W_o}\right] \qquad \text{(E 40)}$$

Let $C_{L_{max}} = 3.0$. By use of reverse propeller thrust or landing skids, deceleration rate may be taken as -10 ft/sec^2 (Ref 7: chap 8, p.10). Although in general (L/D) during approach will be a function of aspect ratio, it may be assumed constant by appropriate use of air-brakes during approach. Assuming an (L/D) of 9 during approach, then $\cos\gamma \approx 1$. Landing field lengths for wing loadings of 5, 10, 15, and 20 are shown in Figure 4. The aspect ratio for maximum range was used to determine W_f/W_o.

Sample Calculation

Let $(W/S)_{t.o.} = 15$

At the aspect ratio for maximum range, 20, the fuel ratio, W_f/W_o, was calculated to be 0.610. Then from (E 40)

$$(W/S)_{land} = 15\ (1-0.610) = 5.85$$

Then from (E 39)

$$V_{50}^2 = \frac{(5.85)}{\frac{1}{2}(2.06)(10^{-4})(3.0)} = 1.89\,(10^4)$$

From (E 38)

$$S_F = (9)\left[(.0102)(1.89)(10^4) + 50\right] + \frac{(0.72)(1.89)(10^4)}{10}$$

$$= 3550 \text{ feet}$$

Using a safety factor of 1/0.7:

Landing field length = 5080 feet

The same calculation for an AR of 16 gives a landing field length of 5030 feet.

Take-Off Field Length

From

$$dv = adt \qquad \text{(E 41)}$$

and

$$ds = vdt \tag{E 42}$$

then

$$S_{T.O.} = \int_0^{V_{TO}} \frac{V\,dV}{a} \tag{E 43}$$

where

$$a = \frac{T-R}{W/g_0} \tag{E 44}$$

T is propeller thrust during take-off

R is total ground and air resistance

Now

$$R = D + \mu\,(W-L) \tag{E 45}$$

where μ is coefficient of rolling friction. Equation (E 45) may be rewritten

$$\begin{aligned} R &= D - \mu L + \mu W \\ &= \tfrac{1}{2}\rho V^2 S\,(C_D - \mu C_L) + \mu W \\ &= \tfrac{1}{2}\rho V^2 S\left(C_{D_0} + \frac{C_L^2}{\pi e\,AR} - \mu C_L\right) + \mu W \end{aligned} \tag{E 46}$$

From the method of Hartman (Ref 26:196), the optimum C_L for minimum resistance during take-off roll is obtained by differentiating (E 46)

$$\frac{dR}{dC_L} = \frac{1}{2}\rho V^2 S\left(\frac{2C_L}{\pi e AR} - \mu\right) = 0 \qquad \text{(E 47)}$$

from which

$$C_L = \frac{\mu \pi e AR}{2} \qquad \text{(E 48)}$$

Ground effect on induced drag can be approximated by assuming a span-efficiency factor, e, of unity (Ref 26:196). Then E 46 becomes

$$R_{min} = \frac{1}{2}\rho V^2 S\left(C_{D_0} - \frac{\mu^2 \pi AR}{4}\right) + \mu W \qquad \text{(E 49)}$$

combining with (E 43) and E 44) gives

$$S_{T.O.} = \frac{W}{g_\delta}\int_0^{V_{TO}} \frac{V\,dV}{(T-\mu W) - \frac{1}{2}\rho V^2 S\left(C_{D_0} - \frac{\mu^2 \pi AR}{4}\right)} \qquad \text{(E 50)}$$

Assuming constant thrust and vehicle weight during take-off roll, equation (E 50) integrates to

$$S_{TO} = \frac{W/S}{g_{\delta}\rho\left(C_{D_0} - \frac{\mu^2 \pi AR}{4}\right)} \log\left[\frac{(T-\mu W)}{(T-\mu W) - \frac{1}{2}\rho S V_{TO}^2\left(C_{D_0} - \frac{\mu^2\pi AR}{4}\right)}\right] \quad \text{(E 51)}$$

For propeller driven aircraft, .9 $C_{L_{max}}$ may be used as take-off C_L (Ref 7: chap 8, p.10). Thus

$$V_{TO}^2 = \frac{W/S}{\frac{1}{2}\rho C_{L_{max}}(0.9)} \quad \text{(E 52)}$$

For a turboprop engine, the static thrust can be approximated by the relation (Ref 23:223)

$$T_{ssl} = 2.5\ (EHP)_{ssl} \quad \text{(E 53)}$$

Substituting (E 52) and (E 53) into (E 51) gives

$$S_{TO} = \frac{W/S}{g_{\delta}\rho\left(C_{D_0} - \frac{\mu^2 \pi AR}{4}\right)} \log\left[\frac{2.5(EHP) - \mu W}{2.5(EHP) - \mu W - \frac{W}{.9 C_{L_{max}}}\left(C_{D_0} - \frac{\mu^2\pi AR}{4}\right)}\right] \quad \text{(E 54)}$$

A reasonable value of μ for a semi-smooth runway is 0.05 (Ref 17: chap 14, p. 62). And

$$C_{D_o} = C_{D_p} + \Delta C_{D_{L.G.}} + \Delta C_{D_{flaps}} \qquad \text{(E 55)}$$

Consider the vehicle with wing loading of 15 and aspect ratio of 16. From Figure 2-68 (Ref 26)

$$\Delta C_{D_{L.G.}} = \frac{6}{667} = 0.009$$

$$\Delta C_{D_{flaps}} = 0.030 \qquad (30^\circ \text{ Fowler Flaps})$$

From the range calculation (p.82), $C_{Dp} = 0.0166$ and $(EHP)_{cruise}$ $= \frac{(445)(1.10)}{(0.8)} = 612$ HP

Then

$$C_{D_o} = 0.0166 + 0.03 + 0.009 = 0.0556$$

From Figure 11

$$(EHP)_{ssl} = (612)\frac{(887)}{(950)} = 571$$

Use of water/alchol injection may increase static EHP by 10% (Ref 23:225). Then

$$(EHP)_{t.o.} = (571)(1.10) = 630$$

Take-off thrust is then

$$T = (2.5)(630) = 1570 \text{ lbs.}$$

and

$$\frac{\mu^2 \pi AR}{4} = \frac{(.05)^2 \pi (16)}{4} = 0.0314$$

Then for $C_{Lmax} = 3.0$ and $g_\sigma = 12.2$ ft/ sec^2, take-off field length from equation (E 54) is

$$S_{T.O.} = \frac{15}{(12.2)(2.06)(10^{-4})(.0556 - .0314)} \times$$

$$\text{Log}\left[\frac{1570 - (.05)(10,000)}{1570 - 500 - \frac{10,000}{(.9)(3.0)}(.0556 - .0314)}\right]$$

$$S_{t.o.} = 21,000 \text{ feet}$$

Appendix F

Layout Calculations

Propeller

From a general propeller chart, it is seen that propeller efficiency is a function of the parameter $J/C_p{}^{3}$ (Ref 26:149), where J is advance ratio and C_p is power coefficient. For rpm of N

$$J = \frac{60V}{ND} \tag{F 1}$$

while

$$C_p = \frac{Power}{\rho N^3 D^5} = \frac{(550)(HP)}{\rho \left(\frac{N}{60}\right)^3 D^5} \tag{F 2}$$

thus

$$\frac{J}{C_p{}^{1/3}} = \frac{\rho^{1/3} V D^{2/3}}{(550\ HP)^{1/3}} \tag{F 3}$$

From Figure 3-20 (Ref 26), for high propeller efficiencies at low speeds let $(J/C_p{}^{1/3})$ be 3.5 at V = 550 fps and 3 Km altitude. From Appendix E, EHP available at cruise is 612 HP, or 306 HP per engine. For a nominal propeller efficiency of 0.8, nozzle specific thrust can be estimated from Appendix B by

$$(T_N)_{sp} = \left[\frac{1 + f/a}{\eta_p} - 1\right] \frac{V_0}{g_c} \tag{F 4}$$

So

$$(T_N)_{sp} = (THP_N)_{sp} = \left[\frac{1.15}{0.8} - 1\right]\frac{550}{32.2} = 7.5 \quad HP/lb$$

But from Figure 9, at 550 fps $(EHP)_{sp}$ is 254 HP/lb. Then turboprop shaft horsepower is given by

$$306\left[1 - \frac{(7.5)}{(254)(0.8)}\right] = 294 \ HP$$

For a reduction gear efficiency of 0.95, then power delivered to the propeller shaft is

$$(294)(0.95) = 280 \ HP$$

From (F 3) the required propeller diameter is

$$D = \frac{(3.5)^{3/2}(550)^{1/2}(280)^{1/2}}{(2.06)^{1/2}(10^{-2})(.90)^{1/2}(550)^{3/2}} = 14.4 \ feet$$

Now the blade tip Mach number is given by

$$M_{tip} = M_{cruise}\sqrt{1 + \frac{\pi^2}{J^2}} \qquad (F\ 5)$$

Then if blade tip speed is limited to 0.9 M,

$$J_{min} = \frac{\pi}{\sqrt{\left(\frac{0.9}{0.51}\right)^2 - 1}} = 2.16$$

Then

$$(ND)_{max} = \frac{(60)(550)}{(2.16)} = 15,300 \text{ fpm}$$

$$N_{max} = \frac{15,300}{14.4} = 1060 \text{ rpm at } 550 \text{ fps}$$

and

$$C_p = \frac{(2.16)^3}{(3.5)^3} = 0.235$$

For a 4-blade single rotation propeller, the total activity factor will be 600, and the power adjustment factor will be 0.8 (Ref 26:150). Then

$$C_{px} = \frac{0.235}{0.8} = 0.294$$

Then from Figure 8:4 (Ref 7), efficiency is 0.86. Correcting for profile drag losses (Figure 8:5), $\Delta\eta = -0.02$. Thus net propeller efficiency is 0.84. Then overall propeller - reduction gear efficiency is

$$\eta_p = (0.84)(0.95) = 0.80$$

which was originally assumed in all previous calculations. Propeller efficiency at all speeds, altitudes and power levels can now be determined immediately from equations (F 3), (F 4), (F 5), Figure 9, and the general propeller charts for the given propeller diameter of 14.4 ft. (Ref 7: chap 8, pp 7-9). Then from EHP values of Figure 11, the thrust horsepower available at all speeds and altitudes may be determined.

<u>Mean</u> <u>Aerodynamic</u> <u>Chord</u>

The length of M.A.C. is given by

$$M.A.C. = 2/3\left(C_r + C_t - \frac{C_r C_t}{C_c + C_t}\right) \qquad \text{(F 6)}$$

where C_r is root chord and C_t is tip chord. For a trapezoidal half-wing of taper ratio 0.4

$$\frac{S}{2} = \tfrac{1}{2}\,(C_r + C_t)\ b/2 \qquad \text{(F 7)}$$

$$\frac{C_t}{C_r} = 0.4$$

For $S = 667\ ft^2$ and $AR = 16$

$$C_r = \frac{2(667)}{\sqrt{(667)(16)}}\left(\frac{1}{1+0.4}\right) = 9.23 \text{ feet}$$

$$C_t = (9.23)(0.4) = 3.69 \text{ feet}$$

Then

$$M.A.C. = 2/3\left[(12.92) - \frac{(3.69)(9.23)}{12.92}\right]$$

$$= 6.87 \text{ feet}$$

Lateral position of M.A.C. is given by

$$\frac{y}{b/2} = \frac{1}{3}\,\frac{(c_r + 2c_t)}{(c_r + c_t)} \qquad \text{(F 8)}$$

Then

$$y = 1/3\,\frac{(9.23 + 7.38)}{(12.92)}\;\frac{(103)}{2} = 22.1 \text{ feet}$$

__Tail__ __Surfaces__

Tail area is selected to provide static stability. Preliminary estimates are obtained from Corning (Ref 7: chap 4, p.38).

$$S_{HT} = C_{HT}\,\frac{(S_{wing})(M.A.C.)_{wing}}{l_{HT}} \qquad \text{(F 9)}$$

$$S_{VT} = C_{VT}\,\frac{(S_{wing})\,b}{l_{VT}} \qquad \text{(F 10)}$$

where C_{ht} is horizontal tail volume based on wing M.A.C. and C_{vt} is vertical tail volume based on wing span.

For $l_{ht} = l_{vt} = b/2$

$C_{ht} = 1$

$C_{vt} = 0.075$

Then

$$S_{ht} = \frac{(1)(667)(6.87)}{(51.5)} = 89.1 \text{ ft}^2$$

$$S_{vt} = \frac{(0.075)(667)(103)}{(51.5)} = 100 \text{ ft}^2$$

Fuel Storage

Space available for fuel storage may be approximated from Corning (Ref 7: chap 2, pp 58-60). For wet wing storage from tip to tip between 15% and 65% chord, from Figure 2:29 (Ref 7), available fuel volume is 2180 gallons. From Table III, the approximate bulk specific gravity of propellants is 1.19. Then the weight of fuel that can be stored in the wing is

$$\frac{(2180)}{(7.48)} (62.4)(1.19) = 22{,}300 \text{ Earth lbs}$$

$$= 8500 \text{ lbs wt on Mars}$$

Gust Load Factor

Load factor due to gusts can be estimated from Corning (Ref 7: chap 12, pp 5-7)

$$n = 1 + \frac{\rho a K u V}{2(W/S)} \qquad \text{(F 11)}$$

where

a is slope of lift curve $(dC_L/d\alpha)$

u is peak gust velocity

V is flight speed

$$K = \frac{0.88\,\mu_g}{5.3 + \mu_g}$$ is gust alleviation factor (F 12)

$$\mu_g = \frac{2\,(W/S)}{\rho\,\bar{c}\,a\,g}$$ is airplane mass ratio (F 13)

$\bar{c}$ is mean geometric chord

Slope of lift curve is given by

$$a = f\,\frac{\bar{a}_o}{1 + \left(\frac{57.3\,\bar{a}_o}{\pi\,AR}\right)} \qquad \text{(F 14)}$$

where $\bar{a}_o$ is slope of 2-D lift curve

f is a correction factor for taper ratio

For the selected airfoil, $\bar{a}_o$ = 0.11 deg-1

From Figure 2-55 (Ref 26, f = 0.997

Then

$$a = \frac{(.997)(0.11)}{1 + \frac{(57.3)(0.11)}{16\pi}} = 0.0975 \text{ deg}^{-1}$$

$$= 5.58 \text{ per radian}$$

$$\bar{c} = \sqrt{\frac{S}{AR}} = \sqrt{\frac{667}{16}} = 6.46 \text{ feet}$$

Then for fully loaded aircraft at surface level

$$\mu_g = \frac{2(15)}{(2.06)(10^{-4})(6.46)(5.58)(12.2)} = 330$$

$$K = \frac{(0.88)(330)}{330 + 5.3} = 0.866$$

$$n = 1 + \frac{(2.06)(10^{-4})(5.58)(.866)}{(2)(15)} u V$$

$$= 1 + 0.332(10^{-4}) u V$$

Table IV shows n as a function of u and V

Weight Estimation

Empirical weight equations used in aircraft design course of the Institute of Technology are

$$W_{wing} = K_1 \sqrt{n}\; W^{1.15} \left(\frac{1+\lambda}{t/c}\right)^{0.4} \frac{1}{(W/S)^{0.6}} \frac{(AR)^{0.5}}{\cos \Lambda} \qquad \text{(F 15)}$$

$$W_{fuse} = K_2 L^{1.15} D^{0.25} (n W)^{0.17} \qquad \text{(F 16)}$$

where

W_{wing}	is wing weight in Earth lbs
W_{fuse}	is fuselage weight in Earth lbs
W	is gross weight of aircraft in lbs force
λ	is taper ratio
Λ	is sweepback
L	is fuselage length in feet
D	is max. fuselage diameter in feet
n	is structural load factor
K_1	= 0.012
K_2	= 0.8

Then

$$W_{wing} = (.012)\sqrt{3.75}\,(10{,}000)^{1.15} \left(\frac{1+0.4}{0.153}\right)^{0.4} \frac{1}{(1.5)^{0.6}} \frac{(16)^{0.5}}{1}$$

$= 1768$ Earth lbs $= 670$ lbs wt. on Mars

$$W_{fuse} = (0.8)(75)^{1.15}(7)^{0.25}(37{,}500)^{0.17}$$

$= 1123$ Earth lbs $= 427$ lbs wt. on Mars

Also

$$\frac{W_{tail}}{S_{tail}} = 2.5 \text{ Earth lb/ft}^2 \qquad \text{(Ref 32:146)}$$

Then $W_{tail} = (2.5)(100 + 89.1) = 472$ Earth lbs

$= 178$ lbs wt on Mars

and $W_{l.g.} = 6\%$ of gross weight before take-off

$= (0.06)(10{,}000 + 300)$

$= 618$ Earth lbs

$= 235$ lbs wt on Mars

$W_{nacelle} = 20\%$ of engine + propeller weight (Ref 7: chap 8, p.22)

$= (0.20)(1320) = 264$ Earth lbs $= 100$ lbs wt on Mars

<u>Structural Wing Mass Required to Support Bending Moment</u>

Consider the maximum bending moment at the wing root for a fully loaded aircraft with no fuel in the wings. Let the moment arm for lift on the half-wing be at the lateral position of the M.A.C. Let the moment arm for dead weight of half-wing, engine, and propeller be 11 ft. Then maximum bending moment M_{max} is

$$M_{max} = \tfrac{1}{2}(10{,}000)(3.75)(22.1) - \tfrac{1}{2}(670 + 1320 + 100)(3.75)(11)$$

$= 372{,}000$ ft-lb

$= 4{,}460{,}000$ in-lb

The bending modulus of rupture, f_{br}, is defined by

$$f_{br} = \frac{M_{max}}{I}(h/2) \qquad \text{(F 17)}$$

where h is height of a rectangular beam cross-section, and I is moment of inertia for a rectangular section.

$$I = \frac{bh^3}{12} = \frac{Ah^2}{12} \qquad \text{(F 18)}$$

where A is cross-sectional area. For a wing thickness ratio of 0.153, the maximum root thickness is

$$h = (0.153)(9.23)(12) = 17 \text{ inches}$$

Assuming sectional stability, f_{br} may be taken as the ultimate tensile strength (Ref 24:13). Aluminum 2024 alloy extrusions have ultimate tensile strength of 70,000 psi (Ref 24:82). Then combining (F 17) and (F 18), the root cross-sectional area of a single aluminum spar of mean height of 16 inches is

$$A_r = \frac{6\ M_{max}}{h_r\ f_{br}} = \frac{6(4,460,000)}{(16)(70,000)} = 24 \text{ in}^2$$

For nearly elliptical spanwise load distribution, assume cross sectional area increases as the square of distance from the wing tip. Then volume of aluminum spar is

$$\text{Vol.} = 1/3\ A_r(b/2) = 1/3\ (24)(51.5)(12)$$

$$= 4950 \text{ in}^3$$

Density of 2024 Al is 0.1 Earth lb/in^3, then

$$\text{Weight of wing supports} = (495)(2) = 990 \text{ Earth lbs}$$

$$= 376 \text{ lbs wt on Mars}$$

Aircraft Center of Gravity

Let X be the distance from nose of aircraft to leading edge of component. Let d be the distance from leading edge of component to component c.g. as suggested by Corning (Ref 7: chap 4, p.41). Weight of fuel, fuel system fixed equipment, landing gear, and payload is assumed centered about aircraft c.g. The wing is then positioned so that the aircraft c.g. is at 0.25 M.A.C. (1.72 ft):

Component	Weight lbs	Location of c.g.	d ft.	x ft.	Moment ft - lbs
wing	670	0.40M.A.C.	2.75	X_w	$670(X_w + 2.75)$
fuselage	427	0.35 L	26.2	0	11,200
tail	178	0.25 M.A.C.	2.60	67.2	11,950
nacelles (2)	100	0.40 l_n	4.40	X_w-3	$100\ (X_w + 1.4)$
engines (2)	1100	0.40 l_n	4.40	X_w-3	$1100\ (X_w + 1.4)$
propellers (2)	217	X_w-3	0	X_w-3	$217\ (X_w - 3)$
crew (2)	175	nose	0	5	875
A.T.O. rocket	15	nozzle	0	60	900
	2882				Σ = total moment

Total moment = $(2882)(X_w + 1.72)$

Solving the above equation for X_w gives

$$X_w = 28.7 \text{ feet}$$

Thus aircraft c.g. is at

$$28.7 + 1.72 = 30.42 \text{ feet ref. nose}$$

For total empty weight of aircraft (including landing gear, fixed equipment, and fuel system) of 3920 lbs wt, the total moment empty is

$$(3920)(30.42) = 119{,}000 \text{ ft-lbs}$$

Flaps and Slots

For the selected airfoil, $C_{l_{max}} = 1.45$ with no flaps. From Figure 2-51 (Ref 26), a ΔC_l of 1.55 can be obtained from Fowler type flaps with a flap area/wing area ratio (S_f/S) of 0.30. Using 0.40 chord Fowler flaps with a flap span b_f on a trapezoidal wing-planform, then

$$S_f = \tfrac{1}{2} (0.4\, C_r + 0.4\, C_f)\, b_f \qquad \text{(F 19)}$$

where C_r is wing root chord and C_f is wing chord at b_f/b of wing span. Let

$$b_f/b = X \qquad \text{(F 20)}$$

Then for a trapezoid with a taper ratio of 0.4

$$S_W = \tfrac{1}{2} (C_r + C_t) b = 0.7\, C_r b \qquad \text{(F 21)}$$

So

$$\frac{S_f}{S} = \frac{0.2\,(C_r + C_f)}{0.7\, C_r} \quad X \qquad \text{(F 22)}$$

But

$$\frac{C_f - C_t}{C_r - C_t} = \frac{b - b_f}{b} = 1 - X \qquad \text{(F 23)}$$

Since $C_t = 0.4\, C_r$ then

$$\frac{C_f - 0.4\, C_r}{0.6\, C_r} = 1 - X \qquad \text{(F 24)}$$

$$C_r + C_f - 1.4\, C_r = 0.6\, C_r (1 - X) \qquad \text{(F 25)}$$

$$\frac{C_r + C_f}{C_r} = 2 - 0.6\,X \qquad \text{(F 26)}$$

Combining with (F 22)

$$\frac{S_f}{S} = (4/7)\,X\,(1 - 0.3\,X) \qquad \text{(F 27)}$$

Then for S_f/S = 0.30, solving for X gives

$$X = b_f/b = 0.65$$

or S_f/S = 0.30 corresponds to 65% span flaps and gives an aircraft $C_{l\,max}$ of 3.0. Use of leading edge slats will inhibit tip stall and decrease L/D for take-off and landing.

For the basic aircraft at low speeds C_{Dp} = .017

From Figure 2-68 (Ref 26) $\Delta C_{D_{L.G.}}$ = .009

and $\Delta C_{D_{flaps}}$ = .055

Estimating drag due to slots from Figure 2-51 a (Ref 26)

$$\Delta C_{D\ slots} = .045$$

Thus C_{D_o} = 0.126

$$C_{D_i} = \frac{(3.0)^2}{\pi(16)} = 0.179 \text{ including ground effect}$$

Then C_D for take-off and landing = 0.305

$$(L/D)_{t.o.\ and\ land} = \frac{3.0}{.305} = 9.85$$

Appendix G

Performance and Stability Calculations

Power Required

Horsepower required is given by

$$HP_{reqd} = \frac{DV}{550} = \frac{\frac{1}{2}\rho S V^3 C_D}{550}$$

$$= \frac{\frac{1}{2}\rho S a^3 M^3 C_D}{550} \qquad (G\ 1)$$

Let $\sigma = \rho/\rho_{SL}$, $\theta = T/T_{SL}$, $\delta = P/P_{SL}$

then

$$HP_{reqd} = \frac{1}{2}\frac{\sigma\theta^{3/2}}{550}\rho_{SL}\, a_{SL}^3\, S M^3 C_D \qquad (G\ 2)$$

But for an adiabatic atmosphere

$$\sigma = \theta^{\frac{1}{\gamma-1}} \qquad (G\ 3)$$

and

$$\theta = \delta^{\frac{\gamma-1}{\gamma}} \qquad (G\ 4)$$

Then for $\gamma = 1.4$

$$\sigma\theta^{3/2} = \theta^{3/2 + 2.6} = \delta^{0.286(1.5+2.5)} = \delta^{1.143} \qquad \text{(G 5)}$$

Substituting values for ρ_{sl}, a_{sl}, and S into (G 2)

$$HP_{reqd} = (2.06)(10^{-4})(1100)^3(667)\delta^{1.143} M^3 C_D$$

$$= 166{,}700\,\delta^{1.143} M^3 C_D \qquad \text{(G 6)}$$

where

$$C_D = C_{D_p} + \Delta C_{D_{comp}} + C_{D_i} \qquad \text{(G 7)}$$

$$C_{D_i} = \frac{C_L^2}{\pi e AR} = \frac{C_L^2}{\pi(.85)(16)} = \frac{C_L^2}{42.7} \qquad \text{(G 8)}$$

$$C_L = \frac{W/S}{\frac{1}{2}\rho V^2} = \frac{W/S}{\frac{1}{2}\sigma\theta\rho_{SL}a_{SL}^2 M^2} \qquad \text{(G 9)}$$

But

$$\sigma\theta = \delta \qquad \text{(G 10)}$$

So

$$C_L = \frac{W}{\frac{1}{2}(2.06)(10^{-4})(667)(1100)^2 \delta M^2} = \frac{W}{83{,}000\, \delta M^2} \qquad \text{(G 11)}$$

Drag rise due to compressibility, $\Delta C_{D_{comp}}$, is a function of $M_{CR_D}-M$, which in turn is a function of aircraft C_L.

Parasite drag, C_{D_p}, is a function of speed and altitude, since

$$C_{D_p} = \frac{\sum C_{f_i} A_i}{S} \qquad \text{(G 12)}$$

An equivalent parasite drag coefficient may be defined as

$$C_F = \frac{\sum C_{f_i} A_i}{\sum A_i} \qquad \text{(G 13)}$$

where $\sum A_i$ is the total wetted area of the aircraft. For this aircraft the total wetted area is 2980 ft^2. At 550 fps and 3 Km altitude $C_{D_{P1}}$ = 0.0166 (Appendix E). Thus

$$C_F = \frac{(0.0166)(667)}{2980} = 0.00372$$

Applying the equation of Prandtl, for turbulent flow, to the equivalent parasite drag

$$C_F = \frac{0.455}{(\log_{10} Re_n)^{2.58}} \qquad \text{(G 14)}$$

where Re_n is Reynolds number of flow based on some fictitious length. Assuming the Sutherland law for variation of viscosity with temperature

$$\left(\mu / \mu_{SL}\right) = \theta^{0.76} \qquad \text{(G 15)}$$

Then Re_n may be expressed as

$$Re_n = \frac{\rho_{SL} a_{SL} \sigma \theta^{1/2} M \ell}{\mu_{SL} \theta^{0.76}}$$

$$= \frac{\rho_{SL} a_{SL} M \ell \theta^{2.24}}{\mu_{SL}} \qquad \text{(G 16)}$$

$$= \frac{\rho_{SL} a_{SL} \ell M \delta^{0.64}}{\mu_{SL}}$$

From $C_F = 0.00372$

$$(\log_{10} Re_{n_1})^{2.58} = \frac{0.455}{0.00372} = 122.4$$

which gives $Re_{n_1} = 2.82\ (10^6)$ at 550 fps and 3 Km. Since at this speed and altitude, M = 0.51 and δ = 0.863, then

$$\frac{\rho_{SL} a_{SL} l}{\mu_{SL}} = \frac{(2.82)(10^6)}{(0.51)(0.863)^{0.64}} = 6.07(10^6)$$

Thus for any speed and altitude

$$Re_N = 6.07(10^6) M \delta^{0.64} \qquad \text{(G 17)}$$

Then from (G 12), (G 13), and (G 14), at any speed and altitude

$$\frac{C_{D_{p1}}}{C_{D_p}} = \frac{C_{F1}}{C_F} = \left[\frac{\log_{10} Re_n}{\log_{10} Re_{n_1}}\right]^{2.58} \qquad \text{(G 18)}$$

$$\frac{\log_{10} Re_N}{\log_{10} Re_{N_1}} = \frac{\log_{10} 6.07(10^6) + \log_{10} M + 0.64 \log_{10} \delta}{6.45}$$

$$= 1.051 - 0.155 \log_{10} \frac{1}{M} - 0.0993 \log_{10} \frac{1}{\delta} \qquad \text{(G 19)}$$

(G 18) and (G 19) define C_{D_p} for any speed and altitude.

Values of horsepower required as function of Mach number were calculated for altitudes of 0, 6, 12, and 18 kilometers, at vehicle weights of 10,000, 8,000, 6,000, and 4,000 lbs wt on Mars. The power required curves are shown in Figures 7, 12, and 13.

Sample Calculation

Weight	8000 lbs
Altitude	6 Km (20,000 ft)
Speed	0.40 Mach

From Appendix A $\delta = 0.742$

From eqn (G 11)

$$C_L = \frac{8}{(83)(0.742)(0.4)^2} = 0.811$$

then

$$C_{D_i} = \frac{(0.811)^2}{42.7} = 0.0155$$

From Figures 2:7 and 2:9 (Ref 6), at $C_L = 0.811$

$$M_{CR_D} = 0.511$$

From Figure 2:23 (Ref 6)

$$\Delta C_{D_{comp}} = .0001$$

From (G 19)

$$\frac{\log_{10} Re_N}{\log_{10} Re_{N_1}} = 1.051 - 0.155 \log_{10} \frac{1}{0.4} - 0.0993 \log_{10} \frac{1}{0.742} = 0.976$$

Then from (G 18)

$$C_{D_p} = \frac{0.0166}{(0.976)^{2.58}} = 0.0176$$

Then

$$C_D = .0155 + .001 + .0176 = 0.0332$$

and from (G 6)

$$HP\ reqd = (166{,}700)(0.742)^{1.143}\ (0.4)^3(0.0332)$$
$$= 252\ HP$$

Range

Range for a turboprop aircraft is best found from

$$Range = \int_0^{W_f} r\,dw \qquad \text{(G 20)}$$

where r is miles per lb. wt. of propellant.

$$r = \frac{(60)\quad V\eta_p}{(88)\ (ESPC)(HP_{reqd})\ g'} \qquad \text{(G 21)}$$

for V in ft/sec and ESPC in Earth lbs/HP-hr. Then

$$r = \frac{(60)(1100)\,\theta^{\frac{1}{2}}\eta_p\, M}{(88)(0.38)(HP_{reqd})(ESPC)}$$

$$= \frac{1975\ \delta^{0.143}\ \eta_p M}{(HP_{reqd})\ (ESPC)} \quad \frac{mi.}{lb.\ wt.} \qquad \text{(G 22)}$$

For a given M, h, and W, HP reqd is obtained from power curves. ESPC for a given M and h is obtained from Figure 10. η_p can be found by the method of Appendix F. Figure 14 shows r for vehicle weights of 10,000, 8000, and 6000 lbs and at altitudes of 0, 6, and 12 Km.

Example:

	Weight	8000 lbs
	Altitude	6 Km
	Speed	0.40 Mach

From Figure 12 — HP reqd = 252

From Figure 10 — ESPC = 1.97 Earth lb/HP-hr

Assuming a propeller-reduction gear efficiency near 0.8

$$\text{EHP reqd/engine} = \frac{252}{0.8} \times \frac{1}{2} = 158$$

From equation (F 3)

$$\frac{J}{(C_p)^{1/3}} = \frac{(14.4)^{2/3}(0.206)^{1/3}\ (0.9^{1/3})(0.4)(1054)}{(550)^{1/3}\ \ (158)^{1/3}} = 3.22$$

From Figures 8:4 and 8:5 (Ref 7), with a reduction-gear efficiency of 0.95

$$\eta_p = 0.95\ (0.86 - .02) = 0.80$$

Then from (G 22)

$$r = \frac{(1975)(0.742)^{.143}(0.8)(0.4)}{(1.97)(252)}$$

$$= 1.21 \text{ mi/lb wt}$$

By inspection of Figure 14, the maximum r for a given aircraft weight may be selected. Figure 15 shows r as a function of weight for a cruise-climb and constant altitude flight. Range for any initial fuel weight is obtained by graphical or approximate integration of Figure 15. Figure 6 shows the integration of Figure 15.

Static Stability

Stick-fixed longitudinal stability is determined by the method of Etkin (Ref 10:23-26, 446-479). From the airfoil data, the aerodynamic center of the wing section (h_w) in fraction of M.A.C. is

$$h_w = 0.266$$

From Figure B.8, 1 (Ref 10), the forward shift of the neutral point due to fuselage effect is

$$\Delta h_n = -0.143$$

From the same figure, an additional forward shift due to nacelles is

$$\Delta h_n = -0.01$$

Thus the neutral point of the wing-body-nacelle combination, reduced by 5% for high-wing configuration is

$$h_{n_{wb}} = 0.266 - 0.95\ (0.143 + 0.01) - 0.121$$

The pitching moment coefficient about a wing section is given by

$$C_{M_{ac}} = -0.11\ \text{deg}^{-1}$$

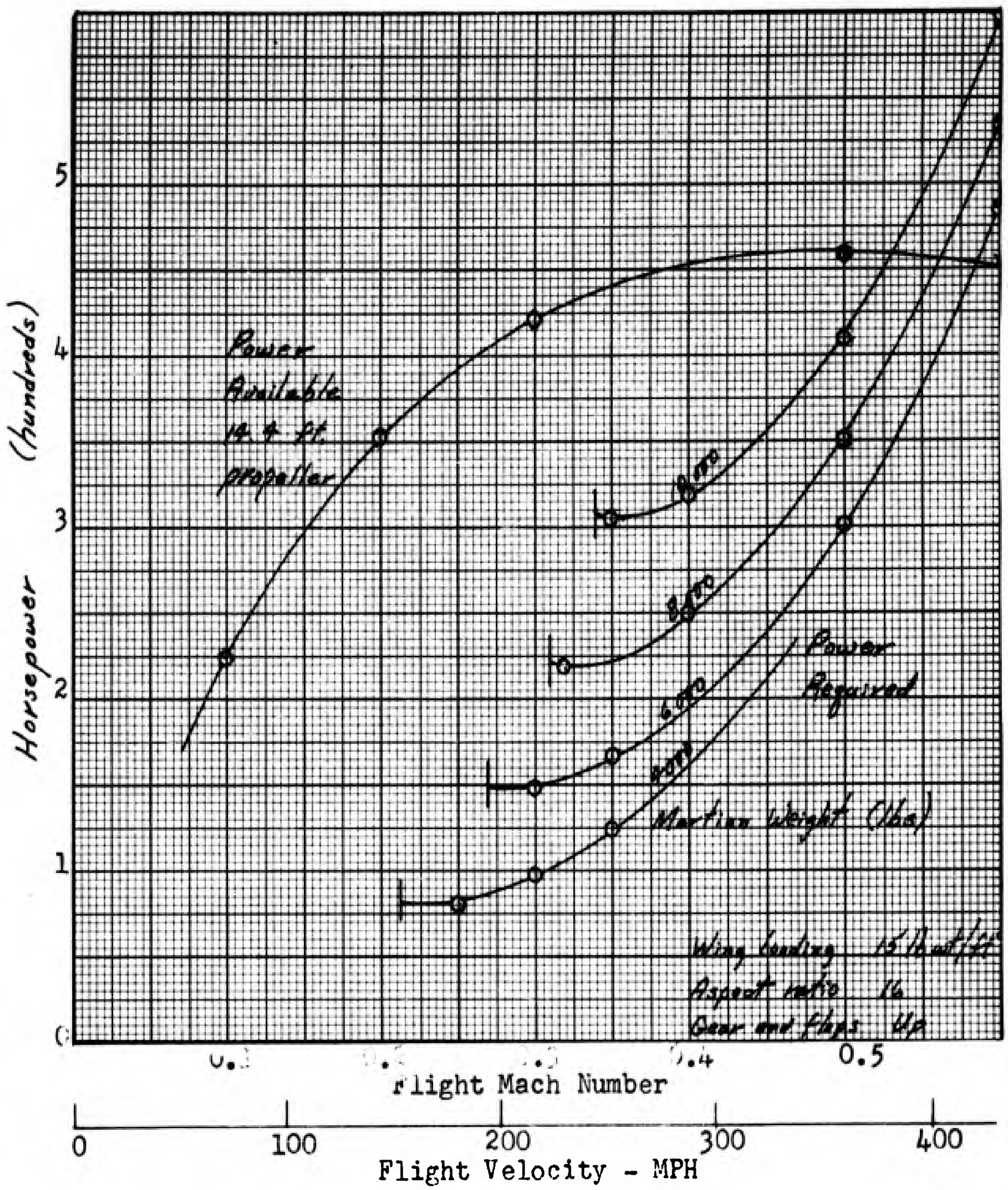


Figure 12

Power Curves for a Martian Turboprop
at 6 Km (20,000 ft.)

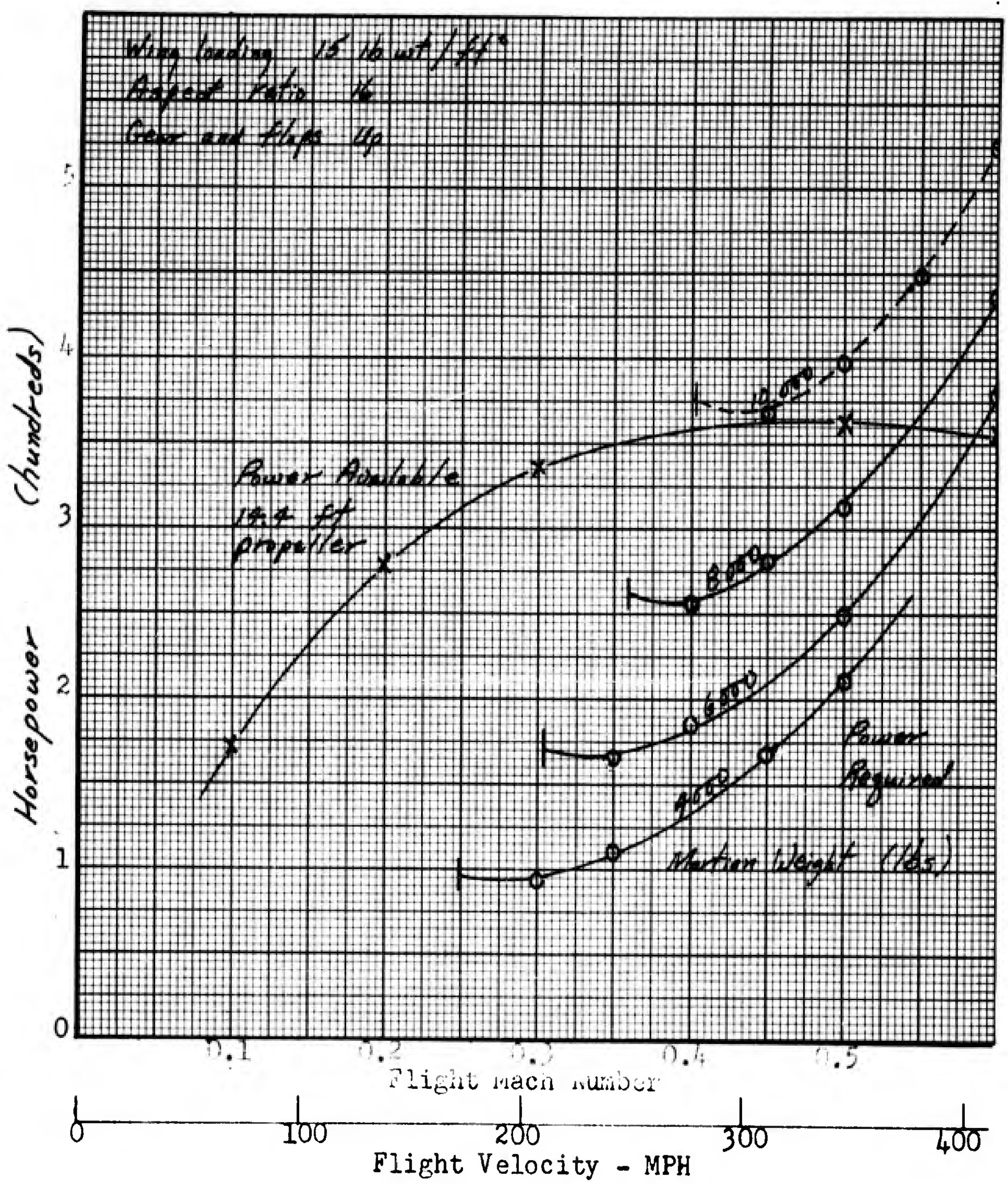


Figure 13

Power Curves for a Martian Turboprop
at 12 Km (40,000 ft.)

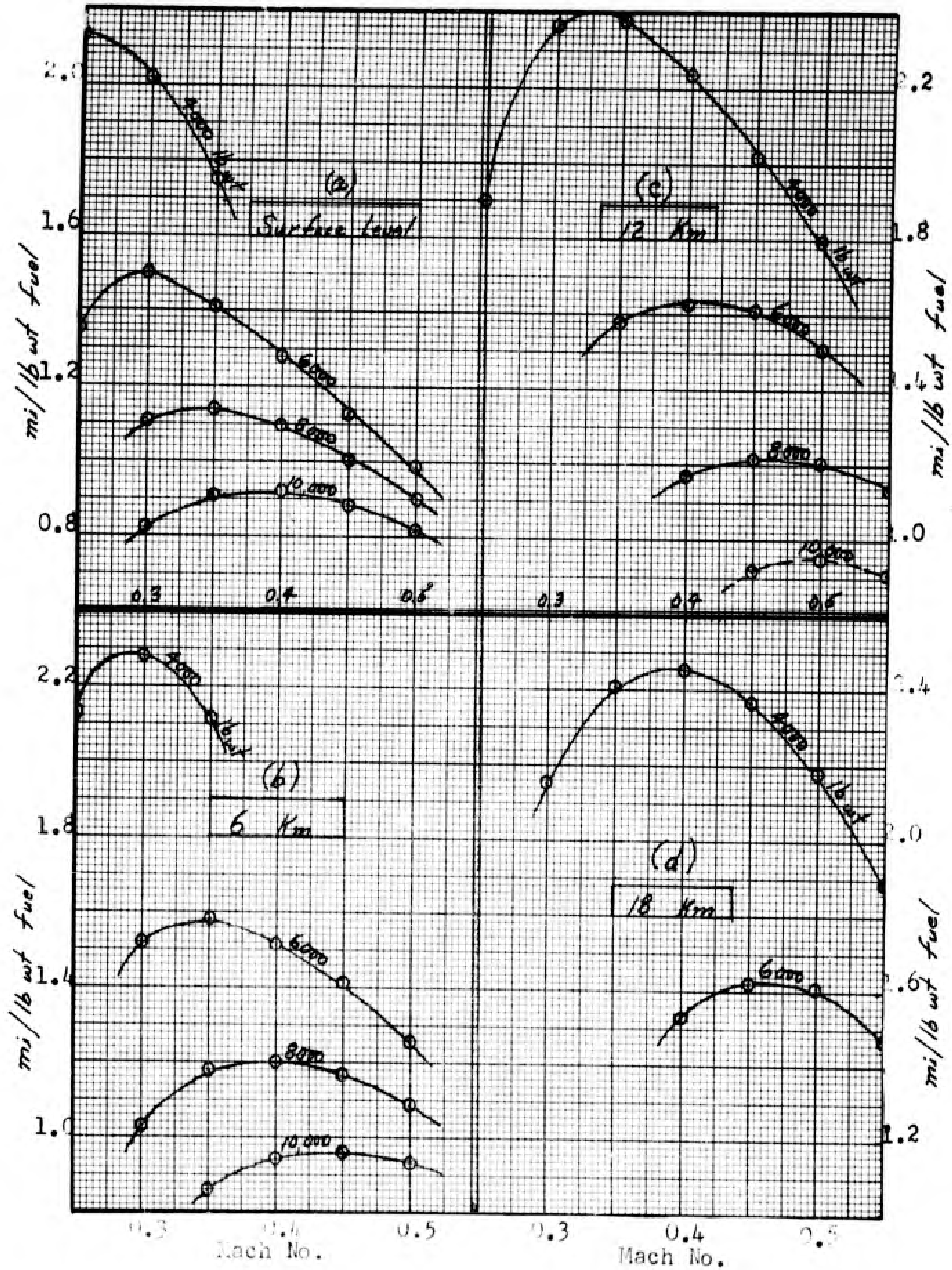


Figure 14

Miles per Pound of Propellant

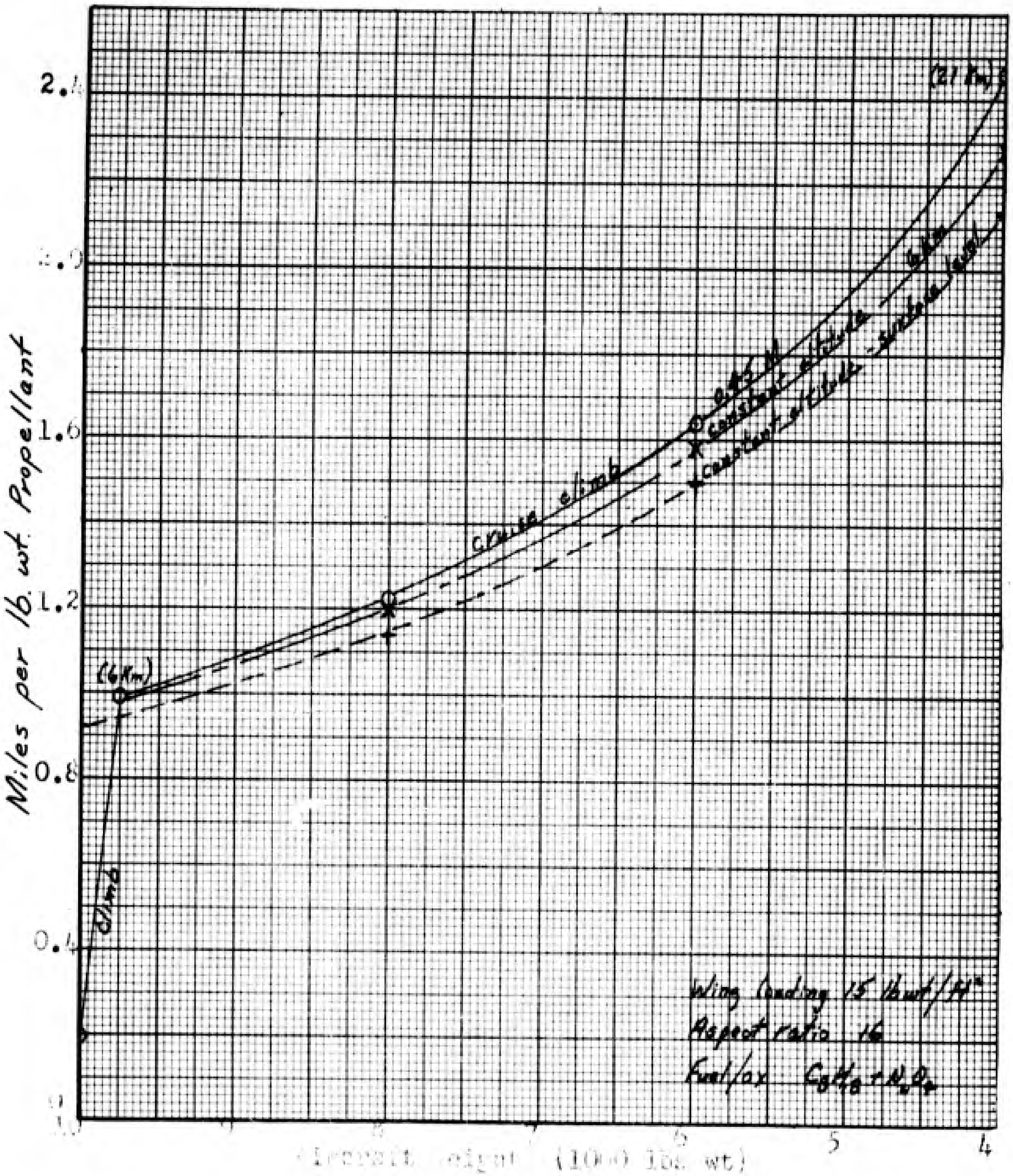


Figure 15

Miles per Pound Weight Fuel/ox as a Function of Flight Plan for a [illegible] Turboprop

Correcting for 3-D effect of aspect ratio and taper ratio (Ref 26:88-89)

$$C_{M_{ac}} = (1.05)(-.11) = -.115 \text{ deg}^{-1}$$

From Figure B.8,2 (Ref 10), the change in pitching moment due to fuselage effect is

$$(C_{M_{o_B}}) = -0.056$$

The correction for high wing configuration is + .004. Thus the pitching moment coefficient of wing-body combination is

$$C_{M_{o_{wb}}} = -.115 -.056 = -0.171 \text{ deg}^{-1}$$

From Appendix F, the lift-curve slope of the wing (a) corrected for aspect ratio is

$$a_w = .0975 \text{ deg}^{-1}$$

From Figure B.1, 4 (Ref 10), the correction factor for fuselage effect is 0.99. Thus the lift-curve slope of the wing-body combination is

$$a_{wb} = (.99)(.0975) = .0966 \text{ deg}^{-1}$$

From Figures B.6, 1-2 (Ref 10), by interpolation for taper ratio and extrapolation for aspect ratio, the downwash angle at the tail is given by

$$\epsilon = \epsilon_o + \frac{\partial \epsilon}{\partial \alpha} \alpha \qquad \text{(G 23)}$$

where

$$\epsilon_0 = 0.12 \text{ deg}$$

and

$$\frac{\partial \epsilon}{\partial \alpha} = 0.28$$

Using a symmetric horizontal tail airfoil with the same lift-curve slope as the wing, then for a tail aspect ratio of 3

$$a_t = \frac{\bar{a}_0}{1 + \frac{57.3\,\bar{a}_0}{\pi AR}} = \frac{0.11}{1 + \frac{(57.3)(.11)}{3\pi}}$$

$$= 0.066 \text{ deg}^{-1}$$

Assuming a tail efficiency factor (power-off) of 0.9, then

$$a_t \text{ in situ} = (.066)(.9) = 0.0593 \text{ deg}^{-1}$$

The horizontal tail volume referred to wing-body neutral point is given by

$$V'_H = \frac{l'_t}{\text{M.A.C.}} \left(\frac{S_t}{S} \right) \qquad \text{(G 24)}$$

where l'_t is distance from tail a.c. to wing-body neutral point. Taking the tail $\frac{1}{4}$ M.A.C. as the position of tail a.c., then from the aircraft layout,

$$l'_t = 69.65 - 28.70 - (0.121)(6.87) = 40.12 \text{ ft}$$

Thus

$$V'_H = \frac{(40.1\text{[illegible]})}{(6.87)} \; \frac{(89.1)}{(667)} = 0.780$$

The lift-curve slope of the aircraft is given by

$$a = a_{wb} + a_t \left(\frac{S_t}{S}\right)\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$

$$a = .0966 + .0593 \left(\frac{89.1}{667}\right)(1 - 0.28) \qquad \text{(G 25)}$$

$$= 0.0972 \ \text{deg}^{-1}$$

The aircraft stick-fixed ne[illegible]ral point is then given in fraction of M.A.C. by

$$h_m - h_{m_{wb}} = V'_H \frac{a_t}{a}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$

$$= 0.780 \left(\frac{.0593}{.0972}\right)(1 - 0.28) \qquad \text{(G 26)}$$

$$= 0.342$$

Thus the stick-fixed neutral point is at

$$h_n = 0.342 + 0.121 = 0.463 \text{ M.A.C.}$$

For the aircraft c.g. at 0.25 M.A.C., the static margin is then

$$(h_n - h) = .463 - .25 = 0.213 \text{ M.A.C.} = 1.46 \text{ feet}$$

The slope of the pitching moment curve is then

$$C_{m_\alpha} = a(h-h_n) = (0.0972)(-.213) = -0.0207 \ \ deg^{-1} \qquad \text{(G 27)}$$

The horizontal tail volume referred to aircraft aerodynamic center is given by

$$V_{H_n} = V_H' - \frac{S_t}{S}(h_n - h_{n_{wb}}) = 0.780 - \frac{89.1}{667}(.342) = 0.734 \qquad \text{(G 28)}$$

The pitching moment coefficient about aircraft a.c. is then

$$C_{M_o} = C_{M_{o_{wb}}} + a_t V_{H_n}(\epsilon_o + i_t) \qquad \text{(G 29)}$$

Tail incidence angle i_t is selected so that

$$C_M = C_{M_o} + C_{M_\alpha}\alpha = 0 \qquad \text{(G 30)}$$

for α corresponding to cruise C_L of 0.8

Substituting values for (G 29) and (G 30) and solving simultaneously gives

$$i_t = 7.6 \text{ deg} \quad \text{(down)}$$

$$C_{m_o} = 0.170$$

Since the zero lift angle of wing section is -4 deg, then for a wing-fuselage incidence angle of 3 deg, the tail-fuselage incidence angle is -4.6 deg.

Stick-fixed directional stability is determined from data of Perkins and Hage (Ref 26: 318-326). The derivative of yawing-moment coefficient with respect to yaw, C_{n_ψ}, is estimated by adding the values due to aircraft components:

contribution from wing (no sweepback)	0
contribution from fuselage	+ .0006 deg^{-1}
contribution from nacelle	+ .00002
correction for high-wing configuration	- .0002
contribution from propellers	+ .00007
Vertical tail aspect ratio	1.5
effective aspect ratio due to end plate effects	3.1
lift-curve slope of V.T. (Ref 26: Fig 8.8)	0.046 deg^{-1}
vertical tail volume	0.0567
vertical tail efficiency (power off)	0.9

Contribution to C_{n_ψ} of vertical tail is given by

$$(C_{n_\psi})_{V.T.} = -a_{VT} V_{VT} \eta_{VT}$$

$$= -(.046)(.0567)(.9) = -0.0023 \quad \text{(G 31)}$$

correction for high wing	+ 0.0005
Net C_{n_ψ} from all components	-0.0011 deg^{-1}

Rudder Control for Single Engine Stability

For single-engine operation at full power with no side-slip, then

$$C_n = C_{n_{\delta_r}} \delta_r + C_{n_{s.e.}} = 0 \qquad \text{(G 32)}$$

where C_n is yawing moment coefficient

δ_r is rudder deflection

$C_{n_{s.e.}}$ is yawing moment contribution of one engine.

$$C_{n_{s.e.}} = \frac{y_m C_D}{b} \qquad \text{(G 33)}$$

where C_D is drag coefficient of aircraft

y_m is moment arm of propeller thrust

Maximum C_D will occur at maximum C_L. Then

$$C_{D_{Max}} = \frac{(1.45)^2}{\pi (0.85)(16)} + 0.017 = 0.066$$

Adding 10% to account for asymmetrical effects gives

$$C_{n_{s.e.\ max}} = \frac{(11)}{(103)} (.066)(1.10) = 0.0080$$

Now

$$C_{m_{\delta_r}} = -a_r V_{VT} \eta_{VT} \qquad \text{(G 34)}$$

where a_r is lift curve slope of vertical tail with rudder.

$$a_r = a_{VT} \tau \qquad \text{(G 35)}$$

where τ is control surface effectiveness. Using a criterion of one degree of yaw per one degree of rudder, then

$$C_{m_{\delta_r}} = C_{m_\psi} = -0.0011 \text{ deg}^{-1} \text{ power off}$$

Combining (G 34), (G 35), and substituting values

$$\tau = \frac{(.0011)}{(.046)(.0567)(.9)} = 0.47$$

From Figure 5-33 (Ref 26)

$$\frac{S_{rudder}}{S_{VT}} = 0.27$$

Correcting $C_{n_{\delta_r}}$ for power-on conditions (tail in propeller slipstream) (Ref 26:238)

$$C'_{m_{\delta_r}} = (-.0011)(1.307) = -0.00144 \text{ deg}^{-1}$$

Then from (G 32)

$$\delta_r \text{ for S.E. control} = \frac{-.008}{-.00144} = 5.6 \text{ degrees}$$

Appendix H

Lists of Symbols Used in Calculations

Because of the nature of this design problem, many of the standard symbols used in aerodynamics, thermodynamics, and thermochemistry may have overlapping meanings. For this reason the symbols are separated into three groups: those related to engine performance calculations, to aerodynamic design, and to aircraft stability.

Engine Performance Symbols: Appendices A, B, C, and D.

a	speed of sound
C_p	heat capacity of compressor gases
C_p'	heat capacity of hot expansion gases
C_{pa}	heat capacity of air in combustion chamber
EHP	equivalent horsepower (turboprop)
ESPC	equivalent specific fuel-ox consumption
f/a	mass ratio of fuel and oxidizer to air
F_s	specific thrust
g	acceleration due to gravity
g_c	unit conversion factor 32.2
h	specific enthalpy
ΔH	enthalpy per mole
ΔH_{form}	heat of formation
HP	horsepower
HPSPC	thrust-horsepower specific fuel-ox consumption
I_{sp}	specific impulse
J	mechanical equivalent of heat
M	Mach number
MW	molecular weight
$\dot{m}$	mass flow rate
P	pressure
$\mathcal{R}$	compressor pressure ratio
R_o	universal gas constant
T	temperature
V	velocity
W	specific power or work
Y	turbofan bypass ratio; mole fraction
γ	ratio of specific heats
γ'	ratio of specific heats of hot expansion gases

ϵ	burner pressure efficiency
η	efficiency
ρ	density

Subscripts:

b	combustion
c	compressor
e	nozzle exit
n	nozzle
o	free stream, ambient conditions
p	propeller
r	ram
sl	surface level
t	turbine
⊕	Earth
♂	Mars

Aerodynamic Design Symbols: Appendices D, E, F, and G.

A	wetted area of component
a	ground acceleration rate
a	slope of aircraft lift curve dC_l/d
a	sonic velocity
$\bar{a}_o$	theoretical two-dimensional lift-curve slope
AR	aspect ratio
b	wing span
b_f	flap span
$\bar{c}$	mean geometric chord
C_D	drag coefficient
ΔC_{Dcomp}	compressibility drag coefficient
C_{Di}	induced drag coefficient
$\Delta C_{Dl.g.}$	increment in drag coefficient due to landing gear
C_{Do}	coefficient of all drag except induced drag
C_{Dp}	parasite drag coefficient
C_f	friction coefficient
cf	wing chord at outboard edge of wing flaps
C_F	equivalent parasite drag coeffient
C_{HT}	horizontal tail volume coefficient
C_L	lift coefficient
C_p	power coefficient
C_{px}	adjusted propeller power coefficient
c_r	root chord
c_t	tip chord

C_{vt}	vertical tail volume coefficient
c.g.	center of gravity
D	drag
D	propeller diameter
d	distance to component c.g.
e	span efficiency factor
EHP	equivalent horsepower
ESPC	equivalent specific propellant consumption
F	retarding forces on landing approach, drag plus reverse power
f	correction factor for taper ratio
f_{br}	bending modulus of rupture
g'	relative gravity
h	altitude, height
I	moment of inertia
J	propeller advance ratio V/ND
K	correction factor, gust alleviation factor
L	lift
L,l	characteristic length of component
M	Mach number
m	mass
M_{CR_D}	critical drag Mach number
M_{max}	maximum bending moment
M.A.C.	mean aerodynamic chord
N	propeller rpm
n	aircraft load factor
R	aircraft resistance during take-off ground run
r	miles per pound of propellant
Re_n	Reynolds number
Re_{nl}	Reynolds number at initial cruise conditions
S	wing area
S_F	landing-field length = $S_{50} + S_G$
S_f	wing-flap area
S_G	ground roll distance from touchdown to full stop
S_{HT}	horizontal tail area
S_{to}	take-off field length
S_{vt}	vertical tail area
S_{50}	distance from clearing 50 foot obstacle to touchdown
T_n	nozzle thrust
T_{ssl}	thrust at static surface conditions
t	time
t/c	wing thickness ratio
THP	thrust-horsepower
u	effective gust velocity
V	velocity
V_{so}	stall speed, power off
V_{td}	touchdown speed
V_{50}	speed clearing a 50 foot obstacle
W	weight in pounds force
W_f	fuel weight

W_o	initial weight
W_{pp}	powerplant weight
W/S	wing loading
X	longitudinal station referred to nose of aircraft
y	lateral station from fuselage center-line
γ	glide angle
δ	pressure ratio P/P_{SL}
η_p	propeller efficiency x reduction gear efficiency
θ	temperature ratio T/T_{SL}
Λ	sweepback
λ	taper ratio
μ	coefficient of rolling friction
μ	viscosity
μ_g	airplane mass ratio
ρ	density
σ	density ratio ρ/ρ_{SL}

Stability Symbols: Appendix G.

a	aircraft lift-curve slope dC_L/d
a_t	lift curve slope of horizontal tail
a_{wb}	lift curve slope of wing-body combination
a.c.	aerodynamic center
b	wing span
C_M	pitching moment coefficient
$C_{M_{ac}}$	pitching moment coefficient about wing a.c.
C_{M_o}	pitching moment coefficient about aircraft a.c.
$C_{M_{o_{wb}}}$	pitching moment coefficient about a.c. of wing-body combination
C_M	slope of pitching moment coefficient curve
C_N	yawing moment coefficient
$C_{N_{s.e.}}$	yawing moment coefficient due to one engine inoperative
C_{n_ψ}	derivative of yawing moment coefficient w.r.t. yaw
h	position of aircraft c.g. in fraction of M.A.C.
h_n	stick fixed neutral point of aircraft in fraction of M.A.C.
$h_{n_{wb}}$	aerodynamic center of wing-body combination in fraction of M.A.C.
h_w	wing a.c. in fraction of M.A.C.
i_t	tail incidence angle
l'_t	distance from tail a.c. to wing-body neutral point
V'_H	horizontal tail volume referred to wing-body neutral point
V_{H_n}	horizontal tail volume referred to aircraft a.c.
V_{vt}	vertical tail volume
y_n	moment arm of propeller thrust

α	angle of attack
δ_r	rudder deflection angle
ϵ	downwash angle
η_{vr}	vertical tail efficiency
τ	control surface effectiveness

Vita

John Ross Stetson was born on [redacted] He matriculated at Stanford University and graduated with distinction in 1955, receiving the degree of Bachelor of Science in Chemical Engineering. He accepted a position with E. I. duPont de Nemours Co., Inc., and for seven months worked in the Engineering Service Division for the Electro-Chemicals Department. He entered active duty in February of 1956 as an Air Force lieutenant and completed flight training in April, 1957. Prior to entering the Institute of Technology (Air University), he was assigned to Mather Air Force Base, California as a pilot, flying both conventional and jet-type aircraft. He is the father of two sons.

Permanent address: [redacted]

This thesis was typed by Miss Elsie V. Miller
and Miss M. Lucille Martin

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