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INSTITUTE OF TECHNOLOGY

AIR UNIVERSITY

UNITED STATES AIR FORCE

SCHOOL OF ENGINEERING

THESIS

WRIGHT-PATTERSON AIR FORCE BASE, OHIO

AF-WP-B-SEP 60 5M

ANALYSIS OF CONTROLLED FIELD CIRCUIT DESIGN

THESIS

Presented to the Faculty of the School of Engineering of
the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

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Preface

This thesis is written in an attempt to further the present day knowledge of circuit design utilizing a distributed resistive network for circuit resistor elements. Although some of the material in this thesis may seem intuitively obvious, such was not the situation when this subject was taken for a thesis study.

My sincere appreciation is extended to Professor Bill Owens of Louisiana State University for his advice on satisfying boundary conditions of the resistive sheet, to Captain Robert V. Hendon and to First Lieutenant John Gamble for their assistance in arriving at a computer solution to the potential field problem, and to Professor Jerzy Lubelfeld, who, as Faculty Thesis Advisor, was a constant source of advice and encouragement.

Carl J. Bossert

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Abstract

A method for calculating the resistance between circular current sources on a distributed resistive sheet is shown. The method uses a finite number of images to satisfy boundary conditions on the plane. The use of distributed resistance in circuits is shown with circuit analysis of a transistor amplifier. A digital computer solution to the static-potential field problem by solving Laplace's equation in difference form is shown. Distributed resistive networks offer an approach to the elimination of circuit resistor connections.

ANALYSIS OF CONTROLLED FIELD CIRCUIT DESIGN

I. Introduction

For electronic circuits it has been the practice to interconnect active elements such as transistors with lumped impedances. This analysis is a study of the two-dimensional potential field of a resistive card on which a transistor multivibrator was constructed. The resistors of the multivibrator circuit were distributed on a single piece of resistance paper.

In solid state semiconductor networks it has been proposed that the circuit elements such as resistors be built into the material on which the transistor itself is formed in order to minimize the difficulties encountered in making reliable connections between separate elements. Circuit connections are difficult to make and present problems as to reliability. The number of connections may be reduced by reducing the number of components. In general, there is a 1:1 correspondence between resistors in a circuit with standard components and physically shaped resistor regions of a semiconductor network (Ref 5:30).

The elimination of discrete circuit components was achieved by allowing circuit functions to be controlled by a potential field on a resistive sheet. The binary multivibrator was constructed with a single resistive sheet providing the resistive paths of the circuit.

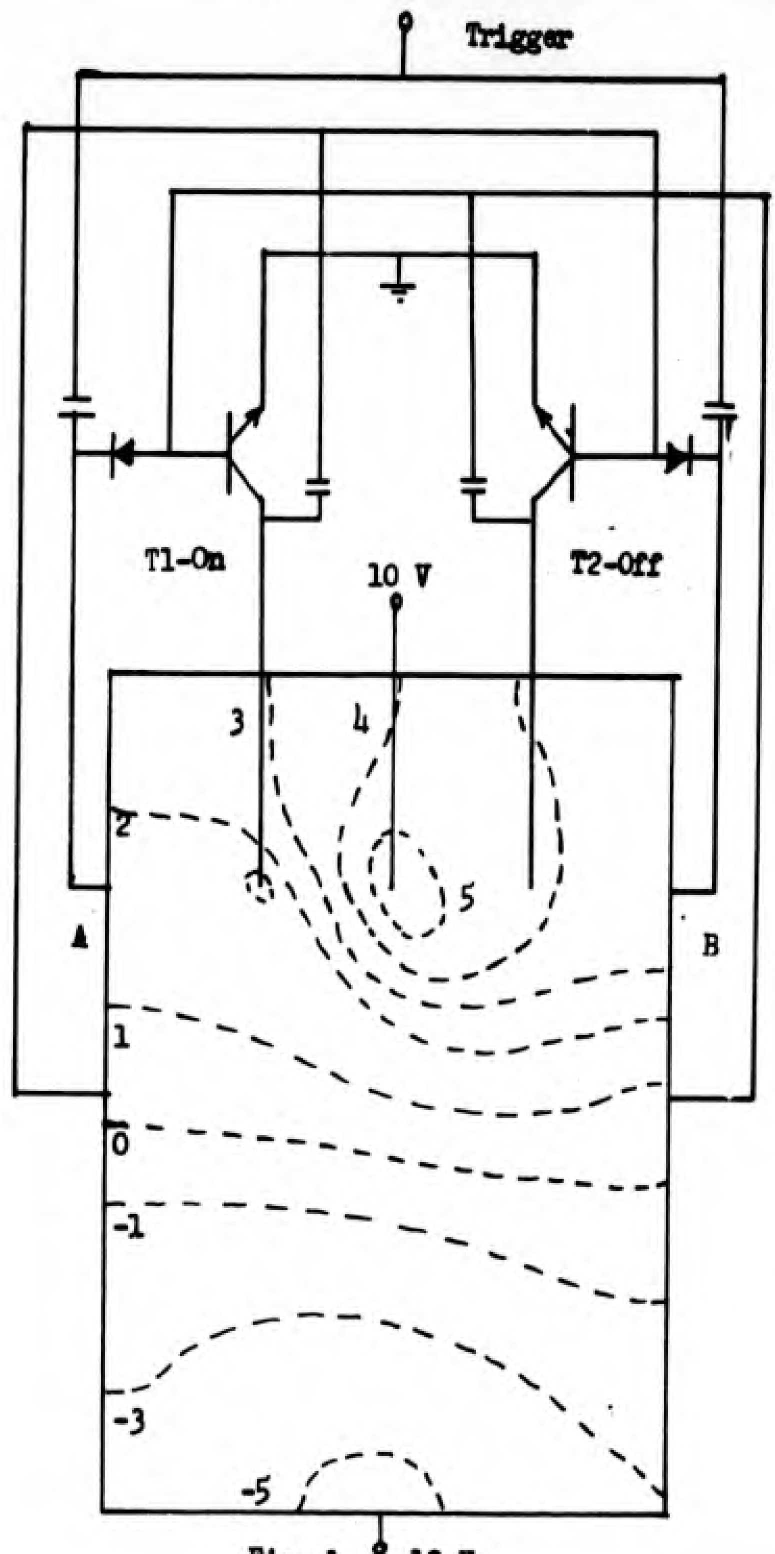


Fig. 1

Controlled Field Bistable Multivibrator

Transistors, diodes, capacitors, and voltage supply were attached to the resistive sheet and multivibrator operation was achieved, as shown in Figure 1. The potential field controlled the voltage at the base of the transistor in common-emitter configuration and controlled the on and off operation of the transistor. By pulsing the potential field at A and B of Figure 1, switching of the circuit was achieved.

The purpose of this investigation was to derive a method for calculating the current-voltage relationship of the distributed resistive network to allow circuit analysis.

The following approach was used in this analysis. First, Laplace's equation was used to calculate the potential at points on the resistive sheet. It was necessary to satisfy boundary conditions for the solution by replacing the boundaries with a system of current source images. The expression for this calculation is derived in Chapter II. This chapter also shows the derivation of Laplace's equation in a difference form. This equation was solved by a digital computer in order to calculate potential at points of the field.

In Chapter III, the expression for the calculation of potential is expanded for a rectangular coordinate system. Dimensions are specified for six calculations. Measurements for these same dimensions are shown.

Next, measurements were made of the resistance between current sources to illustrate the variation of resistance between current sources as the distribution of sources was altered. These measurements are presented graphically in Chapter IV.

Next, a transistor amplifier circuit was selected to illustrate circuit analysis and voltage gain calculation which agrees with measured voltage gain of the circuit. All the resistance elements of the circuit were provided by the resistive sheet and the resistance values were measured.

Finally, Laplace's equation in a difference form was solved by a digital computer to compute potential at points of the transistor multivibrator circuit. A comparison of measured potential values with computed values is shown for the transistor multivibrator circuit.

II. Calculation of Potential - Laplace's Equation

W. R. Smythe in his book, Static and Dynamic Electricity, states a general theorem for current flow in two dimensions (Ref 7:233). "If the normal component of the current density is given over all boundaries of a conductor as well as the size and location of all sources or sinks of current inside, then the value of the potential difference between any two points in the conductor is known."

The resistive sheet under consideration has no specified boundary potential. The normal component of the current density at the boundary is zero since the boundaries allow no current to leave the sheet unless a source or sink is located at the boundary. This investigation assumed no magnetic effects due to current flow. The currents considered are on the order of 20 milliamperes or less. A method for calculating the potential difference between any two points in the conductor is investigated.

The following basic equations can be written for a current source on a resistive sheet of infinite extent. Ohm's Law in scalar and vector quantities can be defined at a point for conductors:

$$\bar{J} = \gamma \bar{E} \tag{1}$$

where, $\bar{J}$ - current density in amperes/meter^2

γ - conductivity in mhos

$\bar{E}$ - electric field intensity in volts/meter

For the conducting medium of the resistive sheet the vector quantities are two-dimensional.

For the conducting medium and a single source:

$$I = \bar{J}A \tag{2}$$

where, I - current in amperes

A - area in square meters presented to the current source

Even though the resistive sheet is considered two-dimensional a finite thickness presents area through which current flows.

Solving for $\bar{J}$,

$$\bar{J} = \frac{I}{A} = \frac{I}{2\pi\, rt}\,\bar{r} \tag{3}$$

where r - distance from the center of the current source in meters

t - thickness of the resistive sheet in meters

$\bar{r}$ - unit vector in radial direction

Substituting in equation 1:

$$E = \frac{1}{\gamma}\,\frac{I}{2\pi\, rt}\,\bar{r} = \frac{1}{2\pi}\frac{p}{t}\,\frac{I}{r}\,\bar{r} \tag{4}$$

where $\frac{p}{t}$ - is resistivity of the conducting sheet per square meter.

Potential V is related to the total current through the line integral. The potential difference is:

$$V_1 - V_2 = \int_{A_2}^{A_1} \bar{E}\cdot dr \tag{5}$$

where r_1, and r_2 are radial distances from the current source shown in Figure 2.

Substituting equation 4 we may write

$$V_{r_1} - V_{r_2} = \int_{A_1}^{A_2} \frac{I}{2\pi}\frac{p}{t}\frac{1}{r}\,\bar{r}\,dr = \frac{I}{2\pi}\frac{p}{t}\ln\frac{r_2}{r_1} \qquad (6)$$

When a circular current sink is added to the sheet, the current point source shifts from the center of the circular current source in the direction of the current sink. The dimension r_1, in equation 6 is not measured from the center of the circular current source as shown in Figure 2, but from the center of influence as shown in Figure 3.

Consider the absolute potential V_p at any field point P as given by the difference of potential between P and the origin 0 of potential zero as shown in Figure 4 (Ref 2:[illegible]),

$$V_p = \frac{I}{2\pi}\frac{p}{t}\ln\frac{h}{r_1} + \frac{-I}{2\pi}\frac{p}{t}\ln\frac{h}{r_2} \qquad (7)$$

$$V_p = \frac{I}{2\pi}\frac{p}{t}\ln\frac{r_2}{r_1} \qquad (8)$$

Solving for

$$\frac{r_2}{r_1} = e^{\left(\frac{2\pi t}{\rho I}V_p\right)} = \text{constant } A \text{ for } V_p \text{ fixed} \qquad (9)$$

This may be written in Cartesian form

where P is in terms of x and y

$$r_2^2 = (h+x)^2 + y^2 \qquad (10)$$

$$r_1^2 = (h-x)^2 + y^2 \qquad (11)$$

$$r_2^2 = A^2 r_1^2 \qquad (12)$$

from equation 9,

$$(h + x)^2 + y^2 = A^2\left[(h - x)^2 + y^2\right] \qquad (13)$$

Rearranging in quadratic form

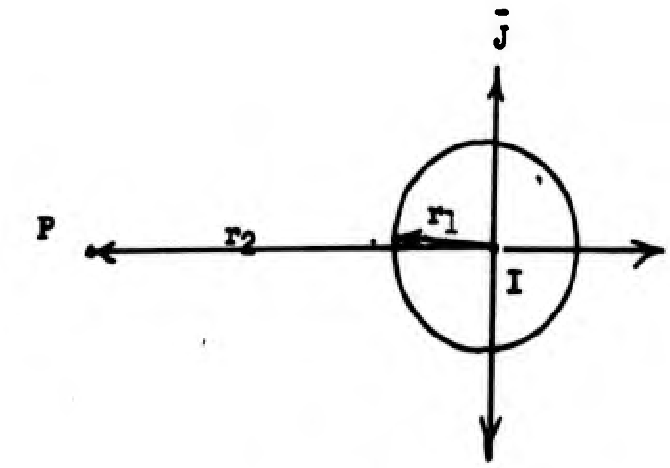


Fig. 2

Single Current Source

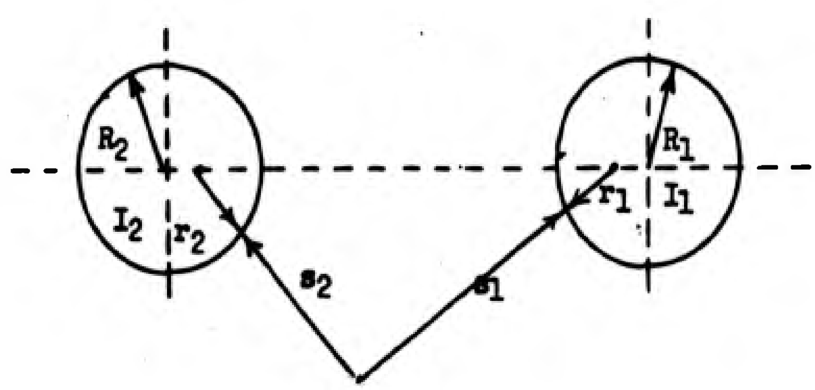


Fig. 3

Two Current Sources

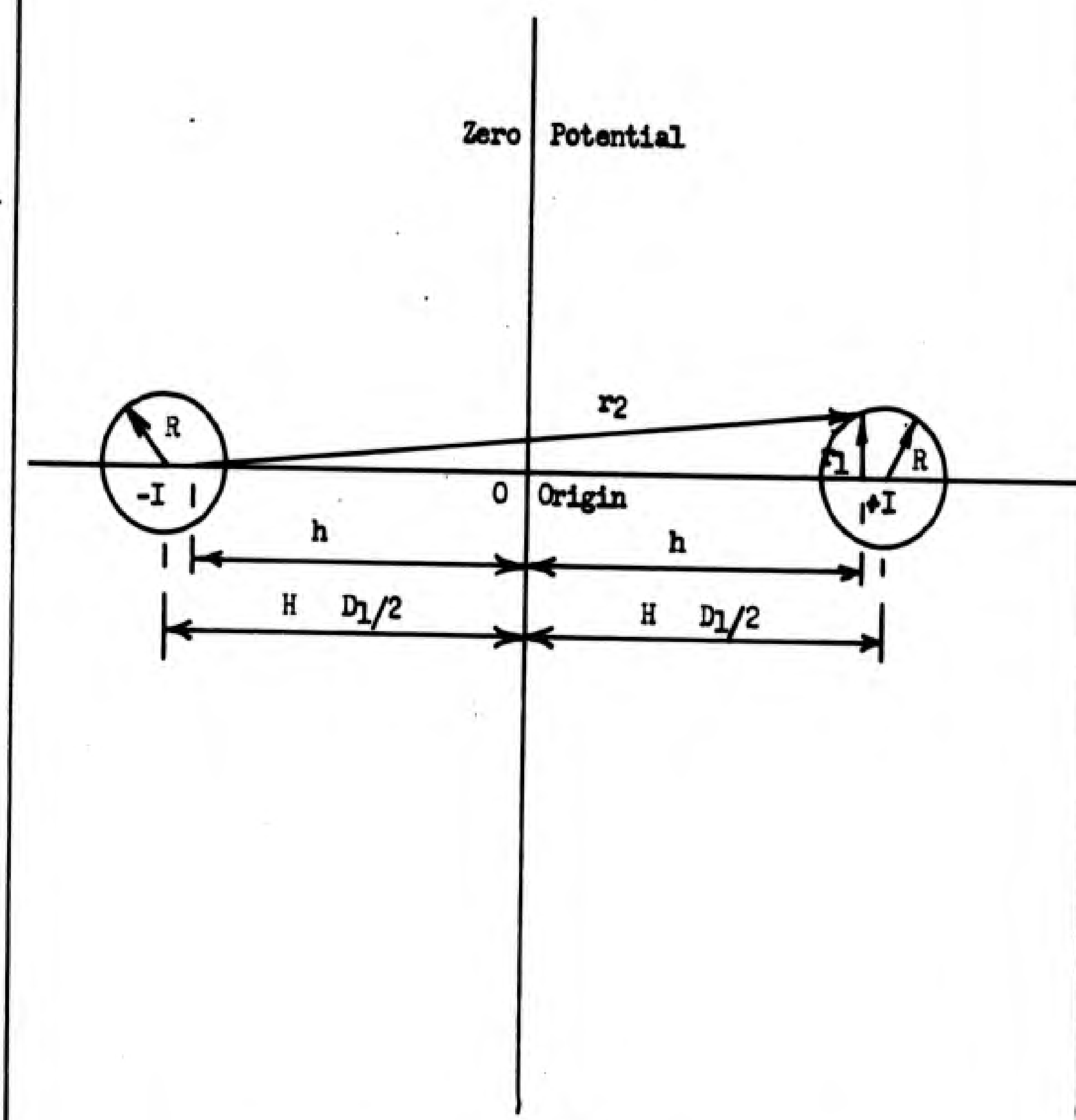


Fig. 4

Two Current Sources with Zero Equipotential

$$\left(x - h\,\frac{A^2 + 1}{A^2 - 1}\right)^2 + (y)^2 = \left(h\,\frac{A\,2}{A^2 - 1}\right)^2 \tag{14}$$

which is the equation of a circle with

Center coordinates $\left(+ h\,\frac{A^2 + 1}{A^2 - 1},\ 0\right)$, or $\frac{D_1}{2} = h\,\frac{A^2 + 1}{A^2 - 1}$ (15)

Radius equal to $h\,\frac{2A}{A - 1} = R_1$ (16)

whence $R_1^2 = h^2\left[\frac{(A^2 + 1)^2}{(A^2 - 1)^2} - 1\right] = h^2\left[\left(\frac{D}{2h}\right)^2 - 1\right] = \left(\frac{D_1}{2}\right)^2 - h^2$ (17)

Also $h = \sqrt{\left(\frac{D_1}{2}\right)^2 - R_1^2}$, $A = \frac{D_1}{2R_1} + \sqrt{\left(\frac{D_1}{2R_1}\right)^2 - 1} = \frac{r_2}{r_1}$

The absolute potential of any point P is then (18)

$$V_p = \frac{I}{2\pi}\frac{p}{t}\ln\frac{r_2}{r_1} = \frac{I}{2\pi}\frac{p}{t}\ln\left[\frac{D_1}{2R_1} + \sqrt{\left(\frac{D_1}{2R_1}\right)^2 - 1}\right] \tag{19}$$

The distance of the center of influence of the current source from the zero potential plane is (Ref 2:142):

$$h = \sqrt{\left(\frac{D_1}{2}\right)^2 - R^2} = \sqrt{H^2 - R^2} \tag{20}$$

where $H = \frac{D_1}{2}$

When small current sources are considered $h \cong H$

and $H = D/2$ for circular sources small compared with distance from the boundary.

For this current source system

$$V_A = \frac{I}{2\pi}\frac{p}{t}\ln\frac{D}{R} \tag{21}$$

In the problem where the points of zero potential are unknown due to an arbitrary location of a current source and sink on a resistive sheet of finite dimensions, the only information we have is about the location and radius of the sources and the size of the resistive sheet. Also, since the zero equipotential points are unknown, the potential at a point on the plane is calculated with reference to the potential established by the current source.

Referring to Figure 3, the following expression can be written for the potential at P(x,y) due to two current sources I_1, and I_2 on an infinite sheet.

$$Vp = \frac{I_1 \rho}{2\pi t} \ln \frac{r_1}{s_1} + \frac{I_2 \rho}{2\pi t} \ln \frac{r_2}{s_2} \tag{22}$$

where r_1, r_2, is distance from a point current source to the equipotential established by the current source, s_1, s_2, is distance from P to the equipotential established by the current source. When additional current sources are attached to the sheet, they contribute to potential at P. By superposition we can write for any number of sources.

$$Vp = \frac{I_1 \rho}{2\pi t} \ln \frac{r_1}{s_1} + \dots + \frac{I_n \rho}{2\pi t} \ln \frac{r_n}{s_n} \tag{23}$$

In the case for two sources $I_1 = -I_2 = I$

$$Vp = \frac{I \rho}{2\pi t} \ln \frac{r_1}{s_1} \times \frac{s_2}{r_2} \tag{24}$$

The fundamental features of current flow in a conservative field, which are considered in this problem are summarized as follows:

1. Current-flow lines and equipotential lines must everywhere intersect at right angles.

2. The application of a potential source to an electrode surface will cause this surface to be an equipotential, and thus all flow lines must meet the source at right angles.

3. At the boundary, current flow is parallel to the boundary.

4. Equipotential lines are normal to the boundary.

As shown in Figure 5, by placing like conductors spaced equally from the boundary, the boundary conditions can be satisfied.

The current source system under consideration is shown in Figure 6a. Only positive current sources are shown in the system of images. Boundary conditions at A are satisfied by two current sources located at the same distances below the boundary as the original sources above the boundary. This is indicated as step 1. Step 2 considers boundary B. The original sources and the images from the first step must be imaged with the distance requirement that the distance of a source below the boundary must equal the distance of a source above the boundary. Step 3 reconsiders boundary A and requires additional images. Thus there are an infinite number of images required to satisfy the boundary conditions for each boundary.

The images indicated in steps 1-4 satisfying only boundaries A and B and are labelled Column C_0. The boundary C must have the boundary conditions satisfied. This is done by adding a column of images, C_1. Columns C_2 and C_4 are required to satisfy boundary conditions at boundary D. Thus a system of columns of images are required to satisfy the boundary conditions. An analytical expression for calculating potential at a point on the resistive sheet for such a system of images will be derived in the next chapter.

Laplace's equation can be written as a difference equation allowing computer solution to the two-dimensional potential field problem.

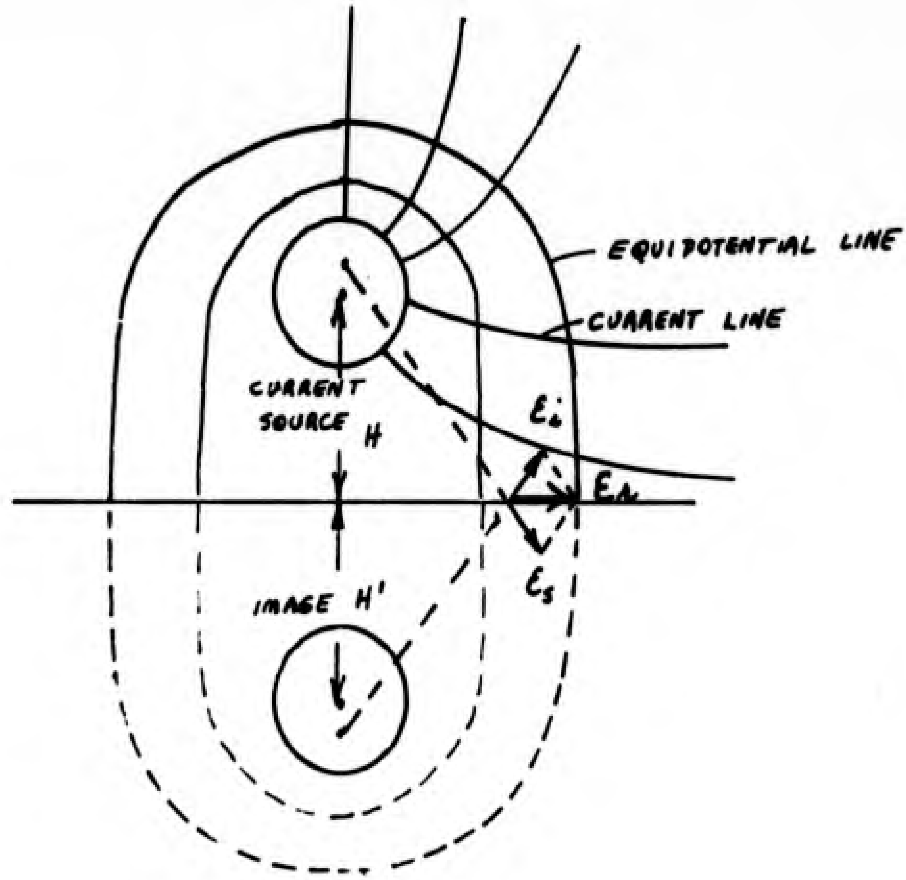


Fig. 5

Boundary Image

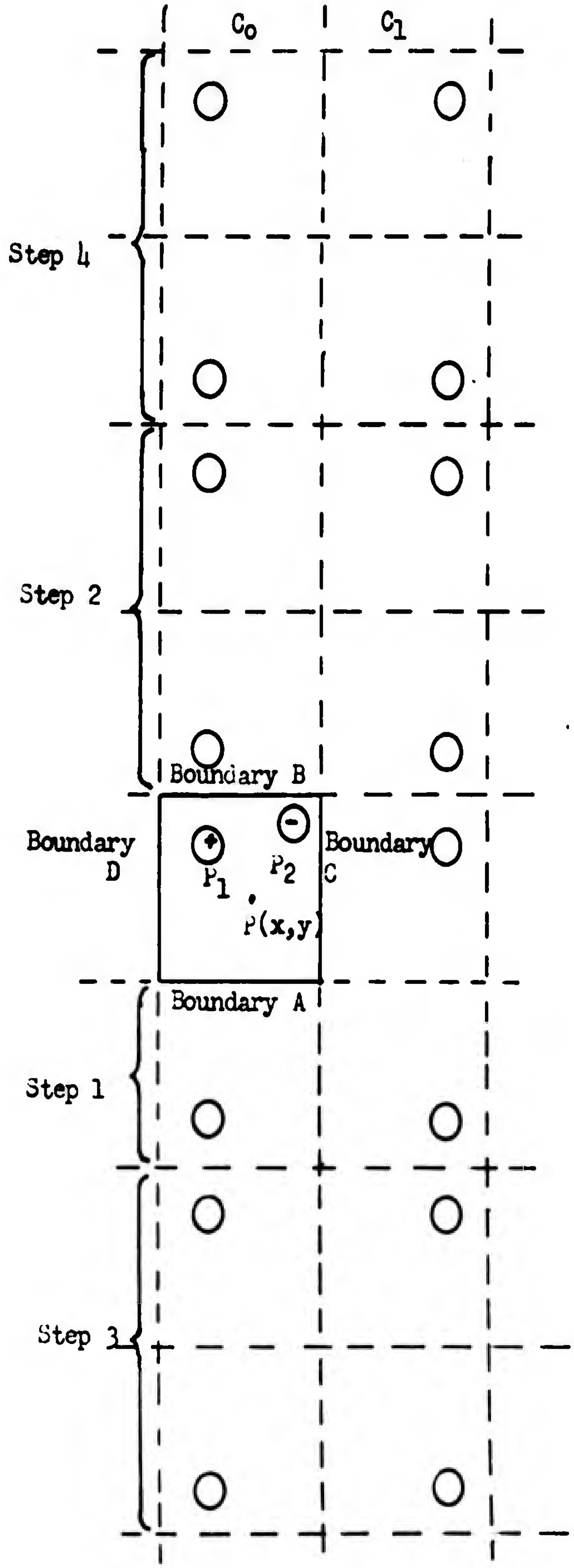


Fig. 6a System of Images

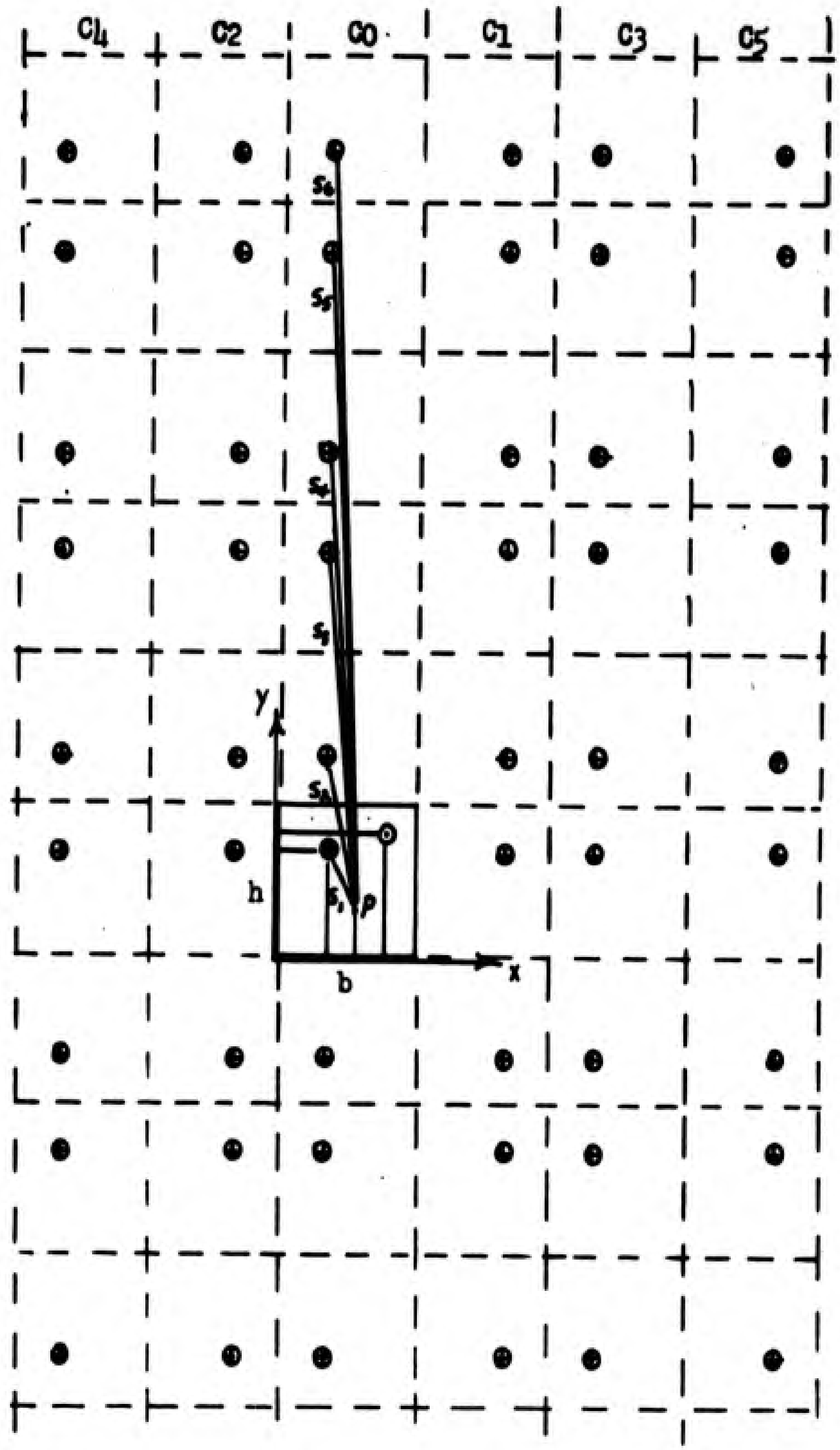


Fig. 6b System of Images for One Source

The first derivative at the point (0) in the xy plane can be written as difference operators (Ref 6:72). The notation is shown in Figure 7.

$$\frac{\partial V}{\partial x} = \frac{1}{h}(V_1 - V_0)$$

$$\frac{\partial V}{\partial x} = \frac{1}{h}(V_0 - V_3)$$

$$\frac{\partial V}{\partial y} = \frac{1}{h}(V_2 - V_0)$$

$$\frac{\partial V}{\partial y} = \frac{1}{h}(V_0 - V_4) \qquad (25)$$

The second derivatives are as follows:

$$\frac{\partial^2 V}{\partial y^2} = \frac{\frac{1}{h}(V_2 - V_0) - \frac{1}{h}(V_0 - V_4)}{h} = \frac{1}{h^2}\left[V_2 + V_4 - 2V_0\right]$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{\frac{1}{h}(V_1 - V_0) - \frac{1}{h}(V_0 - V_3)}{h} = \frac{1}{h^2}\left[V_1 + V_3 - 2V_0\right] \qquad (26)$$

Substituting in Laplace's equati n and solving for V_0:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \qquad (27)$$

$$V_0 = .25\,(V_1 + V_2 + V_3 + V_4) \qquad (28)$$

This equation is the basis for the computer solution to the potential field problem.

The dimension of the distance between points of the array with respect to the size of the array and the size of the current sources determines the accuracy of the com uter solution. Since the equipotential lines are continuous, in order to compute values at points of the array, finite differences are taken between points, and values of the equipotentials are calculated. Many calculations are required to approach the value of equipotential at a point of the lattice. A tolerance is assigned in the computer program. When this tolerance is met the computer stops and prints the calculated potentials.

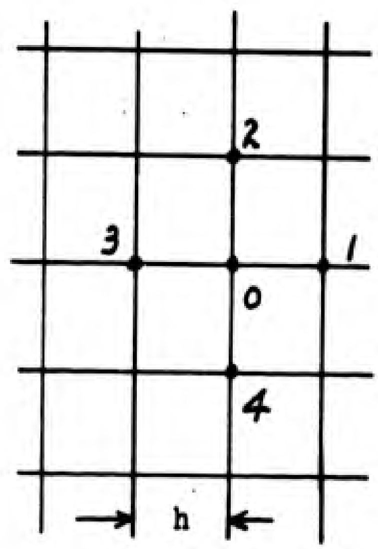


Fig. 7
Lattice Points of Numerical Array

III. Calculations and Measurements

It was shown in Chapter II that in order to satisfy the boundary conditions of the resistive sheet, a system of columns of images were necessary as shown in Figure 6b. In order to write an analytical expression in rectangular coordinates two approximations are made.

1. The coordinates of the center of the circular current source are used as the center of influence of the current source. This allowed $r_1, \ldots, r_n$ of equation 23 to become the radii of the respective sources. Since the system of sources is the image of the original source, $r_1 = r_2 = \ldots = r_n$

2. The radius of the current source is small compared to the distance between sources, $r_1 << s_1, \ldots$ This allows $s_1, \ldots s_n$ terms to be written in terms of cartesian coordinates.

In Figure 6b, only the positive circular current sources of column C are considered initially. Equation 23 becomes

$$Vp = \frac{I_1}{2\pi}\frac{p}{t} \ln \frac{r_1}{s_1} \cdot \frac{r_2}{s_2} \cdot \frac{r_3}{s_3} \ldots \frac{r_n}{s_n} \qquad (29)$$

where $s_1, s_2, \ldots, s_n$ are shown above the original resistive sheet in Figure 6b.

The term $\frac{r_1}{s_1}$ becomes $\frac{r_1}{\left[(x - x_1)^2 + (y_1 - y)^2\right]^{\frac{1}{2}}}$

where x, y, are coordinates of P

x_1, y_1 are coordinates of the positive current source.

The term $\frac{r_2}{s_2}$ becomes $\frac{r_1}{\left[(x - x_1)^2 + (2h - y_1 - y)^2\right]^{\frac{1}{2}}}$

where h is the height of the resistive card.

$$r_2 = r_1$$

Substituting in equation 29

$$Vp = \frac{I_1}{2\pi} \frac{p}{t} \ln \frac{r_1}{\left[(x - x_1)^2 + (y_1 - y)^2\right]^{\frac{1}{2}}} \cdot \frac{r_1}{\left[(x - x_1)^2 + (2h - y_1 - y)^2\right]^{\frac{1}{2}}} \qquad (30)$$

Positive image sources below the original resistive sheet contribute to Vp by superposition. The $\frac{r}{s}$ expressions for the first two images below the card are:

$$\frac{r_1}{\left[(x - x_1)^2 + (y + y_1)^2\right]^{\frac{1}{2}}} \cdot \frac{r_1}{\left[(x - x_1)^2 + (2h - y_1 + y)^2\right]^{\frac{1}{2}}}$$

When the minus current sources are considered (minus defines current leaving the resistive sheet), the $\frac{s}{r}$ expressions for two sources in the center column are:

$$\frac{\left[(x - x_2)^2 + (y_2 - y)^2\right]^{\frac{1}{2}}}{r_2} \qquad \frac{\left[(x - x_2)^2 + (2h - y_2 - y)^2\right]^{\frac{1}{2}}}{r_2}$$

where x_2, y_2 are coordinates of the negative current source

r_2 is the radius of the minus current source and the minus image sources.

By superposition of the plus and minus current sources of the center column, equation 23 can be written as follows:

$$Vp = \frac{p\ I}{2\pi\ t} \ln \left\{ \frac{(r_1^2)^n}{(r_2^2)^n} \frac{\left[(x-x_2)^2 + (y_2 - y)^2\right]}{\left[(x-x_1)^2 + (y_1 - y)^2\right]} \cdot \frac{\left[(x-x_2)^2 + (2h-y_2 - y)^2\right]}{\left[(x-x_1)^2 + (2h-y_1 - y)^2\right]} \right.$$

$$\left. \frac{\left[(x-x_2)^2 + (y + y_2)^2\right]}{\left[(x-x_1)^2 + (y + y_1)^2\right]} \cdot \frac{\left[(x-x_2)^2 + (2h-y_2 + y)^2\right]}{\left[(x-x_1)^2 + (2h-y_1 + y)^2\right]} \cdot \left[\cdots\right] \right\}^{\frac{1}{2}} \qquad (31)$$

where n is the number of current sources used in the calculation. The complete expression to calculate Vp by superposition of the current sources and the images in the center column would contain the $\frac{r_1}{s_1}$ expressions for all the columns of plus images and the $\frac{s_2}{r_2}$ expressions for all the columns of minus images. A list of the expressions for $\frac{r_1}{s_1}$ and $\frac{s_2}{r_2}$ combined for two sources is given in appendix A by columns number where the 1 and 2 subscript is for plus and minus current sources respectively. The number of images used in calculations was that number of images which contributed to the potential Vp significantly. It is shown in the sample calculation of appendix A that, when images were grouped together, the total contribution approached the value one as the distances from the original resistive sheet increased. Additional images then became insignificant in the calculation and were neglected.

In the center column the ratio of distances $\frac{s_2}{s_1}$ can be written for the first two images as follows:

$$\left\{\left[\frac{(x - x_2)^2 + (a_n h - y_2 - y)^2}{(x - x_1)^2 + (a_n h - y_1 - y)^2}\right] \cdot \left[\frac{(x - x_2)^2 + (a_n h + y_2 - y)^2}{(x - x_1)^2 + (a_n h + y_1 - y)^2}\right]\right\}^{\frac{1}{2}}$$

where a increases by 2 for two succeeding terms.

$$a_n = 2n$$

$$n = 1,2,3,\ldots$$

Divide by a_n

$$\lim_{n \to \infty} \left\{\frac{\left[\left(\frac{x - x_2}{a_n}\right)^2 + \left(h - \frac{y_2}{a_n} - \frac{y}{a_n}\right)^2\right]}{\left[\left(\frac{x - x_1}{a_n}\right)^2 + \left(h - \frac{y_1}{a_n} - \frac{y}{a_n}\right)^2\right]}\right\}^{\frac{1}{2}}$$

$$\lim_{n \to \infty} \left\{ \left[\frac{h}{h}\right] \left[\frac{h}{h}\right] \right\}^{\frac{1}{2}} = 1 \qquad (32)$$

This is a necessary condition for convergence.

Equation 31 was used for calculating the potential of the plus and minus current sources, A and B respectively.

RAB was calculated using the equation

$$\frac{VA - VB}{I} = RAB \qquad (33)$$

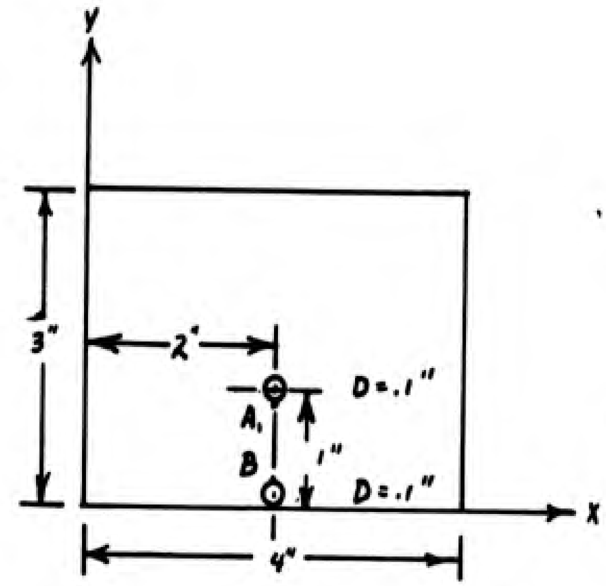
Shown in Figures 8a, and 8b, the dimensions of the variables are shown for six calculations. The variables are tabulated in Table I by problem number. The first three calculations were to find the resistance between two current sources. In problem 3 all dimensions of problem 2 were doubled. The last three calculations were to find the potential at a point due to the current sources shown in Figure 6b. These last three calculations were made considering only the image sources in columns C_0 and C_2. Columns C_0 and C_2 are symmetrical about the points A_4, A_5, and A_6 on the boundary as shown in Figure 8.

Table II shows the results of six calculations. The calculated value of RAB for problem three is the same as RAB of problem two.

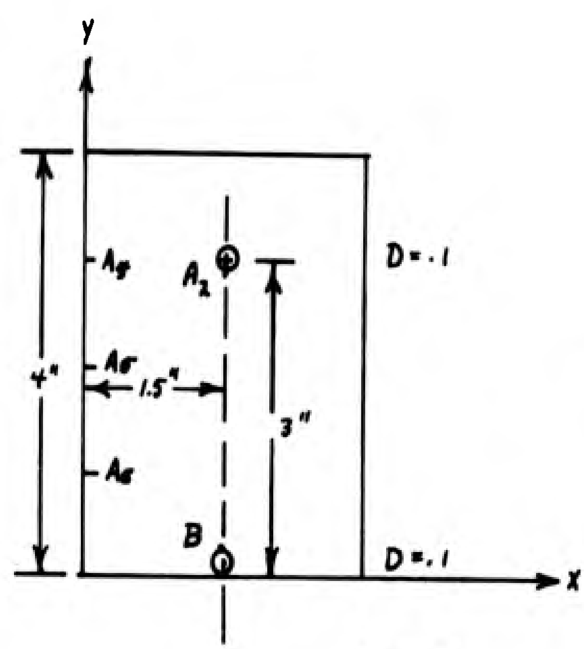
Tables III and IV compare the measured resistance and potential values with the computed values.

Measurements

Type L Teledeltos paper was used for the resistive sheet. This thin electrical conducting paper is .004 inches thick. For two-dimensional field studies this material approaches the ideal plane. The paper is



Problem 1.



Problems 2-6

Fig. 8

Dimensions for Problems 1 - 6

Table I
Table of Dimensions for Six Problems

Variable	Problem No.					
	1	2	3	4	5	6
h	3	4	8	4	4	4
b	4	3	6	3	3	3
x_1	2	1.5	3	1.5	1.5	1.5
y_1	1	3	6	3	3	3
x_2	2	1.5	3	1.5	1.5	1.5
y_2	.05	.05	.1	.05	.05	.05
x_a	2	1.5	3	0	0	0
y_a	.95	2.95	5.90	3.00	2.00	1.00
x_b	2	1.5	3	1.5	1.5	1.5
y_b	.10	.10	.20	.10	.10	.10

1. All Dimensions - Inches
2. h, b - dimensions of resistive sheet
3. x_1, y_1 - coordinates of plus current source
4. x_2, y_2 - coordinates of minus current source
5. x_a, y_a - coordinates of point A
6. x_b, y_b - coordinates of point B

Table II
Table of Calculated Results

Problem No.	RAB	Problem No.	V_{AB}
1	2900	4	7.2
2	4100	5	6.06
3	4100	6	4.69

1. Dimensions of circuit are given in Table I
2. RAB, VAB shown in Figure 8

Table III
Table of Comparison Between Measured and Calculated Resistance Between Two Sources

Problem No.	Samples	Measured R_{ab} (OHMS)	Calculated R_{ab} (OHMS)	Maximum Difference	
				OHMS	Per cent
1	1 2 3	2780 2860 2940	2900	120	4.14
2	1 2 3 4 5 6 7 8	4000 4060 4110 4110 4060 4110 4170 4220	4100	120	2.93
3	1 2 3	4050 4080 4120	4100	50	1.22

1. Measured R_{ab} values were obtained by plotting a volt-ampere relationship where current source system dimensions are defined in Table 1.
2. R_{ab} is slope of V-I curve.

Table IV

Table of Comparison Between Measured and Calculated Voltages Using Two Columns of Images

Prob. No	Measured V	Calculated V	Difference	
			Volts	Per Cent
4	7.5 7.5 7.4	7.2 V	.3	4
5	6.0 6.0 5.9	6.06 V	.06	1
6	4.5 4.5 4.4	4.69 V	.19	4.05

1. V_{A1} = 10 volts
2. V_B = 0
3. R_{AB} = 4100
4. I_{AB} = 2.44 ma

made of carbon or graphite impregnated stock and comes in large rolls, 31 inches wide. The uniformity of resistance of sample areas has been found to vary with ±3 per cent to ±8 per cent from a maximum deviation from a mean value with the paper showing the lower resistance values when measured lengthwise of the roll as compared with the higher values obtained when measuring crosswise. This is a permanent property introduced in the processing of the paper (Ref 4:10).

Since the resistive paper is described to have anisotropic properties, different samples of resistive paper for each calculation were prepared by arranging the current sources along the two axes. For the measurement of problem 2, four samples were prepared with the current sources spaced along the axis of the length of the roll of paper and four samples were prepared with the current sources spaced along the axis of the width of the roll of paper. The measurements of these samples for problem 2 are shown in Table III.

The current sources were brushed on with silver paint by using paper templates of the proper dimension. Silver conducting paint has a resistance of 2 or 3 ohms per square when it is applied thinly. Silver paint was applied to form a conducting source with negligible resistance in comparison with the resistance of the resistive sheet. The .1" diameter sources used were sections of brass welding rods whose dimensions measured 100/1000 inch in diameter. For large current sources for the graphs of the following chapter, circles of brass were used for conducting sources. These circles of brass were placed upon the silver paint sources to insure the measured voltage was at the edge of the current source.

The voltage and current measurements were made from Weston standard laboratory meters. Manufacturer tolerance for the voltmeter is 3% however, calibration of the meter was less than 1% deviation. The ammeter was a Weston Ammeter (0-15 ma) whose manufacturer tolerance is 1%. The meter was calibrated to less than 1% deviation.

Adjustable probes were clamped to the current sources to insure a sturdy source system.

The resistance per square of the resistive paper was also measured. This constant for the material was used in the calculations. The equation for resistance between two circular current sources of radii R_1 and R_2 with a distance D between their centers is (Ref 7:234)

$$R = \frac{\rho}{2\pi t} \cosh^{-1}\left(+\frac{D^2 - R_1^2 - R_2^2}{2R_1R_2}\right) \tag{34}$$

where D is the distance between centers

R_1, R_2 are the radii of the sources

This is an exact expression for the resistance between two sources where the resistive sheet is infinite in extent. The above equation was solved for $\frac{\rho}{t}$ with measured values of R for dimensions D, R_1, and R_2. Table V shows the results of such measurements and calculations. The resistive sheet used in this measurement measured 30 by 40 inches. The effect of the physical boundaries was considered negligible for the measurements in Table IV.

Also two samples of resistance paper two inches and four inches wide were prepared. Conducting metal electrodes were painted on with silver paint and allowed to dry. The electrodes were spaced two inches and four inches apart respectively. The resistance of a square section was obtained by measuring the voltage-current relationship between electrodes, as shown in Figure 9. The resistance values were 2175, and 2170 ohms for the 2 inch and 4 inch squares respectively. Based on these measurements and the measurements of Table V, the resistivity per square used in the calculations was

$$p/t = 2170 \text{ ohms/square}$$

Sample calculation:

Dimensions are in inches.

$D = 3$, $R_1 = .125$, $R_2 = .125$

Substituting in equation 34

$$R = \frac{1}{2\pi}\frac{p}{t}\cosh^{-1}\frac{(3^2 - .125^2 - .125^2)}{2(.125)(.125)} \qquad (35)$$

$$R = \frac{p}{t}\frac{1}{6.28}\cosh^{-1}\left[\frac{9 - .01563 - .01563}{.03126}\right]$$

$$R = \frac{p}{t}\frac{1}{6.28}\cosh^{-1} 287$$

$$\cosh^{-1} x = \log e \left(x + \sqrt{x^2 - 1}\right) \text{ (Ref 3:24)} \qquad (36)$$

$$\cosh^{-1} 287 = \log e \left(287 + \sqrt{287^2 - 1}\right) = \log e\,(574) = 6.35$$

$$R = p/t\,\frac{6.35}{6.28} = 1.01\,p/t$$

$$R(\text{measured}) = 2190$$

$$p/t = \frac{2190}{1.01} = 2170 \qquad (37)$$

Table V
Table for Resistivity of Teledeltos Resistance Paper

D	R_1	R_2	R(calculated)	R(Measured)	p/t
2	.125	.125	.882 p/t	1920	2180
3	.125	.125	1.01 p/t	2190	2170
4	.125	.125	1.105 p/t	2400	2170

1. Dimensions in inches
2. R(measured) in ohms
3. p/t in ohms/square

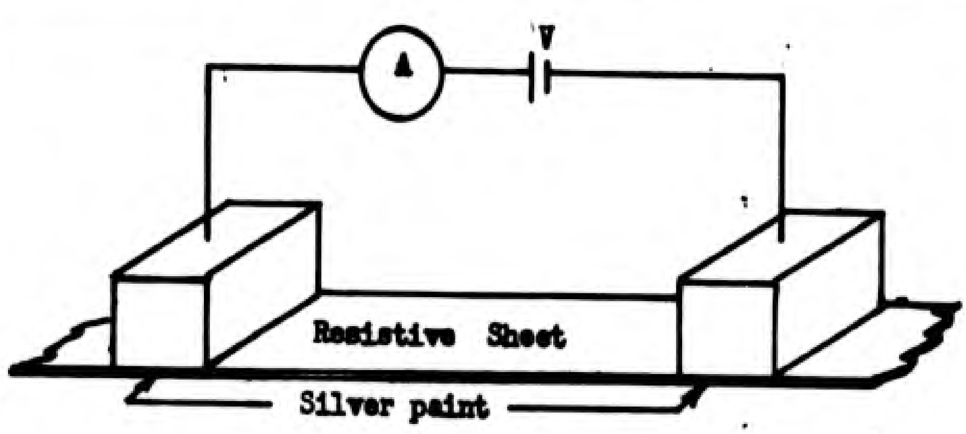


Fig. 9
Electrodes for Resistivity Measurement

Results of Calculations and Measurements

Although approximations were made to facilitate deriving an analytical expression for the resistance between two sources, measurements show that calculations may be performed using images giving accuracies within 2%. The variation of the measured values is believed due to the nature of establishing contact to the resistance sheet material plus the variation of resistance of the samples used for measurement. The samples were cut by hand and slight irregularities at the boundary are assumed negligible. Current sources of small diameter were used in order to validate the approximations of the problem.

The calculation of potential at a point on the plane may be made using symmetry to reduce the number of images required in the calculation. When the symmetry of the problem deviates, however, as shown in problem 6 and fewer images are used, less accurate results are obtained.

When additional current sources are added to the resistive card, the effective resistance between two sources is altered. The effective resistance between two sources on the resistive sheet is a function of the size and distribution of all sources on the resistive sheet plus the dimensions of the sheet. The next chapter illustrates the variation of resistance values between two current sources.

IV. Resistance Between Sources on a Resistance Sheet

Examination of equation 31 shows the potential at a point on the resistance sheet is a function of a constant for the material and the natural logarithm of a ratio of dimensions. In order to illustrate the variation of resistance between current sources, as the size and distribution of the current sources is altered, the following graphs are presented. Resistance values were determined by plotting a range of volt-ampere values and the slope of this plot was used as the resistance value. Where additional current sources were added to the resistive card, these sources were grounded and the voltage-current relationship was measured for the two sources under investigation. Measurements were made on a 3 x 4 inch resistive card except where proportional resistive networks are illustrated.

Graph 1 shows variation of resistance between two sources as the diameter of source 2 increases from .25 to 1.00 inches. As additional sources were added the effective resistance between sources 1 and 2 increased.

Graph 2 shows variation of resistance as a function of stations in the x direction.

Graph 3 shows variation of resistance as a function of stations in the Y direction.

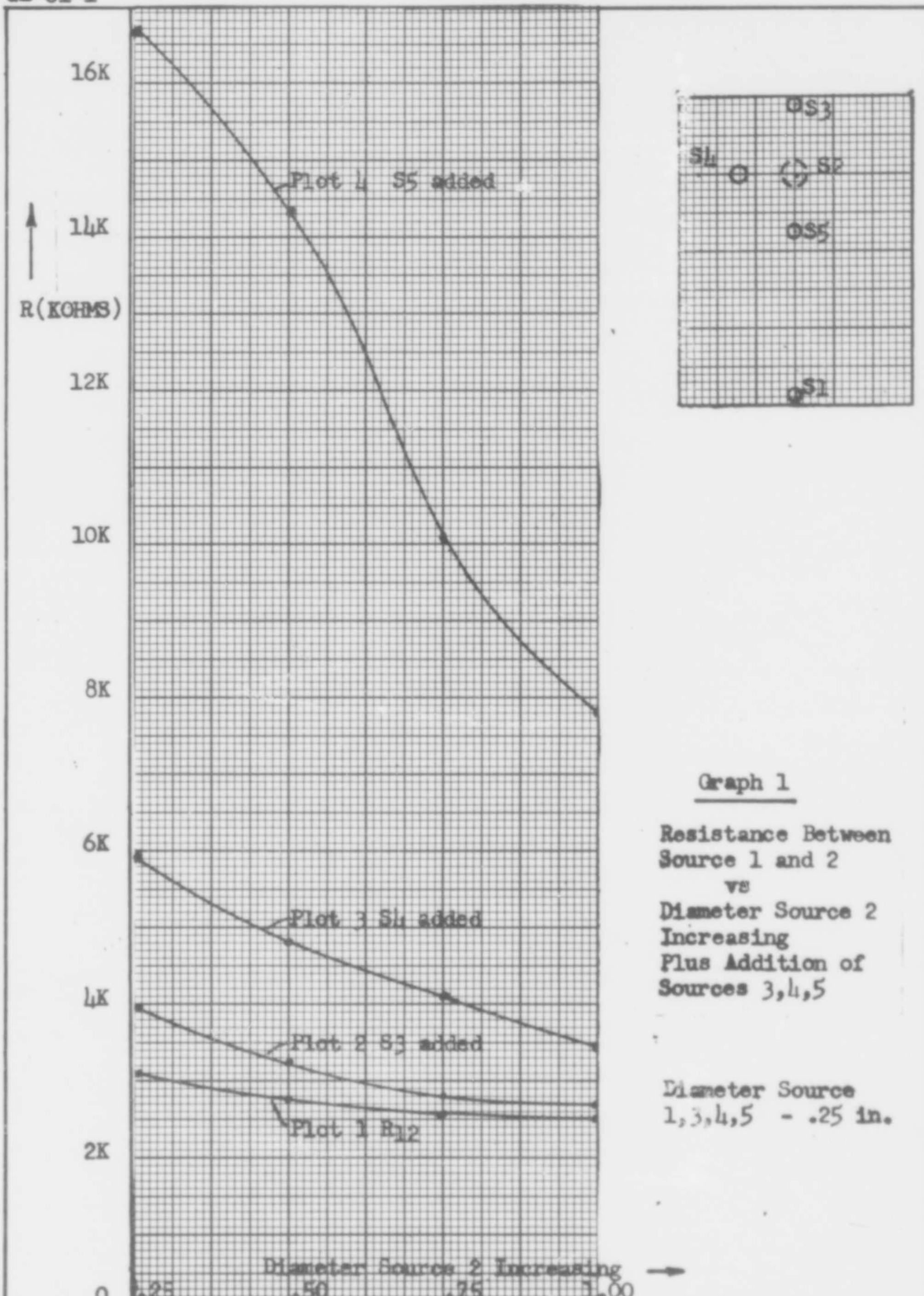
Graph 4 shows variation of resistance as a function of stations in the Y direction where the diameter of source 2 is increased to 1.00 inch.

Graph 5 shows the similarity of resistance values when the size of the resistive sheet and current sources are reduced proportionally.

Figure 10 shows the resistive network for four current sources when the dimensions of the resistive sheet and the current sources are reduced by one half. The accuracy of the resistive values obtained depends on the accuracy of the proportional dimensions and the uniformity of the resistive sheet.

The purpose of these graphs is to show that the effective resistance between sources on the resistive sheet is a function of the size and distribution of the current sources, that proportional resistive networks can yield equivalent resistances, and that a qualitative approach to resistor shaping is possible.

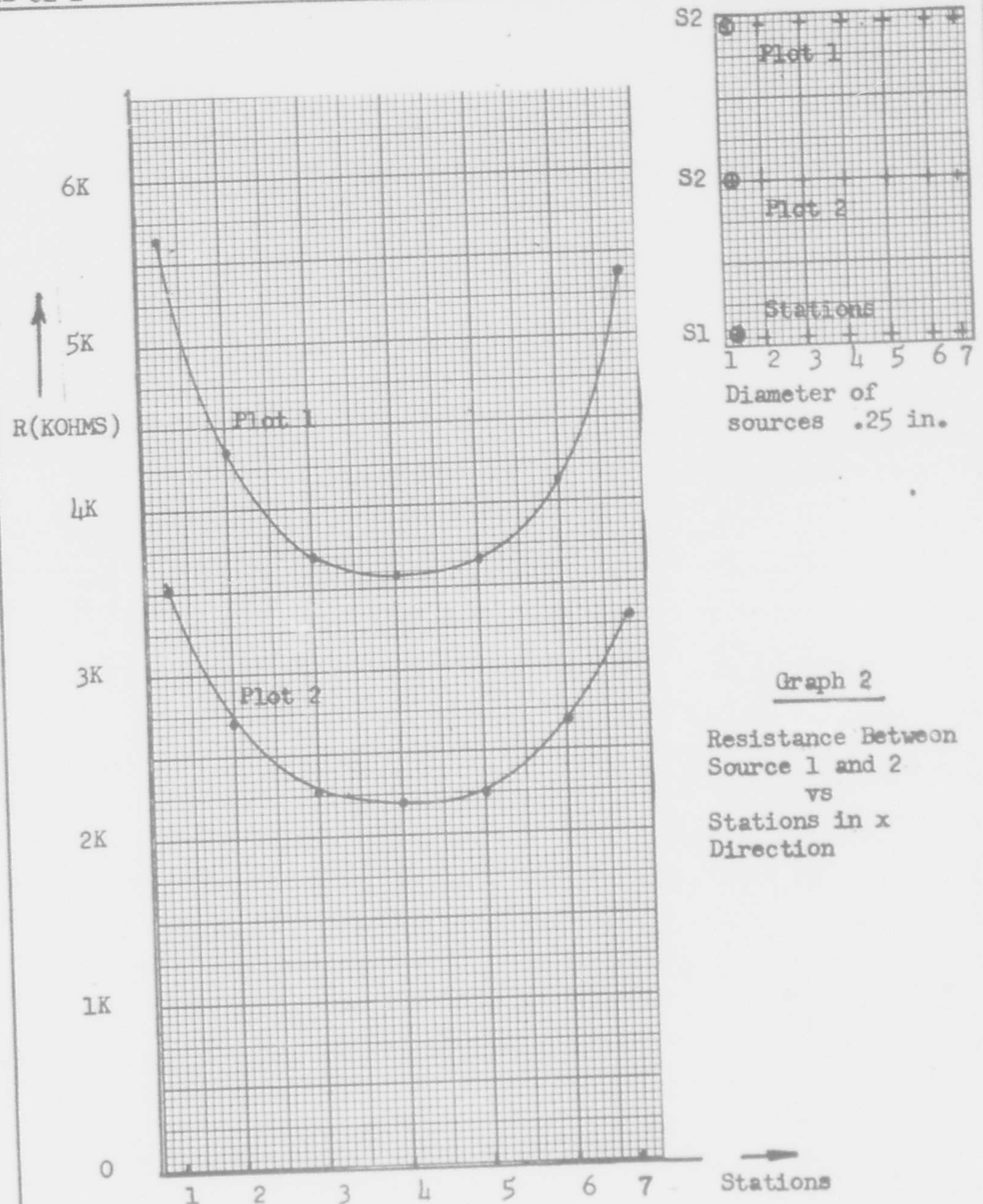
GE-61-1



Graph 1

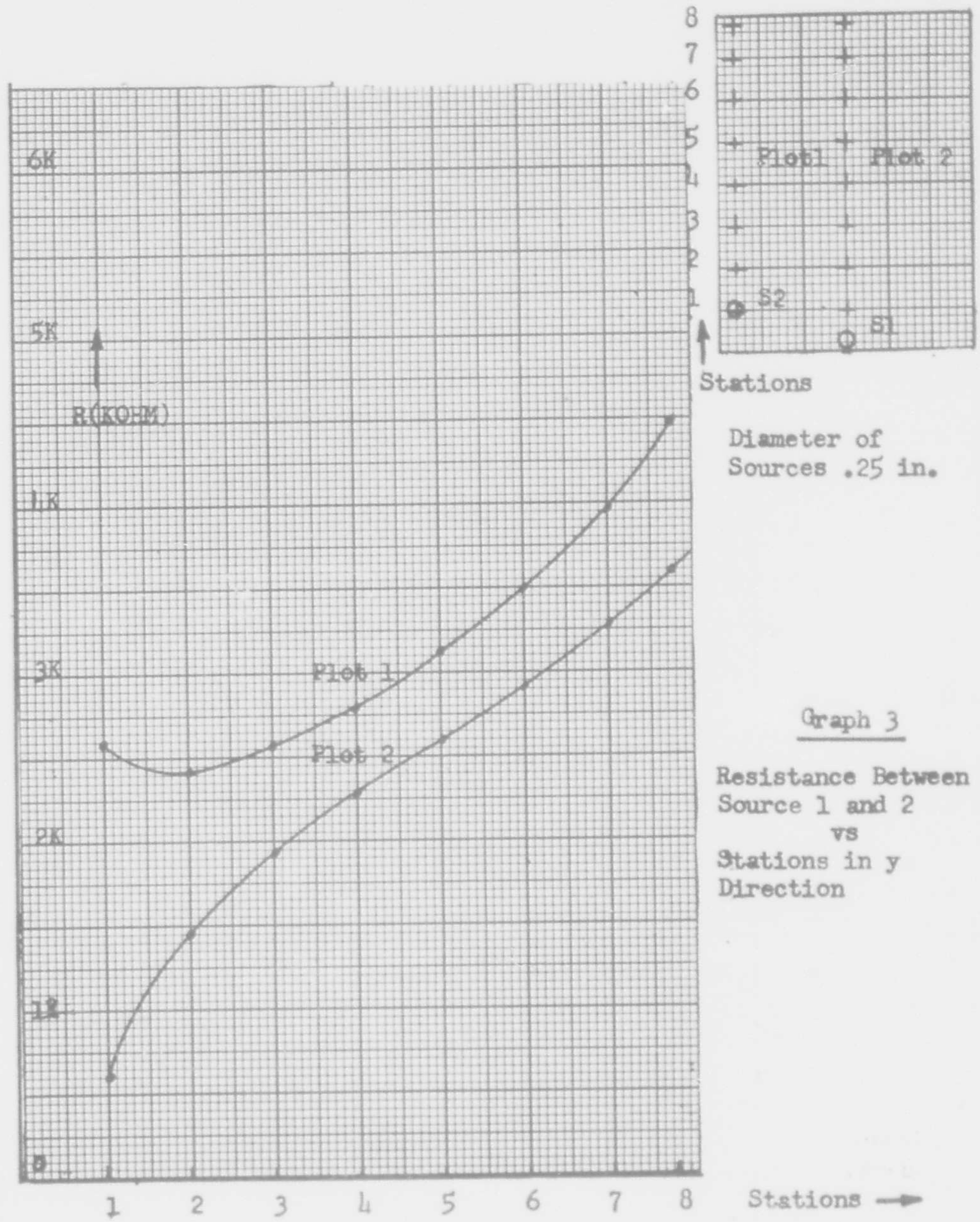
Resistance Between Source 1 and 2 vs Diameter Source 2 Increasing Plus Addition of Sources 3,4,5

Diameter Source 1,3,4,5 - .25 in.



Graph 2

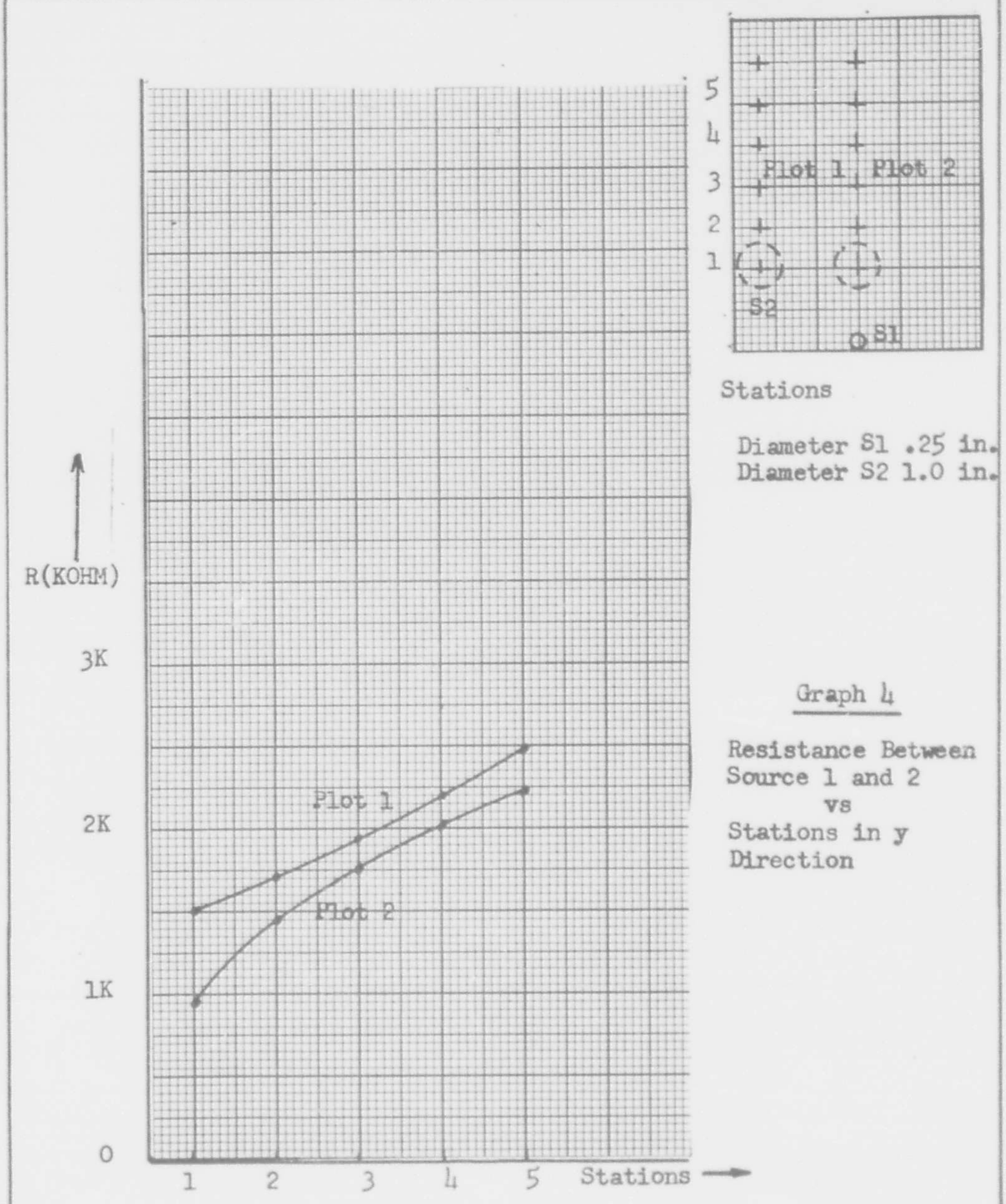
Resistance Between Source 1 and 2 vs Stations in x Direction



Diameter of Sources .25 in.

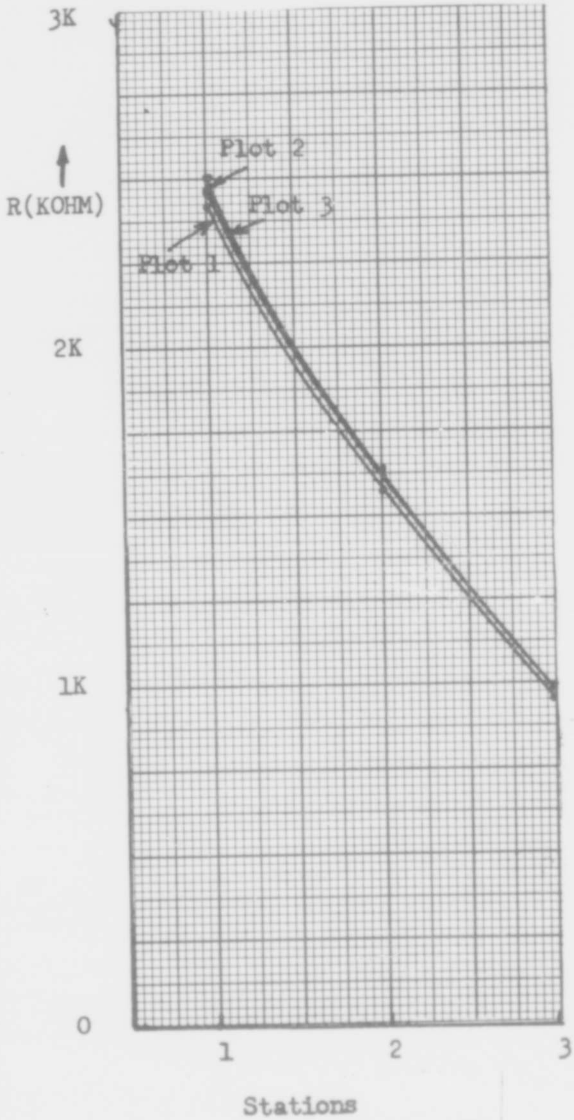
Graph 3

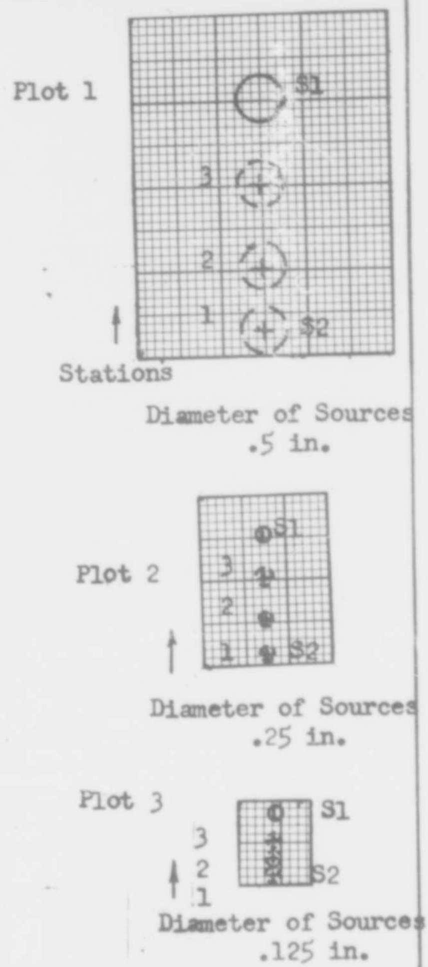
Resistance Between Source 1 and 2 vs Stations in y Direction



Graph 4

Resistance Between Source 1 and 2 vs Stations in y Direction

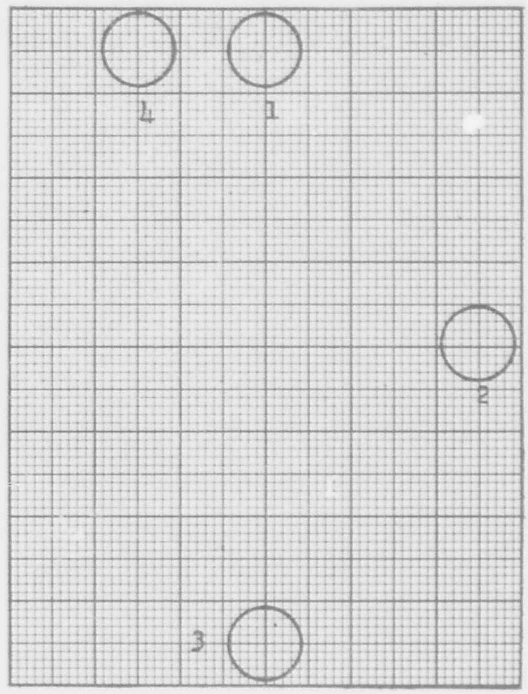




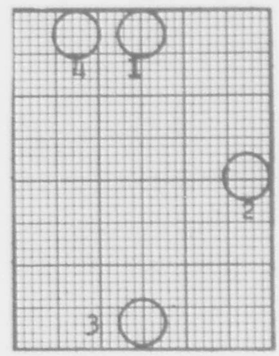
Graph 5

Resistance Between Source 1 and 2 vs Stations in y Directions on Three Proportional Resistance Sheets

A



B



Diameter of sources .5 inch
Resistive sheet 3 by 4 inches

Network A

R_{12}	3450
R_{13}	11780
R_{14}	870
R_{23}	2900
R_{24}	9520
R_{34}	10000

Diameter of sources .25
Resistive sheet 1.5 by 2

Network B		Δ R
R_{12}	3570	120
R_{13}	11400	380
R_{14}	857	13
R_{23}	2860	40
R_{24}	9520	0
R_{34}	10000	0

Fig. 10

Example of Two Proportional Networks

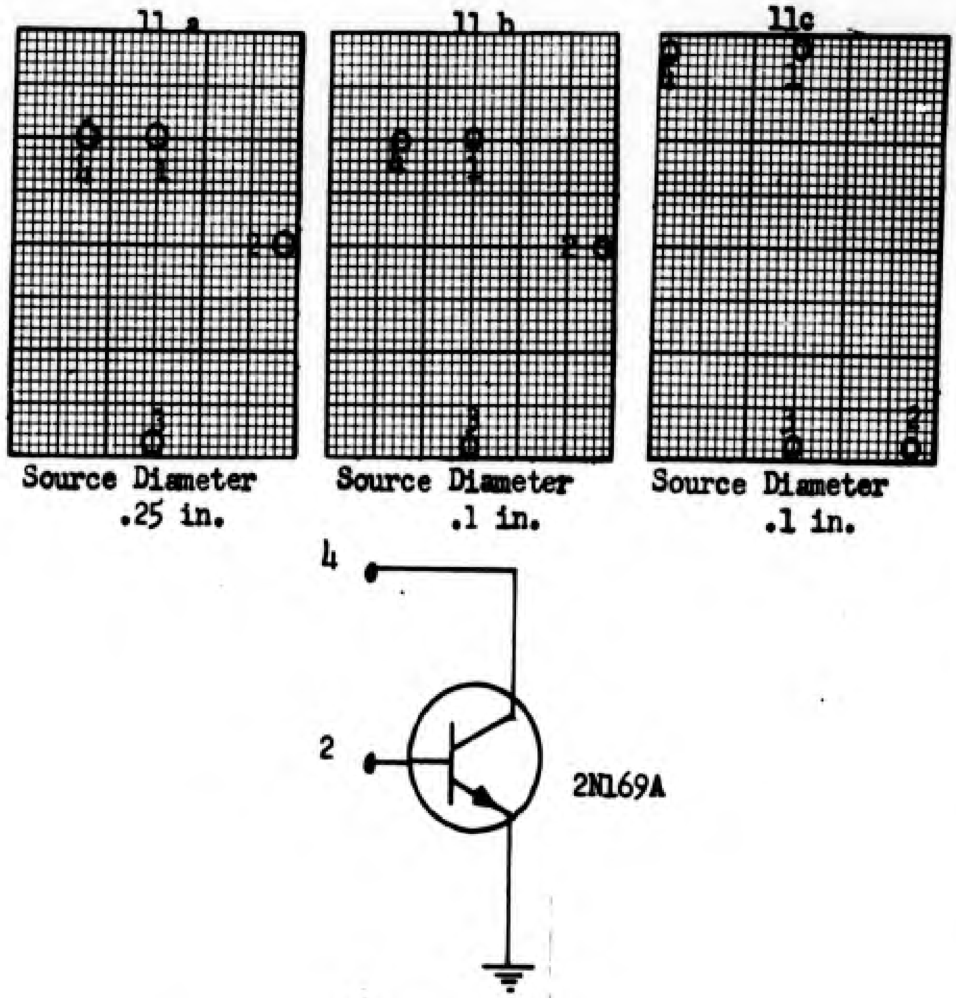
V. Circuit Analysis

Shown in Figure 11 are three resistive networks for a common emitter amplifier circuit. The current sources on the resistive sheet are numbered clockwise. The transistor was attached at points 2 and 4. Battery supply was attached at points 1 and 3. This circuit was selected for circuit analysis since it is the simplest configuration for an active device where all the resistive elements of the circuit are provided by the sheet of resistance paper.

By referring to equation 23, the equations defining the current and potential relationship for this system of current sources may be written as Kirchhoff's equations. The use of node equations facilitates measuring the circuit resistances (Ref 7:37).

$$
\begin{aligned}
I_1 &= Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3 + Y_{14} V_4 \\
I_2 &= Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3 + Y_{24} V_4 \\
I_3 &= Y_{31} V_1 + Y_{32} V_2 + Y_{33} V_3 + Y_{34} V_4 \\
I_4 &= Y_{41} V_1 + Y_{42} V_2 + Y_{43} V_3 + Y_{44} V_4 \\
I_1 &+ I_2 + I_3 + I_4 = 0
\end{aligned}
\tag{38}
$$

Based on these equations the admittance values were computed from measurements of the volt-ampere relationship of the network. These measurements were made by applying known voltage at one current source, applying zero voltage at the remaining three sources, and measuring the current flow. The following admittances were calculated for the circuit of Figure 11a. Sample calculations are shown in Appendix B.



	11 a	11 b	11 c
R_{12}	3570	6350	16660
R_{13}	9090	10530	11100
R_{14}	1500	1910	5500
R_{23}	5400	8330	5890
R_{24}	9510	10000	38400
R_{34}	8000	9100	22700

Fig. 11

Resistance Network for Three
Common Emitter Amplifiers

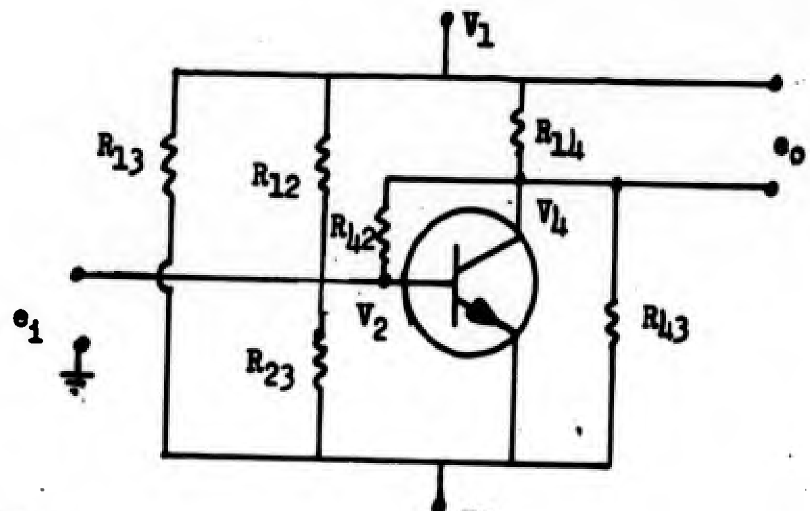


Fig. 12a

Amplifier Circuit

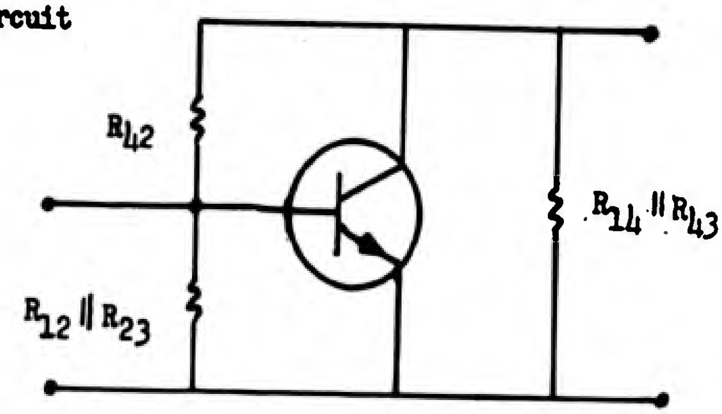


Fig. 12b

AC Circuit

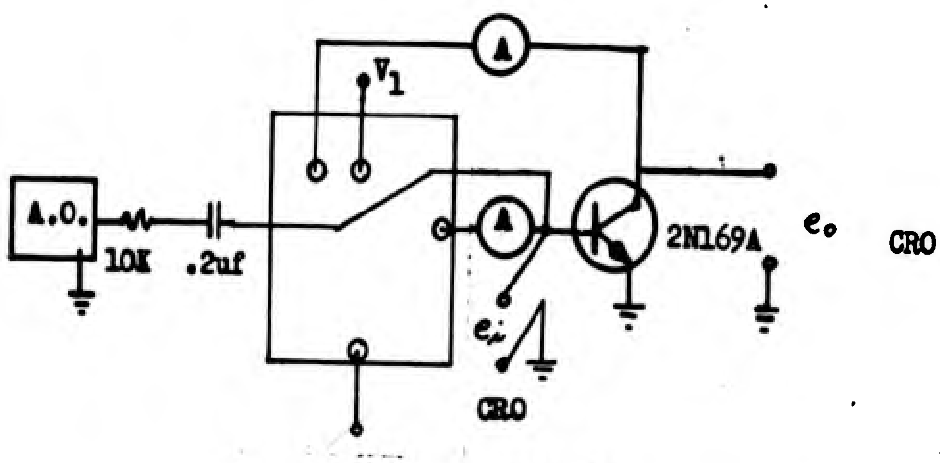


Fig. 13

Experimental Circuit

$$I_1 = .001056\ V_1 - .000280\ V_2 - .000110\ V_3 - .000666\ V_4$$
$$I_2 = -.000280\ V_1 + .000570\ V_2 - .000185\ V_3 - .000105\ V_4$$
$$I_3 = -.000110\ V_1 - .000185\ V_2 + .000420\ V_3 - .000125\ V_4$$
$$I_4 = -.000666\ V_1 - .000105\ V_2 - .000125\ V_3 + .000896\ V_4 \qquad (39)$$

The following Q point was established

$$Ib = -.25 \text{ ma}, \quad VCE = 2.0 \text{ volts}, \quad Ic = -3.3 \text{ ma}$$

The minus denotes current leaving the resistive sheet.

$$V_1 = 10.3$$
$$I_1 = 11.2$$
$$V_2 = V_{BE} = .22$$
$$I_2 = Ib = -.250$$
$$V_3 = -14.6$$
$$I_3 = -7.6$$
$$V_4 = VCE = 2.0$$
$$I_4 = IC = -3.3 \qquad (40)$$

where V is in volts

I is in milliamperes

The above voltages were substituted in equation 39. Table VI shows a comparison between measured and calculated values of I using the measured voltages. Sample calculations are in Appendix B.

In Figure 12a the resistance values are located as they may appear in a circuit. For the incremental circuit, R_{13} is neglected, as shown in Figure 12b.

Measured V gain at 1000 cps = -15.5

Output voltage = .1 volts rms

The gain vs bandwidth is plot 1 of igure 14.

Table VI

Table of Calculated and Measured Currents of Circuit 11a

Current Source	Calculated Values (Milliamps)	Measured Values	Difference
I_1	11.09	11.2	.11
I_2	-.265	-.250	.015
I_3	-7.56	-7.6	.04
I_4	-3.27	-3.3	.03

In order to obtain larger load and feedback resistance values, the sources were reduced in size from .25 to .1 inch diameters as shown in Figure 12b. The following Q point was established:

V_1 = 10.3 volts
V_3 = -13.2 volts

Ib = 85 μa, Ic = 1.6 ma, Vce = 2.1 volts

Measured V gain at 1000 cps = -23.2

Output Voltage = .5 volt rms

The gain vs frequency is shown as plot 2 of Figure 14.

Calculated V gain at 1000 cps = -23.6

In order to further increase the feedback and load resistances the resistive network of Figure 11c was used. The following Q point was established.

V_1 = 10.3 volts
V_2 = -2.8 volts
Ib = 67 ma, Ic = 1.4 ma, V_{CE} = 1.8 volts

Measured V gain at 1000 cps = -43.5

Output Voltage = 1 volt rms

The gain vs frequency is shown as plot 3 of Figure 14.

Substituting a 2N697 transistor in this circuit, the following Q point was established

V = 10.3 V
V = -2.8 V
Ib = 22 ma, Ic = 1.1 ma, V_{CE} = 3 volts

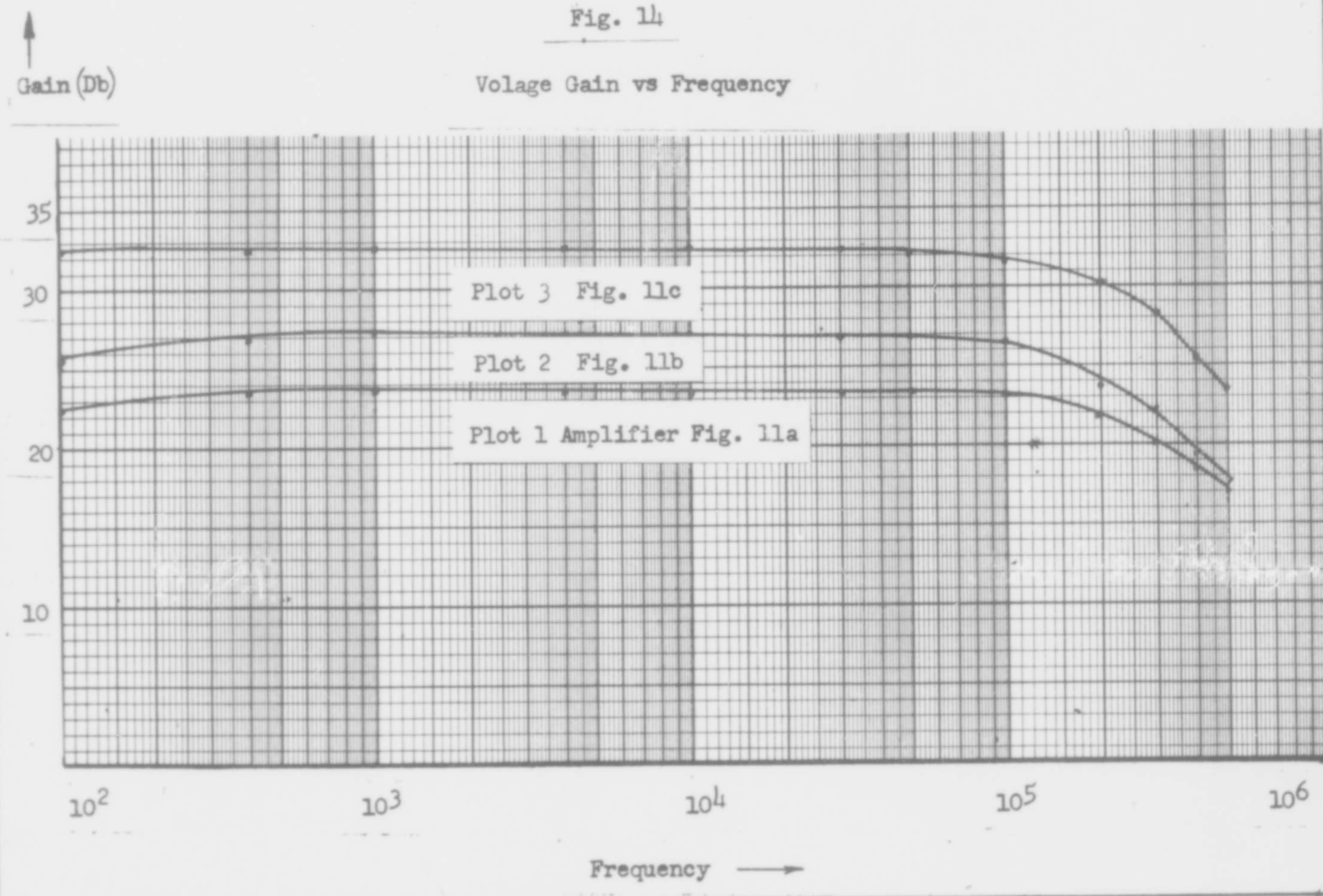
Measured V gain at 1000 cps =-102

Output voltage = 2 volts rms

A constant V_1 was maintained for the amplifier circuits. This value was arbitrarily selected. The Q points were selected to give maximum linear output voltage as viewed on a cathode-ray oscilloscope. The output voltage was measured on a VTVM. The amplifier current gain and power gain were dissipated within the resistive sheet and were not measured. A diagram of the experimental circuit is shown in Figure 13. A list of the equipment used is given in Appendix B. The incremental model is shown in Appendix B. The differences in the calculated currents at the Q point for 11a and voltage gain for amplifier 11b are believed to be the result of instrumentation inaccuracies, and slide-rule calculations. The large variation of resistances shown in Figure 11 is an illustration of "resistance shaping".

Fig. 14

Volage Gain vs Frequency



VI. Computer Solution to Laplace's Equation

The equation $V_0 = .25\ (V_1 + V_2 + V_3 + V_4)$ was derived in Chapter II. This equation is the basis for the iteration process for computing the potential of the field problem. A lattice of equally spaced points is dimensioned for the resistive sheet under investigation. The potential source size is defined in proportion to the lattice of points and is assigned a constant V. By successive computations for succeeding points of the lattice, the specified voltage sources give values to each point of the lattice. This calculation can be performed rapidly by the digital computer.

The purpose of this computer program was to compute potential at points of the field which would assist in circuit analysis. The program was written to arbitrarily locate potential sources and to assign varied potential values to the sources. The use of square potential sources is not a limitation to this method since circular sources can be approximated by a rectangular lattice.

Computer limitations on the Institute of Technology's IBM 1620 Digital Computer became evident as a lattice of approximately 20 by 20 columns and rows were available after specifying only 8 points of the lattice as constant voltage sources. Since 4 points are required to specify a square voltage source, this computer imposes a limitation to the complexity of the field problem.

The IBM 7090 Computer of the Digital Computation Division of the Aeronautical Systems Division imposed no limitation on the complexity of the problem. A large array can be programmed for accurate calculations.

It offers the advantage of a much faster solution to the field problem.

The fortran computer program in Appendix C contains the following general instructions:

1. Dimension the lattice of points. This dimension is reserved in the memory of the computer. Input data of problems may never exceed this dimension.

2. Input data dimensions the problem M and N where M is number of rows and N the number of columns.

3. M1 = M + 1
   N1 = N + 1
   An additional row and column are defined which will be used for the boundary conditions.

4. Do 5. This statement defines subscripts which are the notation for each point of the lattice.

5. Statement 5 clears the memory of any potential values.

6. Read input tape 2, 103, is an input statement containing the coordinates of eight points of the lattice for which potential is specified.

7. Read input tape 2, 104, is an input statement containing the potential values for the eight points of the lattice specified in input tape 2, 103, and a tolerance T.

8. Statement 20 assigns to three additional points of the lattice the same potential which is adjacent to the coordinates specified. In this way a square potential source is defined in the lattice.

9. Statement 17 defines D and K. D is the symbol for difference. K is the symbol for a counter.

10. Do 21 commands that the subscript notation be increased from 2 to M for I and 2 to N for J. The inner Do loop is completed first. Thus, the points of the lattice will be scanned from left to right and then down by rows.

11. Statements 23 thru 50 are branch statements. These statements compare the coordinates of the lattice point under investigation with the coordinates of points of the lattice already specified. If the lattice point has had its potential specified, no calculation is performed. The counter K is set to one which allows the second point adjacent to the first point to be by-passed also.

12. Statement 51. If the point of the lattice under investigation has not had its coordinates specified, then the computation is performed for a new potential value U.

13. D1 = U-V(I,J). A term D1 is defined which is the difference between the potential value U calculated and the potential at the coordinates previous to the calculation.

14. The next two statements take the absolute value of the difference D1 if D1 is negative.

15. D = D + D1. A sum of the differences (D1) calculated at each point of the lattice is made. This term D is a control term.

16. The potential value U calculated is assigned to the point of the lattice.

17. Go to 21. This statement continues the problem.

18. When a point under investigation has been specified the computer by-passes this point and a counter is set to 1. The second point is also by-passed and the counter is reset to 0.

19. After the points of the lattice have been processed, the boundary columns are equated to the next inner columns. The top and bottom rows are equated to the next inner rows. In this way the boundary conditions are approximated since the equipotential lines are normal to the boundary.

20. Statement 56. The tolerance T of the problem controls the number of times the lattice points are calculated. The end of the program requires the term D to be less than the tolerance specified in input data.

21. When D is less than or equal to T the program ends and calculated potential values are printed as specified. In this program 5 columns of potential values are printed as shown in figure
The potential-field problem of the multivibrator of Figure 1 was assigned to the computer with supply voltages of +8, -8 volts.

Figure 15 shows the coordinate system of the problem and the location of the columns for output data.

Figure 16 shows the physical model and location of columns for measured data. Potentials were measured with a VTVM.

Table VII shows input data for 3 computer problems.

Table VIII shows output data for Problem 1. All potential values were specified. T = .5.

Table IX shows output data for Problem 2. All potential values were specified. T = .1.

Table X shows output data for Problem 3. Potential values were specified for one transistor. A tolerance T of .1 was specified.

Table XI shows a comparison between Run 1 and measured values.

Figure 17 shows potential field plot of Problem 1.

The results of the computer Problem 1 shows close correspondence between the computed potential and the measured potential values on the resistance sheet. The exact computed values were within less than 1/8 inch for the model constructed. This degree of accuracy would depend on the proportional size of the circuit. If the resistance sheet circuit were altered in size, it is believed the variations would vary proportionally. Deviations also may reflect on the non-uniformity of the resistance sheet.

Computer Problem 2 shows the increase of potential values at the lower right hand corner as the problem time increases. The potentials specified for the three problems are shown in boxes in the output data tables.

Problem 3 specified four potential values. The purpose of this problem was to calculate the base and collector potential of the off transistor. It was assumed the potentials of the transistor in saturation remain constant. The potential calculated was .10 and 3.63 volts for the base and collector respectively. The voltage measured was .15 and 3.0 volts respectively. This problem is included to show the limitation of this computer solution when only a relatively few points are specified on the potential plane and a lower tolerance T is specified.

If a rubber sheet analogy is made of the potential field, where the value of potential is the third dimension, then the curve of the sheet between a source and the boundary straightens and potentials increase or decrease accordingly where a small tolerance, T, of .1 is programmed.

An improvement to this program can be made by reversing the scan of lattice points for alternate runs or by starting the iteration process alternately from the four corners. This would require a change in the branch statements and the counter K.

Problem 1 required 5 minutes computer time. Problems 2 and 3 required 7 and 4 minutes respectively.

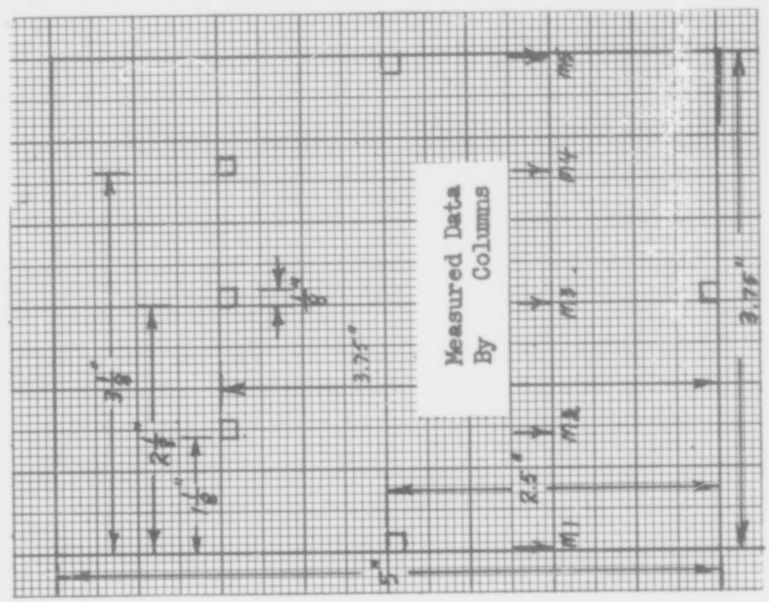


Fig. 16

Dimensions for Measurements

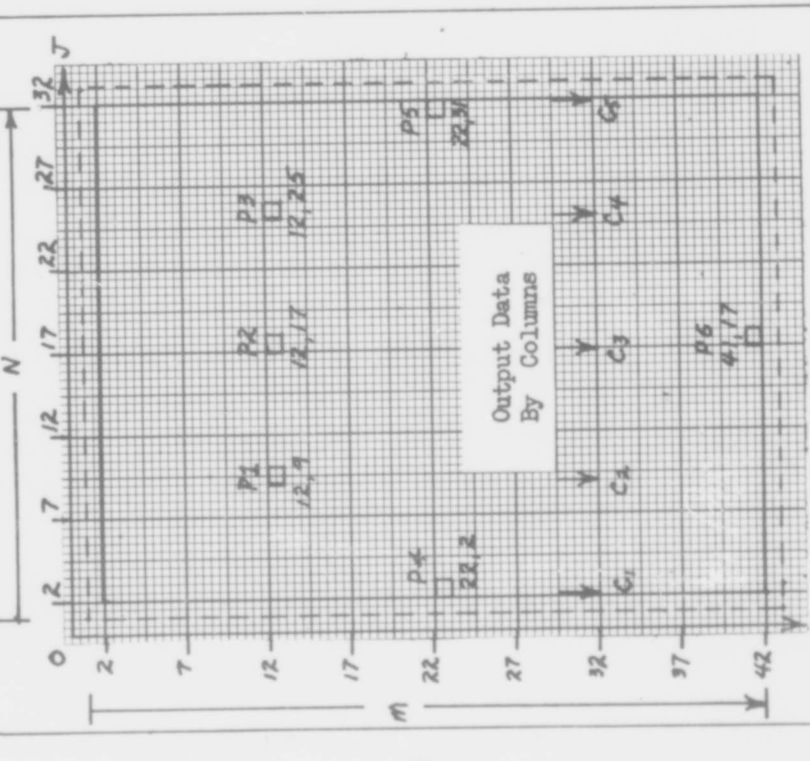


Fig. 15

Coordinate System for Computer Problem

Table VII Input Data for Three Computer Problems

	Problem 1	Problem 2	Problem 3
M,N	42, 32	Same as	42, 32
I1,J1	12, 9	Problem 1	12, 9
I2,J2	12, 17	except	12, 17
I3,J3	12, 25	T = .1	22, 31
I4,J4	22, 2		41, 17
I5,J5	22, 31		41, 17
I6,J6	41, 17		41, 17
I7,J7	41, 17		41, 17
I8,J8	41, 17		41, 17
V1	.47		.47
V2	8.0		8.0
V3	2.85		.35
V4	.16		-8.0
V5	.35		-8.0
V6	-8.0		-8.0
V7	-8.0		-8.0
V8	-8.0		-8.0
T	.5		.1

Table VIII Output Data for Problem 1

C1	C2	C3	C4	C5
1.[illegible]	2.47	3.60	3.54	3.27
1.93	2.44	3.69	3.54	3.24
1.82	2.32	3.72	3.54	3.17
1.65	2.07	4.40	3.49	3.05
1.45	1.58	5.38	3.34	2.87
1.24	0.47	8.00	2.85	2.63
1.05	0.83	6.01	2.64	2.34
0.87	1.14	3.93	2.52	1.97
0.67	1.05	2.73	2.03	1.53
0.44	0.80	1.85	1.49	1.00
0.16	0.45	1.12	0.95	0.35
-0.05	0.07	0.47	0.40	0.17
-0.39	-0.35	-0.15	-0.12	-0.22
-0.76	-0.78	-0.75	-0.65	-0.63
-1.12	-1.23	-1.36	-1.16	-1.05
-1.47	-1.68	-2.01	-1.68	-1.45
-1.79	-2.12	-2.72	-2.18	-1.83
-2.07	-2.53	-3.57	-2.66	-2.15
-2.29	-2.89	-4.67	-3.08	-2.42
-2.45	-3.15	-6.45	-3.40	-2.60
-2.51	-3.20	-8.00	-3.56	-2.67

Table IX Output Data for Problem 2

C1	C2	C3	C4	C5
2.02	2.52	3.64	3.57	3.31
1.97	2.47	3.72	3.57	3.28
1.85	2.35	3.95	3.56	3.20
1.68	2.09	4.41	3.51	3.07
1.47	1.59	5.39	3.35	2.88
1.25	0.47	8.00	2.85	2.64
1.05	0.87	6.00	2.83	2.33
0.86	1.12	3.91	2.50	1.96
0.66	1.02	2.68	2.00	1.52
0.43	0.75	1.78	1.45	0.99
0.16	0.39	1.04	0.89	0.35
-0.06	-0.01	0.37	0.33	0.14
-0.47	-0.45	-0.25	-0.22	-0.28
-0.86	-0.90	-0.86	-0.75	-0.72
-1.25	-1.36	-1.48	-1.28	-1.16
-1.62	-1.82	-2.13	-1.81	-1.58
-1.95	-2.27	-2.85	-2.31	-1.97
-2.25	-2.68	-3.68	-2.80	-2.31
-2.48	-3.05	-4.76	-3.22	-2.57
-2.63	-3.31	-6.47	-3.54	-2.76
-2.70	-3.43	-8.00	-3.69	-2.84

Table X Output Data for Problem 3

C1	C2	C3	C4	C5
2.06	2.58	3.80	3.87	3.65
2.01	2.54	3.88	3.88	3.63
1.89	2.40	4.10	3.89	3.55
1.71	2.14	4.54	3.88	3.43
1.49	1.62	5.48	3.81	3.25
1.26	0.47	8.00	3.63	2.99
1.05	0.88	6.04	3.28	2.64
0.85	1.13	3.97	2.79	2.22
0.64	1.02	2.76	2.21	1.70
0.39	0.74	1.85	1.60	1.09
0.10	0.37	1.09	1.00	0.35
-0.22	-0.04	0.42	0.41	0.16
-0.57	-0.47	-0.22	-0.16	-0.25
-0.94	-0.93	-0.84	-0.71	-0.69
-1.30	-1.39	-1.47	-1.25	-1.13
-1.66	-1.84	-2.12	-1.78	-1.55
-1.99	-2.29	-2.84	-2.29	-1.95
-2.28	-2.70	-3.67	-2.78	-2.29
-2.50	-3.06	-4.75	-3.21	-2.56
-2.66	-3.33	-6.47	-3.53	-2.74
-2.73	-3.45	-8.00	-3.68	-2.82

Table XI

Table of Comparison Between Measured and Calculated Potentials Problem 1

M1	C1	M3	C3	M5	C5
2.1	1.98	3.60	3.60	3.23	3.27
2.0	1.93	3.63	3.69	3.20	3.24
1.86	1.82	3.85	3.92	3.20	3.17
1.75	1.65	4.2	4.40	3.10	3.05
1.52	1.45	5.1	5.38	2.9	2.87
1.30	1.24	8.00	8.00	2.65	2.63
1.08	1.05	5.9	6.01	2.3	2.34
.9	.87	3.8	3.93	2.0	1.97
.70	.67	2.5	2.73	1.6	1.53
.45	.44	1.75	1.85	1.25	1.00
.16	.16	1.05	1.12	.35	.35
-.06	-.03	.45	.47	.20	.17
-.4	-.39	-.18	-.15	-.18	- .22
-.8	-.76	-.73	-.75	-.56	- .63
-1.1	-1.12	-1.33	-1.36	-1.05	-1.05
-1.55	-1.47	-1.95	-2.01	-1.45	-1.45
-1.87	-1.79	-2.70	-2.72	-1.80	-1.83
-2.15	-2.07	-3.40	-3.57	-2.15	-2.15
-2.32	-2.29	-4.4	-4.67	-2.43	-2.42
-2.45	-2.45	-6.20	-6.43	-2.60	-2.60
-2.50	-2.51	-8.00	-8.00	-2.65	-2.67

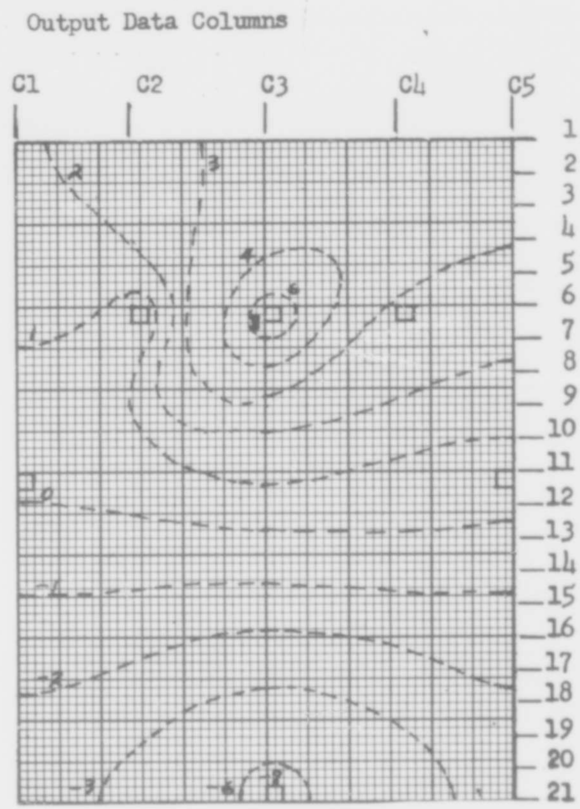


Fig. 17

Potential Field Plot-Computer Problem 1

VII. Summary and Conclusions

An analytic expression for the potential at a point on a plane was derived. This expression was used in calculating the resistance between two current sources on a resistive sheet. When a large number of images is used in calculations and the sources are dimensionally small, accurate calculations may be made. Where symmetry applied at a boundary of the resistive sheet, it was shown that a fewer number of images allowed reducing the amount of numerical calculations required in the calculation for potential.

Measurements of resistance values were presented graphically to illustrate the variation of resistance values between sources as dimensions were altered. It was shown a large variation of resistance values are obtained by dimensional variations. The resistance value between two sources is dependent on the total geometry of sources on the resistive sheet. When dimensions of the current sources and dimensions of the resistive sheet are reduced proportionally, it was shown that identical resistance values can be obtained. The accuracy of these values depends on the accuracy of the fabrication of sources on the resistive sheet and the uniformity of the resistivity of the sheet.

Circuit analysis was shown for a feedback amplifier where all the resistive elements were contained in the resistive sheet. Four connections to the resistive sheet produced six resistance values which were used in circuit analysis. Calculations were made for the Q point and the voltage gain of the amplifier in the common emitter configuration.

By reducing the diameter of the current sources from .25 inches to .1 inches the gain of the amplifier was increased from 15.5 to 23.2. Redistribution of the sources allowed an increase in gain to 43.5. Circuit analysis allowed locating the resistance values which were altered to achieve desirable circuit performance.

The solution to the potential field problem was investigated with the digital computer. This method considers only the potential values of the sources and calculates the potential at points on the plane. The current, resistivity, and the specific dimensions of the plane problem are not used in the computer problem. Measured potential values were shown in comparison with experimental potential values of the transistor multivibrator circuit.

The following conclusions are made:

1. The calculation of resistance between current sources is possible by satisfying boundary conditions with a system of images. The actual calculation for accurate results is a long calculation. Resistance values can be measured readily.
2. Circuit analysis for a given system of sources on a resistive sheet may be performed by measuring resistance values between sources. Resistance values on a resistive sheet vary as the size and distribution of sources is varied. The distribution and size of the sources may be altered to improve circuit performance.
3. Resistive networks may be reduced or increased proportionally to obtain equivalent resistances.

4. The number of resistor connections required for a circuit may be reduced by circuit design using distributed resistance material.

5. The digital computer solution to the potential field problem provides a method of rapidly computing the potential field for a given system of potential sources.

The following recommendations are made:

1. The theory of complex variables allows the transformation of a circular area in the z plane to an area outside of the circle in the w plane. Electrolytic tank procedures use this transformation, $w = \frac{1}{z}$, to effect an infinite medium when actually the boundaries of the tank are finite. A shelf is placed at the bottom of the tank to effect this transformation. If a resistive cylinder could be constructed such that the top, bottom, and sides were of uniform resistive material, calculations and circuit design may be accomplished without the use of images by considering the physical cylindrical surface as the infinite sheet. A study in this area is recommended.

2. The calculation of an impedance matrix may be possible with computer techniques. An expression for a system of sources may lend itself to computer solution.

3. Three-dimensional studies are possible on the IBM 7090. Laplace's equation in three dimensions could be used for three dimensional field problems.

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Appendix A

Image Equations and Sample Calculations

This Appendix contains the algebraic expressions for the ratio of distances of plus and minus current sources. Problem No. 2 is shown in a sample calculation.

The algebraic expressions are those dimensions appearing in the algebraic expression:

$$V_p = \frac{I}{2\pi}\frac{p}{t} \ln \frac{r_1}{s_1} \cdots \frac{s_2}{r_2} \cdots = \frac{I}{2\pi}\frac{p}{t} \ln \frac{s_2 \cdots}{s_1 \cdots} \qquad (41)$$

The r_1, r_2, quantities are the radii of the sources.

The s_1, s_2, quantities are the distances between the (x,y) coordinates of the sources and the point P (x,y).

$\left(\frac{s_2}{s_1}\right)^2$ is given by column number shown in Figure 6.

Column C_o sources above boundary A of Figure 6a, 6b.

$$\left(\frac{s_2}{s_1}\right)^2_{C_o} = \left[\frac{(x-x_2)^2 + (y_2-y)^2}{(x-x_1)^2 + (y_1-y)^2}\right]\left[\frac{(x-x_2)^2 + (2h-y_2-y)^2}{(x-x_1)^2 + (2h-y_1-y)^2}\right]\left[\frac{(x-x_2)^2 + (2h+y_2-y)^2}{(x-x_1)^2 + (2h+y_1-y)^2}\right]$$
$$\left[\frac{(x-x_2)^2 + (4h-y_2-y)^2}{(x-x_1)^2 + (2h-y_1-y)^2}\right]\left[\frac{(x-x_2)^2 + (4h+y_2-y)^2}{(x-x_1)^2 + (4h+y_1-y)^2}\right]\left[\cdots\right.$$

Sources below

$$= \left[\frac{(x-x_2)^2 + (y_2+y)^2}{(x-x_1)^2 + (y_1+y)^2}\right]\left[\frac{(x-x_2)^2 + (2h-y_2+y)^2}{(x-x_1)^2 + (2h-y_1+y)^2}\right]\left[\frac{(x-x_2)^2 + (2h+y_2+y)^2}{(x-x_1)^2 + (2h+y_1+y)^2}\right]$$
$$\left[\frac{(x-x_2)^2 + (4h-y_2+y)^2}{(x-x_1)^2 + (4h-y_1+y)^2}\right]\left[\frac{(x-x_2)^2 + (4h+y_2+y)^2}{(x-x_1)^2 + (4h+y_1+y)^2}\right]\left[\cdots\right. \qquad (42)$$

Column C_1 Sources above

$$\left(\frac{s_2}{s_1}\right)^2_{C_1} = \left[\frac{(2b-x_2-x)^2 + (y_2-y)^2}{(2b-x_1-x)^2 + (y_1-y)^2}\right]\left[\frac{(2b-x_2-x)^2 + (2h-y_2-y)^2}{(2b-x_1-x)^2 + (2h-y_1-y)^2}\right]$$

$$\left[\frac{(2b-x_2-x)^2 + (2h+y_2-y)^2}{(2b-x_1-x)^2 + (2h+y_1-y)^2}\right]\cdots$$

Sources below

$$= \left[\frac{(2b-x_2-x)^2 + (y+y_2)^2}{(2b-x_1-x)^2 + (y+y_1)^2}\right]\left[\frac{(2b-x_2-x)^2 + (2h-y_2+y)^2}{(2b-x_1-x)^2 + (2h-y_1+y)^2}\right]\left[\frac{(2b-x_2-x)^2 + (2h+y_2-y)^2}{(2b-x_1-x)^2 + (2h+y_1-y)^2}\right]\cdots \quad (43)$$

Notice, for each column the distance y is always the same for corresponding images, only the x values change for columns. For a single column of like current images, the x distance once calculated is constant. Thus, the x distances are shown algebraically.

Column C_2 Sources above.

$$\left(\frac{s_2}{s_1}\right)^2_{C_2} = \left[\frac{(x+x_2)^2 + (y-y_2)^2}{(x+x_1)^2 + (y-y_1)^2}\right]\left[\frac{(x+x_2)^2 + \cdots}{(x+x_1)^2 + \cdots}\right]\cdot\left[\cdots\right.$$

Sources below

$$= \left[\frac{(x+x_2)^2 + (y_2+y)^2}{(x+x_1)^2 + (y_1+y)^2}\right]\left[\cdots\right. \quad (44)$$

Column C_3

$$\left(\frac{s_2}{s_1}\right)^2_{C_3} = \left[\frac{(2b+x_2-x)^2 + (y_2-y)^2}{(2b+x_1-x)^2 + (y_1-y)^2}\right]\left[\cdots\right. \quad (45)$$

Column C_4

$$\left(\frac{s_2}{s_1}\right)^2_{C_4} = \left[\frac{(2b-x_2+x)^2 + (y_2-y)^2}{(2b-x_1+x)^2 + (y_1-y)^2}\right]\left[\cdots\right. \quad (46)$$

Column C_5

$$\left(\frac{s_2}{s_1}\right)^2_{C_5} = \left[\frac{(4b+x_2-x)^2 + (y-y_2)^2}{(4b+x_1-x)^2 + (y-y_1)^2}\right]\left[\cdots\right. \quad (47)$$

Column C_6

$$\left(\frac{s_2}{s_1}\right)^2_{C_6} = \left[\frac{(4b-x_2+x)^2 + (y_2-y)^2}{(4b-x_1+x)^2 + (y_1-y)^2}\right]\left[\cdots\right. \quad (48)$$

Once the y distance is calculated for an image this distance is calculated for an image, this distance is the same for corresponding images of each column

Sample calculation. Problem No. 2

$$\begin{array}{lllll} h = 4 & x_1 = 1.5 & x_2 = 1.5 & x_A = 1.5 & x_B = 1.5 \\ b = 3 & y_1 = 3 & y_2 = .05 & y_A = 2.95 & y_B = .10 \end{array} \quad (49)$$

Calculation of V_a substituting above values.

Column C_o sources above resistive sheet boundary A

$$\left(\frac{s_2}{s_1}\right)^2_{C_o} = \left[\frac{0 + (.05-2.95)^2}{0 + (3.00-2.95)^2}\right]\left[\frac{(8-.05-2.95)^2}{(8-3-2.95)^2}\right]\left[\frac{(8+.05-2.95)^2}{(8+3-2.95)^2}\right]\left[\frac{(16-.05-2.95)^2}{(16-3-2.95)^2}\right]$$

$$\left[\frac{(16+.05-2.95)^2}{(16+3-2.95)^2}\right]\left[\frac{(32-.05-2.95)^2}{(32-3-2.95)^2}\right]\left[\frac{(32+.05-2.95)^2}{(32+3-2.95)^2}\right]$$

$$= \frac{(-2.90)^2}{(.05)^2} \cdot \frac{(5)^2}{(2.05)^2} \cdot \frac{(5.10)^2}{(8.05)^2} \cdot \frac{(13)^2}{(10.05)^2} \cdot \frac{(13.10)^2}{(16.05)^2} \cdot$$

$$\frac{(29)^2}{(26.05)^2} \cdot \frac{(29.10)^2}{(32.05)^2}$$

$$= \frac{8.41}{.0025} \cdot \left(\frac{25}{4.21} \cdot \frac{26.1}{65}\right) \cdot \left(\frac{169}{110} \cdot \frac{172}{258}\right) \cdot \left(\frac{840}{680} \cdot \frac{850}{1030}\right) =$$

$$= 3370 \cdot 2.38 \cdot 1.025 \cdot 1.019 = 8380 \qquad (50)$$

Column C_o sources below resistive sheet boundary A

$$\left(\frac{s_2}{s_1}\right)^2_{C_o} = \left[\frac{0 + (3)^2}{0 + (5.95)^2}\right]\left[\frac{(8-.05+2.95)^2}{(8-3+2.95)^2}\right]\left[\frac{(8+.05+2.95)^2}{(8+3+2.95)^2}\right]\left[\frac{(16-.05+2.95)^2}{(16-3+2.95)^2}\right]$$

$$\left[\frac{(16+.05+2.95)^2}{(16+3+2.95)^2}\right]\left[\frac{(32-.05+2.95)^2}{(32-3+2.95)^2}\right]\left[\frac{(32+.05+2.95)^2}{(32+3+2.95)^2}\right]$$

$$= \frac{(3)^2}{(5.95)^2} \cdot \frac{(10.95)^2}{(7.95)^2} \cdot \frac{(11.05)^2}{(13.95)^2} \cdot \frac{(18.90)^2}{(15.95)^2} \cdot \frac{(19)^2}{(21.95)^2} \cdot \frac{(34.9)^2}{(31.95)^2} \cdot \frac{(35)^2}{(37.95)^2}$$

$$= \frac{9}{35.5} \cdot \left(\frac{120}{63.2} \cdot \frac{122}{195}\right) \cdot \left(\frac{356}{255} \cdot \frac{361}{481}\right) \cdot \left(\frac{1220}{020} \cdot \frac{1221}{1440}\right)$$

$$= (.254)(1.19)(1.045)(1.01) = .319 \qquad (51)$$

Column C_1 sources above

$$\left(\frac{s_2}{s_1}\right)^{\lambda}_{c_1} = \left(\frac{9+8.41}{9+.0025}\right)\left(\frac{9+25}{9+4.21}\right)\left(\frac{9+26.1}{9+65}\right)\left(\frac{9+169}{9+110}\right)\left(\frac{9+172}{9+158}\right)\left(\frac{9+840}{9+680}\right)\left(\frac{9+850}{9+1030}\right)$$

$$= \left(\frac{17.41}{9.0025}\right)\left(\frac{34}{13.21}\quad\frac{35.1}{74}\right)\left(\frac{178}{119}\quad\frac{181}{267}\right)\left(\frac{849}{689}\quad\frac{859}{1039}\right)$$

$$= (1.93)(1.22)(1.015)(1.018) = 2.39 \qquad (52)$$

Sources below

$$\left(\frac{s_2}{s_1}\right)^{\lambda}_{c_1} = \left(\frac{9+9}{9+35.5}\right)\left(\frac{9+120}{9+63.2}\right)\left(\frac{9+122}{9+195}\right)\left(\frac{9+356}{9+255}\right)\left(\frac{9+361}{9+481}\right)\left(\frac{9+1220}{9+1020}\right)\left(\frac{9+1220}{9+1440}\right)$$

$$= \left(\frac{18}{44.5}\right)\left(\frac{129}{72.2}\quad\frac{131}{204}\right)\left(\frac{365}{264}\quad\frac{370}{490}\right)\left(\frac{1229}{1029}\quad\frac{1229}{1449}\right)$$

$$= (.405)(1.145)(1.045)(1.015) = .465 \qquad (53)$$

Column C_2 is identical to C_1 due to symmetry

Column C_3 x distance is: $2b+x_2-x = 6+1.5-1.5 = 6$

Column C_3 sources above

$$\left(\frac{s_2}{s_1}\right)^{\lambda}_{c_3} = \frac{(36+8.41)}{(36+.0025)}\quad\frac{(36+25)}{(36+4.21)}\quad\frac{(36+26.1)}{(36+65)}\quad\frac{(36+169)}{(36+110)}\quad\frac{(36+172)}{(36+258)}\quad\frac{(36+840)}{(36+680)}\quad\frac{(36+850)}{(36+1030)}$$

$$= \left(\frac{45}{71.5}\right)\left(\frac{156}{99.2}\quad\frac{158}{231}\right)\left(\frac{392}{291}\quad\frac{397}{517}\right)\left(\frac{876}{716}\quad\frac{886}{1066}\right)$$

$$= (.630)(1.078)(1.032)(1.015) = 1.16 \qquad (54)$$

Sources below

$$\left(\frac{s_2}{s_1}\right)^2_{C_3} = \left(\frac{36+9}{36+35.5}\right)\left(\frac{36+120}{36+63.2}\right)\left(\frac{36+122}{36+195}\right)\left(\frac{36+356}{36+255}\right)\left(\frac{36+361}{36+481}\right)\left(\frac{36+1220}{36+1020}\right)\left(\frac{36+1220}{36+1440}\right)$$

$$= \left(\frac{45}{71.5}\right)\left(\frac{156}{99.2}\ \frac{158}{231}\right)\left(\frac{392}{291}\ \frac{397}{517}\right)\left(\frac{1256}{1056}\ \frac{1256}{1476}\right)$$

$$= (.630)(1.078)(1.032)(1.012) = .709 \tag{55}$$

Column C_4 is identical to C_3 due to symmetry.

Column C_5 sources above

$$\left(\frac{s_2}{s_1}\right)^2_{C_5} = \left(\frac{81+8.4}{81+.0025}\right)\left(\frac{81+25}{81+4.21}\right)\left(\frac{81+26.1}{81+65}\right)\left(\frac{81+169}{81+110}\right)\left(\frac{81+172}{81+258}\right)\left(\frac{81+840}{81+680}\right)\left(\frac{81+850}{81+1030}\right)$$

$$= \left(\frac{89.41}{81.0025}\right)\left(\frac{106}{85.21}\ \frac{102.1}{146}\right)\left(\frac{250}{191}\ \frac{253}{339}\right)\left(\frac{921}{761}\ \frac{931}{1111}\right)$$

$$= (1.101)(.915)(.978)(1.015) = 1 \tag{56}$$

Sources below

$$\left(\frac{s_2}{s_1}\right)^2_{C_5} = \left(\frac{81+9}{81+35.5}\right)\left(\frac{81+120}{81+63.2}\right)\left(\frac{81+122}{81+195}\right)\left(\frac{81+356}{81+255}\right)\left(\frac{81+361}{81+481}\right)\left(\frac{81+1220}{81+1020}\right)\left(\frac{81+1221}{81+1440}\right)$$

$$= \left(\frac{90}{116.5}\right)\left(\frac{201}{144.2}\right)\left(\frac{203}{270}\right)\left(\frac{437}{336}\right)\left(\frac{442}{562}\right)\left(\frac{1301}{1101}\ \frac{1302}{1521}\right)$$

$$= (.772)(1.025)(1.025)(1.01) = .820 \tag{57}$$

Column C_6 is identical to C_5 due to symmetry

The procedure for V_b is the same process. Only the numbers showing convergence will be shown.

Column C_o sources above

$$\left(\frac{s_2}{s_1}\right)^2_{c_o} = (.000298)(1.37)(1.078)(1.02) = .000490 = \frac{1}{2230} \qquad (58)$$

Sources below

$$\left(\frac{s_2}{s_1}\right)^2_{c_o} = (.00234)(1.325)(1.075)(1.025) = .00341 = \frac{1}{293} \qquad (59)$$

Column C_1 sources above:

$$\left(\frac{s_2}{s_1}\right)^2_{c_1} = (.517)(1.213)(1.07)(1.02) = .685 = \frac{1}{1.46} \qquad (60)$$

Sources below:

$$\left(\frac{s_2}{s_1}\right)^2_{c_1} = (.484)(1.19)(1.065)(1.028) = .630 = \frac{1}{1.583} \qquad (61)$$

Column C_2 is same as C_1 due to symmetry.

Column C_3 sources above:

$$\left(\frac{s_2}{s_1}\right)^2_{c_3} = (.810)(1.041)(1.05)(1.02) = .902 = \frac{1}{1.11} \qquad (62)$$

Sources below:

$$\left(\frac{s_2}{s_1}\right)^2_{c_3} = (.789((1.035)(1.045)(1.025) = .884 = \frac{1}{1.13} \qquad (63)$$

Column C_4 same as C_3 due to symmetry.

Column C_5 sources above:

$$\left(\frac{s_2}{s_1}\right)^2_{c5} = (.907)(.980)(1.03)(1.011) = .926 = \frac{1}{1.08} \qquad (64)$$

Sources below:

$$\left(\frac{s_2}{s_1}\right)^2_{c5} = (.894)(.976)(1.025)(1.02) = .911 = \frac{1}{1.095} \qquad (65)$$

$$V_A = \frac{I}{2\pi}\frac{p}{t} \ln\left[\left(\frac{r_1^2}{r_2^2}\right)^n \left(\frac{s_2}{s_1}\right)^2_{C_0} \left(\frac{s_2}{s_1}\right)^2_{C_1} \left(\frac{s_2}{s_1}\right)^2_{C_2} \cdots\right]^{\frac{1}{2}}$$

$$V_A = \frac{I}{2\pi}\frac{p}{t} \ln\left[8380\,(2.43)^2 (1.16)^2 (1)^2 (.319)(.492)^3 (.709)^2 (.820)^2\right]^{\frac{1}{2}}$$

$$= \frac{I}{2\pi}\frac{p}{t} \ln (1732)^{\frac{1}{2}} = \frac{I}{2\pi}\frac{p}{t} \ln 41.6 = \frac{I}{2\pi}\frac{p}{t}(3.72) \qquad (66)$$

$$V_B = \frac{I}{2\pi}\frac{p}{t} \ln\left[\frac{1}{2230}\left(\frac{1}{1.46}\right)^2\left(\frac{1}{1.11}\right)^2\left(\frac{1}{1.08}\right)^2 \frac{1}{293}\left(\frac{1}{1.1588}\right)^2\left(\frac{1}{1.13}\right)^2\left(\frac{1}{1.095}\right)^2\right]^{\frac{1}{2}}$$

$$V_B = \frac{I}{2\pi}\frac{p}{t} \ln\left[\frac{1}{7{,}500{,}000}\right]^{\frac{1}{2}}$$

$$V_B = -\frac{I}{2\pi}\frac{p}{t} \ln 2740 = -\frac{I}{2\pi}\frac{p}{t}(8.15) \qquad (67)$$

$$V_{AB} = V_A - V_B = \frac{I}{2\pi}\frac{p}{t}(3.72) + \frac{I}{2\pi}\frac{p}{t}(8.15) \qquad (68)$$

$$\frac{V_{AB}}{I} = \frac{(11.87)}{2\pi}\frac{p}{t}\ ; \qquad \frac{p}{t} = 2170 \qquad (69)$$

$$R_{AB} = \frac{(11.87)(2170)}{6.28} = 4100 \qquad (70)$$

In this problem six similar cards above the original card were used in each column plus seven image cards below. The original column was imaged by three columns on each side.

Appendix B

Sample Calculations for Chapter V

1. Sample calculation of Y matrix elements

Table XII

Voltage Current Table for Admittance Calculations

V_1 = 10.0 volts	V_2 = 10.0 volts	V_3 = 10.0 volts
$V_2 = V_3 = V_4 = 0$	$V_1 = V_3 = V_4 = 0$	$V_1 = V_2 = V_3 = 0$
$I_1 = 2.80$ $I_3 = 1.10$ $I_4 = 6.65$	$I_3 = 1.85$ $I_4 = 1.05$	$I_4 = 1.25$ ma

Note: the above values were selected from a plotted V-I range of values.

From Table XII the following calculations were made:

$$R_{12} = \frac{10.0}{2.80} = 3570 \text{ ohms}$$

$$R_{13} = \frac{10.0}{1.10} = 9090 \text{ ohms} \qquad (71)$$

Similarly

$R_{14} = 1500$

$R_{23} = 5400$

$R_{24} = 9510$

$R_{34} = 8000$

$$Y_{12} = \frac{1}{R_{12}} = Y_{21} = .000280$$

Similarly

$$Y_{13} = Y_{31} = .000110$$

$$Y_{14} = Y_{41} = .000666$$

$$Y_{11} = Y_{12} + Y_{13} + Y_{14} = .0001056$$

$$Y_{23} = \frac{1}{R_{23}} = Y_{32} = .000185$$

$$Y_{24} = Y_{42} = .000105$$

$$Y_{22} = Y_{21} + Y_{23} + Y_{24} = .000280 + .000185 + .000105 = .000570$$

$$Y_{34} = Y_{43} = .000125$$

$$Y_{33} = Y_{31} + Y_{32} + Y_{34} = .000110 + .000185 + .000125 = .000420$$

$$Y_{44} = Y_{41} + Y_{42} + Y_{43} = .000666 + .000105 + .000125 = .000896 \qquad (72)$$

The above values were substituted in equation 58

2. Sample calculation for Q point for circuit shown in Figure 11a.

$V_1 = 10.3, \quad V_2 = .22, \quad V_3 = -14.6, \quad V_4 = 2.0$

Substituting in equation 38

Solving for I (ma)

$$I_1 = (1.056)(10.3) - (.28)(.22) - (.110)(-14.6) - (.666)(2.0)$$

$$I_1 = 10.88 - .0615 + 1.605 - 1.33$$

$$I_1 = 11.09 \text{ ma.} \qquad (72a)$$

$$I_2 = (-.280)(10.3) + (.570)(.22) - (.110)(-14.6) - (.105)(2.0)$$

$$I_2 = -2.88 + .1253 + 2.70 - .210$$

$$I_2 = -.265 \text{ ma} \qquad (72b)$$

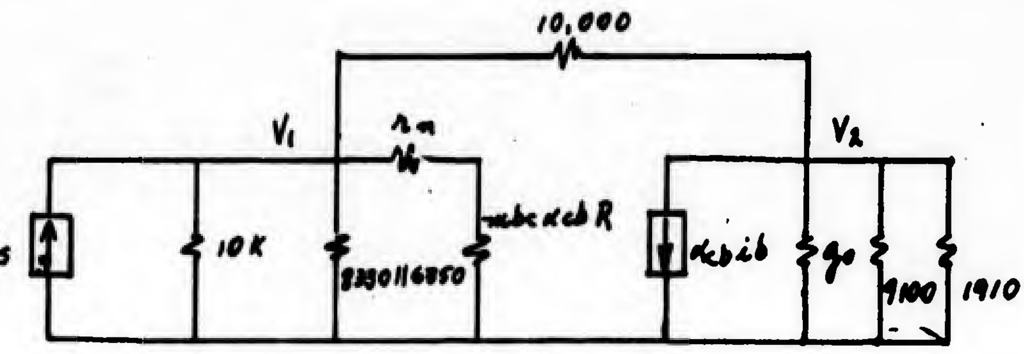

Fig 18

Incremental Model for Amplifier Fig. 11b(Ref 1:180)

3. Gain calculation:

Measured NPN 169A transistor parameters at the Q pt (1000 cps)

$$g_o = h_{oe} = 2\times10^{-5}$$
$$\alpha_{cb} = h_{fe} = 17$$
$$r_\pi = h_{ie} = 985$$
$$\mu_{bc} = h_{re} = 1.36\times10^{-3} \qquad (73)$$

These measurements were provided by the Molecular Electronics Technology Section, Wright Patterson AFB, Ohio.

$$R = R_L \| g_o = 50K \| 9100 \| 1910 = 1530$$
$$-\mu_{bc}\alpha_{cb}R = (-1.36\times10^{-3})(17)(1530) = 35.4$$
$$r_\pi - \mu_{bc}\alpha_{cb}R = 985 - 35.4 = 949\,\Omega$$

$$10K \| 8330 \| 6350 = 2650 \qquad (74)$$

Writing node equations for the above circuit

$$V_1\left(\frac{1}{2650} + \frac{1}{949} + \frac{1}{10,000}\right) - V_2\left(\frac{1}{10,000}\right) = i_s$$
$$-V_1\left(\frac{1}{10,000}\right) + V_2\left(\frac{1}{10,000} + \frac{1}{1530}\right) = -\alpha_{cb}i_b \qquad (75)$$

where $i_b = \dfrac{V_1}{949}$

$$V_1(.000377 + .001055 + .0001053) - V_2(.0001053) = i_s$$
$$-V_1(.0001000) + V_2(.00010 + .000653) = -.0179V_1$$

$$\frac{V_2}{V_1} = \frac{-.0178000}{.000753} = -23.6 \qquad (76)$$

4. Equipment list.

Two Cathode Ray Oscilloscopes, Dumont
Audio-Frequency Oscillator, Telectro, Model TS-382 F/U
Standard Signal Generator, Model 65-B
Vacuum Tube Voltmeter, 410B
Transistor Power Supply Model 210
Weston Micrommeter 0-200 D.C.
Milliameters 0-15 ma
0-1 ma
Weston D.C. Voltmeter 0-30 volts

Appendix C

Computer Program

```
BOSSERT CONTROLLED FIELD ANALYSIS
      DIMENSION V(103,33)
  983 READ INPUT TAPE 2,101,M,N
  101 FORMAT(2I3)
      M1=M+1
      N1=N+1
      DO 5 I=1,M1
      DO 5 J=1,N1
 5    V(I,J)=0.
      READ INPUT TAPE 2,103,I1,J1,I2,J2,I3,J3,I4,J4,I5,J5,I6,J6,I7,J7,I8
     1,J8
  103 FORMAT(16I3)
      READ INPUT TAPE2,104,V(I1,J1),V(I2,J2 ),V(I3,J3),V(I4,J4),V(I5,J5
     1),V(I6,J6),V(I7,J7),V(I8,J8),T
  104 FORMAT(6F12.0)
 20   V(I1+1,J1)=V(I1,J1)
      V(I1,J1+1)=V(I1,J1)
      V(I1+1,J1+1)=V(I1,J1)
      V(I2+1,J2)=V(I2,J2)
      V(I2,J2+1)=V(I2,J2)
      V(I2+1,J2+1)=V(I2,J2)
      V(I3+1,J3)=V(I3,J3)
      V(I3,J3+1)=V(I3,J3)
      V(I3+1,J3+1)=V(I3,J3)
      V(I4+1,J4)=V(I4,J4)
      V(I4,J4+1)=V(I4,J4)
      V(I4+1,J4+1)=V(I4,J4)
      V(I5+1,J5)=V(I5,J5)
      V(I5,J5+1)=V(I5,J5)
      V(I5+1,J5+1)=V(I5,J5)
      V(I6+1,J6)=V(I6,J6)
      V(I6,J6+1)=V(I6,J6)
      V(I6+1,J6+1)=V(I6,J6)
      V(I7+1,J7)=V(I7,J7)
      V(I7,J7+1)=V(I7,J7)
      V(I7+1,J7+1)=V(I7,J7)
      V(I8+1,J8)=V(I8,J8)
      V(I8,J8+1)=V(I8,J8)
      V(I8+1,J8+1)=V(I8,J8)
 17   D=0.
      K=0
      DO 21 I=2,M
      DO 21 J=2,N
      IF(K)22,23,22
 23   IF(I-I1)24,25,24
```

```
      V(I6,J6+1)=V(I6,J6)
      V(I6+1,J6+1)=V(I6,J6)
      V(I7+1,J7)=V(I7,J7)
      V(I7,J7+1)=V(I7,J7)
      V(I7+1,J7+1)=V(I7,J7)
      V(I8+1,J8)=V(I8,J8)
      V(I8,J8+1)=V(I8,J8)
      V(I8+1,J8+1)=V(I8,J8)
17    D=0.
      K=0
      DO 21 I=2,M
      DO 21 J=2,N
      IF(K)22,23,22
23    IF(I-I1)24,25,24
24    IF(I-I1-1)27,25,27
25    IF(J-J1)27,28,27
27    IF(I-I2)29,30,29
29    IF(I-I2-1)31,30,31
30    IF(J-J2)31,28,31
31    IF(I-I3)32,33,32
32    IF(I-I3-1)34,33,34
33    IF(J-J3)34,28,34
34    IF(I-I4)35,36,35
35    IF(I-I4-1)37,36,37
36    IF(J-J4)37,28,37
```

BOSSERT CONTROLLED FIELD ANALYSIS

```
37    IF(I-I5)38,39,38
38    IF(I-I5-1)41,39,41
39    IF(J-J5)41,28,41
41    IF(I-I6)43,44,43
43    IF(I-I6-1)45,44,45
44    IF(J-J6)45,28,45
45    IF(I-I7)46,47,46
46    IF(I-I7-1)48,47,48
47    IF(J-J7)48,28,48
48    IF(I-I8)49,50,49
49    IF(I-I8-1)51,50,51
50    IF(J-J8)51,28,51
51    U=.25*(V(I+1,J)+V(I-1,J)+V(I,J+1)+V(I,J-1))
      D1=U-V(I,J)
      IF(D1)52,53,53
52    D1=(-D1)
53    D=D+D1
      V(I,J)=U
      GOTO21
28    K=1
      GOTO21
22    K=0
21    CONTINUE
      DO 54 I=1,M1
      V(I,1)=V(I,2)
54    V(I,N1)=V(I,N)
      DO 55 J=1,N1
      V(1,J)=V(2,J)
```

```
      DO 21 J=2,N
      IF(K)22,23,22
23    IF(I-I1)24,25,24
24    IF(I-I1-1)27,25,27
25    IF(J-J1)27,28,27
27    IF(I-I2)29,30,29
29    IF(I-I2-1)31,30,31
30    IF(J-J2)31,28,31
31    IF(I-I3)32,33,32
32    IF(I-I3-1)34,33,34
33    IF(J-J3)34,28,34
34    IF(I-I4)35,36,35
35    IF(I-I4-1)37,36,37
36    IF(J-J4)37,28,37
```

BOSSERT CONTROLLED FIELD ANALYSIS

```
37    IF(I-I5)38,39,38
38    IF(I-I5-1)41,39,41
39    IF(J-J5)41,28,41
41    IF(I-I6)43,44,43
43    IF(I-I6-1)45,44,45
44    IF(J-J6)45,28,45
45    IF(I-I7)46,47,46
46    IF(I-I7-1)48,47,48
47    IF(J-J7)48,28,48
48    IF(I-I8)49,50,49
49    IF(I-I8-1)51,50,51
50    IF(J-J8)51,28,51
51    U=.25*(V(I+1,J)+V(I-1,J)+V(I,J+1)+V(I,J-1))
      D1=U-V(I,J)
      IF(D1)52,53,53
52    D1=(-D1)
53    D=D+D1
      V(I,J)=U
      GOTO21
28    K=1
      GOTO21
22    K=0
21    CONTINUE
      DO 54 I=1,M1
      V(I,1)=V(I,2)
54    V(I,N1)=V(I,N)
      DO 55 J=1,N1
      V(1,J)=V(2,J)
55    V(M1,J)=V(M,J)
56    IF(D-T)57,57,17
57    WRITEOUTPUT TAPE 3,102,(V(I,2),V(I,9),V(I,17),V(I,25),V(I,32),
     1I=2,M,2)
102   FORMAT(5F20.2)
      GO TO 983
      END(1,0,0,0,0,0,1,0,0,0,0,0,0,0,0)
```

Vita

Carl J[redacted] Bossert was born on [redacted], [redacted], the son of Karl Bossert and Anna Bossert. After completing his work in 1951 at [redacted], he entered the U. S. Military Academy, West Point, New York. Upon graduation in 1955 he received a commission in the United States Air Force and entered flying training. Upon completion of jet flying training at Laredo AFB, Texas and Fighter-Interceptor School, Tyndall AFB, Florida, he was assigned to the Air Defense Command. His military assignment prior to his coming to the Air Force Institute of Technology was Interceptor Pilot in the Washington Air Defense Sector.

Permanent address: [redacted]

This thesis was typed by Mrs. Judy Adams.