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THE EFFECT OF PHASE-LOCK CIRCUITRY ON RANGE OF COMMUNICATION SYSTEMS

THESIS

Presented to the Faculty of the School of Engineering of
the Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

By

John Milton Kamm, Jr., B.S.
1st Lt. USAF

Graduate Astronautics

August 1961

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Preface

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I wish to express my gratitude to Professor T. L. Regulinski, whose understanding and patience encouraged me and whose recommendations provided me much material. I also wish to acknowledge my indebtedness to my wife and son who have sacrificed as much as I during this educational endeavor.

Preface

This report is an attempt to discover any benefit of phase-lock circuitry that could be applied to space communications. Although my formal training has been quite varied, with little emphasis on any one engineering or scientific field, this has been an opportunity to delve more deeply into one specialized field and to learn for myself what, perhaps, most electrical engineers take for granted.

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List of Symbols

a	vehicle acceleration (ft/sec^2)
b	rate of change of vehicle acceleration (ft/sec^3)
B_1	first order loop bandwidth parameter
B_2	second order loop bandwidth parameter
B_3	third order loop bandwidth parameter
E	error
f_d	doppler frequency (cps)
f_m	modulating frequency range (cps)
f_o	carrier frequency (mc/sec)
f_r	pulse repetition frequency (cps)
Δf	bandwidth
G_R	gain of the receiver
G_T	gain of the transmitter
P_R	power required by the receiver
P_T	power of the transmitter
P-L	subscript referring to a system employing phase-lock circuitry
R	range (ft)
S	La Placian operator
t	time (sec)
v	vehicle velocity (ft/sec)
$\Delta\alpha$	frequency ramp (cycles/sec^2)
$\Delta\beta$	frequency parabola (cycles/sec^3)
θ_1	input signal

List of Symbols

θ_2 output signal

λ_0 wave length of the carrier frequency (ft/cycle)

Υ pulse repetition period (sec)

$\Delta\omega$ frequency step (cps)

$\approx$ approximately equal to

$=$ equal to

Abstract

Phase-lock is a technique which is applied to phase coherent communication systems and which will effectively reduce the bandwidth of those systems. A first order telemeter shows a significant increase in range, as does a second order radar and a third order telemeter, when phase-lock circuitry is incorporated into the system. The magnitude of this range advantage is greatest in the low megacycle range of carrier frequencies. This advantage also depends on the characteristics of the transmitted wave and the equation of motion of the transmitting vehicle.

THE EFFECT OF PHASE-LOCK CIRCUITRY ON RANGE OF COMMUNICATION SYSTEMS

I. Introduction

The purpose of this study is to determine any range advantage that may result from the employment of phase-lock circuitry in communications and to explore this advantage over the communication frequency spectrum. Those communication systems which offer three different types of input signals will be studied. The pulse time modulated telemeter employs a frequency step which will be analyzed in a first order phase-lock loop. The pulse-doppler radar return from an accelerating target employs a frequency ramp which will be imposed on a second order system. Another pulse time modulated telemeter employs a frequency parabola when transmitted from a vehicle traveling at a constant rate of change of acceleration. This will be the input to a third order loop.

Background

Point to point radio communications in the high frequency ranges are governed by the range equation:

$$R = \frac{\lambda_o}{4\pi}\left(\frac{P_T\,G_T\,G_R}{P_R}\right)^{\frac{1}{2}}$$

Any method of increasing the gain of the receiver (G_R) will cause an increase in the range (R) if other factors remain constant. A range increase is of great value for deep space communications systems.

An increase in the gain of the receiver will also be of value in satellite communication, where range varies only slightly, since a proportional decrease in the power required by the transmitter will result. Power supply occupies a large portion of the weight of every satellite, and the weight percentage could be reduced appreciably by a significant decrease in power required by the receiver.

Phase-lock is the term applied to a circuit which could cause a significant decrease in the signal power required by the receiver. This circuit technique employs no new theories, but it is possible at the present time assuming no so-called "scientific breakthroughs."

Assumptions

In this study, the input is assumed to be a sine wave plus uncorrelated white Gaussian noise. The sine wave was chosen since any electrical signal can be defined as a Fourier series of sine terms. White noise is narrowbanded noise about the center frequency of the signal. The spectrum of white noise is flat, which approximates the characteristics of atmospheric noise over a narrow band and allows statistical evaluation of phase jitter.

The optimum loop transfer function is used in this study since it is physically realizable. In a previous study the fixed filter phase-lock loop was preceeded by a band pass limiter. The output of this loop was shown to closely approximate that of a loop containing a variable filter that was continually readjusted to optimum conditions (Ref 2:10).

Statement of the Problem

Every communication system has a characteristic bandwidth through which noise is introduced into the system. This bandwidth can be reduced by the addition of a phase-lock loop to the system in question. Such a reduction in bandwidth reduces the noise introduced into the receiver and this can be interpreted as an increase in the gain of the receiver. The bandwidth reduction method is employed in this study to determine any advantage achieved through addition of a phase-lock loop for various carrier frequencies.

II. Description of the Phase-Lock Loop

In figure 1, the signal input to a loop is assumed to be:

$$\sqrt{2}\, A \sin\left[\omega t + \theta_1(t)\right]$$

where: A = rms signal amplitude

ω = signal center radian frequency

$\theta_1(t)$ = information content of the signal

It is assumed that the noise input to the loop is narrow banded about the signal center frequency and has a flat spectrum over the band. The noise input may be represented by

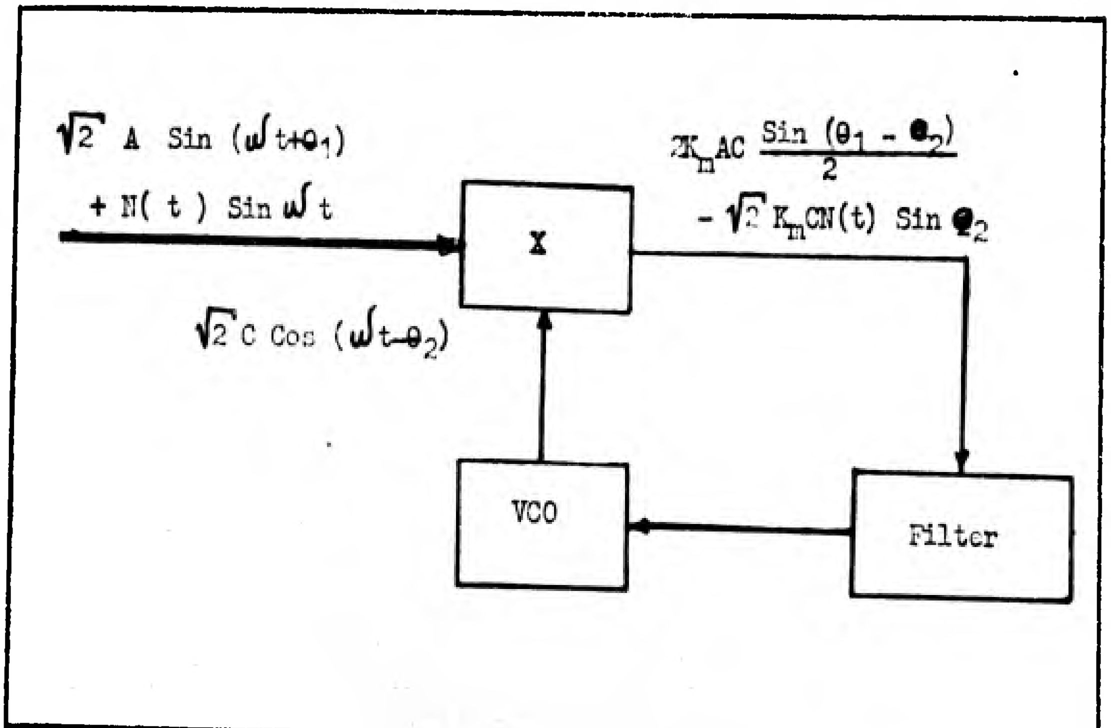
$$N(t) \sin \omega t$$

The output of the voltage-controlled oscillator (VCO) is assumed to have an rms amplitude of C, a center frequency identical to that of the signal of ω, and a phase output of $\theta_2(t)$.

A multiplier beats the signal input and the output of the voltage-controlled oscillator together, giving a low frequency output proportional to $(\theta_1 - \theta_2)$. The loop filter passes only this low frequency term which is applied to the voltage-controlled oscillator and which forces the output signal, θ_2, to be equal to the input signal, θ_1.

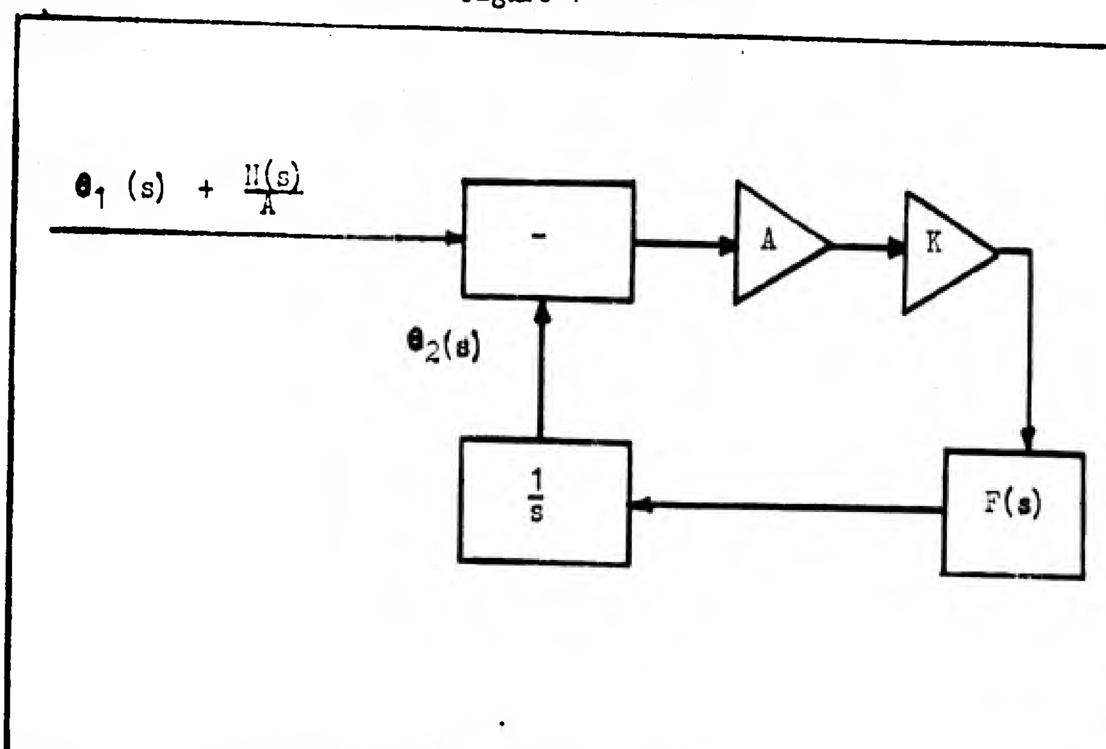
The bandwidth required by the phase-lock loop must be large enough to pass the difference between the input and the output signals. Since this difference varies less than the signal frequency, the reduced bandwidth decreases the amount of noise passed into the loop (Ref 2:2).

According to the experiments conducted at the Jet Propulsion Laboratory by R. Jaffe and E. Rechtin, this circuit can be simplified by linearization. This linear circuit is shown in figure 2. All gain coefficients are combined into a single amplifier of gain K, and all functions are expressed in LaPlace transforms rather than in functions of time. The product AK is defined as the loop gain.



The Phase-Lock Loop

Figure 1



The Linearized Phase-Lock Loop

Figure 2

III. The First Order System

The error in a first order system is

$$E(s) = \left(\frac{s}{s + B_1}\right) \theta_1 \qquad (2.)$$

as shown in Appendix A. If the input to this system is a frequency step, then

$$\theta_1 = \frac{\Delta \omega}{s^2} \qquad (3.)$$

The error then becomes

$$E(s) = \frac{\Delta \omega}{s\,(s + B_1)} \qquad (4.)$$

or

$$E(t) = \frac{\Delta \omega}{B_1} \left(1 - e^{-B_1 t}\right) \qquad (5.)$$

as shown in figure 3. The error is a maximum when

$$\frac{d}{dt}\,(E) = 0 \qquad (6.)$$

or E(t) becomes maximum as time increases without bound, at which time

$$E(t)_{max} = \frac{\Delta \omega}{B_1} \qquad (7.)$$

Jaffee and Rechtin have found it advisable to limit this error to $30^{\circ} = .0833$ cycles (Ref 2:31), thus

$$B_1 = \frac{\Delta \omega}{.0833} \qquad (8.)$$

If a vehicle is transmitting a pulse time modulated signal of $f_o \times 10^6$ cps, there will be no error if the distance between transmitter and receiver is constant. Therefore, only the radial component of velocity will cause the reception of a signal other than $f_o \times 10^6$ cps. If the vehicle is traveling away from the receiver at a constant velocity, the signal received will be a frequency step,

$$\Delta\omega = \frac{v}{\lambda_o} = \frac{v\ f_o}{982} \qquad (9.)$$

thus

$$B_1 = \frac{v\ f_o}{(982)(.0833)} = (.0122)\ v\ f_o \qquad (10.)$$

This variation of B_1 with velocity and carrier frequency is shown in figure 4, where increasing bandwidth results from increasing velocity or increasing carrier frequency.

Since the loop bandwidth parameter is approximately equal to twice the bandwidth (Ref 2:5), the following comparison may be made between a phase-lock system and a system without phase-lock circuitry

$$\Delta f_{(P-L)} \approx \frac{B_1}{2} = (.0061)\ v\ f_o \qquad (11.)$$

$$\Delta f \approx 2\ f_m \quad \text{(Ref 5:31)} \qquad (12.)$$

This comparison implies a difference in gain of the receiver as follows:

$$G_{R\ (P-L)} = \frac{\Delta f}{\Delta f_{(P-L)}}\ G_R$$

$$= 328\ \frac{f_m}{v\ f_o}\ G_R \qquad (13.)$$

The range equation (1) can be modified to become

$$R = \frac{982}{4\pi f_o}\left\{\frac{P_T\ G_T\ G_R}{P_R}\right\}^{\frac{1}{2}} \qquad (14.)$$

for a system not employing phase-lock circuitry. This equation is modified when phase-lock is employed, due to the change in gain of the receiver, and becomes

$$R_{(P-L)} = \frac{982}{4\pi f_o} \left(\frac{\Delta f}{\Delta f_{(P-L)}} \frac{P_T G_T G_R}{P_R} \right)^{\frac{1}{2}} \qquad (15.)$$

Under the following assumptions

$G_T = 1$

$G_R = 1$

$P_R = 1$

$f_m = 3400$ cps

$v = 2000$ ft/sec

solving equation 15 for $\frac{R}{P_T^{\frac{1}{2}}}$ yields

$$\frac{R}{P_T^{\frac{1}{2}}} = \frac{78.2}{f_o} \qquad (16.)$$

$$\frac{R_{(P-L)}}{P_T^{\frac{1}{2}}} = \frac{1850}{f_o^{3/2}} \qquad (17.)$$

These results are compared in figure 5.

In figure 5, the range-power comparison is not always in favor of the phase-lock system as demonstrated with a carrier frequency of 1000 mc. This implies a certain critical carrier frequency, above which the advantage is with the system which does not employ phase-lock circuitry. This is the carrier frequency for which:

$$\frac{R}{P_T^{\frac{1}{2}}} = \frac{R_{(P-L)}}{P_T^{\frac{1}{2}}} = \frac{78.2}{f_o} = \frac{1850}{f_o^{3/2}} \qquad (18.)$$

or where $f_o = 562$ mc in this example.

In general

$$\frac{R}{P_T^{\frac{1}{2}}} = \frac{78.2}{f_o} \qquad (16.)$$

$$\frac{R_{(P-L)}}{P_T^{\frac{1}{2}}} = 1418 \left(\frac{f_m}{v f_o^3} \right)^{\frac{1}{2}} \qquad (19.)$$

In this case, the critical carrier frequency may be expressed as a function of vehicle velocity and modulating frequency range.

$$f_o = 328 \frac{f_m}{v} \qquad (20.)$$

A comparison of these results is given in figure 6.

For the first order system, the phase-lock loop can provide a significant increase in range over a conventional pulse time modulating system. To accomplish the highest range-power advantage modulating frequency range must be as large as possible, and vehicle velocity must be as slow as possible. Once these two parameters are fixed, equation 20 can be solved for the critical carrier frequency. The carrier frequency actually used must be less than that calculated from equation 20, or the phase-lock is a detriment to the communications system.

If vehicle velocity is not constant, this analysis is not valid. The simplest variable velocity is the constant rate of change, or constant acceleration. This vehicle motion is considered in the second order system.

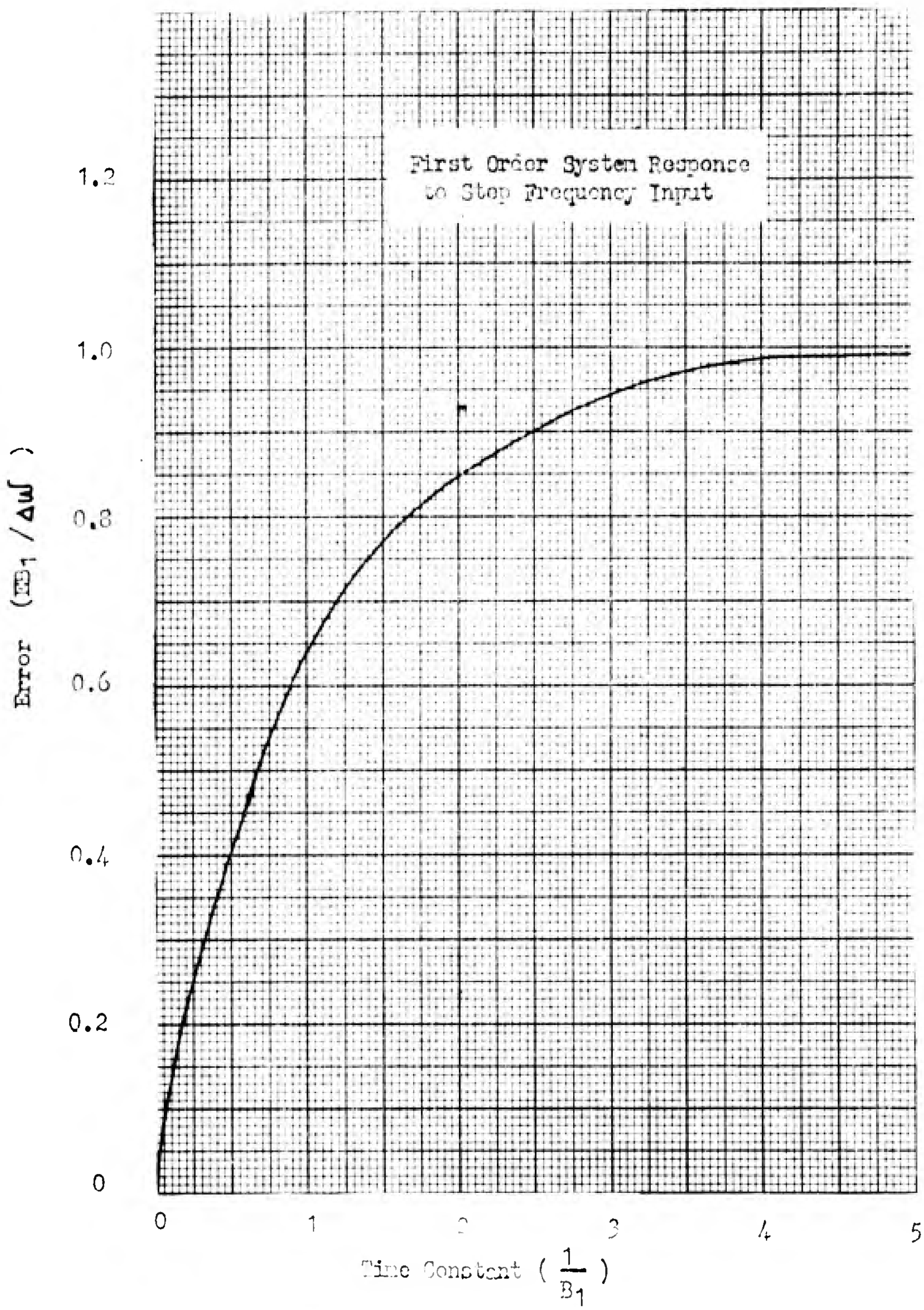

First Order System Response
to Step Frequency Input
Error (E_1 / $\Delta\omega$)
1.2
1.0
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
Time Constant ($\frac{1}{B_1}$)

Figure 3

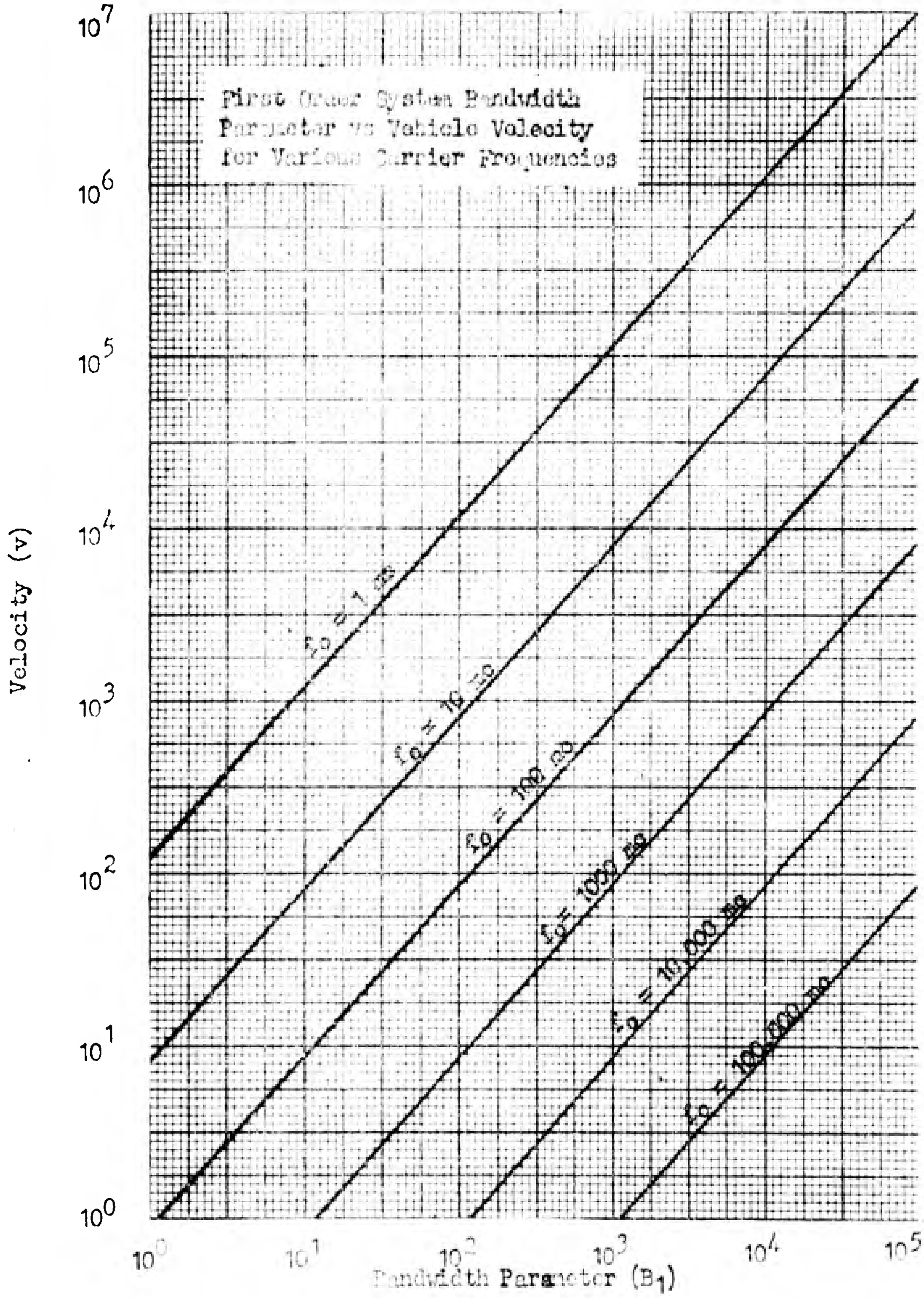

First Order System Bandwidth
Parameter vs Vehicle Velocity
for Various Carrier Frequencies
Velocity (v)
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
f_o = 1 mc
f_o = 10 mc
f_o = 100 mc
f_o = 1000 mc
f_o = 10,000 mc
f_o = 100,000 mc
10^0
10^1
10^2
10^3
10^4
10^5
Bandwidth Parameter (B_1)

Figure 4

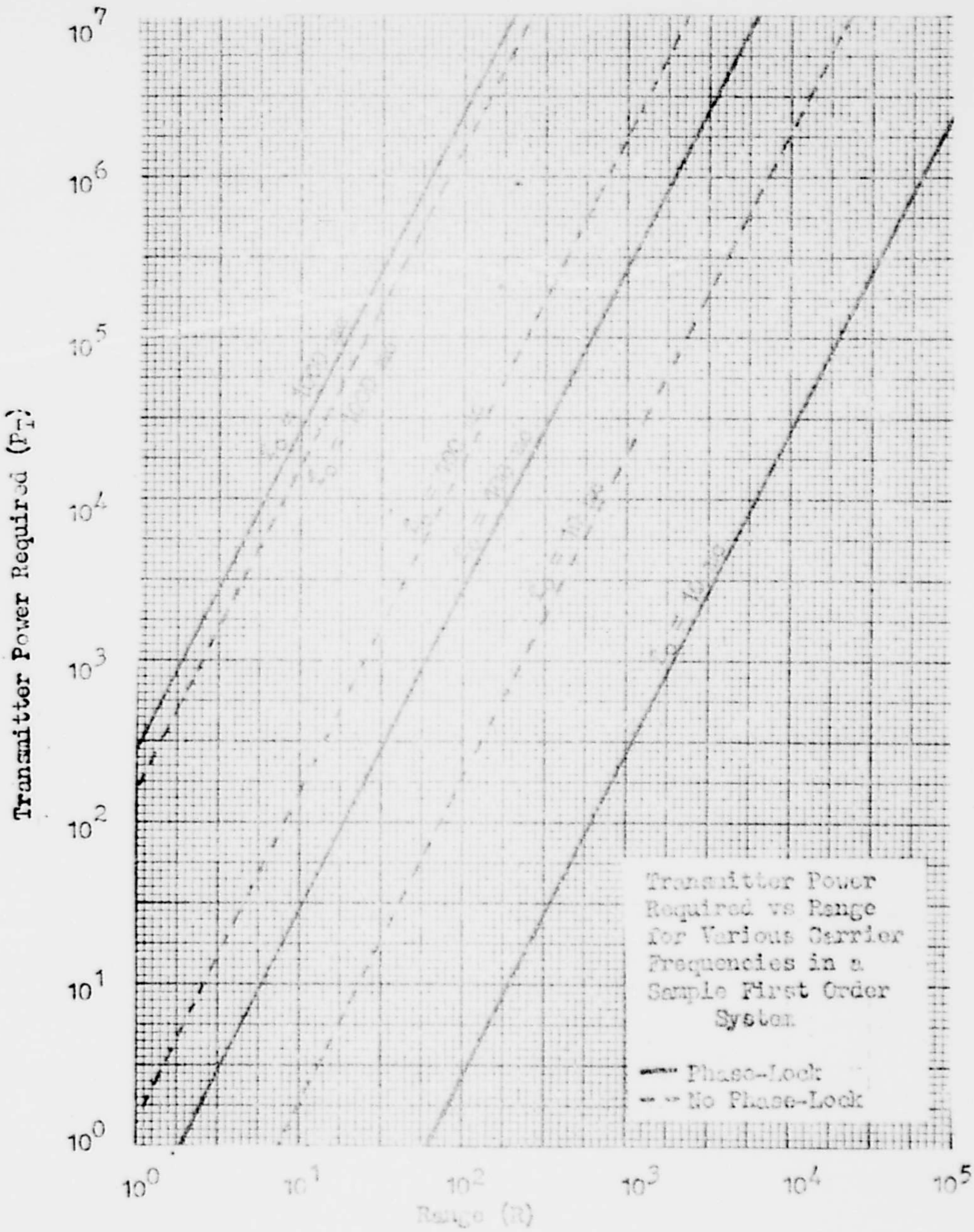

Transmitter Power Required (P_T)
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
Range (R)
Transmitter Power Required vs Range for Various Carrier Frequencies in a Sample First Order System
Phase-Lock
No Phase-Lock

Figure 5

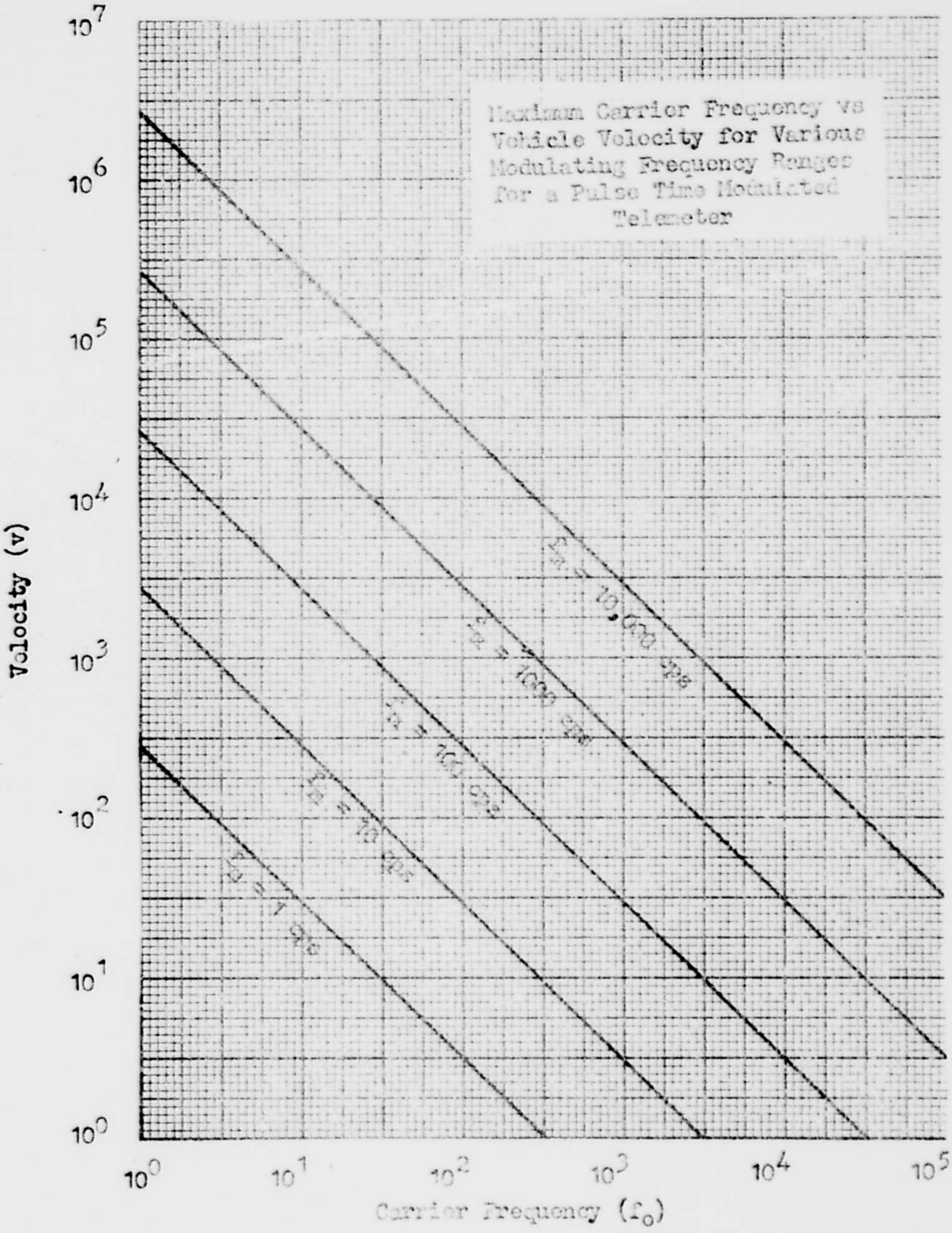

Maximum Carrier Frequency vs
Vehicle Velocity for Various
Modulating Frequency Ranges
for a Pulse Time Modulated
Telemeter
Velocity (v)
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^0
10^1
10^2
10^3
10^4
10^5
Carrier Frequency (f_o)

Figure 6

IV. The Second Order System

The error in a second order system is

$$E(s) \left(\frac{s^2}{B_2^2 + \sqrt{2}\, B_2 s + s^2} \right) \theta_1 \qquad (21.)$$

as shown in Appendix B. If the input to this system is a frequency ramp, then

$$\theta_i = \frac{\Delta\alpha}{s^3} \qquad (22.)$$

The error then becomes

$$E(s) = \frac{\Delta\alpha}{s\left[\left(s + \frac{B_2}{2}\right)^2 + \frac{B_2^2}{2} \right]} \qquad (23.)$$

or

$$E(t) = \frac{\Delta\alpha}{B_2^2}\left[1 + \sqrt{2}\, e^{-\frac{B_2 t}{2}} \sin\left(\frac{B_2 t}{2} - \frac{3\pi}{4} \right) \right] \qquad (24.)$$

as shown in figure 7. This error is a maximum when

$$\frac{d}{dt}(E) = 0 \qquad (6.)$$

or E(t) becomes maximum at $t = \frac{\pi\sqrt{2}}{B_2}$ at which time

$$E(t)_{max} = \frac{\Delta\alpha}{B_2^2}\left[1 + \sqrt{2}\, e^{-\pi} \sin\left(\frac{\pi}{4} \right) \right] \qquad (25.)$$

As recommended by Jaffe and Rechtin, this error is not permitted to exceed 30° = .0833 cycles (Ref 2:31), thus

$$B_2^2 = 12.5\, \Delta\alpha \qquad (26.)$$

If a vehicle, travelling at constant velocity, is being tracked by doppler radar, it will return a frequency step pulse. As time increases without bound, the error will approach zero for a second order system (Ref 1:128). However, if the target vehicle is approaching the receiver at a constant acceleration, a frequency ramp pulse return will result. Since

$$f_d = \frac{2\ f_o\ v}{982} \qquad \text{(Ref 5:52)} \qquad (27.)$$

it may be shown that

$$\Delta\alpha = \frac{d}{dt}\ (f_d) = \frac{2\ f_o\ a}{982} \qquad (28.)$$

Thus

$$B_2 = (.1593)(a\ f_o)^{\frac{1}{2}} \qquad (29.)$$

Figure 8 shows the bandwidth parameter variation with acceleration and carrier frequency.

$$\Delta f_{(P-L)} \approx \frac{B_2}{2} = (.00635\ f_o\ a)^{\frac{1}{2}} \quad \text{(Ref 2:5)} \qquad (30.)$$

$$\Delta f \approx \frac{2.0}{\tau} = 2\ f_r \qquad \text{(Ref 5:59)} \qquad (31.)$$

The difference in receiver gain due to this bandwidth change can be expressed as follows:

$$G_{R(P-L)} = \frac{\Delta f}{\Delta f_{(P-L)}}\ G_R$$

$$= 25.19\ \frac{f_r}{(f_o\ a)^{\frac{1}{2}}}\ G_R \qquad (32.)$$

The range equation is modified due to this change in receiver gain and becomes

$$R_{(P-L)} = \frac{982}{4\pi f_o}\left(\frac{\Delta f}{\Delta f_{(P-L)}}\ \frac{P_T\ G_T\ G_R}{P_R}\right)^{\frac{1}{2}}$$

$$= 393 \frac{f_r^{\frac{1}{2}}}{f_o^{3/2}\, a^{\frac{1}{4}}} \left(\frac{P_T G_T G_R}{P_R}\right)^{\frac{1}{2}} \qquad (33.)$$

Under the following assumptions:

$$G_T = 1$$
$$G_R = 1$$
$$P_R = 1$$
$$f_r = 2000 \text{ cps}$$
$$a = 1000 \text{ ft/sec}^2$$

The range equation can be solved once again for $\frac{R}{P_T^{\frac{1}{2}}}$, where P_T represents the power reflected from the target vehicle.

$$\frac{R}{P_T^{\frac{1}{2}}} = \frac{78.2}{f_o} \qquad (16.)$$

$$\frac{R_{(P-L)}}{P_T^{\frac{1}{2}}} = \frac{3120}{f_o^{5/4}} \qquad (34.)$$

These results are compared in figure 9.

As in the first order system, there is a critical carrier frequency, above which the conventional system has a range-power advantage over the phase-lock system. This is the frequency for which

$$\frac{R}{P_T^{\frac{1}{2}}} = \frac{R_{(P-L)}}{P_T^{\frac{1}{2}}} = \frac{78.2}{f_o} = \frac{3120}{f_o^{5/4}} \qquad (35.)$$

or where $f_o = 2.55 \times 10^6$ mc.

In general, however, the critical carrier frequency may not be this high, since

$$\frac{R}{P_T^{\frac{1}{2}}} = \frac{78.2}{f_o} \qquad (16.)$$

$$\frac{R_{(P-L)}}{P_T^{\frac{1}{2}}} = 392 \left(\frac{f_r^2}{f_o^5\, a}\right)^{\frac{1}{4}} \qquad (36.)$$

In this case the critical carrier frequency, as a function of vehicle acceleration and pulse repetition frequency is

$$f_o = 632 \frac{f_r^2}{a}$$

This result is plotted in figure 10.

For the second order system also, the phase-lock loop can provide a significant range-power advantage. In the doppler radar system this advantage is greatest when pulse repetition frequency is high and vehicle acceleration is low. Once these parameters are fixed, equation 37 will yield the critical carrier frequency. The carrier frequency employed by the system concerned must be lower than this calculated value for the phase-lock system to provide a range power advantage.

If acceleration is not constant, this evaluation is not valid. A third order system must be used if the vehicle travels at a constant rate of change of acceleration.

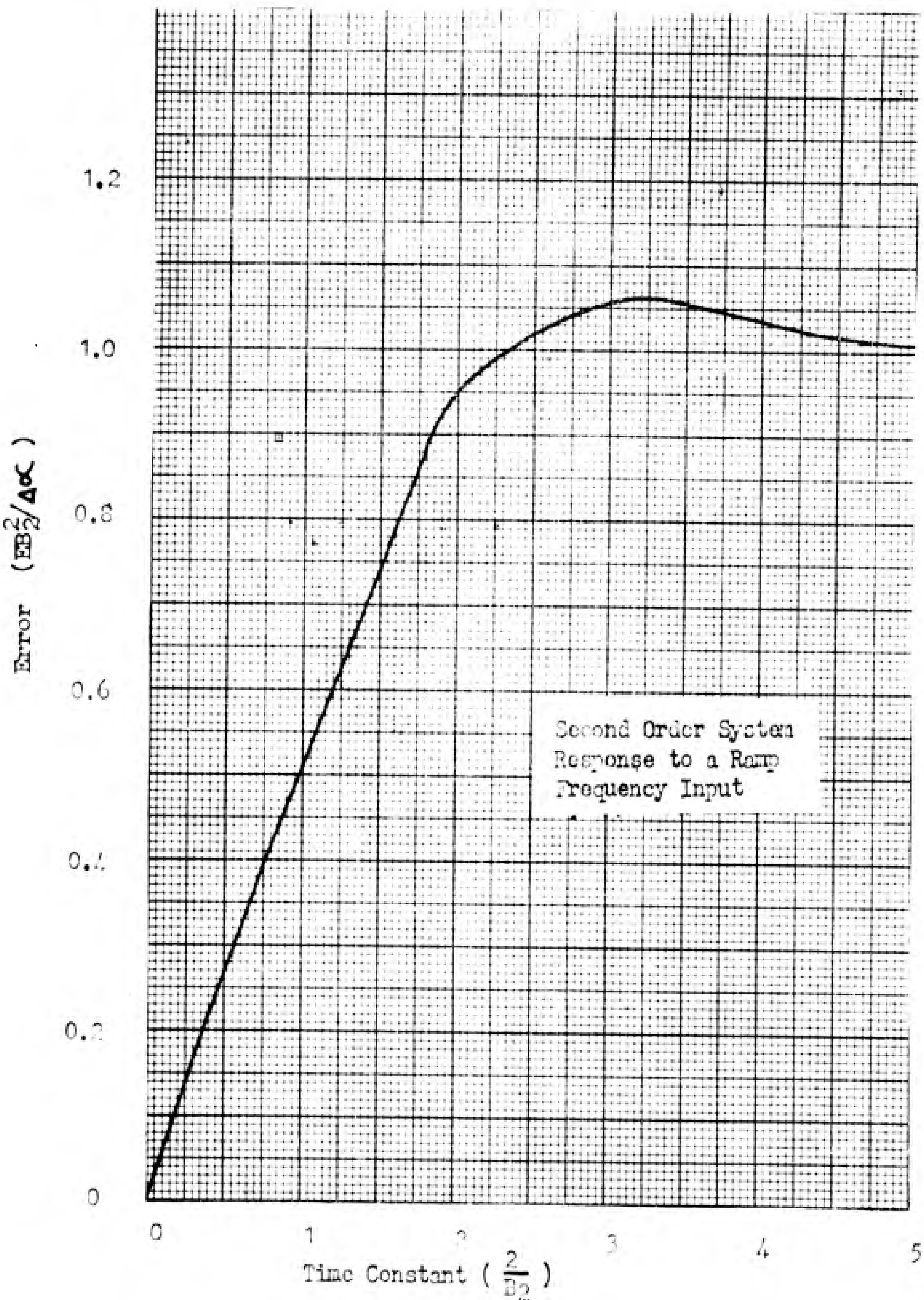

1.2
1.0
0.8
0.6
0.4
0.2
0
Error ($EB_2^2/\Delta\alpha$)
0
1
2
3
4
5
Time Constant ($\frac{2}{B_2}$)
Second Order System
Response to a Ramp
Frequency Input

Figure 7

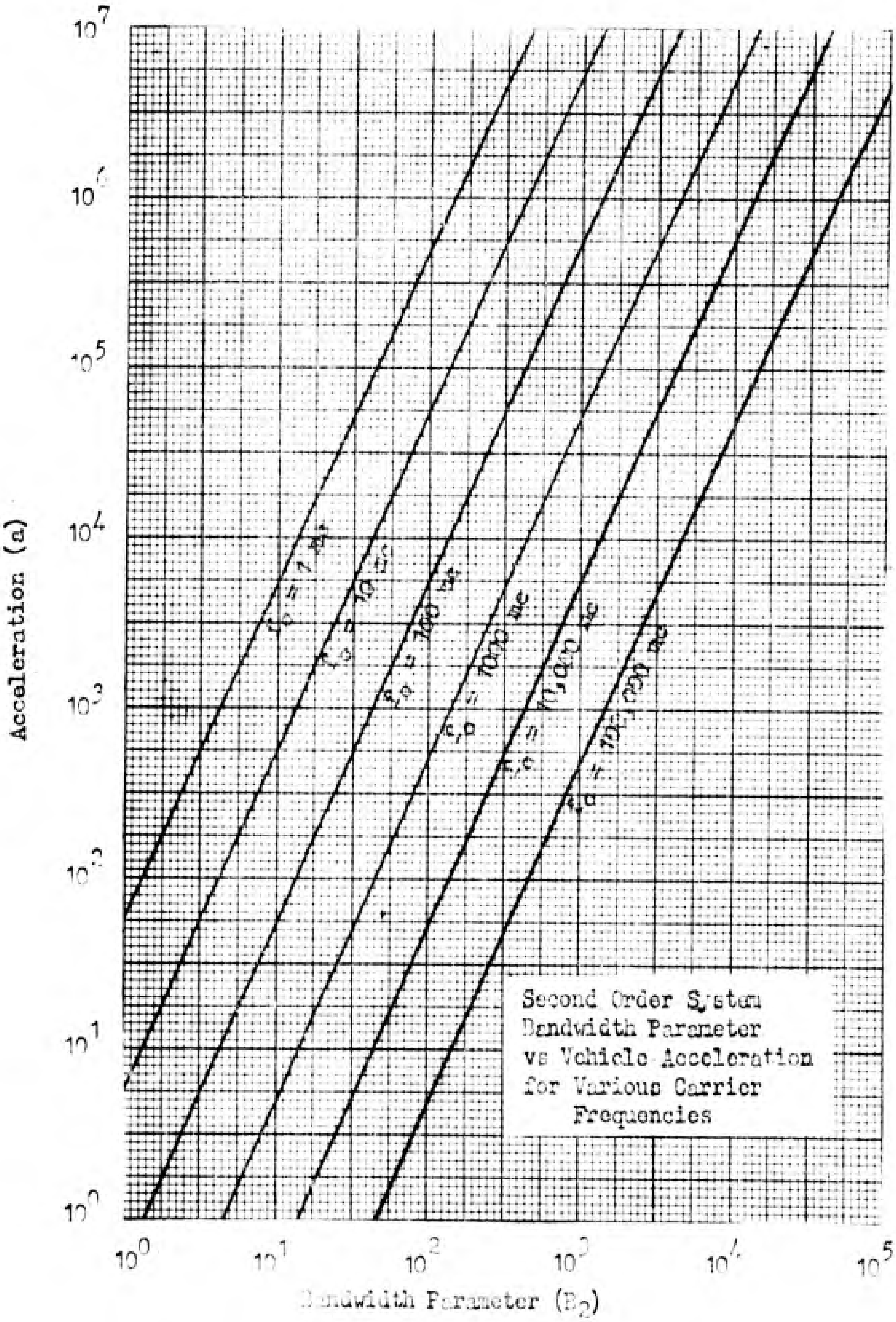
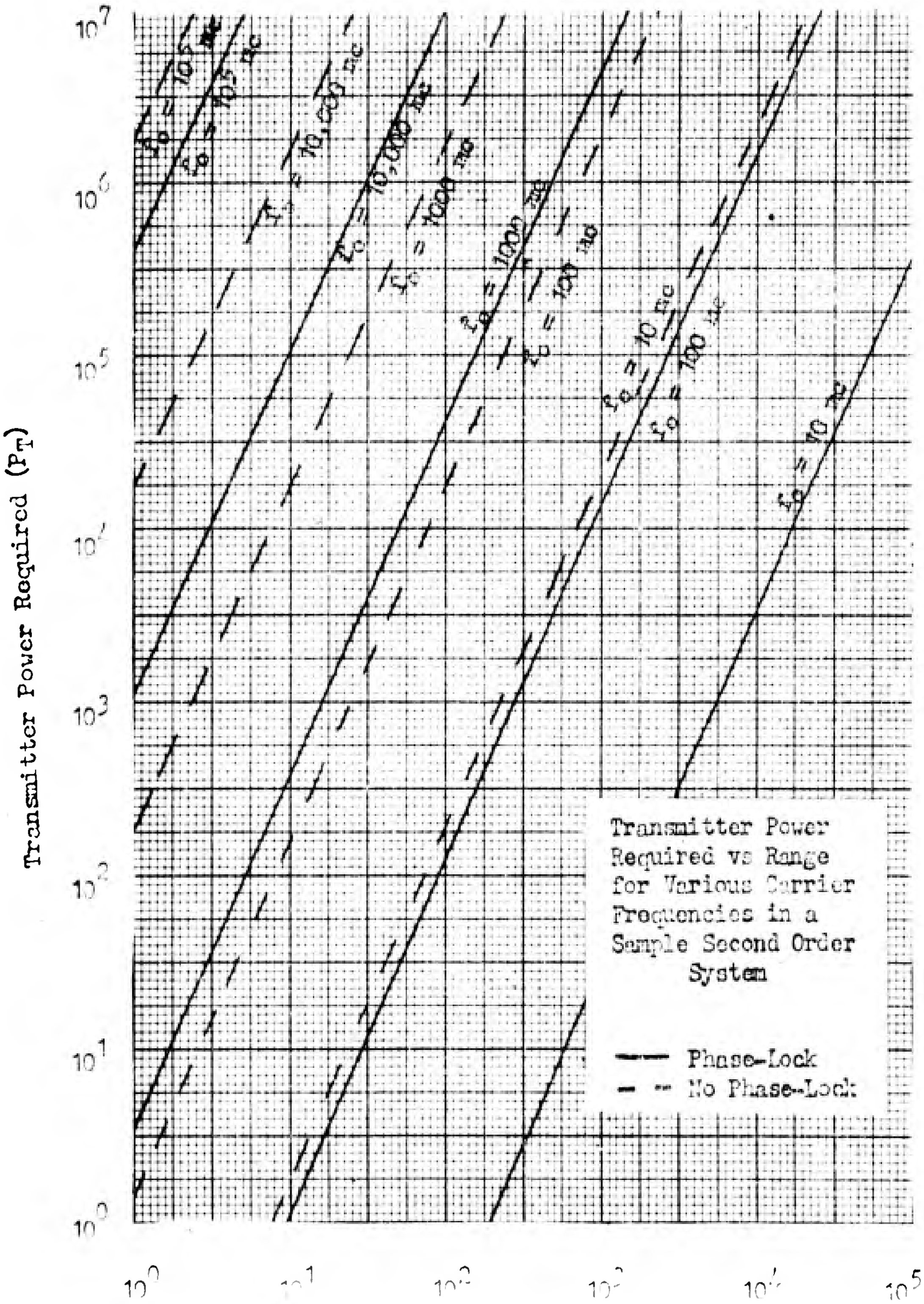
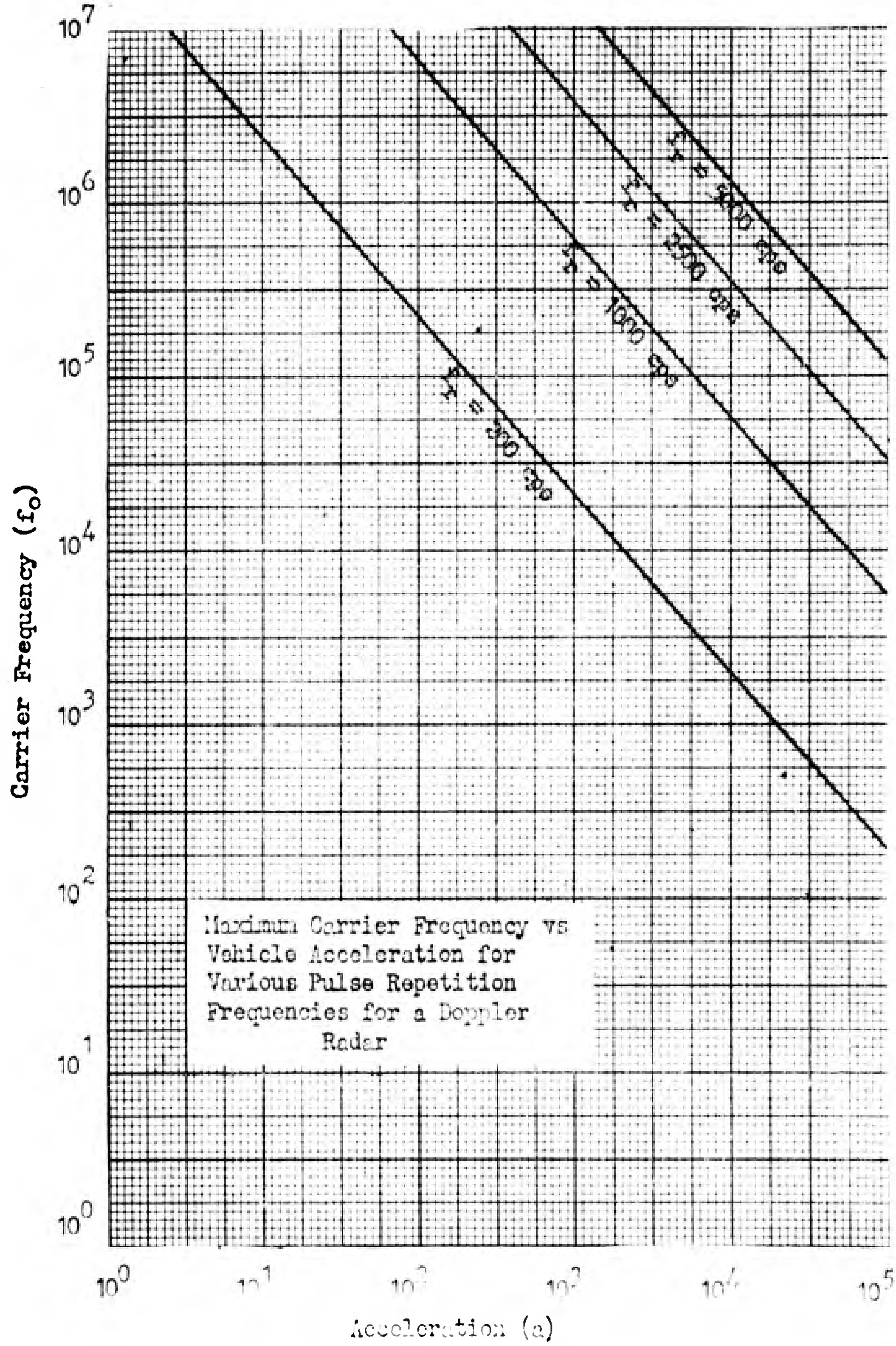

Acceleration (a)
Bandwidth Parameter (B_2)
Second Order System
Bandwidth Parameter
vs Vehicle Acceleration
for Various Carrier
Frequencies

Figure 8



Transmitter Power Required (P_T)
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
Transmitter Power Required vs Range for Various Carrier Frequencies in a Sample Second Order System
Phase-Lock
No Phase-Lock
10^0
10^1
10^2
10^3
10^4
10^5
Range (R)

Figure 9



Carrier Frequency (f_o)
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
f_r = 5000 cps
f_r = 2500 cps
f_r = 1000 cps
f_r = 300 cps
Maximum Carrier Frequency vs Vehicle Acceleration for Various Pulse Repetition Frequencies for a Doppler Radar
10^0
10^5
Acceleration (a)

Figure 10

V. The Third Order System

The error in a third order system is

$$E(s) = \left(\frac{s^3}{B_3^3 + 2B_3^2 s + 2B_3 s^2 + s^3} \right) \theta_1 \qquad (38.)$$

as shown in Appendix C. If the input to this system is a frequency parabola, then

$$\theta_1 = \frac{\Delta\beta}{s^4} \qquad (39.)$$

The error then becomes

$$E(s) = \frac{\Delta\beta}{s\left[s+B_3\right]\left[\left(s + \frac{B_3}{2}\right)^2 + \frac{3B_3^2}{4}\right]} \qquad (40.)$$

or

$$E(t) = \frac{\Delta\beta}{B_3^3}\left[1 - e^{-B_3 t} + \frac{2}{\sqrt{3}} e^{-\frac{B_3 t}{2}} \sin\left(\frac{\sqrt{3}}{2} B_3 t - \pi\right)\right] \qquad (41.)$$

as shown in figure 11.

The error is a maximum when $\frac{d}{dt}(E) = 0$ or $E(t)$ becomes maximum at $t \approx \frac{4.9}{B_3}$ at which time

$$E(t)_{max} = 1.0811 \frac{\Delta\beta}{B_3^3} \qquad (42.)$$

If this error is not permitted to exceed 30° as recommended by Jaffe and Rechtin (Ref 2:31), then

$$B_3^3 = 13.00\, \Delta\beta \qquad (43.)$$

If a vehicle is transmitting a pulse time modulated signal of $f_0 \times 10^6$ cps, the error due to constant velocity or constant acceleration will approach zero for a third order system (Ref 1:128). However, if the transmitting vehicle is approaching the receiver at a constant rate of change of acceleration, a frequency parabola will result from a transmitter square wave. It may be shown that the frequency parabola will be

$$\Delta\beta = \frac{f_o}{982} \frac{da}{dt} = \frac{f_o b}{982} \qquad (44.)$$

thus

$$B_3 = (.2366)\ (f_o b)^{1/3} \qquad (45.)$$

Figure 12 shows the variation in bandwidth parameter with rate of change of acceleration and carrier frequency.

The bandwidth comparison made between the phase-lock system and the system without phase-lock circuitry is as follows:

$$\Delta f_{(P-L)} \approx \frac{B_3}{2} = (.1183)\ f_o^{1/3} b^{1/3} \quad \text{(Ref 2:5)} \qquad (46.)$$

$$\Delta f \approx 2 f_m \qquad \text{(Ref 5:31)} \qquad (12.)$$

The difference in receiver gain due to this change in bandwidth is:

$$G_{R(P-L)} = \frac{\Delta f}{\Delta f_{(P-L)}} G_R$$

$$= 16.9 \left\{ \frac{f_m}{f_o^{1/3} b^{1/3}} \right\} G_R \qquad (47.)$$

The range equation is modified due to this change and becomes

$$R_{(P-L)} = \frac{982}{4\pi f_o} \left(\frac{\Delta f}{\Delta f_{(P-L)}} \frac{P_T G_T G_R}{P_R} \right)^{\frac{1}{2}}$$

$$= 322 \frac{f_m^{\frac{1}{2}}}{f_o^{7/6} b^{1/6}} \left(\frac{P_T G_T G_R}{P_R}\right)^{\frac{1}{2}} \qquad (33.)$$

Under the following assumptions

$$G_T = 1$$
$$G_R = 1$$
$$P_R = 1$$
$$f_m = 3400 \text{ cps}$$
$$b = 100 \text{ ft/sec}^3$$

the range equation can be solved for $\frac{R}{P_T^{\frac{1}{2}}}$

$$\frac{R}{P_T^{\frac{1}{2}}} = \frac{78.2}{f_o} \qquad (16.)$$

$$\frac{R(P\text{-}L)}{P_T^{\frac{1}{2}}} = \frac{8730}{f_o^{7/6}} \qquad (48.)$$

These results are compared in figure 13.

A critical carrier frequency, similar to the first and second order systems, exists in this system also. The phase-lock system does not maintain a range-power advantage above the carrier frequency for which

$$\frac{R}{P_T^{\frac{1}{2}}} = \frac{R(P\text{-}L)}{P_T^{\frac{1}{2}}} = \frac{78.2}{f_o} = \frac{8730}{f_o^{7/6}} \qquad (49.)$$

or where $f_o = 1.55 \times 10^{12}$ mc.

In general, the critical carrier frequency may be determined from the following:

$$\frac{R}{P_T^{\frac{1}{2}}} = \frac{78.2}{f_o} \qquad (16.)$$

$$\frac{R_{(P-L)}}{P_T^{\,1/2}} = 322 \left\{ \frac{f_m^{\,3}}{f_o \, b} \right\}^{1/6} \qquad (50.)$$

The critical carrier frequency becomes

$$f_o = 70 \, \frac{f_m^{\,3}}{b} \qquad (51.)$$

which is plotted in figure 14.

As in the first and second order systems, the third order phase-lock loop can provide a significant increase in range over a conventional system. This advantage is determined by the characteristics of the transmitted wave and the vehicle motion. In the pulse time modulation system a large modulating frequency range and a small rate of change of acceleration of the transmitting vehicle result in the greatest advantage for the phase-lock system. Once these parameters have been determined, a critical carrier frequency can be determined from equation 51. The carrier frequency actually used must be less than the calculated value for phase-lock to provide any increase in range.

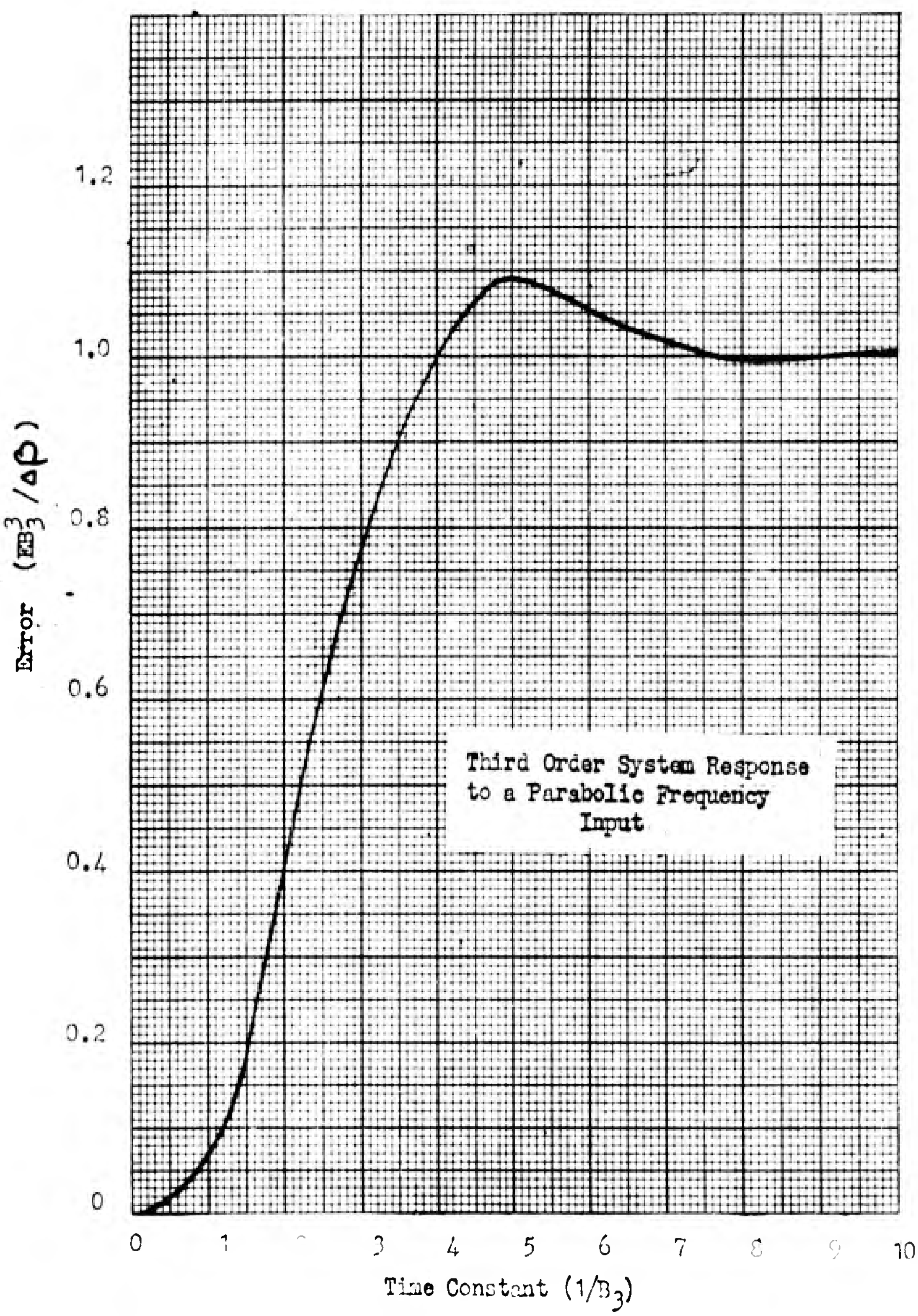

1.2
1.0
0.8
0.6
0.4
0.2
0
Error (EB$_3^3$/Δρ)
Third Order System Response to a Parabolic Frequency Input
0
1
3
4
5
6
7
8
9
10
Time Constant (1/B$_3$)

Figure 11

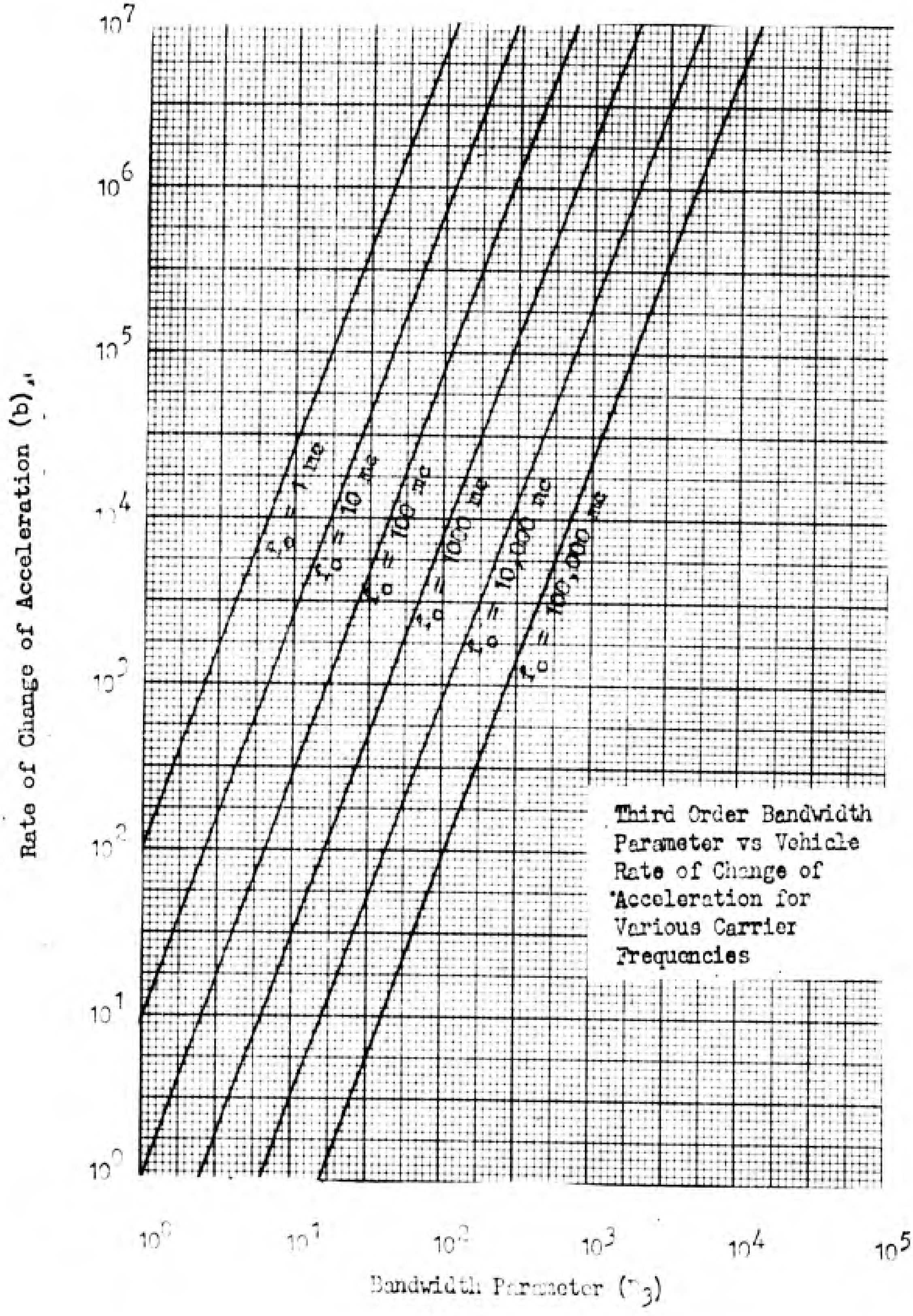

Rate of Change of Acceleration (b)
Bandwidth Parameter (P_3)
f_c = 1 mc
f_c = 10 mc
f_c = 100 mc
f_c = 1000 mc
f_c = 10,000 mc
f_c = 100,000 mc
Third Order Bandwidth Parameter vs Vehicle Rate of Change of Acceleration for Various Carrier Frequencies

Figure 12

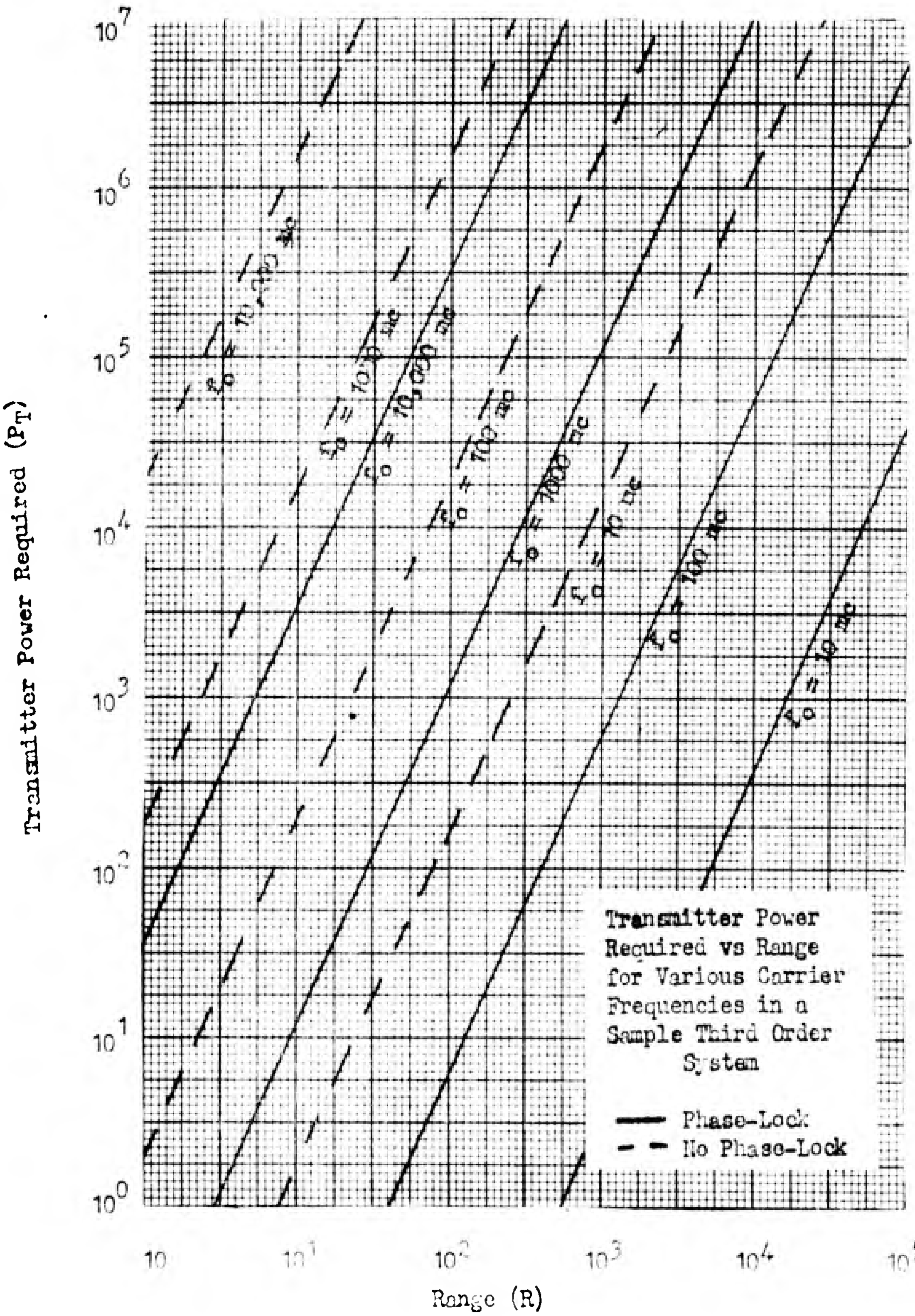

Transmitter Power Required (P_T)
Range (R)
Transmitter Power Required vs Range for Various Carrier Frequencies in a Sample Third Order System
Phase-Lock
No Phase-Lock

Figure 13

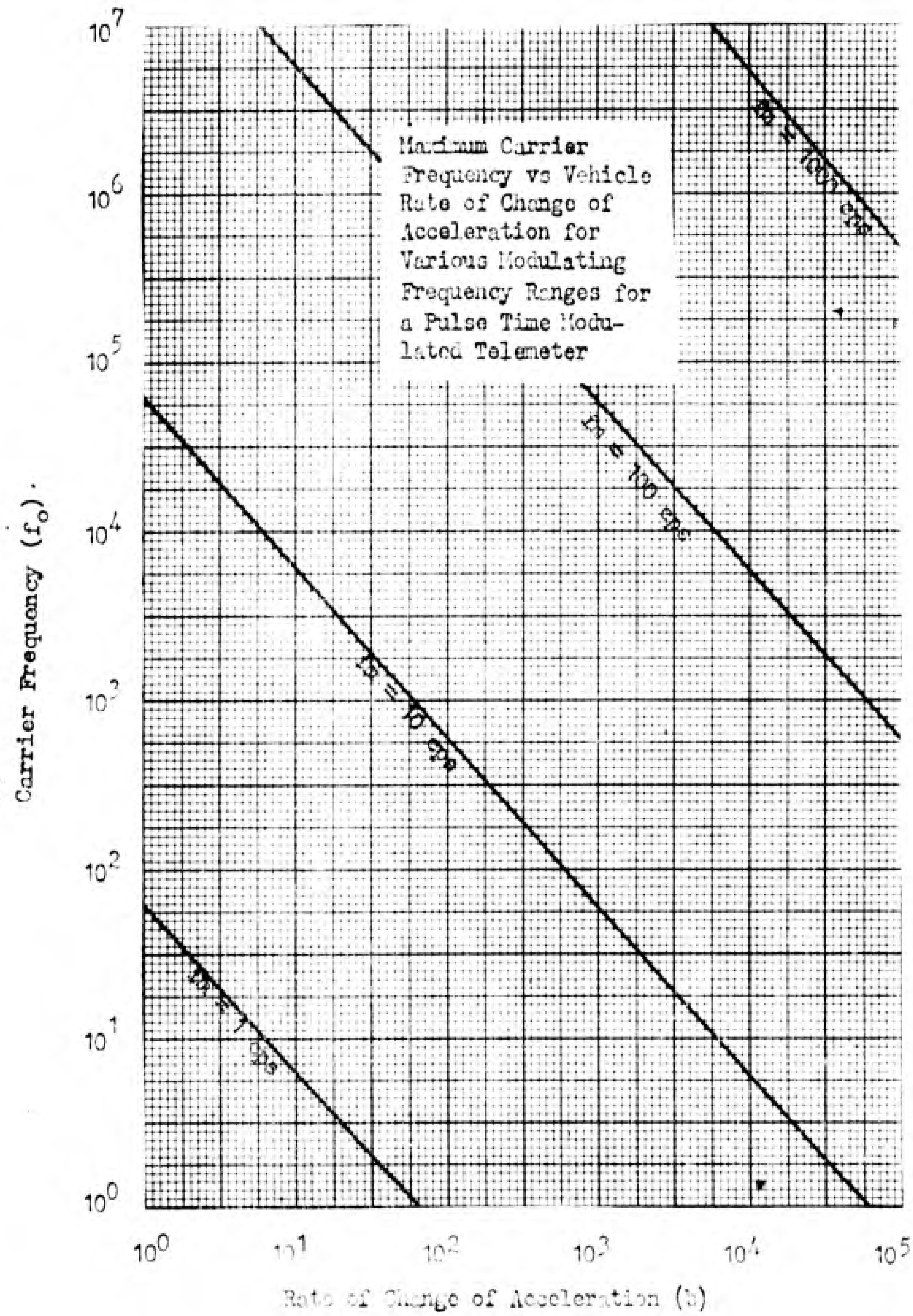

Maximum Carrier Frequency vs Vehicle Rate of Change of Acceleration for Various Modulating Frequency Ranges for a Pulse Time Modulated Telemeter
Carrier Frequency (f_o)
Rate of Change of Acceleration (b)
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
fm = 1 cps
fm = 10 cps
fm = 100 cps
fm = 1000 cps

Figure 14

VI. Summary

This study shows that under various signal characteristics, the use of phase-lock circuitry increases the range of a communication system. In some cases the range is increased by two orders of magnitude with no increase in power transmitted.

A vehicle is travelling at 2000 ft/sec and transmitting a pulse time modulated signal with a modulating frequency range of 3400 cps. With a carrier frequency of 10 mc, the range can be increased 7.6 times with the simple addition of phase-lock circuitry. This can be determined by equations 16 and 17. However, if the carrier frequency is 100 mc, the range is increased only 2.4 times. Although the increase in range is less at the higher carrier frequency, it is still appreciable.

A similar situation exists for the second order system. If a vehicle travelling at constant acceleration of 1000 ft/sec^2 radiates isotropically, the range of a doppler radar with a pulse repetition frequency of 2000 cps can be increased appreciably. This increase can be determined by equations 16 and 34. If the carrier frequency is 10 mc, the phase-lock system has a range advantage of 22.5. If the carrier frequency is 100 mc, this range advantage falls to 12.6. The increase in range in either case makes the phase-lock system far superior to the conventional doppler radar.

A vehicle is travelling at a constant rate of change of acceleration of 100 ft/sec^3 while transmitting a pulse time modulated signal with a modulating frequency range of 3400 cps. At a carrier frequency of 10 mc, a phase-lock loop increases the range of the transmitted signal 76.5 times; and at a carrier frequency of 100 mc, the phase-lock loop increases the range 51.5 times according to equations 16 and 48.

There is, however, a limiting carrier frequency above which the phase-lock system no longer has a range advantage over the conventional system. This limit is fixed by the motion of the vehicle transmitting the signal and by the characteristics of the signal itself.

A vehicle is travelling at 2000 ft/sec and transmitting a pulse time modulated signal under the same conditions as the first order system discussed above except for the carrier frequency. If the carrier frequency is higher than 562 mc as determined by equation 18, the range advantage lies with the conventional system rather than the phase-lock system. In this case the conventional system will have a range advantage of 1.34. Similar calculations can be made for the second and third order systems, and similar results will be obtained.

In all cases a square wave is assumed to have been transmitted and vehicle motion is assumed to have modified the signal. In the first order system a square wave is assumed to have been received since a frequency step is the only signal which will give a finite, non-zero error to this system. Similarly the second order system is assumed to receive a frequency ramp since a frequency step produces zero error and a frequency parabola produces infinite error. In the third order system the frequency parabola produces the only finite, non-zero error.

The bandwidth reduction method is not the only, nor is it necessarily the best, method of analysis of a phase-lock system. Nevertheless, this method suggested by C. L. Nielson (Ref 4) is useful in demonstrating the advantage that may be obtained. Other methods may be used to compare signal-to-noise ratios by statistical studies, and a series solution may be applied to the phase-lock loop (Ref 3) to determine the output signal.

One system worthy of further study is artificial satellite communication. Since a large percentage of the weight of any satellite is devoted to power supply, a great savings in weight might be made due to the decreased power requirement of phase-lock communications.

The equation of motion for an artificial satellite with respect to a fixed point on the earth's surface must first be derived. This will determine the order of the phase-lock system required.

The signal to be transmitted from the satellite must be specified in order to determine the modulating frequency range and the carrier frequency to be used. The maximum range from the satellite to the fixed ground station must be determined. Finally the gain of the transmitter, the gain of the receiver, and the power required by the receiver must be specified. It is then possible to determine the power of the transmitter for phase-lock and conventional systems.

To determine any weight savings of the phase-lock system, the weight of the equipment necessary to generate the calculated power must be determined.

In the third order system described, equations 16 and 48 can be used to determine the ratio of power required by the conventional system to the power required by the phase-lock system. If the carrier frequency is 1000 mc, the conventional system requires 1275 times as much power transmitted as does the phase-lock system.

Phase-lock is the most promising development in long range, point-to-point communication available at the present time. It requires no new discoveries, no new techniques, and no new processes. It can be applied to any phase-coherent system in use today with little modification of equipment and can increase the range of communication of that system ten to one hundred times in many cases.

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Appendix A

Optimum Error Function for a First Order System

$$Y(s) = \frac{AKF(s)}{s + AKF(s)} \qquad \text{(Ref 2:26)}$$

where: $Y(s)$ = loop transfer function
$F(s)$ = optimum filter transfer function
A = signal level
K = loop gain constant

$$F(s) = \frac{B_1}{KA} \qquad \text{(Ref 2:26)}$$

where: B_1 = loop step bandwidth parameter.

Therefore:

$$Y(s) = \frac{B_1}{s + B_1}$$

Since by definition:

$$\theta_2(s) = Y(s)\theta_1(s)$$

where:

$\theta_2(s)$ = output signal

$\theta_1(s)$ = input signal

thus: $E(s) = \theta_1(s) - \theta_2(s)$

where: $E(s)$ = error

$$E(s) = \theta_1(s) - Y(s)\theta_1(s)$$

$$= \left[1 - Y(s)\right]\theta_1(s)$$

$$= \left(\frac{s}{s + B_1}\right)\theta_1(s)$$

Appendix B

Optimum Error Function for a Second Order System

$$Y(s) = \frac{AKF(s)}{s + AKF(s)} \quad \text{(Ref 2:26)}$$

$$F(s) = \frac{B_2^2 + \sqrt{2}\, B_2\, s}{AKs} \quad \text{(Ref 2:25)}$$

thus:

$$Y(s) = \frac{B_2^2 + \sqrt{2}\, B_2 s}{s^2 + B_2^2 + \sqrt{2}\, B_2 s}$$

$$E(s) = \theta_1(s) - \theta_2(s)$$

$$= \left[1 - Y(s)\right] \theta_1(s)$$

$$= \left(\frac{s^2}{B_2^2 + \sqrt{2}\, B_2\, s + s^2}\right) \theta_1(s) \quad \text{(Ref 4:544)}$$

Appendix C

Optimum Error Function for a Third Order System

$$Y(s) = \frac{AKF(s)}{s + AKF(s)}$$

$$F(s) = \frac{B_3^3 + 2\,B_3^2\,s + 2\,B_3 s^2}{AK\ s^2} \quad \text{(Ref 2:28)}$$

thus:

$$Y(s) = \frac{B_3^3 + 2\,B_3^2\,s + 2\,B_3 s^2}{s^3 + B_3^3 + 2\,B_3^2\,s + 2\,B_3 s^2}$$

$$E(s) = \left[1 - Y(s)\right] \theta_1\,(s)$$

$$= \left(\frac{s^3}{(B_3^3 + 2\,B_3^2\,s + 2\,B_3\,s^2 + s^3)}\right)\theta_1\,(s) \quad \text{(Ref 4:545)}$$

Vita

John Milton Kamm, Jr. was born on [redacted] he enrolled at Franklin and Marshall College. He enlisted in the U. S. Army in 1951 and entered the United States Military Academy in 1952 from which he was graduated in 1956 with the degree of Bachelor of Science. His military assignment prior to coming to the Institute of Technology was as tactical pilot in Troop Carrier operations of the Tactical Air Command.

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This thesis was typed by Mrs. Marjorie Burger.

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