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WATERTOWN ARSENAL LABORATORIES

DRIVING EDGE PRESSURE ON A ROTATING BAND

TECHNICAL REPORT NO. WAL TR 760/410-3

BY

KENNETH D. ROBERTSON

and

FRANCIS I. BARATTA

DATE OF ISSUE - JANUARY 1962

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D/A PROJECT 504-03-061

WATERTOWN ARSENAL
WATERTOWN 72, MASS.

Rotating Band

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TITLE

DRIVING EDGE PRESSURE ON A ROTATING BAND

ABSTRACT

An analysis of the pressure exerted on the driving edge of a rotating band during the firing of a projectile is presented. Separate analyses are presented for uniform and variable twist rifling and equations are developed which relate the driving edge force, i.e., the integral of the driving edge pressure, to the other forces acting on the band. Solution of the equations developed must in general be accomplished by numerical methods. Simplification of these equations for the case of uniform twist rifling in the region of rifling beyond the forcing cone permits direct determination of the average driving edge pressure in that region.

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NOMENCLATURE

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
M	Mass of projectile	lb-sec ² /in.
I	Mass moment of inertia of projectile about longitudinal axis	lb-in-sec ²
ρ	Radius of gyration of projectile	in.
r_m	Mean of land and groove radii	in.
x	Cylindrical coordinate defining position along longitudinal axis of gun tube measured from origin of rifling	in.
x_p	Cylindrical coordinate defining position of forward end of band along longitudinal axis of gun tube measured from origin of rifling	in.
x_1	Cylindrical coordinate defining the position of a point on the band at which the driving edge displacement is zero measured along longitudinal axis of gun tube from origin of rifling. Used for variable twist rifling only	in.
Y	Cylindrical coordinate, see Figure 7	
Z	Cylindrical coordinate, see Figure 7	
ϕ_r	Cylindrical coordinate defining the angular rotation of a point on the rifling space curve, see Figure 7	radian
ϕ_p	Cylindrical coordinate defining the angular rotation of a point on the projectile about the longitudinal axis of the gun tube.	radian
b	Constant of proportionality in space curve of rifling $r\phi_r = bx^n$	in. ⁿ⁻¹
n	Exponent of x in space curve of rifling $r\phi_r = bx^n$	
θ_r	The angle between the tangent to the unrolled rifling curve and the longitudinal axis of the gun tube, see Figures 3 and 6. $\tan \theta_r = \frac{dr\phi_r}{dx}$	radian

θ_p	The angle between the tangent to the unrolled path of the projectile and the longitudinal axis of the gun tube, see Figures 3 and 6 $\tan \theta_p = \frac{dr\theta_p}{dx}$	radian
ξ	Rectangular coordinate defining the position of a point P on the band measured from the forward end of the band in the longitudinal direction, see Figure 8	in.
u	Rectangular coordinate defining the position of a point P on the band measured from the periphery of the band in the radial direction, see Figure 8	in.
$P(\xi, u)$	A point on the band defined by ξ and u	
h	Depth of engraving	in.
N	Number of lands or grooves	
l_b	Band length	in.
A_b	Area at base of projectile	sq in.
α	Forcing cone angle, see Figures 3 and 6	radian
δ_1	Displacement of the driving edge of the band land normal to land, see Figures 2, 3, 5 and 6	in.
δ_2	Displacement of the trailing edge of the band land normal to land, see Figures 3 and 6	in.
μ	Coefficient of friction	
K	Physical constant of band material	
t	Time	sec
P_g	Gas pressure, see Figures 1 and 4	psi
P_b	Radial band pressure, see Figures 1 and 4	psi
S_1	Normal pressure on driving edge of band land, see Figures 1 and 4	psi
S_2	Normal pressure on trailing edge of band land, see Figures 1 and 4	psi
S_{ave}	Average normal pressure on driving edge of band land	psi

PURPOSE AND SCOPE

The object of this investigation was to analyze the forces exerted on a rotating band during the firing of a projectile and derive expressions to predict the driving edge pressure on the band as a function of these forces.

INTRODUCTION

The maximum allowable bearing pressure on the driving edge of a rotating band is one of the criteria used to determine adequate band length. General expressions for the force exerted on the driving edge of a rotating band can be obtained as the resultant of all other forces acting on the band. The driving edge pressure can then be obtained as a function of the driving edge force and the driving edge area provided the pressure distribution is known.

Previously derived expressions for driving edge pressure are presented in Reference 1. Those expressions do not include the effects of friction, and in the case of variable twist rifling only the net driving edge pressure is considered.

The analysis presented in this report is a refinement of the previous analysis presented in Reference 1 and represents a revision of Section 3.50, Volume 1 of Reference 1.

RESULTS

An analysis of the pressure exerted on the driving edge of a rotating band during the firing of a projectile is presented in Appendix A. Two sets of simultaneous equations are developed as a result of that analysis and are summarized below. The first set applies to constant twist rifling, the second set applies to variable twist rifling.

Constant Twist Rifling

$$P_g A_b - N[\sin \theta_r + \mu \sin \theta_r] \int_0^h \int_0^{\xi_{\max}} S_1 d\xi du \\ - 2\mu \pi r_m \xi_{\max} P_b \cos \theta_p = M \frac{d^2 x_p}{dt^2} \quad \dots (1)$$

1. WATERTOWN ARSENAL LABORATORIES, Rifling and Rotating Band Design (Preliminary) (U), Volume I, WAL 760/410 (C), 30 April 1951.

$$N r_m [\cos \theta_r - \mu \sin \theta_r] \int_0^h \int_0^{\xi_{\max}} S_1 d\xi du - 2\mu \pi r_m^2 \xi_{\max} P_b \cos \theta_p = I \frac{d^2 \phi_p}{dt^2} \quad \dots (2)$$

where

$$\xi_{\max} = x_p \quad x_p < l_b$$

$$\xi_{\max} = b \quad x_p > l_b$$

$$\tan \theta_p = \frac{dr \phi_p}{dx} \quad \dots (3)$$

$$S_1 = K f(\delta_1) \quad \dots (4)$$

where f denotes some function and $K f(\delta_1)$ must be known or experimentally determined.

$$\delta_1 = r_m [\{\phi_r(x_p - \xi) - \phi_r(u \cot \alpha)\} - \{\phi_p(x_p) - \phi_p(\xi + u \cot \alpha)\}] \quad \dots (5)$$

The pressures P_g and P_b must be known or experimentally determined.

Variable Twist Rifling

$$\begin{aligned} P_g A_b - P_g h \int_{x_1}^{x_p - \xi_{\max}} \sin \theta_r d\xi - N \int_0^h \int_{x_1}^{x_p} S_1 [\sin \theta_r + \mu \cos \theta_r] d\xi du \\ - N \int_0^h \int_{x_p - \xi}^{x_1} S_2 [\sin \theta_r - \mu \cos \theta_r] d\xi du \\ - 2\mu \pi r_m P_b \xi_{\max} \cos \theta_p = M \frac{d^2 x_p}{dt^2} \quad \dots (6) \end{aligned}$$

$$N r_m \int_0^h \int_{x_1}^{x_p} S_1 [\cos \theta_r - \mu \sin \theta_r] d\xi du - N r_m \int_0^h \int_{x_p - \xi}^{x_1} S_2 [\cos \theta_r + \mu \sin \theta_r]$$

$$\begin{aligned} d\xi du - P_g h r_m \int_{x_1}^{x_p - \xi_{\max}} \cos \theta_r d\xi - 2\mu \pi r_m^2 P_b \xi_{\max} \sin \theta_p \\ = I \frac{d^2 \phi_p}{dt^2} \quad \dots (7) \end{aligned}$$

$$\tan \theta_r = \frac{dr_r}{dx} = bn(x_p - \xi)^{n-1} \quad \dots (8)$$

$$\tan \theta_p = \frac{dr_p}{dx} = bnx_1^{n-1} \quad \dots (9)$$

$$\frac{d^2 \phi_p}{dt^2} = \frac{bn(n-1)x_1^{n-2}}{r_m} \frac{dx_1}{dt}^2 + \frac{bnx_1^{n-1}}{r_m} \frac{d^2 x_1}{dt^2} \quad \dots (10)$$

$$\delta_1 = r_m[\{\phi_r(x_p - \xi) - \phi_r(u \cot \alpha)\} - \{\phi_p(x_p) - \phi_p(\xi + u \cot \alpha)\}] \quad (11)$$

for $x_p > \xi + y \cot \alpha$ and $\xi < x_p - x_1$.

$$\delta_2 = r_m[\{\phi_r(x_p - \xi) - \phi_r(u \cot \alpha)\} - \{\phi_p(x_p) - \phi_p(\xi + u \cot \alpha)\}] \quad \dots (12)$$

for $x_p > \xi + y \cot \alpha$ and $\xi > x_p - x_1$.

$$S_1 = Kf_1(\delta_1) \quad \dots (13)$$

$$\text{where } f \text{ denotes some function and } S_2 = Kf_2(\delta_2). \quad \dots (14)$$

$Kf(\delta)$ must be known or experimentally determined. The pressures P_g and P_b must be known or experimentally determined.

A general solution of the above equations 1 to 5 and 6 to 14 does not appear feasible, however, solutions to specific problems can be obtained by numerical methods. Fortunately the equations for constant twist rifling can be simplified in the region of rifling beyond the forcing cone. In this region equations 1 to 5 yield a solution for the average driving edge pressure which can be expressed as follows:

$$S_{ave} = \frac{P_g A_b \tan \theta_r \sec \theta_r + 2\mu r_m \ell_b P_b \left(\frac{r^2}{\rho^2} - 1 \right) \tan \theta_r}{Nh \ell_b \left\{ \frac{r^2}{\rho^2} (1 - \mu \tan \theta_r) + \tan \theta_r (\mu + \tan \theta_r) \right\}}$$

It should be noted, however, that this solution yields only the average driving edge pressure. If the maximum driving edge pressure is desired equations 1 to 5 must be solved.

APPENDIX A

DRIVING EDGE PRESSURE

A general set of equations describing the motion of a projectile in a gun tube of any rifling geometry can be obtained by equating all forces and torques acting on the projectile to zero. The driving edge pressure can then be obtained as a function of the driving edge force and the driving edge area, provided the pressure distribution is known. Differences in loading conditions between constant twist rifling and variable twist rifling, however, make it convenient to analyze each separately. In the analysis that follows, these two types of rifling will be treated separately.

Constant Twist Rifling

The equations of motion of a projectile for constant twist rifling can be derived from the free body diagram of a projectile, Figure A-1, by use of D'Lambert's principle. These equations express the driving edge force on a projectile as a function of all other forces acting on the projectile. Before proceeding with the actual derivation however, it will be advantageous to consider the deformation of the driving edge in the initial stages of engraving, i.e., in the forcing cone, since certain assumptions concerning this deformation are involved in the final result.

A section of the unrolled groove of a gun tube of constant twist rifling is shown by the lines AB and CD, Figure A-2a. The origin of the rifling is along the line BC and the driving edge of the rifling is represented by the line CD. As a band is engraved the lands and grooves of the band are formed by the gun tube rifling which acts as a die. The band land as formed in the initial stages of engraving is shown in Figure A-2a by the outline BCEF. At the start of engraving the motion of the band is along the axis of the gun tube, i.e., the x axis in Figure A-2a, and no rotation occurs. The angle θ_p , Figure A-2b, at the forward end of the band is thus equal to zero. It should be observed from Figure A-2a that interference has occurred between the band land BCEF and the gun tube rifling ABCD. This interference is shown by the shaded area in Figure A-2a. This interference actually represents driving edge deformation. The displacement of the driving edge S, Figure A-2a is accompanied by pressures normal to the driving edge. This pressure causes the projectile to rotate. Consequently the angle θ_p changes due to this rotation. Eventually the angle θ_p approaches the angle θ_r as shown in Figure A-2b. Thereafter the band can be considered to engrave at the angle $\theta_p = \theta_r$. It should be borne in mind that the subsequent increase in driving edge displacement after engraving is completed is uniformly distributed over the length of the band and consequently the point of maximum deformation and thus maximum stress is always at the forward end of the band. This is illustrated in Figure A-2c where the band land as formed is shown at a subsequent travel position. In Figure A-2c the shaded area represents driving edge deformation.

The equation of motion of a projectile for constant twist rifling will now be derived by considering the force and torque equilibrium of the projectile shown in Figure A-1.

$$\Sigma F_x = 0$$

$$P_g A_b - N[\sin \theta_r + \mu \cos \theta_r] \int_0^h \int_0^{\xi_{\max}} S_1 d\xi du - 2\mu\pi r_m \xi_{\max} P_b \cos \theta_p = M \frac{d^2 x_p}{dt^2} \quad \dots (A1)$$

$$N r_m [\cos \theta_r - \mu \sin \theta_r] \int_0^h \int_0^{\xi_{\max}} S_1 d\xi du - 2\mu\pi r_m^2 \xi_{\max} P_b \cos \theta_p = I \frac{d^2 \phi_p}{dt^2} \quad \dots (A2)$$

where

$$\xi_{\max} = x_p \quad x_p < l_b$$

$$\xi_{\max} = l_b \quad x_p > l_b$$

To complete the above set of equations other relations between the variables must be known or experimentally determined. The necessary relations together with some explanatory notes are indicated below. The derivative of the curve $r\phi_p(x_p)$ with respect to x_p represents the tangent of the angle $\theta_p(x_p)$.

$$\tan \theta_p(x_p) = \frac{d r\phi_p(x_p)}{dx_p} \text{ see Figure 3. } \dots (A3)$$

The driving edge pressure S_1 can be expressed as a function of the driving edge displacement as follows:

$$S_1 = Kf(\delta_1) \quad \dots (A4)$$

where $Kf(\delta)$ must be determined experimentally.

The driving edge displacement δ_1 can be expressed as a function of the rotations $r\dot{m}\theta_r$ and $r\dot{m}\theta_p$. As shown in Figure A-3a, the projectile has traveled the distance $x_p = \xi$. The point $P(\xi, u)$ will not engage the rifling until the projectile has moved to the position $x_p = \xi + u \cot \alpha$

shown in Figure A-3b. The rotation of the projectile, Figure A-3c, at position $x_p = \xi + u \cot \alpha$ is $\phi(\xi + u \cot \alpha)$. The rotation of the rifling, Figure A-3c, at the initial point of contact of $P(\xi, u)$ is $\phi(u \cot \alpha)$. These rotations represent the initial position of the rifling and projectile for the point $P(\xi, u)$ and must be subtracted from the total rotations at subsequent travel positions. The rotation of the projectile at a subsequent travel position is $\phi_p(x_p)$. The rotation of the rifling at the point $P(\xi, u)$ is $\phi_r(x_p - \xi)$. The driving edge displacement at travel position x_p is then

$$\delta_l = r_m[\{\phi_r(x_p - \xi) - \phi_r(u \cot \alpha)\} - \{\phi_p(x_p) - \phi_p(\xi + u \cot \alpha)\}] \quad \dots (A5)$$

for $x_p > \xi + u \cot \alpha$. $\delta = 0$ $x_p < \xi + u \cot \alpha$. P_g and P_b must be known or experimentally determined.

A general solution of the above set of equations in closed form does not appear feasible, but solutions to specific problems are possible by numerical methods.

In the region of rifling beyond the forcing cone the above set of equations can be reduced and solved for the average driving edge pressure. In this region the following conditions will prevail:

$$\theta_p \approx \theta_r = \text{constant}$$

$$h = \text{constant}$$

$$P_b \approx \text{constant}$$

$$\frac{d^2 \phi_p}{dt^2} \approx \frac{d^2 \phi_r}{dt^2} = \frac{\tan \theta_r}{r} \frac{d^2 x_p}{dt^2}$$

$$\xi_{\max} = l_b$$

With the above simplifications, equations A1 and A2 reduce to

$$P_g A_b - N h [\sin \theta_r + \mu \cos \theta_r] \int_0^{l_b} S_1 d\xi - 2\mu \pi r_m l_b P_b \cos \theta_r = M \frac{d^2 x_p}{dt^2} \quad \dots (A6)$$

$$\text{and} \quad N h r [\cos \theta_r - \mu \sin \theta_r] \int_0^{l_b} S_1 d\xi - 2\mu \pi r_m^2 l_b P_b \sin \theta_r = I \frac{\tan \theta_r}{r} \frac{d^2 x_p}{dt^2} \quad (A7)$$

The driving edge force can thus be expressed as

$$h \int_0^{l_b} S_1 d\xi = \frac{P_g A_b \tan \theta_r \sec \theta_r + 2\mu\pi r_m l_b P_b \left(\frac{r^2}{\rho^2} - 1 \right) \tan \theta_r}{N \left\{ \frac{r^2}{\rho^2} (1 - \mu \tan \theta_r) + \tan \theta_r (\mu + \tan \theta_r) \right\}} \quad \dots (A8)$$

If it is now assumed that the driving edge pressure is uniformly distributed, the results of equation A8 will yield a conservative estimate of the load-carrying ability of the band. The pressure involved in this assumption is the average driving edge pressure and should not be interpreted as the actual stress on the band which could be much greater. The average driving edge pressure is given by the following formula:

$$S_{ave} = \frac{P_g(t) A_b \tan \theta_r \sec \theta_r + 2\mu\pi r_m l_b P_b \left(\frac{r^2}{\rho^2} - 1 \right) \tan \theta_r}{N h l_b \left\{ \frac{r^2}{\rho^2} (1 - \mu \tan \theta_r) + \tan \theta_r (\mu + \tan \theta_r) \right\}} \quad \dots (A9)$$

Variable Twist Rifling

The equations of motion of a projectile for variable twist rifling can be derived from the free body diagram of a projectile, Figure A-4. Some of the pressures acting on the projectile and band, Figure A-4, such as the gas pressure P_g and the band pressure P_b , are readily apparent and need no further explanation. Other pressures such as S_1 and S_2 which originate in part due to the squeezing action of a continuously changing rifling geometry are not apparent and may require further explanation. Consequently the origin of the pressures S_1 and S_2 will be considered in detail first, and subsequently a set of equations involving the driving edge pressure will be derived.

A section of the unrolled groove of a gun tube of variable twist rifling is shown by the curves AB and CD, Figure A-5. The origin of the rifling is along the line BC and the driving edge of the rifling is represented by the curve CD. As the band is engraved, the lands and grooves of the band are formed by the gun tube rifling which acts as a die. The band land as formed in the initial stages of engraving is shown in Figure A-5a by the outline EFGH. At the start of engraving the motion of the band is along the axis of the gun tube, the x axis in Figure A-5a, and no rotation occurs. The angle θ_p is thus zero at $x_p = 0$. This is shown in Figure A-5b on the trailing edge of the band at $\xi = 0$. It should be observed from Figure A-5a that interference has occurred between the band land as originally formed and the gun tube rifling. This interference is shown by the shaded area in Figure A-5a. The interference actually represents driving edge deformation. The displacement of the driving edge δ_1 is accompanied by pressures normal to the driving edge. These pressures cause the projectile to rotate. Consequently, the angle

θ_p changes due to this rotation. A band land as originally formed is shown at a subsequent travel position in Figure A-5b. It will be observed that interference now occurs on both sides of the band land. The shaded areas of Figure A-5b represent this interference. The interference on the trailing edge is primarily due to the difference between the rifling geometry at x_p and the band land geometry as originally formed. As the band travels through the gun tube, the rifling angle θ_r , Figure A-5c, is continually changing. This continual change in θ_r causes further deformation of the band land since it must conform to the rifling geometry. In addition to the deformation due to a change of rifling geometry there is also a deformation due to applied torque. The total deformation due to the combined effects of torque and a change in rifling geometry is illustrated in Figure A-5c. In Figure A-5c a band land as originally formed is shown at two positions of travel, first at the fully engraved position A and secondly at any subsequent travel position B. The shaded areas of Figure A-5c represent band land deformation. These deformations are the origin of the stresses S_1 and S_2 of Figure A-4.

The equations of motion of a projectile for variable twist rifling will now be developed by considering the force and torque equilibrium of the projectile shown in Figure A-4.

$$\Sigma F_x = 0$$

$$\begin{aligned} & P_g A_b - P_g h \int_{x_1}^{x_p - \xi_{\max}} \sin \theta_r d\xi \\ & - N \int_0^h \int_{x_1}^{x_p} S_1 [\sin \theta_r + \mu \cos \theta_r] d\xi du \\ & - N \int_0^h \int_{x_p - \xi}^{x_1} S_2 [\sin \theta_r - \mu \cos \theta_r] d\xi du \\ & - 2 \mu \pi r_m P_b \xi_{\max} \cos \theta_p = M \frac{d^2 x_p}{dt^2} \end{aligned} \quad \dots (A10)$$

$$\Sigma T_x = 0$$

$$\begin{aligned} & N r_m \int_0^h \int_{x_1}^{x_p} S_1 [\cos \theta_r - \mu \sin \theta_r] d\xi du \\ & - N r_m \int_0^h \int_{x_p - \xi}^{x_1} S_2 [\cos \theta_r + \mu \sin \theta_r] d\xi du \end{aligned}$$

$$- P_g h r_m \int_{x_1}^{x_p - \xi_{\max}} \cos \theta_r d\xi$$

$$- 2\mu\pi r_m^2 P_b \xi_{\max} \sin \theta_p = I \frac{d^2 \phi_p}{dt^2} \quad \dots (A11)$$

To complete the above set of equations other relations between the variables must be known or experimentally determined. The necessary relations together with some explanatory notes are indicated below. The derivative of the space curve $r\phi_r(x_p)$ with respect to x represents the tangent of the angle θ_r at the point $x_p - \xi$ as shown in Figure A-6.

$$\tan \theta_r = \frac{dr\phi_r}{dx} = bn(x_p - \xi)^{n-1} \quad \dots (A12)$$

The distance from the origin of rifling to the point on the band land at which the driving edge displacement is zero is defined as the distance x_1 . This point has no relative rotation with respect to either the rifling or the band land.

Thus

$$r_m\phi_p(x_1) = r_m\phi_r(x_1)$$

and

$$\tan \theta_p(x_1) = \frac{dr\phi_r(x_1)}{dx_1} = bnx_1^{n-1} \quad \dots (A13)$$

Thus

$$\frac{d^2 \phi_p}{dt^2} = \frac{bn(n-1)x_1^{n-2}}{r_m} \left(\frac{dx_1}{dt} \right)^2 + \frac{bnx_1^{n-1}}{r_m} \frac{d^2 x_1}{dt^2} \quad \dots (A14)$$

The driving edge displacement δ can be expressed as a function of the rotations $r_m\phi_r$ and $r_m\phi_p$ as explained for constant twist rifling. Thus,

$$\delta_1 = r_m[\{\phi_r(x_p - \xi) - \phi_r(u \cot \alpha)\} - \{\phi_p(x_p) - \phi_r(\xi + u \cot \alpha)\}] \quad (A15)$$

$$\text{for } x_p > \xi + u \cot \alpha \text{ and } \xi < x_p - x_1$$

$$\delta_2 = r_m[\{\phi_r(x_p - \xi) - \phi_p(u \cot \alpha)\} - \{\phi_p(x_p) - \phi_p(\xi + u \cot \alpha)\}] \quad .(A16)$$

for $x_p > \xi + u \cot \alpha$ and $\xi > x_p - x_1$

The pressure on the driving and trailing edges, δ_1 and δ_2 , can be expressed as a function of the driving edge displacement as follows:

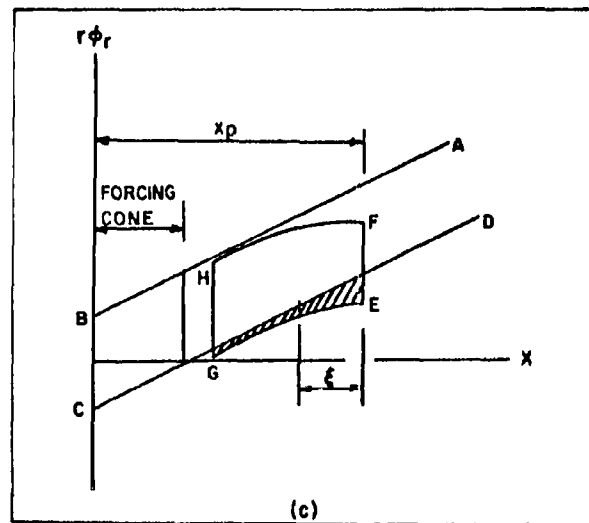
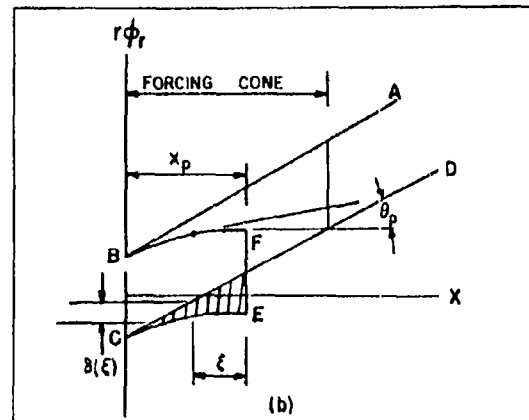
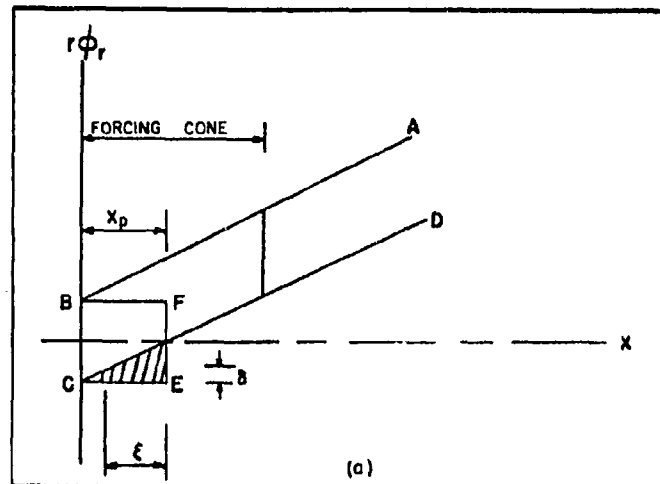
$$S_1 = Kf_1(\delta_1) \quad(A17)$$

$$S_2 = Kf_2(\delta_2) \quad(A18)$$

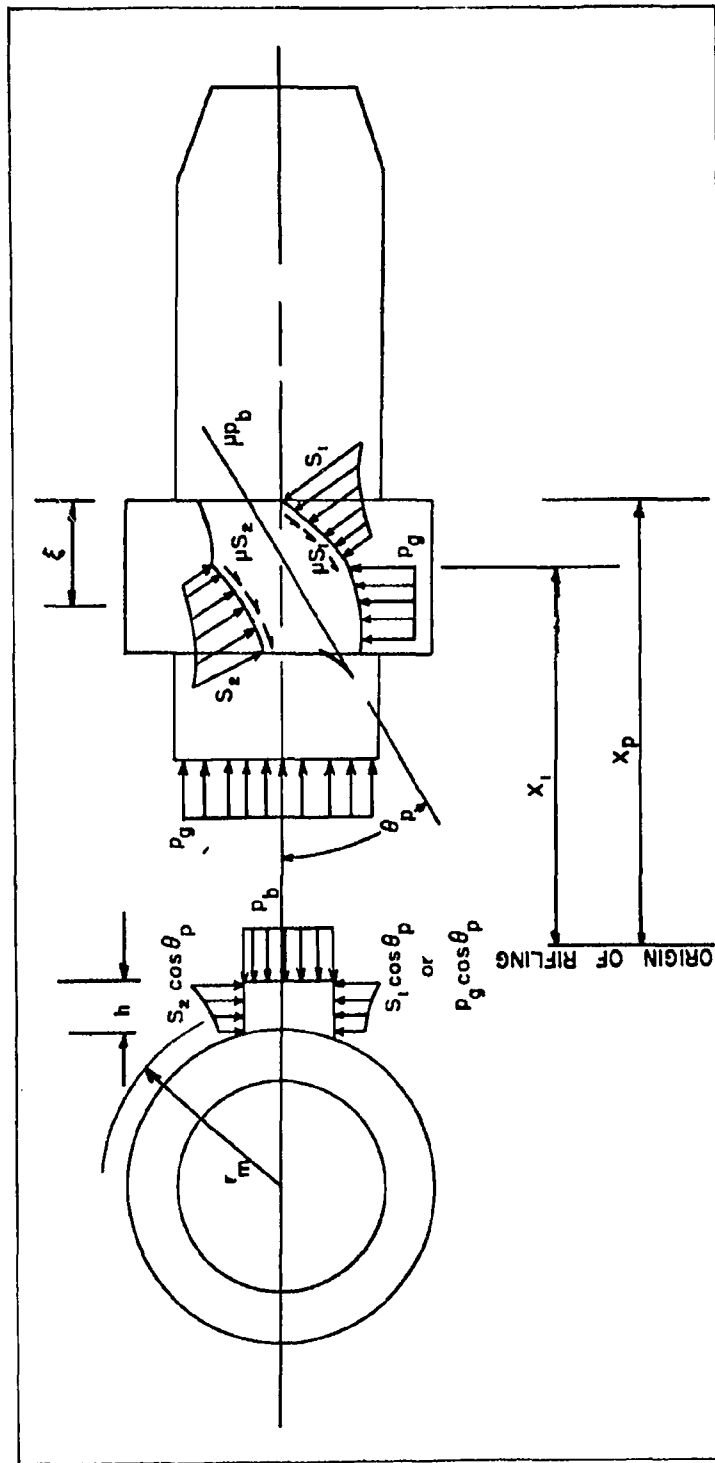
where $Kf(\delta)$ must be determined experimentally.

The gas pressure P_g and the radial band pressure P_b must be known or experimentally determined.

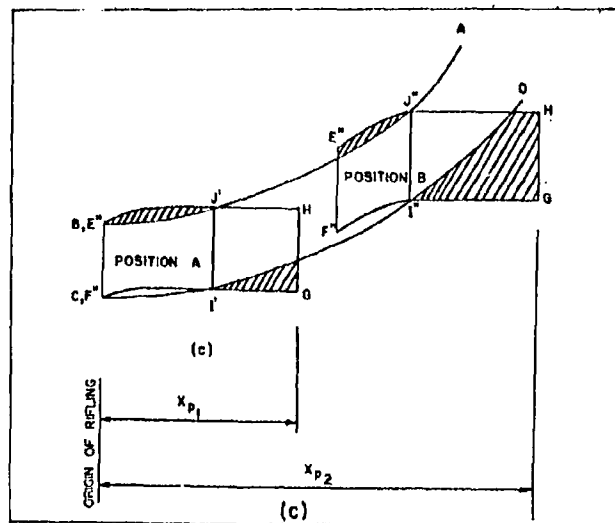
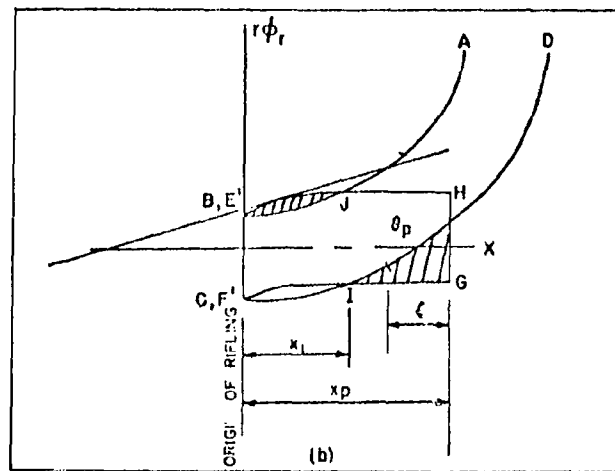
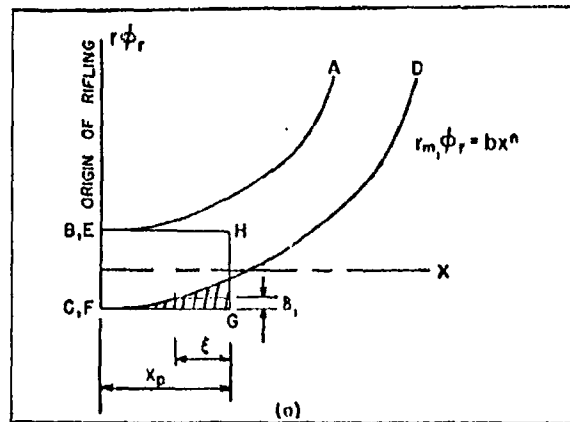
A general solution of the above set of equations in closed form does not appear feasible but solutions to specific problems are possible by numerical methods.



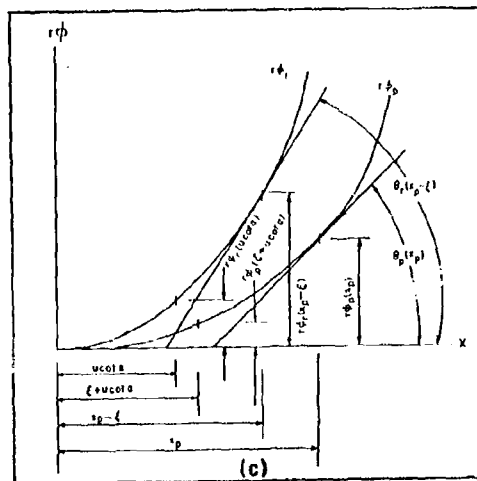
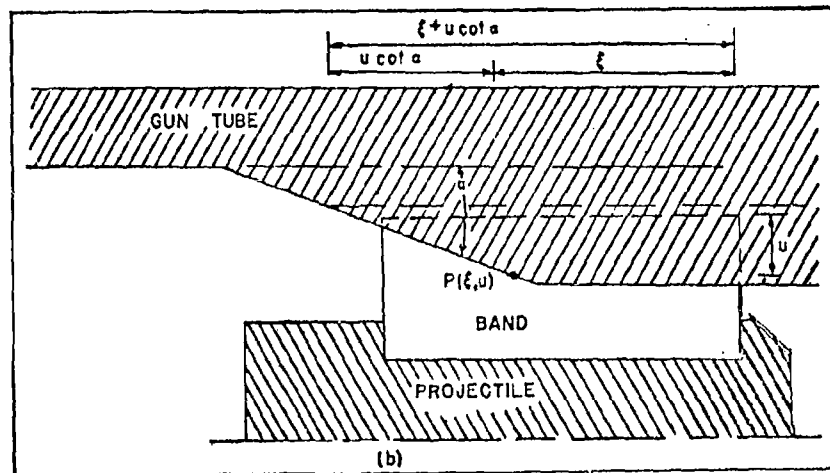
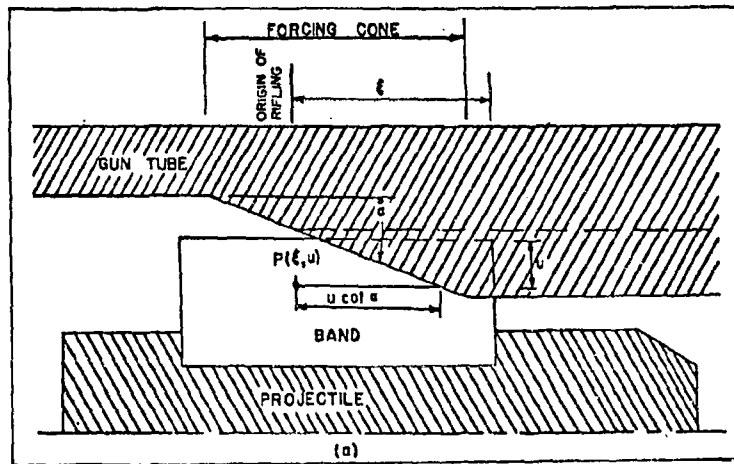
DRIVING EDGE DEFORMATION CONSTANT TWIST RIFLING



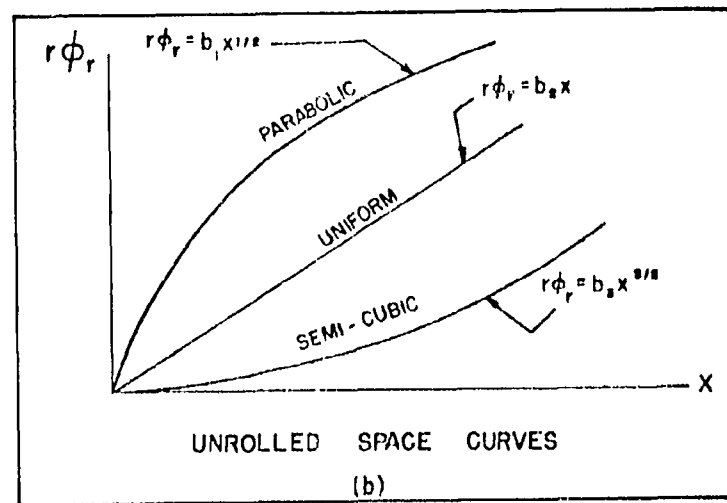
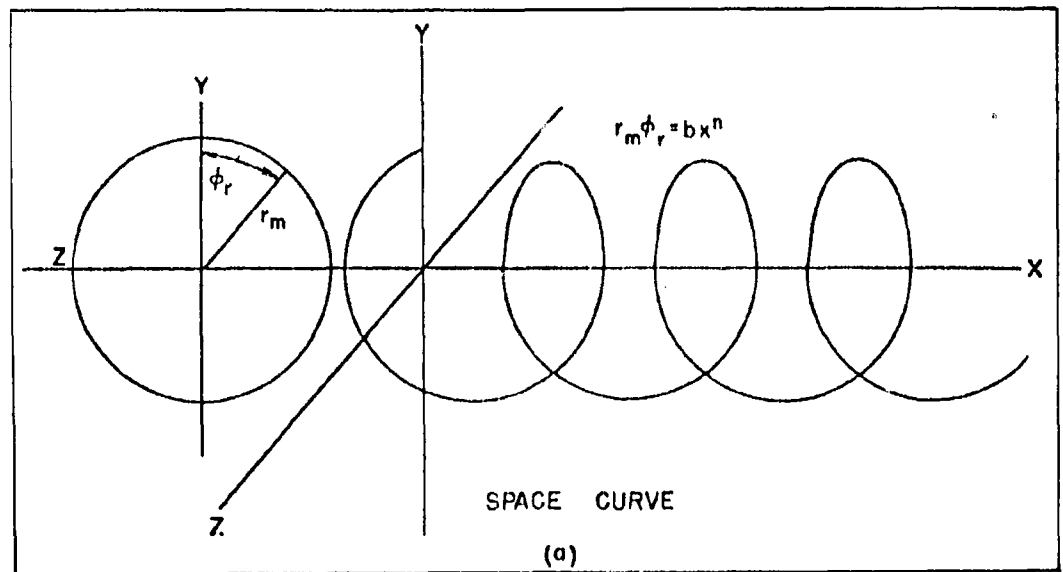
FREE BODY DIAGRAM FOR VARIABLE TWIST



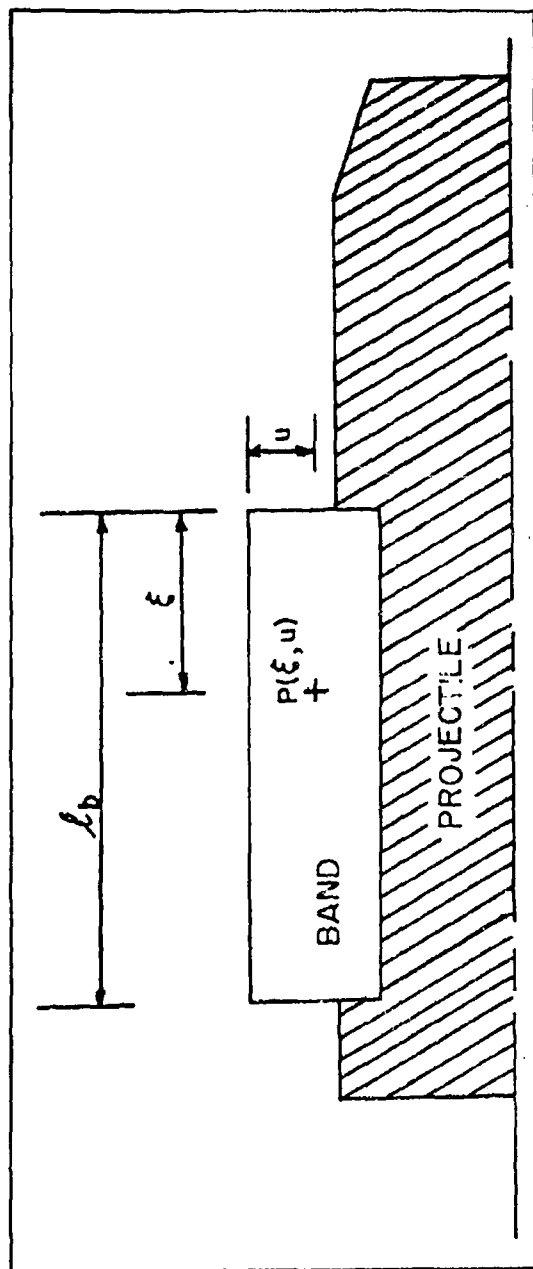
DRIVING EDGE AND TRAILING EDGE DEFORMATION VARIABLE TWIST RIFLING



DRIVING EDGE DISPLACEMENT VARIABLE TWIST RIFLING



RIFLING CURVES



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January 1962

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An analysis of the pressure exerted on the driving edge of a rotating band during the firing of a projectile is presented. Separate analyses are presented for uniform and variable twist rifling and equations are developed which relate the driving edge force, i.e., the integral of the driving edge pressure, to the other forces acting on the band. Solution of the equations developed must in general be accomplished by numerical methods. Simplification of these equations for the case of uniform twist rifling in the region of rifling beyond the forcing cone permits direct determination of the average driving edge pressure in that region.

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