



Division of Engineering
BROWN UNIVERSITY
PROVIDENCE, R. I.

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EQUIVALENT CIRCUIT PARAMETER FOR AN
INHOMOGENEOUS BIFURCATED WAVEGUIDE
BY

H. M. CRONSON

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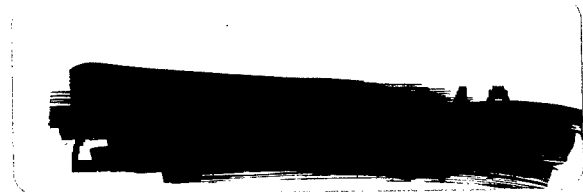
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ABSTRACT

In this report an approximate solution for an equivalent circuit representation of a two dimensional waveguide junction is obtained over several ranges of significant parameters. The junction is formed by a semi-infinite, dielectric filled, parallel plate guide contained symmetrically within an infinite parallel plate guide. The structure is first reduced to a half-space problem and formulated in terms of the appropriate field modes in each of the parallel plate guides forming the junction. These modes are matched at the plane of discontinuity and the resulting equations rearranged to resemble ordinary circuit equations. The circuit elements of the equivalent circuit representation are identified from the equations. The determination of the values of these elements requires the solution of an infinite set of inhomogeneous algebraic equations. Solutions are obtained by approximate methods with the aid of an IBM 7070 computer. The approximate solutions are checked by comparing the results obtained for the degenerate case, in which the dielectric vanishes, with known exact results. This check suggests that the approximate results for the case of non-vanishing dielectric should be within a few per cent of the correct values.

I. Introduction

This report concerns the determination of an equivalent circuit representation for a waveguide junction formed by a semi-infinite, dielectric filled, parallel plate guide contained symmetrically within an infinite parallel plate guide (see Fig. 1). We will assume that the dimensions of the guide are such that only the T.E.M. mode will propagate in each guide at the excitation frequency, and that the exciting source is so polarized that there will be no field variations in the x-direction.

The literature on waveguide discontinuities discloses the closely related problem of a homogeneous E-plane bifurcation, which can be solved exactly using integral transform methods.¹ However a preliminary investigation of these transform procedures applied to the problem considered in this report revealed that the inhomogeneity introduced by the dielectric in guide B made these methods impractical to apply. Another approach, better suited to the type of discontinuity considered here, is an approximate method originally due to Whinnery and Jamieson.² Their approach is based on mode matching at the discontinuity. It has the advantage of being relatively straightforward, inherently providing a clear physical picture of the propagating and non-propagating waves in the guide. Moreover, it is particularly adaptable to the two dimensional equivalent

¹N. Marcuvitz, "Waveguide Handbook", M. I. T. Radiation Laboratory Series, Vol. 10 (McGraw-Hill Book Company, Inc., 1951), pp. 160-167.

²J. R. Whinnery and H. W. Jamieson, "Equivalent Circuits for Discontinuities in Transmission Lines", Proc. I.R.E., 32, 98-114, Feb., 1944.

circuit formulation of the problem.

In the structure examined here, the method consists of finding the evanescent mode amplitudes excited in the guide of height $2a$. After considerable manipulation of the field equations at the plane $Z = 0$, we obtain an expression for the equivalent capacitances of the junction in terms of these non-propagating mode amplitudes which, in turn, are defined by an infinite set of non-homogeneous equations. An approximate solution to these equations over a convenient range of parameters is obtained by machine calculation using an IBM 7070 computer. The procedure is checked by comparing the results for the degenerate case, in which $\epsilon_b = \epsilon_0$, with the values given by Marcuvitz.¹

II. Field Equations

A. Reduction to a Half-Space Problem.

The given waveguide configuration can be simplified by the following considerations.

(1) Due to the method of excitation and the nature of the discontinuity, there will be no H-modes in the guide.

(2) The symmetry of the structure allows us to place an infinite perfectly conducting plane at the plane $y = 0$ without disturbing the fields.

Thus instead of solving the original problem, it is sufficient to solve the half-space problem shown in Fig. 2. (See Appendix of reference 3 for the justification of these statements.)

B. Field Representation in the Half-Space Guide.

The field in each of the guides shown in Fig. 2 can be represented by an incident and reflected T.E.M. mode plus an infinite sum of evanescent E modes. Using harmonic time dependence, $e^{j\omega t}$, and assuming T.E.M. sources at frequency in guides B and C at $Z = -\infty$ and in guide A at $Z = \infty$, the y component of the electric field in guide B is given by:

$$E_{yB}(y, z) = B_0' e^{-jk_B z} + B_0'' e^{jk_B z} + \sum_{n=1}^{\infty} B_n e^{\gamma_n^B z} \cos \frac{n\pi y}{b} \quad (1)$$

for $0 \leq y < b$ and $z \leq 0$

where $k_B = \omega \sqrt{\mu_0 \epsilon_B}$ and $\gamma_n^B = \sqrt{\left(\frac{n\pi}{b}\right)^2 - k_B^2}$ are always real.

³H. M. Cronson, "Equivalent Circuit Parameters for an Inhomogeneous Bifurcated Waveguide", Master's thesis, Brown University, (1961).

The x component of magnetic field in guide B can be obtained from Maxwell's Equation $\nabla \times \underline{H}_B = j\omega \epsilon_B \underline{E}_B$, using the fact that since only E modes are present, $H_y = H_z = 0$. This yields

$$H_{xB}(y,z) = -\sqrt{\frac{\epsilon_B}{\mu_0}} \left[B_o^i e^{-jk_B z} - B_o^r e^{jk_B z} \right] + j\omega \epsilon_B \sum_{m=1}^{\infty} \frac{B_m}{\gamma_m^B} e^{\gamma_m^B z} \cos \frac{m\pi}{b} y \quad (2)$$

for $0 \leq y < b$ and $z \leq 0$

Similarly, for the fields in Guide A we can write

$$E_{yA}(y,z) = A_o^i e^{jkz} + A_o^r e^{-jkz} + \sum_{m=1}^{\infty} A_m e^{-\gamma_m^A z} \cos \frac{m\pi}{a} y \quad (3)$$

$$H_{xA}(y,z) = \sqrt{\frac{\epsilon_0}{\mu_0}} \left[A_o^i e^{jkz} - A_o^r e^{-jkz} \right] - j\omega \epsilon_0 \sum_{m=1}^{\infty} \frac{A_m}{\gamma_m^A} e^{-\gamma_m^A z} \cos \frac{m\pi}{a} y \quad (4)$$

for $0 \leq y \leq a$ and $z > 0$

where $k = \omega \sqrt{\mu_0 \epsilon_0}$ and $\gamma_m^A = \sqrt{\left(\frac{m\pi}{a}\right)^2 - k^2}$

Also, in Guide C, letting $y' = y - b$,

$$E_{yC}(y',z) = C_o^i e^{-jkz} + C_o^r e^{jkz} + \sum_{g=1}^{\infty} C_g e^{\gamma_g^C z} \cos \frac{g\pi}{c} y' \quad (5)$$

$$H_{xC}(y',z) = -\sqrt{\frac{\epsilon_0}{\mu_0}} \left[C_o^i e^{-jkz} - C_o^r e^{jkz} \right] + j\omega \epsilon_0 \sum_{g=1}^{\infty} \frac{C_g}{\gamma_g^C} e^{\gamma_g^C z} \cos \frac{g\pi}{c} y' \quad (6)$$

for $0 < y' \leq C$ and $z \leq 0$

where $\gamma_g^E = \sqrt{\left(\frac{g\pi}{C}\right)^2 - k^2}$

For subsequent calculations it is convenient to define the following quantities,

$$\begin{aligned} A_0 &= A_0^i + A_0^r & A_0 Y_{A_0} &= \sqrt{\frac{\epsilon_0}{\mu_0}} [A_0^i - A_0^r] \\ B_0 &= B_0^i + B_0^r & B_0 Y_{B_0} &= -\sqrt{\frac{\epsilon_0}{\mu_0}} [B_0^i - B_0^r] \\ C_0 &= C_0^i + C_0^r & C_0 Y_{C_0} &= -\sqrt{\frac{\epsilon_0}{\mu_0}} [C_0^i - C_0^r] \end{aligned} \quad (6a)$$

$$Y_{A_m} = -\frac{j\omega\epsilon_0}{\gamma_m^A} \quad m = 1, 2, 3, \dots \quad (7)$$

$$Y_{B_n} = -\frac{j\omega\epsilon_B}{\gamma_n^B} \quad n = 1, 2, 3, \dots \quad (8)$$

$$Y_{C_g} = \frac{j\omega\epsilon_0}{\gamma_g^C} \quad g = 1, 2, 3, \dots \quad (9)$$

Hence equations (1) through (6) can be written, at $z = 0$, as

$$E_{yB}(y, 0) = B_0 + \sum_{n=1}^{\infty} B_n \cos \frac{n\pi}{b} y \quad (10)$$

$$H_{xB}(y, 0) = B_0 Y_{B_0} + \sum_{n=1}^{\infty} Y_{B_n} B_n \cos \frac{n\pi}{b} y \quad (11)$$

$$E_{yA}(y,0) = A_0 + \sum_{m=1}^{\infty} A_m \cos \frac{m\pi}{a} y \quad (12)$$

$$H_{xA}(y,0) = A_0 Y_{A_0} + \sum_{m=1}^{\infty} Y_{A_m} A_m \cos \frac{m\pi}{a} y \quad (13)$$

$$E_{yC}(y',0) = C_0 + \sum_{q=1}^{\infty} C_q \cos \frac{q\pi}{c} y' \quad (14)$$

$$H_{xC}(y',0) = C_0 Y_{C_0} + \sum_{q=1}^{\infty} Y_{C_q} C_q \cos \frac{q\pi}{c} y' \quad (15)$$

We now employ the condition of continuity of the tangential E and H fields at $Z = 0$ to give,

$$B_0 + \sum_{n=1}^{\infty} B_n \cos \frac{n\pi}{b} y = A_0 + \sum_{m=1}^{\infty} A_m \cos \frac{m\pi}{a} y \quad 0 \leq y < b \quad (16)$$

$$C_0 + \sum_{q=1}^{\infty} C_q \cos \frac{q\pi}{c} y' = A_0 + \sum_{m=1}^{\infty} A_m \cos \frac{m\pi}{a} y \quad \begin{matrix} b < y \leq a \\ 0 < y' \leq c \end{matrix} \quad (17)$$

$$Y_{B_0} B_0 + \sum_{n=1}^{\infty} Y_{B_n} B_n \cos \frac{n\pi}{b} y = Y_{A_0} A_0 + \sum_{m=1}^{\infty} Y_{A_m} A_m \cos \frac{m\pi}{a} y \quad 0 \leq y < b \quad (18)$$

$$Y_{C_0} C_0 + \sum_{q=1}^{\infty} Y_{C_q} C_q \cos \frac{q\pi}{c} y' = Y_{A_0} A_0 + \sum_{m=1}^{\infty} Y_{A_m} A_m \cos \frac{m\pi}{a} y \quad \begin{matrix} b < y \leq a \\ 0 < y' \leq c \end{matrix} \quad (19)$$

The above relationships between the A_m , B_n , and C_q , will be utilized later to provide an equivalent circuit representation for the waveguide junction.

III. Equivalent Circuit Representation

A. The Transmission Line Analogy of a Parallel Plate Waveguide.

Consider an infinite parallel plate waveguide of height d in which the T.E.M. mode is propagating in the positive Z direction (see Fig. 3). The T.E.M. wave may be written for harmonic time dependence $e^{j\omega t}$ as

$$E_y = D e^{-jkz} \quad H_x = -\sqrt{\frac{\epsilon}{\mu}} D e^{-jkz}$$

where D is a constant determined by the strength of the source.

Maxwell's equations within the waveguide may be written in the form

$$\nabla \times \underline{E} = -j\omega\mu \underline{H} \quad \nabla \times \underline{H} = j\omega\epsilon \underline{E}$$

or as

$$\frac{\partial E_y}{\partial z} = j\omega\mu H_x \quad (20)$$

$$\frac{\partial H_x}{\partial z} = j\omega\epsilon E_y \quad (21)$$

We want to define a "voltage" depending on E_y and a "current" depending on H_x so that equations (20) and (21) can be viewed as transmission line equations. We define

$$V(z) = -\int_0^d E_y dy \quad (22)$$

For the parallel plate guide

$$V(z) = -E_y d \quad (23)$$

We also define

$$I(z, d) = w \underline{\ell} \cdot \hat{z}_0 \quad (24)$$

where $\underline{\ell}$ is the current per unit length on the surface of the top plate ($y = d$), w is a length in the x direction, and $\hat{z}_0$ is the unit vector in the z direction, (see Fig. 3). On the surface of a perfect conductor $\hat{n} \times \underline{H} = \underline{\ell}$ where for this case $\hat{n}$ is the normal pointing into the guide. Similarly

$$I(z, 0) = w \underline{\ell} \cdot \hat{z}_0$$

We note

$$\underline{\ell} \Big|_{y=d} = (-\hat{y}_0) \times H_x \hat{x}_0 = H_x \hat{z}_0$$

$$\underline{\ell} \Big|_{y=0} = (\hat{y}_0) \times H_x \hat{x}_0 = -H_x \hat{z}_0$$

Therefore

$$I(z, d) = w H_x$$

$$I(z, 0) = -w H_x$$

We further define

$$I(z) = I(z, d) = w H_x = -I(z, 0) \quad (25)$$

Substitution of equations (23) and (25) into (20) and (21) yields

$$\frac{dV(z)}{dz} = -j\omega\mu \frac{d}{w} I(z) \quad (26)$$

$$\frac{dI(z)}{dz} = -j\omega\epsilon \frac{w}{d} V(z) \quad (27)$$

The appropriate transmission line equations are given by Marcuvitz (op. cit.) in the form

$$\frac{dV}{dz} = -j k Z I \quad (28)$$

$$\frac{dI}{dz} = -j k Y V \quad (29)$$

where V and I are the voltage and current respectively in the transmission line, $k = \omega \sqrt{\mu\epsilon}$, and $Z = 1/Y$ is the characteristic impedance. In comparing equations (26) and (27) with (28) and (30), we see that the T.E.M. mode in a parallel plate waveguide can be completely represented by a transmission line with characteristic impedance.

$$Z = \frac{d}{w} \sqrt{\frac{\mu}{\epsilon}} = Z_d \quad (29-a)$$

Note that the power flow in the guide through a cross section $w \times d$ is

$$P = \iint_0^d \int_0^w \frac{1}{2} \operatorname{Re} [\underline{E} \times \underline{H}^*] \cdot \hat{Z}_0 dx dy = -\frac{wd}{2} E_y H_x^* = \frac{1}{2} V(z) I^*(z)$$

which is the expression for the power flow in a transmission line.

B. The Waveguide Junction as an Equivalent Circuit.

Since a parallel plate waveguide can be represented by a transmission line, it is convenient to represent the junction by a lumped network. The junction in Fig. 2 stores energy with no dissipation in a local region close

to the plane $Z = 0$. Thus it is reasonable to represent the junction by a lossless, lumped network. In this section we will show that the field equations at the junction, expressed in terms of our defined voltage and current, correspond to the nodal equations for a certain lumped, lossless, and passive network.

The voltage from 0 to b in guide B at the plane $Z = 0$ is determined using equations (10) and (22):

$$V(0) \Big|_{\text{guide B}} = V_{0B} = - \int_0^b E_{yB}(y, 0) dy = - \int_0^b \left[B_0 + \sum_{n=1}^{\infty} B_n \cos \frac{n\pi y}{b} \right] dy = -B_0 b$$

The voltage from 0 to a in guide A and from 0 to c in guide C at $Z = 0$ are found to be

$$V_{0A} = -A_0 a \qquad V_{0C} = -C_0 c \qquad (29-b)$$

We note that, since the cosine dependent modes integrate to 0, the voltages at $Z = 0$ are due only to the electric field of the dominant T.E.M. mode.

If equation (16) is integrated with respect to y from 0 to b and added to the result obtained when equation (17) is integrated with respect to y from b to c, one obtains

$$B_0 b + C_0 c = A_0 a$$

or in terms of voltages,

$$V_{0B} + V_{0C} = V_{0A} \qquad (30)$$

The defined current on the top plate in guide B at $Z = 0$ can be found from equations (25) and (11) as

$$\left. I(0) \right|_{\text{guide B}} = I_B = w H_{xB}(b, 0) = w \left[B_0 Y_{B0} + \sum_{m=1}^{\infty} Y_{Bm} B_m (-1)^m \right]$$

Similarly, the defined current on the top plate in guide C and guide A at $Z = 0$ can be written as

$$\begin{aligned} I_C &= w \left[C_0 Y_{C0} + \sum_{g=1}^{\infty} Y_{Cg} C_g (-1)^g \right] \\ I_A &= w \left[A_0 Y_{A0} + \sum_{m=1}^{\infty} Y_{Am} A_m (-1)^m \right] \end{aligned}$$

We designate the currents of the dominant mode as

$$I_{0A} = w A_0 Y_{A0} \quad I_{0B} = w B_0 Y_{B0} \quad I_{0C} = w C_0 Y_{C0}$$

Integrating equation (18) with respect to y from 0 to b we obtain

$$Y_{B0} B_0 = Y_{A0} A_0 + \frac{a}{\pi b} \sum_{m=1}^{\infty} \frac{Y_{Am} A_m}{m} \sin m\pi \frac{b}{a}$$

Substituting equation (7) we have

$$Y_{B0} B_0 = Y_{A0} A_0 + \frac{a}{\pi b} \sum_{m=1}^{\infty} \left[-\frac{j\omega\epsilon_0}{\gamma_m^A} \right] \frac{A_m}{m} \sin m\pi \frac{b}{a} \quad (31-a)$$

We can also show that equation (19) can be written

$$Y_{C0} C_0 = Y_{A0} A_0 - \frac{a}{\pi c} \sum_{m=1}^{\infty} \left[-\frac{j\omega\epsilon_0}{\gamma_m^A} \right] \frac{A_m}{m} \sin m\pi \frac{b}{a} \quad (32-a)$$

Suppose the summation term in equations (31-a) and (32-a) is multiplied by $\frac{B_o - C_o}{B_o - C_o}$ and we define

$$F = \frac{\alpha \epsilon_o}{\pi} \sum_{m=1}^{\infty} \frac{A_m \sin m\pi \frac{b}{a}}{[B_o - C_o] \gamma_m^A m} \quad (33)$$

We may then write equations (31-a) and (32-a) as

$$Y_{B_o} B_o = Y_{A_o} A_o - j\omega \frac{F}{b} B_o + j\omega \frac{F}{b} C_o$$

$$Y_{C_o} C_o = Y_{A_o} A_o + j\omega \frac{F}{c} B_o - j\omega \frac{F}{c} C_o$$

Using these equations and the definitions of the currents in terms of the admittances, we obtain

$$I_{oB} = I_{oA} + j\omega \frac{F}{b^2} \omega V_{oB} - j\omega \frac{F}{bc} \omega V_{oC} \quad (31-b)$$

$$I_{oC} = I_{oA} - j\omega \frac{F}{bc} \omega V_{oB} + j\omega \frac{F}{c^2} \omega V_{oC} \quad (32-b)$$

Substituting $V_{oC} = V_{oA} - V_{oB}$ and $V_{oB} = V_{oA} - V_{oC}$ in equations (31-b) and (32-b) and remembering that $a = b + c$, we have

$$I_{oB} = I_{oA} + j\omega \frac{aF}{b^2c} \omega V_{oB} - j\omega \frac{F}{bc} \omega V_{oA}$$

$$I_{oC} = I_{oA} + j\omega \frac{aF}{bc^2} \omega V_{oC} - j\omega \frac{F}{bc} \omega V_{oA}$$

Define

$$\frac{aFw}{b^2c} = C_B \quad (34)$$

$$\frac{aFw}{bc^2} = C_C \quad (35)$$

$$-\frac{Fw}{bc} = C_A \quad (36)$$

Substituting these definitions in the previous equations yields

$$I_{OB} = I_{OA} + j\omega C_B V_{OB} + j\omega C_A V_{OA}$$

$$I_{OC} = I_{OA} + j\omega C_C V_{OC} + j\omega C_A V_{OA}$$

which are the nodal equations of the network shown in Fig. 4. Thus, the waveguide junction can be represented by a lumped, lossless, and passive network.

IV. Determination of the Network Capacitances

From equations (34), (35), (36), we see that the capacitances could be determined if F were known. To find F we must find $\frac{A_m}{B_0 - C_0}$. We shall now determine equations for this unknown.

We multiply equation (16) by $\cos \frac{m'\pi y}{a}$ and integrate with respect to y from 0 to b . This equation is then added to the result obtained when equation (17) is multiplied by $\cos \frac{m'\pi y}{a}$ and integrated with respect to y from b to c . These operations result in

$$A_m = \frac{2 \sin m\pi \frac{b}{a}}{m\pi} [B_0 - C_0] + \frac{2m}{\pi} \left(\frac{b}{a}\right)^2 \sin m\pi \frac{b}{a} \sum_{\substack{n=1 \\ n \neq \frac{b}{a}m}}^{\infty} \frac{(-1)^n B_n}{n^2 \left\{ \left(\frac{mb}{na}\right)^2 - 1 \right\}} + \frac{b}{a} \sum_{n=1}^{\infty} \delta_{m, \frac{b}{a}n} B_n \\ - \frac{2m}{\pi} \left(\frac{c}{a}\right)^2 \sin m\pi \frac{b}{a} \sum_{\substack{q=1 \\ q \neq \frac{c}{a}m}}^{\infty} \frac{C_q}{q^2 \left\{ \left(\frac{mc}{qa}\right)^2 - 1 \right\}} + \frac{c}{a} (-1)^{\frac{mb}{a}} \sum_{q=1}^{\infty} \delta_{q, \frac{c}{a}m} C_q \quad (37)$$

where

$$\delta_{m, \frac{b}{a}n} = \begin{cases} 0 & \text{if } n \neq \frac{b}{a}m \\ 1 & \text{if } n = \frac{b}{a}m \end{cases}$$

Next we multiply equation (18) by $\cos \frac{m'\pi y}{b}$ and integrate with respect to y from 0 to b . The result is

$$Y_{Bn} B_n = \frac{2b(-1)^m}{n^2 a \pi} \sum_{\substack{m=1 \\ m \neq n \frac{a}{b}}}^{\infty} \frac{m Y_{Am} A_m \sin m\pi \frac{b}{a}}{\left\{ \left(\frac{mb}{na}\right)^2 - 1 \right\}} + \sum_{m=1}^{\infty} \delta_{m, \frac{na}{b}} Y_{Am} A_m \quad (38)$$

Equation (19) is multiplied by $\cos \frac{q'\pi y}{c}$ and integrated with respect

to y' from 0 to C . This gives

$$Y_{C_0} C_0 = -\frac{2C}{g^2 a \pi} \sum_{\substack{m=1 \\ m \neq \frac{g a}{2}}}^{\infty} \frac{m Y_{Am} A_m \sin m \pi \frac{b}{a}}{\left\{ \left(\frac{mC}{ga} \right)^2 - 1 \right\}} + \sum_{m=1}^{\infty} \delta_{m, \frac{g a}{2}} (-1)^{\frac{m b}{a}} Y_{Am} A_m \quad (39)$$

Substituting the B_m and C_q from equations (38) and (39) into (37) yields

$$\begin{aligned} A_m = & \textcircled{1} \frac{2 \sin m \pi \frac{b}{a}}{m \pi} \textcircled{2} [B_0 - C_0] - \frac{4 m b^3 \sin m \pi \frac{b}{a}}{\epsilon_B \pi^2 a^3} \sum_{\substack{m=1 \\ m \neq \frac{b m}{a}}}^{\infty} \frac{\gamma_m^B}{m^2 \left\{ \left(\frac{m b}{a} \right)^2 - 1 \right\}} \textcircled{3} \sum_{\substack{p=1 \\ p \neq \frac{a}{b} m}}^{\infty} \frac{p A_p \sin p \pi \frac{b}{a}}{\gamma_p^A \left\{ \left(\frac{p b}{a} \right)^2 - 1 \right\}} \\ & - \frac{2 m b^2 \sin m \pi \frac{b}{a}}{\epsilon_B \pi a^2} \sum_{\substack{m=1 \\ m \neq \frac{b m}{a}}}^{\infty} \frac{(-1)^m \gamma_m^B}{m^2 \left\{ \left(\frac{m b}{a} \right)^2 - 1 \right\}} \sum_{p=1}^{\infty} \frac{\delta_{p, \frac{m a}{b}} A_p}{\gamma_p^A} \\ & - \frac{2 b^2}{\epsilon_B a^2 \pi} \sum_{m=1}^{\infty} \frac{\delta_{m, \frac{b m}{a}} (-1)^m \gamma_m^B}{m^2} \sum_{\substack{p=1 \\ p \neq \frac{a}{b} m}}^{\infty} \frac{p A_p \sin p \pi \frac{b}{a}}{\gamma_p^A \left\{ \left(\frac{p b}{a} \right)^2 - 1 \right\}} \\ & - \frac{b}{\epsilon_B a} \sum_{m=1}^{\infty} \delta_{m, \frac{b m}{a}} \gamma_m^B \sum_{p=1}^{\infty} \frac{\delta_{p, \frac{m a}{b}} A_p}{\gamma_p^A} \\ & - \frac{4 m C^3 \sin m \pi \frac{b}{a}}{\pi^2 a^3} \sum_{\substack{g=1 \\ g \neq \frac{a}{C} m}}^{\infty} \frac{\gamma_g^C}{g^2 \left\{ \left(\frac{m C}{g a} \right)^2 - 1 \right\}} \sum_{\substack{p=1 \\ p \neq \frac{g a}{C} m}}^{\infty} \frac{p A_p \sin p \pi \frac{b}{a}}{\gamma_p^A \left\{ \left(\frac{p C}{g a} \right)^2 - 1 \right\}} + \end{aligned}$$

(equation continued on next page)

$$+ \frac{2mc^2 \sin m\pi b/a}{\pi^2 a^2} \sum_{\substack{g=1 \\ g \neq \frac{c}{a}m}}^{\infty} \frac{\eta_g^c}{g^2 \left\{ \left(\frac{mc}{ga} \right)^2 - 1 \right\}} \sum_{p=1}^{\infty} \frac{\delta_{p, \frac{a}{c}} (-1)^{p \frac{b}{a}} A_p}{\eta_p^A} \quad (8)$$

$$+ \frac{2c^2 (-1)^{mb/a}}{\pi a^2} \sum_{g=1}^{\infty} \frac{\delta_{g, \frac{c}{a}m} \eta_g^c}{g^2} \sum_{\substack{p=1 \\ p \neq g \frac{a}{c}}}^{\infty} \frac{p A_p \sin p\pi b/a}{\eta_p^A \left\{ \left(\frac{pc}{ga} \right)^2 - 1 \right\}} \quad (9)$$

$$- \frac{c(-1)^{mb/a}}{a} \sum_{g=1}^{\infty} \delta_{g, \frac{c}{a}m} \eta_g^c \sum_{p=1}^{\infty} \frac{\delta_{p, \frac{a}{c}} (-1)^{p \frac{b}{a}} A_p}{\eta_p^A} \quad (10) \quad (40)$$

where $\epsilon_B = \frac{\epsilon_B}{\epsilon_0}$ and the encircled numbers designate the terms of the equation for subsequent identification.

Next, each term is rearranged so that A_p appears only under the first summation. For instance term (3) is rearranged by noting that

$$\gamma_m^B = \frac{\pi}{b} \left[m^2 - \left(\frac{2b}{\lambda} \right)^2 \right]^{\frac{1}{2}}$$

where λ is the wavelength in guide B and also by changing the order of summation

Term (3) becomes

$$- \frac{4m}{\epsilon_B \pi} \left(\frac{b}{a} \right)^2 \sin m\pi \frac{b}{a} \sum_{p=1}^{\infty} \frac{p A_p \sin p\pi \frac{b}{a}}{a \gamma_p^A} \sum_{\substack{m=1 \\ m \neq \frac{b}{a} p}}^{\infty} \frac{\left[m^2 - \left(\frac{2b}{\lambda} \right)^2 \right]^{\frac{1}{2}}}{m^4 \left\{ \left(\frac{mb}{na} \right)^2 - 1 \right\} \left\{ \left(\frac{pb}{na} \right)^2 - 1 \right\}}$$

In order to group all the A_m terms together in equation (40), we also separate term (3) into two parts. One including all the A_p save A_m and the other just containing A_m . When all the terms have been suitably rearranged, equation (40) becomes

$$\begin{aligned} \frac{2 \sin \pi m \frac{b}{a}}{m\pi} &= X(m) \left\{ \pi \left[m^2 - \left(\frac{2a}{\lambda} \right)^2 \right]^{\frac{1}{2}} + \pi \delta \left(\frac{b}{a} m, \text{int} \right) \left[\frac{1}{\epsilon_B} \left[\left(\frac{bm}{a} \right)^2 - \left(\frac{2b}{\lambda} \right)^2 \right]^{\frac{1}{2}} + \left[\left(\frac{cm}{a} \right)^2 - \left(\frac{2c}{\lambda} \right)^2 \right]^{\frac{1}{2}} \right] \right. \\ &\quad \left. + \frac{4m^2 \sin^2 \left(m\pi \frac{b}{a} \right)}{\pi} \left[\frac{1}{\epsilon_B} \left(\frac{b}{a} \right)^2 \sum_{\substack{m=1 \\ m \neq \frac{b}{a} m}}^{\infty} \frac{\left[m^2 - \left(\frac{2b}{\lambda} \right)^2 \right]^{\frac{1}{2}}}{m^4 \left\{ \left(\frac{mb}{na} \right)^2 - 1 \right\}^2} + \left(\frac{c}{a} \right)^2 \sum_{\substack{m=1 \\ m \neq \frac{cm}{a}}}^{\infty} \frac{\left[m^2 - \left(\frac{2c}{\lambda} \right)^2 \right]^{\frac{1}{2}}}{m^4 \left\{ \left(\frac{mc}{na} \right)^2 - 1 \right\}^2} \right] \right\} + \end{aligned}$$

(equation continued on next page)

$$\begin{aligned}
& + \sum_{\substack{p=1 \\ p \neq m}}^{\infty} X(p) \left\{ \frac{2m \sin(m\pi \frac{b}{a}) \delta(\frac{pb}{a}, \text{int}) (-1)^{\frac{pb}{a}}}{m^2 - p^2} \left[\frac{1}{\epsilon_B(\frac{b}{a})} \left[\left(\frac{pb}{a} \right)^2 - \left(\frac{2b}{a} \right)^2 \right]^{\frac{1}{2}} - \frac{1}{(\frac{c}{a})} \left[\left(\frac{pc}{a} \right)^2 - \left(\frac{2c}{a} \right)^2 \right]^{\frac{1}{2}} \right] \right. \\
& \quad + \frac{2p \sin(p\pi \frac{b}{a}) \delta(\frac{mb}{a}, \text{int}) (-1)^{\frac{mb}{a}}}{p^2 - m^2} \left[\frac{1}{\epsilon_B(\frac{b}{a})} \left[\left(\frac{mb}{a} \right)^2 - \left(\frac{2b}{a} \right)^2 \right]^{\frac{1}{2}} - \frac{1}{(\frac{c}{a})} \left[\left(\frac{mc}{a} \right)^2 - \left(\frac{2c}{a} \right)^2 \right]^{\frac{1}{2}} \right] \\
& \quad \left. + \frac{4mp \sin(m\pi \frac{b}{a}) \sin(p\pi \frac{b}{a})}{\pi} \left[\frac{1}{\epsilon_B(\frac{b}{a})} \sum_{\substack{n=1 \\ n \neq \frac{b}{a}m, \frac{b}{a}p}}^{\infty} \frac{\left[m^2 - \left(\frac{2b}{a} \right)^2 \right]^{\frac{1}{2}}}{n^4 \left\{ \left(\frac{mb}{na} \right)^2 - 1 \right\} \left\{ \left(\frac{pb}{na} \right)^2 - 1 \right\}} + \frac{1}{(\frac{c}{a})} \sum_{\substack{n=1 \\ n \neq \frac{c}{a}m, \frac{c}{a}p}}^{\infty} \frac{\left[m^2 - \left(\frac{2c}{a} \right)^2 \right]^{\frac{1}{2}}}{n^4 \left\{ \left(\frac{mc}{na} \right)^2 - 1 \right\} \left\{ \left(\frac{pc}{na} \right)^2 - 1 \right\}} \right] \right\} \\
& \hspace{15em} (41)
\end{aligned}$$

$m = 1$ to ∞

where $X(m) = \frac{A_m}{a \gamma_m^A [B_0 - C_0]}$

and $\delta\left(\frac{b}{a}m, \text{int}\right) = \begin{cases} 1 & \text{if } \frac{b}{a}m \text{ is an integer} \\ 0 & \text{if } \frac{b}{a}m \text{ is not an integer} \end{cases}$

now
We now write equation (33) as

$$F = \frac{a^2 \epsilon_0}{\pi} \sum_{m=1}^{\infty} \frac{X(m)}{m} \sin m\pi \frac{b}{a} \quad (42)$$

Using equations (36) and (42) we write

$$C_A = - \frac{a^2 \omega \epsilon_0}{bc \pi} \sum_{m=1}^{\infty} \frac{X(m)}{m} \sin m \pi \frac{b}{a} \quad (43)$$

From equations (35) and (36) we note

$$C_B = - \frac{a}{b} C_A \quad (44)$$

$$C_C = - \frac{a}{c} C_A \quad (45)$$

The remainder of this report will be concerned with determining C_A by solving equation (41) by approximate methods.

One check on the correctness of equations (41) and (43) is provided by noting in Fig. 4 that for the case $\epsilon_B = \epsilon_0$ C_A should not change when the dimensions of guides B and C are interchanged. In accordance with this physical observation it can be shown (see Appendix of reference 3) using equations (41) and (43) for the case $\epsilon_B = 1$ that C_A is invariant to the transformation replacing b by c and c by b. Thus in this respect the equations check.

V. Solution of the Equations

A. Parameters

The main objective of this report is to determine the guide capacitances for various combinations of $\frac{b}{a}$, ϵ_B , and $\frac{2a}{\lambda_a}$. Since the analysis is based on only the T.E.M. modes propagating in each guide, there is the restriction that $\frac{2a}{\lambda_a}$, $\frac{2b}{\lambda_b}$, and $\frac{2c}{\lambda_c}$ are not greater than 1. The values chosen for this report are

$$\frac{b}{a} = 0.1, 0.5, 0.9$$

$$\epsilon_B = 1, 2.5, 10, 100$$

$$\frac{2a}{\lambda_a} = 0, 0.6, 1.0$$

in order that results may be obtained over a wide range of parameters.

Since it can easily be shown that $\frac{2b}{\lambda_b} = \frac{b}{a} \sqrt{\epsilon_B} \frac{2a}{\lambda_a}$, those permutations of the parameters for which $\frac{2b}{\lambda_b} > 1$ will not be allowed.

B. Method of Approximation

The infinite number of unknowns $X(m)$ can, in principle, be determined from the infinite set of simultaneous equations represented by equation (41). The $X(m)$ are then used to calculate the capacitance C_A , by employing equation (43). The other capacitances immediately follow from equations (44) and (45). In practice, however, equation (41) must be solved by some method of approximation. The method utilized in this report attempts to approximate the exact answer by solving only finite sets of simultaneous linear equations.

We will solve equation (41) as a set of simultaneous equations of rank 4, 5, and 6 and from the behavior of these solutions determine an approximate capacitance. The validity of this approximation will be examined by a comparison with the exact results for the case $\epsilon_B = 1$.¹

The quantity we are interested in obtaining is the capacitance C_A given by equation (43). We define

$$C = -\frac{bc\pi}{\alpha^2 \omega \epsilon_0} C_A = \sum_{m=1}^{\infty} \frac{X(m)}{m} \sin m\pi \frac{b}{a} \quad (46)$$

where $X(m)$ are the solutions to the infinite set of equations represented by (41). Let the index m in equation (41) be restricted to the integers from 1 to 6. The solutions to this 6 x 6 set will be called $X_6(m)$. Thus one approximation to C could be

$$C^{(1)} = \sum_{m=1}^6 \frac{X_6(m)}{m} \sin m\pi \frac{b}{a} \quad (47)$$

This approximation has 2 sources of error. The first is the error arising from the fact that $X_6(m) \neq X(m)$ for $m = 1, 6$. The second is the error due to the exclusion of

$$\sum_{m=7}^{\infty} \frac{X(m)}{m} \sin m\pi \frac{b}{a}$$

It is our aim to determine a refinement of the approximation given by equation (47) based on the computed results, which will reduce the errors mentioned above.

The computer program was designed to solve the system of equations of rank 4, 5, and 6. It was felt that a plot of these results for a few

typical cases and an extrapolation would indicate the probable solutions for the first four unknowns ($X(m)$, $m = 1, 4$) and also serve to exhibit the behavior of the unknowns for $m > 4$. As an example of this procedure for one set of parameters, Figs. 5, 6, and 7 show respectively the dependence of the solutions on the equation set size, a rough estimate of the values of $\Delta X(m)$ for $m = 1, 4$ extrapolated from the finite sets, and the extrapolated values of $X(m)$. An examination of the computed and graphical results reveals the following information:

- (1) $X(m) \sin(m\pi \frac{b}{a})$ is always positive
- (2) The extrapolated value of $X(m)$ is very close to $x_6(m)$
- (3) The extrapolated value of $x(m)$ for $m > 5$ decreases

roughly as $\frac{1}{m}$. Here we must make an exception for

$$\frac{b}{a} = 0.5 \text{ because the results show } |x(2m+1)| > > |x(2m)|.$$

But in this case the odd values of m decrease as $\frac{1}{m}$ and these are the only values that enter into the computation of C .

Based on the above observations a suitable refinement to equation (47) is

$$C^{(2)} = C^{(1)} + 5|x_6(5)| \sum_{m=7}^{\infty} \left| \frac{\sin m\pi \frac{b}{a}}{m^2} \right| \quad (48)$$

The series

The series $\sum_{m=7}^{\infty} \left| \frac{\sin m\pi \frac{b}{a}}{m^2} \right|$ is roughly the same if $\frac{b}{a}$ is 0.1, 0.5, or 0.9.

We note for $\frac{b}{a} = 0.5$ the value of this series can be determined from the well known result

$$\sum_{m=1}^{\infty} \left| \frac{\sin \frac{m\pi}{2}}{m^2} \right| = \sum_{m=1}^{\infty} \frac{1}{(2m-1)^2} = \frac{\pi^2}{8}$$

From this we find

$$\sum_{m=7}^{\infty} \left| \frac{\sin \frac{m\pi}{2}}{m^2} \right| = 0.0826144$$

Our approximation is now given by

$$c^{(2)} = c^{(1)} + 0.413 |\kappa_6(5)|$$

or using equation (48)

$$-\frac{a^2 c^{(2)}}{bc\pi} = \frac{C_A^{(2)}}{WE_0} = \frac{C_A^{(1)}}{WE_0} - \frac{a^2}{bc} \frac{0.413}{\pi} |\kappa_6(5)| \quad (49)$$

where

$$\frac{C_A^{(1)}}{WE_0} = -\frac{a^2}{bc\pi} \sum_{m=1}^6 \frac{\kappa_6(m)}{m} \sin m\pi \frac{b}{a}$$

The computed results are tabulated below. Figures 8 through 13 show these results in graphical form.

TABLE OF COMPUTED RESULTS

$\frac{b}{a}$	$\frac{2a}{\lambda_b}$	ϵ_B	$\frac{2a}{\lambda_b}$					
			0.0	0.6	1.0	1.0	1.0	1.0
			$ K_6(s) $	$-\frac{C_A^{(1)}}{w\epsilon_0}$	$ K_6(s) $	$-\frac{C_A^{(1)}}{w\epsilon_0}$	$ K_6(s) $	$-\frac{C_A^{(1)}}{w\epsilon_0}$
0.1	1	1	.00698	.09252	.103	.00708	.1018	.112
0.5	1	1	.01032	.2083	.214	.0112	.2451	.251
0.9	1	1	.00698	.09252	.103	.00708	.1018	.112
0.1	2.5	2.5	.0075	.09475	.106	.00762	.1042	.115
0.5	2.5	2.5	.0101	.2258	.231	.01074	.2705	.276
0.9	2.5	2.5	.00712	.1112	.122	.00706	.1292	.139
0.1	10	10	.00778	.09596	.107	.00792	.1055	.117
0.5	10	10	.00951	.2387	.244	.00977	.2916	.297
0.9	10	10	.00688	.1250	.135	*	-	-
0.1	100	100	.00787	.09634	.108	.008	.1059	.118
0.5	100	100	.00922	.2434	.248	-	-	-
0.9	100	100	.00672	.1302	.140	-	-	-

*The dash (-) indicates that $\frac{2b}{\lambda_b} = \frac{b}{a} \sqrt{\epsilon_B} \frac{2a}{\lambda_b} >$ and thus solutions with these sets of parameters are excluded from this analysis.

C. Comparison with the Results Obtained by Marcuvitz for $\epsilon_B = \epsilon_0$.

On pages 353-355 of Marcuvitz (op. cit.) the E-plane bifurcation problem is discussed and the results given graphically for an exact solution of this problem using integral transform methods. These results can be compared to those of this report for the case $\epsilon_B = 1$. We use the notation on page 353 but to avoid confusion; since b denotes different dimensions in this report and in Marcuvitz, we let the b of Marcuvitz be called b' . The dimension Marcuvitz labels b is the dimension of guide A in this report. Comparing Fig. 4 in this report to Figure 6.4 - 2 in Marcuvitz we observe

$$X = -\frac{1}{\omega C_A}$$

where $X = Z_0 \cot \frac{2\pi d}{\lambda}$

For this problem $\lambda = \lambda_0$ and the characteristic impedance is given by

$$Z_0 = \frac{b'}{w} \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{a}{w} \sqrt{\frac{\mu_0}{\epsilon_0}}$$

Therefore

$$C_A = \frac{1}{\omega X} = -\frac{w}{\omega b' \sqrt{\frac{\mu_0}{\epsilon_0}} \cot \frac{2\pi d}{\lambda}}$$

Using $\omega \sqrt{\mu_0 \epsilon_0} = \frac{2\pi}{\lambda}$ we have

$$\frac{C_A}{w \epsilon_0} = -\frac{\tan \frac{2\pi d}{\lambda}}{\frac{2\pi b'}{\lambda}} \quad (50)$$

$$\text{When } \frac{1}{\lambda} \rightarrow 0 \quad \text{we have } \lim_{\frac{1}{\lambda} \rightarrow 0} \left[\frac{C_A}{wE_0} \right] = - \frac{\alpha}{b'} \quad (51)$$

Using the graph on page 354 we can construct the table below and compare

$\frac{C_A}{wE_0}$ with the computed $\frac{C_A^{(2)}}{wE_0}$ of this report.

TABLE OF COMPARED RESULTS FOR $\epsilon_b = 1$.

$\epsilon_b = 1$	$\frac{b'}{b} = \frac{b}{a}$	$\frac{2b' - 2a}{\lambda}$	$\frac{\pi d}{b'}$	$\frac{2\pi d}{\lambda}$	$\tan\left(\frac{2\pi d}{\lambda}\right)$	$-\frac{CA}{w\epsilon_0}$	$-\frac{CA^{(2)}}{w\epsilon_0}$	$\left[\frac{CA - CA^{(2)}}{CA}\right] 100\%$
	0.1	0.0	0.32	0.0	0.0	0.102	0.103	-0.9%
	0.5	0.0	0.69	0.0	0.0	0.219	0.214	2.28%
	0.1	0.6	0.35	0.21	0.213	0.113	0.112	0.9%
	0.5	0.6	0.76	0.456	0.49	0.260	0.251	3.46%
	0.1	1.0	0.69	0.69	0.824	0.262	0.256	2.29%
	0.5	1.0	1.245	1.245	2.95	0.938	0.858	8.52%

From the above results we see that in all but one case the error is within 4% of Marcuvitz' results. Thus $\frac{C_A^{(2)}}{\omega \epsilon_0}$ is a fairly good approximation to $\frac{C_A}{\omega \epsilon_0}$ for $\epsilon_B = 1$. Since the structure of the equations is the same for $\epsilon_B \neq 1$, and the computed results manifest no drastic change for $\epsilon_B \neq 1$, we may expect that $\frac{C_A^{(2)}}{\omega \epsilon_0}$ will be within a few per cent of the correct value for all ϵ_B .

VI. Scattering Description

This section will be concerned with transforming the impedance description of the guide to a scattering description. Since we know now the values of the capacitances in the equivalent circuit, all other waveguide quantities such as reflection coefficient, standing wave ratio, etc., may be determined. We will assume, for the sake of simplicity, that only guide B is excited and that guides A and C are terminated in their characteristic impedance. To calculate the reflection coefficient at the junction, $\Gamma(0)$, we use equation (30) given on page 14 of "Waveguide Handbook" (op. cit.).

$$\Gamma(z) = \frac{1 - Y'(z)}{1 + Y'(z)} \quad (52)$$

where $Y'(z) = Y(z)/Y^0$

and $Y(z)$ is the admittance of the transmission line at z and Y^0 is the characteristic admittance of the transmission line.

We note that the circuit shown in Fig. 4 corresponds to the half-space problem. However, due to symmetry, the reflection coefficient of the original guide in Fig. 1 is the same as the half-space guide. The circuit we consider is shown in Fig. 14, where the characteristic admittances are found from equation (29-a). We make the following definitions:

$$Y_A = Y_A^0 + j\omega C_A \quad (53)$$

$$Y_B = j\omega C_B \quad (54)$$

$$Y_C = Y_A^0 + j\omega C_C \quad (55)$$

We then have

$$Y'(0) = \frac{Y}{Y_B^0} = \frac{Y_B}{Y_B^0} + \frac{Y_A Y_C}{Y_B^0 (Y_A + Y_C)} \quad (56)$$

Using equations (29-a) and (45) we write equation (55) as

$$Y_C = \frac{\omega}{C} \sqrt{\frac{\epsilon_0}{\mu_0}} - j\omega \frac{a}{C} C_A \quad (57)$$

We make use of the quantity $-\frac{C_A}{\omega \epsilon_0}$ computed in part V by defining the positive number C_V

$$C_V = -\frac{C_A}{\omega \epsilon_0} \quad (58)$$

We now substitute equation (58) into (57) to yield

$$Y_C = \frac{\omega}{C} \sqrt{\frac{\epsilon_0}{\mu_0}} \left[1 + j\omega \sqrt{\mu_0 \epsilon_0} a C_V \right] \quad (59)$$

but since $\omega \sqrt{\mu_0 \epsilon_0} = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda}$ equation (59) can be written as

$$Y_C = \frac{\omega}{C} \sqrt{\frac{\epsilon_0}{\mu_0}} \left[1 + j \left(\frac{2\pi a}{\lambda} \right) C_V \right] \quad (60)$$

Similarly equation (53) can be written as

$$Y_A = \frac{\omega}{a} \sqrt{\frac{\epsilon_0}{\mu_0}} \left[1 - j \left(\frac{2\pi a}{\lambda} \right) C_V \right] \quad (61)$$

After a few of these substitutions equation (56) may be written as

$$Y'(0) = \frac{A(1+C_L^2) + jC_L(A^2-1)}{\sqrt{\epsilon_B} (A^2 + C_L^2)} \quad (62)$$

where

$$A = \frac{2a}{b} - 1$$

and

$$C_L = \frac{2a\pi}{\lambda} C_V$$

Substituting (62) into (52) we can calculate the reflection coefficient $\Gamma(0)$.

Let us now examine the behavior of the reflection coefficient for two simple cases.

Case 1. $\frac{2a}{\lambda} = 0$

If $\frac{2a}{\lambda} = 0$, then $C_L = 0$ and equation (62) becomes

$$Y'(0) = \frac{1}{\sqrt{\epsilon_B} A} \quad (63)$$

Substituting equation (63) into (52) we obtain

$$\Gamma(0) = \frac{\sqrt{\epsilon_B} A - 1}{\sqrt{\epsilon_B} A + 1} \quad (64)$$

We note when $A \rightarrow 1$

$$\Gamma(0) = \frac{\sqrt{\epsilon_B} - 1}{\sqrt{\epsilon_B} + 1}$$

and when

$$\sqrt{\epsilon_B} \rightarrow 1 \quad \Gamma(0) \rightarrow 0$$

Also if $\epsilon_B = 1$, equation (64) becomes

$$\Gamma(0) = 1 - \frac{b}{a} \quad (65)$$

This is in agreement with the physical result that as $\frac{b}{a} \rightarrow 1$, the guide ceases to become bifurcated and no reflections occur.

Case 2. $C_L^2 \ll 1$

A study of the Table of Computed Capacitance in section V-C shows that this is a fairly good approximation when $2a/\lambda_a \leq 0.6$. Since $A = \left[\frac{2a}{b} - 1 \right] \geq 1$, we also have $C_L^2 \ll A$. With this approximation equation (62) can be written

$$Y'(0) = \frac{1}{\sqrt{\epsilon_B} A} + j \frac{C_L(A^2 - 1)}{A^2} \quad (66)$$

This tells us the obvious fact that the best match, i.e. $Y'(0) = 1$ occurs when $A \rightarrow 1$ and $\sqrt{\epsilon_B} \rightarrow 1$.

It is also interesting to inquire how the wave entering from guide B is divided between guides A and C. Using equations (6-a) and (29-b) we obtain

$$V_{0A} = -A_o^r a \quad V_{0C} = -C_o^r c \quad (67)$$

where A_o^r and C_o^r are the complex amplitudes of the outgoing waves propagating in guides A and C respectively. We have the relations

$$V_{0A} = \frac{I}{Y_A} \quad V_{0C} = \frac{I}{Y_C} \quad (68)$$

where I is shown in Fig. 14. Combining equations (60), (61), (67), and (68) we get

$$\frac{A_o^r}{C_o^r} = \frac{1 + jC_L}{1 - jC_L} \quad (69)$$

and

$$\left| \frac{A_o^r}{C_o^r} \right| = 1 \quad (70)$$

which gives the result that the magnitudes of the transmitted waves in guides A and C are always the same. To find the ratio of the powers flowing into guides A and C, assuming them terminated by their characteristic impedance we employ the equation

$$P = \frac{1}{2} |V|^2 Y^0$$

$$\therefore \frac{P_A}{P_C} = \frac{|V_{0A}|^2 Y_A^0}{|V_{0C}|^2 Y_C^0} = \frac{|A_o^r a|^2 / c}{|C_o^r c|^2 / a} = \frac{a}{c} \quad (71)$$

This is a general result independent of ϵ_B and subject to the restriction that $\frac{2a}{\lambda} \leq 1$

We now give a table of computed $|r(0)|$ for the range of parameters considered in this problem.

Table of Computed $|r(0)|$

$\frac{b}{a}$	ϵ_B	$\frac{2a}{\lambda}$	0.0	0.6	1.0
			$ r(0) $	$ r(0) $	$ r(0) $
0.1	1		.90	.90	.90
0.5	1		.50	.50	.37
0.9	1		.10	.10	.10
0.1	2.5		.94	.93	.92
0.5	2.5		.65	.61	.35
0.9	2.5		.32	.32	-
0.1	10		.97	.98	.95
0.5	10		.81	.77	-
0.9	10		.59	-*	-
0.1	100		.99	.99	.99
0.5	100		.94	-	-
0.9	100		.85	-	-

These results show that increasing ϵ_B and decreasing $\frac{b}{a}$ and $\frac{2a}{\lambda}$ tends to increase $|r(0)|$.

*The dash (-) indicates that $\frac{2b}{\lambda} = \frac{b}{a} \sqrt{\epsilon_B} \frac{2a}{\lambda} > 1$ and thus solutions with these sets of parameters are excluded from this analysis.

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3. H. M. Cronson, "Equivalent Circuit Parameters for an Inhomogeneous Bifurcated Waveguide", Master's thesis, Brown University, (1961).

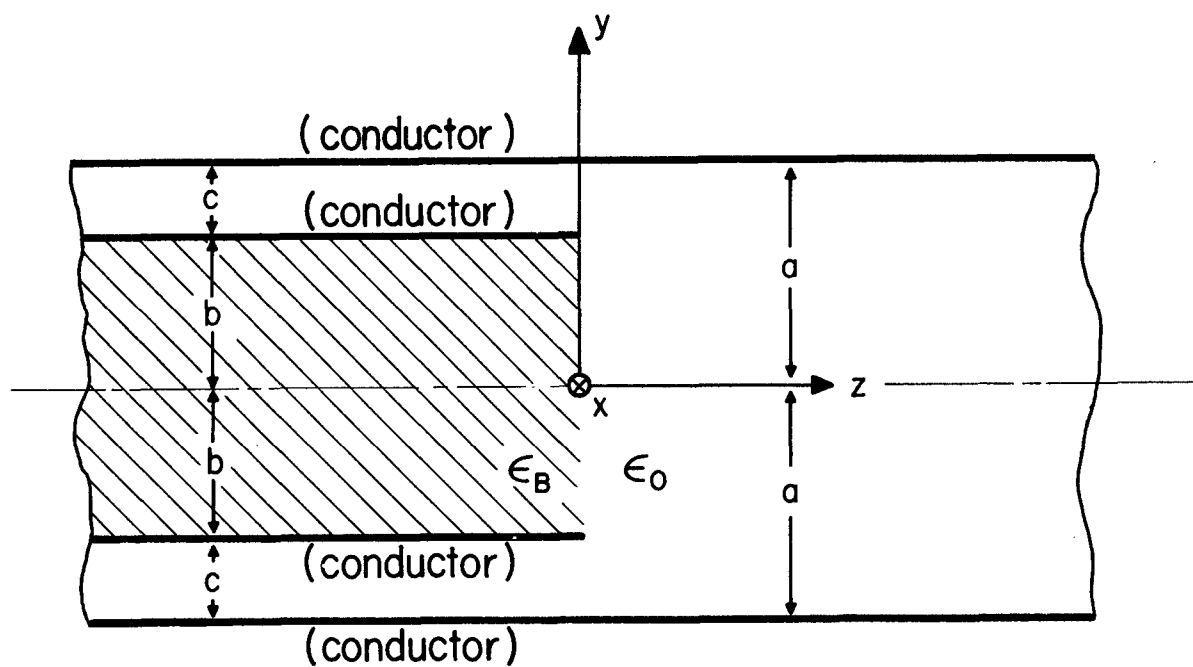


FIG. 1 A SEMI-INFINITE, DIELECTRIC FILLED, PARALLEL PLATE GUIDE CONTAINED SYMMETRICALLY IN AN INFINITE PARALLEL PLATE GUIDE.

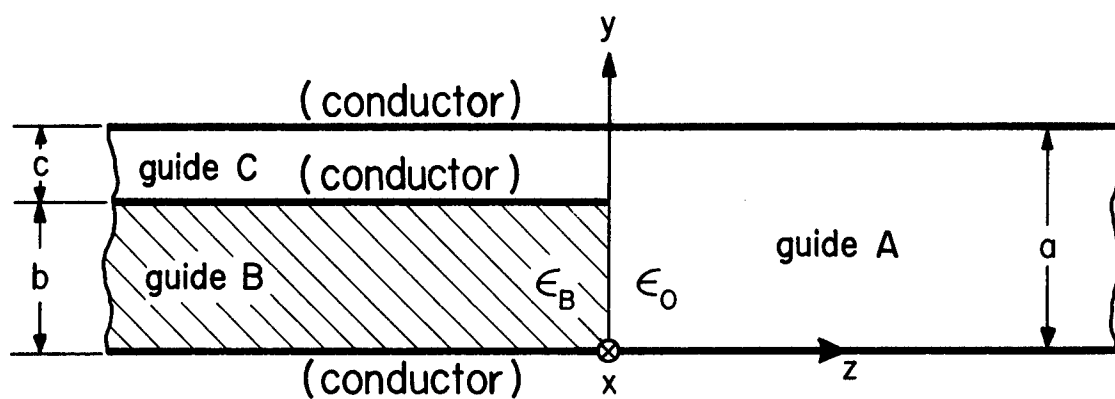


FIG. 2 THE INHOMOGENEOUS BIFURCATED GUIDE

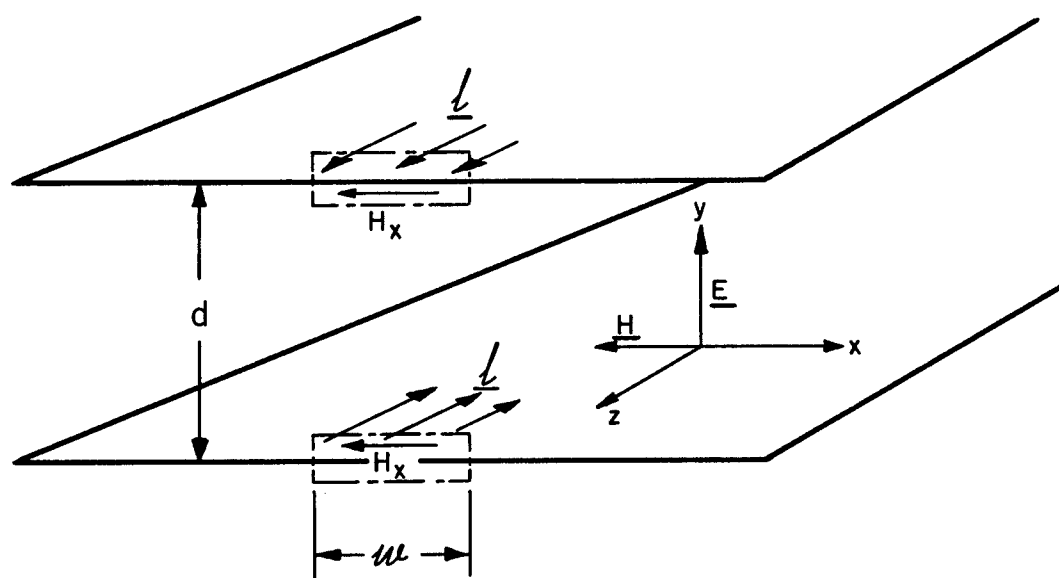


FIG. 3 ORIENTATIONS OF $\underline{l}$, $\underline{w}$, AND H_x

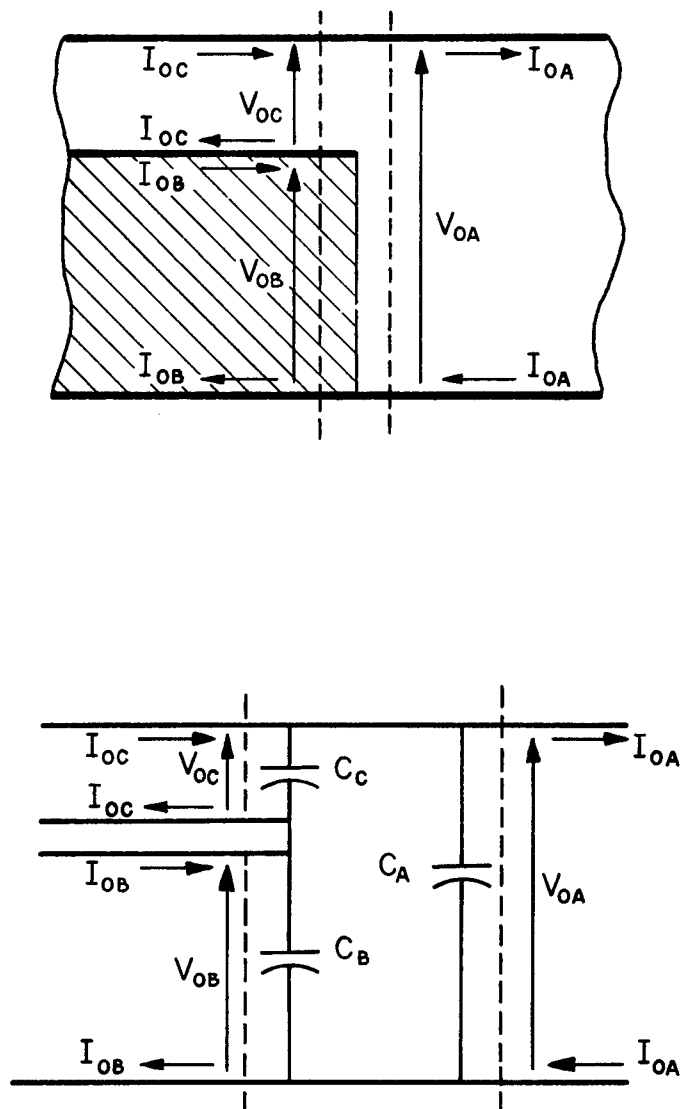


FIG. 4 EQUIVALENT CIRCUIT REPRESENTATION

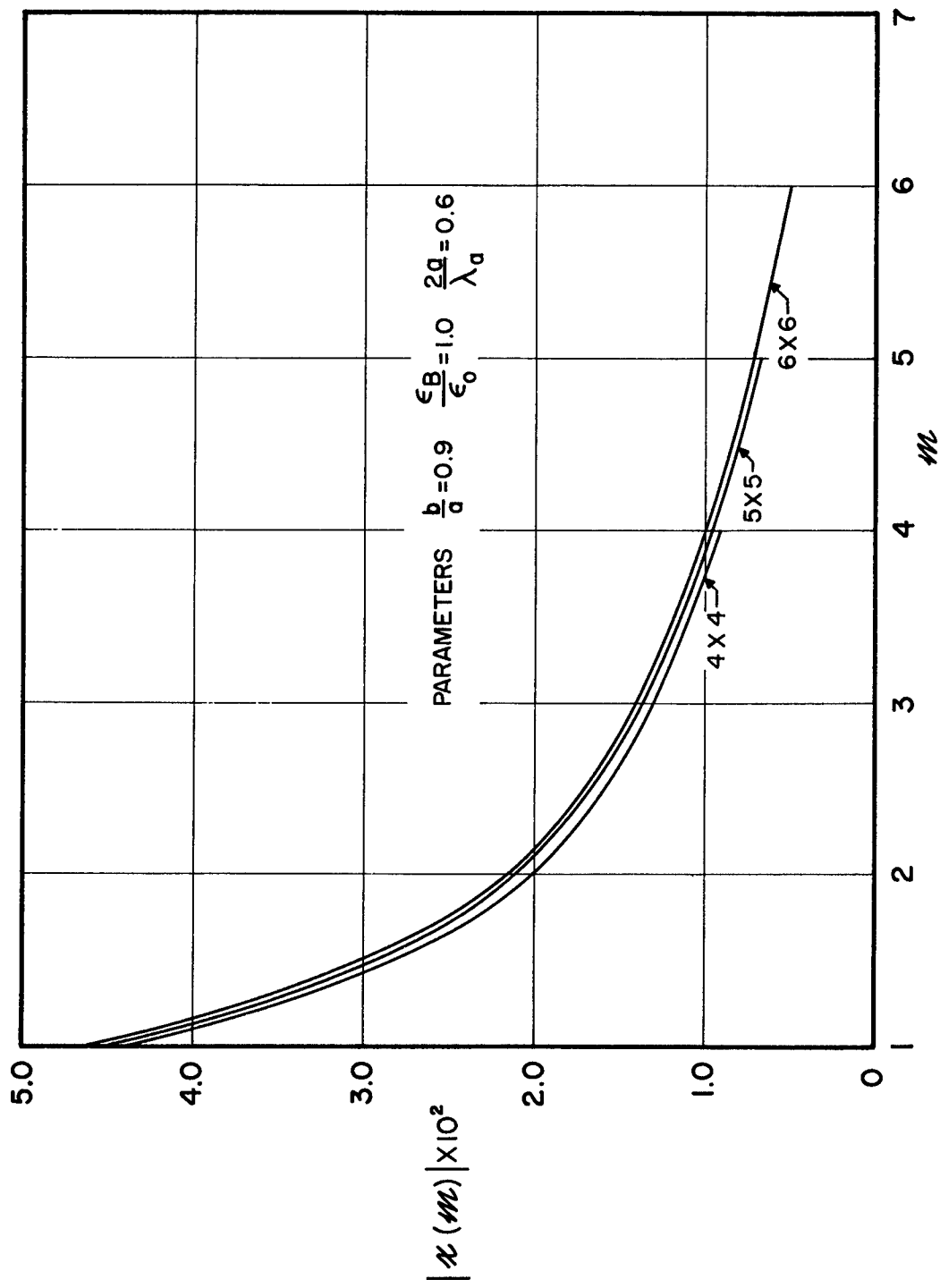


FIG. 5 THE DEPENDENCE OF THE SOLUTIONS ON THE EQUATION SET SIZE

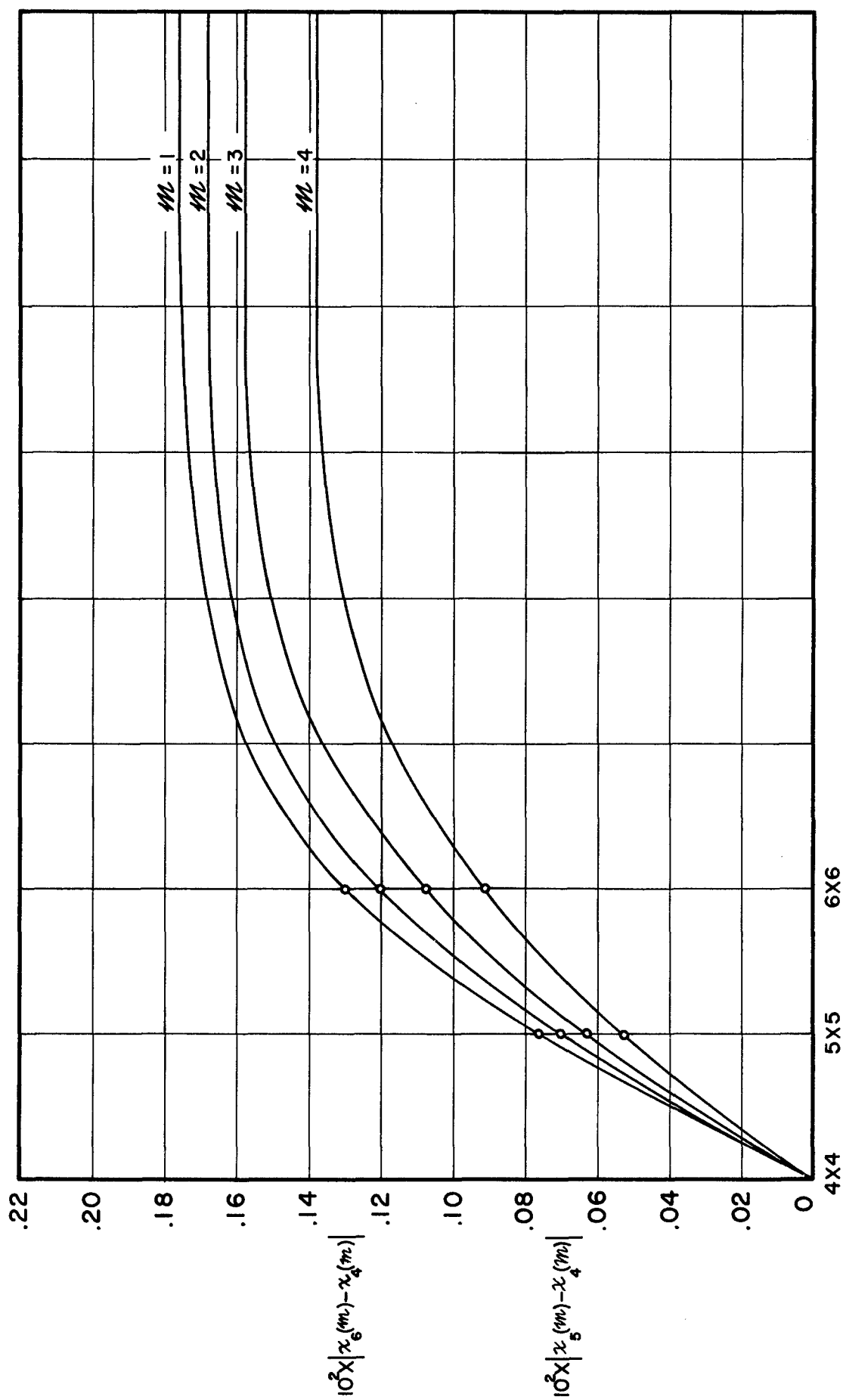


FIG. 6 EXTRAPOLATING VALUES OF $\kappa(m)$ FOR $m=1$ TO 4 FROM THE FINITE SETS. PARAMETERS $\frac{b}{a} = 0.9$ $\frac{\epsilon_B}{\epsilon_0} = 1.0$ $\frac{2a}{\lambda_a} = 0.6$

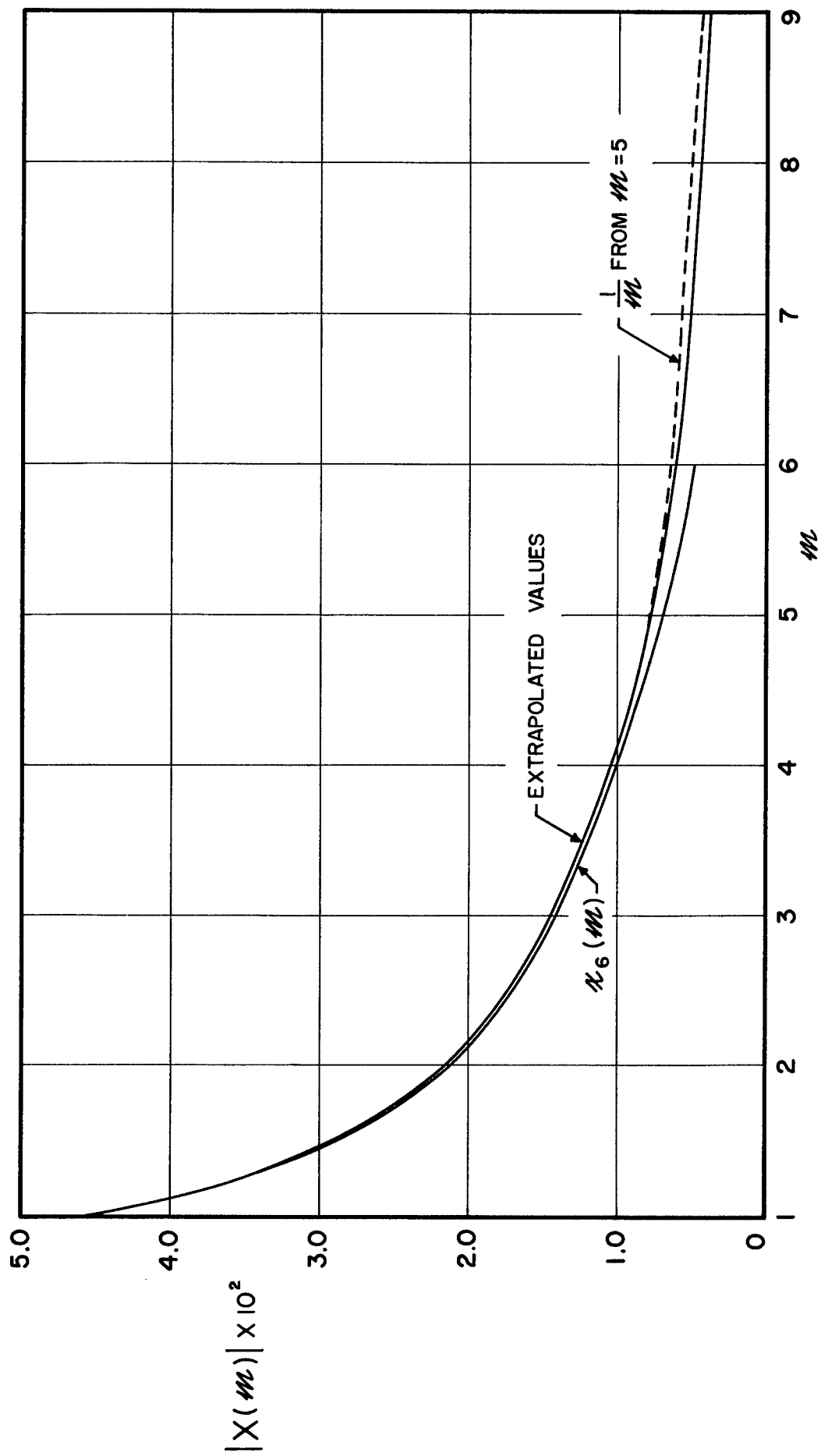


FIG. 7 EXTRAPOLATED VALUES OF $\kappa(m)$ PARAMETERS $\frac{b}{a} = 0.9$ $\frac{\epsilon_B}{\epsilon_0} = 1.0$ $\frac{2a}{\lambda_0} = 0.6$

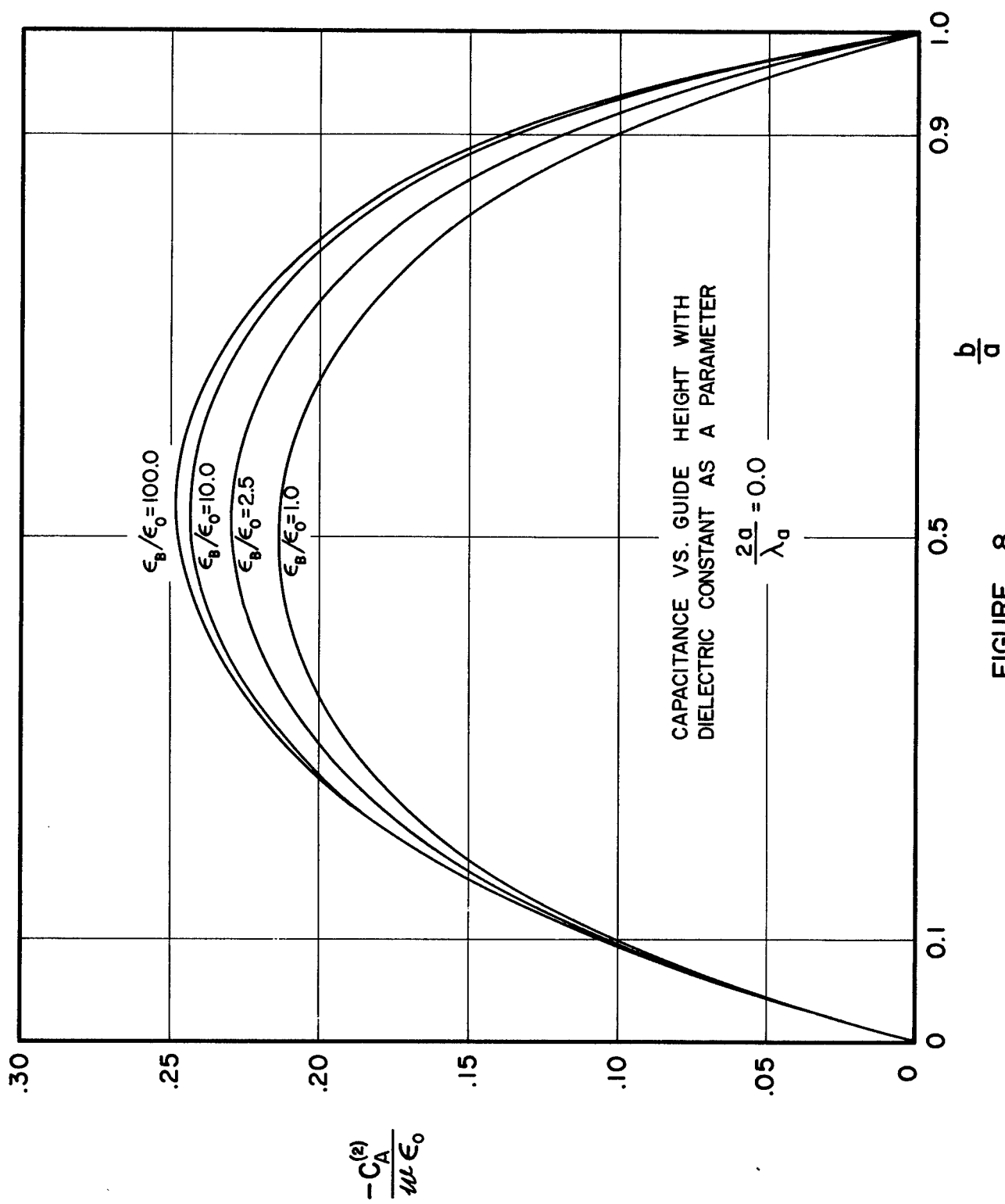


FIGURE 8

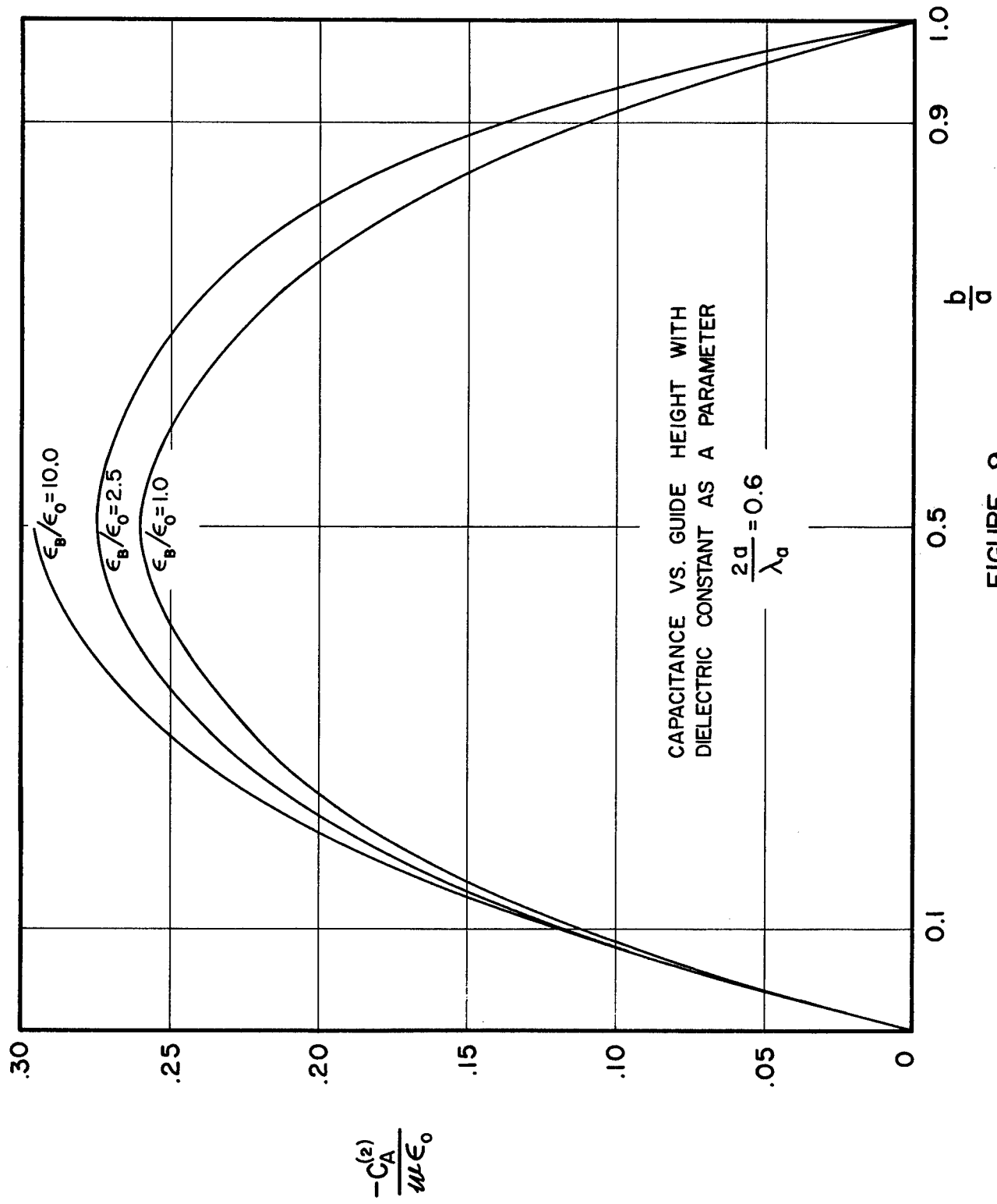


FIGURE 9

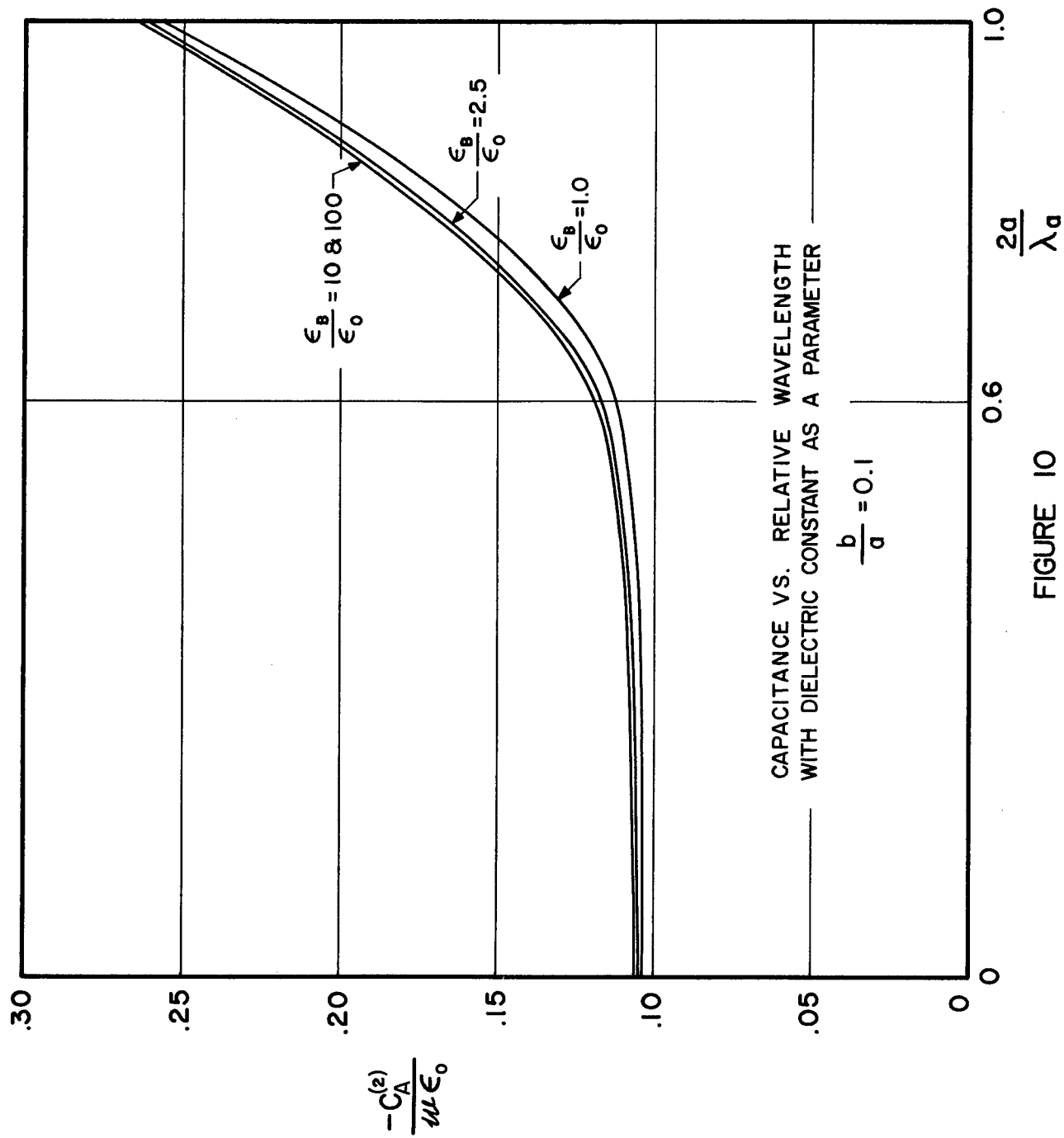


FIGURE 10

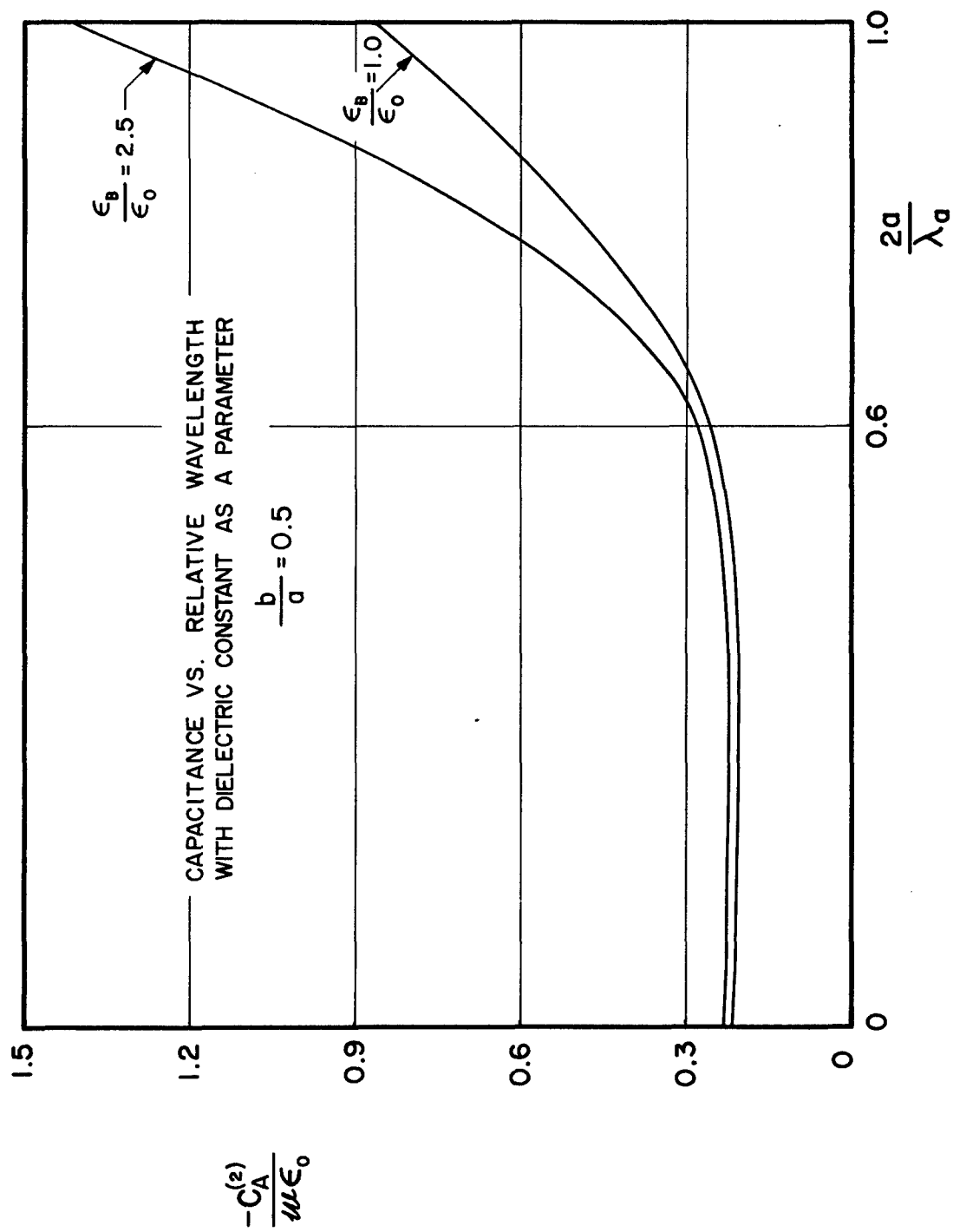


FIGURE 11

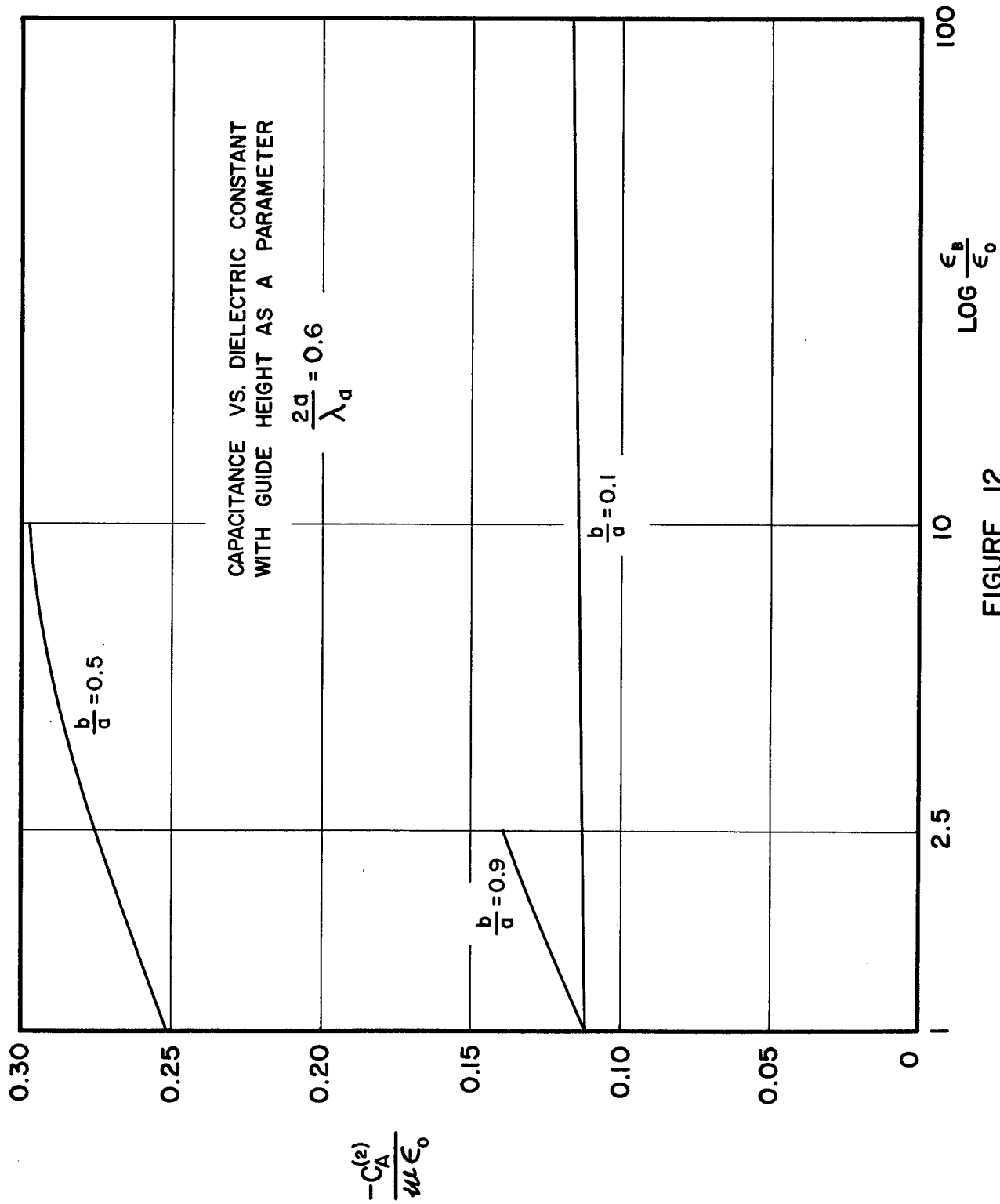


FIGURE 12

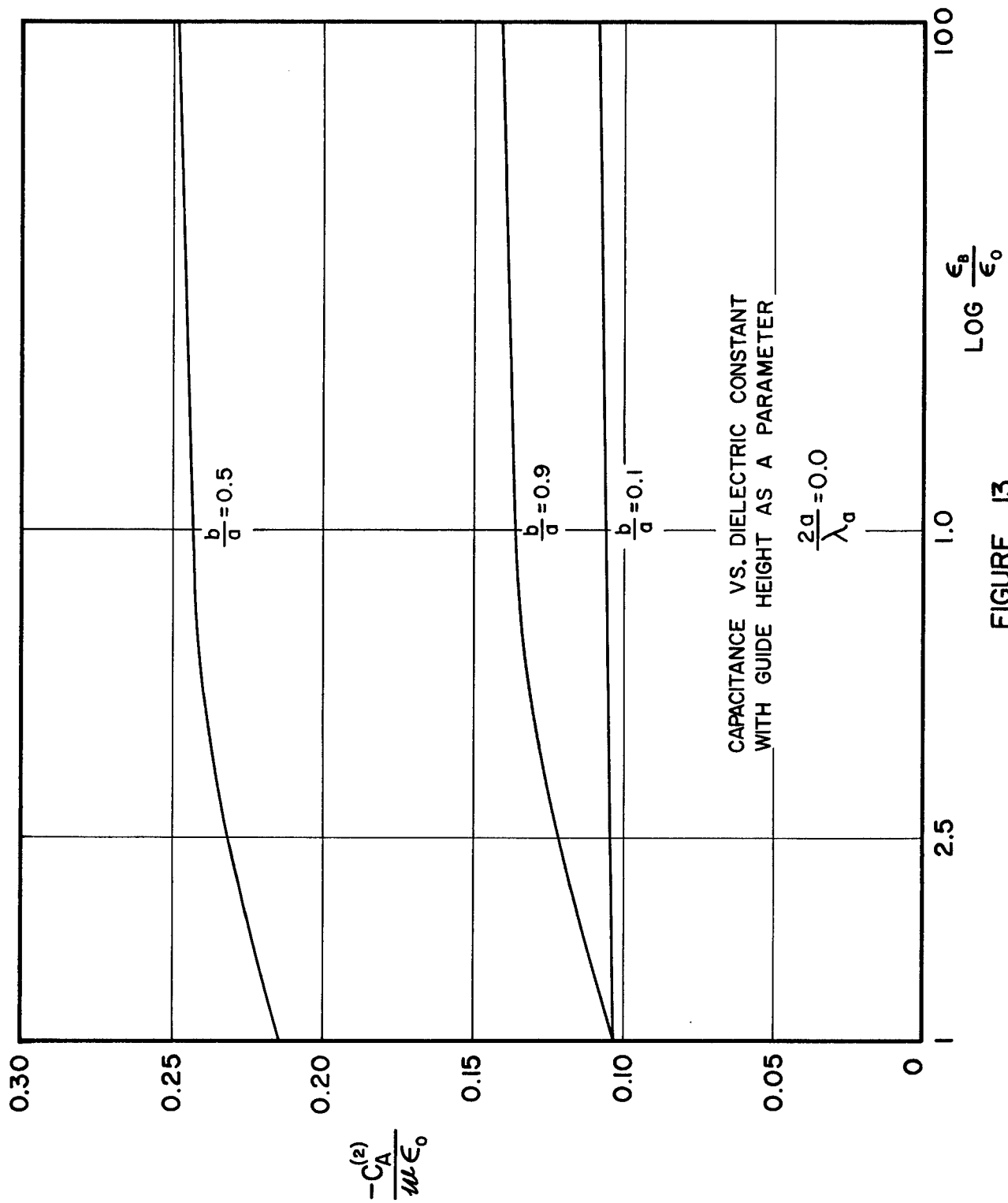


FIGURE 13

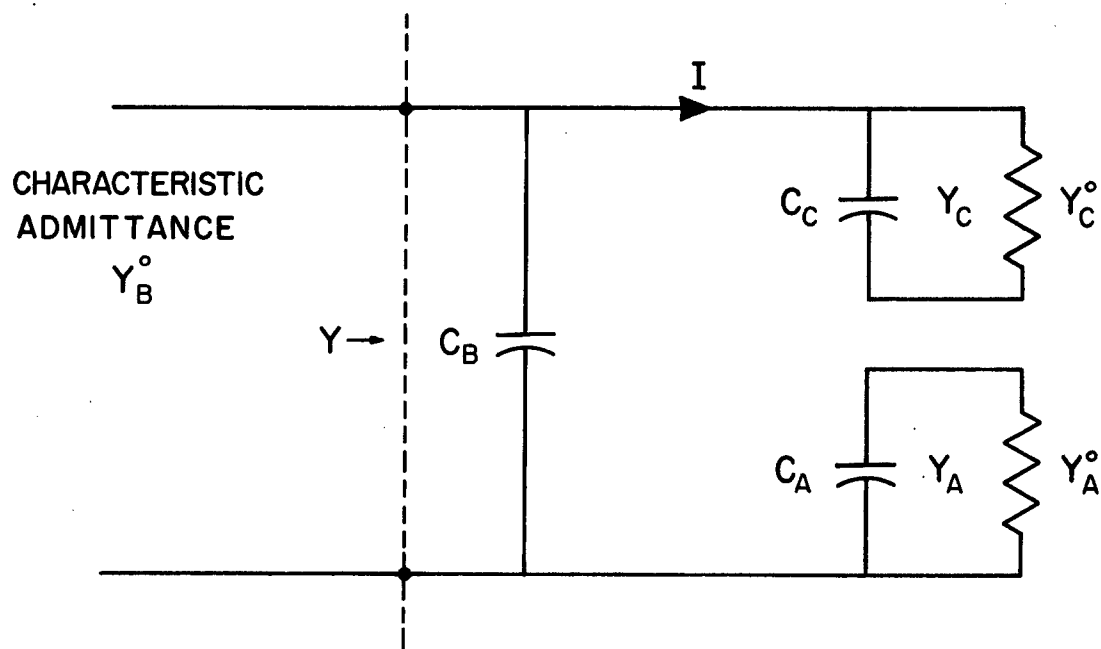


FIG. 14 EQUIVALENT CIRCUIT OF THE HALF SPACE GUIDE WITH GUIDES A AND C TERMINATED IN THEIR CHARACTERISTIC ADMITTANCES

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