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CRDL Special Publication 1-30

BREAKUP OF A LIQUID MASS IN FREE FALL BY CANOPY FORMATION: A THEORETICAL STUDY

by

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FOREWORD

The authority for the work described in this report was Project 4C08-03-016, CW Agent Research (U), Task 4C08-03-016-10, Agent Physicochemical Research (U). The work was started in November 1959 and completed in December 1960.

Acknowledgments

The experimental work of James D. Wilcox and Joseph Pistritto, Colloid Branch, Physicochemical Research Division, on liquid-mass breakup was used as a guide for a large portion of the theory. Technical points were clarified and some of the mathematical derivations were expanded at the suggestion of Mr Paul Kolavski of the Operations Research Group, Mr W. R. Van Antwerp, Physicist, Nuclear Defense Laboratories, Army Chemical Center, Maryland, assisted the authors in review of this manuscript.

Notice

Disposition

Chemical Research and Development Laboratories Special Publication 1-30
BREAKUP OF A LIQUID MASS IN FREE FALL BY CANOPY FORMATION:
A THEORETICAL STUDY

Task No.: 4C08-03-016-10
Notebook: None

Date Started: November 1959
Date Completed: October 1960

APPROVED:

Carl M Herget
CARL M. HERGET, Ph. D.
Director of Research

DIGEST

A theory explaining the breakup of large liquid drops by canopy formation has been derived. The theory explains the deformations that take place during the disintegration of a drop in terms of aerodynamic, hydrostatic, and surface-tension pressures. The effect of liquid viscosity has been assumed to be negligible. Therefore, the theory will not apply to liquids of high viscosity. This theory, insofar as it has been possible to verify it, is in good agreement with breakup experiments performed with liquid drops.

CONTENTS

BREAKUP OF A LIQUID MASS IN FREE FALL BY CANOPY FORMATION: A THEORETICAL STUDY

I. INTRODUCTION.

When a liquid mass disintegrates in free fall in nonturbulent air, three different modes of breakup may be observed because of varying conditions of instability of the mass. These methods of breakup are the following:

1. Canopy formation or "bursting-bag effect," experimentally described by Magarvey and Taylor[1] and Blanchard.[2]

2. Surface stripping, investigated by Wilcox, Pistritto, and Palmer.[3]

3. Rotation and oscillation, experimentally observed by Blanchard.[4]

The first mechanism, canopy formation, refers to the deformation of a liquid mass in such a way that the mass first becomes flattened on its leading side and subsequently inflated somewhat like a soap bubble until it bursts. The second mechanism, surface stripping, refers to small amounts of liquid being continuously stripped off the surface of the mass by aerodynamic forces. The third mechanism, rotation and oscillation, consists of rotation of the entire mass until centrifugal force causes it to split. Two smaller masses of approximately equal size usually are formed. This same type of splitting occurs when the mass oscillates into ellipsoidal shapes undulating first in one plane and then in a plane perpendicular to it.

In order to derive a generalized theory of liquid-mass breakup that will account for all of the mechanisms involved, a series of idealized situations will be presented to make it possible to analyse one mechanism at a time.

II. ANALYSIS OF CANOPY-FORMATION MECHANISM IN LIQUID BREAKUP.

The most fundamental mechanism of breakup appears to be canopy formation. By fundamental it is meant that this mechanism can be analyzed independently of the others without imposing many restrictions on the idealized experiment and without making many broad assumptions. An analysis of this mechanism will be the first step in deriving a generalized theory of liquid-mass breakup.

Assume that a spherical mass (of radius r_0) of a nonviscous inelastic liquid of density ρ_ℓ, viscosity μ_ℓ, and surface tension γ, is at rest at time zero (t = 0) when it begins to undergo free fall in a gravitational field g through a nonturbulent still atmosphere of density ρ_a and of viscosity of μ_a. It is assumed that the total mass of the liquid remains constant during free fall until the time when the canopy bursts; i.e., the amount of liquid surface stripped[3] or evaporated is negligible. In the case of a 1-gm spherical mass, the evaporation is calculated to be less than 0.01% by Langmuir's equation.[5]

As will be shown later, this assumption of constant mass is only valid for spheres in a limited size range. At time zero the idealized liquid-mass profile appears circular as in figure 1. The mass is always described as a surface of revolution.

III ANALYSIS OF LIQUID-MASS DEFORMATION.

As the sphere begins to fall, the aerodynamic pressure on the leading hemisphere will begin to increase; the entire sphere will begin to be flattened. The flattening of the leading hemisphere will be more rapid than that of the trailing hemisphere. This can be visualized by considering the movement of the leading point as the aerodynamic pressure increases.

A. Development of Oblate Spheroid.

The profile of the liquid mass, which is always described as a surface of revolution, may be represented on the original coordinate axes (figure 1), which are moving with the same velocity as the entire mass relative to the air. As the aerodynamic pressure increases with increasing velocity, the leading point will move toward the trailing point (figure 2). The distance traveled by the leading point toward the trailing point will be called "s." As the leading point moves toward the trailing point, the liquid in the lower hemisphere will be displaced outward from the waist of the liquid mass along the path of least resistance. This will continue to occur until the hydrostatic pressure at the waist of the mass plus the surface-tension pressure caused by the curvature at the trailing point is equal to the surface tension at the waist of the mass. This pressure relationship of the liquid mass may be expressed by the following mathematical model:

$$2P_{sw} = P_h + P_{st} \tag{1}$$

where

P_{sw} = surface-tension pressure at any point on waist

P_h = hydrostatic pressure at any point on waist

P_{st} = surface-tension pressure at trailing point

Using the equations derived in appendix A, these parameters may all be expressed terms of s and r_o:

$$P_{sw} = \gamma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \gamma\left[\frac{3(2r_o - s)^2}{8r_o^4 + 2r_o(2r_o - s)^3 - 3r_o^2(2r_o - s)^2}\right]^{\frac{1}{2}} +$$

$$\gamma r_o^{-2}\left[\frac{8r_o^4 + 2r_o(2r_o - s)^3 - 3r_o^2(2r_o - s)^2}{3(2r_o - s)^2}\right]^{\frac{1}{2}}$$

$$P_h = \rho_\ell (2r_o - s) g$$

$$P_{st} = \frac{2\gamma}{r_k} = 2\gamma\left[\frac{3(2r_o - s)^2}{4r_o^3 + (2r_o - s)^3}\right]$$

where

r_k = radius of curvature of trailing point

r_1 = major radius of curvature at waist

r_2 = minor radius of curvature at waist

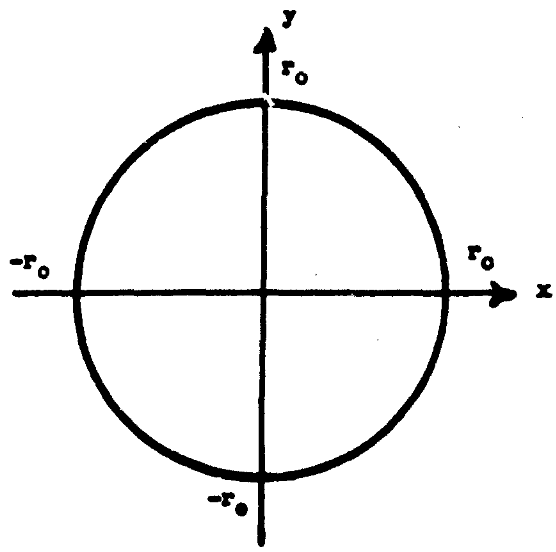


FIGURE 1

PROFILE OF LIQUID MASS AT TIME ZERO
(Coordinate axes x and y move with velocity = V)

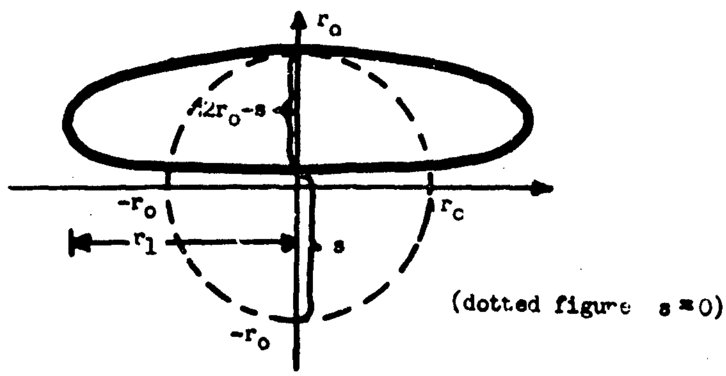


FIGURE 2

PROFILE OF LIQUID MASS WHEN $s_a > s > r_o$

The pressure relationship of the liquid mass, equation (1), may now be written:

$$2\gamma\left(\frac{1}{r_1}+\frac{1}{r_2}\right)=\rho_l\,(2r_o-s)\,g+\frac{2\gamma}{r_k} \tag{1a}$$

$$2\gamma\left(\left[\frac{3(2r_o-s)^2}{8r_o^4+2r_o\,(2r_o-s)^3-3r_o^2\,(2r_o-s)^2}\right]^{\frac{1}{2}}+\right.$$

$$\left.r_o^{-2}\left[\frac{8r_o^4+2r_o\,(2r_o-s)^3-3r_o^2\,(2r_o-s)^2}{3(2r_o-s)^2}\right]^{\frac{1}{2}}\right)$$

$$=\rho_l\,(2r_o-s)\,g+2\gamma\left[\frac{3(2r_o-s)^2}{4r_o^3+(2r_o-s)^3}\right] \tag{1b}$$

In order for the liquid mass to be stable, an aerodynamic pressure of P_a must be equal to the lesser of the two total pressures shown to be a surface-tension pressure at the waist of P_{sw} and a hydrostatic pressure of P_h plus a surface-tension pressure at the trailing point of P_{st}. The aerodynamic pressure, P_a, at the leading point can be expressed by the equation $P_a = 1/2\rho_a v^2$, where v is the velocity of the air relative to the mass. By solving equation (1b) for s, the general appearance of the liquid mass can be determined. The terminal velocity of the mass can then be determined from the terminal-velocity equation derived in appendix B. If the aerodynamic pressure at terminal velocity is less than the hydrostatic pressure when equation (1b) is satisfied, the mass will not break up by canopy formation, although it might disintegrate by the surface-stripping mode.

In the first stage of mass deformation, an oblate spheroid is formed. When $r_o \geq s \geq 0$, the radius of curvature, r_c, of the leading point will go to infinity. When the acceleration at the leading point, s'', is expressed in terms of the surface-tension pressure and aerodynamic pressure, an equation is obtained giving the distance the leading point travels as a function of time:

$$s'' = \frac{1}{\rho_l (2r_o - s)} \left[\frac{\rho_a v^2}{2} + \frac{2\gamma}{r_c} - \gamma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right] \qquad (2)$$

Rewriting the equation for v^2, derived in appendix B in terms of t, r, and r_o, the following equation is obtained:

$$v^2 = \frac{8\rho_l r_o^3 g}{3\rho_a C_D} \left[\frac{8r_o^4 + 2r_o(2r_o - s)^3 - 3r_o^2(2r_o - s)^2}{3(2r_o - s)^2} \right]^{-1} \times$$

$$\tanh^2 \left[\frac{3\rho_a g C_D}{4\rho_l r_o^3}\right]^{\frac{1}{2}} \left[\frac{8r_o^4 + 2r_o(2r_o - s)^3 - 3r_o^2(2r_o - s)^2}{3(2r_o - s)^2} \right] t \qquad (2a)$$

The radius of curvature of the leading point, r_c, is expressed in terms of s and r_o in equation (4), appendix A, as:

$$r_c = \frac{-4r_o^5 - r_o^2(2r_o - s)^3 + 5(2r_o - s)^2 r_o^3 + (r_o - s)^3(2r_o - s)^2}{3(2r_o - s)^2(r_o - s)^2} \qquad (2b)$$

Equation (2), which shows the acceleration on the leading point, can now be rewritten in terms of s, r_o, and t, to obtain t as a function of s'' and s.

The time for the development of the oblate spheroid, the first stage of deformation, can be determined in this manner. If equation (2) is rewritten in terms of s and t, the following equation is obtained:

$$
s'' = \frac{4r_o^3 g}{3(2r_o - s)C_D} \left[\frac{8r_o^4 + 2r_o(2r_o - s)^3 - 3r_o^2(2r_o - s)^2}{3(2r_o - s)^2}\right]^{-1} \times
$$

$$
\tanh^2\left(\left[\frac{3\rho_a g C_D}{4\rho_l r_o^3}\right]^{\frac{1}{2}} \left[\frac{8r_o^4 + 2r_o(2r_o - s)^3 - 3r_o^2(2r_o - s)^2}{3(2r_o - s)^2}\right] t\right) +
$$

$$
\frac{2\gamma}{\rho_l(2r_o - s)} \left[\frac{3(2r_o - s)^2(r_o - s)^2}{-4r_o^5 - r_o^2(2r_o - s)^3 + 5r_o^3(2r_o - s)^2 + (r_o - s)^3(2r_o - s)^2}\right] -
$$

$$
\frac{\gamma}{\rho_l(2r_o - s)} \left(\left[\frac{3(2r_o - s)^2}{8r_o^4 + 2r_o(2r_o - s)^3 - 3r_o^2(2r_o - s)^2}\right]^{\frac{1}{2}} +
r_o^{-2}\left[\frac{8r_o^4 + 2r_o(2r_o - s)^3 - 3r_o^2(2r_o - s)^2}{3(2r_o - s)^2}\right]^{\frac{1}{2}}\right) \qquad (2c)
$$

By solving the above equation for t when $r_o \geq s \geq 0$, the time to reach any stage of deformation in this interval of s can be estimated.

B. Development of Disk.

The development of the disk, the second stage of deformation, occurs when $s_a \geq s \geq r_o$, where s_a equals the value of s when the pressure relationship of the liqu[illegible] equation (1b) is satisfied. In this stage of deformation, the bottom of the mass has become flattened and $r_c = \infty$. The second term in equation (2) drops out, and the following equation is obtained:

$$
s'' = \frac{\rho_a v^2}{2\rho_l(2r_o - s)} - \gamma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \frac{1}{\rho_l(2r_o - s)} \qquad (3)
$$

Equation (3) can now be rewritten in terms of s, r_o, and t, and is analogous to the acceleration equation (2[illegible]). By solving these equations for t, the time to reach any stage of deformation in the interval $s_a \geq s \geq 0$ can be calculated.

C. Development of Canopy.

The canopy will begin to form (the third stage of deformation) after $s = s_a$, as indicated in figure 3. The minor radius at the waist, r_2, causes in effect a torus of thickness $2\,r_T$. This will determine the geometry of the canopy. The outer diameter of the torus is $2\,r_1$. The canopy can then be visualized as an inflating soap bubble formed from a soap film stretched across a ring. The ring in the case of the canopy is the torus. The body radius of the torus, r_T, is given by the following equations, which are derived in appendix C:

$$r_T = \left(\frac{r_o}{r_1}\right) r_2 \qquad \text{when } r_o \geq s \geq 0$$

$$r_T = \left(\frac{2r_o - s}{r_1}\right) r_2 \qquad \text{when } 2r_o \geq s \geq r_o$$

The inner diameter of the torus, $2r_w$, is given by the following equation:

$$2r_w = 2\left(r_1 - r_T\right)$$

The inner radius of the torus, r_w, is constant until $s_a = r_w$ as indicated in figures 2, 3, and 4, where s_a is the distance that the central point of the canopy travels after $s = s_a$. When $\infty \geq s_a \geq r_w$, then $r_w = r_c$.

When the latter condition has been met, the torus itself will expand as the canopy expands. The inner radius of curvature of the canopy, r_c, can now be expressed in terms of r_w and s_a to obtain the following equation:

$$r_c = \frac{r_w^2}{2s_a} + \frac{s_a}{2} = \frac{r_w^2 + s_a^2}{2s_a}$$

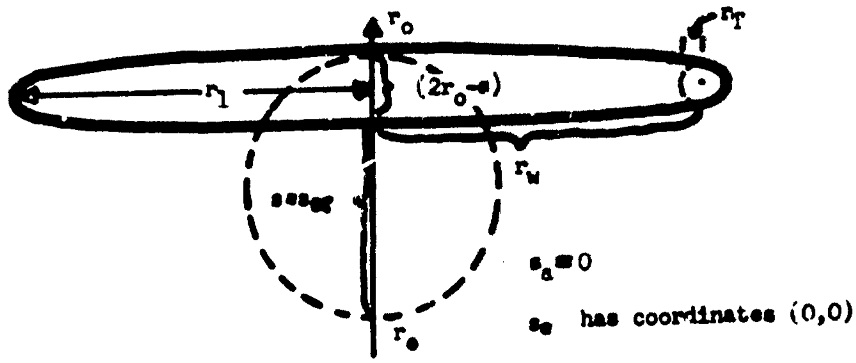
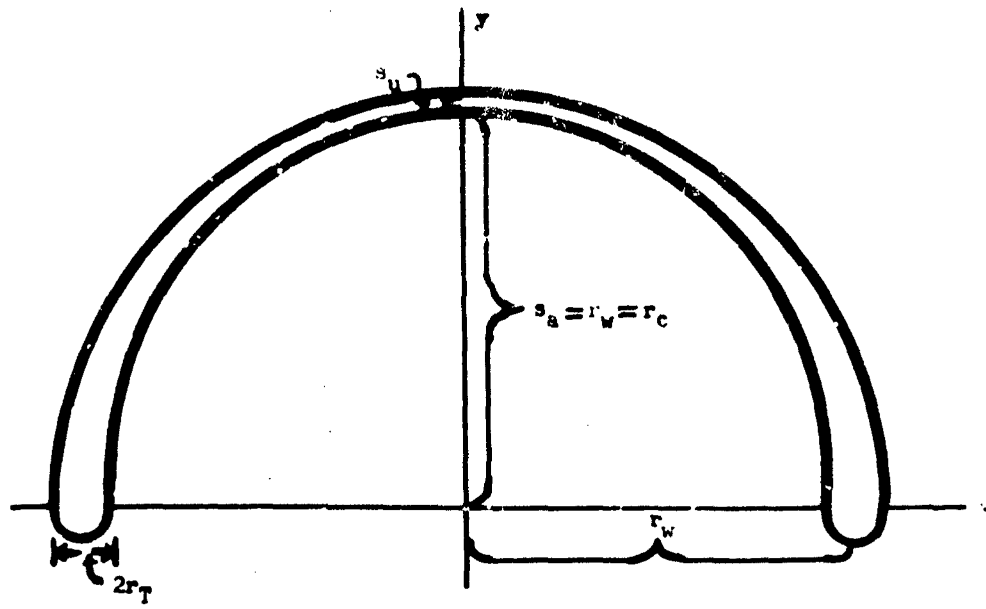


FIGURE 3

PROFILE OF LIQUID MASS WHEN $s = s_a$



(Canopy is a hemisphere $r_K \approx r_c$)

s_u = average thickness of canopy

FIGURE 4

PROFILE OF LIQUID MASS WHEN $s_a = r_w$

(Original axes transformed so that $y = s_a$)

The preceding equation shows that r_c will be a minimum when $s_a = r_w$. Figures 2, 3, and 4 show the general appearance of the liquid mass when $s_a > s > r_o$, $s = s_a$, and $s_a = r_w$, respectively.

An equation similar to acceleration equations (2) and (3) must now be found to describe the movement of the central point of the canopy.

The acceleration of the central point of the canopy is a function of aerodynamic pressure and the changing radius of curvature. During the major portion of the interval $r_w \geq s_a \geq 0$, the canopy has two surfaces with almost equal curvatures. The acceleration of the central point is now approximated by the following equation:

$$s''_a = \frac{\rho_a v^2 \pi r_w^2}{2M_c} - \frac{4\gamma \pi r_w^2}{M_c r_c} \tag{4}$$

where

M_c = mass of the canopy (constant)

$$M_c = \rho_\ell \left[\frac{4}{3} \pi r_o^3 - 2 \pi^2 (r_1 - r_T) r_T^2\right]$$

and

$$v^2 = \frac{4r_o^3 g \rho_\ell}{3\rho_a r_w^2 C_D} \tanh^2 \left(\frac{3\rho_a g C_D}{4r_o^3 \rho_\ell}\right)^{\frac{1}{2}} r_w(t_i + t)$$

where

t_i = value of t when $s = s_a$

Substituting the equations for M_c and v^2 into equation (4), the following equation is obtained:

$$s_a'' = \frac{2r_o^3 g}{C_D(2r_o^3 - 3r_w \pi r_T^2)} \tanh^2\left[\left(\frac{3\rho_a g C_D}{4r_o^3 \rho_\ell}\right)^{\frac{1}{2}} r_w(t_i + t)\right] - \frac{1}{\rho_\ell}\left[\frac{12\gamma r_w^2}{2r_o^3 - 3r_w \pi r_T^2}\right]\left[\frac{s_a}{r_w^2 + s_a^2}\right] \tag{4a}$$

By solving equations (2), (3), and (4), it is possible to determine the time it takes the liquid mass to deform up to the time when $s_a = r_w$.

D. Development and Expansion of Torus and Canopy.

The maximum radius of the liquid mass, r_w, is no longer constant after $s_a = r_w$, but is equal to r_c, and the torus begins to expand. In this fourth stage of deformation, the canopy will continue to inflate until the aerodynamic forces are greater than the cohesive forces of the canopy and the canopy ruptures.

The volume of the canopy remains constant if the total mass remains constant. The canopy will become increasingly thin as it undergoes expansion. The average thickness of the canopy, s_u, as shown in figure 4, is given by the following equation:

$$s_u = \frac{\text{volume of canopy}}{\text{area of canopy}} = \frac{V_c}{A_c}$$

The term A_c is a function of r_c, shown in appendix D, where the following equations are derived:

$$A_c = 2\pi\left[\frac{r_w^2 + s_a^2}{2s_a}\right]\left[\frac{r_w^2 + s_a^2}{2s_a} - \left(\frac{r_w^4 + 2s_a^2 r_w^2 + s_a^4}{4s_a^2} - r_w^2\right)^{\frac{1}{2}}\right]$$

when $r_w \geq s_a \geq 0$.

$$A_c = 2\pi\left[\frac{r_w^2 + s_a^2}{2s_a}\right]\left[\frac{r_w^2 + s_a^2}{2s_a} + \left(\frac{r_w^4 + 2s_a^2 r_w^2 + s_a^4}{4s_a^2} - r_w^2\right)^{\frac{1}{2}}\right]$$

when $\infty \geq s_a \geq r_w$.

When the conditions of the following equation, derived in appendix E, are met, the canopy will rupture (figure 5):

$$4\pi r_c s_u C_h = \pi r_c^2 \rho_a v^2 \tag{5}$$

where C_h is the tensile strength of the liquid.

Simplifying equation (5), expressing it in terms of s_a and r_w, and substituting M_c/ρ_a for V_c in $V_c = s_u A_c$, the following equation, derived in appendix E, is obtained:

$$\frac{8C_h\pi}{3\rho_a v^2}\left(2r_o^3 - 3\pi r_w r_T^2\right) = \left(\frac{r_w^2 + s_a^2}{2s_a}\right)^2 \left(\frac{r_w^2 + s_a^2}{2s_a} - \left[\left(\frac{r_w^2 + s_a^2}{2s_a}\right)^2 - r_w^2\right]^{\frac{1}{2}}\right) \tag{5a}$$

when $r_w \geq s_a \geq 0$, and

$$\frac{8C_h\pi}{3\rho_a v^2}\left(2r_o^3 - 3\pi r_w r_T^2\right) = \left(\frac{r_w^2 + s_a^2}{2s_a}\right)^2 \left(\frac{r_w^2 + s_a^2}{2s_a} + \left[\left(\frac{r_w^2 + s_a^2}{2s_a}\right)^2 - r_w^2\right]^{\frac{1}{2}}\right) \tag{5b}$$

when $\infty \geq s_a \geq r_w$ and where r_T is taken at $s = s_a$ and v is the velocity of the mass just prior to breakup.

$\therefore$ when $s_a \geq r_w \leq \infty$, $r_w = r_c$ and:

$$s_a'' = \frac{\rho_a v^2 \pi r_c^2}{2M_c} - \frac{4\gamma\pi r_c}{M_c} \tag{6}$$

where

$$v^2 = \frac{8r_o^3 g\rho_l}{3\rho_a r_c^2 C_D} \tanh^2 \left(\frac{3\rho_a g C_D}{4r_o^3\rho_l}\right)^{\frac{1}{2}} r_c(t_{ii} + t)$$

where t_{ii} is the total time elapsed from $s = 0$ to $s_a = r_w$

$$\therefore s_a'' = \frac{2r_o^3 g}{(2r_o^3 - 3r_w \pi r_T^2)C_D} \tanh^2 \left[\left(\frac{3\rho_a g C_D}{4r_o^3\rho_l}\right)^{\frac{1}{2}} \left(\frac{r_w^2 + s_a^2}{2s_a}\right)(t_{ii} + t)\right] - \frac{6\gamma}{2r_o^3 - 3r_w \pi r_T^2}\left(\frac{r_w^2 + s_a^2}{2s_a}\right) \qquad (6a)$$

IV. SUMMATION OF ANALYSIS.

Previous sections of this report show that from the acceleration equations (2), (3), (4), and (6) it is possible to determine the time it takes for the mass to reach any stage of deformation from time zero to the time when the canopy bursts.

If one assumes the shock of the liquid disintegration does not produce oscillations that would cause the torus to break up (since, at this point in the study, it appears that considerable energy is expended during breakup of the canopy), then, after the canopy has burst, the torus will remain intact until the aerodynamic and surface-tension pressures break it up into smaller more stable pieces. Since the velocity of the mass is known at the time of canopy rupture, it is possible to calculate the kinetic energy of the torus at this instant. The surface energy of the torus also may be calculated. The torus will break up into the number of pieces that will give a minimum of surface energy. The most probable number of pieces will be found when the torus breaks at intervals of 4.5 diameters, since this will give a minimum of surface area and, therefore, a minimum surface energy.[6]

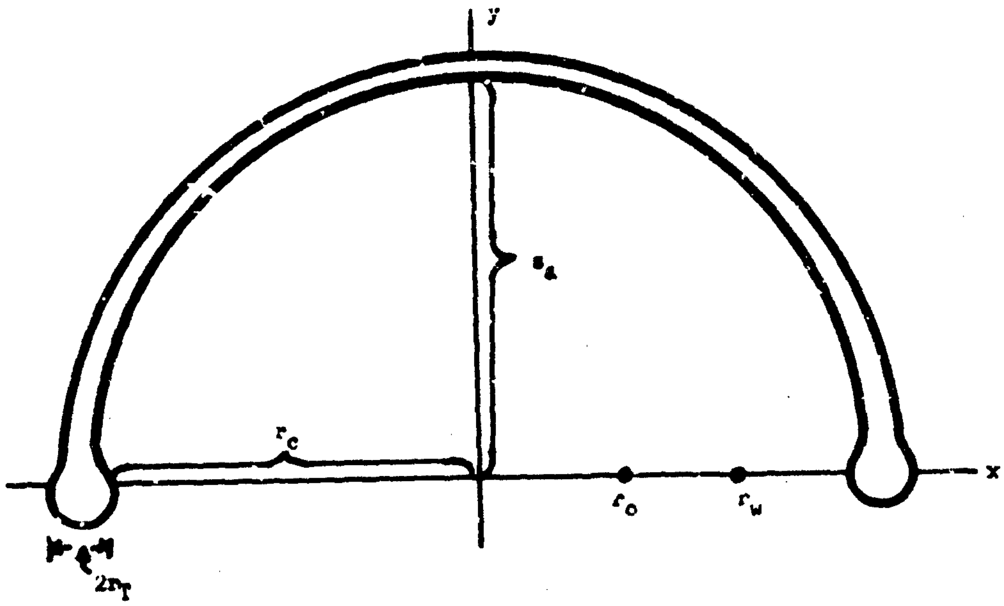


FIGURE 5

PROFILE OF LIQUID MASS JUST PRIOR TO CANOPY RUPTURE WHEN $\infty > s_a > r_w$

The radius of the spheres into which the torus breaks (r_s) is given by the following equation:

$$\frac{4}{3} \pi r_s^3 = \frac{9}{2} r_T \pi r_T^2 ; \qquad r_s = \frac{3}{2} r_T$$

where

$$2 r_c r_T^2 - r_T^3 - \frac{\text{volume of torus}}{\pi^2} = 0 \text{ and } r_T \text{ is taken just}$$

before disintegration.

If r_s is less than the maximum stable drop size determined by the acceleration equations (2), (3), (4), and (6), the system has become stable. If r_s is more than the maximum stable size, the whole process outlined in this paper must be repeated for each drop, taking into consideration that the initial velocity of the air is no longer zero.

V. DISCUSSION.

When deriving this theoretical treatment of canopy-formation breakup, two assumptions were made. First, it was assumed that the mass of the liquid remained constant throughout the deformation process. This assumption is valid only for relatively small liquid spheres. It follows from the definition of surface-tension pressure that, as a liquid mass becomes distorted into a larger shape, the surface-tension forces holding the outer layers of liquid will decrease with increasing surface area. The time required to reach any given free-fall velocity and the related aerodynamic force will also decrease as the sphere becomes larger. Consequently, at any given instant in the deformation process, there will be a greater probability of stripping off of surface layers of liquid from the mass. For spherical liquid masses in the diameter range of 1.2 cm to 4.0 cm, there is a negligible amount of surface stripping.[1,3] For diameters of over 4.0 cm, surface stripping can no longer be neglected,[3] and the theory derived in this paper must be combined with a theory of liquid-mass breakup caused by surface stripping. A future study is contemplated to show that very large liquid masses break up almost entirely by surface-stripping effects.

The second assumption was that the effect of the viscosity of the liquid was negligible. The viscosity may be neglected in the case of small masses of relatively nonviscous liquids of high surface tension such as water, but cannot be neglected in general. The effect of increased viscosity will be to slow down the rate of deformation by decreasing the acceleration of the leading point and, possibly, to slow down the acceleration to the point where the mass will be stable; i. e., the mass will have reached terminal velocity before the aerodynamic force can overcome the viscosity and surface tension of the liquid. Consequently, the rate at which a liquid mass will break up can be decreased by increasing the viscosity, or it can be increased by decreasing the surface tension. As a liquid mass becomes sufficiently large so that viscosity must be considered, the main mechanism of breakup is no longer canopy formation but surface stripping.

This theory was checked and found to be in agreement with the studies reported by Magarvey and Taylor[1] and Blanchard.[2] These agreements are shown in appendixes F and G. The difficulty of solving acceleration equations (2), (3), (4), and (6) analytically has made it impractical to check the times of deformation. The predicted general geometry assumed by the liquid mass is, however, in very good agreement with experimental results. The coefficient of cohesion between the liquid surfaces is uncertain since it depends on the impurities in the liquid. The coefficient of cohesion can be calculated by combining this theory with data obtained in canopy-breakup experiments. This cohesion coefficient is shown in appendix G to be considerably less than 1 atmosphere, which is much lower than would be predicted from molecular considerations. The effect of impurities on the tensile strength of a liquid is still not known in a quantitative sense. Before this theory can be tested fully or put to some practical use, more work on the tensile strengh of liquids is necessary. A satisfactory method for solving nonlinear differential equations of the second order, such as those describing the acceleration of the trailing point of the liquid mass, must also be found. For the time being, a numerical approximation must suffice.

In conclusion, it should be mentioned that the mathematical model postulated for the deformation process is certainly not an exact one. It appears to be a very close approximation as shown by figures and examples in appendixes C and F. If ellipsoids of revolution had been postulated as mathematical models instead of sections of spheres, this theory would not have been in as good agreement with experimental results as it is.

LITERATURE CITED

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GLOSSARY

A	- maximum cross-sectional area of liquid mass
A_c	- area of inner surface of canopy
C_D	- coefficient of drag
$F_{a_{max}}$	- maximum aerodynamic force acting on liquid mass
g	- acceleration caused by gravity
M	- total mass of liquid
M_c	- mass of canopy
P_a	- aerodynamic pressure on leading point of liquid mass
P_h	- hydrostatic pressure at any point or at waist of liquid mass
P_{sw}	- surface-tension pressure at any point on waist of liquid mass
P_{st}	- surface-tension pressure at trailing point
P_σ	- pressure of stability
r_o	- original radius of spherical liquid mass (figure 1)
r_1	- major radius of curvature at waist of mass (figure 2)
r_2	- minor radius of curvature at waist of mass
r_c	- radius of curvature of central leading point of liquid mass
r_k	- radius of curvature of central trailing point of liquid mass
r_T	- one-half thickness of torus; i.e., radius of body of torus
r_w	- inner radius of torus

r_s - radius of spheres into which torus breaks

R_e - Reynolds number

s - distance leading point travels toward trailing point in moving coordinate system where trailing point moves with same velocity as coordinate axes (figures 2 and 3)

s_α - value of s <u>when</u> equation (1b) is satisfied

s_a - distance central point on leading surface of canopy travels <u>after</u> equation (1b) is satisfied

s_u - average thickness of canopy (figure 3)

t - time elapsed between any given intervals of s or s_a

t_i - value of t when $s = s_\alpha$

t_{ii} - total time elapsed from $s = 0$ to $s_a = r_w$

v - velocity of atmosphere relative to liquid mass

v_t - terminal velocity of mass

V_1 - volume of liquid above waist of mass

V_2 - volume of liquid below waist of mass

V_c - volume of canopy

ρ_a - density of air

ρ_ℓ - density of liquid

μ_a - viscosity of air

μ_ℓ - viscosity of liquid

γ - surface tension of liquid

APPENDIXES

Appendix | Page

APPENDIX A

DERIVATION OF RADII OF CURVATURE AT WAIST AND AT LEADING AND TRAILING CENTRAL POINTS

In the range $r_o \geq s \geq 0$, where r_o is the original radius of the spherical mass and s is the distance that the central leading point travels toward the central trailing point in a moving coordinate system where the central trailing point remains stationary relative to the coordinate axes, there exists the following relationship:

$$\frac{4}{3} \pi r_o^3 = V_1 + V_2$$

where

V_1 = volume of liquid above waist of mass

V_2 = volume of liquid below waist of mass

V_1 and V_2, described as sections of spheres, are volumes of revolution and may be visualized in figure 1.

The volumes V_1 and V_2 then are calculated readily in terms of the radius of curvature, r_k, of the central trailing point and the radius of curvature, r_c, of the central leading point.

In general:

$$V_1 = \pi \int_{(r_k - r_o)}^{r_k} \left(r_k^2 - y^2\right) dy = \pi \left[r_k^2 y - \frac{y^3}{3} \right]_{(r_k - r_o)}^{r_k}$$

When $r_o \geq s \geq 0$:

$$V_1 = \pi \left(r_k r_o^2 - \frac{r_o^3}{3} \right) \tag{1}$$

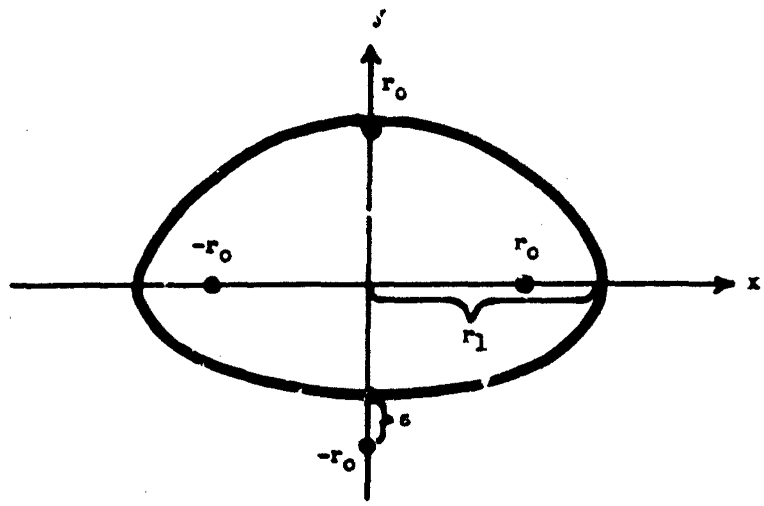

FIGURE 1

APPEARANCE OF LIQUID MASS IN INTERVAL $r_o \geq s \geq 0$

In general:

$$V_2 = \pi \left(r_c^2 - y^2 \right) dy = \pi \left[r_c^2 y - \frac{y^3}{3} \right]_{r_c - (r_o - s)}^{r_c}$$

When $r_o \geq s \geq 0$:

$$V_2 = \pi \left[r_c \left(r_o - s \right)^2 - \frac{(r_o - s)^3}{3} \right] \qquad (2)$$

Since the mass of the liquid is assumed constant and $\frac{4}{3} \pi r_o^3 = V_1 + V_2$, the following relationship exists:

$$\frac{4}{3} r_o^3 = r_k r_o^2 - \frac{r_o^3}{3} + r_c \left(r_o - s \right)^2 - \frac{(r_o - s)^3}{3} \text{ as } V_2 \longrightarrow 0,\ V_1 \longrightarrow \frac{4}{3} \pi r_o^3$$

When $2r_o \geq s \geq r_o$:

$$\frac{4}{3} \pi r_o^3 = \pi \int_{\left[r_k - (2r_o - s) \right]}^{r_k} \left(r_k^2 - y^2 \right) dy = \pi \left[r_k (2r_o - s)^2 - \frac{(2r_o - s)^3}{3} \right]$$

$$\therefore r_k = \frac{4r_o^3 + (2r_o - s)^3}{3(2r_o - s)^2} \qquad (3)$$

$$\therefore \frac{4}{3} r_o^3 = \frac{4r_o^5}{3(2r_o - s)^2} + \frac{(2r_o - s)\, r_o^2}{3} - \frac{r_o^3}{3} + r_c (r_o - s)^2 - \frac{(r_o - s)^3}{3}$$

$$r_c = \frac{-4r_o^5 - (2r_o - s)^3 r_o^2 + 5r_o^3 (2r_o - s)^2 + (r_o - s)^3 (2r_o - s)^2}{3(2r_o - s)^2 (r_o - s)^2} \quad (4)$$

r_1 = maximum radius of the mass (figure 1)

By expressing r_k in terms of its position in the moving coordinate system, the following relationships are obtained:

$$x^2 + y^2 = r_k^2 \qquad x^2 = r_1^2 \qquad y = r_k - r_o$$

$$\therefore r_1^2 = r_k^2 - (r_k - r_o)^2 = 2r_k r_o - r_o^2$$

$$r_1 = \left(2r_k r_o - r_o^2\right)^{\frac{1}{2}}$$

After substituting equation (3) for r_k, the following expression for r_1 is derived:

$$r_1 = \left[\frac{8r_o^4 + 2r_o (2r_o - s)^3 - 3r_o^2 (2r_o - s)^2}{3(2r_o - s)^2}\right]^{\frac{1}{2}} \quad (5)$$

r_2 = minor radius of curvature at the waist of the mass

When $r_1 = r_o$, $r_2 = r_o$ and as $r_1 \longrightarrow \infty$, $r_2 \longrightarrow 0$.

Therefore, by symmetry, the following expression for r_2 is derived (figure 2):

$$r_2 = \frac{r_o^2}{r_1} = \frac{r_o^2}{(2r_k r_o - r_o^2)^{1/2}} \quad (6)$$

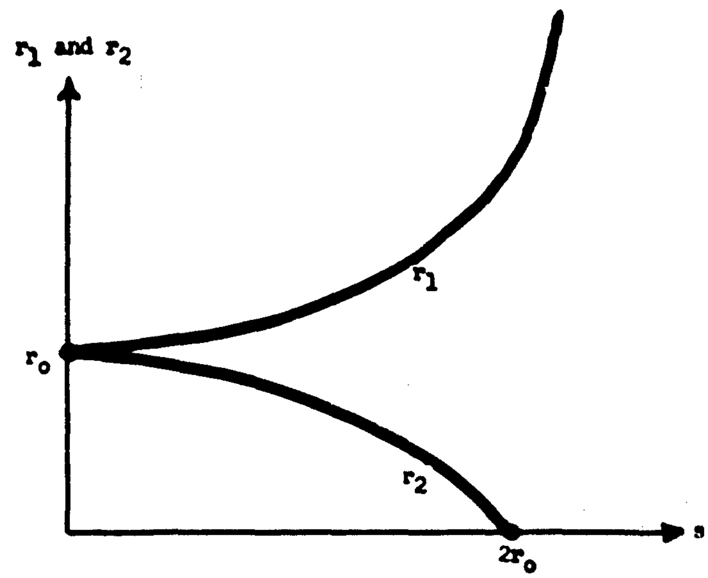

FIGURE 2

PLOT OF r_1 AND r_2 RELATIVE TO s

APPENDIX B

DERIVATION OF VELOCITY OF AIR RELATIVE TO FREE-FALLING MASS

The total acceleration acting on a mass may be expressed by the following equation:

$$\frac{dv}{dt} = g - \frac{\rho_a v^2 A C_D}{2 M} \tag{1}$$

where

ρ_a = density of air

g = acceleration due to gravity

v = velocity of air relative to mass

A = πr^2 = maximum cross-sectional area

t = time elapsed after beginning of free fall

C_D = coefficient of drag of mass (assume constant)

M = $\rho_1 \frac{4}{3} \pi r_o^3$ = total mass of sphere (assume constant)

By solving equation (1) for v, the following expression is obtained:

$$v = a\left[\frac{e^{2\,at/k} - 1}{e^{2\,at/k} + 1}\right] \tag{2}$$

where

$$a^2 = \frac{2\,Mg}{\rho_a A C_D}$$

$$k = \frac{2\,M}{\rho_a A C_D}$$

By rewriting and squaring equation (2), the following expression is obtained:

$$v^2 = a^2 \tanh^2(at/k) \tag{3}$$

By substituting the original values for a and k in equation (3), the following expression for the square of the velocity is obtained:

$$v^2 = \frac{8\rho_\ell r_o^3 g}{3\rho_a r_1^2 C_D} \tanh^2\left[\left(\frac{3\rho_a g C_D}{8r_o^3}\right)^{\frac{1}{2}} r_1 t\right] \tag{4}$$

where

$$r_1^2 = \frac{8r_o^4 + 2r_o(2r_o - s)^3 - 3r_o^2(2r_o - s)^2}{3(2r_o - s)^2}$$

r_o = original radius of spherical mass

s = distance traveled by leading point toward trailing point

ρ_ℓ = density of liquid

The coefficient of drag, C_D, is not constant for a sphere, but decreases with increasing Reynolds number at low Reynolds numbers. The mass, however, does not remain spherical. It assumes the shape of a section of a sphere and then becomes parachute-shaped. It is, therefore, convenient to assume a constant coefficient of drag of about 0.4, which should give a fairly good approximation of the terminal velocity of masses whose Reynolds number is about 10,000 at terminal velocity. The assumption of a drag coefficient of about 0.4 is justified and can be seen by checking a graph of drag coefficients versus the Reynolds numbers of spheres, cylinders, hemispheres, and disks: shapes that approximate the geometry of the deforming liquid mass. The greatest drag coefficient error will be encountered when the Reynolds number is less than 100. (This error is very small at Reynolds number greater than 100.) Consequently, the equation derived above should yield reasonable values for masses that have a Reynolds number of less than 100 for a relatively short period of time (less than 10%) during free fall.

APPENDIX C

DERIVATION OF BODY RADIUS OF TORUS

In figure 1, r_T is the radius of the body of the torus and r_2 the effective radius of curvature at the waist of the mass.

By trigonometric analogy, when $r_o \geq s \geq 0$

$$r_T = \left(\frac{r_o}{r_1}\right) r_2$$

When $s > r_o$, the liquid-mass profile has the appearance shown in figure 2.

Figure 2 may be approximated by the mathematical model shown in figure 3.

By trigonometric analogy r_T may be expressed as:

$$r_T = \left(\frac{2r_o - s}{r_1}\right) r_2 \qquad \text{when } 2r_o \geq s \geq r_o$$

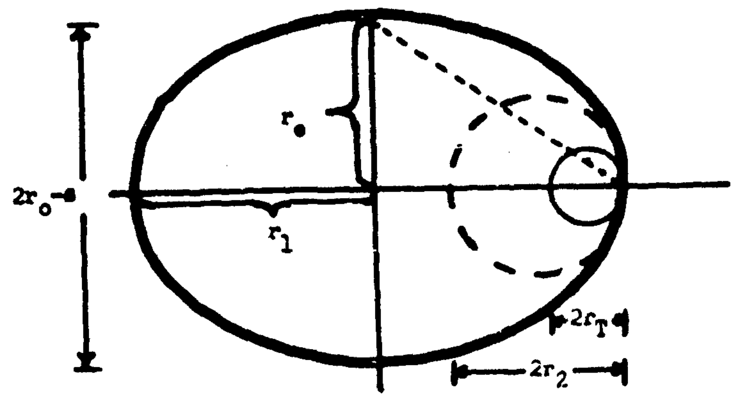

FIGURE 1

PROFILE OF LIQUID MASS WHEN $r_o \geq s \geq 0$

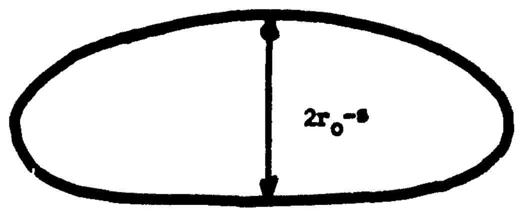

FIGURE 2

ACTUAL APPEARANCE OF LIQUID MASS WHEN $2r_o > s > r_o$

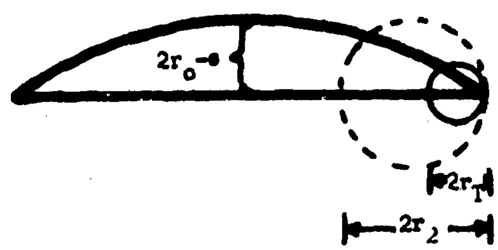

FIGURE 3

APPROXIMATE APPEARANCE OF FIGURE 2

APPENDIX D

DETERMINATION OF CANOPY AREA

To determine the area of the canopy, the profile of the canopy is given in figure 1 as a section of a circle of radius r_c.

$r_c = \dfrac{r_w^2 + s_a^2}{2s_a}$, where $r_w = r_1 - r_T$ when $s_a \geq 0$ (s_a is the distance traveled by the central point of the canopy in the moving coordinate system; $s_a = y$ after $s = s_a$).

The area of the canopy, A_c, is obtained by rotating a section of the circle around the y-axis.

$$dA_c = 2\pi x\, ds, \quad ds = \sqrt{dx^2 - dy^2}$$

$$x^2 + (y - s_a + r_c)^2 = r_c^2 \qquad \text{(equation of a circle)}$$

$$\therefore \quad x^2 + y^2 = 2s_a r_c - s_a^2 - 2y(r_c - s_a)$$

$$\text{and } -dy^2 = \frac{x^2 dx^2}{r_c^2 - x^2}, \text{ so that } ds^2 = \frac{r_c^2\, dx^2}{r_c^2 - x^2}$$

$\therefore$ When $r_w \geq s_a \geq 0$

$$A_c = 2\pi r_c \int_0^{r_w} x(r_c^2 - x^2)^{-\frac{1}{2}}\, dx$$

$$A_c = 2\pi r_c \left[-(r_c^2 - x^2)^{\frac{1}{2}} \right]_0^{r_w}$$

$$A_c = -2\pi r_c\left[(r_c^2 - r_w^2)^{\frac{1}{2}} - r_c\right]$$

$$A_c = 2\pi\left[\frac{r_w^2 + s_a^2}{2s_a}\right]\left[\frac{r_w^2 + s_a^2}{2s_a} - \left[\left(\frac{r_w^2 + s_a^2}{2s_a}\right)^2 - r_w^2\right]^{\frac{1}{2}}\right]$$

When $s_a \geq r_w$

$$A_c = 2\pi r_c^2 + 2\pi r_c \int_{r_w}^{r_c} x(r_c^2 - x^2)^{-\frac{1}{2}}\, dx$$

$$A_c = 2\pi r_c^2 + 2\pi r_c(r_c^2 - r_w^2)^{\frac{1}{2}}$$

$$A_c = 2\pi r_c\left[r_c + (r_c^2 - r_w^2)^{\frac{1}{2}}\right]$$

$$A_c = 2\pi\left[\frac{r_w^2 + s_a^2}{2s_a}\right]\left[\frac{r_w^2 + s_a^2}{2s_a} + \left[\left(\frac{r_w^2 + s_a^2}{2s_a}\right)^2 - r_w^2\right]^{\frac{1}{2}}\right]$$

A_c is the area of the inner surface of the canopy. The area of both surfaces is approximately $2\,A_c$. As $s_a \rightarrow \infty$, this approximation becomes exact.

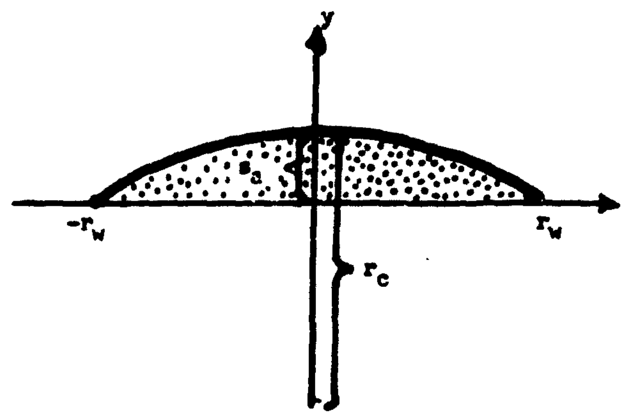

FIGURE 1

PROFILE OF CANOPY WHEN $r_w > s_a > 0$

APPENDIX E

DETERMINATION OF DISINTEGRATION POINT

The canopy will rupture when the aerodynamic force becomes greater than the tensile-strength force holding the canopy intact. The point of equilibrium between the two forces is given by the following expression:

$$2\pi r_c s_u C_h = \pi r_c^2 \frac{\rho_a v^2}{2} \quad \text{when } s_a \geq r_w \tag{1}$$

where

r_c = radius of curvature of canopy

s_u = average thickness of canopy

C_h = tensile strength of liquid

ρ_a = density of atmosphere

v = velocity of atmosphere relative to mass

Simplifying and solving equation (1) for r_c, the following equation is obtained:

$$r_c = \frac{4 s_u C_h}{\rho_a v^2}$$

Since $r_c = \dfrac{s_a^2 + r_w^2}{2 s_a}$, it is possible to find the value of s_a at which the canopy will rupture if s_u is also expressed as a function of s_a.

Since $s_u = \dfrac{V_c}{A_c} = \dfrac{\text{volume of canopy}}{\text{area of canopy}}$

$$A_c r_c = \frac{4V_c C_h}{\rho_a v^2}, \quad \text{or } A_c r_c = L, \quad \text{letting } \frac{4V_c C_h}{\rho_a v^2} = L.$$

From appendix D, when $s_a \geq r_w$:

$$r_c A_c = 2\pi \left[\frac{r_w^2 + s_a^2}{2s_a}\right]^2 \left[\frac{r_w^2 + s_a^2}{2s_a} + \left(\frac{r_w^4 + 2r_w^2 s_a^2 + s_a^4}{4s_a^2} - r_w^2\right)^{\frac{1}{2}}\right]$$

$$\therefore \quad r_c A_c = f(s_a)$$

By solving equation (1) in terms of r_c, the following equation is obtained:

$$r_c^4 - \frac{2 L r_c^3}{r_w^2} + \frac{L^2}{r_w^2} = 0 \qquad (2)$$

If equation (2) is solved for r_c and then substituted in the equation for s_a, the point of canopy rupture is obtained. The velocity, v, can be assumed constant at this point and is approximated by the following equation:

$$v = \left(\frac{8r_o^3 g \rho_\ell}{3\rho_a r_w^2 C_D}\right)^{\frac{1}{2}}$$

APPENDIX F

SAMPLE CALCULATIONS OF CRITERION FOR STABILITY OF DROPS

Consider a spherical drop of water for which $r_o = 0.45$ cm, γ = surface tension = 72 dynes/cm, and $\rho_\ell = 1$ gm/cc. Solving the acceleration equation (1b) of the text for s_a and then substituting s_a into the appropriate equations derived in appendixes A and C, the following values are obtained.

$s_a = 0.435$ cm	$(2r_o - s_a) = 0.465$ cm	$r_k = 0.719$ cm
$r_1 = 0.656$ cm	$r_T = 0.216$ cm	$r_w = 0.440$ cm

$$2\gamma\left[\frac{1}{r_1} + \frac{1}{r_2}\right] = \rho_\ell(2r_o - s_a)g + \frac{2\gamma}{r_k} \qquad (1)$$

$$\text{or } 144\left[\frac{1}{0.656} + \frac{0.656}{(0.45)^2}\right] = 0.465\,g + \frac{144}{0.719} = 656 \text{ dynes/cm}^2$$

All the spherical masses being considered have Reynolds numbers in the neighborhood of 10,000. Therefore, insofar as a spherical shape is concerned, $C_D = 0.4$ is a good general value for the coefficient of drag. By using the equation derived in appendix B, the terminal velocity for a spherical mass may be determined. This velocity is not the terminal velocity that the mass actually reaches, but is the velocity that the mass would reach if it retained its original spherical shape. The criterion for stability is that the hydrostatic pressure, P_h, at the leading point of the spherical mass be greater than the aerodynamic pressure, P_a, of the spherical mass at terminal velocity. This relationship may be expressed in the following manner:

$$\rho_\ell(2r_o)\,g \geq \frac{4r_o\rho_\ell g}{3C_D} \qquad \text{(stability criterion)}$$

where

r_o = radius of sphere

ρ_ℓ = density of liquid

C_D = 0.4 drag coefficient (when R_e = 10,000)

The stability criterion has the following appearance when r_o = 0.45 ρ_ℓ = 1: $\rho_\ell(2r_o)g$ = 883 dynes/cm^2 = P_h

$$\frac{4r_o^2\rho_\ell g}{3C_D} = 756 \text{ dynes/cm}^2 = P_a$$

A 9-mm-diameter mass of water is then seen to be stable and will not disintegrate by canopy formation in agreement with the finding of Blanchard[2] and Magarvey and Taylor.[1] This does not, however, preclude disintegration by rotation or oscillation.[2]

Magarvey and Taylor experimentally determined that the minimum waterdrop size that would disintegrate by canopy formation was about 1.2 cm diameter. Applying the stability criterion to a 1.2-cm drop, the following relationships are obtained:

$$\rho_\ell 2r_o g = 1{,}175 \text{ dynes/cm}^2 = P_h$$

$$\frac{4r_o^2\rho_\ell g}{3C_D} = 1{,}175 \text{dynes/cm}^2 = P_a$$

This is in excellent agreement with Magarvey and Taylor.

APPENDIX G

SAMPLE CALCULATION OF SIGNIFICANT PARAMETERS FOR WATERDROPS

Using the formulas derived in the previous appendixes and in the body of the report and applying them to a waterdrop 15 mm in diameter for which $r_o = 0.75$ cm, the following values for the significant parameters are obtained:

$2r_o - s_a = 0.490$ cm	equation (1b)
$r_k = 2.5$ cm	appendix A
$r_1 = 1.79$ cm	appendix A
$r_2 = 0.314$ cm	appendix A
$r_T = 0.086$ cm	appendix C
$r_w = r_1 - r_T = 1.704$ cm	

The torus will begin to expand when $s_a = 1.704$ cm.

When $s_a = 6$ cm, $r_c = \dfrac{s_a^2 + r_w^2}{2s_a} = 3.242$ cm

When $r_c = 3.242$ cm, $v_{max} = \left(\dfrac{8r_o^3 g\rho_\ell}{3\rho_a r_c^2}\right)^{\frac{1}{2}} = 298$ cm/sec

Maximum aerodynamic force ($F_{a_{max}}$) is given by the following expression:

$$F_{a_{max}} = \frac{1}{2}\rho_a v_{max}^2 \pi r_c^2 = 1.8 \times 10^3 \text{ dynes}$$

This is the total force available for disrupting the canopy.

From appendix F, $F_{a_{max}} = 2\pi r_c s_u C_h$

$$\therefore C_h = \frac{F_{a_{max}}}{2\pi r_c s_u} \quad \text{dynes/cm}^2$$

$$s_u = \frac{\text{volume of canopy}}{\text{area of canopy}} = \frac{V_c}{A_c}$$

$$V_c = \frac{4}{3}\pi r_o^3 = 2\pi^2 r_w r_T^2$$

$$V_c = 1.77 \text{ cc} - 0.242 \text{ cc} = 1.528 \text{ cc}$$

$$A_c = 2\pi r_c \left[r_c + (r_c^2 - r_w^2)^{\frac{1}{2}} \right] \qquad \text{appendix E}$$

$$A_c = 122 \text{ sq cm}$$

$$s_u = \frac{V_c}{A_c} = \frac{1.528}{1.22 \times 10^2} = 1.25 \times 10^{-2} \text{ cm}$$

$$C_h = \frac{1.8 \times 10^3 \text{ dynes}}{[6.28\ (3.242)]\ (0.0125) \text{ sq cm}} = 7.06 \times 10^3 \text{ dynes/sq cm}$$

This value of C_h is seen to be much lower than would commonly be predicted by quantum mechanics for water. It might be explained that a t[illegible] in the canopy begins around a nucleus that could be an impurity in the liquid.

The point of breakup was chosen from the photographs in Magarve and Taylor.[1] The other calculated parameters are in very good agreement with the photographs reproduced in the above-mentioned article.

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John D. Garcia and James D. Wilcox

CRDLSP 1-30, November 1961

Task 4C08-03-016-10, UNCLASSIFIED REPORT

A theory explaining the breakup of large drops by canopy formation has been derived. The theory explains the deformations that take place during the disintegration of a drop in terms of aerodynamic, hydrostatic, and surface-tension pressures. The effect of liquid viscosity has been assumed to be negligible. Therefore, the theory will not apply to liquids of high viscosity. This theory, insofar as it has been possible to verify it, is in good agreement with breakup experiments performed with liquid drops.

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3. Free-Fall Liquid Breakup
4. Drop Breakup
5. Terminal Velocity of Liquids
6. Breakup, liquid drops

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