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COLLISION DAMPING OF PLASMA OSCILLATIONS

D. F. DuBois, V. Gilinsky, and M. G. Kivelson

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The RAND Corporation
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PREFACE

This study is one aspect of our continuing work in basic and applied physics. It deals with the electromagnetic properties of plasmas, specifically with the absorption of longitudinal electromagnetic waves in a plasma. This short paper will be submitted to Physical Review Letters, and a more complete treatment of the problem of plasma oscillations will appear in a future report by the same authors.

SUMMARY

Calculations have been made of the absorption of longitudinal electromagnetic waves in a plasma (plasma oscillations). The results show that collision damping, which we calculate here, dominates the "collisionless" Landau damping for long wavelengths in high temperature, low density plasmas.

It is well known that the Vlasov equation, i.e., the collisionless Boltzmann equation with a self-consistent electric field, predicts a damping of plasma oscillations[1] (Landau damping) which vanishes exponentially for wave numbers k much less than the Debye wave number k_D. We have calculated the collision damping of long wavelength, small amplitude plasma oscillations in the high temperature weak coupling limit where $4\pi e^2 n^{1/3}/kT = (k_D^3/n)^{2/3} \ll 1$. For both the electron gas with a smeared background of positive charge and the electron-icn plasma the collision damping is considerably larger than the Landau damping for $k \ll k_D$.

In an electron-ion plasma the collision damping rate $\gamma_c(k)$ divided by the classical plasma frequency Ω_p is

$$\frac{\gamma_c(k)}{\Omega_p} = \frac{1}{12\pi}\left(\frac{2}{\pi}\right)^{1/2} \frac{k_D^3}{n} \ \ell n\left(\alpha \frac{4}{\sqrt{3}} \frac{k_T}{k_D}\right) \tag{1}$$

where $\beta = 1/kT$, n is the particle density and $\alpha = e^{-C}$, C is Euler's constant, k_T is the thermal de Broglie wave number $\hbar^2 k_T^2 = 3M/\beta$. This damping does not vanish in the k = 0 or long wavelength limit. On the other hand in the uniform electron gas the result is

$$\frac{\gamma_c(k)}{\Omega_p} = \frac{4}{15\pi^{3/2}}\left(\frac{k}{k_D}\right)^2 \frac{k_D^3}{n} \ \log\left[\alpha \frac{4}{\sqrt{3}} \frac{k_T}{k_D}\right] \tag{2}$$

which vanishes as $k \rightarrow 0$. At T = 0 and high densities γ_c/Ω_p (which includes exchange effects) is also of order k^2 for long wavelengths.[2]

These results are to be compared with the Landau damping rate which for both systems is

$$\frac{\gamma_L(k)}{\Omega_p} = (1 + \Lambda)\sqrt{\frac{\pi}{2}}\ \left(\frac{k_D}{k}\right)^2\ e^{-\frac{1}{2}\left(\frac{k_D}{k}\right)^2} \tag{3}$$

where Λ is of higher order in k_D^3/n. Both Eqs. (1) and (2) give collision damping rates considerably larger than Eq. (3) when $k \ll k_D$. It is particularly interesting that in the electron-ion plasma there is a residual damping even when $k = 0$. The vanishing of the damping in this limit in an electron gas is related to the fact that electron-electron scattering does not effect the long wavelength conductivity since total current and momentum are proportional. We also note that the results diverge in the classical, $\hbar \rightarrow 0$, limit. This is a well known short wavelength divergence[3] which in the classical theory is usually corrected by using a minimum impact parameter equal to $1/k_T$.

We can give here only an outline of our calculational methods which will be discussed in detail in a later communication. Briefly, the equation of continuity, Poisson's equation and the linear Ohm's law equation connecting the local current density to the local electric field are combined to obtain the following equation for the local charge density fluctuation $\rho(x,t)$

$$\frac{\partial\rho(x,t)}{\partial t} = 4\pi \int d^3x' \int dt'\ \sigma(x - x',\ t - t')\ \rho(x',\ t') \tag{4}$$

Here $\sigma(x - x',\ t - t')$ is the local electrical conductivity for the homogeneous system. The normal modes of oscillation for the charge density are then seen to have the following frequency dispersion equation

$$1 = 4\pi \frac{\sigma(k,\omega)}{\omega}$$

where $\sigma(k,\omega)$ is the complex Fourier transform of the conductivity. From this we find the plasma frequency Ω and the damping γ in the long wavelength limit when $\frac{\gamma}{\Omega} \ll 1$:

$$1 = 4\pi \text{ Re } \frac{\sigma(k,\Omega)}{\Omega}$$

$$\gamma(k) = 4\pi \text{ Im } \sigma(k,\Omega)$$

The calculation of the conductivity is reduced to a calculation of its imaginary part by use of the dispersion relation

$$\sigma(k,\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \frac{1}{\omega - \omega' + i\epsilon} \text{ Im } \sigma(k,\omega')$$

Starting with the recently developed techniques of quantum statistical perturbation theory in which the conductivity, expressed as a trace, is represented by a series of closed Feynman diagrams[(4)(5)] we have reduced the calculation to a consideration of open diagrams. This technique is analogous to the use of dispersion relations and the unitary diagram cutting techniques used in vacuum field theory.

The diagrams are conveniently ordered according to the number of particle-hole pairs (each pair representing a single particle excitation from the equilibrium medium) in the final state as in Fig. 1. The matrix elements for all contributions to a given process are then squared and averaged over final states in essentially the same manner as in calculating a transition probability. The Landau damping, which is the decay of the collective state into a single particle excitation, is given by diagram 1a while higher order diagrams such as 1b give rise to the correction factor Λ in Eq. (3). Diagrams 1c and 1d represent the decay into a single particle excitation which then scatters with another particle to form a final state of two single particle excitations, i.e., collision damping. In addition to diagrams 1c and 1d, two diagrams arise in which the labels of both final pairs are interchanged. The contribution from these four diagrams is

$$\operatorname{Im}\sigma_c(k,\omega) = \frac{1}{2} e^2 \frac{\omega}{k^2} \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \int \frac{d^3r}{(2\pi)^3} \int \frac{d^3s}{(2\pi)^3}$$

$$(2\pi)^2\, \delta^{(3)}(\vec{k}-\vec{p}+\vec{q}-\vec{r}+\vec{s})\; 2\pi\delta(\omega-\epsilon_p+\epsilon_q-\epsilon_r+\epsilon_s)$$

$$\left[f(\epsilon_r) - f(\epsilon_s)\right] \left[f(\epsilon_p) - f(\epsilon_q)\right] \left[g(\epsilon_p-\epsilon_q) - g(\epsilon_s-\epsilon_r)\right]$$

$$\left| \frac{4\pi e^2}{|\vec{r}-\vec{s}|^2} \left[\frac{1}{\omega-(\epsilon_{q+k}-\epsilon_q)} - \frac{1}{\omega-(\epsilon_p-\epsilon_{p-k})}\right] + \frac{4\pi e^2}{|\vec{p}-\vec{q}|^2} \left[\frac{1}{\omega-(\epsilon_{s+k}-\epsilon_s)} - \frac{1}{\omega-(\epsilon_r-\epsilon_{r-k})}\right] \right|^2$$

where $f(x) = \left[1 + e^{\beta x}\right]^{-1}$, $g(x) = \left[1 - e^{\beta x}\right]^{-1}$, $\epsilon_p = \frac{p^2}{2m} - \mu$, μ is the chemical potential.

When the masses are equal, as in the electron gas, there is a cancellation between the two terms within the absolute value sign for small k which explains why Eq. (2) vanishes when $k \longrightarrow 0$. For electron-ion scattering the first term in the absolute value is of order m/M compared with the second term and the result does not vanish as $k \rightarrow 0$.

The integrals are finite without including the screening of the interparticle interaction since conservation of energy and momentum prohibit momentum transfers in scattering which are less than $\hbar k_D$. The effect of screening is to change the argument of the logarithm in Eqs. (1) and (2) by a factor of the order of one.[5] Corrections of higher order in (k_D^3/n) arise from diagrams not considered here.

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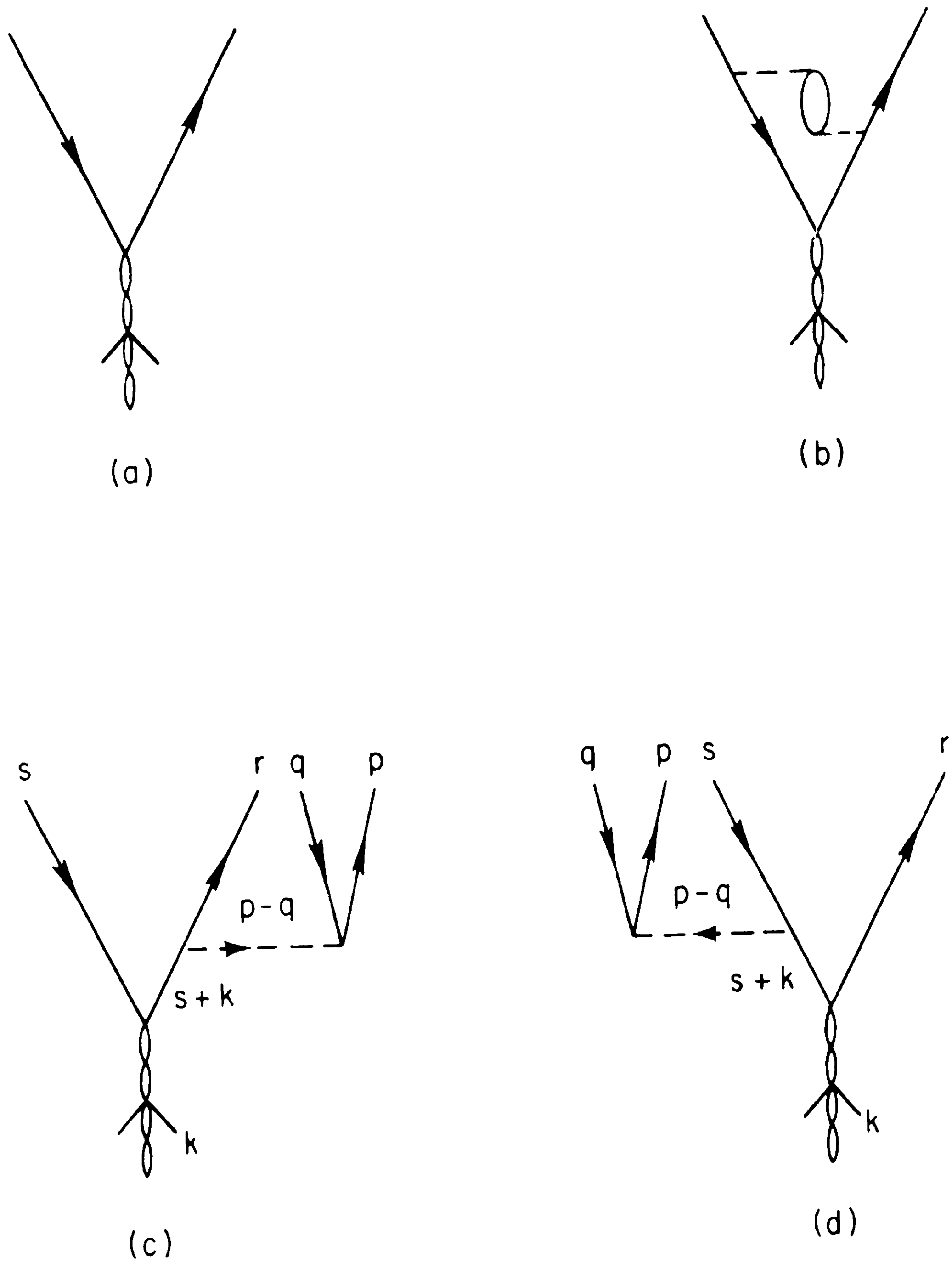

Fig. 1

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