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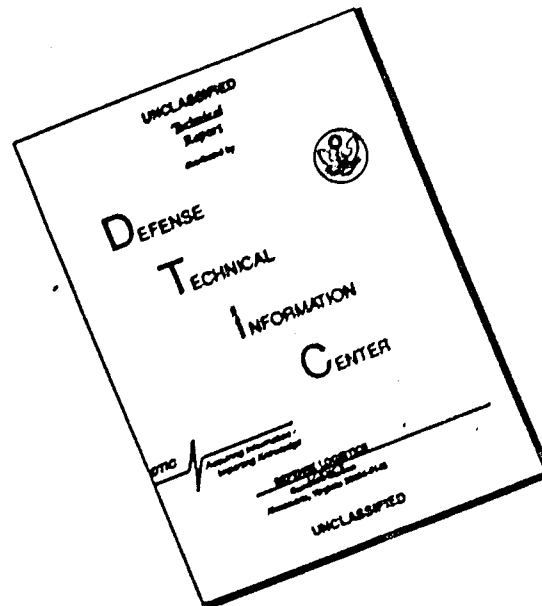
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**RESEARCH
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A NOTE ON THE SURFACE INTEGRAL OF LAMINAR HEAT
FLUX TO SYMMETRIC BODIES AT ZERO INCIDENCE

Nelson H. Kemp

RESEARCH REPORT 116

Contract No. AF 04(647)-278

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HEADQUARTERS
BALLISTIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE

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ABSTRACT

In the study of heating problems of missiles it is frequently necessary to obtain the surface integral of the heat transfer. This is usually obtained by a numerical integration of the heat transfer rate distribution. The purpose of this note is to show that, for laminar heat transfer to symmetric bodies at zero angle of attack, this integration can be done analytically, if only a few reasonable approximations are made. The result is expressible entirely in terms of quantities which also enter the heat transfer rate distribution expression, so performing the surface integral becomes a very simple matter of algebraic combination of known quantities.

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The "local similarity" theory of laminar heat transfer to bodies at zero angle of attack gives the heat transfer rate as¹

$$q = \frac{r^j \rho_w \mu_w u_e H_e}{\sqrt{2\xi}} \frac{g_{\eta w}}{\sigma_w} \quad (1)$$

Here r is the body radius in the cross-section plane, u_e and H_e velocity and stagnation enthalpy at the edge of the boundary layer, ρ_w , μ_w and σ_w the density, viscosity and Prandtl number at the wall, $g_{\eta w}$ the non-dimensional enthalpy gradient at the wall, and ξ a transformed coordinate related to the coordinate x parallel to the body surface by

$$\xi(x) = \int_0^x \rho_w \mu_w u_e r^{2j} dx. \quad (2)$$

The index j is zero for two-dimensional and unity for axisymmetric flow.

To integrate q over the body surface up to station x_i we multiply by the element of surface area $dA = 2(\pi r)^j dx$ and integrate from $x = 0$ to $x = x_i$, obtaining

$$Q_i = \int_0^{x_i} q dA = 2(\pi)^j \int_0^{x_i} \left[\frac{\rho_w \mu_w u_e r^{2j}}{\sqrt{2\xi}} \right] H_e \frac{g_{\eta w}}{\sigma_w} dx. \quad (3)$$

It is this integral which we will approximate.

First note that H_e is a constant. Second, note from Eq. (2) that the numerator of the term in the square brackets is $d\xi/dx$ so we can write

$$Q_i = 2 (\pi)^j H_e \int_0^{x_i} \frac{g_{\eta w}}{\sigma_w} d(\sqrt{2\xi}). \quad (4)$$

The Prandtl number may be taken to be a constant, σ . The non-dimensional enthalpy gradient $g_{\eta w}$ does not vary much around the body either¹, since it has a weak dependence on pressure gradient and a roughly square root dependence on $\rho\mu$ ratio. In fact, Lees² took it to be a constant at the zero pressure gradient, constant $\rho\mu$, value in his heat transfer theory. Thus, a reasonable approximation is to take a suitable average value, say $g_{\eta wo}$. We may then integrate Eq. (4) to obtain

$$Q_i = 2 (\pi)^j H_e g_{\eta wo} \sqrt{2\xi(x_i)} / \sigma. \quad (5)$$

This formula expresses the surface integral of the heat transfer rate in terms of the transformed body coordinate ξ at the end station of the integration interval x_i . However, ξ can be expressed in terms of q from Eq. (1), so Q_i can be related to $q(x_i)$ by

$$Q_i = 2 (\pi r_i)^j H_e^2 \left(\frac{\rho_w \mu_w u_e}{q} \right)_i \left(\frac{g_{\eta wo}}{\sigma} \right)^2. \quad (6)$$

A more useful form for blunt bodies is obtained by introducing the stagnation point heat rate by the appropriate limiting form of Eq. (1):

$$q_s = \left[(1+j) \rho_{ws} \mu_{ws} (du_e/dx)_s \right]^{1/2} H_e g_{\eta ws} / \sigma \quad (7)$$

where $(du_e/dx)_s$ is the stagnation point velocity gradient. Under the approximation that $g_{\eta w}$ is constant, division of Eq. (6) by Eq. (7), and use of Eq. (1) for q yields

$$\int_0^{x_i} \frac{q}{q_s} dA = \frac{\rho_{wi} \mu_{wi}}{\rho_{ws} \mu_{ws}} \frac{A_i u_{ei}/r_i}{(du_e/dx)_s} \left[\frac{q(x_i)}{q_s} \right]^{-1}, \quad (8)$$

where A_i is the body cross-sectional area at $x = x_i$. This is the principal result of the present note, showing that the surface integral of q/q_s is inversely proportional to the value of q/q_s at the end point of the integration interval. Since the total heat input to the body up to station x_i is proportional to the boundary layer thickness there, which in turn is inversely proportional to the local heat transfer rate, the result in Eq. (8) is physically reasonable.

Further simplification of Eq. (8) is obtained if the body can be taken to have a cool wall at which the perfect gas law is applicable. Then we have

$$\frac{\rho_{wi} \mu_{wi}}{\rho_{ws} \mu_{ws}} = \frac{p_{ei}}{p_{es}} \left(\frac{\mu_{wi}/T_{wi}}{\mu_{ws}/T_{ws}} \right), \quad (9)$$

where p_e is the pressure in the boundary layer. Finally, if the wall can be taken as constant temperature, the factor in parenthesis in Eq. (9) is unity and we have the simple result

$$\int_0^{x_i} \frac{q}{q_s} dA = \frac{p_{ei}}{p_{es}} \frac{A_i u_{ei}/r_i}{(du_e/dx)_s} \left[\frac{q(x_i)}{q_s} \right]^{-1}. \quad (10)$$

As an example, for a hemispherical nose of radius R at hypersonic speeds, u_e is practically a linear function of $x = R\theta$, and $r = R \sin \theta$. The area and velocity factor is then

$$\frac{A_i u_{ei}/r_i}{(du_e/dx)_s} = \pi R^2 \theta \sin \theta. \quad (11)$$

In the constant, low temperature wall case, Ref. 2, Eq. (15) or Ref. 3, Eq. (7) (with the correction of a square root in the denominator) gives

$$\frac{q}{q_s} = \frac{p_e}{p_{es}} \frac{\theta \sin \theta}{[F_1(\theta) + \beta F_2(\theta)]^{1/2}},$$

$$\frac{p_e}{p_{es}} = 1 - \beta \sin^2 \theta, \quad (12)$$

$$F_1(\theta) = \theta^2 - \theta \sin 2\theta + \sin^2 \theta,$$

$$F_2(\theta) = -3/4 \left[\theta^2 - \theta \sin 2\theta (1 + 2(\sin^2 \theta)/3) + \sin^2 \theta (1 + (\sin^2 \theta)/3) \right].$$

Thus, the integrated heat transfer becomes, from Eqs. (10)-(12),

$$\int_0^{\theta_i} \frac{q}{q_s} dA = \pi R^2 \left[F_1(\theta_i) + \beta F_2(\theta_i) \right]^{1/2}, \quad (13)$$

a result which agrees with a direct integration of Eq. (12).

If a direct integral formulation is desired, instead of an expression in terms of q , we can restore ξ in place of q in Eq. (8) by means of Eqs. (1) and (7):

$$\int_0^{x_i} \frac{q}{q_s} dA = \left(\frac{8}{1+j} \right)^{1/2} (\pi)^j \left[\int_0^{x_i} \frac{\rho_w \mu_w}{\rho_{ws} \mu_{ws}} \frac{u_e r^{2j}}{(du_e/dx)_s} dx \right]^{1/2} \quad (14)$$

Further simplification for a constant, low temperature wall leads to the formula analogous to Eq. (10):

$$\int_0^{x_i} \frac{q}{q_s} dA = \left(\frac{8}{1+j} \right)^{1/2} (\pi)^j \left[\int_0^{x_i} \frac{p_e}{p_{es}} \frac{u_e r^{2j}}{(du_e/dx)_s} dx \right]^{1/2} \quad (15)$$

If the hemispherical nose example of Eqs. (11) and (12) is used in Eq. (15), the integration can be performed, and the result agrees, of course, with Eq. (13).

The advantage of Eqs. (8) and (10), over Eqs. (14) and (15), is that the integrals in the latter are directly related to the heat transfer rate distribution, and so have already been calculated if that distribution is known. In this case the surface integral can be found from Eqs. (8) and (10) with no further integration. If, however, one desires to find the integrated heat input without first finding the heat flux distribution, Eqs. (14) and (15) are the useful ones.

If the integral of q itself is desired, the value of q_s should be found from the formulas given in Ref. 4.

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