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ESSENTIAL FACTORS OF THUNDERSTORM FORECASTING

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J. D. Sartor

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PREFACE

This Memorandum is one of a number of general studies of geophysical phenomena that may be of practical or theoretical interest to the Air Force. It points a way toward combining empirical analysis with mathematical formulation that could help in predicting thunderstorms. One such combination is effected. Although no further effort on thunderstorm prediction is to be made at RAND, it may prove useful if means can be found elsewhere for testing the proposed formulation. Moreover, the further exploration and utilization of the method for deriving such expressions may itself prove to be of considerable value.

SUMMARY

A method for formulating a mathematical relationship between the occurrence of thunderstorms and several essential prior conditions is demonstrated. A winnowing of 24 thunderstorm-forecasting parameters leads to the isolation of five essential factors and an expression demonstrating their possible relationship to thunderstorms.

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I. INTRODUCTION

The prediction of thunderstorm occurrence has proved over the years to be a difficult forecasting problem. Many papers have been written on the problem with almost as many approaches to the solution; none have offered a completely satisfactory solution.

In this investigation, an attempt is made to extract the essence of experience in forecasting thunderstorms from a number of papers and reports that have appeared in the recent literature. The distilled experience is then compared with kinematic and dynamic parameters to uncover similarities or differences that may prove interesting or instructive.

II. PROCEDURE

The procedure was to survey a representative number of empirical studies of thunderstorm forecasting and to prepare a composite list of the empirical forecasting parameters. The list was then studied critically with the objective of reducing the empirical forecasting parameters to the common essential factors found to be useful for forecasting thunderstorms. Finally, the distilled list of forecasting parameters was evaluated by physical reasoning and was compared with the analytical expressions of the kinematic motions and dynamic processes of the atmosphere.

III. SURVEY OF PAPERS ON EMPIRICAL FORECASTING OF THUNDERSTORMS

Papers dealing with all aspects of thunderstorm forecasting from empirical data have been included regardless of the type or origin of the thunderstorms. This may appear to be somewhat arbitrary, as many of the studies themselves purport to deal with one type of thunderstorm, e.g., air mass, frontal, or orographic. The value of including all types of data became obvious when the distillation of the empirical parameters was begun. In the first place, one cannot truly objectively distinguish between types. In the second place, common factors appear in all types, and it is the common factors that are sought here. Finally it appears that it is the separation into types that is arbitrary, not the combination of all types. Papers on severe storm and tornado forecasting are included in the study, since purportedly they investigate phenomena which, while extreme, are mainly special cases of thunderstorm development.

The choice of papers to include in the study was more or less random, but certain papers were chosen to broaden the list of forecast parameters or to ensure that a parameter of special interest would be included. Titles of all the papers included in the survey of forecasting parameters are given in the references.

Table 1 presents the composite list of parameters obtained from the survey of papers on thunderstorm forecasting. The survey started with Chapter IX of The Study of Air Mass and Isentropic Analysis by Jerome Namias (1940). This paper is the first chronologically and probably the most comprehensive of the papers surveyed in considering all types of thunderstorm activity; it contains a larger number of empirical parameters than most of the subsequent studies.

Table 1

FORECASTING PARAMETERS

| Parameter | Use of Parameter by Author Referenced | | | | |
|--|---|--|--|---|--|
| | Winston (Ed.) (1956) Nemias (Frontal) (1940) Nemias (Air Mass) (1940) Nemias (Isentropic) (1940) Fawbush, et al. (1951) | Means (1945) Miller & Panofsky (1958) MacDonald (1952) Crawford (1950) Bailey (1955) | Doege (1953) 2nd Weather Wing (1954) Whiting (1957) Telfer (1958) Neumann (1951) | Eyers and Rodebush (1948) Riehl (1949) | |
| 1. Air-Mass History. | x | | | | |
| 2. Convective Lifting Alone. | x | | | | |
| 3. Cyclonic Curvature. | | | x | | |
| 4. Cyclonic Shear. | | | | | |
| 5. Frontal Lifting (including troughs) . | x x | x | | x x | |
| 6. High Temperature. | x x | | | | |
| 7. Horizontal Convergence. | x x | | | x x | |
| 8. Horizontal Temperature Gradient . . . | | x x | | | |
| 9. Instability Indices | x x x | | x x x | x | |
| 10. Intersection of Wind and Moisture Axes | x | x | | | |
| 11. Low Pressure. | x | | | | |
| 12. Mechanical Lifting. | x x x | | x | x | |
| 13. Mechanical Turbulence | x | | | | |
| 14. Moisture. | x x x x | x x | x x | | |
| 15. Moisture and Isentropic Surfaces. . . | x x x | | | | |
| 16. Relative Advection. | x x x | | | | |
| 17. Small-Scale Perturbations | x x x | | x x | x | |
| 18. Strong Winds. | x x | | | | |
| 19. Turning of Wind | | | x | | |
| 20. Transport of Cold Air | x x* | | | | |
| 21. Upward Vertical Motion. | | x | | | |
| 22. Warm-Air Advection. | x x | x x | x | | |
| 23. Winds and Isentropic Surfaces | x x x | | | | |
| 24. Wind Shear. | x x | | | | |

*Mentioned implicitly.

In Table 1, the principal author of each of the papers is listed, and under his name an "x" is used to indicate each forecast parameter he employed. In preparing this list, the forecast parameters have already been digested to some extent in order to keep the list from being completely unwieldy. However, this has been done only where the intent of an author was obviously the same as that of another author who used a different terminology. An exception to this rule was made in connection with "instability indices." All "instability indices" were considered as having the same intent. This has the desired effect of considerably reducing the total number of parameters, since there are many ways of evaluating instability among the different authors.

Some of the papers like those by Namias (1940), Fawbush, Miller and Starrett (1951), and Winston (1956), consider large fractions of the total list of parameters. Other papers — for example, those of Means (1945), Crawford (1950), and Neumann (1951) — mention only one. This is because their studies dealt with a special situation and not because other important parameters were overlooked. For example, where Means and Crawford found explanations for thunderstorm activity in broad-scale atmospheric motion, Bailey (1955) and Telfer (1958) find the analysis of vertical soundings sufficient. To some extent this is explained in Bailey's paper. In the first place, his study is confined to forecasting local thunderstorm activity, thereby effectively eliminating explicit consideration of any variation in topography. Secondly, 189 soundings were chosen by Bailey for analysis out of an original total of 1000. The remaining 811 soundings were eliminated for various reasons. A group of 445 soundings were eliminated as being too dry above 850 mbs, using a very complex criterion. Another

group of 184 soundings was eliminated because of fronts, squall lines, and strong winds (25 knots at the 850 mb and 700 mb levels). A last group of 167 soundings at Lake Charles and Burrwood were eliminated when it appeared that tropical cumuli were diluting the good results being obtained at inland stations. With the large number of (811) soundings removed by so many criteria, some of which are mainly subjective, "instability" may not truly be a primary indicator.

The importance of atmospheric motions in organizing convection or producing the required vertical motion is discussed by Byers and Rodebush (1948). They point out, for example, that conditional and convective instability is present day after day over the tropical oceans but is maintained continuously with about the same degree of instability by means of convective overturning represented by only moderate-sized cumulous clouds. Cumulo-nimbus clouds and substantial quantities of rain are observed only when a disturbance producing large-scale horizontal convergence arrives. They conclude that thermal instability is a necessary but not sufficient condition for thunderstorm formation. Similar conclusions were reached by Riehl (1949) and the Second Weather Wing (1954). These considerations were weighed in the process of distilling the list of empirical thunderstorm forecasting parameters.

One may note the absence of two parameters prominently considered in forecasting. The first of these is the vorticity, which, although discussed in some of the papers, does not appear as a forecast parameter. This is probably due to the difficulty with which it is visualized on a synoptic map and the labor involved in its computation. Actually, its measurement might prove to be the most accurate method of obtaining the divergence and consequently the vertical motion.

Missing also is the pressure-jump hypothesis. This is mentioned by Winston (1956) but is shown to be of little utility, since the jump is not uniquely associated with thunderstorm activity, and where associations have been found, they are almost simultaneous. Since pressure jumps are not easily forecastable, they have not been included.

IV. EXTRACTING THE ESSENTIAL FACTORS

Often the short title given in Table 1 does not accurately or adequately convey the meaning of the original author. However, in considering the sufficiency of the factors to be included in a list of essential forecast factors, the author's defined or implied meaning was used to the greatest extent possible. A number of tentative lists of essential factors were prepared and, as additional studies were included, the lists were continually reviewed and revised until the list of factors exhibited in Table 2 was finally settled upon. The essential factors might have been expressed differently, but it appears that this list is suitable as a distillation of the total list of parameters presented. It is not possible, with complete objectivity, to place each of the parameters from the long list opposite one and only one of the essential factors. For example, "Warm Air Advection" contains concepts of wind shear and low-level warming. Generally speaking, moisture parameters have been considered as part of the concept of instability indices, since these are largely a function of the moisture distribution. The long list has, however, been regrouped so that, insofar as possible, the grouping will conform generally to the parameter of the composite list appearing on the left.

Table 2

ESSENTIAL FACTORS OF THUNDERSTORM FORECASTING PARAMETERS

| Essential Factor | Parameters |
|--------------------------------------|--|
| a. Instability Indices | Instability Indices (9) Moisture (14) Moisture and Isentropic Surfaces (15) |
| b. Horizontal Divergence | Horizontal Convergence (7) Low Pressure (11) Small-Scale Perturbation (17) Cyclonic Curvature (3) |
| c. Vertical Shear of Horizontal Wind | Strong Winds (18) Wind Shear (24) Wind and Moisture Axes (10) Turning of Wind (19) Winds and Isentropic Surfaces (23) Cyclonic Shear (4) Relative Advection (16) |
| d. Low-Level Warming | Convective Lifting (2) High Temperatures (6) Horizontal Temperature Gradient (8) Warm-Air Advection (22) Transport of Cold Air (20) Air Mass History (1) |
| e. Mechanical Lifting | Mechanical Lifting (12) Upward Vertical Motion (21) Mechanical Turbulence (13) Frontal Lifting (5) |

V. PHYSICAL EVALUATION OF ESSENTIAL COMMON FACTORS

The assumption is now made that the five essential common factors,

- a. Instability indices,
- b. Horizontal divergence,
- c. Vertical shear of horizontal winds,
- d. Low-level warming, and
- e. Mechanical lifting,

do indeed contain all the essentials necessary for forecasting thunderstorms.

As a check on this assumption, the role of each of these factors will be evaluated by physical reasoning and an attempt made to find a suitable means of combining them into a scheme for a practical forecast study.

An example of a similar approach to practical forecasting studies is contained in a study of precipitation amounts at Washington, D. C., by R. R. Rapp (1949). For a hypothesis, Rapp takes an expression for the precipitation rate derived by Fulks (1935), which is discussed and reanalyzed by Holmboe, Forsythe and Gustin (1945). The hypothetical expression makes it possible to evaluate the role of each of its variables by statistical analysis of meteorological data and to use those variables to forecast thunderstorms in the manner suggested by the theory.

In the present procedure we have evaluated a substantial number of empirical studies, isolating the five physical concepts listed above, which we have called essential factors. We now attempt to find a theoretical hypothesis linking the five factors and suitable for testing in the manner employed by Rapp (1949).

INSTABILITY INDICES

The Second Weather Wing (1954) attempted to determine the relative value of three types of instability indices as an objective forecasting tool for European thunderstorms. None of the instability indices performed satisfactorily. Added refinements to the procedures involving large-scale isobaric curvature and relative advection failed to improve the forecasting skill significantly.

The failure of these techniques, found to be generally useful for forecasting thunderstorms in the United States, was thought to be due to the more homogeneous air masses of Europe. In an attempt to uncover some relationship between forecasting parameters and thunderstorm activity in Europe, the Second Weather Wing prepared a series of daily charts covering the period of one summer by plotting thunderstorm occurrences at all stations. They analyzed the surface and upper air charts for all levels independently, taking care to incorporate all minor perturbations appearing in two or more adjacent contours. Superimposing the thunderstorm observation chart on the corresponding synoptic charts two levels at a time, they investigated the relationship of thunderstorm activity with features of the charts. As a result, the following conclusions were obtained:

a. Thunderstorms formed most frequently in areas indicated as unstable according to the Showalter index; however they frequently occurred in neutral areas and occasionally in stable regions.

b. With one exception the thunderstorms formed along lines of fronts or minor trough perturbations in the general flow and were associated with sectors of warm-air advection as deduced from the contour orientation at successive levels.

c. Most of the minor perturbations maintained continuity for several days, the associated thunderstorm activity exhibiting a diurnal cycle.

This work has been discussed in some detail because it suggests some significant considerations. Dynamic motions act to organize the convection resulting from surface heating in a manner necessary for the production of thunderstorms. This corroborates the findings of Byers and Rodebush in connection with Florida summer thunderstorms. They conclude that in each case of widespread thunderstorm activity previously attributed to insolation there must be also a dynamically induced source of low-level horizontal convergence, perhaps in some cases related to diurnal heating, and that, since high moisture content throughout a deep layer results from low-level convergence, the conclusion is in agreement with the well known correlation between thunderstorm areas and the locations of moist tongues on isentropic or related charts. Similar conclusions appear in the papers by Neumann (1951), Riehl (1949), and Means (1945). Since a measure of the moisture content of the air is implicit in an instability index and is highly correlated with local changes in convergence, it is not included as a primary factor.

HORIZONTAL DIVERGENCE

Experimental confirmation of the necessity for strong low-level convergence and high-moisture wedges in areas of active thunderstorms appears in The Thunderstorm, Byers (1949). These results connect the moisture with horizontal convergence and instability indices, thus providing further justification for the omission of moisture as an explicitly essential factor from the composite list.

Byers and Rodebush (1948) attribute the day-to-day changes in the thunderstorm activity of the Florida peninsula to changes in the degree of low-level convergence. The degree varies as the differential heating over land and over water is modified by the broader-scale circulation. Riehl (1949) comments on the Byers-Rodebush paper and shows the similarity between their results and earlier studies of his own, which demonstrate the insufficiency of instability analysis for forecasting thunderstorms and emphasize the importance of large-scale horizontal divergence and convergence. Neumann (1951) attributes the same role to the land breezes in the Mediterranean in the formation of nocturnal thunderstorms in that area.

The importance of horizontal divergence is, of course, the resulting vertical motion. When low-level convergence is associated with upper-level divergence, there must be an upward motion of the intermediate air in order to maintain continuity. The difference between the convergence at the lower level and the divergence at an upper level results in surface-pressure changes which have been shown to be a small difference between the total convergence and divergence integrated throughout the vertical. Miller and Panofsky (1958) discuss five different methods for estimating the vertical velocity, which they call the precipitation, the kinematic, the adiabatic, the vorticity, and the NWP methods. The precipitation and adiabatic methods measure the results of vertical motion and therefore are not forecasting parameters. The remaining methods, kinematic, vorticity, and NWP, involve directly or indirectly the computation of the divergence.

Miller and Panofsky show the relation between weather probability in fixed areas, dew-point depression at 850 mbs and vertical motion near 700 mbs. From their results it is apparent that clear weather can be

forecast quite well with these variables, since the probability varies between 92 per cent for dry air with downward motion to two per cent for moist air with upward motion. The range is much smaller for precipitation: from zero for dry air and downward motion to 59 per cent for moist air and upward motion. These results clearly establish an important role for the divergence in producing the vertical motion necessary for precipitation. Their explanation for the low probability of precipitation with upward motion and moist air is of interest in connection with the purpose of this study. As reasons for this they give: first, the high-moisture concentration at 850 mb may not always persist to higher levels. Second, the vertical motion and the moisture are measured locally, and drier air may be advected into the region of consideration from the outside. Third, the convective cells, superimposed on the generally upward moving air, may prevent rain from falling everywhere.

Similarly, the existence of a few cases of cloudy weather in subsiding dry air may be accounted for by advection from moister areas, or by errors in analysis of cloud type. However, without contradicting their conclusions, it might also be pointed out that an additional parameter increasing in magnitude along with the factors responsible for upward motion but having the opposite effect on precipitation may be acting to reduce the likelihood of precipitation.

VERTICAL SHEAR OF HORIZONTAL WINDS

Wind shear is considered to have two main implications. It is considered that it provides a measure of relative advection through the warm- and cold-air advection evidenced by the turning of the winds with height. It is also thought of as a mechanism for producing high-level

turbulence. The turbulence on the one hand could initiate overturning, which under appropriate initial stability conditions may be conducive to thunderstorm conception. On the other hand, the turbulence may be responsible for tearing apart growing cumulous clouds, thus preventing their ever reaching the mature stage. The correlation of warm-air advection with cloudiness and precipitation has been demonstrated many times. Warm-air advection appears as a parameter in a number of the reference studies, some dealing almost exclusively with it. The highly significant paper by Means (1945) explains the nocturnal maximum of thunderstorms by warm-air advection. MacDonald (1952) demonstrates how squall lines can be predicted and followed along the line of maximum warm-air advection at 850 mb. Namias (1940) gives warm-air advection a prominent role in the formation of frontal thunderstorms. Many of the other studies surveyed include parameters implicitly involving the advection of warm air into the lower levels where thunderstorms or tornadoes are to be forecast.

The dual role of strong vertical shear for thunderstorm initiation under one set of stability conditions and for their inhibition under another set is suggested by contrasting the papers of Bailey (1955) and Byers and Rodebush (1948) with those by Fawbush, Miller and Starrett (1951) and Winston (1956). The first two contain statements to the effect that thunderstorms are inhibited by strong winds aloft and pronounced vertical shear. The others give, as one criterion for tornadoes and severe thunderstorms, a band of strong winds aloft intersecting a moisture wedge in the lower levels that is apparently oriented with the general flow near the surface.

LOW-LEVEL WARMING

Low-level warming may be produced by warm-air advection in the lower levels and, in this respect, is not entirely divorced from the vertical wind shear. It must be retained as a separate factor, however, in order to include the effects of insolation heating and the modification of cold air moving rapidly over warm surfaces.

MECHANICAL LIFTING

Frontal lifting, orographic lifting, and turbulence or frictional drag produced by rough terrain are considered together in the factor of mechanical lifting. Professor T. Bergeron, at a cloud-physics seminar sponsored by the University of Stockholm in Törsta, Sweden, during the summer of 1955, exhibited a series of rainfall charts which clearly depicted zones of alternating maximum and minimum 24-hour precipitation amounts oriented parallel to the low-lying coasts of the Netherlands and southeastern Sweden. He attributed this pattern of rainfall to wave disturbances produced by the frictional drag on the on-shore moving air due to the greater roughness of the land surface in contrast to that of the sea surface. Namias (1940) mentions the same feature in connection with thunderstorm formation over rough, hilly terrain in the United States.

SUMMARY OF PHYSICAL EVALUATION

Summarizing the above evaluation, it appears that all of the five essential factors require consideration to some extent, even though in some cases two or more appear to have at least partially overlapping meaning. Instability indices seem to be best suited to the role of initial conditions, primarily in that they provide a measure of the potential energy of the atmosphere, and secondarily because the prognosis of vertical

soundings appears to be a time consuming step of considerable subjectivity that may be eliminated by allowing the other factors to act upon the initial instability to produce a more objective forecast of thunderstorm activity. At any one particular place, one or more of these essential factors may be sufficient to provide useful forecasts. This is due to the overriding importance of those one or two factors as a result of the special physiographic conditions at the particular place in question. No forecasting technique is known that is completely satisfactory. Perhaps the inclusion of some of the other factors could offer improvement. The present study is involved with thunderstorm forecasting in general, and as many as possible or all of the factors must be considered in an attempt to provide a coordinated hypothesis for testing with the data. We will, therefore, consider instability as an initial condition and seek a theoretical formulation of the remaining factors: horizontal divergence, wind shear, low-level warming and mechanical lifting, that could be applied to the forecasting of thunderstorms.

Because of the observed correspondence of thunderstorm activity to smaller-scale perturbations in the air flow and the generally irregular geographical distribution of precipitation, minor perturbations must not be masked by analytical smoothing. This will be kept in mind during the attempt to find a theory embracing the essential empirical factors.

VI. THEORETICAL APPLICATION OF EMPIRICAL FACTORS

This study does not attempt to offer a solution for forecasting thunderstorms but only to go as far as attempting to find a theoretical expression relating thunderstorms to the essential factors uncovered in the survey of a much larger number of parameters used in empirical forecasting studies. It is hoped that the theoretical expression could be used as hypothesis for statistical testing with suitable meteorological data.

A search of the literature for a theory relating horizontal divergence, wind shear, low-level warming, and mechanical lifting that could be used in thunderstorm forecasting uncovered a paper by S. M. Neamtan (1944), which discusses vertical motion in relation to precipitation processes and which provides a theoretical expression relating the individual pressure change to the horizontal divergence and the wind shear. Neamtan's development is as follows:

Writing,

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} \quad (1)$$

where p represents pressure, u , v , w are the components of the wind in the x , y , and z directions (z being vertical), and t indicates time.

Differentiating with respect to z , it is found after some rearrangement that

$$\frac{\partial}{\partial z} \left(\frac{dp}{dt} \right) = \frac{d}{dt} \left(\frac{\partial p}{\partial z} \right) + \frac{\partial u}{\partial z} \frac{\partial p}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial p}{\partial y} + \frac{\partial w}{\partial z} \frac{\partial p}{\partial z} \quad (2)$$

(The rearrangement involves the assumption that the partial derivatives are continuous. Since we wish to accept small-scale perturbations and

would like to investigate frontal zones in detail, the assumption is valid for our purposes.) Neglecting vertical accelerations,

$$g\rho = - \frac{\partial p}{\partial z} \quad (3)$$

Where g is the acceleration of gravity and ρ the air density. Substituting in the equation of continuity in the form

$$\frac{1}{\rho} \frac{d\rho}{dt} = - \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \quad (4)$$

we have

$$\frac{d}{dt} \left(\frac{\partial p}{\partial z} \right) = - \frac{\partial p}{\partial z} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \quad (5)$$

Substituting Eq. (5) into Eq. (2),

$$\frac{\partial}{\partial z} \left(\frac{dp}{dt} \right) = - \frac{\partial p}{\partial z} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \frac{\partial u}{\partial z} \frac{\partial p}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial p}{\partial y} \quad (6)$$

Integrating between levels 1 and 2,

$$\begin{aligned} \left(\frac{dp}{dt} \right)_2 &= \left(\frac{dp}{dt} \right)_1 - \int_{z_1}^{z_2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \frac{\partial p}{\partial z} dz + \\ &\int_{z_1}^{z_2} \left(\frac{\partial u}{\partial z} \frac{\partial p}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial p}{\partial y} \right) dz \end{aligned} \quad (7)$$

Or

$$\begin{aligned} \left(\frac{dp}{dt} \right)_2 &= \left(\frac{dp}{dt} \right)_1 + g \int_{z_1}^{z_2} \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz + \\ &\int_{z_1}^{z_2} \left(\frac{\partial u}{\partial z} \frac{\partial p}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial p}{\partial y} \right) dz \end{aligned} \quad (7a)$$

This development suggests that either of these equations, (7 or 7a), expresses, in a functional form, a relationship between most of the essential factors used for forecasting thunderstorms. Instability indices do not appear explicitly in the equations but may be incorporated indirectly according to the previous implication that, in objective studies, the value of the instability index is used much as an initial condition. The vertical motion derived from the calculation of $\frac{dp}{dt}$ may be used to modify the initial instability index to obtain a forecast instability index, and it is this parameter that would be related empirically to thunderstorm occurrence. The value of $\left(\frac{dp}{dt}\right)_1$, can be evaluated at the surface by considering the component of the surface wind normal to the contours of the surface of the earth or to those of a meteorological front, thus providing a measure of mechanical lifting. In a like manner, low-level warming contributes to $\left(\frac{dp}{dt}\right)_1$ by transforming the resulting density change due to insolation heating or advective warming into pressure change. With these suggestions in mind, the author presents the relation of the five essential factors in thunderstorm forecasting to terms in equation (7a) in the following table.

Table 3

RELATION OF ESSENTIAL FACTORS TO TERMS OF EQUATION (7a)

| Factor | Relationship to Eq. (7a) |
|-----------------------|--|
| Instability indices | Implicit in $\left(\frac{dp}{dt}\right)_2$ (See text) |
| Horizontal divergence | $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ |
| Wind shear | $\frac{\partial u}{\partial z} \frac{\partial p}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial p}{\partial y} = \frac{\partial \tilde{v}}{\partial z} \cdot \nabla_H P$ |
| Low-level warming | Implicit in $\left(\frac{dp}{dt}\right)_1$ (See text) |
| Mechanical lifting | Implicit in $\left(\frac{dp}{dt}\right)_1$ (See text) |

The actual feasibility of this expression (from equations 7 or 7a) for practical forecasting cannot be evaluated at this time. It is presented to demonstrate the possibility of obtaining a theoretical formulation of the empirical parameters that could be applied to meteorological data for the purposes of an objective thunderstorm-forecasting study.

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