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AVERAGE NUMBER OF NUCLEONS IN COSMIC RAYS INDUCED NUCLEON CASCADES

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This is a preliminary report on studies which have been undertaken dealing with various effects of cosmic rays on matter. In particular, this report will be concerned with the development of nucleon cascades especially the similarities and differences in their development in various materials. The report's motivation is further described in the conclusion.

Introduction

Many properties of cosmic rays are now fairly well established. For example, it is known that particles of energies from about a hundred Mev to about 10^{10} Gev are present in the flux with an energy distribution which has been roughly determined.[1] It is known that protons are the primary constituent ($\approx$87%), alpha particles being about 13% of the incoming beam with minor contributions from heavier nuclei making up the remainder.[2] It is also approximately valid that the cosmic ray flux varies little as a function of time and is nearly isotropically distributed in space.

A study of the effects of cosmic rays on matter, then, must concern itself with an extremely large energy range if completeness is to be obtained. Certainly an exact treatment valid over such a range would be very difficult to find and will undoubtedly never be accomplished. It would seem far better to treat the problem by dealing with various energy ranges individually using, in those ranges, appropriate approximations. At least three energy ranges come immediately to mind differentiated by characteristically different processes. We shall call these regions*: the low energy range ($E < 10$ Gev), medium ($1 \text{ Gev} < E < 100$ Gev) and high energy ($E > 100$ Gev).

* Of course, these energy divisions are only approximate and are only meant to indicate general areas of division.

In the low energy range, the processes are primarily those well known from previous nuclear studies. Thus, the primary mode of energy loss is by ionization, losses by nuclear interactions becoming less important as the energy decreases. The production of mesons, although energetically allowed for energies greater than 290 Mev, plays a minor role because of a still small cross section at these energies.

In the medium energy range, however, one finds a new phenomena coming into importance, the production of nucleon cascades. Consider a proton of about 10 Gev energy which impinges upon matter. For these energies, loss of energy by ionization is usually negligible so that the proton will pass through the material until it makes a direct collision with some nucleus. Since the DeBroglie wavelength of protons at this energy is small compared to the size of the nucleons, the proton can be considered to interact with a single nucleon in the nucleus. In general, both the incident proton and the recoiling nucleon will leave the collision with a large amount of energy compared to a nucleon's binding energy. Thus, not only is the incident proton free to leave the nucleus, but so may the struck nucleon. However, instead of leaving the nucleus immediately there is a large probability that these nucleons will each collide with other nucleons because the mean free path of nucleons in nuclear matter is small at these energies. Thus, it is possible for nucleon cascade to begin; one nucleon incident on the nucleus with several leaving it. This phenomena is not possible at lower energies because the amount of energy transferred in a collision would not be enough to sustain a cascade.

Another very important effect manifests itself in the medium energy range, the production of mesons. Although energetically possible at lower energies, it is not until these medium energies are reached that mesons are produced in appreciable numbers. Thus, one must now also consider the possible production of mesons and their interaction with matter. This is a great complication not only because of the new degrees of freedom introduced, but also because many of the necessary facts are either unknown or known only poorly.

Because of the presence of π^0 mesons which are now produced in this energy range, one can also find large electron-photon showers. (Extensive air showers, EAS, is the name commonly reserved for these showers when observed in the atmosphere.) These showers have been studied extensively, but no entirely successful treatment has been given which considers their development in connection with the parent nucleon cascade. There is no question that these showers are an important consideration when dealing with the effects of cosmic rays on matter.

In the high energy range all the above phenomena take place but in addition K mesons and hyperons are produced with more frequency. The electron-photon showers become much larger and, most important from our viewpoint here, the nature of the nucleon cascade changes somewhat.

In the medium energy nucleon-nucleon collision, the recoiling particles separate at such an angle in the laboratory system that when the secondary collisions are made, the original two nucleons are well separated and the secondary collisions can be considered as independent of each other. This is not so at high energies. In this case, the center of mass velocity is quite large so that when the problem is transformed back to laboratory coordinates all CM angles become collimated into very nearly the forward direction. Thus, after the first collision the recoil nucleons do not separate appreciably so that their further collisions cannot be considered separately. Thus, a somewhat different description of the cascade in this region should be employed*.

The remainder of this report will concern itself with the description of a preliminary attach on the problems encountered in the medium energy range.

Homogeneous Nuclear Matter

As a beginning towards consideration of the medium energy problem this report will concern itself with a determination of the average number of cascade nucleons to be expected as a function of the depth below the material surface when a proton of appropriate energy strikes the material. In this respect, we shall not consider the effect of mesons upon the nucleon cascade except insofar as one allows nucleon-nucleon collisions to be inelastic. Thus, it will be assumed that in a collision some energy is radiated as mesons, but those mesons will be considered no farther. No distinction will be made between neutrons and protons in the following so that when one speaks of a cross-section for nucleon-nucleon interaction what is actually meant is the interaction averaged over spin and isobaric spin.

Let us consider at first a semi-infinite slab of homogeneous nuclear matter and calculate the average number of nucleons with energy $\geq E$, $N(\frac{E}{E_o},x)$, present at a depth x below the surface due to the interactions of a single nucleon of energy E_o incident on the surface. We shall derive a diffusion-like equation for N.

Let $w(E_o;E',E'')\,dE'dE''dx$ be the probability that a nucleon of energy E_o suffers a collision with a second nucleon in dx with the result that

* The "tunnel" model proposed by McCusker and Roessler[3] and modified by Cocconi[4] would seem to be appropriate although somewhat crude.

the scattered nucleons* have energies in the ranges E' to $E' + dE'$ and E'' to $E'' + dE''$. We shall assume that the collision may be inelastic so that $E' + E''$ need not equal E_o but may be less (the excess energy being radiated as mesons).

The functional form of w is almost completely unknown so that it has beem customary to assume some reasonable dependence in order to simplify calculations.[5] Thus, we shall assume

1) $$w(E_o; E', E'')\, dE' dE'' = w\left(\frac{E'}{E_o}, \frac{E''}{E_o}\right) \frac{dE'}{E_o} \frac{dE''}{E_o} \quad ,$$

a form which seems reasonable when one considers the analogy between this process and that of bremsstrahlung radiation, the latter having a cross-section of this form. The theory can be carried through using only equation 1), however, in order to obtain numerical results one must be more specific. This will be considered later.

Conservation of energy restricts the values of E' and E'' such that $E' + E'' \leq E_o$. If one defines $\varepsilon' = \frac{E'}{E_o}$ and $\varepsilon'' = \frac{E''}{E_o}$ one can put

2) $$w(\varepsilon', \varepsilon'') = 0 \ ; \quad \varepsilon' + \varepsilon'' > 1 \quad .$$

The total collision probability per unit path length, α, is given by

3) $$\int_0^1 \int_0^{1-\varepsilon'} w(\varepsilon', \varepsilon'')\, d\varepsilon'\, d\varepsilon'' = \alpha \quad .$$

The differential equation for the development of the nucleon cascade can be obtained by considering Figure 1. The number of nucleons of energy

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* This treatment is entirely one dimensional neglecting any direction changes in the scattering process. At these energies where most of the laboratory scattering is in the forward direction, this assumption may not be too poor.

$\geq E$ which appear at a depth x below the incidence of a particle of energy E_o is denoted by $N(E/E_o,x) = N(\varepsilon,x)$.

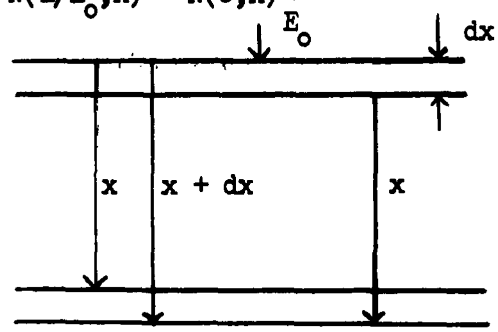

Figure 1

The number present at a depth x + dx can be found by first considering the contribution from those particles entering the material at a depth dx from top surface. Each particle of energy E_o present at depth dx will give a contribution at depth x + dx equal to $N(\frac{E}{E_o},x)$. Since there is one incident particle, one may consider $(1 - \alpha dx)$ to be the number of particles to reach the depth dx with the primary energy E_o. Thus, the total number of particles reaching depth x + dx due to particles whose first collisions were at depths greater than dx is $(1 - \alpha dx)\, N(\frac{E}{E_o},x)$. One must still add in the contribution due to particles which interacted in dx giving rise to secondaries of energy E' and E'' which themselves may cascade to contribute at x + dx.

The probability that a collision takes place in the distance dx giving rise to secondaries of energies E' and E'' in the ranges dE' and dE'' respectively is $w(\frac{E'}{E_o},\frac{E''}{E_o})\, \frac{dE'dE''}{E_o\,E_o}\, dx$.

The particle of energy E' may now be considered as a primary which will produce at a depth x below its point of production $N(\frac{E}{E'},x)$ nucleons of energy greater than E. The average number of nucleons (of energy $\geq E$) than, at x + dx due to the cascade of the particle of energy E' will be

$$\int_{E''=0}^{E_o-E'} N(\frac{E}{E'},x)\; w(\frac{E'}{E_o},\frac{E''}{E_o})\; \frac{dE'}{E_o}\; \frac{dE''}{E_o}\; dx$$

with a similar expression for the contribution of E". Summing the two and integrating over all possible values of E' and E" one finds (making use of 2)

$$4)\quad N\left(\frac{E}{E_o},x+dx\right) = (1-\alpha dx)\, N\left(\frac{E}{E_o},x\right) + \int_o^\infty\int_o^\infty \left[N\left(\frac{E}{E'},x\right) + N\left(\frac{E}{E''},x\right)\right] \cdot w\left(\frac{E'}{E_o},\frac{E''}{E_o}\right)\frac{dE'}{E_o}\frac{dE''}{E_o}\, dx \quad .$$

This equation can be written by rearranging, dividing by dx, and substituting ε' for E'/E_o etc.

$$5)\quad \frac{dN}{dx}(\varepsilon,x) + \alpha\, N(\varepsilon,x) = \int_o^\infty\int_o^\infty N\left(\frac{\varepsilon}{\varepsilon'},x\right) + N\left(\frac{\varepsilon}{\varepsilon''},x\right) w(\varepsilon',\varepsilon'')\, d\varepsilon'\, d\varepsilon'' \quad .$$

The solution of equation 5) will then give, for homogeneous nuclear matter, the average number of nucleons with energy greater than $E = \varepsilon E_o$ at a depth x below a surface due to the incidence of a nucleon of energy E_o. Then defining

$$w(\varepsilon) = \int_o^{1-\varepsilon} \left[w(\varepsilon,\varepsilon'') + w(\varepsilon'',\varepsilon)\right] d\varepsilon'' \quad \text{where} \quad w(\varepsilon) = 0,\ \varepsilon > 1$$

one has

$$6)\quad \frac{dN(\varepsilon,x)}{dx} + \alpha\, N(\varepsilon,x) = \int_o^\infty N\left(\frac{\varepsilon}{\varepsilon'},x\right) w(\varepsilon')\, d\varepsilon' \quad .$$

This equation may be solved by the use of Mellin transforms.[6] Defining the transform pairs

$$M(s,x) = \int_o^\infty \varepsilon^{s-1}\, N(\varepsilon,x)\, d\varepsilon$$

$$N(\varepsilon,x) = \frac{1}{2\pi i}\int_{s_o-i\infty}^{s_o+i\infty} \frac{M(s,x)dx}{\varepsilon^s} \quad \text{where } s_o > 0$$

and applying these to 6) one finds

$$7)\quad \frac{dM(s,x)}{dx} + \alpha\, M(s,x) = M(s,x)\, w(s)$$

where

8) $$w(s) = \int_0^\infty \varepsilon^s\, w(\varepsilon)d\varepsilon \quad .$$

Equation 7) is easily solved to give

9) $$M(s,x) = M(s,0)\; e^{-\left[\alpha - w(s)\right]x}$$

but $M(s,0) = \int_0^\infty \varepsilon^{s-1}\, N(\varepsilon,0)d\varepsilon = 1/s$ because $N(\varepsilon,0) = 1$ for $\varepsilon < 1$ and is zero otherwise. Substituting into 9) and taking the inverse transform we find our answer.

10) $$N(\varepsilon,x) = \frac{1}{2\pi i}\int_{s_0 - i\infty}^{s_0 + i\infty} \frac{\varepsilon^{-s}\; e^{-\left[\alpha - w(s)\right]x}}{s}\, ds\ , \quad s_0 > 0 \quad .$$

An exact analytic evaluation of this integral does not seem feasible so that one must either resort to machine calculations or else use some approximate method. The integral may readily be evaluated using the method of steepest descent. We write equation 10)

11) $$N(\varepsilon,x) = \frac{1}{2\pi i}\int_{s_0 - i\infty}^{s_0 + i\infty} e^{f(s)}\, ds$$

with $$f(s) = -\left\{\left[\alpha - w(s)\right]x + \ln\varepsilon + \ln s\right\} \quad .$$

The evaluation of the integral then gives

12) $$N(\varepsilon,x) = \frac{1}{\sqrt{2\pi f''(s_0)}}\; e^{f(s_0)}$$

where the saddle point, $f''(s_0)$ is given by $f'(s_0) = 0$. The accuracy of the saddle point method in this case has been tested by comparing the results with exact machine calculations[7]. The results indicate an accuracy of about 5% which should be quite satisfactory for our purposes.

In order to actually obtain numerical results it is necessary to state more definitely the form of the interaction cross section of equation 1). Since the data are very scarce we can only postulate a form not in variance with the data. This has been done by Messel[8] and we shall merely give his result.

$$13)\qquad w(\varepsilon',\varepsilon'') = \frac{120}{\alpha}\,\varepsilon'\varepsilon''(1-\varepsilon'-\varepsilon'') \text{ for } \varepsilon' + \varepsilon'' \leq 1 \quad .$$

$1/\alpha$ is the mean free path in nuclear matter which we assume to be of the order of the nucleon Compton wavelength, $1/\alpha \simeq 1.4 \times 10^{-13}$ cm. With this, one can evaluate equation 12. This has been done and the results are presented in Figure 2 for several values of ε.

Knowing the results for nuclear matter, one can than calculate results for given nuclei. Assume a nucleus of spherical shape and diameter d_A and calculate the probability of hitting the nucleus at a distance r of closest approach from the center. This is a geometrical factor which may easily be obtained by considering Figure 3. This probability is given by

$$\frac{2\pi r dr}{\frac{\pi}{4} d_A^{\,2}} = \frac{8\, rdr}{d_A^{\,2}}$$

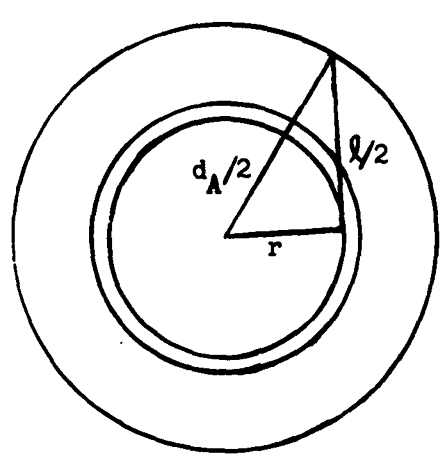

Figure 3

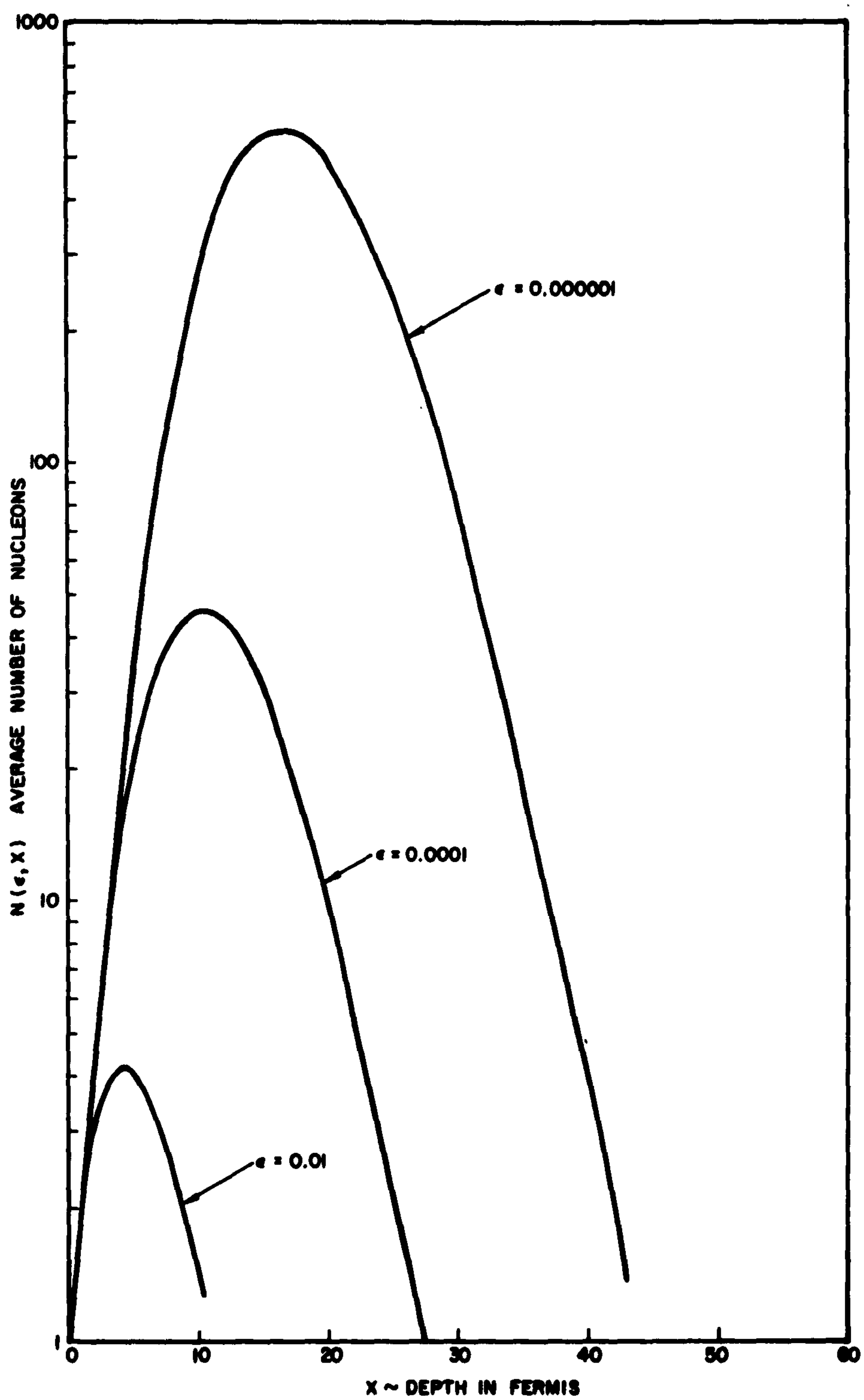

Figure 2

The average number of nucleons $N(\varepsilon,x)$ with energies greater than εE_0 produced at a depth x in homogeneous nuclear matter by an accident nucleon of energy E_0.

If the particle strikes with the impact parameter r, the path length through the nucleus will be $\ell = 2\sqrt{\frac{d_A^2}{4} - r^2}$. The probability of obtaining a given path length ℓ when striking a nucleus of diameter d_A is then given by

14) $$P(\ell)\, d\ell = \frac{2}{d_A^2}\, \ell\, d\ell \, .$$

One could then compute the average number of nucleons, $s(\varepsilon)$ with energies greater than $E = \varepsilon E_0$ which exit from a nucleus of diameter d_A, produced by an incident nucleon of energy E_0, by averaging the results over the possible path lengths of homogeneous nuclear matter in the nucleus.

15) $$S(\varepsilon) \int_0^{d_A} N(\varepsilon,x)\, P(x)\, dx$$

which upon using equations 10 and 13) and interchanging orders of integration becomes

16) $$S(\varepsilon) = \frac{1}{2\pi i}\, \frac{2}{d_A^2} \int_{s_0 - i\infty}^{s_0 + i\infty} \frac{ds}{s}\, \frac{1}{\varepsilon^s} \int_0^{d_A} e^{-\alpha(s)x}\, x dx$$

after defining $\alpha(s) = \alpha - w(s)$.

Upon performing the integration one has the final expression

17) $$S(\varepsilon) = \frac{1}{2\pi i} \int_{s_0 - i\infty}^{s_0 + i\infty} \frac{ds}{s}\, \frac{1}{\varepsilon^s}\; 2 \left\{ 1 - \frac{\left[1 + d_A \alpha(s)\right] e^{-d_A \alpha(s)}}{\left[d_A\, \alpha(S)\right]^2} \right\}$$

which may also be evaluated by the method of steepest descents. The results of equation 17) by themselves are not of great usefulness except for use as a stepping stone for calculations involving discrete matter. Thus, the problem now becomes a cascade wherein the cross section for the production of nucleons is not related to the nucleon-nucleon collision probability given in equation 1) but to equation 17).

Discrete Matter

The problem of real physical importance is that of the propagation of the nucleon cascade in discrete matter. One wishes to obtain an expression for the number of nucleons $T(\varepsilon,\theta)$ of energy greater than εE_o present at a depth θ below a surface upon which is incident a nucleon of energy E_o. The previous considerations have indicated an approach to the problem. Thus, the only change which is necessitated is to replace the properties of a nucleon-nucleon collision with the properties of a nucleon-nucleus collision. Let L be the mean free path of a nucleon in discrete matter and $w_n(\varepsilon_1,\varepsilon_2 \cdots \varepsilon_n)$ $d\varepsilon_1\, d\varepsilon_2, \cdots d\varepsilon_n\, d\theta$ the probability that a nucleon of energy Eo collides with a nucleus in $d\theta$ such that n nucleons are produced having energies in the range $\varepsilon_1 E_o$ to $(\varepsilon_1 + d\varepsilon_1)E_o, \cdots \varepsilon_n E_o$ to $(\varepsilon_n + d\varepsilon_n)E_o$, respectively. Then the diffusion equation derived in a manner very similar to that used to find equation 5 becomes

$$18)\quad \frac{dT(\varepsilon,\theta)}{d\theta} + \frac{1}{L}\, T(\varepsilon,\theta) = \sum_{n=2} n \int_0^1 \int_0^{1-\varepsilon} \int_0^{1-\varepsilon_1-\cdots-\varepsilon_{n-1}} w_n(\varepsilon_1,\varepsilon_2,\cdots\varepsilon_n)\, T\left(\frac{\varepsilon}{\varepsilon_1},\theta\right) d\varepsilon_1\, d\varepsilon_2 \cdots d\varepsilon_n$$

where complete symmetry of w_n with respect to all of its variables has been assumed.

One can now define

$$19)\qquad w(\varepsilon) = \sum_n n \int_0^{1-\varepsilon} \int_0^{1-\varepsilon-\cdots\varepsilon_{n-1}} w_n(\varepsilon,\varepsilon_2,\cdots\varepsilon_n)\, d\varepsilon_2 \cdots d\varepsilon_n \qquad \varepsilon \leq 1 \quad ,$$

$$= 0 \qquad \varepsilon \leq 1 \quad .$$

Inserting 19) into 18) one has

$$20)\qquad \frac{dT}{d\theta}(\varepsilon,\theta) + \frac{1}{L}\, T(\varepsilon\theta) = \int_0^{\infty} T\left(\frac{\varepsilon}{\varepsilon_1},\theta\right) w(\varepsilon_1)\, d\varepsilon_1 \quad .$$

Equation 20) has exactly the same form as does equation 6), therefore, its solution can immediately be written down from equation 10).

21) $$T(\varepsilon,\theta) = \frac{1}{2\pi i}\int_{s_0-i\infty}^{s_0+i\infty} ds\,\frac{e^{-s}}{s}\,e^{-\left[1-LW(s+1)\right]\frac{\theta}{L}}$$

where

22) $$W(s+1) = \int_0^\infty \varepsilon^s\,w(\varepsilon)d\varepsilon \quad .$$

The solution for discrete matter, equation 21), can be related to equation 17). $S(\varepsilon)$ is the average number of particles with energies $\geq \varepsilon E_o$ which exit a nucleus struck by a nucleon of energy E_o. This number can be calculated by considering w_n. The average number of particles produced by a collision in $d\theta$ whose energies are $\geq E$ is given by

$$\sum_n n \int_\varepsilon^1 \cdots \int_0^{1-\varepsilon_1-\cdots\varepsilon_{n-1}} w_n(\varepsilon_1,\varepsilon_2,\ldots\varepsilon_n)d\varepsilon_1\ldots d\varepsilon_n d\theta$$

where the symmetry of w_n has again been used. The quantity $S(\varepsilon)$ can now be found by

$$S(\varepsilon) = \frac{\text{ave. num. particles of energy} > E}{\text{per collision}} = \frac{\text{ave. num. particles of energy} > E \text{ produced}}{\text{per unit length}}$$

$$\times \frac{\text{average distance}}{\text{per collision}} \quad .$$

23) $$S(\varepsilon) = L\sum_n n\int_\varepsilon^1\int_0^{1-\varepsilon_1}\int_0^{1-\varepsilon_1-\cdots-\varepsilon_{n-1}} w_n(\varepsilon_1,\ldots\varepsilon_n)d\varepsilon_1\ldots d\varepsilon_n \quad .$$

The Mellin Transform of $S(\varepsilon)$ is

24) $$S(s) = \int_0^\infty \varepsilon^{s-1}S(\varepsilon)d\varepsilon = L\sum_n n\int_0^\infty \varepsilon^{s-1}\,d\varepsilon \int_\varepsilon^1 \int_0^{1-\varepsilon_1-\cdots-\varepsilon_{n-1}} w_n(\varepsilon_1,\ldots\varepsilon_n)d\varepsilon_1\ldots d\varepsilon_n$$

which may be integrated by parts to give

$$25) \qquad S(s) = L \sum_n n \int_0^\infty d\varepsilon \frac{\varepsilon^s}{s} \int_0^{1-\varepsilon} \int_0^{1-\varepsilon-\varepsilon_2-\cdots-\varepsilon_{n-1}} w_n(\varepsilon,\varepsilon_2\cdots\varepsilon_n)d\varepsilon_2\cdots d\varepsilon_n \quad .$$

Comparing 25) with 22) and 19) we see

$$26) \qquad sS(s) = LW(s+1)$$

so that the solution equation 21) can be expressed now in terms of the average numbers of particles which leave a nuclear collision. Then by combining 17), 26) and 21), we find

$$27) \qquad T(\varepsilon,\theta) = \frac{1}{2\pi i} \int_{s_o-i\infty}^{s_o+i\infty} \frac{ds}{s} \frac{1}{\varepsilon^s} \quad e^{-h\left[d_A \alpha(S)\right] \frac{\theta}{L}}$$

where

$$28) \qquad h(x) = 1 - \frac{2\left[1-(1+x)e^{-x}\right]}{x^2} \quad .$$

At these energies the mean free path may be calculated using the geometrical cross section

$$29) \qquad L = \frac{1}{\sigma N} = \frac{A^{1/3}}{N_o \rho \pi r_o^2}$$

where A is the atomic weight, ρ the density, N_o Avogadro's number and $r_o\, A^{1/3}$ the radius of the nucleus. Thus

$$30) \qquad L(\text{cm.}) = \frac{100}{.6025\pi} \quad \frac{A^{1/3}}{r_o^2 \rho}$$

where ρ is in g/cm^3 and r_o in fermis (10^{-13}cm.).

The integral in 27) can then be evaluated, as before, using the method of steepest descents and the results are presented in Figures 4, 5 and 6 for several values of $\varepsilon(10^{-2}, 10^{-4}, 10^{-6})$ and of A(235, 55.8, 27). For all curves r_o = 1.4f, so that one has L_{235} = 8.9 cm, $L_{55.8}$ = 13.1 cm and

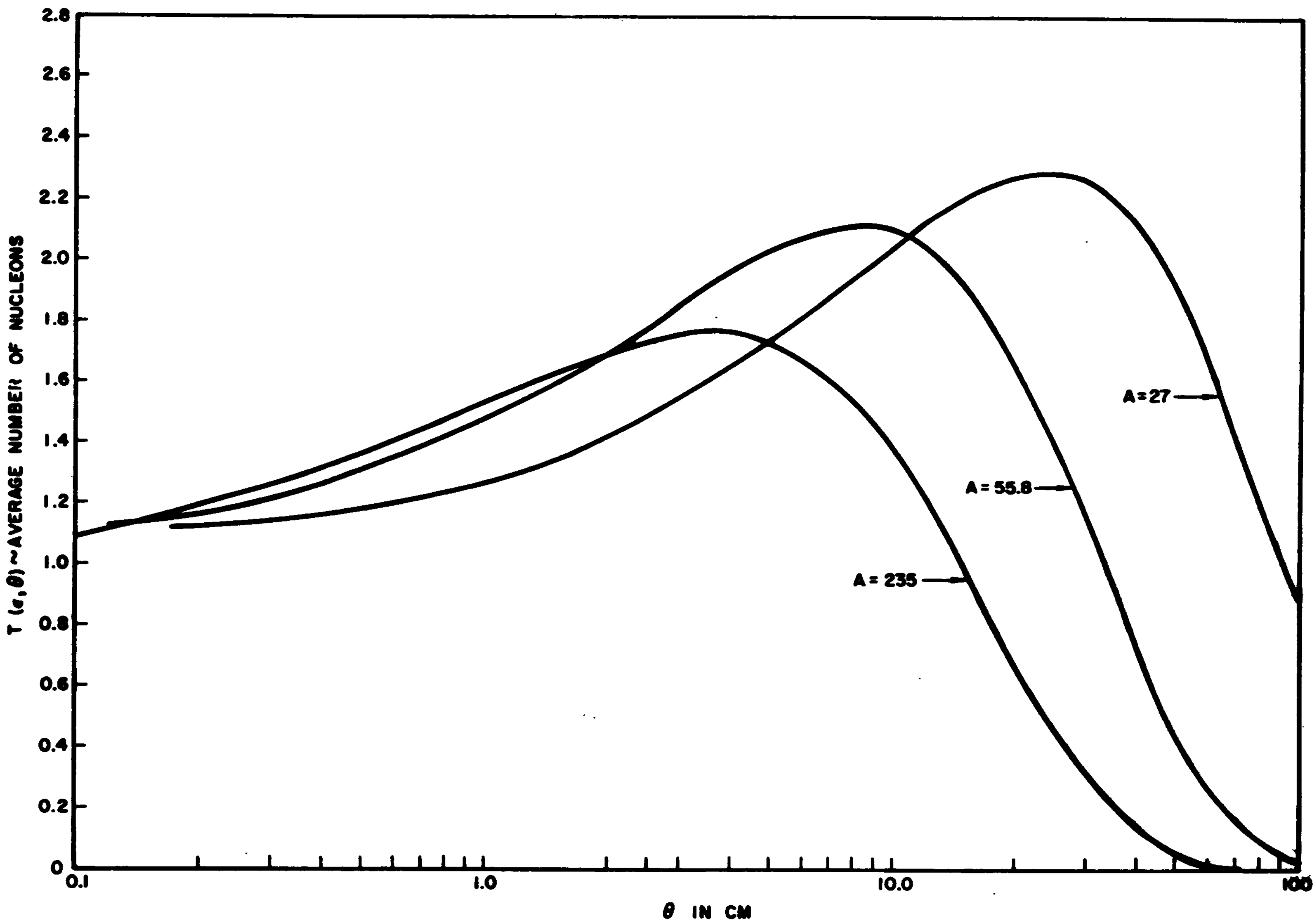

Figure 4
The average number of nucleons $T(\varepsilon,\theta)$ with energies greater than εE_0 produced at a depth θ in discrete matter by an incident nucleon of energy E_0. $\varepsilon = .01$, curves are given for atomic weights, A = 235, 55.8 and 27.

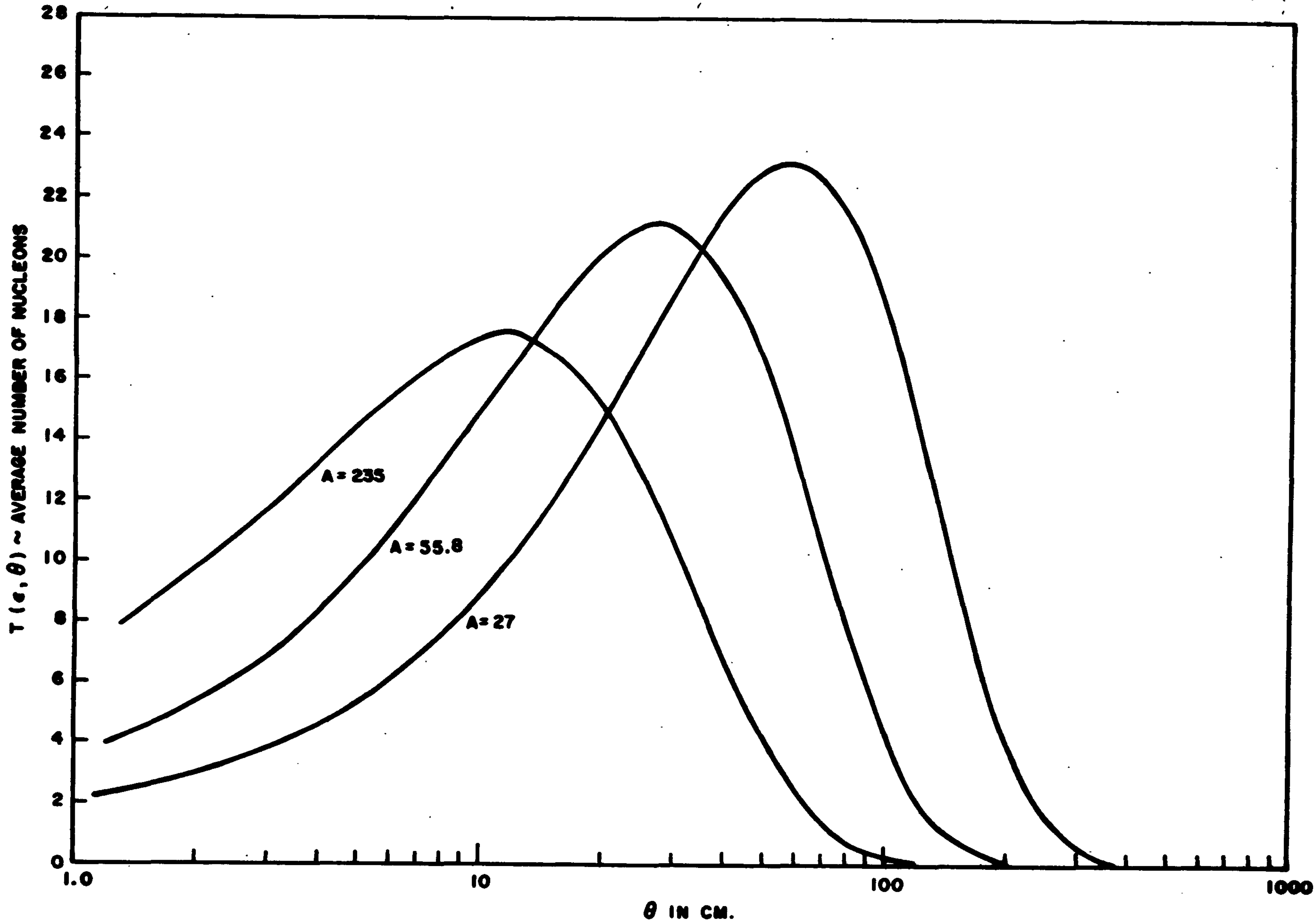

Figure 5

The average number of nucleons $T(\varepsilon,\theta)$ with energies greater than εE_0 produced at a depth θ in discrete matter by an incident nucleon of energy E_0. $\varepsilon = .0001$, curves are given for atomic weights, A = 235, 55.8, and 27.

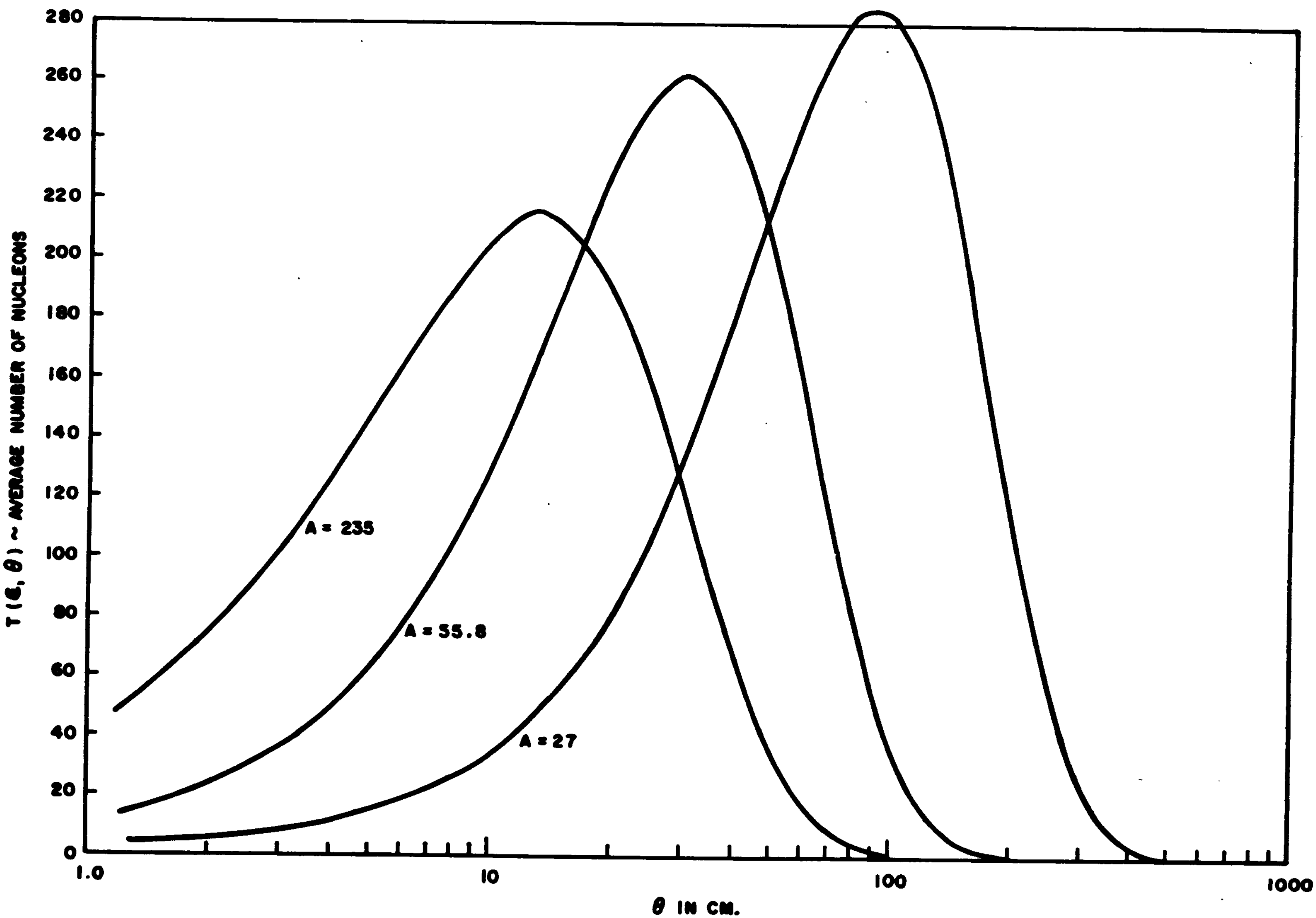

Figure 6

The average number of nucleons $T(\varepsilon,\theta)$ with energies greater than εE_0 produced at a depth θ in discrete matter by an incident nucleon of energy E_0. ε = .000001, curves are given for atomic weights, A = 235, 55.8 and 27.

$L_{27} = 29.95$ cm assuming densities appropriate to uranium($\rho = 18.7 \frac{g}{cm^3}$), iron ($\rho = 7.86 \frac{g}{cm^3}$) and aluminum ($\rho = 2.70 \frac{g}{cm^3}$). These elements were chosen so as to obtain results representative of light, medium and heavy nuclei in order to find any qualitative differences between the behavior of the cascades in various materials.

Two important facts can be seen from the figures. First, the depth dependence of T on the atomic number is not simply contained in the dependence on the mean free path. Thus, the ratio of the depths corresponding to the cascade maximum for two materials is not the same as the ratio of the mean free paths, the dependence of $h(d_A \alpha)$ on A seems to have importance. The second observation is that the average number of nucleons at the maximum varies only about 25% from element to element. The reason for this would seem to lie in the approximation which was made when it was assumed that the interaction in a nucleus was the same as for nuclear matter averaged over all possible path lengths. Thus, when one takes into account the conservation of energy, the theory should predict a maximum nearly equal to that of homogeneous nuclear matter for every element and this is nearly what is observed.

Discussion

Results have been presented for a very simplified picture of the nucleon cascade. It should be pointed out here some of the most important simplifications made and their possible effect on the results.

First and probably most important is the role of radiated mesons on the development of the cascade. At these energies many mesons[9,10] will be produced in a nucleon-nucleon collision and these mesons themselves may take part in secondary production of nucleons by collisions with other nuclei.

This possibility has been entirely neglected although there seems to be no justification for doing so and, indeed, the next step must be to include the meson component into the formalism. This is a great theoretical complication necessitating the solution of two coupled diffusion equations instead of the one equation which we have considered above.

The second most important consideration concerns the fluctuations involved in the cascade process. Thus, although we have found an expression for the average number of nucleons nothing has been said about possible deviations from this average which of course must occur. Some work has been done on this problem[8] and the results indicate that the fluctuations are greater than that given by a Poisson distribution (i.e., random fluctuations) everywhere except at the maximum where the deviations nearly follow a Poisson distribution. Thus, the deviations from the average values of the curves given in Figures 4, 5 and 6 would be large enough to mask any differences in the maximum of the curves.

Also neglected has been the differences between protons and neutrons, the main effect being that of ionization loss. For energies above about 3 Gev, this is probably not too important but will become significant at lower energies considerably reducing the number of protons of energy greater than εE_o at greater depths. The neutrons, not being ionizing, themselves, will be affected only indirectly by the loss of protons which could produce neutrons in the cascade. The result will be a reduction of the proton neutron ratio.

These are probably the most important simplifications which have been made and indicate the approximate validity of the results and also difficulties which stand in the way of improvements on them.

Conclusion

The motivation for this work lay in an attempt to determine the nature of the material in which a cascade has developed by observation of the nucleons produced. Under ideal circumstances, i.e., knowledge of the energy of the initial nucleon, the thickness of the material and small fluctuations from the average, the curves in Figures 4, 5 and 6 would enable a reasonable estimate of the nature of the material to be made. But this would be possible only under these very ideal circumstances. In more practical circumstances, not only would E_o be unknown, but the thickness of the material and hence the fluctuations would not be known. In addition , the task of detecting all nucleons of energy greater than εE_o and excluding other particles such as the high energy mesons would appear to be formidible.

It would appear that some quantities other than the average numbers of nucleons must be considered. Possibilities might be any variables which depend upon the atomic number of the material. Such quantities might be the proton to neutron ratio or the properties of the photon-electron cascade. Further investigations will be needed to determine whether such possibilities are plausible.

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