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PROJECT SPACE TRACK

On the Representation of Air Density in Satellite Deceleration Equations by Power Functions with Integral Exponents

by

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FOREWORD

This work was performed for the 496 L System Program Office (Project SPACE TRACK), Deputy for Systems Management, Electronic Systems Division, AFSC, L. G. Hanscom Field, Bedford, Massachusetts.

It is a pleasure to acknowledge our deep indebtedness to Dr. Eberhard W. Wahl, Technical Director of Project Space Track, without whose continuing interest and enthusiastic support this work would not have been possible. We are further indebted to him for many stimulating discussions and criticisms and for making available to us certain unpublished information. We are pleased to acknowledge also the assistance of the Mathematical Services Laboratory, Eglin A. F. B. , through the particular contributions of Mrs. F. W. Biele, Mr. C. W. Davy and Mr. M. Stephenson.

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ABSTRACT

The scale height in height-bands between 150 and 400 km is assumed to vary linearly with height. Integration of the hydrostatic equation for an ideal gas above a spherical earth then leads to a power function representation of the air density over the band. With integral exponents such power laws give better fits to several proposed model atmospheres over altitude ranges of several hundred kilometers than those provided by the usual exponential representation of air density.

The representation of air density in satellite deceleration equations by power functions with integral exponents reduces them to elementary forms. It has been possible with such density distributions to obtain simplified formulas which may be useful for (a) computing atmospheric densities from satellite accelerations, (b) comparing proposed model atmospheres with observations, and (c) developing further the theory of satellite orbits in the presence of air drag. As is possible with the exponential form, these power functions may be modified to take account of the effect of an oblate, rotating atmosphere. Their use may, therefore, permit the development of a simplified, accurate orbit theory for satellites with perigee heights below 300 km. Certain preliminary results are discussed and compared with previous theory and observations.

PUBLICATION REVIEW

This technical documentary report has been reviewed and is approved.

Robert H. Warren

ROBERT H. WARREN
Major General, USAF
Commander

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INTRODUCTION

The orbit of a close earth satellite is continuously contracting. This is because the main effects of air drag are to reduce the eccentricity e and the length of the semimajor axis a. The contraction, which is most pronounced when the perigee height is below 300 km, is well described by two equations [King-Hele, Cook and Walker, 1959; Parkyn, 1960]. These equations express the changes Δa and Δe in the semimajor axis and eccentricity during one revolution from perigee to perigee in the form

$$\Delta a = -\delta a^2 \int_0^{2\pi} (1 + e \cos E)^{3/2} (1 - e \cos E)^{-1/2} \rho dE \qquad (1)$$

$$\Delta e = -\delta a (1 - e^2) \int_0^{2\pi} \cos E (1 + e \cos E)^{1/2} (1 - e \cos E)^{-1/2} \rho dE \qquad (2)$$

where

$\delta = FC_D A/M$

F = factor to account for rotation of atmosphere

C_D = drag coefficient

A = effective cross section of satellite

m = mass of satellite

E = eccentric anomaly

ρ = atmospheric density.

Equation (1) has been employed in the past as a basis for the development of formulas for computing atmospheric densities from observations of satellite decelerations [Sterne, 1958; Groves, 1958; King-Hele, Cook and Walker, 1959]. When coupled with (2), (1) also provides a logical starting point for the development of equations which relate the variations of a and e with each other, with the time, and with the revolution number. To make use of (1) and (2), an explicit functional dependence of air density on the height $r - q_0$ above a reference level q_0 must be assumed. The reference level q_0 and r are to be measured from the geocenter.

In many previous investigations [e.g. - King-Hele, Cook and Walker, 1959; Parkyn, 1960] an exponential form of the vertical distribution of air density $\rho(r) = \rho(q_0) \exp[-(r - q_0)/H']$ has been assumed in which the density scale height H' is treated as constant in the integrals of (1) and (2). Evaluation of these satellite deceleration integrals then leads to formulas which involve modified Bessel functions of the first kind of argument ae/H'. If, as Jacchia has pointed out [Jacchia, 1960], H' exhibits an altitude dependence, a systematic error is introduced into the development.

An examination of the analyses and data presented in Nicolet[Nicolet, 1959] and several preliminary or proposed model atmospheres [Kallmann, 1959; Minzner, Champion, and Pond, 1959: Paetzold, 1961] suggests a linear fit to a given model as a first approximation to the variation of the atmospheric scale height H with altitude in a height band of interest within the altitude range considered here 150-400 km. These height bands can be fairly broad, covering a range of from 100 to 200 km. For example, in the height band 200-400 km, of greatest importance in satellite orbit theory in the presence of air drag, the scale height over the entire band may, for the Nicolet data and for two of the models cited, be reasonably well approximated by constant gradients. The third model, due to Paetzold, can be approximated by a linear fit over the height band 220-400 km.

In the discussion in the following section it is seen that the assumption of a linear variation of scale height H over a given height band leads, through the equation of hydrostatic equilibrium, to a modified power function representation of air density over the band. Similarly, the assumption of a linear variation of H' with altitude leads to a pure power function representation. In either form, the exponent n of the power law depends only upon the value of the constant scale height gradient and, in general, it will not turn out to be an integer. The introduction of such a vertical distribution of atmospheric density into (1) and (2) leads to integrands for which no simple integration is apparent. On the other hand, power functions with integral exponents reduce the satellite deceleration integrals to elementary forms and, hence, may provide a base for developing formulas which may be useful for (a) computing atmospheric densities from satellite accelerations, (b) comparing proposed model atmospheres with observations, and (c) developing further a simplified satellite orbit theory in the presence of air drag. The possibility of so representing the air density over the interval 150-400 km will be considered, and certain preliminary results derived from the use of such a representation will be introduced and discussed.

ANALYTICAL REPRESENTATIONS OF AIR DENSITY BASED ON A VARIABLE SCALE HEIGHT

Values of atmospheric density and scale height taken from Nicolet, [Nicolet, 1959] and the several model atmospheres previously cited are presented in Table 1 for the altitude interval 150-400 km. The same data are shown in Figs. 1 and 2. In Fig. 1 it is seen that, except for the band 180-220 km in the Paetzold data, there is a significant steepening of the slope d log ρ /dr below 300 km which increases (negatively) as the altitude decreases. Thus, an exponential distribution of air density, which would appear as a straight line on Fig. 1, could not reasonably be expected to represent the data well below 300 km except over limited intervals.

The curves depicted in Fig. 2 have been constructed by the rather arbitrary procedure of connecting the published data points with straight lines. Thus, they indicate only the gross character of the data. Reasonable linear approximations to these data may be obtained by treating the two height bands 150-200 km and 200-400 km separately for all but the Paetzold model. For each of the two bands and for each set of data, a gradient of scale height may be assigned. For the Paetzold data three bands 150-180, 180-220, and 220-400 km may be treated similarly.

Assume then a linear variation in scale height

$$H = H_0 + \beta (r - q_0) \tag{3}$$

where β is the scale height gradient and H_0 is the value of H at the reference level q_0. Let the intercept

$$s = q_0 - H_0 / \beta \tag{4}$$

of the line (3) with the r-axis be introduced into (3) and the result substituted into the equation of hydrostatic equilibrium adapted to a terrestrial gas [Nicolet, 1954]

$$Hdp = - pdr. \tag{5}$$

Equation (5) may be integrated to yield

$$p = p_0 [(q_0 - s)/(r - s)]^n \tag{6}$$

where p_0 is the value of the atmospheric pressure p at the reference level q_0 and $n = 1 + \beta^{-1}$. The air density may be introduced into (6) from the perfect gas law

$$p = \rho g H = \rho g_0 (q_0 / r)^2 H$$

TABLE 1. SCALE HEIGHTS AND ATMOSPHERIC DENSITIES FROM SEVERAL SOURCES

Height h (km)	Air densities ρ (gm/cm^{-3})*				Scale heights H (km)*			
	ρ_A	ρ_B	ρ_C	ρ_D	H_A	H_B	H_C	H_D
150	1.5 (-12)	6.1 (-12)	1.8 (-12)	3.4 (-12)	38.4	20.2	32.4	20.5
160	1.1 (-12)	3.2 (-12)	1.1 (-12)	1.5 (-12)	40.6	24.3	38.4	31.5
180	6.4 (-13)	1.2 (- 12)	6.0 (-13)	5.8 (-13)	45.2	32.4	44.9	51.0
200	3.8 (-13)	5.9 (-13)	3.7 (-13)	4.0 (-13)	49.9	37.8	48.2	52.0
220	2.4 (-13)	3.2 (-13)	2.4 (-13)	2.8 (-13)	54.7	42.4	50.8	50.0
240	1.6 (-13)	1.9 (-13)	1.5 (-13)	1.7 (-13)	56.0	46.4	53.1	54.0
300	4.7 (-14)	4.8 (-14)	4.8 (-14)	5.2 (-14)	60.9	58.1	60.3	74.0
350	1.9 (-14)	1.9 (-14)	2.0 (-14)	2.4 (-14)	66.3	67.8	66.4	84.5
400	8.0 (-15)	8.7 (-15)	8.9 (-15)	1.2 (-14)	72.6	74.2	72.5	99.0

* The subscripts A, B, C, and D on the symbols ρ and H refer to data taken from Nicolet, Kallmann, Minzner et al., and Paetzold, respectively. For brevity the symbol (-x) is used to represent 10^{-x}.

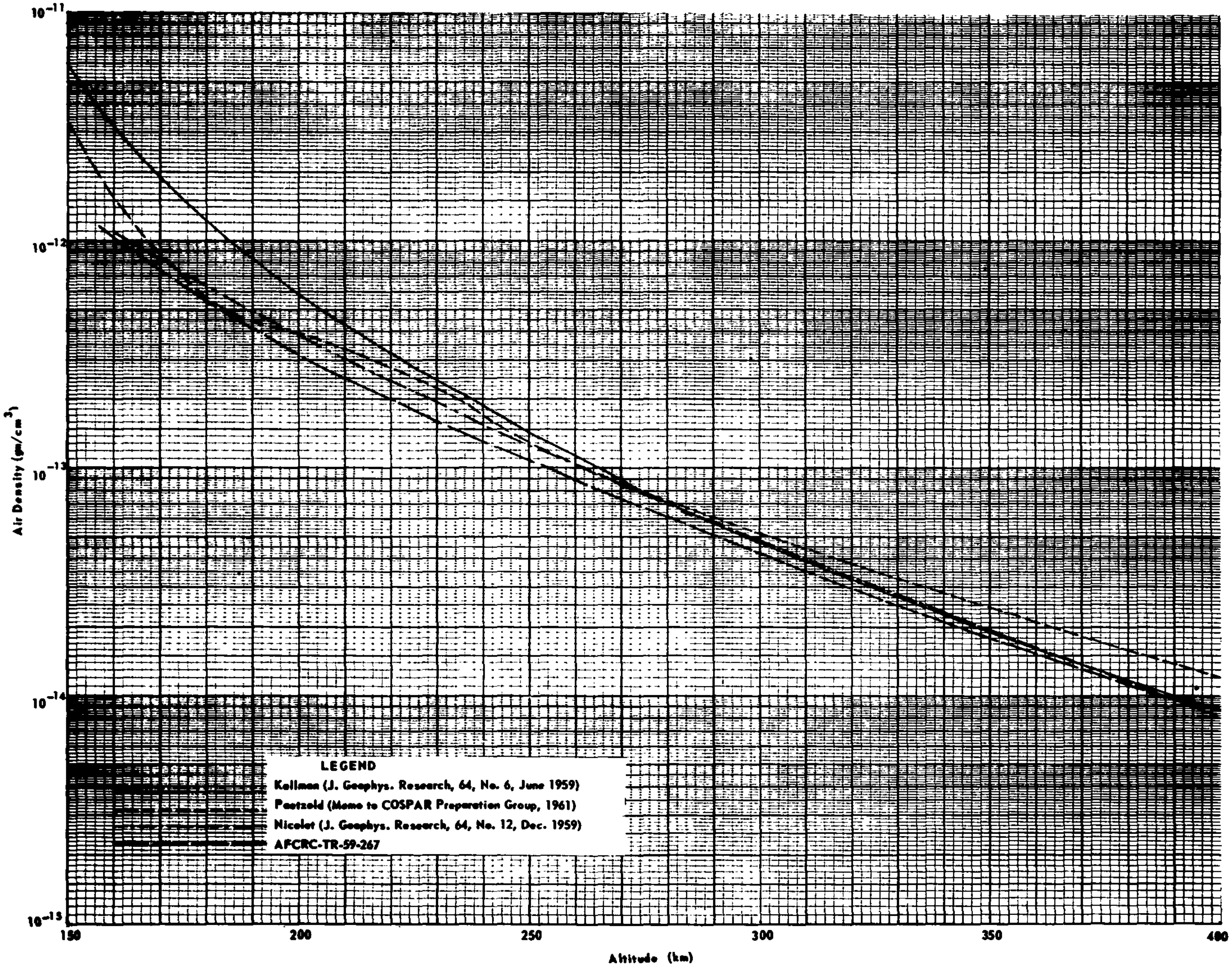

Fig. 1. Variation of ρ with Height Above the Earth's Surface for Various Atmospheric Models.

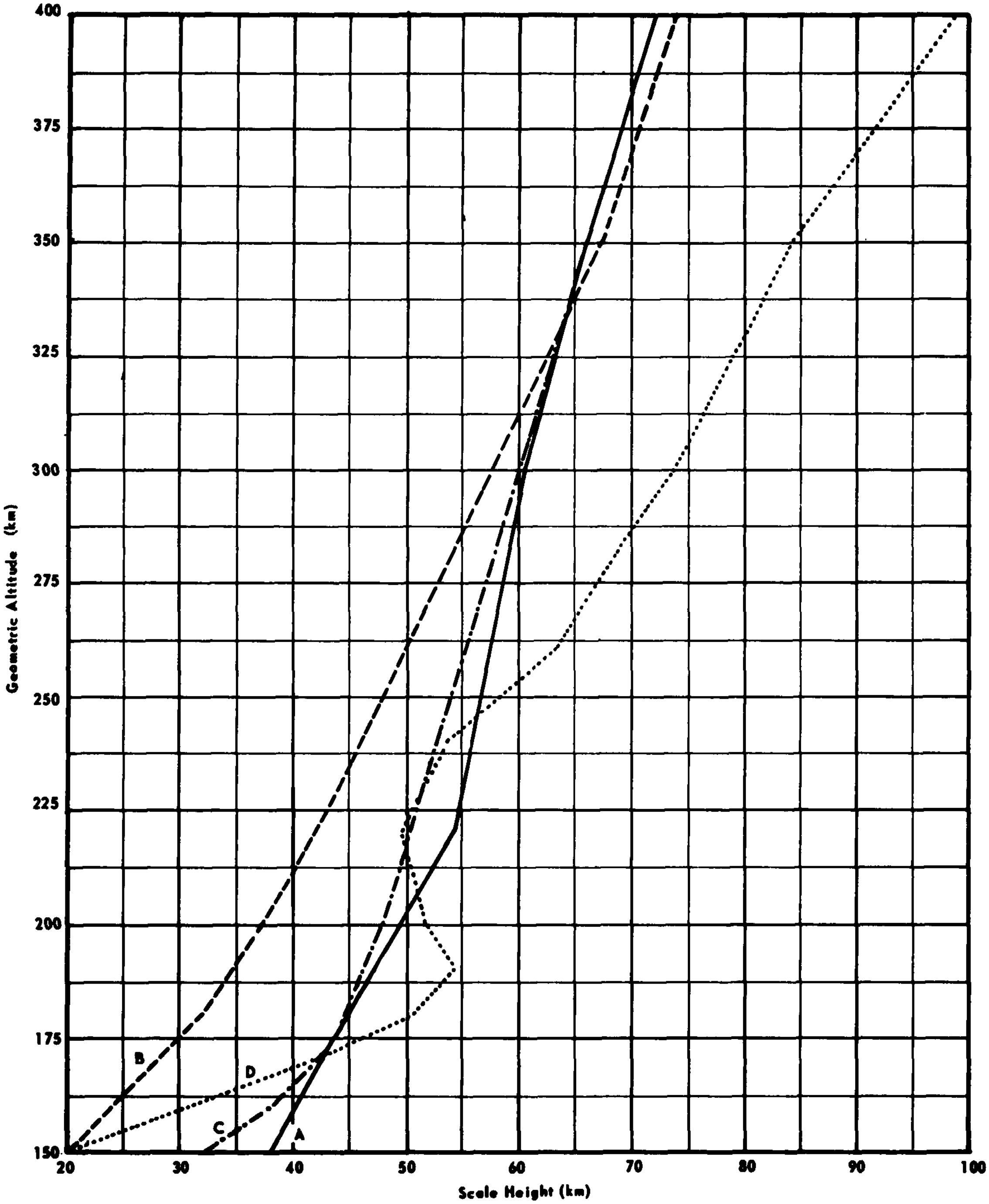

Fig. 2: Variation of H with Height Above the Earth's Surface for Various Atmospheric Models.

where g_0 is the value of the acceleration of gravity g at the reference level q_0. This substitution yields the modified power function for the air density

$$\rho^{(2)} = \rho_0 (r/q_0)^2 [(q_0 - s) /(r - s)]^n . \quad (7)$$

The superscript notation on the symbol for air density is introduced into (7), and elsewhere, for convenience in distinguishing among the different air density functions to be discussed. For the special case that H is constant (7) reduces to the modified exponential form

$$\lim_{\beta \to 0} \rho^{(2)} = \rho_0 (r/q_0)^2 \exp [-(r - q_0)/H] \quad (7a)$$

Alternative forms of (7) suggest themselves since the factor r/q_0 is close to unity in the region of interest. For example, the functions

$$\rho^{(1)} = \rho_0 (r/q_0) [(q_0 - s) /(r - s)]^n \quad (8)$$

$$\rho^{(0)} = \rho_0 [(q_0 - s)/(r - s)]^n \quad (9)$$

give values of air density which differ, in general, by less than three per cent over the entire band 200-400 km from those given by (7).

The form of (8) will find application later in developing a relationship between the semimajor axis and the eccentricity. The form of (9) may be derived from the definition of density scale height [Jacchia, 1959; King-Hele, Cook and Walker, 1960]

$$H'd\rho = -\rho dr. \quad (10)$$

If a linear variation of density scale height with altitude $H' = \beta'(r - s')$ is assumed, integration of (10) yields $\rho^{(0)} = \rho_0 [(q_0 - s')/(r - s')]^{n'}$. Here $s' = q_0 - H'_0 / \beta'$, H'_0 is the density scale height at q_0 and $\beta = 1/n'$ is the density scale height gradient. If, however, H' is taken to be constant (10) leads to the more familiar exponential representation of air density

$$\rho^{(3)} = \lim_{\beta' \to 0} \rho^{(0)} = \rho_0 \exp [-(r - q_0)/H']. \quad (11)$$

If a linear variation of $\gamma = Hg$ is assumed another air density distribution which combines features of the power and exponential laws may be obtained. Take

$$\gamma = \gamma_0 + K (r - q_0) = K (r - S) \tag{12}$$

where γ_0 is the value of γ at q_0, K is the γ gradient, and $S = q_0 - \gamma_0/K$. Substitution of (12) into (5) leads to

$$\rho^{(4)} = \rho_0 (r/q_0)^{\nu} [(q_0 - S)/(r - S)]^{\nu + 1} \exp [-\nu S (r^{-1} - q_0^{-1})] \tag{13}$$

where

$$\nu = \mu/KS^2$$

μ = earth's gravitational constant $\approx 1.5362 \times 10^{-6}$ (earth radii)^3 sec^{-2}.

COMPARISON OF POWER AND EXPONENTIAL LAW FITS TO ATMOSPHERIC DENSITY DATA FROM SEVERAL SOURCES

If (7) is to be fitted to a given atmospheric model at a point (ρ_0, q_0) in a height band of interest, an integral exponent $n = 1 + \beta^{-1}$ may be determined by an approximate linear fit to the scale height data. Values of n determined in this way (see Fig. 2) are given in Table 2 for several atmospheric models for the height bands noted.

In principle, the intercept with the r-axis of the line (3) fitted to the scale height data is the value of s to associate with each n. If s is determined in this way, however, it is sensitive to uncertainties in the values of H, particularly for the smaller values of β. For this reason a different procedure was adopted here. The air density function (7) was first identified with each model at a reference level q_0, near the midpoint of the height band for which n had been determined. An average value of s was computed from the two values obtained by equating the function to the air densities of the model at the end points of the band. The results are shown in Table 2.

For any given height band, the quality of the fit of an air density distribution such as (7) to an atmospheric model may be expressed in terms of the ratios of the respective air densitites. Calculations of this

TABLE 2. VALUES OF n AND s IN SELECTED HEIGHT BANDS FOR VARIOUS ATMOSPHERIC MODELS

Model	Height-Band (km)	n	s
A	150-200	5	0.9993
	200-400	11	0.9580
B	150-200	4	1.0135
	200-400	6	1.0010
C	150-200	4	1.0064
	200-400	9	0.9716
D*	150-180	2	1.0203
	220-400	5	1.0026

* The scale height in the Paetzold standard model is roughly constant in the interval 180-220 km, and the corresponding air density distribution may be represented either by (7a) or a power function adapted to this interval.

type, based upon the data in Table 2, have been made and the results are given in Tables 3 and 4 for the reference points 180 and 300 km, respectively. It can be seen that the power laws, in general, represent their respective models to within a few parts per hundred over the height band of the fit. Thus, the reference point (ρ_0, q_0) may be chosen anywhere within the height bands studied without materially changing the quality of the fit over the band.

The fits of the power laws to the Nicolet data and the several proposed model atmospheres may be compared to the fits obtained with the exponential law (11) by referring to Tables 5 and 6. These data were calculated on the basis of the published values of scale height at the reference levels indicated in the tables. It is apparent that for height bands of any appreciable width the power functions with integral exponents give closer approximations to the models than do the exponential functions. Several illustrative examples are shown in Figs. 3 and 4.

TABLE 3. COMPARISON OF SEVERAL AIR DENSITY FUNCTIONS FITTED TO VARIOUS ATMOSPHERIC MODELS WITH REFERENCE LEVEL 180 KM

Height h (km)	Ratio of air densities							
	n = 5 ; s = 0.9993		n = 4 ; s = 1.0135		n = 4 ; s = 1.0064		n = 2 ; s = 1.0203	
	$\rho^{(0)}/\rho_A$	$\rho^{(2)}/\rho_A$	$\rho^{(0)}/\rho_B$	$\rho^{(2)}/\rho_B$	$\rho^{(0)}/\rho_C$	$\rho^{(2)}/\rho_C$	$\rho^{(0)}/\rho_D$*	$\rho^{(2)}/\rho_D$*
150	1.0313	1.0220	0.9264	0.9179	0.8856	0.8772	1.0006	0.9957
160	1.0291	1.0227	0.9828	0.9769	1.0182	1.0118	1.0150	1.0137
180	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9857	0.9902
200	1.0095	1.0155	0.9359	0.9415	0.9451	0.9508	0.7446	0.7527
220	1.0042	1.0163	0.9013	0.9122	0.9054	0.9167		
240	0.9844	1.0025	0.8679	0.8837	0.9473	0.9647		
300	1.1228	1.1643	0.9177	0.9517	1.0285	1.0665		
350	1.3016	1.3700	0.9984	1.0511	1.2170	1.2810		
400	1.6013	1.7100	1.0634	1.1621	1.4989	1.4250		

* The Paetzold standard model was fitted with reference level 165 km, the mid-point of the interval for which n = 2. It is apparent that the values of the ratios for other reference levels are simply related to the values given in this table. For example, $\rho^{(2)}/\rho_A$ and $\rho^{(2)}/\rho_B$ for a reference level of 200 km are obtained by multiplying columns 3 and 5 of this table 0.985 and 1.06, respectively.

TABLE 4. COMPARISON OF SEVERAL AIR DENSITY FUNCTIONS FITTED TO VARIOUS ATMOSPHERIC MODELS WITH REFERENCE LEVEL 300 KM

Height h (km)	Ratio of air densities							
	$n = 11$; $s = 0.9580$		$n = 6$; $s = 1.0010$		$n = 9$; $s = 0.9716$		$n = 5$; $s = 1.0026$	
	$\rho^{(0)}/\rho_A$	$\rho^{(2)}/\rho_A$	$\rho^{(0)}/\rho_B$	$\rho^{(2)}/\rho_B$	$\rho^{(0)}/\rho_C$	$\rho^{(2)}/\rho_C$	$\rho^{(0)}/\rho_D$	$\rho^{(2)}/\rho_D$
150	0.9113	0.8707	0.5651	0.5400	0.7661	0.7322		
160	0.9582	0.9182	0.7206	0.6906	0.9609	0.9209		
180	0.9972	0.9617	0.9258	0.8925	1.0560	1.0183		
200	1.0392	1.0084	0.9812	0.9520	1.0551	1.0241	1.1524	1.1182
220	1.0388	1.0142	1.0053	0.9813	1.0263	1.0017	0.9763	0.9529
240	1.0013	0.9831	0.9916	0.9737	1.0600	1.0413	0.9781	0.9608
300	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
350	0.9784	0.9932	0.9853	1.0000	0.9865	1.0015	0.9387	0.9526
400	0.9888	1.0186	0.9545	0.9833	0.9870	1.0167	0.9526	0.9695

TABLE 5. COMPARISON OF EXPONENTIAL AIR DENSITY FUNCTION $\rho^{(3)}$ WITH NICOLET AND KALLMANN DATA FITTED AT VARIOUS REFERENCE LEVELS*

Height h (km)	$\rho^{(3)}/\rho_A$				$\rho^{(3)}/\rho_B$			
	160 km	200 km	240 km	350 km	160 km	200 km	240 km	350 km
150	0.9976	0.7621	0.6249	0.3489	0.9060	0.4730	0.3186	0.1074
160	1.0000	0.8338	0.7003	0.4036	1.0000	0.6564	0.4694	0.1716
180	0.9289	0.9224	0.8127	0.4973	0.8940	0.9277	0.7462	0.3212
200	0.8454	1.0000	0.9240	0.6017	0.6096	1.0000	0.9054	0.4581
220	0.7233	1.0193	0.9875	0.6836	0.3768	0.9772	0.9956	0.5926
240	0.5865	0.9840	1.0000	0.7362	0.2127	0.8723	1.0000	0.7008
300	0.3150	0.8935	1.0471	0.9263	0.0317	0.5139	0.8400	0.9591
350	0.1668	0.7348	0.9701	1.0000	0.0052	0.2655	0.5831	1.0000
400	0.0853	0.5803	0.8632	1.0364	0.0007	0.1186	0.3498	0.9013

* The density scale heights used for the calculations appearing in this table were deduced from the relationship $H = (1 + \beta) H'$ [Nicolet, 1961].

TABLE 6. COMPARISON OF EXPONENTIAL AIR DENSITY FUNCTION $\rho^{(3)}$ WITH ARDC AND PAETZOLD DATA FITTED AT VARIOUS REFERENCE LEVELS*

Height h (km)	$\rho^{(3)}/\rho_C$				$\rho^{(3)}/\rho_D$			
	160 km	200 km	240 km	350 km	160 km	200 km	240 km	350 km
150	0.8638	0.6612	0.5608	0.3297	0.8310	0.3077	0.4016	0.1360
160	1.0000	0.8565	0.7427	0.4545	1.0000	0.5755	0.7223	0.2658
180	0.9177	0.9840	0.8913	0.5942	0.7294	1.0132	1.1757	0.5117
200	0.7449	1.0000	0.9461	0.6868	0.2981	1.0000	1.0727	0.5520
220	0.5749	0.9661	0.9548	0.7548	0.1202	0.9725	0.9646	0.5863
240	0.4603	0.9686	1.0000	0.8602	0.0559	1.0902	1.0000	0.7185
300	0.1805	0.7452	0.8766	0.9724	0.0048	1.1242	0.8150	0.9670
350	0.0768	0.5559	0.7296	1.0000	0.0004	0.9313	0.5551	1.0000
400	0.0306	0.3885	0.5681	0.9629	0.0000	0.7121	0.3488	0.9547

* The density scale heights used for the calculations appearing in this table were deduced from the relationship $H = (1 + \beta) H'$ [Nicolet, 1961].

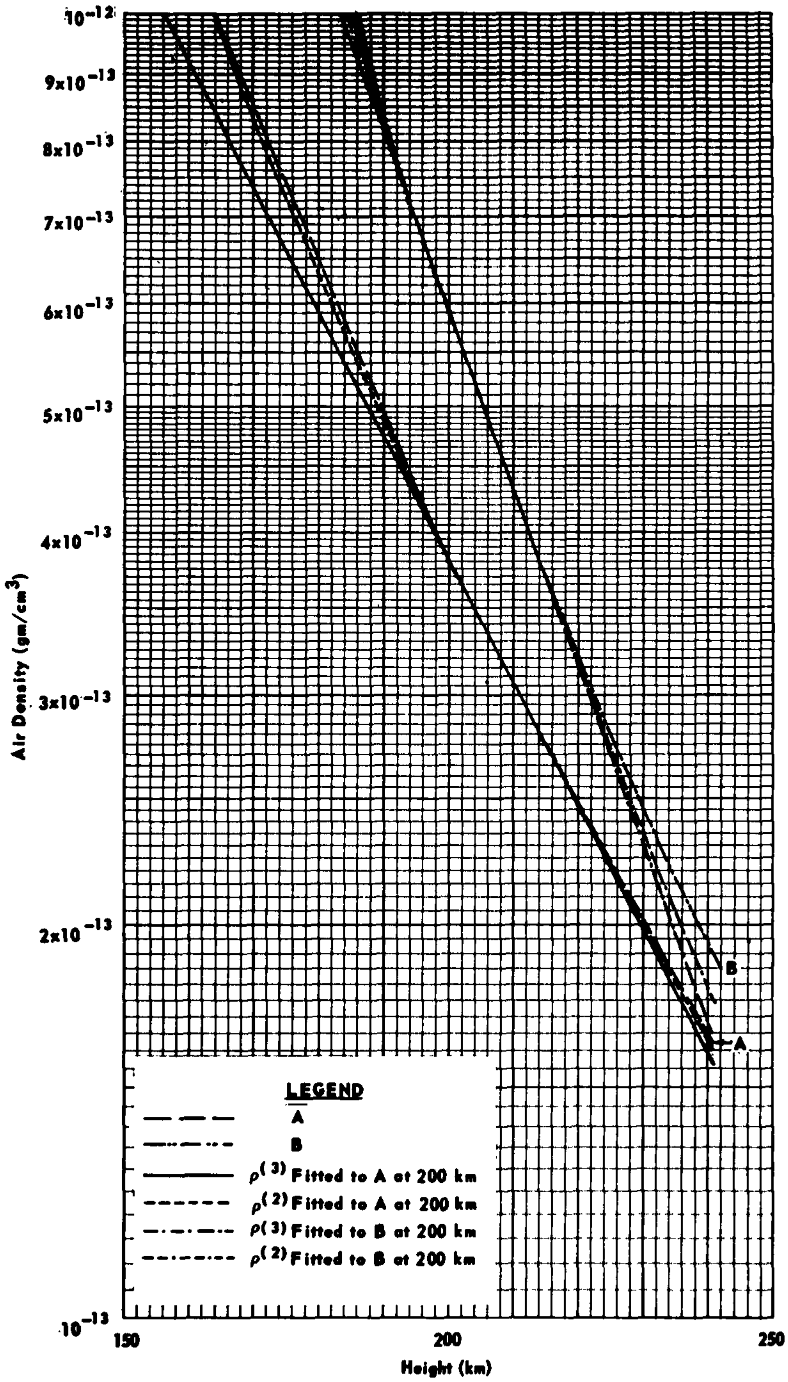

Fig. 3: Comparison of Exponential and Power Function Air Density Distributions Fitted to Models A and B.

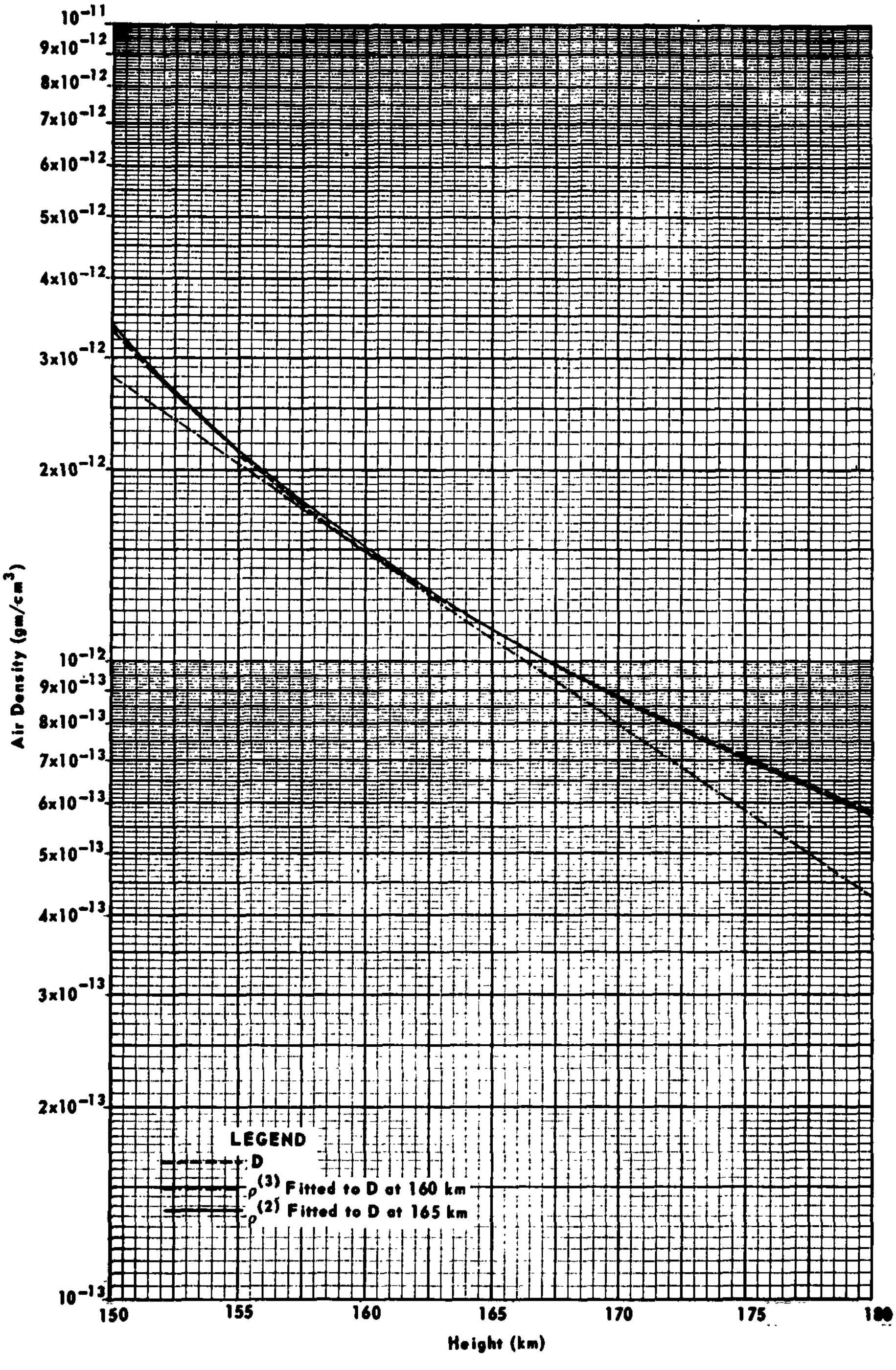

Fig. 4: Comparison of Exponential and Power Function Air Density Distributions Fitted to Model D.

RELATIONSHIP BETWEEN a AND e DERIVED FROM A MODIFIED POWER LAW REPRESENTATION OF THE ATMOSPHERIC DENSITY

Several important previous studies have been carried out in which the vertical distribution of air density in the integrals (1) and (2) has been represented by the exponential form (11) [e. g. - King-Hele, Cook and Walker, 1960; Parkyn, 1960]. In this paper, a modified power function with integral exponent will be used to represent the air density ρ. In addition to giving generally closer fits to the model atmospheres than do the exponential distributions, such power functions also permit a more ready development of the integrals (1) and (2). The algebraic forms which result from the integration of (1) and (2) depend in a simple way upon the value of the exponent and may easily be tabulated for various choices of n [Peirce, 1929]. These forms should find application in computing atmospheric densities and comparing proposed model atmospheres with air densities deduced from observations.

In the treatment to follow, a simple series relationship (21) between a and e will be obtained from (1) and (2) using the air density function $\rho^{(1)}$. The development will contain the parameters n and s and will be carried out by integrating, with suitable approximations, the ratio $\Delta a/\Delta e$. The later evaluation of (21) will be performed for n = 6, s = 1 for which $\rho^{(1)}$ will give an acceptable representation of the shape of the Kallmann model from 150 to 400 km (See Table 4). These values have been chosen in order to present certain preliminary results. In general, the parameters ρ_0, n and s are to be adjusted to fit the power function to the best available atmospheric density data.

The definite integral in (1) may be written

$$I^{(i)} = \int_0^{2\pi} (1 - e^2 \cos^2 E)^{3/2} (1 - e \cos E)^{-2} \rho^{(i)} \, dE,$$

$$i = 0, 1, 2, 3, 4. \tag{14}$$

For values of eccentricity greater than 0.02, the major contributions to the integrals $I^{(i)}$ occur at or near perigee q_0. This suggests the use of the approximation

$$I^{(i)} \simeq I^{(i)*} = (1 - e^2)^{3/2} \int_0^{2\pi} (1 - e \cos E)^{-2} \rho^{(i)} \, dE. \tag{15}$$

As an illustration (14) and (15) have been programmed for the IBM 7090 computer and evaluated for n = 6, s = 1 for typical values of the parameters e and q. The computed values of $I^{(i)}$ and $I^{(i)*}$, given in Table 7 in units of ρ_0, permit a comparison of the sensitivity of the integrals to the choice of air density function and to the approximation noted.

TABLE 7. VALUES OF $I^{(i)}$ AND $I^{(i)*}$ IN UNITS OF ρ_0† FOR TYPICAL VALUES OF e and q_0

q_0 (earth radii)	e††	$I^{(0)}$	$I^{(0)*}$	$I^{(1)}$	$I^{(1)*}$	$I^{(2)}$	$I^{(2)*}$	$I^{(3)}$	$I^{(3)*}$	$I^{(4)}$	$I^{(4)*}$
1.0470	0.2	0.6793	0.6777	0.6827	0.6810	0.6861	0.6844	0.6266	0.6255	0.6766	0.6750
(H'_0 = 48.4 km)	0.1	0.8478	0.8468	0.8521	0.8510	0.8565	0.8554	0.7808	0.7800	0.8444	0.8434
	0.05	1.135	1.134	1.141	1.140	1.147	1.146	1.040	1.039	1.130	1.129
	0.01	2.655	2.655	2.669	2.669	2.682	2.682	2.443	2.443	2.643	2.643
	0.003	4.442	4.442	4.453	4.453	4.463	4.463	4.337	4.337	4.434	4.434
1.0314	0.2	0.5598	0.5589	0.5617	0.5608	0.5636	0.5627	0.5073	0.5067	0.5487	0.5478
(H'_0 = 31.5 km)	0.1	0.6974	0.6968	0.6998	0.6992	0.7022	0.7016	0.6312	0.6308	0.6834	0.6828
	0.05	0.9284	0.9280	0.9316	0.9312	0.9349	0.9345	0.8375	0.8372	0.9091	0.9088
	0.01	2.137	2.137	2.145	2.145	2.153	2.153	1.1940	1.1940	2.088	2.088
	0.0003	3.890	3.890	3.899	3.899	3.907	3.907	3.685	3.685	3.839	3.839

†The value of ρ_0 is referenced to the particular value of perigee at which the integration was performed.

††For e = 0, $I^{(i)} = I^{(i)*} = 2\pi$.

The substitution of

$$\rho^{(1)} = C(n, s) r (r-s)^{-n}; \; C(n, s) = \rho_0 (q_0 - s)^n / q_0$$

$$r = a(1 - e \cos E)$$

$$(1 - e^2 \cos^2 E)^{1/2} \simeq (1 - e^2)^{1/2}$$

into (1) and (2) leads to the equations

$$\Delta a = - \delta \, C \, a^3 (1 - e^2)^{1/2} (a - s)^{-n} [F(\eta;n) + e \, G(\eta;n)] \qquad (16)$$

$$\Delta e = - \delta \, C \, a^2 (1 - e^2)^{3/2} (a - s)^{-n} G(\eta;n) \qquad (17)$$

where

$$\eta = ae \, (a - s)^{-1}, \; 0 \leqq \eta \leqq 1$$

$$F(\eta;n) = \int_0^{2\pi} (1 - \eta \cos E)^{-n} dE = 2\pi (1-\eta^2)^{-(2n - 1)/2} f(\eta;n)$$

$$G(\eta;n) = \int_0^{2\pi} \cos E (1 - \eta \cos E)^{-n} dE = 2\pi (1 - \eta^2)^{-(2n - 1)/2} k(\eta;n).$$

The functions f and k are polynomials in η whose coefficients are readily deduced from tables [Peirce, 1929]. The order of f is n - 2, n even; n - 1, n odd. The order of f is n - 1, n even; n - 2, n odd. Explicitly, for $2 \leqq n \leqq 11$,

$$f(\eta;n) = 1 + \frac{1}{2} C_{n-3}^{n-1} \eta^2 + \frac{3}{8} C_{n-5}^{n-1} \eta^4 + \frac{5}{16} C_{n-7}^{n-1} \eta^6 + \frac{35}{128} C_{n-9}^{n-1} \eta^8 + \frac{63}{256} C_{n-11}^{n-1} \eta^{10}$$

$$k(\eta;n) = \frac{n\eta}{2} \left[1 + \frac{1}{4} C_{n-4}^{n-2} \eta^2 + \frac{1}{8} C_{n-6}^{n-2} \eta^4 + \frac{5}{64} C_{n-8}^{n-2} \eta^6 + \frac{7}{128} C_{n-10}^{n-2} \eta^8 \right]$$

where the C_{n-p}^{n-j}, j = 1, 2; p = 3, 4, ..., 11, are the binomial coefficients.

Although the use of $\rho^{(0)}$ or $\rho^{(2)}$ in (1) and (2) will lead to relationships similar to (16) and (17), the particular forms of (16) and (17) are convenient for the following development.

The quotient $\Delta a / \Delta e$ may be treated as a derivative to provide the first order, nonlinear equation

$$\frac{d \ln ae}{d \ln e} = (1 + ef/k)/(1 - e^2). \tag{18}$$

The product e (f/k) is small compared to unity for the entire lifetime of the satellite. This suggests that the function f/k be expanded in a Taylor series in powers of $\eta - \eta_0$, where η_0 is the initial value of η. In turn, $\eta - \eta_0$ may be expanded in powers of $e - e_0$ under the approximation $\eta \simeq q_0\, e/(q_0 - s + e)$. When these expansions are introduced into (18) it may be reduced to

$$\frac{d \ln a}{de} = A_0 (1 + e^2) + A_1 (e - e_0) + A_2 (e - e_0)^2 + A_3 (e - e_0)^3 + e + e^3 + 0 (e^4). \tag{19}$$

If the symbols $f^{(i)} = i!f_i$ and $k^{(i)} = i!k_i$, $i = 0, 1, 2, 3$, designate the functions f and k and their first three derivatives evaluated at η_0, the constants A_0, A_1, A_2, and A_3 may be written

$$\begin{aligned}
A_0 &= y_0 && \text{(a)}\\
A_1 &= y_1/(q_0 - s + e_0) && \text{(b)}\\
A_2 &= (y_2 - y_1)/(q_0 - s + e_0)^2 && \text{(c)}\\
A_3 &= (y_1 - 2y_2 + y_3)/(q_0 - s + e_0)^3 && \text{(d)}
\end{aligned} \tag{20}$$

where

$$y_0 = f_0/k_0$$

$$y_1 = \frac{q_0 (q_0 - s)}{q_0 - s + e_0} \cdot \frac{f_1\, k_0 - f_0\, k_1}{k_0^2}$$

$$y_2 = \frac{q_0^2\, (q_0 - s)^2}{(q_0 - s + e_0)^2} \cdot \frac{f_0\, (k_1^2 - k_0\, k_2) - f_1\, k_0\, k_2 + f_2\, k_0^2}{k_0^3}$$

$$y_3 = \frac{q_0^3\, (q_0 - s)^3}{(q_0 - s + e_0)^3} \cdot \frac{f_0(2k_0k_1k_2 - k_1^3 - k_0^2\, k_3) + f_1(k_0k_1^2 - k_0^2\, k_2) - f_2k_0^2\, k_1 + f_3k_0^3}{k_0^4}.$$

The exponential function obtained from the integration of (19) may be expanded. The resultant series may then be truncated and collected in the form

$$a = \sum_{j=0}^{4} B_j (e - e_0)^j \tag{21}$$

where the B_j are functions of the initial values a_0, e_0 through the relations

$$
\begin{aligned}
B_0 &= a_0 && \text{(a)}\\
B_1 &= a_0 (A_0 + e_0 + A_0 e_0^2 - A_1 e_0^3) && \text{(b)}\\
B_2 &= (a_0/2)(1 + A_0^2 + A_1 + 4 A_0 e_0 - 2 A_1 e_0^2) && \text{(c)}\\
B_3 &= (a_0/6)(A_0^3 + 3A_0A_1 + 5A_0 + 2 A_2 - 2 A_1 e_0) && \text{(d)}\\
B_4 &= (a_0/12)(4 A_0 A_2 + 3 A_3) \; . && \text{(e)}
\end{aligned} \tag{22}
$$

In the preceding development it has been tacitly assumed that the earth's atmosphere is spherically symmetrical. For the purpose of developing a satellite orbit theory in the presence of air drag for perigee heights below 300 km a spheroidally symmetrical model of the earth's atmosphere is to be preferred. If the spherically symmetric atmospheric model is referenced to the earth's equator, a latitude dependent factor $\exp(-q_0 \epsilon \sin^2 \Phi / H'_0)$ [King-Hele, Cook and Walker, 1960] can be introduced to take account of the earth's oblateness. Here Φ is the geocentric latitude and ϵ the earth's ellipticity ($\epsilon \simeq 0.003353$). Modified in this way the air density function $\rho^{(1)}$ may be written

$$\rho^{(1)}(r, \Phi) = \rho(q_0, 0)(r/q_0)[(q_0 - s)/(r - s)]^n \exp(-q_0 \epsilon \sin^2 \Phi / H'_0).$$

The factor $\exp(-q_0 \epsilon \sin^2 \Phi\, H'_0)$ may be expanded in terms of the small parameter $\epsilon / H'_0 < 0.6$ and the theory developed in terms of a solution due to a spherical atmosphere plus perturbative terms arising from the oblateness.

The angle Φ is related to the orbital inclination i (essentially constant), the argument of perigee ω and the true anomaly

$$v = \cos^{-1}[(\cos E - e)/(1 - e \cos E)]$$

through the relation $\sin^2 \Phi = \sin^2 i \sin^2 (v + \omega)$. Thus, the oblateness effect depends on the parameters ϵ, H'_0, q_0, i, ω, e and E, although in (1) and (2), the integrations are carried out under the assumption that only E varies during a single revolution from perigee to perigee. Consequently the gross influence of oblateness may be conveniently interpreted in terms of "external" contributions, independent of E, due to the precession of the perigeal line over an oblate earth and "internal" contributions, dependent on E, due to a single orbital passage of the satellite over an oblate earth. For orbits of moderate eccentricity ($e > 0.02$) both the "external" and "internal" contributions are suppressed in ratios such as (18). The "external" contributions cancel directly and the "internal" contributions effectively cancel because most of the drag occurs at or very near perigee. Hence expression (21) is relatively insensitive to oblateness effects for eccentricities $e > 0.02$ [King-Hele, Cook and Walker, 1960].

DISCUSSION OF RESULTS

Equation (21) has been programmed for the IBM 7090 computer and calculations of a ($n = 6$; $s = 1$), based upon orbital data compiled by Walker [Walker, 1961], have been made for all of the satellites of interest launched through 1960. Certain representative results are shown in Tables 8 and 9. Values of a computed from (21) and the corresponding values computed from the K-H C W [King-Hele, Cook and Walker] theory for an oblate atmosphere are tabulated. Also given are values of a deduced from observations of the periods of the satellites [Walker, 1961]. Values of a computed from (21) using Smithsonian mean eccentricities for 1958 γ (Izsak, 1960) are given in Table 9 along with the counterpart Smithsonian data.

The agreement of the predictions of (21) with the K-H C W theory, with the Smithsonian mean elements and with the observed values is quite good over the range of eccentricities ($e > 0.02$) for which (21) is expected to hold. The differences in the lengths of the semimajor axis given either by the K-H C W theory or by Smithsonian data and those given by (21) are less than 5 km with a single exception. The exception, a difference of 13 km, occurs for 1957β for the small value of eccentricity $e = 0.022$ at which (21) might be expected to yield poorer results, particularly for orbits with a near-critical inclination angle (see below). General agreement of a similar quality between the values computed from (21) and the observed values may also be noted.

No simple well ordered analysis of the effects of atmospheric oblateness is possible since they depend on a medley of parameters. There should be, however, a strong dependence on both the orbital inclination and the argument of perigee due to the appearance of $\sin^2\phi$ in the latitude-dependent factor [King-Hele, Cook and Walker, 1960]. The close agreement of

TABLE 8. COMPARISON OF VALUES OF a COMPUTED FROM (21) WITH PREVIOUS THEORY AND OBSERVATIONS.

NAME	DATE	THEORY			OBSERVED		
		Eccentricity (K-H C W)	Semi-major Axis (Earth Radii) (K-H C W)	Semi-major Axis (Earth Radii) (21)	Semi-major Axis (Earth Radii)	Inclination (Degrees)	Argument of Perigee (Degrees)
Sputnik 2 1957	1957 Nov. 3.1	0.099	1.1467	1.1467	1.1467	65.33	59
	1958 Jan. 4.00	0.080	1.1225	1.1221	1.1227	65.29	35
	1958 Feb. 21.00	0.060	1.0970	1.0971	1.1002	65.26	14
	1958 Mar. 25.00	0.040	1.0720	1.0727	1.0715	65.23	359
	1958 Apr. 9.00	0.022	1.0490	1.0510	1.0490	65.21	352
Atlas (1958ζ)	1958 Dec. 18.96	0.090	1.1307	1.1307	1.1299	32.3	130
	1959 Jan. 1.78	0.070	1.1057	1.1055	1.1049	32.3	249
	1959 Jan. 17.0	0.036	1.0647	1.0643	1.0638	32.3	37
Discoverer 5 ($1959\ \epsilon_2$ Capsule)	1960 Feb. 15.1	0.0101	1.1501	1.1501	1.1505	79	47
	1960 June 8.0	0.083	1.1260	1.1268	1.1265	79	100
	1960 Dec. 2.3	0.042	1.0766	1.0758	1.0770	79	320
Sputnik 3 ($1958\ \delta_1$) (Rocket)	1958 May 15.3	0.111	1.1623	1.1623	1.1625	65.19	58
	1958 Aug. 15.1	0.089	1.1336	1.1334	1.1338	65.14	26
	1958 Oct. 11.25	0.066	1.1040	1.1043	1.1040	65.09	5
	1958 Nov. 13.66	0.041	1.0737	1.0737	1.0737	64.04	347
	1958 Nov. 30.59	0.017	1.0429	1.0451	1.0430	65.00	339
Explorer 4 (1958ϵ)	1958 July 26.63	0.128	1.1940	1.1940	1.1936	50.3	50
	1959 Mar. 21.02	0.085	1.1368	1.1359	1.1365	50.25	60
	1959 Aug. 22.02	0.048	1.0894	1.0891	1.0889	50.25	252
	1959 Oct. 19.5	0.011	1.0432	1.0441	1.0430	50.25	120

TABLE 9. COMPARISON OF VALUES OF a COMPUTED FROM (21) WITH PREVIOUS THEORY AND OBSERVATIONS.

NAME	DATE	THEORY			OBSERVED		
		Eccentricity (K-H C W)	Semi-major Axis (Earth Radii) (K-H C W)	Semi-major Axis (Earth Radii) (21)	Semi-major Axis (Earth Radii)	Inclination (Degrees)	Argument of Perigee (Degrees)
Explorer 3	1958 Mar. 26.73	0.166	1.2338	1.2338	1.2332	33.3	90
(1958 γ)	1958 May 15.0	0.110	1.1551	1.1549	1.1544	33.3	70
	1958 June 14.13	0.063	1.0958	1.0952	1.0949	33.3	326
		0.16099*		1.2264	1.2261*	33.347*	130.19*
		0.14537*		1.2037	1.2030*	33.332*	221.84*
		0.13337*		1.1867	1.1862*	33.245*	305.78*
		0.11161*		1.1570	1.1568*	33.335*	422.91*
		0.08884*		1.1273	1.1280*	33.336*	532.71*
		0.06102*		1.0928	1.0932*	33.239*	692.32*

* Smithsonian mean elements [Special Report No. 40-R, Smithsonian Astrophysical Observatory, pp. 21-23, 1960].

the data from (21) with the K-H C W theory for satellites such as 1957 β, 1958 δ and 1959 ϵ_2 which have significantly different values of the oblateness parameters illustrates the fact that atmospheric oblateness has little influence on the variation of a with e for $e > 0.02$. The influence of atmospheric oblateness becomes more important for the less eccentric orbits particularly for orbits with inclination near the critical angle (63.4°) where the precessional rate of the argument of perigee is small, as with 1957 β and 1958 δ_1. If ω sweeps out one or more revolutions, as with 1958 ϵ, an averaging effect is present which tends to make the oblateness effect small. This is illustrated by the fact that the differences in a between the values given by (21) and the K-H C W theory is greater for the satellites 1957 β and 1958 δ_1 with $e = 0.022$ and $e = 0.017$, respectively, than for the satellite 1958 ϵ with $e = 0.011$.

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The scale height in height-bands between 150 and 400 km is assumed to vary linearly with height. Integration of the hydrostatic equation for an ideal gas above a spherical earth then leads to a power function representation of the air density over the band. With integral exponents such power laws give better fits to several proposed model atmospheres over altitude ranges of several hundred kilometers than those provided by the usual exponential representation of air density. The representation of air density in satellite deceleration equations by power functions with integral exponents reduces them to elementary forms. It has been possible with such density distributions to obtain simplified formulas which may be useful for (a) computing atmospheric densities from satellite accelerations (b) comparing proposed model atmospheres with observations, and (c) developing further the theory of satellite orbits in the presence of air drag. As is possible with the exponential form, these power functions may be modified to take account of the effect of an oblate, rotating atmosphere. Their use may; therefore, permit the development of a simplified, accurate orbit theory for satellites with perigee heights below 300 km. Certain preliminary results are discussed and compared with previous theory and observations.

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