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A NEW METHOD OF SHF AMPLIFICATION BY FERRITES

V. A. Fabrikov

A new method of SHF amplification by ferrites is examined, connected with nonlinear gyromagnetic nutation effects.

1. The nonlinear gyromagnetic effects in ferrites at SHF may be divided according to the principles of their use into two groups; one is connected with the frequency and the other with the angular characteristics of the precessional motion of spin angular moments. The first group of effects, used for creating various types of parametric ferrite SHF amplifiers, has been examined in detail in the literature [1]. Effects of the second type, called nutation effects, have been studied comparatively little.

It has been shown that nonlinear nutation effects may be used for amplification of intermediate-frequency signals [2] and for increasing the effectiveness of SHF mixers [3]. In my earlier work [3], I also mentioned the possibility of obtaining a new effect — amplification of one of the SHF signals to be mixed due to another, more powerful signal. THis effect, which may be examined as a new method of SHF amplification by ferrites (its special feature is that amplification holds only in the presence of a magnetic-loss medium), is analyzed in the present work. The analysis is made on the basis of the solution of equations of motion of the gyromagnetic moments of a magnetized ferrite material under the action of two transversely polarized SHF signals of close frequency and of a longitudinal fluctuating reaction field. The solution of the problem is sought in a coordinate system which rotates synchronously with a constant (in magnitude) magnetic moment of the sample.

2. Let us examine a ferrite sample in a constant magnetic field $\vec{H}_0 = \vec{i}_z H_0$ strong enough to bring the material into a state of magnetic saturation. Let H_0 be the effective value of the field within the sample, taking into account the demagnetizing factors of the shape of the sample. Let us assume that together with the constant field $\vec{H}_0$ there is an arbitrarily oriented small field $\vec{h}$ acting on the sample, i.e., $\vec{H} = \vec{H}_0 + \vec{h}$ and $h \ll H_0$. Then the magnetic moment of the sample $\vec{M}$ may be examined as the total and its equation of motion written in vector form as [4]:

$$\dot{\vec{M}} = \gamma\, [\vec{H}\vec{M}] - \vec{R}, \qquad (1)$$

where the term taking losses into account $\vec{R}$ has the components $R_x = M_x/T_2$, $R_y = M_y/T_2$, $R_z = (M_z - M_0)/T_1$ [5, 6]. Here $\gamma = 1.76 \cdot 10^7$ oersted$^{-1}\cdot$second^{-1} is the absolute value of the gyromagnetic ratio of electron spin; T_1 and T_2 the longitudinal and transverse relaxation times, which describe the intensity of damping in the material; M_0 the equilibrium value of the magnetic moment in the constant field H_0. The dot above the symbol denotes differentiation with respect to time t and the brackets denote the vector product.

The above form of writing the equations of motion of the magnetic momentum does not assume the imposing of any limits on the values of T_1 and T_2, and the three scalar equations corresponding to vector Eq. (1) are in general independent. However, in most practical applications the absolute value of the magnetic moment M may apparently be considered unchanged under the action of a fluctuating field, assuming $M = M_0$. This condition is equivalent to the existence of a definite link between the values T_1 and T_2 ($T_2 \simeq 2T_1$) and between the values m_x, m_y, m_z, where $\vec{m} = \vec{M} - \vec{M}_0$. Of the three equations only two are found to be independent, which makes convenient the geometric examination of the problem in a rotating coordinate system which is stationary relative to the vector of the magnetic moment $\vec{M}$.

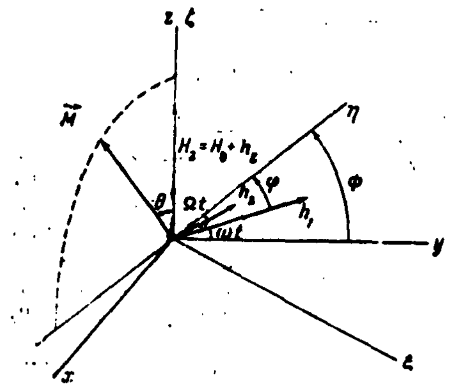

Fig. 1. Diagram of precessional magnetic moment $\vec{M}$ and the magnetic fields acting on it. $H_z = H_0 + h_z$ and $h_x + ih_y = h_1e^{i\omega t} + h_2e^{i(\omega \mp \Omega)t}$ in the rotating system of coordinates ξ, η, ζ.

Together with the cartesian coordinates x, y, z, let us introduce an orthogonal system of axes ξ, η, ζ with unit vectors $\vec{i}_\xi$, $\vec{i}_\eta$, $\vec{i}_\zeta$ fixed relative to the moving vector of the magnetic moment $\vec{M}$. The ζ-axis is in the direction of the constant magnetic field $\vec{i}_z H_0$, and η-axis is in the plane of the vectors $\vec{M}$ and H_0 such that the angle between $\vec{M}$ and $\vec{i}_\eta$ exceeds 90° (Fig. 1). Let Φ denote the angle between

ξ - and x -axes (or η and y), and θ the angle between $\vec{M}$ and $\vec{i}_\zeta = \vec{i}_z$. Obviously, $\Phi = \Phi(t)$. In general $\theta = \theta(t)$.

So far not imposing any additional limits on the longitudinal component of the fluctuating field $\vec{h}$, let us assume that the transverse components are harmonic and polarized circularly, assuming

$$h_x = h_1 \cos \omega t, \quad h_y = h_1 \sin \omega t$$

or, in complex notation,

$$h_x + ih_y = h_1 e^{i\omega t} \tag{2}$$

where i is an imaginary unit. The examination of fluctuating fields of only circular transverse polarization in the mathematical analysis of phenomena occurring under conditions of ferrite resonance has little effect on the generality of the results obtained. The form of notation of (2), which is convenient for describing the circular-polarization fields, should not be confused with the symbolic representation of values having harmonic dependence upon time.

Limiting by the circular-polarization fields allows the transverse components of the field $\vec{h}$ to be represented by the vector $\vec{h}_1$ in the plane xy, rotating in a fixed coordinate system with angular frequency ω. The angle $\varphi = \Phi - \omega t$, determining the position of the vector $\vec{h}_1$ in the rotating coordinate system ξ, η, ζ, in the general case may not be assumed constant.

Let ω' denote the instantaneous angular velocity of rotation of the vector $\vec{M}$ in a fixed coordinate system. Geometrically, it is easy to see that the vectors $\vec{H}$, $\vec{M}$ and $\vec{\omega}'$ in the coördinate system ξ, η, and ζ may be represented in the form

$$\vec{H} = \vec{i}_\xi h_1 \sin\varphi + \vec{i}_\eta h_1 \cos\varphi + \vec{i}_\zeta (H_0 + h_z),$$
$$\vec{M} = -M(\vec{i}_\eta \sin\Theta - \vec{i}_\zeta \cos\Theta),$$
$$\vec{\omega}' = -\vec{i}_\xi \dot{\Theta} + \vec{i}_\zeta \dot{\Phi}. \qquad (3)$$

In this

$$\frac{\dot{\vec{M}}}{M} = \frac{1}{M}[\vec{\omega}'\vec{M}] = \vec{i}_\xi \sin\Theta\dot{\Phi} - \vec{i}_\eta \cos\Theta\dot{\Theta} - \vec{i}_\zeta \sin\Theta\dot{\Theta},$$
$$\frac{[\vec{H}\vec{M}]}{M} = \vec{i}_\xi\{h_1\cos\Theta\cos\varphi + \sin\Theta(H_0 + h_z)\} - \vec{i}_\eta h_1 \cos\Theta \sin\varphi -$$
$$- \vec{i}_\zeta h_1 \sin\Theta\sin\varphi. \qquad (4)$$

Substituting (4) into (1), we obtain a system of two nonlinear differential equations relative to the independent variables φ and θ [7]

$$\dot{\varphi} = (\gamma H_0 - \omega) + \gamma h_z + \gamma h_1 \cot\Theta\cos\varphi,$$
$$\dot{\Theta} = \gamma h_1 \sin\varphi - \frac{\tan\Theta}{T_2}. \qquad (5)$$

The connection between the new variables φ, θ and the old M_x, M_y, M_z is given by the relationships

$$M_x + iM_y = -M\sin\Theta e^{i\Phi},$$
$$M_z = M\cos\Theta. \qquad (6)$$

3. For further examination it will be convenient to introduce a special symbol for the dimensionless parameter

$$x = (\omega - \gamma H_0)T_2 = \frac{H_{res} - H_0}{\Delta H}, \qquad (7)$$

which describes the nearness to the state of ferromagnetic resonance. Here $H_{res} = \omega/\gamma$ is the resonance of the magnetizing field relative to signal of frequency ω and $\Delta H = 1/\gamma T_2$ is the half-width of the line of ferromagnetic resonance.

Equation (5) can be easily solved when $h_z = 0$ and, therefore,

$\varphi = \theta = 0$. The solution taking the new symbols into account has the form

$$\tan \varphi_0 = \frac{1}{x},$$
$$\tan \theta = \frac{1}{\sqrt{1 + x^2}} \frac{h_1}{\Delta H} \qquad (8)$$

At $h_z \neq 0$, the accurate solution of (5) is rather complicated. However, an approximate solution of these equations may be found, examining h_z as the low perturbation of the system whose unperturbed state is described by the equations of (8)*. Expanding the right-hand members of (5) into a Taylor series at the point φ_0 and θ and dropping all terms of the second and higher order of smallness, we obtain two linear differential equations relative to the values $\varphi_1 = \varphi - \varphi_0$ and $\theta_1 = \theta - \theta_0$:

$$T_2\dot{\varphi}_1 + \varphi_1 = \frac{h_z}{\Delta H} - x\sqrt{1+x^2}\,\frac{\Delta H}{h_1}\,\theta_1,$$
$$T_1\dot{\theta}_1 + \theta_1 = \frac{x}{\sqrt{1+x^2}}\,\frac{h_1}{\Delta H}\,\varphi_1. \qquad (9)$$

Strictly speaking, $1 + x^2 + (\frac{h_1}{\Delta H})^2$ should have been written under the radicals in (9), but we ignored the value $(h_1/\Delta H)^2$, assuming

$$\left(\frac{h_1}{\Delta H}\right)^2 \ll 1. \qquad (10)$$

The solution of (9) at given h_z is sufficiently simple and allows the reaction of the harmonic medium to the action of a weak modulating signal to be determined, described in the direction of the z-axis in the case of a sinusoidal field h_z by the complex magnetic susceptibility of the medium at the frequency of this field [7].

* Perturbation may be assumed small if $h_z \ll \Delta H$. See appendix for case when h_z is always small in comparison with ΔH.

If two transverse magnetic fields with circular polarization act simultaneously on the ferrite material, i.e.,

$$h_x + ih_y = h_1 e^{i\omega t} + h_2 e^{i(\omega+\Omega)t}, \qquad (11)$$

then the expression for the field $\vec{H}$ in a rotating coordinate system takes a more general form:

$$\vec{H} = \vec{i}_\xi \{h_1 \sin\varphi - h_2 \sin(\Omega t - \varphi)\} + \vec{i}_\eta \{h_1 \cos\varphi + h_2 \cos(\Omega t - \varphi)\} + \vec{i}_\zeta (H_0 + h_z). \qquad (12)$$

For simplicity, the initial phase shift in formulas (11) and (12) was made equal to zero, which is unessential. Examining only those cases when $h_2 \ll h_1$, formula (8) can be left unchanged, and Eq. (9) should be replaced by more general equations

$$T_2\dot{\varphi}_1 + \varphi_1 = \frac{h_z}{\Delta H} - x\sqrt{1+x^2}\,\frac{\Delta H}{h_1}\,\Theta_1 + \frac{h_2}{h_1}\sqrt{1+x^2}\cos(\Omega t - \varphi_0),$$
$$T_2\dot{\Theta}_1 + \Theta_1 = \frac{x}{\sqrt{1+x^2}}\,\frac{h_1}{\Delta H}\,\varphi_1 - \frac{h_2}{\Delta H}\sin(\Omega t - \varphi_0). \qquad (13)$$

Having determined the parameters φ and Θ_1 from (13), we can find the expressions for the fluctuating components of the magnetic moment m_x, m_y, m_z by formulas

$$m_x + im_y = -M\sin\Theta e^{i\Phi} \simeq -M(\sin\Theta_0 + \nu)\,e^{i(\omega t+\varphi_0)},$$
$$m_z = M(\cos\Theta - \cos\Theta_0) \simeq -M\sin\Theta_0\Theta_1, \qquad (14)$$

where

$$\sin\Theta_0 = \frac{1}{\sqrt{1+x^2}}\,\frac{h_1}{\Delta H};$$
$$\nu = \Theta_1 + i\sin\Theta_0\varphi_1. \qquad (15)$$

Thus, we shall be interested in the variables ν and Θ_1. Let us rewrite (13) relative to these values in the form

$$T_2\dot{\nu} + (1 + ix)\,\nu = i\frac{1}{\sqrt{1+x^2}}\frac{h_1h_z}{(\Delta H)^2} + i\frac{h_1}{\Delta H}e^{i(\Omega t-\varphi_0)},$$

$$T_2^2\ddot{\theta}_1 + 2T_2\dot{\theta}_1 + (1 + x^2)\,\theta_1 = \frac{x}{\sqrt{1+x^2}}\frac{h_1h_z}{(\Delta H)^2} +$$

$$+\frac{h_z}{2\Delta H}\{(x + i - \Omega T_2)\,e^{i(\Omega t-\varphi_0)} + (x - i - \Omega T_2)\,e^{-i(\Omega t-\varphi_0)}\}, \qquad (16)$$

which is the most convenient for analysis. The right-hand member of the latter expression contains the sum of two complex-congugate values.

4. So far we have imposed no limitations on the value of h_z, entering (16), except the requirement of its smallness ($h_z \ll H_0$), where h_z is an arbitrary fluctuating component of the magnetic field $H_z = H_0 + h_z$ with respect to spectral composition, determining the resonance characteristic of the material relative to transversely polarized fields of the SHF at frequencies close to the natural frequency of the material γH_z. Modulation of the magnetizing field H_z may be accomplished by the action of an external signal on the ferrite sample, the magnetic field of which is polarized in the direction of the z-axis, but may also be dependent upon the reaction of the sample or of the interaction of the resonance circuit with the sample to change in the intensity of magnetization of the material in the direction of this axis.

In the present work we shall treat the latter case, which has already been examined [8] from the point of view of the possibility and conditions of forming natural oscillations of intensity of magnetization in the ferrite sample under the action of a sufficiently powerful, transversely polarized SHF signal. When two transversely polarized SHF signals, one of which considerably exceeds the other in magnitude, act on a ferrite sample simultaneously, the presence of a reaction field may lead to the formation, together with the combination effect, of another nonlinear effect — the amplification of a weak SHF signal at the expense of a strong one. Further examination reduces to the analysis of Eqs. (14) and (16) for the two most characteristic cases of links between the values h_z and Θ_1 entering Eq. (16).

Case 1. The field h_z is the reaction field of the resonance circuit which interacts with the ferrite sample (Fig. 2). If the circuit is tuned to the frequency Ω, which coincides with the difference between the two SHF frequencies acting on the sample, then, ignoring the high harmonic oscillations of m_z, the reaction field of the circuit can be represented in the form [8]

$$h_z = -\eta \frac{L}{R} \dot{m}_z = \eta Q M \sin \Theta_0 \dot{\Theta}_1 T_2, \tag{17}$$

where Q is the Q-factor of the circuit at the frequency $1/T_2$; and η is the coupling factor of the circuit with the sample, close to the duty factor. Substituting (17) into (16) and introducing the additional symbols

$$y = \Omega T_2, \quad a = \frac{M h_1^2}{(\Delta H)^2} \tag{18}$$

for the frequently encountered complex values, we obtain the equations

$$\begin{aligned} T_2 \dot{v} + (1 + ix) v &= i\eta Q \frac{a}{1+x^2} \dot{\Theta}_1 T_2 + i \frac{h_2}{\Delta H} e^{i(\Omega t - \varphi_0)}, \\ T_2^2 \ddot{\Theta}_1 + \left(2 - \eta Q a \frac{x}{1+x^2}\right) \dot{\Theta}_1 T_2 + (1 + x^2) \Theta_1 &= \\ = \frac{h_2}{2\Delta H} \{(x - y + i) e^{i(\Omega t - \varphi_0)} &+ (x - y - i) e^{-i(\Omega t - \varphi_0)}\}. \end{aligned} \tag{19}$$

The solution of these equations (under steady conditions) has the form

$$\Theta_1 = \frac{h_2}{2\Delta H}\left\{\frac{x-y+i}{1+x^2-y^2+2iy\left(1-\eta Q\frac{a}{2}\frac{x}{1+x^2}\right)}e^{i(\Omega t-\varphi_0)} + \right.$$
$$\left. + \frac{x-y-i}{1+x^2-y^2-2iy\left(1-\eta Q\frac{a}{2}\frac{x}{1+x^2}\right)}e^{-i(\Omega t-\varphi_0)}\right\},$$
$$\nu = \frac{h_2}{\Delta H}\left\{\frac{1-\eta Q\frac{a}{2}\frac{y}{1+x^2}\left(\frac{1-i(x-y)}{1+x^2-y^2+2iy\left(1-\eta Q\frac{a}{2}\frac{x}{1+x^2}\right)}\right)}{x+y-i}e^{i(\Omega t-\varphi_0)} + \right.$$
$$\left. + \frac{Q\, a\, y}{2(1+x^2)}\frac{1-i(x+y)}{(x-y-i)\left(1+x^2-y^2-2iy\left(1-\eta Q\frac{a}{2}\frac{x}{1+x^2}\right)\right)}e^{-i(\Omega t-\varphi_0)}\right\}. \qquad (20)$$

The latter expression allows the complex magnetic susceptibility of the sample χ_2 relative to a field of frequency $\omega + \Omega$ to be determined immediately. In fact, substituting (20) into (14), it is easy to determine that χ_2 coincides with the coefficient in the exponent $\exp i(\Omega t - \varphi_0)$ in this expression multiplied by $-M/h_2$, i.e.,

$$\chi_2 = -\frac{M}{\Delta H}\,\frac{1-\eta Q\frac{a}{2}\frac{y}{1+x^2}\left(\frac{1-i(x-y)}{1+x^2-y^2+2iy\left(1-\eta Q\frac{a}{2}\frac{x}{1+x^2}\right)}\right)}{x+y-i}. \qquad (21)$$

If

$$\chi_2 = \chi_2' - i\chi_2'', \qquad (22)$$

then

$$\chi_2'' = \frac{M}{\Delta H(1+(x+y)^2)}\left\{1-\eta Q\frac{a}{2}\frac{y}{1+x^2}\,\frac{1-(x^2-y^2)^2-4xy\left(1-\eta Q\frac{a}{2}\frac{x}{1+x^2}\right)}{(1+x^2-y^2)^2+4y^2\left(1-\eta Q\frac{a}{2}\frac{x}{1+x^2}\right)^2}\right\}. \qquad (23)$$

When there are definite relationships between the values x, y, and a sufficiently high value a, the imaginary member of the complex susceptibility χ_2'' may become negative, which corresponds to amplification of the weak signal at a frequency of $\omega + \Omega$. In fact, let

$x = 1$, $y = -2$ and $\eta aQ = 6$. Then $\chi_2'' = -1.75 \frac{M}{\Delta H}$. The graph of the dependence of the value χ_2'' upon y at $aQ = 6$ is shown in Fig. 3.

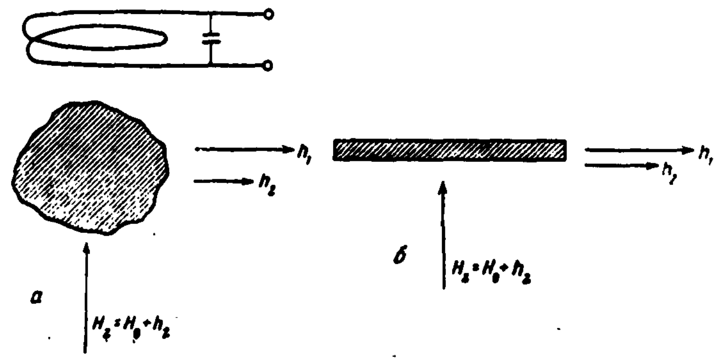

Fig. 2. Two characteristic cases of connection between transversely polarized fluctuating field h_z and the corresponding component of intensity of magnetization of the sample m_z: a) the field h_z is the reaction field of the resonance circuit interacting with the ferrite sample; b) the field h_z is a demagnetizing field of the shape of the sample.

This graph can serve as the characteristic frequency band of the effect under the condition that the resonance circuit is retuned to agree with the change in frequency Ω.

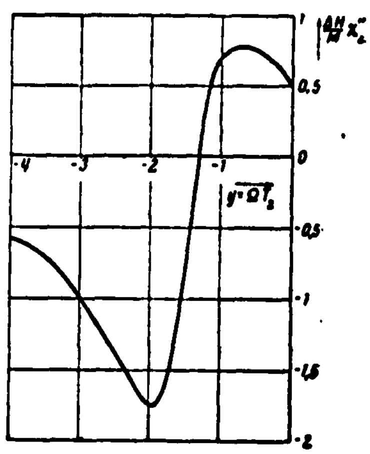

Fig. 3. The dependence of the imaginary member of the complex magnetic susceptibility of the sample χ_2'' relative to a weak signal of frequency $\omega + \Omega$ upon the magnitude of $y = \Omega T_2$ for the case when the field h_z is created by a resonance circuit tuned to the frequency Ω ($\omega = \gamma H_0$, $\eta aQ = 6$).

<u>Case 2.</u> The field h_z is a determining field of the shape of the sample. If the sample is a thin plate magnetized in the short direction, then

$$h_z = - m_z . \tag{24}$$

The substituting of (24) into (16) gives

$$\begin{gathered} T_2\dot{v} + (1 + ix)\,v = i\frac{a}{1+x^2}\,\Theta_1 + i\,\frac{h_2}{\Delta H}\,e^{i(\Omega t - \varphi_0)}, \\ T_2^2\ddot{\Theta}_1 + 2T_2\dot{\Theta}_1 + \left(1 + x^2 - a\,\frac{x}{1+x^2}\right)\Theta_1 = \\ = \frac{h_2}{2\Delta H}\{(x - y + i)\,e^{i(\Omega t - \varphi_0)} + (x - y - i)\,e^{-i(\Omega t - \varphi_0)}\}, \end{gathered} \tag{25}$$

hence, we find

$$\chi_2 = -\frac{M}{\Delta H}\,\frac{1 - \dfrac{a}{2(1+x^2)}\,\dfrac{y - x - i}{1 + x^2 - y^2 - a\dfrac{x}{1+x^2} + 2iy}}{x + y - i}, \tag{26}$$

i.e.,

$$\begin{gathered} \chi_2' = \frac{M}{\Delta H}\,\frac{\dfrac{a}{2(1+x^2)}\,\dfrac{1 + x^2 - y^2 - a\dfrac{x}{1+x^2} - 4xy + (y^2 - x^2)(1 + x^2 - y^2) - x - y}{\left(1 + x^2 - a\dfrac{x}{1+x^2} - y^2\right)^2 + 4y^2} + (x+y)}{1 + (x+y)^2}, \\ \chi_2'' = \frac{M}{\Delta H}\,\frac{1 - \dfrac{a}{1+x^2}\left\{\dfrac{x\left(y^2 - x^2 - 1 + a\dfrac{x}{1+x^2}\right) - y(1 + y^2 - x^2)}{\left(1 + x^2 - y^2 - a\dfrac{x}{1+x^2}\right)^2 + 4y^2}\right\}}{1 + (x+y)^2} \end{gathered} \tag{27}$$

At x = 0 the latter expressions take a more simple form:

$$\begin{gathered} \chi_2' = -\frac{M}{\Delta H}\,\frac{y + \dfrac{y^2 - 1}{y^2 + 1}\,\dfrac{a}{2}}{1 + y^2}, \\ \chi_2'' = \frac{M}{\Delta H}\,\frac{1 + \dfrac{y}{1 + y^2}\,a}{1 + y^2}. \end{gathered} \tag{28}$$

At a = 6, x = 0 and y = — 0.5 we have $\chi_2'' = - 1.12\,\frac{M}{\Delta H}$.

The frequency dependence of χ_2' and χ_2'' for a = 6 is shown in Fig. 4. For comparison, the frequency dependences are given for a = 0. A somewhat larger range can be obtained when x = 0.5.

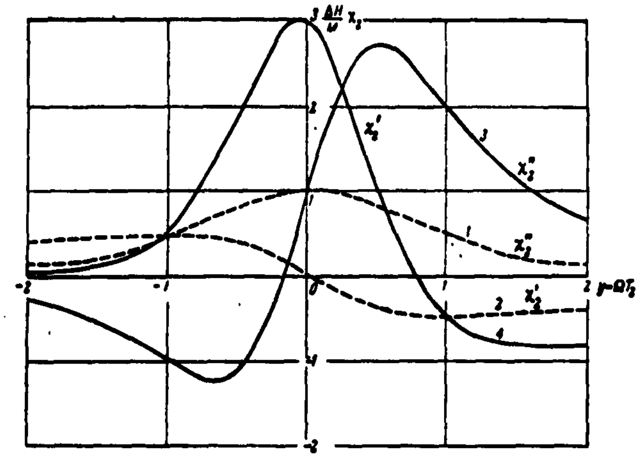

Fig. 4. The dependence of the complex magnetic susceptibility of the sample $\chi_2 = \chi_2' - i\chi_2''$ relative to a weak signal of frequency $\omega + \Omega$ upon the magnitude of $y = \Omega T_2$ when the field h_z is created due to the demagnetizing action of the shape of the sample: 1), 2) $H_0 = H_{res}$, $a = 0$; 3), 4) $H_0 = H_{res} - \Delta H$, $a = 6$.

5. Let us approximately determine P_1 — the power to the SHF necessary to attain $a = 6$, and the amplification factor of the weak signal obtained in this. Let us assume that a sample of yttrium ferrite is used having dimensions 3 x 3 x 0.3 mm, a magnetization saturation M = 1750 gauss and half-width of the resonance line ΔH = 1 oersted. Then

$$P_1 = r_1 h_1^2 = a \frac{(\Delta H)^3}{M} r_1 = 3.43 \cdot 10^{-3} r_1 \qquad (29)$$

If the sample is placed in a waveguide at a distance of four wavelengths from the short-circuited end, then at a wavelength of 3 cm the coupling factor r_1 between P_1 and h_1^2 may be made to equal 25 w/oersted^2 [2]. In this P_1 = 86 mw.

The amplification of the weak signal may be determined by the coefficient

$$K = 1 + \frac{P_r}{P_2}, \qquad (30)$$

where $P_r = -\mu_0 \frac{\chi_2'' h_2^2}{4} (\omega + \Omega) V$ is the power generated by the sample at a frequency of $\omega + \Omega$ under the action of a more powerful signal ω*. Substituting the values $\omega + \Omega = 2\pi \cdot 10^{10}$, $h_2^2 = P_2/r_2$, $V = 2.7 \cdot 10^{-8} \mu^3$, $\mu_0 = 4\pi \cdot 10^{-7}$ henry/m, into the formula, we obtain

$$K = 1 - 0.34 \frac{\chi_2''}{r_2} . \qquad (31)$$

If $r_2 = r_1 = 25$ w/oersted^2 and $\chi_2'' = - M/\Delta H = - 1750$, then $K = 25$.

Thus the actual creation by the method under examination of a ferrite SHF power amplifier operating at powers less than 100 mw is represented, where the amplification factor is above 10 in the frequency band for which the difference frequency of the signal and local oscillator is within a few Mc.

The author thanks A. A. Pistol'kors for examining the paper and making a number of remarks about the work.

APPENDIX

The problem of the interaction of electromagnetic signals with a modulated gyromagnetic medium for high amplitudes of the modulating field.

The initial equations are Landau and Lifshits equations in the form

$$\dot{m}_x + \frac{m_x}{T_2} = -\gamma (H_z m_y - h_y M_z), \qquad \text{(I)}$$

$$\dot{m}_y + \frac{m_y}{T_2} = \gamma (H_z m_x - h_x M_z),$$

$$\dot{m}_z + \frac{m_z}{T_1} = \gamma (h_x m_y - h_y m_x). \qquad \text{(II)}$$

* The factor K describes the effectiveness of the operation of the ferrite element in the power amplification of a weak SHF signal, determining the signal amplification obtainable at an optimum matching of load with the amplifier components. It should be distinguished from the current (or voltage) amplification factor at unknown impedance load.

The symbols are the same as in the text of the article. Introducing a new variable $N = m_x + im_y$ and substituting $N = Se^{\int\left(i\gamma H_z - \frac{1}{T_2}\right)dt}$, Eq. (I) takes the form

$$N = -i\gamma e^{\int\left(i\gamma H_z - \frac{1}{T_2}\right)dt} \int M_z (h_x + ih_y)\, e^{-\int\left(i\gamma H_z - \frac{1}{T_2}\right)dt} dt. \qquad (III)$$

m_z is small in comparison with M and taking it into account in Eq. (III) is unessential. Substituting $M_z \simeq M$ into (III) and

$$h_x + ih_y = (h_1 + h_2 e^{i(\Omega t + \delta)})\, e^{i\omega t}, \quad h_z = h_0 \cos \Omega t \qquad (IV)$$

and using known formulas of expansion into a series [9]

$$e^{\pm iz \cos \Omega t} = \sum_{k=-\infty}^{\infty} (\pm 1)^k J_k(z)\, e^{ik\Omega t}, \qquad (V)$$

we obtain

$$N = -i \frac{M}{\Delta H} e^{i\omega t} \sum_{p=-\infty}^{\infty} J_p(z)\, e^{ip\Omega t} \sum_{k=-\infty}^{\infty} (-1)^k \frac{J_k(z)\, h_1 - J_{k-1}(z)\, h_2 e^{i\delta}}{1 + iX_k} e^{ik\Omega t}. \qquad (VI)$$

Here $X_k = x + ky$; $J_k(z)$ a Bessel function of the first type of the k-th order from the argument $z = \frac{h_0}{\Delta H} \frac{1}{y}$. The initial phase angle δ in formulas (IV) and (VI) is determined with respect to the field h_z.

Expression (VI) by a simple regrouping of terms, takes the form

$$N = -i \frac{M}{\Delta H} \sum_{l=-\infty}^{\infty} A_l e^{i(\omega + l\Omega)t}, \qquad (VII)$$

where

$$A_l = \sum_{k=-\infty}^{\infty} (-1)^l \frac{J_{k-l}}{1 + iX_k} (J_k h_1 - J_{k-1} h_2 e^{i\delta}). \qquad (VIII)$$

These relationships allow the susceptibility of the medium χ_1 and χ_2 relative to signals of frequency $\omega + \Omega$ to be calculated by the formulas

$$\chi_1 = -i\frac{M}{\Delta H}\frac{A_0}{h_1} = -i\frac{M}{\Delta H}\sum_{k=-\infty}^{\infty}\frac{J_k^2 - J_kJ_{k-1}\frac{h_2}{h_1}e^{i\delta}}{1+ix_k}, \qquad (IX)$$

$$\chi_2 = -i\frac{M}{\Delta H}\frac{A_1}{h_2}e^{-i\delta} = -i\frac{M}{\Delta H}\sum_{k=-\infty}^{\infty}\frac{J_{k-1}^2 - J_kJ_{k-1}\frac{h_1}{h_2}e^{i\delta}}{1+iX_k}. \qquad (X)$$

Rewriting Eq. (II) in the form

$$m_z + \frac{m_z}{T_1} = \gamma\,\mathrm{Im}\,\{N(h_x - ih_y)\}$$

and using formulas (VIII) and (VII), we obtain

$$m_z = -\frac{M}{(\Delta H)^2}\,\mathrm{Re}\sum_{l=0}^{\infty}\frac{B_l}{\frac{T_2}{T_1}+ily}e^{il\Omega t}, \qquad (XI)$$

where

$$B_0 = A_0h_1 + A_1h_2e^{-i\delta};$$

$$B_l = (A_l + A_{-l}^*)h_1 + (A_{l+1}e^{-i\delta} + A_{1-l}^*e^{i\delta})h_2, \quad l = 1, 2, \ldots. \qquad (XII)$$

The component of m_z at the frequency Ω, in accordance with formulas (XI) and (XII), is described by the expression

$$(m_z)_\Omega = -\frac{M}{\Delta H}\,\mathrm{Re}\sum_{k=-\infty}^{\infty}[h_1h_2(J_k^2e^{i\delta} + J_{k-1}J_{k+1}e^{-i\delta}) -$$

$$-(h_1^2J_kJ_{k+1} + h_2^2J_kJ_{k-2})]\frac{\xi}{(1-iX_k)(1+iX_{k+1})}. \qquad (XIII)$$

The coefficient $\xi = \dfrac{2+iy}{\frac{T_2}{T_1}+iy}$ in (XIII) at $T_2 = 2T_1$ (the condition of preservation of the value M) reverts to unity.

In practice, formulas (IX), (X), and (XIII) are valid for any h_0, h_1 and h_2 (the only limit is the condition $m_z \ll M$).*

* The examination reduces comfortably to a uniform precession. The possibility of generating higher types of oscillations is not taken in the calculation.

In particular, they explain the widening of the ferromagnetic resonance line under the action of the field h_z [10]. However, these formulas are somewhat cumbersome for practical application.

If $h_0/2\Delta H \ll y$, formulas (IX), (X), and (XIII) ar. simplified, using in the first approximation with respect to the value $\frac{z}{2} = \frac{h_0}{2\Delta H}\frac{1}{y}$ the form

$$\chi_1 = -i\frac{M}{\Delta H}\frac{1}{1+ix}\left\{1+\frac{h_2}{h_1}\frac{h_0 e^{i\delta}}{2\Delta H}\frac{i}{1+i(x+y)}\right\}, \quad \text{(XIV)}$$

$$\chi_2 = -i\frac{M}{\Delta H}\frac{1}{1+i(x+y)}\left\{1+\frac{h_1}{h_2}\frac{h_0 e^{-i\delta}}{2\Delta H}\frac{i}{1+ix}\right\}, \quad \text{(XV)}$$

$$m_z = -\frac{M}{(\Delta H)^2}\,\mathrm{Re}\left\{\frac{h_1 h_2 e^{i\delta}}{(1-ix)[1+i(x+y)]}+\right.$$
$$\left.+\sum_{k=0}^{1}\frac{h_{k+1}^2 h_0}{\Delta H}\frac{X_k}{1+X_k^2}\frac{1}{[1+i(X_k+y)][1-i(X_k-y)]}\right\}. \quad \text{(XVI)}$$

When h_z is the reaction field of the circuit or the sample and is connected with m_z by a linear relationship, expression (XVI) can be used to determine the magnitude of $h_0 e^{-i\delta}$. Substituting the thus-found expression for $h_0 e^{-i\delta}$ into formula (XV) one can obtain, at corresponding values of the coupling factor between h_z and m_z, the expression for χ_2 coinciding at $h_2 \ll h_1$ with those derived in the text (formulas (21) and (27)).

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