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THE NATURE OF THE MOTION OF A GYROSCOPE IN THE PRESENCE OF DISPLACED CENTER OF GRAVITY AND FRICTION*

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A study of the effect of displaced center of gravity in a gyroscopic system in the presence of friction and correction momenta upon the motion of a gyroscope with a horizontal external axis. The problem is solved using the concept of the "mapping point." The effects of displaced center of gravity, correction and fluid friction on the motion of a gyroscope with a constant correction characteristic when the cantilever is fixed are examined.

An examination is made of the effect of displaced center of gravity in a gyroscopic system in the presence of friction and correction momenta on the motion of a gyroscope with a horizontal external axis. An accurate solution to the problem on the effect of forces of dry friction on the behavior of a gyroscope mounted on a fixed cantilever was obtained by Ye. L. Nikolai [1, 2]. This problem was

* Reported at the III Joint Conference of the Institute of Higher Learning on Gyroscopic Technology. The report was called "The Motion of a Gyrovertical in the Presence of Friction and Gaps in the Support Bearings."

solved using the notion of the "mapping point." D. M. Klimov, using this concept, examined the problem on the behavior of a gyroscope on a fixed cantilever under the assumption that friction momenta in the support axes were proportional to the dynamic reactions [3].

N. V. Butenin examined this problem in the presence of fluid as well as dry friction by a more general method; in addition, he solved some new problems for a gyroscope with a vertical external axis [4, 5].

In the present work the problem is solved for a gyroscope with a horizontal external axis by using a "mapping point."

An examination is made of the effect of displaced center of gravity, correction and fluid friction on the motion of a gyroscope with a constant correction characteristic when the cantilever is fixed.

The Effect of Displaced Center of Gravity, Correction and Fluid Friction on the Motion of a Gyroscope

The equations of motion of a free gyroscope on a fixed cantilever have the form [6]:

$$\left.\begin{aligned} I_B\ddot{\theta} + I\dot{\Omega}\cos\theta\cdot\dot{\Psi} &= M_B \\ I_C\ddot{\Psi} - I\Omega\cos\theta\cdot\dot{\theta} &= M_C \end{aligned}\right\}. \qquad (1)$$

Let M_B and M_C, the momenta relative to the axes of the internal and external frames, respectively, be determined by the displacements of the center of gravity of a gyroscopic system and by its weight [7], by the presence of fluid driction in the support axes, and also by correction momenta with a constant correction characteristic [8].

Hence, let us represent the right-hand members of Eqs. (1) in the form

$$\left.\begin{aligned} M_{\text{в}} &= M_{\text{вз}} + M_{\text{вт}} + M_{\text{вк}} \\ M_{\text{c}} &= M_{\text{cз}} + M_{\text{cт}} + M_{\text{cк}} \end{aligned}\right\}, \qquad (2)$$

where

$$\left.\begin{aligned} M_{\text{вз}} &= (a_p + b_p)\cos\theta \cdot G_r \cos\Psi + \\ &+ [a - (a_p + b_p)\sin\theta]\, G_r \cos\Psi \cdot \sin\theta \\ M_{\text{cз}} &= (a \sin\Psi + b)\, G_r \end{aligned}\right\}; \qquad (3)$$

a_p is the radial displacement on the main axis;

b_p the radial displacement on the internal axis;

a the axial displacement along the main axis;

b the axial displacement along the interal axis;

G_g the weight of the gyroscope;

Ψ and θ the angles of turning of the external and internal frames, respectively;

$M_{BT} = -\alpha\dot{\theta}$ the moment of fluid friction relative to the axis of the internal frame;

$M_{CT} = -\beta\dot{\Psi}$ the moment of fluid friction relative to the axis of the external frame;

α and β the coefficients of fluid friction;

$M_{BK} = -k_B$ sign Ψ and $M_{CK} = k_C$ sign θ the correction momenta;

k_B and k_C the coefficients of the correction momenta.

Assuming that the angles Ψ and θ are small and at $\theta = 0$ the external and internal frames are perpendicular, we may write

$$\left.\begin{aligned} M_{\text{в}} &= aG\theta + B - \alpha\dot{\theta} - k_{\text{в}}\,\text{sign}\,\Psi \\ M_{\text{c}} &= aG\Psi + A - \beta\dot{\Psi} + k_{\text{c}}\,\text{sign}\,\theta \end{aligned}\right\}, \qquad (4)$$

where G is the weight of the gyroscope rotor;

$B = a_pG + b_p(G + G_{BK})$;

G_{BK} the weight of the internal cardan ring of the support;

$A = b(G + G_{BK})$.

Substituting the found values from (4) of the momenta M_B and M_C into Eq. (1), we may write

$$\left.\begin{aligned} I_B\ddot{\vartheta} + I\Omega\dot{\Psi} - aG\vartheta &= -\alpha\dot{\vartheta} - k_B \operatorname{sign}\Psi + B \\ I_C\ddot{\Psi} - I\Omega\dot{\vartheta} - aG\Psi &= -\beta\dot{\Psi} + k_C \operatorname{sign}\vartheta + A \end{aligned}\right\}. \qquad (5)$$

Using the concept of the mapping point, in order to study System (5) in general form, it follows to operate in four-dimensional space, which represents considerable difficulties and deprives one of the clearness of a geometrical picture of the motion of the mapping point, therefore, let us examine System (5) without inertial terms

$$\left.\begin{aligned} I\Omega\dot{\Psi} - aG\vartheta &= -\alpha\dot{\vartheta} - k_B \operatorname{sign}\Psi + B \\ I\Omega\dot{\vartheta} + aG\Psi &= \beta\dot{\Psi} - k_C \operatorname{sign}\vartheta - A \end{aligned}\right\}. \qquad (6)$$

Let us transform (6) to the form:

$$\left.\begin{aligned} \frac{d\Psi}{dt} &= \frac{1}{\alpha\beta + (I\Omega)^2}\Big[I\Omega B + \alpha A + \alpha k_C \operatorname{sign}\vartheta - I\Omega k_B \operatorname{sign}\Psi + aG(I\Omega\vartheta + \alpha\Psi)\Big] \\ \frac{d\vartheta}{dt} &= \frac{1}{\alpha\beta + (I\Omega)^2}\Big[\beta B - I\Omega A - I\Omega k_C \operatorname{sign}\vartheta - \beta k_B \operatorname{sing}\Psi - aG(I\Omega\Psi - \beta\vartheta)\Big] \end{aligned}\right\}. \qquad (7)$$

Let $k_B > B$ and $k_C < A$. Let us examine the plane $\vartheta\Psi$ (the coordinate plane) of System (6). The coordinate axes divide the plane $\vartheta\Psi$ into four quadrants, in each of which Eqs. (7) have different forms. From (7) it follows that the differential equation of the phase trajectories is

$$\frac{d\vartheta}{d\Psi} = \frac{\beta B - I\Omega A - I\Omega k_C \operatorname{sign}\vartheta - \beta k_B \operatorname{sign}\Psi - aG(I\Omega\Psi - \beta\vartheta)}{I\Omega B + \alpha A + \alpha k_C \operatorname{sign}\vartheta - I\Omega k_B \operatorname{sign}\Psi + aG(I\Omega\vartheta + \alpha\Psi)}. \qquad (8)$$

The singular points of Eq. (8) on the plane $\vartheta O\Psi$ will be the same points at which the following conditions are fulfilled simultaneously:

$$\left.\begin{array}{l} aG\beta\vartheta - aGl\Omega\Psi + \beta B - l\Omega A - l\Omega k_c \operatorname{sign}\vartheta - \beta k_s \operatorname{sign}\Psi = 0 \\ aGl\Omega\vartheta + aG\alpha\Psi + l\Omega B + \alpha A + \alpha k_c \operatorname{sign}\vartheta - l\Omega k_s \operatorname{sign}\Psi = 0 \end{array}\right\}. \tag{9}$$

These points (singular points) for the initial system correspond to equilibrium states. The coordinates of the singular points are found from the expressions

$$\left.\begin{array}{l} \vartheta_\kappa = \dfrac{k_s \operatorname{sign}\Psi - B}{aG} \\ \Psi_\kappa = -\dfrac{k_c \operatorname{sign}\vartheta + A}{aG} \end{array}\right\}, \tag{10}$$

where K = 1, 2, 3, 4 is the number of the corresponding quadrant.

For the $\vartheta 0 \Psi$ coordinate system

$$\vartheta_1 > 0,\ \Psi_1 > 0;\ \vartheta_2 > 0;\ \Psi_2 < 0;\ \vartheta_3 < 0;\ \Psi_3 < 0;\ \vartheta_4 < 0,\ \Psi_4 > 0.$$

From (10) we find

$$\vartheta_1 > 0;\ \vartheta_2 < 0;\ \vartheta_3 < 0;\ \vartheta_4 > 0,$$
$$\Psi_1 < 0;\ \Psi_2 < 0;\ \Psi_3 < 0;\ \Psi_4 < 0.$$

Hence it follows that beyond the coordinate axes Eq. (8) has only one singular point in third quadrant, which corresponds to the equilibrium position of the gyroscope frames after their deflection at angles equal to respectively

$$\left.\begin{array}{l} \vartheta = -\dfrac{k_s + B}{aG} \\ \Psi = \dfrac{k_c - A}{aG} \end{array}\right\}. \tag{11}$$

The nature of the singular point in this case is determined by the Eqs. [9, 10]

$$S^2 - (n_1 + n_2)S + \lambda^2 + n_1 n_2 = 0, \tag{12}$$

where

$$n_1 = \frac{aG\alpha}{\alpha\beta + (l\Omega)^2}; \quad n_2 = \frac{aG\beta}{\alpha\beta + (l\Omega)^2}; \quad \lambda = \frac{aGl\Omega}{\alpha\beta + (l\Omega)^2}. \tag{13}$$

Here $p = -(n_1 + n_2) < 0$ and $q = n_1 n_2 + \lambda^2 > 0$, therefore, the roots of Eq. (12) S_1 and S_2 will be complex conjugates; at $R_e[S] > 0$ the singular point is an unstable focal point.

Integrating Eq. (8), we find sets of integral curves. Let us first of all substitute the variables in Eq. (8)

$$\begin{aligned} \xi &= \Psi - \Psi_\kappa, \\ \eta &= \vartheta - \vartheta_\kappa. \end{aligned} \tag{14}$$

Equation (8) takes the form

$$\frac{d\eta}{d\beta} = \frac{-\lambda\xi + n_2\eta}{\lambda\eta + n_1\xi}. \tag{15}$$

After integration we obtain

$$\xi^2 + 2h\xi\eta + \eta^2 = c\,e^{\frac{2a_1}{\sqrt{1-ha}} \arctan \frac{\eta + h\xi}{\xi\sqrt{1-h^2}}}, \tag{16}$$

where

$$a_1 = \frac{n_1 + n_2}{2\lambda}; \quad h = \frac{n_1 - n_2}{2\lambda};$$

C is the arbitrary constant of integration.

In the old variables Ψ and ϑ we shall have:

$$\begin{aligned} &(\Psi - \Psi_\kappa)^2 + 2h(\Psi - \Psi_\kappa)\cdot(\vartheta - \vartheta_\kappa) + (\vartheta - \vartheta_\kappa)^2 = \\ &= c\,e^{-\frac{2a_1}{\sqrt{1-h^2}} \arctan \frac{(\vartheta - \vartheta_\kappa) + h(\Psi - \Psi_\kappa)}{(\Psi - \Psi_\kappa)\sqrt{1-h^2}}}. \end{aligned} \tag{17}$$

The set of curves of (17) represent a spiral issuing from the corresponding point with coordinates $\Psi = \Psi_k$, $\vartheta = \vartheta_k$ in each quadrant.

In order to judge the behavior of the angles of deflection Ψ and ϑ as a function of the initial conditions, integral curves may be constructed on the plane $\Psi\vartheta$ and the motion of the mapping point traced. Let us construct a set of integral curves of (17). For the third quadrant the curves will have the form:

$$\left(\Psi-\frac{k_c-A}{aG}\right)^2+\frac{\alpha-\beta}{\gamma\Omega}\left(\Psi-\frac{k_c-A}{aG}\right)\cdot\left(\vartheta+\frac{k_B+B}{aG}\right)+\left(\vartheta+\frac{k_B+B}{aG}\right)^2=$$
$$=ce^{-\frac{\alpha+\beta}{\gamma\Omega}\arctan\frac{\left(\vartheta+\frac{k_B+B}{abG}\right)+\frac{\alpha-\beta}{\gamma\Omega}\left(\Psi-\frac{k_c-A}{aG}\right)}{\Psi-\frac{k_c-A}{aG}}}. \tag{18}$$

Let us find the integral curve in the third quadrant which passes through the point $\Psi=\frac{k_c-A}{aG}$, $\vartheta=0$. For this curve

$$\left(\frac{k_B+B}{aG}\right)^2=ce^{-\frac{\alpha+\beta}{\gamma\Omega}\arctan\infty}=ce^{-\frac{\alpha+\beta}{2\gamma\Omega}\pi} \tag{19}$$

and therefore, the constant of integration

$$c=\left(\frac{k_B+B}{aG}\right)^2e^{\frac{\alpha+\beta}{2\gamma\Omega}\pi}. \tag{20}$$

Substituting the value of (20) into Expression (18), we find the equation of the curve sought after in the form

$$\left(\Psi-\frac{k_c-A}{aG}\right)^2+\frac{\alpha-\beta}{\gamma\Omega}\left(\Psi-\frac{k_c-A}{aG}\right)\cdot\left(\vartheta+\frac{k_B+B}{aG}\right)+\left(\vartheta+\frac{k_B+B}{aG}\right)^2=$$
$$=23\left(\frac{k_B+B}{aG}\right)^2e^{-\arctan\frac{\left(\vartheta+\frac{k_B+B}{ab}\right)+\frac{\alpha-\beta}{\gamma\Omega}\left(\Psi-\frac{k_c-A}{aG}\right)}{\Psi-\frac{k_c-A}{aG}}}. \tag{21}$$

Let us determine under what conditions this integral curve will be tangent to the ϑ-axis at the point $\Psi = 0$; $\vartheta = -\frac{k_B + B}{aG}$. This condition is

$$\left(-\frac{k_c-A}{aG}\right)^2=23\left(\frac{k_B+B}{aG}\right)^2e^{-\arctan\frac{\alpha-\beta}{\gamma\Omega}},$$

that is

$$(-k_c + A)^2 = 23\,(k_B + B)^2\, e^{-\arctan\frac{\alpha-\beta}{\gamma\Omega}}. \qquad (22)$$

Let us note that in the absence of fluid friction this condition would have the form

$$\frac{k_C - A}{k_B + B} \cong 4.8.$$

Expression (22) may be rewritten as

$$A = k_C - 4.8\,(k_B + B)\, e^{-\frac{1}{2}\arctan\frac{\alpha-\beta}{\gamma\Omega}}. \qquad (23)$$

Obviously, if

$$A < k_c - 4{,}8\,(k_B + B)\, e^{-\frac{1}{2}\arctan\frac{\alpha-\beta}{\gamma\Omega}}, \qquad (24)$$

(A is the constant component of the momentum about the external axis, which is dependent upon displacement of the center of gravity), then the curve of (21) does not intersect the ψ-axis; if

$$A > k_c - 4{,}8\,(k_B + B)\, e^{-\frac{1}{2}\arctan\frac{\alpha-\beta}{\gamma\Omega}},$$

then the curve of (21) will intersect the ψ-axis.

The equation of the integral curve passing through the point $Y = 0$, $\psi = -\frac{k_B + B}{aG}$ (in the third quadrant) is found as before:

$$\left(\frac{A - k_c}{aG}\right)^2 = c\, e^{-\frac{\alpha+\beta}{\gamma\Omega}\arctan\frac{\alpha-\beta}{\gamma\Omega}}. \qquad (25)$$

Hence the value of the arbitrary constant

$$C = \left(\frac{A - k_c}{aG}\right)^2 e^{\frac{\alpha-\beta}{\gamma\Omega}\arctan\frac{\alpha-\beta}{\gamma\Omega}}, \qquad (26)$$

and the equation of the curve sought after will have the form

$$\left(\Psi-\frac{k_c-A}{aG}\right)^2+\frac{\alpha-\beta}{l\Omega}\left(\Psi-\frac{k_c-A}{aG}\right)\left(\vartheta+\frac{k_B+B}{aG}\right)+\left(\vartheta+\frac{k_B+B}{aG}\right)^2=$$

$$=\left(\frac{A-k_c}{aG}\right)^2 e^{\arctan\frac{\alpha-\beta}{l\Omega}-\arctan\frac{\left(\vartheta-\frac{k_B+B}{aG}\right)+\frac{\alpha-\beta}{l\Omega}\left(\Psi-\frac{k_c-A}{aG}\right)}{\Psi-\frac{k_c-A}{aG}}}. \qquad (27)$$

The condition under which this curve will be tangent to the Ψ-axis at the point $\Psi=\frac{k_c-A}{aG}$, $\vartheta=0$ (tangency on the strength of Eq. (8)) has the form of (23). This means that when condition (24) is fulfilled, the curve of (27) will intersect the Ψ-axis; when condition (25) is fulfilled, the curve (27) will not intersect the Ψ-axis. In the plane $\Psi\vartheta$ there will be on the Ψ and ϑ axes, segments of the junction of the motion of the mapping point along the integral curves. On the ϑ-axis this section is from $-\frac{k_c+A}{aG}$ to $\frac{k_c-A}{aG}$. On the ϑ-axis the section is from $-\frac{B+k_B}{aG}$ to $-\frac{B-k_B}{aG}$. When the mapping point hits the section of the junction of motion on the Ψ-axis, the equations of motion (7) will be

$$\dot{\Psi}=n_1\Psi+M_{BA}-l\Omega m_B+\frac{\alpha M_{c\alpha}}{\alpha\beta+(l\Omega)^2}>0;$$

(we assume that $M_{BA}+\frac{\alpha M_{c\alpha}}{\alpha\beta+(l\Omega)^2}>l\Omega m_B$)

$$\dot{\vartheta}=-\lambda\Psi-\frac{l\Omega M_{c\alpha}}{\alpha\beta+(l\Omega)^2}+M_{AB}-\beta m_B=0, \qquad (28)$$

where

$$\left.\begin{array}{l} n_1=\frac{aG\alpha}{\alpha\beta+(l\Omega)^2};\quad M_{BA}=\frac{l\Omega B+\alpha A}{\alpha\beta+(l\Omega)^2} \\ m_B=\frac{k_B}{\alpha\beta+(l\Omega)^2};\quad M_{AB}=\frac{\beta B-l\Omega A}{\alpha\beta+(l\Omega)^2};\quad -k_c<M_{c\alpha}<k_c \end{array}\right\} \qquad (29)$$

From Eq. (28) it follows that the mapping point, having hit the Ψ-axis in the section in question, will move along the Ψ-axis in the direction of increasing Ψ to the point $\Psi=\frac{k_c-A}{aG}$, $\vartheta=0$, at which point it leaves the Ψ-axis.

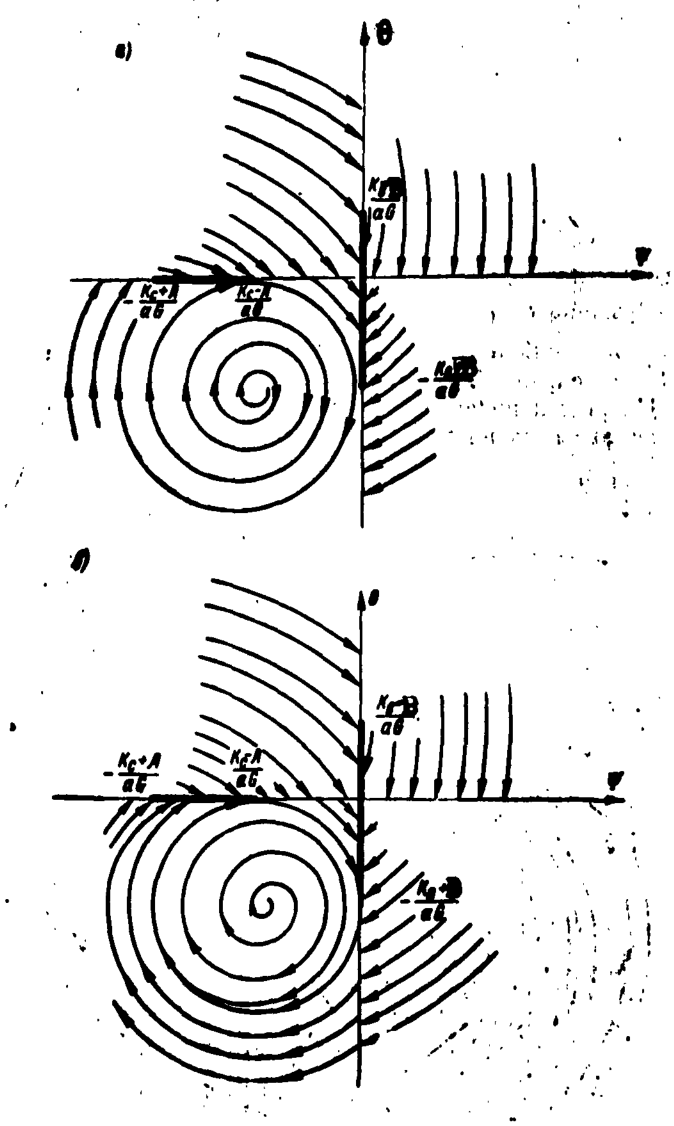

Fig. 1.

When the mapping point strikes a similar section of the ϑ-axis, Eq. (7) will be

$$\left.\begin{aligned} \dot{\Psi} &= \lambda\vartheta + M_{BA} \pm a m_c - \frac{I\Omega M_{sx}}{a\beta + (I\Omega)^2} = 0 \\ \dot{\vartheta} &= n_1\vartheta + M_{AB} + I\Omega m_c - \frac{\beta M_{sx}}{a\beta + (I\Omega)^2} < 0 \end{aligned}\right\}, \qquad (30)$$

where

$$\left.\begin{aligned} m_c = \frac{k_c}{a\beta + (I\Omega)^2}; \quad n_2 = \frac{aG\beta}{a\beta + (I\Omega)^2} \\ -k_s \leqslant M_{sx} \leqslant k_s \end{aligned}\right\}. \qquad (31)$$

Therefore, motion along this section will go to the side of decreasing Φ to a point with coordinates $\Phi = -\frac{k_s+B}{aG}$, $\Psi = 0$, after which the mapping point will leave the Φ-axis. The qualitative picture of the plane $\Psi\Phi$ for Case (24) is shown in Fig. 1a and for Case (25) in Fig. 1b. Figure 2 shows the motion of the mapping point under various starting conditions.

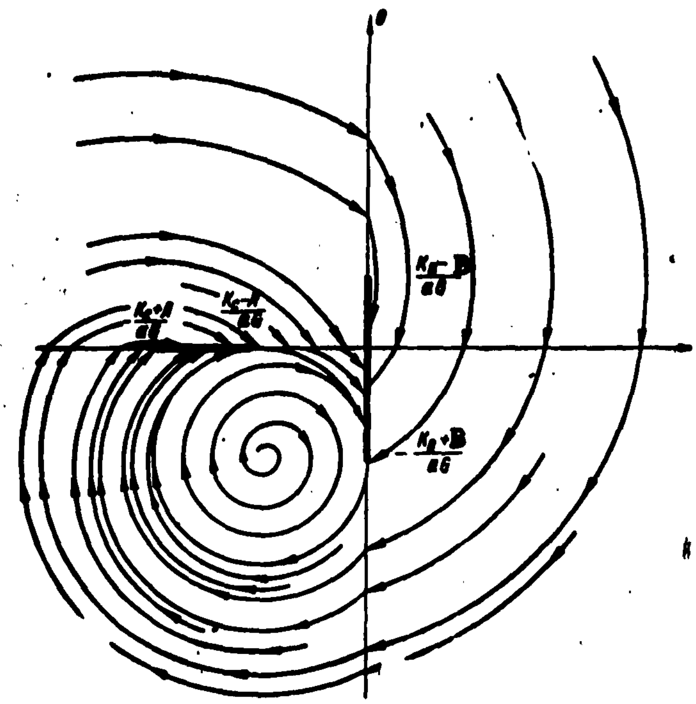

Fig. 2.

From an examination of Fig. 2 it follows that at sufficiently large initial deflections, the motion of the system will be damped and, conversely, at sufficiently small initial deflections in a certain vicinity near a singular point, the motion of the point will be rising. Starting from this, conclusions may be made about the essentialness of a closed phase trajectory, with respect to which all remaining phase trajectories are coiled. This closed phase trajectory corresponds to the periodic motion of the system, wherein the parameters of this motion are not functions of the initial conditions and are determined by the parameters of the dynamic system. It is obvious that this is a stable limited cycle, corresponding to selfoscillations.

Let us examine the physical picture of the motion of the axis of the gyroscope at zero starting conditions, i.e., assuming

$$\vartheta|_{t=0} = \Psi|_{t=0} = 0. \tag{32}$$

In this case the mapping point will at first move along the ϑ-axis to the point $\Psi = 0, \vartheta = -\frac{k_B + B}{aG}$. This means that the external frame will be fixed, while the internal frame will move according to the law

$$\vartheta = \frac{B(B - M_{\text{BB}}) + I\Omega(k_c - A)}{aG\beta}\left(e^{\frac{aGp}{ea\beta + (I\Omega)^2}t} - 1\right). \tag{33}$$

The motion time of the interal frame when the external frame is fixed is found from the condition

$$\vartheta = -\frac{k_B + B}{aG},$$

i.e.,

$$\frac{Q}{n^2}\left(1 - e^{n_2 t}\right) = \frac{k_B + B}{aG}, \tag{34}$$

hence

$$t = \frac{1}{n_2}\ln\left(1 - \frac{k_B + B}{aGQ}n_1\right), \tag{35}$$

where

$$Q = M_{AB} + I\Omega m_c - \frac{M_{\text{BB}}\beta}{a\beta + (I\Omega)^2}.$$

Further, the mapping point will move along the spiral of (21). The deflection angles of the frames will vary according to the law found by integrating Expressions (7). These expressions are more conveniently reduced to the form

$$\left.\begin{aligned}\dot{\vartheta} &= n_2\vartheta - \lambda\Psi + M_{AB} - l\Omega\, m_c \operatorname{sign}\vartheta - \beta\, m_s \operatorname{sign}\Psi \\ \dot{\Psi} &= \lambda\vartheta + n_1\Psi + M_{BA} + \alpha\, m_c \operatorname{sign}\vartheta - l\Omega\, m \operatorname{sign}\Psi\end{aligned}\right\}. \tag{36}$$

Thus at zero initial conditions

$$\left.\begin{aligned}\vartheta &= \frac{q}{k^2}\left[1 + e^{h_1 t}\left(\cos t\sqrt{k^2-h_1^2} - \frac{h_1}{\sqrt{k^2-h_1^2}}\sin t\sqrt{k^2-h_1^2}\right)\right] \\ \Psi &= \frac{1}{\lambda}\left[\frac{h_2 q}{k^2}\left(1 + e^{h_1 t}\cos t\sqrt{k^2-h_1^2}\right) + \frac{q e^{h_1 t}}{k^2\sqrt{k^2-h_1^2}}(k^2 - \right. \\ &\left. - n_2 h_1)\sin t\sqrt{k^2-h_1^2} + M_{AB} + l\Omega\, m_c + \beta\, m_s\right]\end{aligned}\right\}, \tag{37}$$

where

$$q = (\alpha - n_1 l\Omega)\, m_c - (n_1\beta + l\Omega)\, m_s - n_1 M_{AB} + M_{BA};$$

$$h_1 = \frac{n + n_2}{2}; \quad k^2 = n_1 n_2 + \lambda^2.$$

Thus, at any initial values of ϑ and Ψ the motion of the mapping point, and for the case under examination the motion of the vertex of the gyroscope in the plane $O\Psi\vartheta$, reduces to the setting up of selfoscillations.

At $k_B > B$ and $k_C > A$, from Expression (10) it follows that Eq. (7) has no singular points outside of the coordinate axes. The intersection of the sections of the junction of motions along the integral curves will be at the origin of the coordinates $O\Psi\vartheta$. At any initial values of Ψ and ϑ the mapping point will arrive at some section of motion junction.

Figure 3 shows the position of integral curves on the plane $\Psi\vartheta$ for the case in question.

Taking the inertia of the support frames into account, the equations of motion of a free gyroscope set up on the ground have the form of (7)

$$
\left.\begin{array}{l} I_B\ddot{\vartheta}+I\Omega(\dot{\Psi}+\Omega_3\sin\varphi)=M_B \\ I_C\ddot{\Psi}-I\Omega(\dot{\vartheta}-\Omega_3\cos\varphi\cdot\Psi)=M_C \end{array}\right\}. \qquad (38)
$$

Substituting the values of the momenta M_B and M_C into Eqs. (38), we obtain

$$
\left.\begin{array}{l} I_B\ddot{\vartheta}+I\Omega\dot{\Psi}-aG\vartheta+\alpha\dot{\vartheta}=-I\Omega\Omega_3\sin\varphi-k_B\,\mathrm{sign}\,\dot{\Psi}+B \\ I_C\ddot{\Psi}-I\Omega\dot{\vartheta}+(I\Omega\Omega_3\cos\varphi-aG)\Psi+\beta\dot{\Psi}=k_C\,\mathrm{sign}\,\dot{\vartheta}+A \end{array}\right\}. \qquad (39)
$$

Without inertial terms, Eqs. (39) have the form

$$
\left.\begin{array}{l} I\Omega\dot{\Psi}-aG\vartheta+\alpha\dot{\vartheta}=B_1-k_B\,\mathrm{sign}\,\dot{\Psi} \\ I\Omega\dot{\vartheta}+(aG)_1\Psi-\beta\dot{\Psi}=-k_C\,\mathrm{sign}\,\dot{\vartheta}-A \end{array}\right\}, \qquad (40)
$$

where

$$
\left.\begin{array}{l} B_1=B-I\Omega\Omega_3\sin\varphi \\ (aG)_1=aG-I\Omega\Omega_3\cos\varphi \end{array}\right\}. \qquad (41)
$$

Formally, Eqs. (40) have the form of Eqs. (6). Carrying out all operations successively with Eqs. (40) done earlier for the coordinates of the singular points, we obtain equations, similar to (10), in the form

$$
\left.\begin{array}{l} \vartheta_K=\dfrac{k_B\,\mathrm{sign}\,\dot{\Psi}-B_1}{aG-I\Omega\Omega_3\cos\varphi} \\ \Psi_K=-\dfrac{k_C\,\mathrm{sign}\,\dot{\vartheta}+A}{aG-I\Omega\Omega_3\cos\varphi} \end{array}\right\}. \qquad (42)
$$

Let us note that the case of $aG=I\Omega\Omega_3\cos\varphi$ is not examined here. Here, taking the conditions $aG > I\Omega\Omega_3\cos\varphi$; $k_B > B - I\Omega\Omega_3\sin\varphi$; $k_C < A$, we arrive at the previous case with substitution in Expressions (11 to 37) of B_1 and $(aG)_1$, determined by Formulas (41) for B and aG. Naturally, the zones of motion junction in Figs. 1a and 1b and 2 will have boundaries at which B_1 and $(aG)_1$ act instead of B and aG. Here, as in the case when the gyroscope is mounted on a fixed cantilever at any initial values of ϑ and Ψ the motion of the mapping point as well as that of the vertex of the gyroscope in the plane reduces to the setting up of selfoscillations.

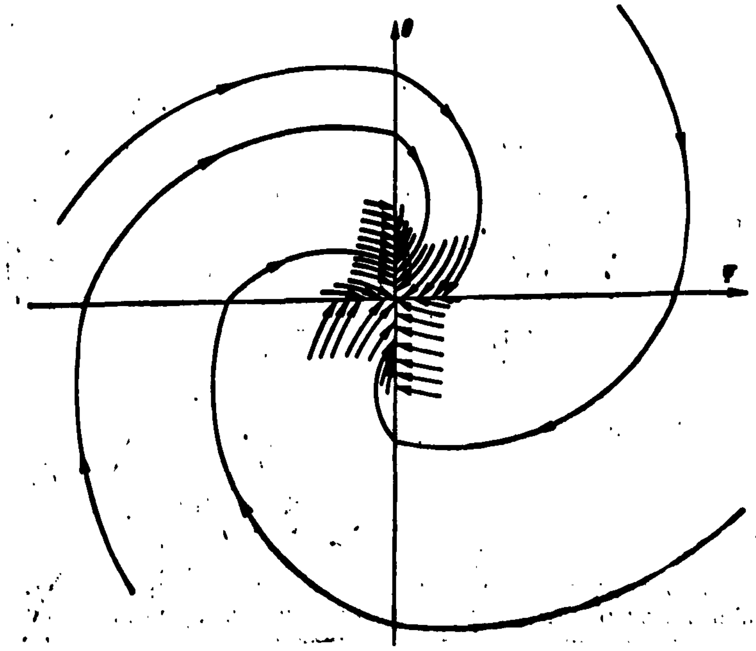

Fig. 3.

As is known [11, 12, 13], equations of the form of (5) written without correction components k_B sign Ψ and k_C sign ϑ represent an unstable system. In fact, in view of the fact that the center of gravity of our gyroscopic system is above its support point (<u>a</u> is negative), stability may be obtained only artificially, for example, by introducing correction momenta, as was done above.

Obviously, without correction momenta, selfoscillation conditions would not set in and this dynamic system would be unstable.

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