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NOTICE

ABSTRACT

A previous report showed that a long-code decision feedback system can be made fail-safe, with as low an error probability as desired at any channel signal-to-noise ratio, and without excessive decoding complexity. This report shows that the same can be done in an information feedback system by following either of two policies.

Various possible decision policies are enumerated for both decision and information feedback subject to the restriction that communication be by binary sequences and that a binary decision be made for individual digits, as well as several other restrictions. It is then shown that, within the allowed class of systems, only the one decision feedback policy described previously and the two information feedback systems described herein can have the desired high-reliability fail-safe properties. The first information feedback system, which uses decision policies on the forward and feedback channels such that erasure-to-confirmation errors are always very rare on both channels, is shown to be nearly equivalent, but slightly inferior, to the decision feedback system. An exception to this statement holds when the feedback channel is extremely reliable in itself. The second information feedback system, which uses

decision policies such that confirmation-to-erasure errors are always very rare on both channels, has the advantage that reliability depends on the number of information digits per code word, rather than the number of check digits as in the other systems. ~~Thus,~~ a saving in the number of forward channel check digits is available. ~~On the other hand~~, the system has the disadvantage that the receiver must store tentatively a large number of words.

GLOSSARY

A	- acceptance region
A,B,M,X,Y	- message words
c	- number of digits, number of check digits
C	- confirmation region
d_i	- erase d_i words
E	- erasure region, erasure instruction
E_i	- erasure instruction i
f	- digit of feedback channel code word
G_A, G_Y	- feedback groups containing words A,Y
i	- information digit of forward channel word
k	- number of information digits in a forward channel message word
k"	- number of information digits in a feedback channel word group
[M], [M(,)]	- matrix of 1's and o's
M✓	- feedback channel indicates a received message word which checks with what was sent
MX	- feedback channel indicates a received message word which does not check with what was sent
n	- sum of n' and n"
n'	- number of digits in a forward channel message word

n'' - number of digits in a feedback channel word group

$[P_1], [P_2]$ - matrices of 1's and o's

$P[\ ], P_e$ - error probability functions

P_c - error probability contribution of case c

P_f - error probability contribution of case f

R - reject region

S - number of sequences associated with message word, decision feedback system

S' - number of sequences associated with message word, information feedback system, forward channel

S'' - number of sequences associated with message word, information feedback system, feedback channel

S_2 - state 2

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I. INTRODUCTION

For a feedback communication system to be fail-safe, complete failure ("blackout") of either the forward or feedback channel or both must have a very low probability of causing a final error or omission in the received message. Also, any other channel condition should have at least an equally low probability of causing an error.

It has been shown[1,2,3] that a long-code decision feedback system can be made fail-safe, with as low an error probability as desired at any channel signal-to-noise ratio, including the case of complete blackout. However, since there are situations which would appear to favor information feedback over decision feedback, it is useful to determine whether an _information_ feedback system can also be made fail-safe. If fail-safe operation can be accomplished, one would then like to know the behavior and requirements of such a system. The purpose of this report is to supply this desired information.

In an information feedback system, as defined previously,[4,5] the receiver reports back in whole or in part the received information, and the sender then transmits corrective information when necessary. The corrective signal will here be restricted to the confirmation or erasure-plus-repetition of some previously transmitted message words.

Information feedback has an advantage over decision feedback when a highly-reliable feedback channel of large capacity is available. In fact, error-free communication without error-protection encoding is possible in an information feedback system if an error-free feedback channel is available of capacity greater than the forward channel.[4,5] However, when extremely low error probabilities are desired, the assumption of an error-free feedback channel should be avoided. Although information feedback procedures previously described[4,5,6,7] have shown how to greatly improve reliability under most circumstances, they do not provide much protection during a "blackout" of the feedback channel. The procedures described in this report are designed to provide this protection.

Although this report is concerned primarily with information feedback, some aspects of decision feedback will also be discussed in order to put the results in better perspective.

II. ENUMERATION OF DECISION POLICIES

The specification of "decision" or "information" feedback does not uniquely define the feedback procedure. There are in fact a great variety of possible procedures. This report is restricted to a discussion of a limited class of systems and decision policies. First, communication by binary digits is assumed. Secondly, in order to keep the logical structure of the systems from becoming too complex, the following procedures will not be permitted:

1. Cumulative Feedback

In cumulative feedback,[4] received information is not completely rejected or erased, but is used in conjunction with additional repetitive information to arrive at a final decision. It is assumed here instead that message words which do not meet a certain criterion are rejected, or erased, and repeated.

2. Iterated Feedback

In iterated feedback[6,7] there is more than one level of feedback; i.e., individual message words may be tentatively accepted or repeated as a result of one check, but a group of these words may later be rejected, or erased, and repeated as a result of a check over this group. The various levels of checking might be all decision feedback, all information feedback, or some combination. In the system considered here, however, all acceptances and confirmations are final.

3. Multi-digit Labeling

Some decision policies can be made fail-safe only if a large number of labeling digits are used. Thus, if long coded message words are used, it is possible to assign a separate label to each word in a long message, reserving some information digits for labeling. These labels could be used to improve reliability in some systems. Since, for example, only 30 binary digits are sufficient to separately label a billion message words, this is not entirely impractical, although the increased complexity and decreased channel capacity are serious disadvantages. The label digits might have additional value in the processing of the information, such as for addressing, sorting, etc. However, the systems considered here will be limited to the use of a small number of label digits.

With the above restrictions, the receiver in a decision feedback system divides received signal space into two regions: A and R. If the signal corresponding to a message word is in A, an indication of acceptance is sent over the feedback channel; if it is in R, a rejection (request for repeat) is sent back. Signal space for the received feedback signal is also divided into two regions A and R, corresponding to acceptance and rejection. In information feedback, the receiver sends back a signal which is some function of the received message word. Depending on whether or not this checks at the original transmitter with that which had been sent, the transmitter sends either a confirmation or an erasure instruction. This can be done in one of two ways: the transmission of a new message may imply confirmation, in which case only the erasure instruction requires a separate signal; or the erasure-confirmation information can be sent entirely separate from the main message. In either case, when a confirmation or erasure instruction is expected, received signal space must be divided into two regions: C (confirmation) and E (erasure). Also, the transmitter must decide whether an erasure or confirmation has been received, so that there is also a received signal space divided into C and E regions on the feedback channel. For compound feedback (combined decision and information feedback), received signal space on both forward and feedback channel may have C, E, R, and A regions.

Now, even when channels are poor or completely blacked out, at least one of the received signal regions on each channel must have a high probability of occurrence. Exactly which regions have high probability depends on the decision policy of the system. Table I enumerates the various possibilities for

Type of Feedback		Regions of High Probability During Blackout		Comments
		Feedback Channel	**Forward Channel**	
Decision Feedback	1.	R	R	This is the long-code fail-safe system.[1,2,3]
	2.	R	A	These will work only if a large number of label digits are used.
	3.	R	R,A	
	4.	A	R	This requires a series of repeat instructions and many label digits.
	5.	A,R	R	
	6.	A	A	Wrong messages are received, and some others are lost.
	7.	A,R	A	
	8.	A,R	A,R	
	9.	A	A,R	
Information Feedback	1.	C	E	A C-E-C (or message, M) chain causes a final error by permanently erasing a good message.
	2.	C,E	C,E	
	3.	C	C,E	
	4.	C,E	E	
	5.	E	E	This is a possible fail-safe system.
	6.	C	C	This is a possible fail-safe system.
	7.	E	C	An E-C-E chain causes a final error, unless a large number of label digits are used.
	8.	E	C,E	
	9.	C,E	C	

Table I.
ENUMERATION OF DECISION POLICIES

decision and information feedback. In the enumeration, "high probability" is taken to mean any probability considerably higher than the desired fail-safe system error probability.

Of the nine decision feedback policies, only the first permits a fail-safe system under the above-mentioned restrictions. The long-code decision feedback system described in a previous report[1] is of this type. In the second and third policies, all messages are nearly certain to be received correctly with the proper procedure, but a good number of extraneous "messages" would be interspersed among them in the final receiver record. The use of a large number of label digits could allow the receiver to "weed out" these extraneous messages so as to produce near-perfect reception. In the fourth and fifth policies, nearly all accepted messages would be correct, but a series of repeat instructions would be needed in order to make the transmitter go back and repeat messages thought to have been accepted. Also, many label digits might be required so that the receiver could distinguish desired repeats from new messages.

Of the nine information feedback policies, only the fifth and sixth permit a fail-safe system under the above-mentioned restrictions. These systems are discussed in sections III and IV, respectively. In the first four procedures, a confirmation can easily be falsely received as an erasure. When this erasure is fed back, it can easily be misinterpreted that a confirmation has been received. This C-E-C error chain results in the permanent erasure of a good message, and thus a final error. In the last three policies, an E-C-E (E→C on forward channel, C→E on feedback channel) error chain is likely. In the case in which a new message is used to imply confirmation, this introduces an extraneous message, and thus a final error unless a large number of label digits are used. If the confirmation-erasure information is separate, then a word which should have been erased is likely to be retained, and there is a final error. The proportion of messages which should be erased is quite appreciable in the latter case, as the erasure is the system's only protection against blackouts and poor conditions.

III. INFORMATION FEEDBACK WITH ERASURE-TO-CONFIRMATION ERRORS RARE

The case discussed in this section corresponds to the fifth on the list of information feedback policies. Since the C-region is of low probability, E→C errors are rare.

Consider first that messages are transmitted by coded binary sequences, and that one of the sequences is reserved to indicate erasure, while the transmission of an actual message implies confirmation. In order that the C-region be of low probability when channel conditions are poor, most of received message space must be interpreted as an erasure instruction in this situation. One way of assuring this is to have most binary sequences interpreted as erasure in the decision scheme, and only a small proportion as any message word. An improvement of this procedure is possible if the receiver is able to monitor channel conditions in some way, and vary the fraction of message space interpreted as erasure according to the estimate of channel conditions. When it is judged that the signal-to-noise ratio is very poor, any received signal could be interpreted as an erasure, while when channel conditions appear to be good, an appreciable fraction of possible received sequences could be interpreted as messages. Such a procedure could reduce somewhat the number of check digits required for a fail-safe system with a prescribed reliability. However, it is assumed in what follows that the decision criterion for a received binary sequence must be maintained constant, and must be preceded by binary decisions on each received digit.

III.A. Description of the System Logic

The high-probability-erasure decision policy can be used to construct a fail-safe information feedback system by following the procedure to be shown here. In the description, it is assumed for simplicity that the transmitter waits for a return signal before sending the next transmission. If a fail-safe system can be found for this case, it can also be found for the situation in which there is no waiting for a return signal. One way this can be done is by dividing the information to be transmitted into groups, and interlacing, just as was described previously[1,2,3] for the fail-safe decision feedback system. Also, it should be possible to do it without interlacing, by the use of several additional labeling digits, as was also described previously.

Since undesired erasures may occur quite frequently at times, it would not be wise, in this case, to use a procedure recommended previously,[4,5] in which j consecutive erasures were interpreted as an instruction to erase the j messages received prior to the first erasure. Rather, it is found more efficient to allow an erasure to erase an erasure, so that an odd number of consecutive erasures is interpreted by the receiver to be an instruction to erase one message word, while an even number of consecutive erasures is ignored.

The trickiest situation to handle is that in which the transmitter has sent a message word, but then receives an erasure via the feedback channel. There are three possibilities in this case:

a. The receiver has been instructed to erase a word which should not have been erased, and also has not received the most recent intended message word. By transmitting another erasure, it is possible to "erase the erasure." It is also necessary to retransmit the most recent intended word. An alternative approach is to require the transmitter to repeat the two words last transmitted, rather than to erase the erasure. This latter approach was studied first, but was discarded in favor of the former, which is more direct and effective. It is described in Appendix I for reference.

b. The receiver has received the message word correctly, but there has been a feedback channel C→E error. In this case the subsequent erasure will erase a good message word, but, since this word will be repeated, it is not lost.

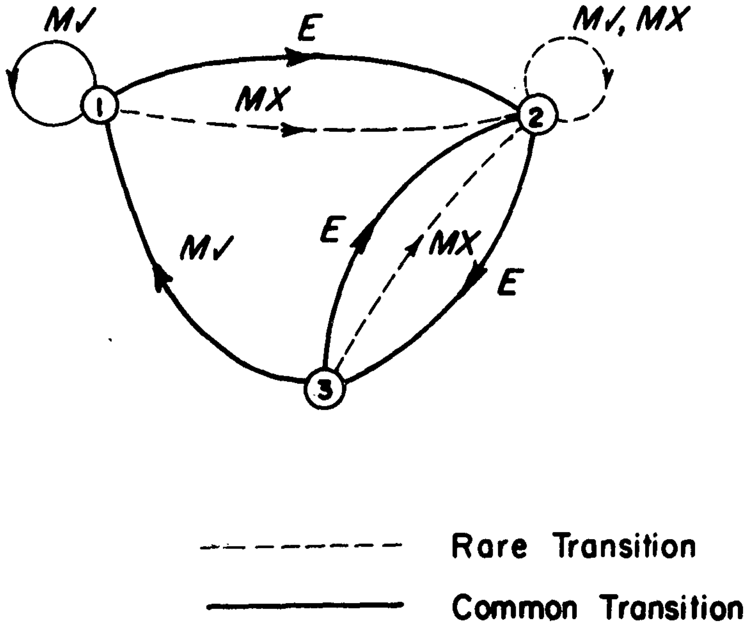
c. The receiver has received an incorrect word, and there has been a feedback channel C→E error. In this case the subsequent erasure will erase an erroneous word, so that the system again operates safely.

The logic of the system is probably best understood by reference to a state diagram. Figure 1 shows the state diagram for the transmitter. The labels for the transitions refer to what has been received on the feedback channel: an E indicates an erasure; the symbol M✓ means that a message word (and confirmation) appears to have been received and the return signal checks; and MX means that a message word appears to have been received but the return signal does not check.

III.B. Error Probability Analysis

If the only errors which occur are C→E errors, the final received record is error-free and complete. Figure 2 shows a sample chain of transmissions when only C→E errors occur. Erroneous transmissions are indicated by dashed lines, and an X below a message indicates that it has been erased by the receiver. The symbol G_A indicates that the returned signal is interpreted as reception of a word in the group containing message word A. (In total information feedback, A is the only word in the group, but in partial information feedback there is more than one word in each group.)

The effects of rarer types of errors are shown in



State	*Operation*
1	Send a New Message
2	Send an Erasure
3	Send a Repeat of The Most Advanced of Prior Transmissions

Fig. 1

State Diagram For The Transmitter

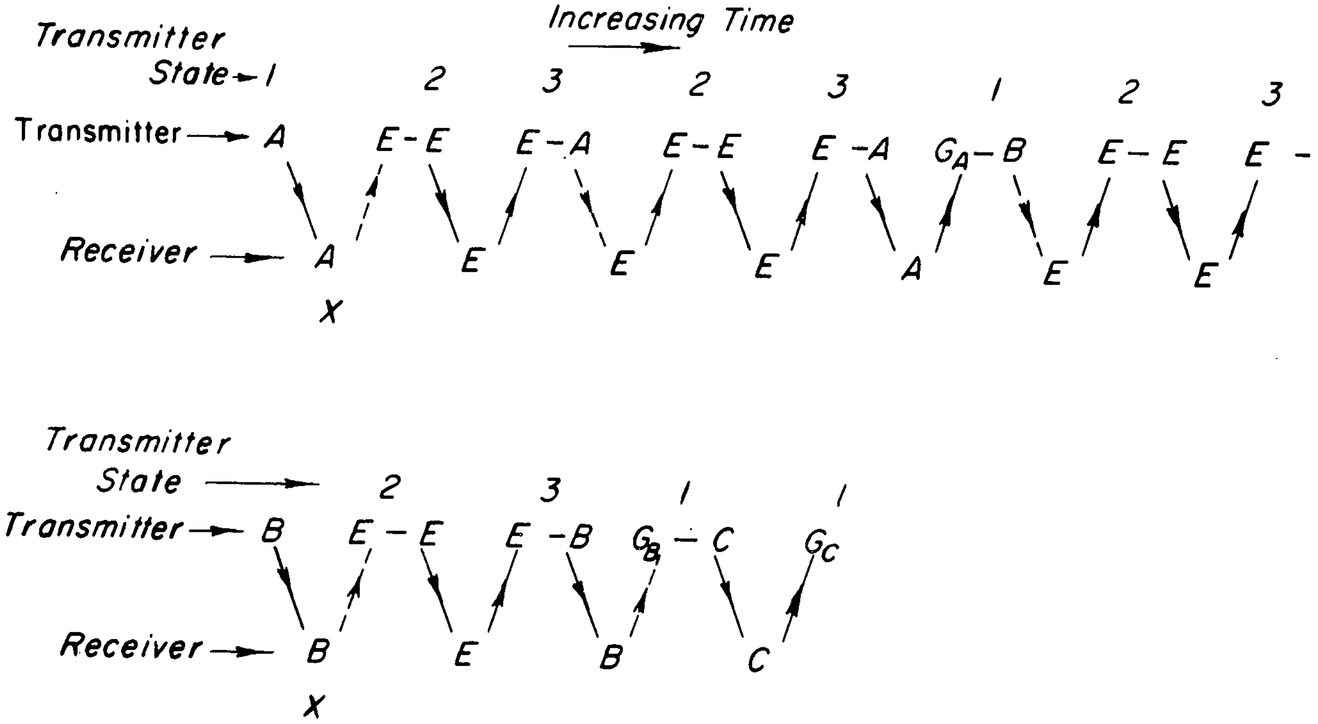


Fig. 2

Effect Of C → E Errors

Fig. 3. In each case, only one rare error (plus one or more of the commoner C→E errors is considered. It can be seen from cases a. and b. that an A→Y type error on the forward channel is always corrected if not followed shortly thereafter by another rare error. On the other hand, from c), an E→Y error on the forward channel causes a final error if followed by a Y→E error on the feedback channel. If a Y→E error does not follow (case d), the result is that A is recorded twice by the receiver. By using one label digit which alternates from word to word, the receiver would be able to detect this as an undesired repeat, and thus cross out one of the A's. Whether this is worth doing depends on whether this case would contribute much to the overall error probability if it were not done.

Cases (e)→(h) deal with feedback channel errors. An E→G_Y feedback error does not cause a final error if preceded by an A→E forward error (case e), but it does lead to a final error if E actually was the previous forward transmission (case f). An E→G_A feedback error (case g) causes a final error, where A was the previous forward transmission, but this case is much rarer than (f), since it requires a return to the exact group containing A. Finally, an A→G_Y feedback error (case h) does not lead to a final error.

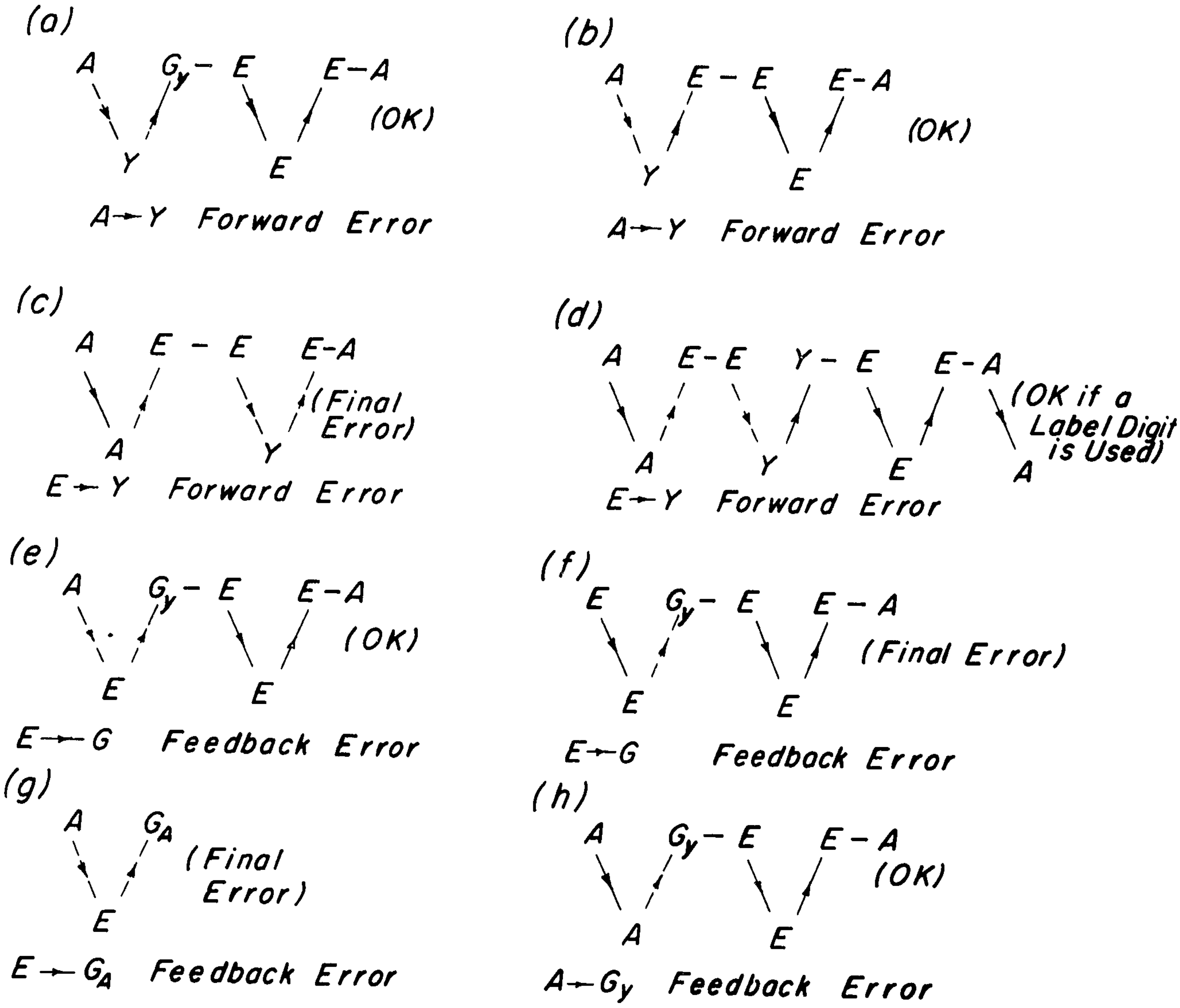
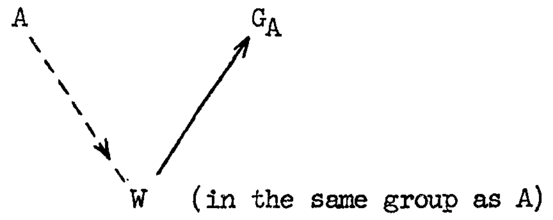


Fig. 3
Effects Of Rarer Types Of Errors

For the case of partial information feedback only, there is also a possible final error caused by the chain:



However, since a word-to-any other word error is rare in this procedure, such an error is completely negligible so long as there are a reasonably large number of different feedback groups.

A transmission must contain at least one "rare" error in order to cause a final error. Therefore, define error probability, P_e, as the probability that a particular transmission loop (one forward transmission plus one feedback transmission) contains a rare error which leads to a final error. This definition is chosen to avoid the presently unneeded consideration of the weighting of the various types of final errors and their effects, which is a rather subjective weighting unless a very specific system is postulated.

The error probability due to case (c) error chains alone is

$$P_c \doteq P(S_2)\; P\left[(E \rightarrow \{Y\})_{FWD},\ (Y \rightarrow E)_{FB}/S_2\right], \qquad (1)$$

where $P(S_2)$ is the probability that the transmitter is in state 2, and $\{Y\}$ represents the set of all message words (except the erasure word). When either channel is blacked out, the transmitter is in state 2 about one-half the time, so that, in this case,

$$P(S_2) = 1/2 \; . \tag{2}$$

Also, when the feedback channel is blacked out, the probability of a Y→E error is close to unity. Thus, in this case,

$$P_c = \frac{1}{2} P\left[(E \rightarrow \{Y\})_{FWD}\right] \; . \tag{3}$$

For case (f),

$$P_f \doteq P(S_2) \; P\left[(E \rightarrow G_Y)_{FB}/S_2\right] \; . \tag{4}$$

During a blackout of either or both channels,

$$P_f \doteq \frac{1}{2} P\left[(E \rightarrow \{G_Y\})_{FB}\right] \; . \tag{5}$$

Case (g) is much rarer than (f), as mentioned before, and case (d) does not cause a final error if we use a label digit. Thus, to a good approximation,

$$P_e = P_c + P_f \; . \tag{6}$$

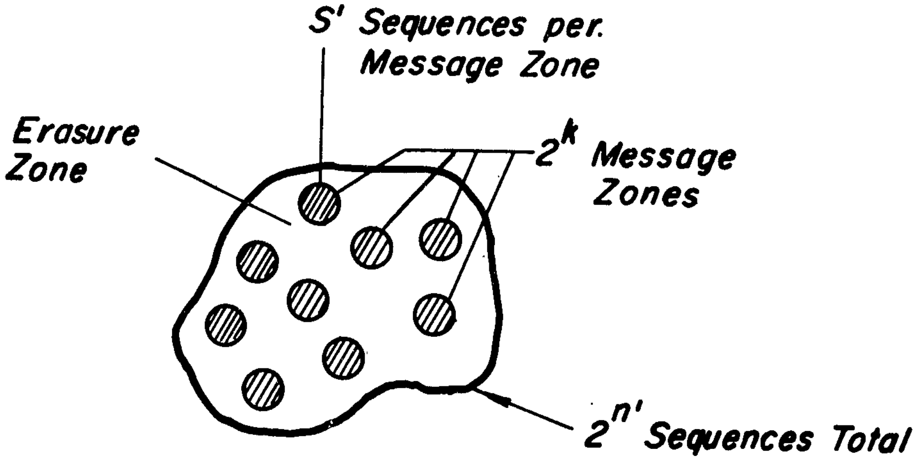
The above discussion is fairly general, and does not specify the exact form of the message words. Consider now the situation of principal interest, that in which transmission is by coded binary sequences. It was shown[6,7] that a coded information feedback system employing parity check (group alphabet[8]) codes contains an (n,k) code composed of an (n',k) code in the forward direction and an (n'',k'') code in the feedback direction, where $k'' \leq k$ and $n' + n'' = n$. (In total information feedback, $k'' = k$.) Figure 4 shows a pictorial representation of decision space for the forward and feedback channels. The exact error probability for such codes cannot be determined without specifying an exact code and the channel noise statistics. However, the error probabilities can be calculated for the case in which the noise is so great that all sequences are equally likely to be received, regardless of what was transmitted. (This is usually the worst case.) If just the forward channel is blacked out in such a manner, then

$$P_e \doteq \frac{1}{2} S' \, 2^{-(n'-k)} \, P\left[(Y \rightarrow E)_{FB}\right] . \tag{7}$$

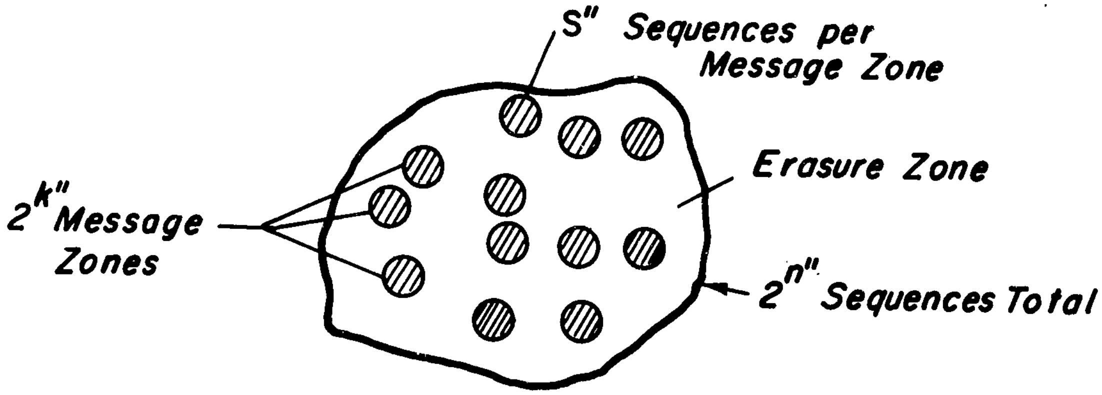
If the feedback channel alone is blacked out,

$$P_e \doteq \frac{1}{2} S'' \, 2^{-(n''-k'')} . \tag{8}$$

If both channels are blacked out,



FORWARD CHANNEL
DECISION SPACE



FEED BACK CHANNEL
DECISION SPACE

Fig. 4
Decision Space

$$P_e \doteq \frac{1}{2} S' \, 2^{-(n'-k)} + \frac{1}{2} S'' \, 2^{-(n''-k'')} \qquad (9)$$

If the system is designed so that operation is normally at a rather good signal-to-noise ratio, then S' (or S") may be made small without getting many erasures except when channel conditions are poor. Typically, S' (or S") = 1 for error detection, or S' = n' + 1 (S" = n" + 1) for single-error correction. Thus, reliability may be made as high as desired by choosing a sufficient number of check digits over each channel (n'-k over the forward channel and n"-k" over the feedback channel).

III.C. Evaluation and Comparisons

If one compares the system described above with the long-code fail-safe decision feedback system,[1,2,3] some interesting observations can be made. In a decision feedback system using an (n',k) code on the forward channel, and having S sequences associated with each message sequence in decision space, the error probability during a blackout of the forward channel is

$$P_e \doteq S \, 2^{-(n'-k)} , \qquad (10)$$

and it would normally be about twice this value if both channels were blacked out. Thus, for the same code structure and decision criterion in the forward channel, the decision feedback system is essentially as reliable as this information feedback system when both channels are blacked out, and poorer only by the factor $\frac{1}{2}\,P\left[(Y\rightarrow E)_{FB}\right]$ when just the forward channel is blacked out. Also, the number of message words repeated is about the same for both systems. Since the decision feedback system need not waste a comparable amount of feedback channel capacity, this strongly favors the decision feedback system. On the other hand, even though the very critical exponential behavior is the same in both cases, $P\left[(Y\rightarrow E)_{FB}\right]$ can be quite small if the feedback channel is extremely good, and this could favor the information feedback system if wastage of feedback channel capacity were a minor factor.

The somewhat unfavorable performance of this information feedback system in comparison to decision feedback can also be explained with the aid of the following reasoning. By checking the received message against what had been transmitted, it is possible to reduce the probability of an uncorrected word-to-other word error as much as desired. This is a valuable property of information feedback. However, $E\rightarrow Y$ errors cannot be corrected with consistency, because compensating $Y\rightarrow E$ errors may occur frequently on the feedback channel as a result of the fact that $E\rightarrow Y$ errors must be kept rare on this channel as well as the forward channel. Thus, since only one of the two important classes of errors (word-to-other word and $E\rightarrow Y$) receives full protection from the check at the transmitter, there is little to be gained, if fail-safeness is the main criterion, by having the transmitter check whether the message agrees or not. Thus, the receiver would do almost as well by sending back just an indication of whether a message or an erasure was received. Then, if an erasure instruction is ignored, as it should be in this case, the erase symbol plays the role of a reject symbol, and the system is identical to decision feedback.

The above conclusions are subject to the assumption of a received signal space of binary sequences only. This is not essential, however, even if the message words are binary sequences. The erasure signal could be entirely non-binary. For example, it could consist of complete suppression of the modulation or transmitted signal, much as has been done in a particular commercial decision feedback system.[9] Suppose that a binary received signal space were used for decision only if positive recognition of binary digits in the signal were first obtained; otherwise, a transmitted erasure would be assumed. Then, an $E \rightarrow Y$ error could be made virtually impossible, even with little or no coding in the forward channel. The same could be done for $E \rightarrow G_Y$ errors on the feedback channel. The possibilities of such a system are well worth further investigation.

III.D. Separate Confirmation-Erasure Signal

An alternative to the approach described thus far in this section is to indicate confirmation or erasure via a separate signal. If we are again restricted to binary received signal space, and if S sequences are associated with a confirmation interpretation, the black-out probability of an $E \rightarrow C$ error is no better than $S \cdot 2^{-c}$, where c is the number of digits in the binary sequence carrying the confirmation-erasure information. However, it is possible to do just as well as this by using a decision feedback system with these c digits as check digits, so that this approach does not offer any improvement over the first approach, at least if decisions must be based on binary sequences.

IV. INFORMATION FEEDBACK WITH CONFIRMATION-TO-ERASURE ERRORS RARE

The case discussed in this section corresponds to the sixth on the list of information feedback policies. Since the E-region is of low probability, C→E errors are rare.

Assume again the transmission of binary sequences, with a fixed decision criterion at the receiver preceded by binary decisions on each digit. Also assume that transmission of a new message word implies confirmation. Then, since the E-region must be of low probability when channel conditions are poor, most of the possible received sequences must be interpreted as some message word.

IV.A. Description of the System Logic

The high-probability-confirmation decision policy can be used to construct a fail-safe information feedback system by following the procedure to be shown here. Again, it is assumed for simplicity that the transmitter waits for a return signal before sending the next transmission. This is not a basic limitation, however, for the reasons mentioned in section III.A.

A basic consequence of the postulated decision policy is that, during a blackout, many erroneous message words will be recorded at the receiver, and thus there must be a way of erasing these words in order to have a fail-safe system. This can be done by means of a eries of erasure instructions, E_1, E_2, ... E_r, where erasure instruction E_i is, "erase the previous d_i words." It is not necessary and may not be desirable to have all $d_i = i$. Typically, d_i might have the value 1, 2, 3, 5, 10, 20, 50, 200, 1,000, 50,000. If, say, the previous 16 words should be erased, one would erase the previous 20.

There is more than one possible arrangement of erasure signals in the set of possible feedback signals. The receiver may attempt to give some information about which erasure instruction has been received, or may even specify it exactly. However, it will be seen from the error probability analysis to follow that the probability of an erasure-to-other erasure instruction error can be made so small as not to need additional protection from the information feedback process. Therefore, the best procedure appears to be to put all the erasures into one group in the feedback transmission, and thus just indicate that some erasure instruction was received.

The logic of this system is fairly simple. When the transmitter gets back a message which does not agree with what was sent, it sends erasure instruction E_1. If anything but the erasure group E is returned via the feedback channel, instruction E_2 is sent. When the number of erasures to be indicated does not correspond to an available instruction, the next greater number is requested to be erased. When an erasure instruction finally results in the return of an erasure signal via the feedback channel, the transmitter goes back to the appropriate word in the message chain. Because not all $d_i = i$, this sometimes necessitates erasing a number of correctly received and confirmed words.

In obeying the erasure instruction, the number of words to be erased is taken to include any previous received erasure instructions. Thus, the reception of

$$\begin{array}{ccccccccc} M_1 & M_2 & M_3 & M_4 & M_5 & E_3 & M_6 & M_7 & E_6 \\ & & X & X & X & X & X & X & \end{array}$$

would require erasure of the six positions indicated by X, though only five actual message words are erased.

IV.B. Error Probability Analysis

Figure 5 shows a sample chain of transmissions when only E→C and message-to-other-message errors occur. The notation is as in Figure 2. If only errors of the type shown occur,

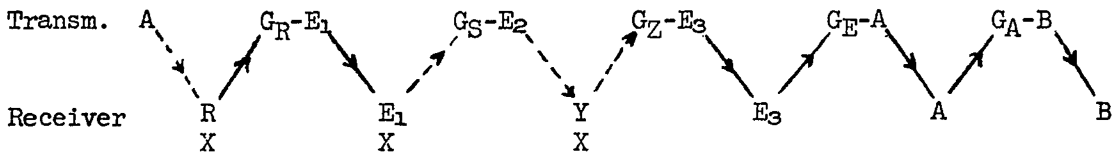


Figure 5

Effect of E→C and word-to-other-word errors.

the final received record is error-free and complete.

A final error usually occurs if one has an $M_i \rightarrow M_j$ error on the forward channel, $(j \neq i)$, and a compensating $G_j \rightarrow G_i$ error on the feedback channel. Also, an erasure-to-other erasure instruction error nearly always causes a final error, and we shall assume that a C→E error on the forward or feedback channel causes a final error. Special procedures **might eliminate** something like half of the latter final errors under some circumstances, but this is of little significance.

Assume that each message word consists of a sequence of coded (or possibly uncoded) binary digits, of which k are information bits. The 2^k possible words include r erasure instructions. Each received sequence is interpreted as one of the messages or erasures, and it is assumed that each message or erasure instruction has about the same number of interpretable sequences associated with it (just one each in the uncoded case). Then, during a blackout, the following transition probabilities exist on the forward channel:

$$P(E_i \longrightarrow E_{j\ j \neq i}) = (r-1)2^{-k} \; ; \tag{10}$$

$$P(M_s \longrightarrow E_{j\ \text{all } j}) = r2^{-k} \; ; \tag{11}$$

$$P(E_i \longrightarrow M_{s\ \text{all } s}) = 1 - r2^{-k} \doteq 1 \; ; \tag{12}$$

$$P(M_k \rightarrow M_s) = 2^{-k} \; . \tag{13}$$

On the feedback channel there are $2^{k''}$ possible transmitted sequences ($k'' \leq k$), one of which represents the set of erasure instructions. During a blackout, the following transition probabilities exist on the feedback channel:

$$P(E \rightarrow \{G_r\}_{\text{all } r}) = 1 - 2^{-k''} \doteq 1; \qquad (14)$$

$$P(G_r \longrightarrow E) = 2^{-k''} ; \qquad (15)$$

$$P(G_r \longrightarrow G_s) = 2^{-k''} . \qquad (16)$$

When the forward channel alone is blacked out, the error probability is approximately the sum of (10) and (11); when both channels are blacked out, it is approximately the sum of (10), (11), (15), and (16). (Equation (16) represents the case in which an $M_s \rightarrow M_r$ error has occurred on the forward channel, and a $G_r \rightarrow G_s$ error follows on the feedback channel.)

By choosing k and k" sufficiently large, the blackout error probability can be made as small as possible. No coding is necessary for this purpose.

In the fail-safe decision feedback system, the case of complete blackout was shown to be the case of highest error probability. Because of the similar structure, this is also true of the information feedback system described in section III. However, for the information feedback system described in this section, there is one more thing which must be done before it can be assured that the system will be equally free of errors under other than blackout conditions.

Suppose that there were no coding, and that the receiver simply sent back each group of k digits exactly as received. Assume also that conditions on both channels are such that the occurrence of exactly one error in k is quite common. Then, in 1/k of the cases in which exactly one digit error per block occurs on each channel, the returned message agrees with what had been sent, even though the received message contains a single digit error. This would not be a satisfactory situation if extremely high reliability communication were desired.

The system can be made virtually error-free under all conditions as well as just during blackout conditions by the simple expedient of applying a linear transformation to the received sequence in order to obtain the feedback sequence. This solution is based chiefly on the idea of Chang[4,5] that sequences close together in the forward channel should be far apart (or at least not close) in the feedback channel.

Consider the case of total information feedback without coding. Let the k forward digits be called i_1, i_2, ... i_k, and let the corresponding feedback digits be f_1, f_2, ... f_k. Then,

$$\begin{bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ \cdot \\ f_k \end{bmatrix} = M \begin{bmatrix} i_1 \\ i_2 \\ \cdot \\ \cdot \\ \cdot \\ i_k \end{bmatrix} , \qquad (17)$$

where M is a $k \times k$ matrix of 1's and o's. (Multiplication is ordinary, but addition is taken modulo two.) The matrix $[M]$ may be chosen at random, provided only that it possesses an inverse. The latter property ensures a one-to-one correspondence between the forward and feedback sequences.

Now, if some digit i_j should be altered in the forward transmission, the sequence $f_1, \ldots f_k$ will differ in many positions from what it would otherwise be. (More precisely, it will differ in as many digits as there are ones in column j of $[M]$.) Thus, for almost any choice of $[M]$, there will be a negligible chance of a compensating feedback error under any channel conditions. The reasoning to support this statement is almost identical to that given in the decision feedback report[1] when averages over randomly chosen codes were considered.

The same procedure can be carried out in the case of partial information feedback, with or without coding. More details are given in Appendix II. For partial information feedback, however, there is one additional error which must be considered for non-blackout conditions. This is the possibility of a forward channel error in which the erroneous word is in the same feedback group as the correct word. The probability of such an error can be estimated by considering a randomly-chosen (k, k-k") parity check code. Figure 6 shows the Slepian array[8] for such a code. Let each of the 2^k sequences in the array represent a forward channel message word. If the feedback groups are chosen as the cosets, the $2^{k''}$ parity sequences of the code

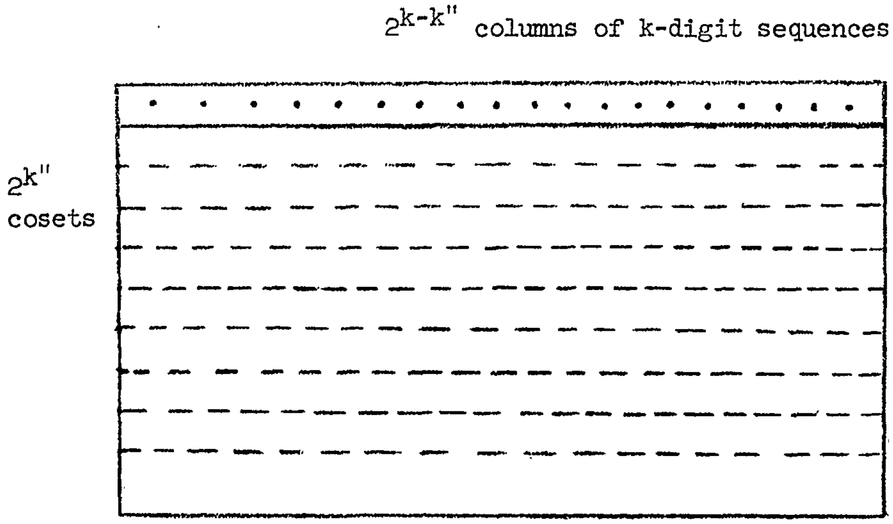


Figure 6

The Slepian Array

become the feedback information digits. Also, it can readily be seen that the probability of an error to another message in the same group is identical to the probability of an undetected error in a zero-error correction $(k, k-k'')$ code. This has already been shown to be about $2^{-k''}$ or less for most codes at any channel signal-to-noise ratio. Thus, the error probability for this case is also negligible for sufficiently large k''.

IV.C. Evaluation and Comparisons

Unlike the approach of section III, the number of check digits (or redundant digits) in the forward direction need not be large, since reliability is dependent on the number of information digits in the message word, rather than the number of check digits. In fact it is possible to have no coding at all, as was mentioned, although one might use some simple error correction to reduce the frequency of erasures.

However, in addition to advantages, there are also disadvantages. Although no coding is needed, the requirement of a matrix operation on the received word, while not especially complex, is hardly much simpler than the coding and decoding scheme used in the fail-safe decision feedback system or in the other information feedback system described here. Also, it is required that the receiver be able to store tentatively a large number of message words, and that the erase capability be larger than the longest blackout period. In some situations, this latter disadvantage may not be too great, since the d_i may be chosen to have convenient values. If, say, the data were printed 10 words to a row, 10 rows per "page", one could have the instructions:

d_1, d_2, d_3, d_4, d_5 - erase the previous 1, 2, 3, 4, 5 words.

d_6 - erase the current row.

d_7 - erase the current row plus the row before.

d_8, d_9, d_{10} - erase the current row plus the 2, 3, 4 previous rows.

d_{11} - erase the current page.

etc.

V. CONCLUSIONS

An enumeration of the possible decision and information policies subject to the restrictions listed in section II revealed that there are only two different information feedback policies and one decision feedback policy which permit fail-safe operation. The decision feedback policy is the one described in a previous report.[1]

The information feedback system with a policy such that erasure-to-confirmation errors are rare was found to have no advantage over the decision feedback system, except possibly when the feedback channel is extremely good and wastage of feedback capacity is not a serious disadvantage. In several respects, this system is almost exactly equivalent, but slightly inferior, to a decision feedback system.

The information feedback system with a policy such that confirmation-to-erasure errors are rare has the feature that error probability is a function of the number of information digits in a message word rather than the number of check digits. Thus, coding technically is not required for reliability, although a transformation from forward received digits to feedback transmitted digits is required, and the operations involved in this operation are somewhat equivalent to coding. A disadvantage of this system is that, during a blackout, the receiver keeps recording erroneous message words. These are all erased eventually, but it requires that the receiver (and also the transmitter) be able to retain <u>tentatively</u> a large number of words.

It must be emphasized that the above conclusions are based on the assumption of a strictly binary channel. If, for example, the erasure signal in section III could consist of complete suppression of the modulation or transmitted signal, or of some waveform far different from a binary sequence, and if a transmitted erasure is assumed whenever positive recognition of binary digits in the received signal is not obtained, then coding might not be needed for reliability (except that a transformation from forward to feedback may be needed, as in the system of section IV.) The comparison with decision feedback might then be more favorable for the information feedback system of section III.

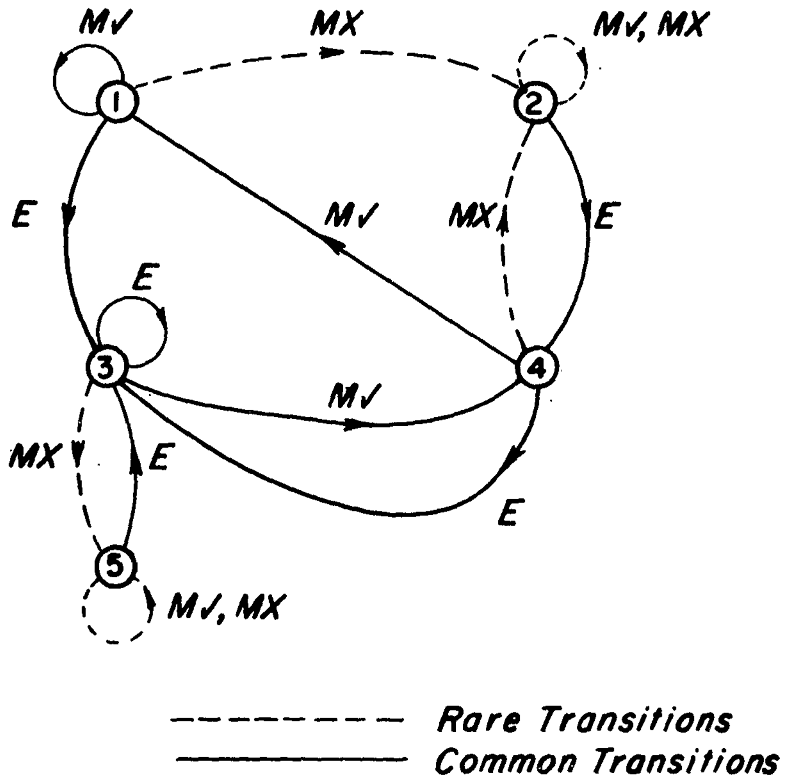
The possibilities of an extension of the theory to allow non-binary erasure or confirmation signals and word decisions based on more than just prior binary digit decision are well worth further investigation.

APPENDIX I: Alternate Logic for the System Described in Section III.

Instead of attempting to "erase the erasure", suppose that the two last transmitted messages are repeated whenever the sending of a message is followed by the reception of an erasure signal on the feedback channel. Since it is not at all certain whether an erasure or a message word had actually been received on the feedback channel, labeling is needed to permit the receiver to distinguish between a new message, a repeat of the most advanced message, and a repeat of the second most advanced message. This requires the use of two labeling digits, just as in the decision feedback system it was necessary to use one label digit to distinguish a new message from a repeat. Successive message words are thus given labels 00, 01, 10, 11, 00, 01, etc.

The transmitter state diagram for this alternate approach is given in Figure 7. Symbols have the same meaning as in Figure 1.

As a result of the transmitter policy, the receiver initially records a sequence which includes the intended messages, some repeated two or more times, some incorrect messages which need erasing, and some erasure instructions. To obtain a



STATE	OPERATION
1	Continue To Next Message
2	Send an Erasure
3	Send Repeat Of Second Most Advanced Of Prior Transmissions
4	Send Repeat Of Most Advanced Of Prior Transmissions
5	Send an Erasure

Fig. 7

State Diagram For The Transmitter

final, probably error-free, reproduction of the intended communication, the following two rules are applied:

a. If there are an even number of consecutive erasures, do not erase the previous message. If there are an odd number, do erase it.

b. When label digits are not consecutive, cross out after the jump as many as are needed to return to the consecutive order.

An example of the application of these rules is shown in Figure 8. In the figure, decoded messages are represented by their labels, and erasures by the letter E. An X indicates that a message is crossed out. Note that, even in this case, an erasure following an erasure is effectively interpreted as an instruction to erase the first erasure instruction.

$$/00/01/10/11/00/\overset{X}{11}/\overset{X}{00}/\overset{X}{00}/\overset{X}{00}/01/10/\overset{X}{10}/11/\overset{X}{00}/E/E/E/00/01/$$

$$\overset{X}{11}/E/10/11/E/E/00/01/10/11/00/01/\overset{X}{00}/\overset{X}{01}/10/\ldots$$

Figure 8

Receiver Erasure Rules

APPENDIX II: Matrix Representation for Information Feedback.

Consider again that an (n', k) code is used in the forward channel, and an (n'', k'') code in the feedback channel $(n' + n'' = n,\ k'' \leq k)$. The feedback sequence can be determined entirely from the forward information digits by means of the equation

$$\begin{bmatrix} f_1 \\ \cdot \\ \cdot \\ \cdot \\ f_{n''} \end{bmatrix} = M \begin{bmatrix} i_1 \\ \cdot \\ \cdot \\ \cdot \\ i_k \end{bmatrix} , \qquad \text{(II-1)}$$

where $[M]$ is an $n'' \times k$ matrix of 1's and o's, f_i is a feedback digit, and i_j is a forward information digit. The dimension of $[M]$ is k''.

Rearrangement of rows and columns of $[M]$ only amounts to a renaming of digits. Let $[M]$ be arranged so that the upper $k'' \times k''$ left corner has an inverse. Let this corner be called $M(k'', k'')$, and let the remaining rows in the first k'' columns be called $M(n'' - k'', k'')$. Then, $f_1 \ldots f_{k''}$ may be treated as information digits in the feedback code, and $f_{k''+1} \ldots f_{n''}$ as check digits. All possible combinations of $i_1 \ldots i_k$ generate all $f_1 \ldots f_{n''}$ sequences. When

$$\begin{bmatrix} f_1 \\ \cdot \\ \cdot \\ \cdot \\ f_{k''} \end{bmatrix} = \begin{bmatrix} M(k'', k'') \end{bmatrix} \begin{bmatrix} i_1 \\ \cdot \\ \cdot \\ \cdot \\ i_{k''} \end{bmatrix} , \tag{II-2}$$

i.e., when the last k-k" forward information digits are zero, it is simultaneously true that

$$\begin{bmatrix} f_{k''+1} \\ \cdot \\ \cdot \\ \cdot \\ f_{n''} \end{bmatrix} = \begin{bmatrix} M(n'' - k'', k'') \end{bmatrix} \begin{bmatrix} i_1 \\ \cdot \\ \cdot \\ \cdot \\ i_{k''} \end{bmatrix} . \tag{II-3}$$

Since $\begin{bmatrix} M(k'', k'') \end{bmatrix}$ has an inverse,

$$\begin{bmatrix} i_1 \\ \cdot \\ \cdot \\ \cdot \\ i_{k''} \end{bmatrix} = \begin{bmatrix} M(k'', k'') \end{bmatrix}^{-1} \begin{bmatrix} f_1 \\ \cdot \\ \cdot \\ \cdot \\ f_{k''} \end{bmatrix} . \tag{II-4}$$

Thus,

$$\begin{bmatrix} f_{k''+1} \\ \cdot \\ \cdot \\ \cdot \\ f_{n''} \end{bmatrix} = \left[M(n''-k'', k'')\right] \left[M(k'', k'')\right]^{-1} \begin{bmatrix} f_1 \\ \cdot \\ \cdot \\ \cdot \\ f_{k''} \end{bmatrix} . \qquad \text{(II-5)}$$

The matrix for the feedback code is then $\left[M(n''-k'', k'')\right]\left[M(k'', k'')\right]^{-1}$

The matrix $[M]$ consists of the components as follows:

$$\left[\begin{array}{c|c} M(k'', k'') & M(k'', k-k'') \\ \hline M(n''-k'', k'') & M(n''-k'', k-k'') \end{array}\right]$$

Given a choice of an (n", k") feedback code, $[M]$ is determined by the following steps:

a. Pick M(k", k") at random, subject to the restriction that it have an inverse. Since the code fixes $\left[M(n''-k'', k'')\right] \cdot \left[M(k'', k'')\right]^{-1}$, $\left[M(n''-k'', k'')\right]$ is then determined.

b. Pick a $(k-k'') \times k''$ matrix $[P_1]$ at random. This determines

$$\left[M(k'', k-k'')\right] = \left[M(k'', k'')\right]\left[P_1\right] . \qquad \text{(II-6)}$$

c. Pick a $(n''-k'') \times k''$ matrix $[P_2]$ at random. This determines

$$\left[M(n''-k'',\ k-k'')\right] = \left[P_2\right]\left[M(k'',\ k-k'')\right] \quad . \qquad \text{(II-7)}$$

The resulting matrix is k'' - dimensional, as desired, since every component is expressible in terms of M(k'', k'').

BIBLIOGRAPHY

1. New York University. "Reliable Fail-Safe Binary Communication." Second Scientific Report, contract AF 19(604)-6168 (July 1960), Unclassified.

2. Metzner, J. J., and Morgan, K. C. "Reliable Fail-Safe Binary Communication." IRE Wescon Convention Record, vol. 4, part 5 (1960), pp. 192-206.

3. Metzner, J. J., and Morgan, K. C. "Coded Feedback Communication Systems." Proceedings NEC, vol. 16 (1960), pp. 250-257. Also AIEE Transactions Paper 61-1050

4. New York University. "Evaluation Theory for Communication Systems." Final Scientific Report, sec. F, contract AF 19(604)-1049 (February 15, 1957), SECRET.

5. Chang, S. S. L. "Theory of Information Feedback Systems." Transactions IRE (PGIT), vol. 2, no. 3 (1956), pp. 29-40.

6. New York University. "Improvement of Two-Way Communication by Means of Feedback." Fifth Scientific Report, contract AF 19(604)-6168 (March 1961), Unclassified.

7. Chang, S. S. L. "Improvement of Two-Way Communication by Means of Feedback." IRE International Convention Record, vol. 9, part 4 (1961), pp. 88-104.

8. Slepian, D. "A Class of Binary Signalling Alphabets." Bell System Technical Journal, vol. 35 (1956), pp. 203-34.

9. Six, W. "The T. O. R. Circuits for the Argentine Radio Links." Communication News, vol. 15, no. 4 (1955), pp. 108-19.

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