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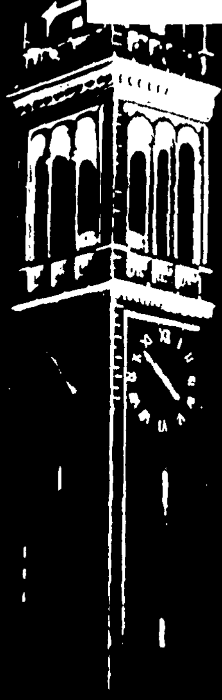

Consolidated Quarterly Progress Report No. 4

February 15, 1962

ELECTRONICS RESEARCH LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA

CONSOLIDATED QUARTERLY PROGRESS REPORT

No. 4

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ELECTRONICS RESEARCH LABORATORY
UNIVERSITY OF CALIFORNIA
Berkeley, California

TABLE OF CONTENTS

INTRODUCTION

This progress report contains a review of the research projects conducted within the Electronics Research Laboratory, University of California, Berkeley, and of the progress made during the three month period ending February 15, 1962.

The research work is separated into the areas:

Circuits
Computer Theory and Programming
Microwave Electronics and Plasmas
Radiation and Propagation
Solid-State Electronics
Systems
Miscellaneous

The research topics within each area are listed alphabetically with regard to the professor in charge of the research.

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R. M. Saunders
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A. J. Thomasian
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Angelakos, D.J.*	Radiation and Propagation
Bergen, A.R.	Systems
Bevensee, R.M.	Microwave Electronics and Plasmas
	Radiation and Propagation
Birdsall, C.K.	Microwave Electronics and Plasmas
Bourne, H.C.	Miscellaneous
Colgate, S.A.	Microwave Electronics and Plasmas
Dalziel, C.F.	Miscellaneous
Desoer, C.A.	Systems
English, A.C.	Solid-State Electronics
Everhart, T.E.	Circuits
	Microwave Electronics and Plasmas
Gill, A.	Systems
Huskey, H.D.	Computer Theory and Programming
Jury, E.I.	Systems
Kuh, E.S.	Circuits
Pai, M.A.	Systems
Pederson, D.O.	Circuits
Pepper, R.S.	Circuits
Polak, E.	Systems
Rumsey, V.H.	Radiation and Propagation
Saunders, R.M.	Miscellaneous
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Sloan, D.H.	Microwave Electronics and Plasmas
Smith, O.J.M.	Circuits
	Systems
Studer, J.J.	Systems
Susskind, C.*	Microwave Electronics and Plasmas
	Miscellaneous
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Turin, G.L.	Systems
Van Duzer, T.	Microwave Electronics and Plasmas
Wang, S.	Solid-State Electronics
Wattenburg, W.	Computer Theory and Programming
Welch, W.J.	Radiation and Propagation
Whinnery, J.R.	Microwave Electronics and Plasmas
Woodyard, J.R.	Microwave Electronics and Plasmas
Zadeh, L.A.	Systems

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GOVERNMENT CONTRACTS AND GRANTS

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Contract No.	ERL No.[1]	
AF 49(638)-1043	02	Joint Tri Service support by the Air Force Office of Scientific Research, the Department of the Army Research Office and Signal Corps, and the Office of Naval Research
AF 19(604)-2270	11	Air Force Cambridge Research Laboratories
AF 19(604)-5466	12	Air Force Cambridge Research Laboratories
AF 33(616)-6139	13	Aeronautical Systems Division
AF 41(657)-114	14	Rome Air Development Center
AF 49(638)-102	15	Air Force Office of Scientific Research
AF 33(616)-7553	16	Aeronautical Systems Division
AFOSR-62-70	17	Air Force Office of Scientific Research
DA 36-039 SC-85278	22	U. S. Army Signal Corps
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G-15965	43	National Science Foundation
D-603513	51	Bell Telephone Laboratories
	01	University Sponsored Research

[1]The ERL No. appears as the first two digits of the numbers of each research topic in the lists which follow.

LIST OF REPORTS PUBLISHED DURING THE QUARTER ENDING 15 FEBRUARY 1962

All reports are published in Series 60 of the University of California's Institute of Engineering Research, and are further designated by issue number (as listed below):

No. 416: R.E. Drews and A.C. English, "Electrical and optical properties of silicon carbide single-crystals," 16 November 1961.
No. 417: E. Polak, "On the equivalence of discrete systems in time-optimal control," 17 November 1961.
No. 418: I. Gorog, "Coherent optical emission from molecular beams," 20 November 1961.
No. 419: R.N. Miller, "A modification of Hamming codes for use in error-detection," 21 November 1961.
No. 420: G.H. Wilson, "Nonlinear analysis of sinusoidal oscillators," 18 December 1961.
No. 421: J.F.A. Ormsby, "Existence and relationship of a class of perfect predictors," 18 December 1961.
No. 422: J.F.A. Ormsby, "Analytic representation of a perfect predictor for a class of stationary processes," 18 December 1961.
No. 423: J.R. Whinnery, C. Susskind, C.K. Birdsall, S.A. Colgate, T.E. Everhart, A.W. Trivelpiece and J.R. Woodyard, "Electron physics of traveling wave tube devices," April 1961.
No. 424: A.R. Boothroyd, "Transistor bandpass amplifiers of well-defined performance," 19 December 1961.
No. 425: E.I. Jury and J. Blanchard, "On the roots of a real polynomial inside the unit circle and a stability criterion for linear discrete systems," 26 December 1961.
No. 426: C.A. Desoer, "A geometric interpretation of the pseudo inverse of a matrix," 28 December 1961.
No. 427: A.J. Thomasian, "The metric structure of codes for the binary symmetric channel," 8 January 1962.
No. 428: B.R. Cheo, V.H. Rumsey and W.J. Welch, "A solution to the frequency-independent antenna problem," 8 January 1962.
No. 429: B.J. Maxum and A.W. Trivelpiece, "Cyclotron wave instabilities," 8 January 1962.
No. 430: Y. Sekine, "Optimum expansion policies for multi-component stochastic systems, I," 17 January 1962.
No. 431: M.A. Breuer, "The minimization of Boolean functions containing unequal and nonlinear cost functions," 22 January 1962.
No. 432: I. Cederbaum, "Applications of graph theory to solution of some systems of linear algebraic equations," 23 January 1962.
No. 433: C.Y. Fong, "The measurements of the scattered patterns of the ferrite post," 25 January 1962.
No. 434: H. Kwakernaak, "Analysis of errors in the estimation of the impulsive response," 6 February 1962.
No. 435: C.K. Birdsall, S.A. Colgate, T.E. Everhart, A.J. Lichtenberg, C. Susskind, A.W. Trivelpiece, J.R. Whinnery and J.R. Woodyard, "Growing wave phenomena in electron beams and plasmas," 1 March 1962.
No. 436: "Notes on system theory, II," 15 February 1962.

CIRCUITS

16-1-01 SOLID CIRCUIT MINIMAL SIZE ANALYSIS. D.A. Waterman (Prof. Everhart)

This problem is concerned with finding the minimum size of a tunnel diode as a function of its physical parameters, such as negative resistance, doping levels, temperature and junction voltage. The method of analysis consists of formulating a mathematical expression for negative resistance in terms of these parameters, and then using this expression to determine the smallest allowable size, i.e., junction area, for any particular value of negative resistance.

The tunneling current I is proportional to the integral, from the top energy level of the conduction band W_{cn} to the bottom energy level of the valence band W_{vp}, of the tunneling probability Z, the difference in the fermi distribution functions $f_c - f_v$, and the product of energy state densities in the conduction and valence bands, and can be written:

$$I = K \int_{W_{cn}}^{W_{vp}} (Z)\left(f_c - f_v\right) \left[C_1\left(W - W_{cn}\right)^{1/2}\right]\left[C_2\left(W_{vp} - W\right)^{1/2}\right] dW \quad (1)$$

Under the assumption that the fermi distribution functions are essentially the same at absolute zero as at room temperature equation (1) becomes:

$$I = \int_{W_{cn}}^{W_{vp}} C_1C_2KZ\left(W - W_{cn}\right)^{1/2}\left(W_{vp} - W\right)^{1/2} dW \quad \text{for } qV \geq W_{fn} \quad (2)$$

and

$$I = \int_{W_{fp}}^{W_{fn}} C_1C_2KZ\left(W - W_{cn}\right)^{1/2}\left(W_{vp} - W\right)^{1/2} dW \quad \text{for } qV \leq W_{fn} \quad (3)$$

where

V = applied junction voltage
W_{fn} = fermi level in N-type material
W_{fp} = fermi level in P-type material

Equations (2) and (3) when integrated lead to the following expressions for tunneling current:

$$I = C_1C_2KZ\,\frac{\pi}{8}\left[2.4\,kT\left(\frac{m}{m_e}\right)\left(\frac{300}{T}\right)\left(\frac{Nd}{2.5\times10^{19}}\right)^{2/3} - qV\right]^2 \quad \text{for } qV \geq W_{fn} \quad (4)$$

and

$$I = C_1C_2KZ\left(\frac{1}{2}qV\right)\left(W_{fn}\right)^{1/2}\left(W_{fn} - qV\right)^{1/2} + \frac{\left(2W_{fn} - qV\right)^2}{4} \tag{5}$$

$$\cdot\left(\tan^{-1}\sqrt{\frac{W_{fn}}{W_{fn} - qV}} - \tan^{-1}\sqrt{\frac{W_{fn} - qV}{W_{fn}}}\right) \quad \text{for } qV \leq W_{fn}$$

The figure below is a comparison of equations (4) and (5) and a known tunnel diode I-V curve. Since the negative slopes are essentially the same, equation (4) can be differentiated with respect to voltage to obtain:

$$G = \frac{\partial I}{\partial V} = K_A\left(K_B\ Nd^{2/3} - 1\right) \tag{6}$$

where

$$K_A = K\frac{32\pi^2q^2V}{h^6}\left(m_em_h\right)^{2/3} e^{-\frac{\pi^2K\epsilon_G am_e}{h^2\ q\ C_0}}$$

$$K_B = 8.37\text{x}10^{-11}\ \frac{k}{qV}\ \frac{m}{m_e}$$

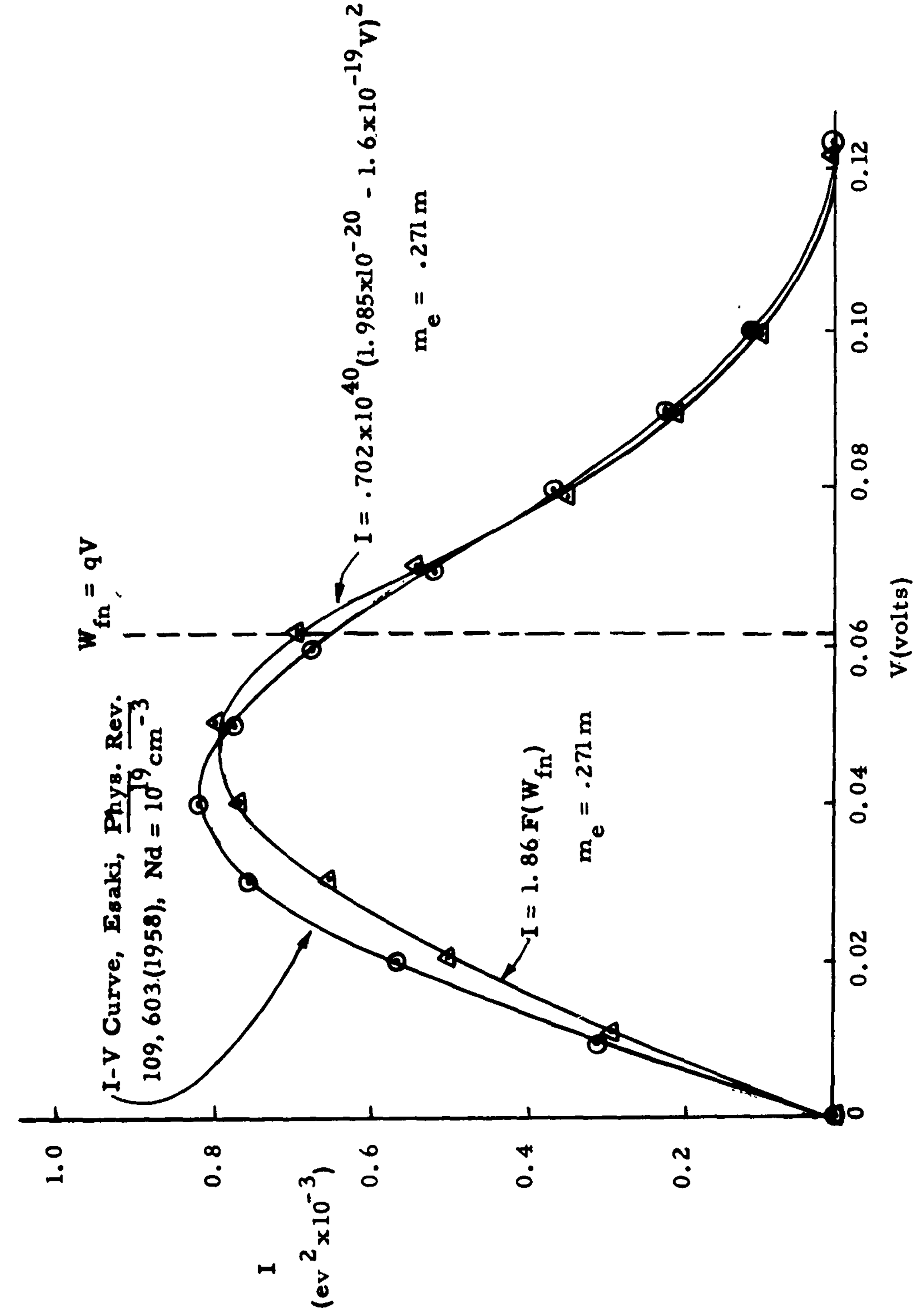
Using equation (6) an expression for negative resistance will next be derived in terms of the doping levels and the junction area.

16-1-07 ELECTRON MICROPROBE FOR SEMICONDUCTOR INTEGRATED CIRCUIT ANALYSIS AND FABRICATION. T. Ota (Prof. Everhart)

An electron microprobe is currently being designed, and experiments designed to test its performance are being devised. The 5 to 30 kV electron beam with a minimum spot size variable from 100 Å to over 1 micron can be scanned over a maximum area of approximately 4 square millimeters.

A simple method of quickly measuring the astigmatism of the microprobe spot and correcting it has been devised, and will enable the instrument to be quickly set to give optimum performance. Essentially, the spot is projected onto a phosphor screen of high resolution and the stigmator adjusted until the projected spot is round. The instrument should then be free from astigmatism until the final aperture needs replacing and possibly until the pole-piece of the final lens is cleaned.

Spot size measurements can be made by sweeping the probe across a straight-edge, or by exposing a colloidion film in the method discussed by Möllenstedt. Other methods, chemical in nature, can also be used to determine spot size. By sweeping the spot in a raster and pulsing on the beam at



Tunnel Diode I-V Curves

discrete times, a series of spots can be developed and examined in the electron microscope for evaluation purposes. This will enable the objective lens astigmatism and curvature of field to be measured in exactly the way they will effect instrument performance. A meaningful estimate of the maximum number of raster lines which can be scanned with a given degradation in resolution can also be obtained experimentally. This is an important parameter when contemplating the use of electron probes for automatic integrated circuit production.

A magnetic-lens pole-piece design for the objective lens has been carried through, and with the exception of pole-piece saturation effects, is complete. This pole-piece will be used with commercially obtainable magnetic lenses.

Saturation effects in the objective lens are being studied using an electrolytic tank; unfortunately, much time has been spent repairing the circuits connected with the tank. This was deemed worthwhile since other static fields will probably be investigated later using the same tank.

42-0-03 GAIN BANDWIDTH LIMITATIONS OF ACTIVE DEVICES.
J.D. Patterson (Prof. Kuh)

We are considering the general problem of the gain bandwidth limitations which may exist for an amplifier consisting of an active device imbedded in an arbitrary passive network.

The specific problem under consideration at present is that of cascade structures, i.e., amplifiers consisting of a single active device with passive input and output networks. As a first step we have studied the limitations imposed by a fixed lossless 2-port connected in cascade with arbitrary passive networks. Preliminary results indicate that gain bandwidth limitations can be found for lossless 2-ports having two or more transmission zeros. Work is now in progress on the limitations imposed by lossy passive 2-ports and it is hoped that the results of this study can be extended to include active 2-ports.

42-1-06 SYNTHESIS OF ACTIVE RC NETWORKS. S.K. Mitra (Prof. Kuh)

Work has been completed and a thesis is being written on this research.

42-1-05 WIDE BAND PARAMETRIC AMPLIFIER STUDY. M. Fukada (Prof. Kuh)

Extension of the design theory of optimum wide band parametric amplifiers has been continued to include the effect of the diode dissipation. In addition to the previous result for the shunt diode amplifier, the results are obtained for the series diode amplifier and inverting converter, and the shunt diode inverting converter.

Present work centers on the design of practical parametric amplifiers in the microwave region to give experimental verification of our theory.

02-1-04 TIME VARYING RC NETWORKS. A. Paige (Prof. Kuh)

This investigation is concerned with determining the physical realizability conditions for the time varying analog of a driving point impedance for an RC network. The immediate problem has been that of determining stability of such networks. Several new approaches were tried with only slight success. Certain special cases have been investigated.

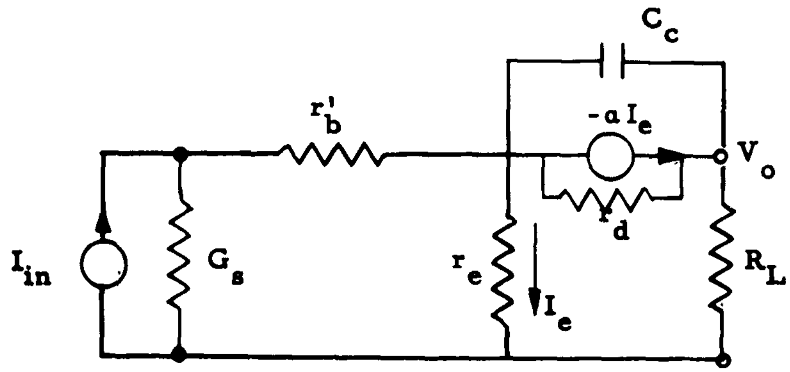
As a by-product of this investigation the problem of realizing an nxn impedance matrix (for time invariant networks) has been studied. It is believed that the necessary and sufficient conditions that such a network can be realized with RC elements and n + 1 nodes has been determined. This will be investigated in greater detail and also an attempt will be made to extend this to time varying networks.

02-1-29 TIME VARYING NETWORKS. R.A. Rohrer (Prof. Kuh)

Investigation of general dissipative time varying circuits has been continued in terms of energy functions. Time dependent Lagrangians and Hamiltonians have been obtained, but thus far they have been of little practical value.

01-1-64 ANALYSIS OF DELAY TIME IN A TRANSISTOR PULSE AMPLIFIER. K. Hatch (Prof. Pepper)

The large signal analysis of a transistor pulse amplifier is complicated since the parameters in the equivalent model are highly nonlinear. For example, in the usual high frequency transistor model shown in the figure below, the emitter resistance, r_e, varies as a function of the emitter current. The purpose of this study is to determine the effect of the nonlinear emitter resistance on the circuit response.



Another important nonlinear element is C_c, but an equivalent, linear value for C_c, as developed by Bashkow, will be used for this project. The problem is to obtain the complete circuit response for the exact, nonlinear representation of r_e (to be accomplished with the aid of the IBM 704 computer). By comparing this solution with the response obtained for fixed values of r_e, one should be able to choose an equivalent linear value (or values) for r_e which will give the correct 0-90% rise time.

For a large output pulse, the value of r_e will be quite small except for the initial part of the pulse. The average value of r_e is an adequate value to use in calculating the 10 to 90% rise time. However, r_e seems to influence the delay time of the output pulse more severely, i.e., the 0-10% rise time. With use of the dominant pole method of analysis, the circuit model of the figure should lead to the correct solution for the 10-90% rise time and the output delay. The problem is then to find an appropriate value of r_e to use in the equations of output delay due to C_c. Furthermore, it appears that the nonlinear nature of r_e may introduce an additional delay which is independent of C_c.

Since the output delay due to the excess phase of alpha is not a function of r_e (to first order), the preliminary analysis of the pulse amplifier will not include the excess phase of alpha.

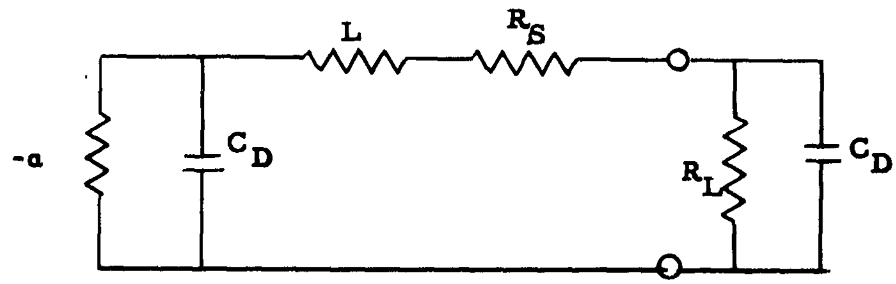
The preliminary investigation will study the effect of r_e alone. A one-pole approximation for alpha will be used, and C_c will be neglected. The result should indicate roughly what effects the nonlinear nature of r_e will have on the zero order response. The second step will be to include the effects of C_c and determine the influence of a nonlinear r_e on the output delay due to C_c.

02-1-06 OSCILLATOR STUDIES. D.K. Lynn, W.R. Ennis (Profs. Pederson and Pepper)

A study is being made of the properties of oscillators which are describable by nonlinear differential equations of order three or higher. Among the properties being studied are the amplitude, minimum period and available power of oscillation. These properties will be compared with those obtained previously for oscillators described by nonlinear, second order differential equations.

Some of the oscillators being studied are a simplified transistor Colpitts, a transistor Hartley and a tunnel diode with a parallel RC load. These oscillators can be described by third order, nonlinear differential equations.

From a linear analysis the maximum frequency of harmonic oscillation has been found for a tunnel diode with an RC load. The circuit is shown in the figure below. This maximum occurs when two of the natural frequencies are on the $j\omega$-axis and the third is at the origin.



Harmonic oscillations require

$$\frac{a}{C_D} > \frac{R_S}{L}$$

$$\frac{1}{LC_D} > \frac{a^2}{C_D^2}$$

Combining these two conditions gives

$$1 - aR_S > 0$$

The maximum frequency of harmonic oscillation is

$$\Omega = \left[\frac{2}{LC_D} - \frac{a^2}{C_D^2} - \frac{R_S}{aL^2}\right]^{1/2}$$

and is obtained with the load

$$R_L = \frac{1 - aR_S}{a}$$

$$C_L = \frac{a}{\left(1 - aR_S\right)\left(\frac{a}{C_D} - \frac{R_S}{L}\right)}$$

The fact that Ω is the maximum frequency of harmonic oscillation is proved by root locus arguments.

For the conditions stated above (which are satisfied for most tunnel diodes), Ω is always greater than the maximum frequency of harmonic oscillation for a tunnel diode with a series RL load, i.e., $\Omega > \omega_L$.

The nonlinear v-i characteristic of the tunnel diode is approximated by

$$i = f(v) = -\alpha v + \beta v^3$$

and a perturbation analysis is performed. The unperturbed equation is

$$\frac{d^3v}{dt^3} + \Omega^2\frac{dv}{dt} = 0$$

This is the equation that gives the maximum frequency of harmonic oscillation, Ω. For this type of analysis there are serious questions as to the validity of the perturbation expansions. Solutions to the oscillator equation are being found with the aid of a computer. Perturbation solutions will then be checked against the computer solutions.

16-1-02 HARMONIC OSCILLATION IN A SEMICONDUCTOR SOLID.
G. Hachtel, D.A. Hodges (Prof. Pederson)

Effort continues on the project of establishing necessary conditions for near harmonic oscillation in a semiconductor integrated circuit. We seek to express these conditions in the physically meaningful terms of charge flow patterns and time variation of stored charge. The UJT oscillator realization of Figure 1 continues to be used as a vehicle in the study. In previous work, physical arguments, based on the dc interpretation of conductivity modulation, were employed to obtain relations between the UJT inductance and negative dynamic resistance and the physical parameters of the device.

In the past quarter the effect of conductivity modulation in the case of small signal sinusoidal excitation superposed on a dc quiescent level was investigated. In particular, an expression was obtained for ac power flow into unit volume of the filamentary bulk. Complex phasor notation is used. The magnitude of the phasor multipliers are assumed small compared to their dc counterparts,

$$\text{Power Flow} = E_\omega\left(J^*_{in\omega}\right) = \frac{|J_{in\omega}|^2}{q\left(u_p + u_n\right)P_{dc}(x)} \left[\underset{(I)}{1} + \underset{(II)}{\frac{q\left(D_n - D_p\right)\nabla P_\omega}{J_{in\omega}}} - \underset{(III)}{\frac{q\left(u_p + u_n\right)E_{dc}(x)P_\omega(x,\omega)}{J_{in\omega}}}\right]$$

where

$$J_{in} \triangleq J_{indc} + R_e\left[J_{in\omega} e^{j\omega t}\right]$$

$$P \triangleq P_{dc} + R_e\left[P_\omega e^{j\omega t}\right]$$

$$E \triangleq E_{dc} + R_e\left[E_\omega e^{j\omega t}\right]$$

Term (I) represents a resistance. P_ω and ∇P_ω can be shown to lag the complex phasor $J_{in\omega}$ by less than 90°. Therefore, term (II) is a positive resistance with a negative reactance. The negative sign in term (III) converts it to the sum of a negative real power flow and a positive reactive part, which thus represents a -R, L element. Term (III) reveals the essence of conductivity modulation, showing how power in the interbase field is modulated by the injected ac excess density. Thus conductivity modulation supplies power to the infinitesimal volume element. In view of these considerations, the circuit model of Figure 2 is proposed for the infinitesimal volume element.

P_ω is a phasor quantity representing the phase and magnitude of the ac excess minority carrier density and thus constitutes a stored charge. Its presence affirms the belief that the oscillation phenomena can be interpreted in terms of charge flow. It is interesting to note the simultaneous presence of real and reactive power terms of both signs. It is now believed that the UJT oscillation can be realized without external elements besides biasing circuits. Work will proceed on establishing the necessary conditions for this oscillation.

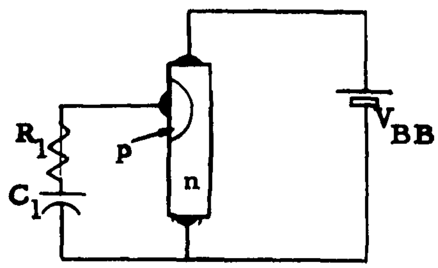


Figure 1
UJT Oscillator Realization

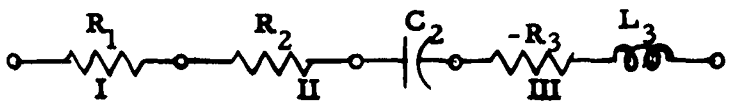


Figure 2
Circuit Model for Infinitesimal Volume Element

Also under study is a realization of an integrated symmetric three phase oscillator. A lumped model of a proposed realization was simulated using three transistors and lumped passive components. The design procedure proposed by J. R. Freeman of this department was modified to include the feedback effects of C_c, the transistor collector capacitance. In this circuit feedback effects, rather than transistor f_T, dominate in determining the maximum frequency of oscillation. It was found that a design using silicon planar transistors with $f_T = 100$ mcs. and $C_c = 25$ pf. is limited to oscillation below about 1.0 mcs. Experimentally, a design for 150 kcs. oscillated at 135 kcs. Gain must be stabilized against variations in temperature in order to obtain sine wave oscillations over even a 20°C temperature range.

16-1-04 BISTABILITY IN A SEMICONDUCTOR SOLID. L.O. Hill (Prof. Pederson)

A technical report summarizing research effort on this topic is now being completed.

16-1-06 INTEGRATED REALIZATIONS OF BANDPASS AMPLIFIERS. D.A. Hodges (Prof. Pederson)

Further experiments have been conducted using a lumped simulation of an integrated bandpass amplifier similar to that shown in the Consolidated Quarterly Progress Report No. 3. The circuit uses a Darlington connection of two transistors and a distributed RC feedback network. In final form the experimental circuit had a center frequency of 2.5 mcs. With no net feedback at the center frequency, power gain of 17 db and fractional bandwidth of 20% were measured. Using 18 db of positive feedback at the center frequency, power gain was 35 db and fractional bandwidth was 2%. In the latter case the power gain and bandwidth varied considerably with changes in temperature and supply voltage.

16-1-08 INTEGRATED MONOSTABLE CIRCUITS. D.A. Hodges, B. Hunts (Profs. Pederson and Pepper)

Various monostable multivibrator configurations are being studied in order to determine which are best suited to realization in integrated form. Experiments are being conducted on lumped simulations of integrated circuits in order to determine the accuracy with which pulse widths can be predicted analytically. Also of interest is the dependence of output pulse properties on supply voltage and input pulse characteristics.

To date the conventional emitter coupled and collector coupled circuits have been analyzed. Experimental measurements verify the results to within 5% in most cases.

A new circuit being studied uses one each of PNP and NPN transistors. Both are off in the stable state and on in the quasi-stable state, reducing power dissipation in applications where duty cycle is small. Short output p pulses are obtained using a single timing capacitor of size practical for realization in integrated form. Preliminary experiments indicate that turn-on is fast for this circuit relative to the speed of the transistors used. This is expected since neither transistor is saturated in the stable state.

02-1-28 ANALYSIS OF INTEGRATED BANDPASS AMPLIFIERS. A.G. Thiele, K.Y. Tsai (Prof. Pederson)

Analysis of two types of bandpass transistor amplifiers is currently being performed. Amplifiers of the first type employ passive RC feedback, negative and/or positive, to produce a bandpass gain-function. Amplifiers of the second type employ tuned LC circuitry, where the inductances are to be realized as the inductive input admittance of unijunction transistors under suitable biasing conditions.

All circuits to be considered must be suitable for integrated realization.

RC feedback amplifiers: All circuits of this type considered to date have been of the form shown in Figure 1, where the matrix (y_{ij}^N) represents the negative feedback circuit, N, and (y_{ij}^P) the positive feedback circuit, P.

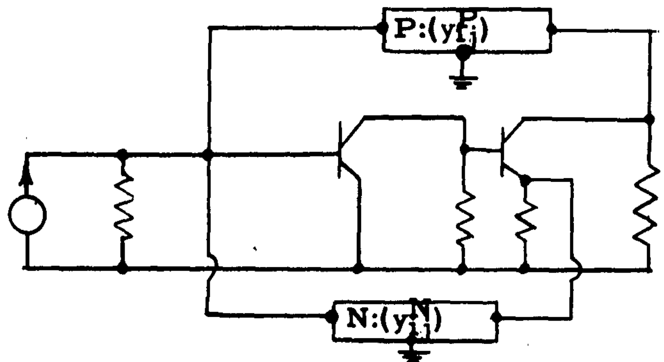


Figure 1

While various combinations of circuits for the networks N and P in Figure 1 have been considered, particular attention has been given to that combination shown in Figure 2. For this case, a simple algebraic procedure has been developed for the approximate determination of the closed-loop gain-function poles. Also optimal relations for the pole and zero locations of the network N have been developed, both by root-locus techniques and analytically, for the gain-limited case where the positive feedback network, P of Figure 2, is not included. In particular, suppose a dominant closed-loop gain-function pole is desired at $P_o = \sigma_o + j\omega_o$, $|\sigma_o| << \omega_o$, as shown in Figure 3. Then for a minimum required T(0) (dc loop-transmission), it is found that the "phantom zeros", Z_1, $\bar{Z}_1 = \sigma_1 \pm j\omega_1$ of y_{12}^N, should be placed such that $\sigma_1 = \sigma_o/2$, provided the angle of approach of the T(p) root-locus at Z_1 is of the order of 180°, say 180° ± 30°. A typical situation is illustrated in Figure 3.

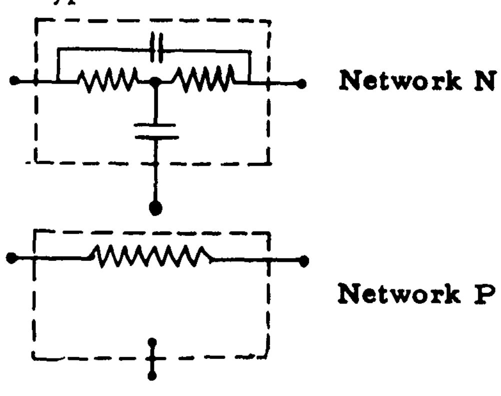


Figure 2

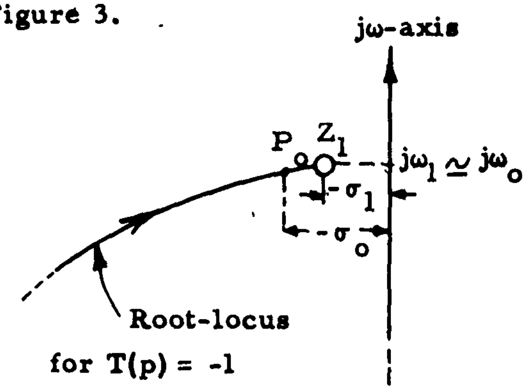


Figure 3

Future work in this area will treat additional feedback network-pairs in the circuit of Figure 1 and also will consider amplifier configurations other than that of Figure 1; for example, two-stage Darlington amplifiers. Emphasis will be placed on a comparison of the sensitivity of Q and center frequency to variations in component values and amplifier gain for the various circuits.

Amplifiers with tuned LC circuits: Circuits of this type have been considered to be of secondary importance to the RC feedback amplifiers discussed above and hence have not yet been studied in detail. The input characteristics of unijunction transistors themselves have been treated analytically, however, and it is found that, under suitable biasing conditions and over a somewhat limited frequency range, the driving-point admittance at the input terminals consists of a negative conductance and an inductive susceptance.

Hence, conventional shunt-peaked and series-peaked LC bandpass amplifiers should be realizable in integrated form through the inclusion of unijunction transistors.

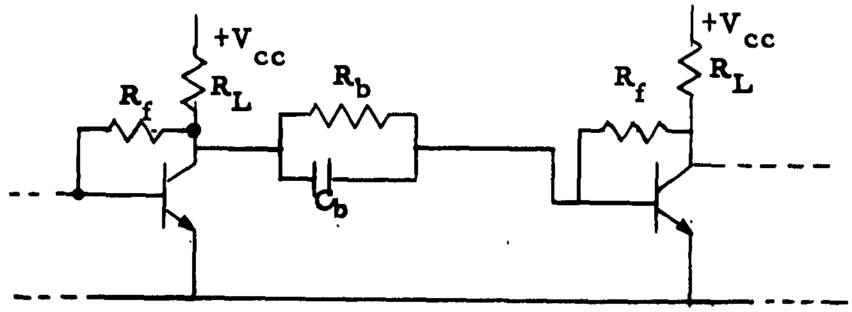
Future work in this area will treat such circuits, again with emphasis on the sensitivity considerations mentioned previously.

02-1-38 INTEGRATED LOW PASS AMPLIFIERS. A. Brodersen (Prof. Pepper)

A study is being undertaken to determine the feasibility of obtaining integrated, low pass, single stage amplifiers or a cascade of such amplifiers. The study is considering four circuit configurations. These circuits were chosen because of the absence of inductance, an element hard to achieve at present in integrated form. The circuits are shown in Figures 1, 2, 3 and 4. Bias circuitry is omitted for simplicity. These circuits will be referred to as the R_b - C_b scheme, the common collector-common emitter pair (CC-CE pair), the shunt series feedback pair and the R_E - C_E scheme, respectively.

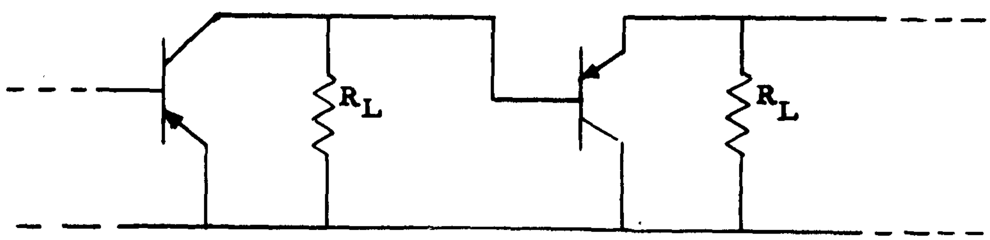
Previous studies of these circuits have been concerned with establishing gain and bandwidth expressions and establishing design procedures for the circuits. This study is to determine which circuit will give the optimum gain bandwidth product with "low sensitivity". "Low sensitivity" is defined as a proportionally small change in gain bandwidth product with respect to a change in an element value of the circuit. "Low sensitivity" is important in this study because of the problems in fabrication of an integrated element of exactly the required value.

The optimum gain bandwidth product of each circuit has been obtained. To make a valid comparison of the four circuits the transistor (or transistors) in all circuits was chosen to be the same,typical, junction transistor. In addition all passive elements were treated as lumped elements. Results indicate that either the R_b - C_b scheme or the shunt-series feedback pair will give the greatest gain bandwidth product.



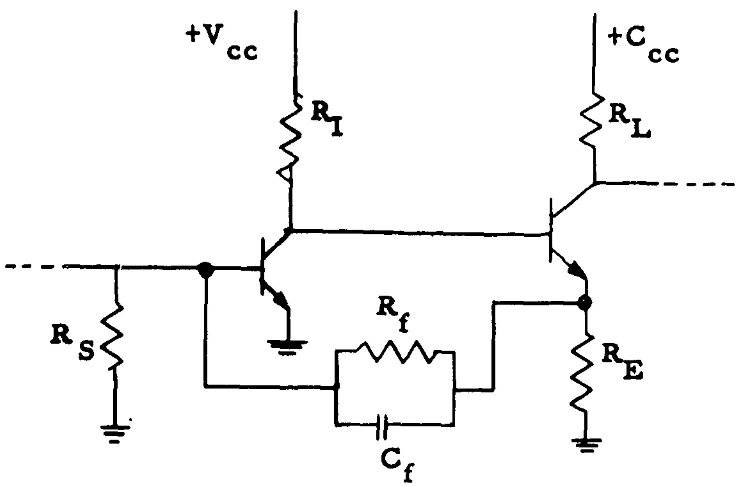
R_b-C_b Scheme

Figure 1



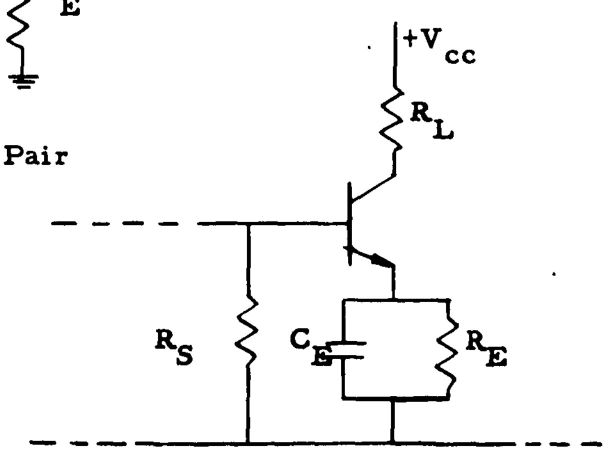
CC-CE Pair

Figure 2



Shunt-Series Feedback Pair

Figure 3



R_E-C_E Scheme

Figure 4

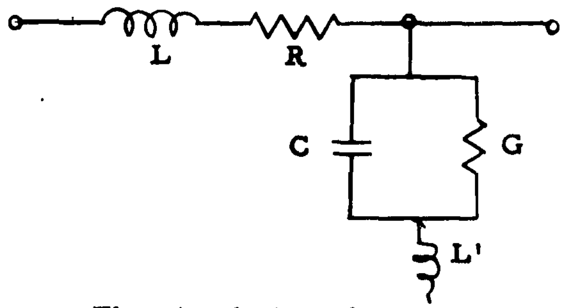
Study is now being undertaken to determine which circuit will give the "least sensitivity".

01-1-60 COMPUTER STUDY OF AN ARTIFICIAL TRANSMISSION LINE.
S.B. Fine (Prof. Smith)

Objective: It is desired that a pulse of current 10^{7} amperes in magnitude be passed through an inductive-resistive load in 10^{-7} seconds. The load resistance and inductance is 10^{-1} ohms and 10^{-7} henries, respectively. This device will be used at Livermore Laboratories to create the magnetic "pinch" effect for nuclear fusion studies.

Means of realization: Current is made to flow in a short circuited transmission line, whose natural impedance matches that of the load. After a sufficient amount of "charging" time has elapsed the load is switched on to the line, replacing the source. The load current is expected to build up rapidly, be maintained for a short period of time while there is energy stored in the line and then die out rapidly with no reflections.

The transmission line is being simulated on the Digital Differential Analyser (D.D.A.), an accessory of the Bendix G-15 computer. The line consists of ten lumped-parameter sections, each section appearing as follows:



$L = 10^{-10}$ hy $R = 2 \times 10^{-4}$ ohm
$L' = 10^{-4}$ hy $G = 2 \times 10^{-2}$ mho
$C = 10^{-8}$ hy

The simulation of the line uses 30 integrators, 40 multipliers and 10 decision integrators (which act like infinite gain amplifiers). The simulation of the load will require an additional 2 integrators, 3 multipliers and at least 1 decision integrator. Thus the D.D.A. is being used to near capacity.

The problem at this point is that of scaling the D.D.A. program so that its decision integrators will function properly during all transients, yet run fast enough to yield results in a reasonable amount of time. Once this "hardware" problem is overcome results will be forthcoming.

COMPUTER THEORY AND PROGRAMMING

COMPUTER THEORY AND PROGRAMMING

01-1-49 IDENTIFIER PREPROCESSOR FOR ALGOL-TYPE LANGUAGES. G. Anderson (Prof. Huskey)

Work has been done on the completion of the identifier preprocessing system for the DIALGOL language. The main emphasis has been applied to preprocessing a DIALGOL source language problem and transferring the preprocessed information to the second stage of the DIALGOL compiler.

Future plans call for the inclusion of some of the more sophisticated features of the ALGOL language into the preprocessing system. As soon as the system proves successful it will be rewritten in the DIALGOL language so that a complete package will be available for bootstrapping the compiler to produce systems of more complexity and also systems for use on machines other than the IBM 704.

01-1-50 AUTOMATIC DECLARATION. J. Spitze (Prof. Huskey)

The general format of the automatic declaration algorithm has been completed. The input and output routines have operated properly for approximately one month with no major revisions. The central portion of the program presently analyzes simple DIALGOL programs containing FOR statements and generates the proper declarations for the recognized single dimensional non-dynamic arrays.

A major revision of the central analyzer has recently been completed but has not been tested as yet. This new version will automatically declare all previously undeclared variables and will assign to them the TYPE (i.e., basic arithmetic form -- REAL, INTEGER or BOOLEAN) which the entire program has been declared to be.

The next field of study will be automatic declaration of multidimensional non-dynamic arrays.

01-1-51 AN ALGORITHM FOR THE TRANSLATION OF ALGOL. W.M. Keese (Prof. Huskey)

A new translator has been written intended to be compatible with the preprocessor and assembler. It also involves some simplification in dealing with brackets and some increased generality in dealing with function designators which appear disguised as variable names (i.e., without any parameters). This is not yet checked out.

Thinking about the possibility of better IL languages has produced the conclusion that the total three-stage effort seems no greater if the IL language, in its algebraic parts, is a traditional Polish string. A translator to produce this has been begun, and it appears that it will prove significantly shorter and faster than previous efforts. Further, its output seems well adapted to direct employment of the assembler in place of output routines.

It has been discovered that some increased generality in source language (to wit: allowing the comma to be used either in the NELIAC or ALGOL guise without ambiguity) occurs if the preprocessor is called as a procedure by the translator, rather than simply preceding it.

01-1-52 INTERMEDIATE LANGUAGE ASSEMBLY PROGRAM.
B.G. Middleditch (Prof. Huskey)

1. An Intermediate Language Assembly Program is being written for the IBM 704. This program will assemble IL commands, expanding them into 704 machine language. This program is being specified in NELIAC language. It provides for real and integer modes of arithmetic and should eventually handle all aspects of IL generated by the DIALGOL translator. At present subroutine transfers and subscripting are being incorporated in the assembler. The objective of this phase is to provide a three phase operating translator on the 704.

2. In parallel with the above effort an attempt is being made to develop an assembler, for a minimal set of IL instructions, which could be used to bootstrap other, more comprehensive, assemblers. This assembler should be as machine independent as possible and is provided with a table of specifications of the object machine. At present the minimum set of IL instructions generated by a minimum set of NELIAC statements is being determined.

01-1-53 OPTIMIZATION OF ULTIMATE OUTPUT BOOLEAN FUNCTIONS.
P. White (Prof. Huskey)

The operating procedure for determining the most economical p-input, q-output switching circuits required for the implementation of q Boolean switching functions of p variables each has been written up as an M.S. thesis and will also soon appear as a UCRL report.

The NELIAC problem automating the procedure is partially debugged and should be available soon.

02-1-31 A NELIAC COMPILER FOR IPL-V. R. Love (Prof. Huskey)

Research is being conducted on developing a program to compile Information Processing Language V routines. IPL-V is a symbol and list-structure manipulating language presently implemented as an interpretive system. To study the operation of the interpreter a Symbolic Assembly Program listing of IPL-V has been obtained. From this listing the interpreter has been rewritten in the NELIAC language. This transformation allows one to more easily visualize the operation of the IPL-V system. Tests are presently being run on the NELIAC program to insure its accuracy. The knowledge from this NELIAC version of the IPL-V interpreter will be used to construct a compiler of IPL-V. To build the compiler three major steps will be performed: 1) development of an indexable list, 2) program to compile instructions to perform the operations of the interpreter and 3) writing basic processes to manipulate the lists and symbols. Once the IPL-V compiler is written it may be combined with the NELIAC compiler to allow efficient algebraic operations during list processing.

02-1-32 AUTOMATION OF ABSTRACTING. J. White (Prof. Huskey)

The primary aim of the project is to determine the possibly significant value of the application of general Linguistic theory to the field of information retrieval. To this end the class work, seminar work and consultations in Linguistics which had been started in September 1961 have been continued. A brief summary of these investigations has been written up in a paper entitled "Proposed Methods of Application of General Linguistic Theory to Information Retrieval".

02-1-33 SIMPLIFICATION OF BOOLEAN FUNCTIONS USING MATHEMATICAL PROGRAMMED TECHNIQUES. M.A. Breuer (Prof. Huskey)

A technique has been developed for minimizing Boolean functions by employing linear programming. The method takes into consideration lead length, loading, fan-in restrictions and the nonlinear cost functions derived from the fan-out problem. In essence the technique simplifies the Boolean function to a form which is minimal in cost to implement. Further use of linear programming in logical design will be investigated.

02-1-40 APPLICATIONS OF NELIAC. R. Love (Prof. Huskey)

Successful tests have been made on the IPL-V interpreter written in the NELIAC language. The IPL-V primitive processes are now being examined to determine the feasibility of writing them in NELIAC. By having the complete IPL-V system written in NELIAC instead of SAP greater insight may be gained in the operation of a symbol and list-structure manipulating language.

The NELIAC load source program for the IBM 704 and 709 computers has been rewritten to improve its speed and to include some ALGOL words. With the inclusion of these special words in the NELIAC source language more readable programs can be developed.

A load source program is being developed for FORTRAN. This program, written in NELIAC, will recognize all the special words used in the FORTRAN source language. New features have been added to this load source to make it faster than previous load source programs used with NELIAC.

A program is being written for the IBM 704 computer to simulate the INTERCOM 500 language used on the Bendix G-15 computer. This program will be useful for programs which become too long to be run on the G-15. A considerable time saving will be available through the use of this program. The INTERCOM 500 language will be written in NELIAC as both an interpreter and compiler.

This activity will test NELIAC as a suitable language for describing a computing system.

02-1-41 THE ADDITION OF SYMBOL MANIPULATION OPERATIONS TO THE NELIAC LANGUAGE. N. Wirth (Prof. Wattenburg)

With the purpose of rendering the NELIAC language less machine-oriented, a new class of operations has been introduced. These operations simplify expressions involving character manipulations. The associated operands (words) are supposed to consist of a chain of characters or symbols, the length of which may be declared in the program.

At present the following two operators have been created:
1. Catenate a character B at the end of a chain B.
2. Obtain the first character of a chain A.

In order to avoid the creation of new special symbols for these operators, a way has been found to express them by means of the + and * symbols, such that any ambiguity with their previous meaning is avoided. Similar to partial word structures, the chain structure of an operand may be declared either in the dimensioning statement preceding the program, or in the program itself.

51-1-01 AUTOMATIC PROGRAMMING SYSTEMS FOR LARGE-SCALE DIGITAL COMPUTERS. C. Conn (Prof. Wattenburg)

The 704 NELIAC system has been completed and is now being used in programming systems studies. The 704 NELIAC system is now being expanded for operation on the IBM 7090 computer. The increased speed and input-output facilities of the 7090 are being utilized in the 7090 NELIAC system.

Several other investigators are now using the NELIAC system in their research.

01-1-47 A HIGH SPEED BINARY SUBTRACTOR UTILIZING TUNNEL DIODES AND MAJORITY LOGIC. J.A. Lukes (Prof. Wattenburg)

A binary subtractor circuit constructed with tunnel diodes as bistable elements is proposed. Instead of the familiar AND and OR operations of ordinary Boolean algebra, a new, more basic operation shall be used for the circuit. This operation is called the majority decision operation and includes, as a special case, the operations mentioned above.

It is thought that an inexpensive and compact unit may be realized that is capable of operation with clock frequencies of 1 mc or higher.

01-1-67 LOGICAL DESIGN. A. Miyoshi (Prof. Wattenburg)

Various logical designs have been considered for a high-speed parallel digital comparator (a circuit which will determine whether a number lies between two preassigned limits). The circuit is of primary interest in high-speed data acquisition systems which require fast and efficient digital filters. Designs are being implemented which should process 10^6 to 10^7 samples per second.

01-1-68 SELF-ORGANIZING SYSTEMS. G.D. Hornbuckle (Prof. Wattenburg)

Self-organizing techniques are being studied on a discrete-state system designed to recognize certain input variables. The jth input, u_j, consists of a finite integer which is transformed to a single output, y_j, where

$$y_j = \begin{cases} 1 \text{ if } \Sigma r_i \leq k \\ 0 \text{ if } \Sigma r_i > k \end{cases} \quad \text{for } r_i = u_j \bmod M_i, \; i = 1, 2, \ldots, m,$$

k is a constant. In the learning mode, the correct response is known for each input and the internal state, $S = (M_1, M_2, \ldots M_m, m)$ is adjusted to give the correct output via an internal feedback sub-system. Of major interest is the problem of determining a) how to most efficiently choose S while b) minimizing m and c) maximizing the number of inputs for which the system will correctly respond.

It has been determined that for all cases tried, a state S exists which will cause the system to correctly respond to a set of integers. Of course, m increases with the number of inputs desired. For these reasons the system is thought to provide a good model on which to study self-organizing techniques.

MICROWAVE ELECTRONICS AND PLASMAS

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01-1-43 STAGGER-TUNED CAVITY CHAIN BEAM AMPLIFIER.
Y. Satoda (Prof. Bevensee)

The equations for an N-cavity chain interacting with a beam passing through the centerholes, coupled to the generator and passive load by means of loops, have been written and programmed for the IBM 704 computer. A mistake in the program is now corrected so that we can analyze the effects upon gain and bandwidth of staggering the cavity parameters. The basic equations for this work, together with preliminary results, are contained in an E. E. 299 report by Y. Satoda. Efficient stagger-tuned chains will be discussed presently.

01-1-44 EQUIVALENT CIRCUITS FOR WAVEGUIDE OBSTACLES.
D. Tuma (Prof. Bevensee)

The equivalent shunt susceptance of a round hole in a rectangular waveguide is being closely examined from the variational point of view to see if one can simplify Marcuvitz' expression for this susceptance.

01-1-17 DISPERSION IN SPECIAL SLOW WAVE STRUCTURES.
(Prof. Bevensee)

A perturbation formula for general helical structures has been developed from the variational expression quoted in the Consolidated Quarterly Progress Report No. 2, August 15, 1961. This perturbation formula should help one to modify the dispersion curves of helices in the same way that Slater's perturbation formula serves for resonant cavities. The details of this work will be discussed presently.

02/13-9-01 FAST-WAVE INTERACTION. L. Haas (Prof. Birdsall)

Objective: Investigation of microwave generation and amplification by fast-wave interaction using no magnetic field.

A tube utilizing double-period focusing has been constructed such that the stream crosses the slit in the ridged waveguide four times. It is called a double-period focus device because the transit time dispersion is large between succeeding interactions but is small between every other interaction. Spatial focusing also occurs in two periods. The tube has not yet been operated with good stream transmission because the operating voltages required appear to cause the gun electrodes to give off some material which poisons the cathode. However, the cathode emission is improving with aging and the rate of poisoning is decreasing. With the low stream currents obtained so far, approximately 300 microamperes, only very small stream-circuit interaction has been obtained.

02/13-0-11 FAST-WAVE INTERACTION. R.E. Lundgren (Prof. Birdsall)

Objective: The generation and amplification of microwave signals by devices using fast-wave interaction is under investigation.

Analysis: We are currently studying the interaction mechanism of the helical-beam device described in the Consolidated Quarterly Progress Report No. 1. The previously reported efficiency of the device, from calculations based on the simple model, was incorrect. The indicated efficiency is actually about 5%.

A second set of calculations was made using a different model. This new model assumed that the interaction region consisted of three sections. In the two outer sections the rf field was purely radial, whereas that of the middle section was purely azimuthal. Further, it was assumed that the magnitude of the field was the same in all three sections and independent of radial position. Calculations based on this model produced results not much different from those of the first model. In this case an efficiency of about 4% was indicated for a tube having 17 beam-wave interactions.

A third set of calculations was made for a tube in which the waveguide and interaction gap follow a contrawound helical path, wound such that beam-wave interactions occur at 254° intervals of azimuthal electron travel. This angle was chosen since it is the focal angle for the electrostatically focused cylindrical system. In this case the indicated efficiency was 5% for 20 beam-wave interactions. However, with this scheme the small-signal conditions were not fully met in that the ratio of amplitude of radial oscillation to equilibrium radius became appreciable, as large as 16% in one case.

13-1-01 ELECTRON STREAM INSTABILITIES IN DRIFT TUBES OF FINITE LENGTH. J. Frey (Prof. Birdsall)

Objective: To study instabilities in electron (or ion) streams in drift tubes, initially to obtain limiting current for a drift tube of finite length.

This study was undertaken to ascertain the static and dynamic behavior of an electron stream injected into a drift tube which has gridded ends. The grids will tend to maintain the potential throughout the stream at a level somewhat above that given by the analytical solution to the problem without ends, and thus to raise the maximum value of current that can be passed through the tube. Also, the model is somewhat more realistic than the one without ends, which implies infinite length.

The potential problem has been programmed and run with various injected currents on a digital computer. For small currents the results agree with the classical results of Haeff (Proc. IRE, December 1939) for the infinite drift tube. For largercurrents the results do not agree; it appears that the potential formulation does not allow rapid convergence so that the initial formulation must be done over.

Future work will include further solutions of the two-dimensional problem, for varying length-to-width ratios and beam widths. It is expected that the work will also be extended to cylindrical drift tubes containing solid and annular beams, and to streams with ion backgrounds.

02-1-37 EXPERIMENTAL INVESTIGATION OF MICROWAVE PROPAGATION IN A PLASMAGUIDE. R.N. Carlile (Prof. Everhart)

Objective: To determine experimentally the phase characteristics and field configurations of the modes of propagation in a cylindrical waveguide partially filled with an ion-neutralized plasma in the presence of a finite constant magnetic field parallel to the waveguide axis (plasmaguide). The modes to be investigated are those which have 1) no azimuthal variation; 2) have an azimuthal variation of period 2π.

Experimental results: Initial measurements of the $n = 0$ forward wave mode at zero magnetic field shows that for discharge currents of .6 amps, the plasma frequency is about 3.5 Gc. The pressure of the Hg vapor is 1 micron. This is a sufficiently high plasma frequency for most projected measurements. These measurements showed the $n = 0$ mode to have the characteristic shape predicted by Trivelpiece. As Trivelpiece found, the upper cut-off is much lower than the theoretical value of $f_p/\sqrt{2}$ (=2.5 Gc), namely about 1.1 Gc.

In addition to the passband due to the $n = 0$ mode, other sharply defined non-overlapping passbands exist between 1.1 and 2.0 Gc (no measurements were made above 2.0 Gc). The nature of these passbands could not be investigated because of tube failures which have persisted until the present time (these failures are not fundamental in nature). The zero temperature model used by Trivelpiece shows passbands corresponding to $n = 1, 2$, etc., modes but they all have a common upper cut-off, namely, $f_p/\sqrt{2}$, and are all, therefore, overlapping.

Measurement of propagation constant: A method of measuring the propagation constant additional to the resonant plasmacavity method discussed in previous progress reports is hereby proposed. This method which consists of the probing of the fields of a wave traveling in a terminated plasmaguide, has the advantage that the experimental apparatus is much simpler since there are no moving parts inside the vacuum as there would be with the plasmacavity. The plasmacavity provides mode selectivity. This may be achieved in a traveling wave system by employing two devices:

1) Selective coupler. By placing identical loop exciting probes (each of which is theoretically capable of exciting any mode) 180° apart spacially but at the same radius and exciting them in phase, they will cause constructive interference of all $n = 0$ modes and destructive interference of all odd numbered modes. Similarly, two identical in-phase probes 90° apart spacially will cause destructive interference of $n = 2$

modes. We conclude that four identical in-phase probes each spaced from the next by 90° will allow the $n = 0$ to exist but destroy all other modes between $n = +3$ and $n = -3$. Similarly, by phasing these four probes so that the probe at $\phi = 90^\circ$ leads the probe at $\phi = 0$ by 90° phase angle, etc., a $n = 1$ mode enjoys constructive interference and the other modes in the range $n = +4$ to $n = -3$ are destroyed. This coupler then allows modes with only a predetermined azimuthal mode number to propagate.

2) Analysis of data. Even using the above coupler, modes with different radial mode numbers may propagate simultaneously. It can be shown that if n such modes coexist, if one probes the fields longitudinally (+z direction) and measures complex amplitudes at 2n equally spaced points, the spacing being L, then the n propagations constants, $\gamma_1 \ldots \gamma_n$, may be determined from

$$e^{-ni\gamma L} + C_n e^{-(n-1)i\gamma L} + \ldots + C_2 e^{-i\gamma L} + C_1 = 0,$$

$$\begin{pmatrix} C_1 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{pmatrix} = (-1) \begin{pmatrix} A_1 & A_2 & \ldots & A_n \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ A_n & A_{n+1} & \ldots & A_{2n-1} \end{pmatrix}^{-1} \begin{pmatrix} A_{n+1} \\ \cdot \\ \cdot \\ \cdot \\ A_{2n} \end{pmatrix},$$

where $A_1 \ldots A_{2n}$ are the measured complex amplitudes.

02/11-1-01 PLASMA MAGNETRON. M. Chamran, Y. Ikeda (Profs. Sloan and Susskind)

Objective: To study the diffusion of charged particles across magnetic flux lines, when aided by electric fields.

This process is at the heart of fusion containment and MHD generator efficiency, at one end of the volt-ampere characteristic curves. At the other end of the curves, magnetron behavior occurs. The intermediate region has been explored for several years, on this project, with the stated objective emphasizing the magnetron end of the curves, which is understood better.

We have succeeded in operating over almost the entire range of phenomena of the desired curves with a single probe during this quarterly report period. In place of a plasma, water cooled magnesium, slightly oxidized, served as the source of electrons by secondary emission.

Diagnosis of the edge of the plasma can be done in an electron sheath between metal surfaces such as in a smooth bore gaseous magnetron. Charge wavelengths and field frequencies are most readily accessible by magnetron interaction techniques. Motion of ions during a cycle is negligible, and will be freely allowed only after processes in a rigid geometry are understood.

Probes which measure n wavelengths of turbulent charge with a drift velocity producing those frequencies which will pass the filter are being studied as a function of cyclotron frequency and plasma frequency.

The properties of interest are described by Hartman's equations for the turbulent diffusion current and drift current. These currents are perpendicular, and their ratio should be very large for MHD and fusion, but should not be much larger than unity for magnetrons.

An extensive survey of frequency response of probes, using methods of microwave tube slow wave structure design, indicates that our old narrow band probes for analyzing the spectrum, point by point, will couple to the turbulence far better than a broad band probe needed for observing simultaneously all frequencies on a spectrum analyzer.

Further improvement in narrow band probes appears probable. After finding an optimum design, a group of them should be built, with perhaps four turbulent charge wavelengths accommodated within each, for one passband, or vice versa.

Cross plots from these families of curves will relate the intensity of turbulent diffusion to cyclotron frequency, turbulent wavelength and sheath thickness.

The new magnet will be limited to 10,000 gauss with cooling by city water. In this field the secondary emission surface will be replaced by a true neutral plasma, with ions traveling on radii too small to reach the cathode surface directly. It is expected that PIG discharges can be made harmless by use of cathode surfaces with secondary emission ratio less than unity.

02/11-1-03 DOUBLE-STREAM CYCLOTRON WAVE INTERACTIONS.
B.J. Maxum (Prof. Trivelpiece)

Objective: The objective of this project is the study of the interactions between drifting electron streams in a finite magnetic field.

A final report on the theoretical and computational aspects of this project is in progress. This will be ERL Report Series No. 60, Issue No. 429, "Cyclotron Wave Instabilities," January 8, 1962.

02/13-1-03 INTERACTION OF A DRIFTING ELECTRON STREAM WITH THE BACKWARD-WAVE MODE OF PROPAGATION IN A FERRITE ROD.
J. Spector (Prof. Trivelpiece)

Objective: To investigate theoretically and experimentally the interaction between a drifting electron stream and the magnetostatic modes in longitudinally magnetized ferrite.

The experimental tube is now being reassembled. Two changes have been made with respect to the previous experiment. First, the ferrite is being coated with germanium so as to provide a low resistance leakage path for intercepted electrons to be conducted away. Second, a new coaxial feed through is being used. This should eliminate much of the rf leakage which has plagued previous experiments.

A technical report is in preparation.

02/11-1-04 INVESTIGATION OF A BEAM PRODUCED PLASMA. C.E. Kuivinen (Prof. Trivelpiece)

A final report is in preparation and will be available in the near future.

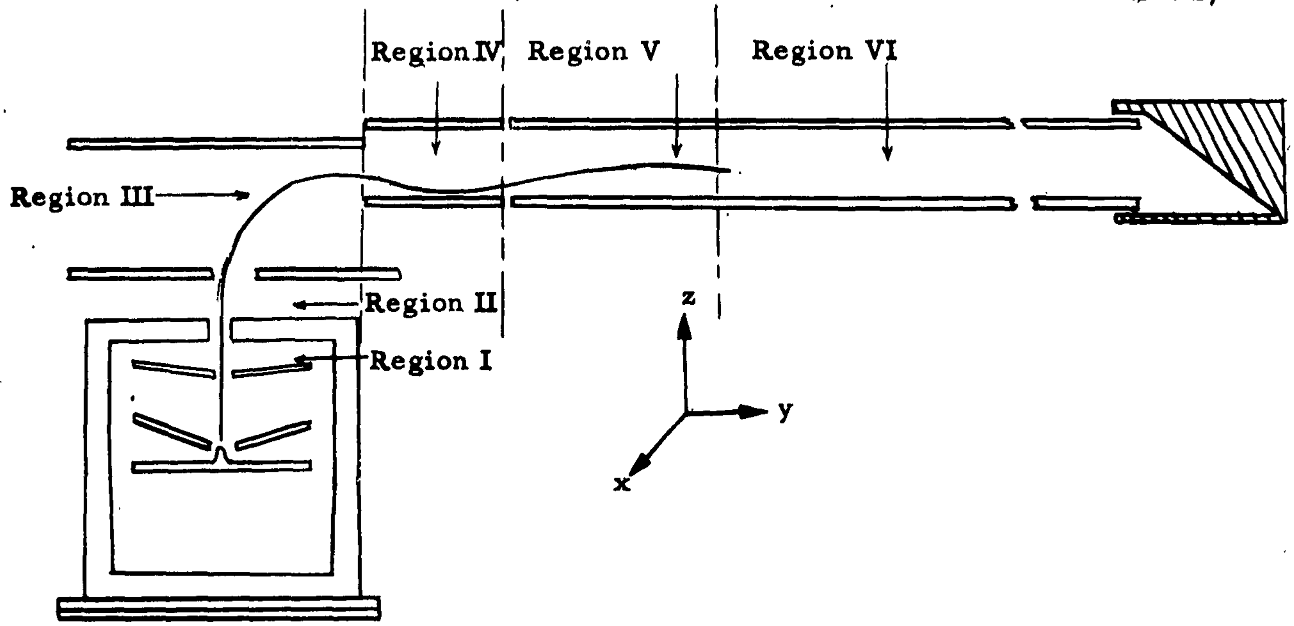
22-1-01 SHIELDED-GUN CROSSED-FIELD AMPLIFIER. R.A. Rao (Prof. Van Duzer)

The dc model reported in the Consolidated Quarterly Progress Report No. 3 developed a leak in one of its vacuum seals before all the anticipated data could be obtained. However, some measurements were made and are reported here. The electron gun worked according to design. At a heater voltage of 7.0 volts and an anode voltage of 600 volts, the beam current was 35 ma. The beam was observed by letting in a small amount of hydrogen through the palladium leak. The beam appeared well formed before it entered the interaction region.

Now the tube is in the shop being reprocessed. Meanwhile, the trajectory of an electron at the center of the beam was calculated for the more realistic situation including the effect of fringing electric fields near the entrance to the interaction region. For the purpose of trajectory computation, the tube is divided into six regions.

Region I:	$B \equiv 0$; E is along negative Z and is constant
Region II:	B is nonuniform; $E \equiv 0$
Regions III & IV:	Both B and E are nonuniform
Region V:	B is nonuniform; E = constant and along negative Z
Region VI:	Both B and E are uniform

The electron trajectory in regions I through V was computed using the Bendix G-15 computer and is shown in the figure below. The modified tube is expected to be ready in three weeks.



Electron Trajectory in Shielded-Gun Tube

22-1-03 NOISE FIGURE MEASUREMENTS. M.N. Raju (Prof. Van Duzer)

The suitability for measuring the noise figures in the region of 30 db has been studied using 1) the noise generator method and 2) the signal generator method. The signal generator method has been found to be superior for this range. Rando's expression for the noise figure of forward and backward wave amplifiers showed that the noise figure is directly proportional to the factor F which depends on the position of the beam relative to the sole. F increases with distance from the circuit. We propose to verify this experimentally. For this purpose the beam must be capable of being shifted in the sole circuit space without being defocused. One obvious way of doing this is to move the entire "Kino" gun (which includes the cathode, beam-forming electrodes and the anode) relative to the fixed sole-circuit system. In doing this, it is to be kept in mind that for successful operation of the "Kino" gun two conditions are to be satisfied: 1) From the "Kino" design values the beam would leave at a certain potential in the gun region. The sole-circuit voltage must be such that the beam may not see a jump in the potential at the exit plane; 2) The electric field in the sole-circuit space should be twice its value at the exit plane in the gun region. It also is desirable that a few equipotential lines immediately above and below the beam be as straight as possible at the gun exit. In order to see that these requirements are satisfied, it is planned to construct an electrolytic-tank model of the various electrodes to give the information about the voltages to be applied to the various electrodes for several beam positions.

In addition, the following noise figure measurements are planned.
1. To study the noise figure variation with the magnetic field B and the anode potential ϕ_A for constant value of ϕ_A/B^2 (this assures a well focused beam) in the limited region where the equipotential lines are parallel to the one corresponding to the "Kino" design value.
2. Biasing the cathode with respect to the beam forming electrodes would change the cathode current without changing the shape or transmission characteristics of the beam appreciably. The noise figure will be measured for this case.

22-1-02 CROSSED-FIELD NOISE IN THE LOW-VELOCITY REGION. M.A. Pollack (Prof. Whinnery)

The objective of this project is the study of the effects of the low-velocity region, and the potential minimum in particular, on noise transport in the crossed-field diode.

During the past quarter a computer program to evaluate self- and cross-power density spectra of current and velocity fluctuations was completed and debugged. It will be used to calculate the spectra of fluctuating quantities at the cathode, potential minimum and anode of the diode being simulated using the Monte Carlo method. Initial checks have been made to compare the results with those obtained previously by others for the zero magnetic field case, and close agreement has been found.

The experimental, crossed-field, space-charge-limited triode to be used for noise measurements was completed. However, before it could be tested, the glass envelope was fractured, distorting the internal structure. The remainder of the experimental set-up, including a shielded, filtered enclosure, has been assembled.

In the next quarter computations to determine the effect of various magnetic field intensities on the fluctuating quantities will be performed using the Monte Carlo computer programs. The experimental tube will be rebuilt, and noise measurements will be made to study the propagation of fluctuations.

22/13-0-03 PLASMA MACHINE PROJECT. D.B. Cummings (Profs. Birdsall, Colgate and Trivelpiece)

Objective: To construct a magnetic mirror configuration to contain a high-temperature plasma (ion energies on the order of 1000 ev) for a useful time; to perform experiments with this hot plasma which will increase knowledge of stability of containment means, of plasma compression, heating and transport means, of plasma dynamical behavior and of energy recovery means.

A. Vacuum: The epoxy-fibreglas vacuum chamber was baked for nearly two months at 100°C with an ultimate pressure of about 2×10^{-4} Torr while hot. The limitation appeared to be the pumping speed of the trap.

A new liquid-nitrogen trap with a 4-inch tubulation was built and installed. An automatic trap filler was built and tested. Baking has been resumed with a new limit of $2\text{-}3 \times 10^{-6}$ Torr while hot.

The modified high vacuum system diffusion pump is finished. Using polyphenyl ether fluid and a "nude" ionization gauge the results are as follows:

Trap Temperature (°C)	Pressure (Torr)
20°	3×10^{-6}
-20°	3×10^{-7}
-100°	5×10^{-9}

This confirms the notion that the base pressure is limited by heavy hydrocarbon fractions from cracked oil rather than light gases. Meanwhile work is continuing at the Lawrence Radiation Laboratory to see what materials can be used to inhibit cracking of the oil.

B. Electronics: The capacitor rack is installed and lights mounted. The energy storage capacitors were tested and installed. The cable troughs for both control and high power pulse cables were installed. The remainder of the equipment racks were put in place and ground cables connected. The majority of the control chassis and sub-equipments have been made, installed and wired in. The ignitron tube mounts are almost finished.

Most of the microwave and counting systems equipment is on order and much of it has been received. Planning is continuing on diagnostic experiments and equipment. A prototype time delay generator is made. It is an improved version of a unit developed at the Lawrence Radiation Laboratory. It will be used for programming the pulsed sequence of the multistage capacitor bank and the diagnostics systems.

C. Mirror coils: (L.J. Demeter) A single-layer model coil of the final stage compression coil set was made in order to measure the magnetic field distribution. Based on these measurements, drawings of a preliminary coil design have been made. A paper mock-up of the coil construction has been prepared. Stress calculations have been started. A series of experiments is being prepared in order to design the transfer coil sets.

D. Plasma sources: (J. Spector) Two phases of this work are under way. First, a source and its pulsing circuits have been designed and partially built for initial operation of the plasma experiment. Second, a source test stand is under construction in order to learn more about the source in use and to develop the theory of operation of this type of device. This should in turn lead to improved sources. Work is continuing on both the source theory and source diagnostic experiments.

02/11-0-05 INTRINSIC OSCILLATIONS IN GASEOUS CONDUCTORS.
B.E. Dobratz (Prof. Woodyard)

Work of this quarter was interrupted by the necessity of replacing a faulty ion pump. Post-repair work was initiated by a lengthy pump-down of the ultrapure hydrogen device to the pressure range of a few millimicrons of mercury.

Introduction of ultrapure hydrogen at pressures of 10 to 100 microns frequently produced the phenomenon of very sharply defined stationary striations. The occurrence of these stationary striations was always noted to be accompanied by the lack of any oscillatory effects and a marked reduction of noise. Signal pickups employed were an electrostatic pickup on the outside of the glass tube and a collimated photomultiplier. Low-frequency oscillations (approximately one kilocycle) were found only when the positive column of the discharge appeared uniform or slightly striated.

Electrical schemes of phase measurement of an oscillation signal from the tube anode and the signal pickups were partially defeated by the time jitter of the phenomena. Currently, a rotating mirror assembly is under construction and will be used in the coming quarter's work. It is hoped that the use of the rotating mirror will give accurate measurements of the velocities of the moving striations, which are suggested by the partial results of phase measurement attempts.

RADIATION AND PROPAGATION

RADIATION AND PROPAGATION

01-1-46 THE RESONANT SLOT ANTENNA. J. Baker (Prof. Bevensee)

In the initial investigation of a longitudinally-slotted rectangular waveguide, a waveguide magnetic field was assumed and the slot voltage was derived from transmission line equations for the slot. The waveguide face was assumed to extend to infinity. Using the solution for the slot voltage, the radiation pattern was calculated and it was noted that the pattern is similar to that of an elementary dipole antenna.

02-1-42 STUDY OF FREQUENCY-INDEPENDENT ANTENNAS. M. Gans (Prof. Rumsey)

Objective: To develop frequency-independent antennas which have any prescribed radiation pattern, polarization and impedance.

Construction of a self complementary plane sheet of sinusoidal wires was previously undertaken. The purpose of this construction is described below. A method of deriving the fields radiated by a finite source in the presence of the sinusoidal wire surface was investigated. The method consists of expanding the true field in terms of the homogeneous solutions previously obtained by Professor V. H. Rumsey.

Theoretical investigation of waves propagating over a plane sheet of sinusoidal wires, when excited with uniform phase along the ordinate of the sinusoids, has indicated that, at the proper frequency, the waves will be attenuated at the high rate of 3.52 nepers per wavelength of the sinusoidal wires.

In order to check the properties of the structure, an experiment is being constructed using a thin sheet of teflon (0.01 in. thick) with parallel closely spaced, sinusoidal strips photoengraved on one side. One edge of this sheet will be placed in a transverse slot in the top of a parabolic pillbox antenna, with an absorber placed in the aperture of the antenna, so that the sheet will be excited with uniform phase. The amplitude, phase and polarization of the field will then be measured with respect to position along the sheet.

02-1-36 SURFACE WAVE INVESTIGATION. D. E. Norton (Prof. Rumsey)

This project deals with the electromagnetic field due to a line magnetic current over a grounded dielectric slab which is placed between two parallel conducting planes. The reader is referred to page 22 of the previous report for the introduction to the problem. The entire field can be expressed in terms of a two component electric vector potential, **F**.

$$\vec{F} = \hat{a}_x f(x,z)\sin\left(\frac{\pi}{2a}y\right) + \hat{a}_y g(x,z)\cos\left(\frac{\pi}{2a}y\right). \tag{1}$$

The y-component of the electric field can be expressed in terms of F_x only. For $x > 0$, E_y is given by

$$E_y = \frac{1}{\epsilon_o} \frac{\partial f(x, z)}{\partial z} \sin(\frac{\pi}{2a} y). \qquad (2)$$

$f(x, z)$ is the solution of equations (4) through (7) of the previous report. One method for solving those equations is to apply the Fourier cosine transform over the z variable to those equations. When this is done the transforms of f can be found as a function of x, and γ, the transform variable. The inverse transform is then expressed as an integral, in terms of which E_y is given by

$$E_y = \frac{E_o}{2a} (K-1) \int_0^{\infty} \frac{\gamma e^{-jh(x+b)} \sin \gamma z \, d\gamma}{(jh + \ell \cot \ell t)(jKh - \ell \tan \ell t)} \sin(\frac{\pi}{2a} y). \qquad (3)$$

h and ℓ are defined by

$$h^2 = k_o^2 - \gamma^2 - (\frac{\pi}{2a})^2$$
$$\ell^2 = k^2 - \gamma^2 - (\frac{\pi}{2a})^2. \qquad (4)$$

The integration of this expression is difficult. It has been found, however, that it may be treated approximately by shifting the contour of integration and using the method of steepest descents.

02-1-39 FAR FIELDS FROM NEAR ZONE MEASUREMENTS. V. Galindo (Prof. Rumsey)

Objective: To investigate a proposed method for directly measuring the far field pattern of an antenna in the near zone of the antenna.

The method utilizes the $(\frac{1}{r})^n$ Sommerfeld expansion for the field by truncating the expansion after a suitable approximation and inverting a linear set of such equations to determine the far field, the coefficient of the $(\frac{1}{r})$ term. The set of equations is formed by a set of measurements taken along a radial line from the antenna.

An upper bound to the error of truncation was found so that for any specified antenna size and distance from the antenna the number of terms, N, necessary for a specified accuracy could be determined.

The inversion for an arbitrary number of linear equations, N, of the type obtained here was accomplished exactly so the magnitude of N was not an inhibiting factor. Furthermore, it was shown possible to construct an antenna array which would simultaneously perform the necessary measurements and compute the far field (invert the linear set) directly in both phase and amplitude.

The possible extrapolation errors inherent in the measuring method were investigated and found to be prohibitively large. This may be explained as follows:

If H_ϕ = measured field column matrix = $L_N a$ where L_N = NxN matrix and a = column matrix of coefficients of $(\frac{1}{r})^n$ expansion, then $a = L_N^{-1} H_\phi$, and for a set of errors in H_ϕ, H_ϕ^ϵ, the error in a, $a^\epsilon = L_N^{-1} H_\phi^\epsilon$. When appropriately normalized, the error in the first element of a^ϵ, $a_{o\epsilon}$, the far field, is found approximately as

$$|a_{o\epsilon}| = \left| \sum_{p=0}^{N-1} \frac{(\frac{R_o}{\Delta r} + p)^{N+1}}{(N-1-p)!\,p!} H_{\phi p}^\epsilon \right|$$

where R_o = distance of the first measurement from the antenna and where Δr = the spacing between measurements of H_ϕ (spacing between probes of the array). It is a straightforward procedure to show that if a normal distribution of errors for H_ϕ^ϵ (with uniformly random phase) is assigned, then the variance of $|a_{o\epsilon}|$ is prohibitively larger than any reasonably assigned value to the $H_{\phi p}^\epsilon$. That is, where the probability for very large errors in the measurements of $H_{\phi p}^\epsilon$ may be kept very small, the probability for the same error in $a_{o\epsilon}$ is very large. Specifically, for a particular practical case, this amplification of errors increases quite rapidly with N as follows:

N = 1 -- Amplification = 1
N = 2 -- Amplification ≈ 2
N = 3 -- Amplification ≈ 12
N = 4 -- Amplification ≈ 130.

In practice much larger N would be required.

In view of the prohibitive extrapolation errors, the measurements method is impractical as it is presently proposed. Because of the importance of this problem, however, some further effort will be directed towards investigating possible modifications of the present scheme of measurements to obtain a practical method.

02-1-11 ELECTROMAGNETIC SCATTERING FROM FERRITE CYLINDERS.
C. Fong (Prof. Welch)

The results of this research have been written up in an ERL Report, Series No. 60, Issue No. 433.

SOLID STATE

SOLID STATE

32-9-02 OSCILLISTOR PHENOMENA. S. Kakihana (Prof. English)

Observations of oscillistor phenomena were made on several samples of varying dimensions and of varying degree of doping. Generally it is noted that the oscillation frequency and amplitude increase as the dimensions of the sample are made smaller. The most interesting observations were made with the smallest sample (1.0 mm x 0.85 mm x 6.0 mm). The results observed are quite different from the previously reported results.

Ivanov and Ryvkin (Soviet Physics - Technical Physics, Vol. 3, No. 4) report that there exists a threshold value of the applied electric field and the magnetic field, and once the oscillation sets in, the amplitude as well as the frequency of oscillation increase monotonically (almost linearly) with the above mentioned quantities.

We find that there exist threshold values of both magnetic field and electric field but the general trend of the amplitude and frequency of oscillation plotted against the longitudinal magnetic field is far from a monotonic increase with the magnetic field. The curve of frequency vs. the magnetic field for a fixed value of the applied voltage shows two maxima and minima indicating that there are more than one mode of oscillation. At the minimum point of the frequency vs. magnetic field curve the signal was seemingly incoherent which may very well be due to the instability of the applied voltage and the magnetic field.

The separation of the two maxima of the frequency vs. magnetic field curve was greater for a smaller applied voltage. The amplitude of oscillation plotted against the magnetic field showed a similar trend. Both the amplitude and frequency of oscillation are larger for a given value of the magnetic field when the applied voltage is higher.

02-1-26 ANOMALOUS PHOTOVOLTAIC EFFECT IN ZnS (APE). F. Junga (Prof. English)

Attempts were made this past quarter to introduce additional impurities into crystals of ZnS. Two methods were tried. Firstly, a number of crystals were irradiated with Co^{60} gamma-rays (energy about 1.25 Mev). This would not, of course, introduce impurities, per se, but would produce vacancies and interstitials, which, under the proper conditions may act like impurities. The spectral response of the APE was measured before and after irradiation. Unfortunately no consistency in the effects of irradiation on different crystals was found (dosages 10^5 roentgens and 10^6 roentgens).

The second method used to introduce impurities was a diffusion technique. Crystals were placed in contact with the doping agent (AgS) and heated to and held at 450° C for about 24 hours. This method, too, proved fruitless since it was not possible to separate the effect of the addition of impurities from the effects of the accompanying heat treatment.

32-1-03 SELENIUM BREAKDOWN PHENOMENA. H. Power (Prof. English)

To tracethe development of hot-shots of molten selenium, which lead to the stabilization mechanism, a rectifier was operated under breakdown conditions, for controlled lines, and the transient current observed when switched to a low voltage supply. Electronics switching was used to minimize the delay in changeover. Examination of the resultant decaying transient showed that the initial value, I_s, could be related to its maximum observed value, I_o, and to the line, t, under stabilized breakdown by a double exponential relation:

$$I_s = I_o - A e^{-be^{ct}},$$

A, b and c being constants.
However, due to finite discharge time of the selenium rectifier capacitance, switching was being delayed for periods varying from 60 to 300 microseconds. The delay was a decreasing function of time in stabilized breakdown, consistent with greater bending of the rectifier leading to lower shunting resistance. Since significant compositional changes may occur in the rectifier during this period, a circuit has been devised to reduce it as much as possible. It is hoped to construct this shortly.

Continued variation in I_s and the delay time was noted for periods of stabilized breakdown long enough that detectable creep had ceased before switch over. This would indicate that changes in the rectifier condition continue after their visible manifestation has ended.

02-1-15 MICROWAVE MODULATION OF LIGHT. G. C. Alexander (Prof. Singer)

This project has as its objective the continuous wave or pulse-code modulation of light by electrical means. The physical phenomenon to be used is the variation of optical Faraday rotation in a rare earth salt when pumped by appropriate microwave power.

When a rare earth ion is imbedded in an asymmetrical crystal field, the degenerate ground state is split into doublets separated in energy which corresponds to optical frequencies. Cooling the rare-earth-doped crystal to liquid hydrogen or helium temperatures depopulates all but the lowest doublet of the paramagnetic rare earth ion. At such low temperatures, only one strong optical absorption band is obtained, which corresponds to electric dipole transitions for the paramagnetic ions.

Application of a constant magnetic field to the crystal results in a splitting of the doublets upon which the optical transitions terminate as sketched in Figure 1 below. With the paramagnetic ions under consideration, ω_1 and ω_2 result from changes in angular momentum of opposite sign. Hence, ω_1 and ω_2 correspond to absorption of opposite types of circularly polarized light.

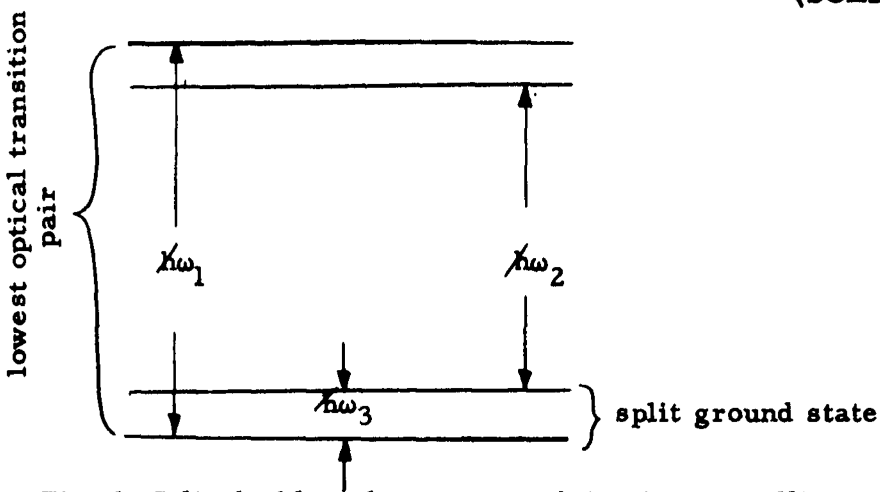


Fig. 1 Split doublets for rare earth ion in crystalline and external magnetic fields

Figure 2, below, shows the absorption and dispersion curves for the energy level scheme of Figure 1. ω_1 is assumed sensitive to right circularly polarized light for explanation purposes. The difference in amplitude of the curves for ω_1 and ω_2 is caused by the difference in population of the two ground state levels in thermal equilibrium. Away from the absorption frequencies, a rotation of the plane of polarization of linearly polarized light is obtained because of the different propagation constants (dispension values) for right and left hand circularly polarized light.

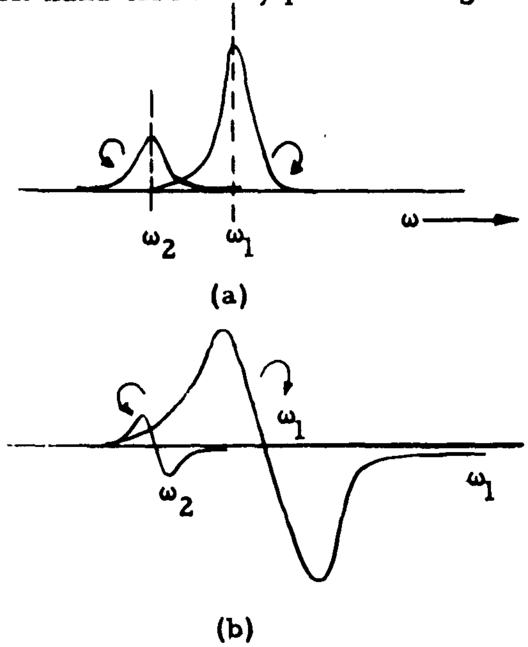


Fig. 2 (a) Relative absorption curves for energy level scheme of Fig. 1;
(b) Relative dispersion curves for energy level scheme of Fig. 1.

Pumping with microwave power tends to equalize the population of the two ground state levels, thereby reducing the Faraday rotation produced by the paramagnetic ions in the crystal.

The project is now at the point of determining the maximum frequency at which the relative population levels can be made to follow variations in microwave power levels for neodymium ethylsulfate. No data has yet been obtained.

Pulse code systems in which series of pi-pulses are used to control the Faraday rotation are under consideration as are different types of rare earth ions and host crystals.

02-1-22 OPTICAL MASER BY THE METHOD OF ELECTRON EXCITATION. L. Lin (Prof. Singer)

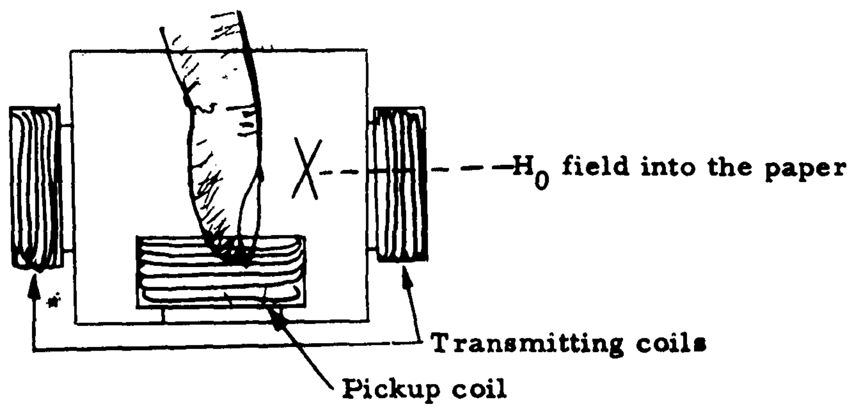
The oscillation of an optical maser must result from the amplification of electromagnetic waves in an "active medium", in which there is a population inversion or negative temperature. In this research we are investigating the possibility of using gases or vapors excited by a stream of electrons as active media.

A major difficulty in our work is to obtain a large stream of electrons of desired energy. The work in this period has been devoted to the construction and testing of an electron source for this purpose. The first model of this electron source is a cylindrical thermionic cathode of 2" dia. x 5" long, inside which are placed grids and an anode to accelerate and collect the electrons. At present the cathode and its power supply are being tested.

A Fahry-Perot type resonator, in which the maser oscillation is to take place, is also being constructed and is near completion.

02-1-35 NUCLEAR MAGNETIC RESONANCE BLOOD FLOW METER. S. Levine (Prof. Singer)

This quarter was spent on the design and testing of the detection system described in the last report. The system is shown in the figure.

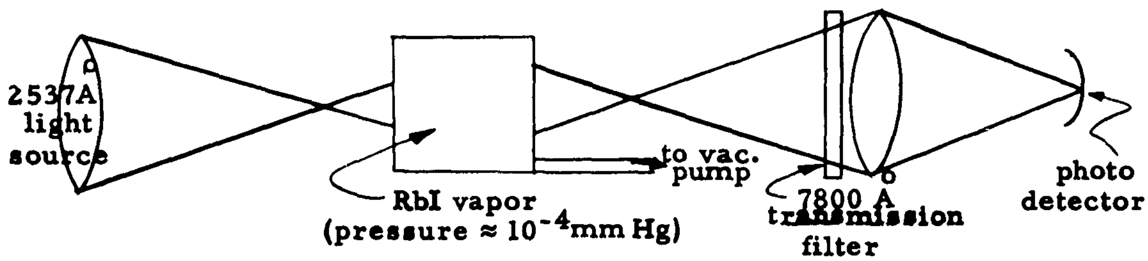


Two units were constructed. The first did not perform as was expected. The reason was that the coefficient of coupling between the transmitting and receiving coils was far too large. This induced voltage, when added to the signal voltage, produced an output signal with an extremely low percentage amplitude modulation. This signal was too weak to be detected by the receiver.

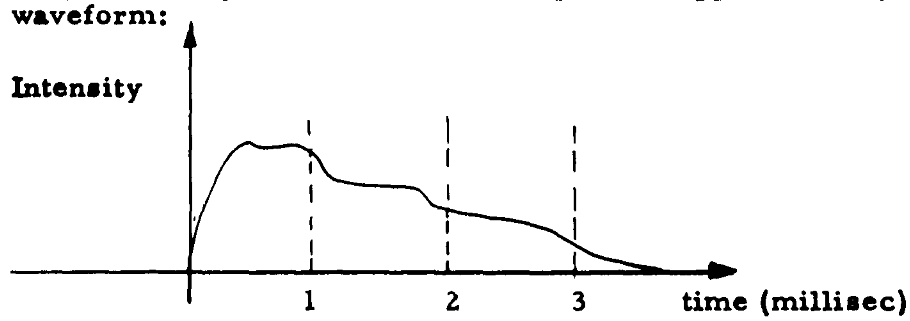
A second unit was constructed and is now under test. The first tests of this improved unit were successful. NMR signals were observed at 3.35 mc. We were not able to hold the signal stable because our power supply would not regulate at the low current we were using. We are now rebuilding the unit to operate at about 20 mc.

24-1-01 RbI OPTICAL MASER. C. W. Hudelson (Prof. Singer)

Experiments are continuing in an attempt to measure the energy of coherent optical emission from $Rb^1(5p\,^2P_{3/2} \longrightarrow 5s\,^2S_{1/2})$.



A cell containing Rubidium Iodide is heated in a furnace to approximately 1000°K where RbI vapor is formed. The pressure of the RbI vapor is lowered to approximately 10^{-4} mm Hg. The 2537 Å light source is then pulsed. The pulse of light exciting the RbI vapor has approximately the following waveform:



as measured by a 7201 photomultiplier tube connected to an oscilloscope.

A short time after the u.v. has dissociated (10^{-8} sec.) the Rubidium Iodide there will be two fluorescent lines corresponding to the $Rb^1(5p \rightarrow 5s)$ transition. $Rb^1(5p^2P_{3/2} \rightarrow 5s^2S_{1/2})$ @ 7800 Å and $Rb^1(5p^2P_{1/2} \rightarrow 5s^2S_{1/2})$ @ 7948 Å. Due to the broadness of our filtering system it is expected that both of these lines will be present at the photo detecter, although the 7800 Å has been calculated to be two times the intensity of the latter.

Once this emission has been detected, the calibration and calculation of its absolute intensity remains.

02/24-1-02 ELECTRIC DIPOLE MOMENTS. A. Pine (Prof. Singer)

Optical dipole inversion in ruby: Preliminary investigations into the feasibility of inverting electric dipole-linked states in solids by the use of optical masers indicate certain restrictions. The photon yield from presently available optical masers is too small to totally invert populations in a typical sized crystal. Note that a 1 cm^3 pink ruby (0.05% Cr) has about 10^{18} - 10^{19} Cr^{3+} ions in the ground state. To invert this, one would require about three joules of energy, which is 10^{19} photons at the required R_1 line transition. This energy must be available in a few spikes of the primary laser output so that the relaxation mechanism does not strongly affect the population in the secondary laser. Typically, available primary sources give only 10^{-3} joules in the required time.

Three alternatives present themselves: 1) One can achieve higher input power by operating the primary source in a pulsed reflector mode. This has given the required intensity by population inversion on a three level scheme. The power input here approximates what one wants to achieve in the output of the secondary crystal. In other words there is no significant advantage in pulsed field inversion over pulsed reflector inversion. 2) One could limit the number of ground state ions by decreasing the size of the crystal or the concentration of ions. In the first case, a crystal of about 1 mm^3 need be used--an impractical size from the standpoint of gain or oscillation measurement. In the second case, lowering the concentration would lower the absorption of the crystal so that there would be a waste of light flux. 3) One may attempt electric dipole inversion at microwave frequencies. This will be explained more fully in the next section.

Microwave electric dipole inversion: Note first that the ratio of optical to microwave frequencies is between 10^4 and 10^5. This means that the required energy per photon for a microwave transition is 10^{-4} times that for an optical transition. Corresponding relaxation times are much greater for the microwave transition. Thus for the same number of ground state ions (low temperatures would be necessary to depopulate equilibrium upper state ions) much less pulse power need be applied to invert populations in a microwave maser. Furthermore, microwave sources with coherent output are more easily available than in the optical case where another laser must be used. Magnetic dipole inversion at microwave frequencies has already been demonstrated, so the impetus and equipment exist.

The drawback to this state of affairs is that most materials have only magnetic dipole transitions at microwave frequencies, or at least have enough of magnetically allowed mixed states to cloud other results. (The magnetic fields in the pumping source cannot be discriminated against according to James Clerk Maxwell.) Notable exceptions to this circumstance may be found in an ammonia gas transition at 23 kmc and in the Cr^{3+} ion in distorted cubic fields such as in ruby. The former example is one of symmetric to antisymmetric transitions in the position of the nitrogen ion relative to the triple hydrogen base. This is strictly an electric dipole. Pulse inversion would enable one to dispense with beam focusers and separators and even the beam itself unless Doppler broadening were a hindrance.

In ruby one observes a zero field splitting of 11 kmc arising from Stark splitting of a 4-fold degenerate ground state. This splitting, if 1st order, indicates electric dipole transitions are allowed although little has been found on the relative effects of electric dipoles, magnetic dipoles (due to mixed states), and electric quadripoles (the Stark splitting may be second order or quadratic). If ruby could be pulse inverted between the zero field split states, one could eliminate the need for a magnet. Tunability however would suffer because of the difficulty in getting static electric fields inside a tuned cavity. Also one would have to apply a field in the crystal field direction for enhancement so that perfect crystals would be necessary. Tuning may be accomplished by applying mechanical stress.

Polarization studies on a 0° oriented ruby laser: It has been previously published that a ruby laser with its c-axis parallel to the direction of propagation produced unpolarized light. These were the findings of experiments which integrated the total ruby laser pulse through suitable polarizers. However, with the aid of a Rochon prism (which separates the two orthogonal polarizations of a beam) and time resolution in the detection apparatus, one notices that the spiked structure of a 0° ruby showed definite polarization. A series of spikes would show various orientations of the plane of polarization within a given output pulse. 90° oriented rubies give polarized light in one plane only. Changes in the plane of polarization for the 0° ruby seem random and certain spikes are unpolarized. There is a higher proportion of unpolarized spikes as the pumping energy is increased. Tests for some pattern or correlation in the changes of rotation among spikes may shed new light on the little-understood mode behavior of masers in general.

02-1-20 STUDY OF NONLINEAR BEHAVIOR OF GARNET AT HIGH MICROWAVE POWERS. G. Bodway and G. Kemanis (Prof. Wang)

Restabilization of an unstable spin system by modulating the d. c. magnetic field is being studied. The sinusoidal modulation case is governed by a differential equation of the Mathieu-Hill type. Its solution entails the evaluation of an infinite determinant, which has not been attempted. Instead, the differential equation was converted to an integral equation, which turns out to be of the Volterra type. An approximate solution by means of iteration has been obtained. Experimental verification of this result will be attempted next.

The necessary equipment for the investigation of ferrimagnetic resonance at high power levels has been assembled.

One component consisted of a microwave cavity designed so that a d. c. magnetic field and two a. c. magnetic fields could co-exist simultaneously at some prescribed region in the cavity. A relation between the diameter of the coupling hole into the cavity and the coupling constant was derived with the cavity Q as a parameter. The relation was checked experimentally and the results agreed with the theoretical curves reasonably well over the region of interest. The results will be of considerable use in subsequent work.

Some adjustments and calibration curves were made on the controls of a constant current power supply, which will enable us to measure ferrimagnetic resonance line widths of the order of 1 gauss at several thousand gauss with an accuracy of better than 15%. The absolute value is accurate to about 3%.

02-1-14 INVESTIGATION OF TUNNELING PHENOMENON IN AN INSULATING THIN FILM. S. Yee (Prof. Wang)

Tunneling through the thin film of aluminum oxide sandwiched between aluminum metal films was not observed. Part of the difficulties probably originated in the microscopic "unclean" aluminum metal films which were evaporated onto a glass substrate with the present available low vacuum system. Another part of the difficulties is probably caused by the non-uniformity of the insulating Al_2O_3 films prepared as described in the last quarterly report.

A high vacuum system is being constructed jointly with the Tube Laboratory Group hopefully to remedy the microscopic "unclean" aluminum metal films. The non-uniformity is hoped to be eliminated by anodizing the aluminum metal surface. The high vacuum system is designed to provide a pressure down to 10^{-6} to 10^{-7} mm Hg.

SYSTEMS

SYSTEMS

41-1-06 OPTIMAL CONTROL AND THE CALCULUS OF VARIATIONS. A. Larsen (Prof. Bergen)

The explicit connection between the classical theory of the calculus of variations and Pontriagin's maximum principle has been investigated. The study has shown that the classical theory includes Pontriagin's results as a special case, and is applicable to a much wider class of problems provided inequality constraints on the control are accounted for by the method of Valentine[1]. A report is now being prepared.

43-1-04 STABILITY OF NONLINEAR CONTROL SYSTEMS. I. Williams (Prof. Bergen)

This project is concerned with the methods used to investigate the stability of nonlinear control systems. Of particular interest is the investigation of techniques for the generation of Lyapunov functions. An investigation has been made of the technique of generating Lyapunov functions for nonlinear control systems by considering the total stored energy of a physical system whose dynamic behavior is governed by the same set of differential equations as that of the system in question. It has been found that this technique gives good results for special types of systems, but because of the fact that the total energy of a system is not always a useful Lyapunov function, this technique is not applicable to many types of systems.

The technique of generating the Lyapunov function through geometric considerations of the physical behavior of the linearized model of the nonlinear system which was previously used to verify Aizerman's conjecture for the class of third order systems having three poles and no zeros has now been successfully applied to a class of third order systems having one zero and three poles.

02-1-16 STUDY OF NONLINEAR DISCRETE-TIME SYSTEMS. S. Kodama (Prof. Desoer)

A practical sufficient condition had been obtained by the Lyapunov direct method for a class of nonlinear systems, $\underline{x}_k = [\underline{B} + g(\underline{x}_k)\,\underline{H}]\,\underline{x}_k$, as stated in the previous Consolidated Quarterly Progress Report, where $\underline{B}$ and $\underline{H}$ are nxn constant matrices and $g(\underline{x}_k)$ denotes a nonlinear gain-function.

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[1] Valentine, F.A., "The Problem of Lagrange with Differential Inequalities as Added Side Conditions," in Contributions to the Calculus of Variations, 1933-37, Chicago, Illinois: University of Chicago Press, 1937, pp. 407-448.

A further study has proved that the Lyapunov approach furnishes also a practical and unified analytical method for the study of the boundedness and the unboundedness of motions of discrete systems. Thus, an auxiliary scalar valued function of the state variables has been used to obtain sufficient conditions for 1) the boundedness of the state sequence $\{\underline{x}_k\}_0^\infty$ generated by a time-varying nonlinear system $\underline{x}_{k+1} = \underline{f}(\underline{x}_k, k)$, and for 2) the unboundedness of the limiting state of $\underline{x}_{k+1} = \underline{f}(\underline{x}_k)$. The $\underline{f}$'s are continuous in all their arguments in both cases. For illustration the above general results have been used to establish 1) the boundedness of a class of nth order saturating servo having a single integrator and 2) the unboundedness of a second and a third order servo each having a nonlinear gain-element and a double integrator.

43-1-21 THE MINIMAL TIME PROBLEM FOR LINEAR DISCRETE SYSTEMS. J. Wing (Prof. Desoer)

Consider a single input time invariant discrete system S whose state transition equation is given by

$$\underline{x}_{k+1} = \underline{A}\,\underline{x}_k + u_{k+1}\,\underline{a} \tag{1}$$

where $\underline{A}$ = nxn constant nonsingular matrix, $\underline{x}_k$ is an n-rowed state vector of the system at t = kT, and $\underline{a}$ is an n-rowed control vector. The control u_{k+1} is restricted to be an admissible control, i.e., $|u_{k+1}| \leq 1$ for $k = 0, 1, 2, \ldots$. It is assumed that the system is completely controllable by admissible control. The minimal time problem may be stated as follows: given any arbitrary initial state $\underline{x}_o$ of the system and any arbitrary target state $\underline{x}_t$, find an admissible control $u_1, u_2 \ldots$ which will transfer the system from $\underline{x}_o$ to $\underline{x}_t$ in the minimum number of sampling periods. Furthermore, it is required to determine an optimal strategy, i.e., determine a scalar valued function $u^o(\cdot\,;\underline{x}_t)$ whose domain is the whole state space such that if the system is at $\underline{x}$ at some sampling instant, then $u^o(\underline{x};\underline{x}_t)$ is an optimal control for the next sampling period.

If it is assumed that there exists a nonempty set $\Delta(\underline{x}_o, \underline{x}_t)$ of admissible controls which transfers $\underline{x}_o$ to $\underline{x}_t$, then an optimal control of the set can be computed in the following manner:

i) Determine the minimum N, say M, such that $(\underline{x}_o - \underline{A}^{-M+1}\,\underline{x}_t) \notin R'_{M-1}$ but $(\underline{x}_o - \underline{A}^{-M}\,\underline{x}_t) \in R'_M$, where R'_N is the set of all initial states that can be brought to the origin $\underline{x}_t = \underline{0}$ by an admissible control in N sampling periods or less.

ii) The state $\underline{x}_o - \underline{A}^{-M}\,\underline{x}_t$ can be considered as the initial state of the system for the minimum time regulator problem.

iii) The optimal strategy has the property that $u^o(\underline{x};\underline{x}_t) = u^o(\underline{x} - \underline{A}^{-M}\,\underline{x}_t;\underline{0})$, for all $\underline{x}$.

Thus once M is known, the minimal time control problem reduces to the minimal time regulator problem.

The two difficulties which are encountered in solving the minimal time problem when $\underline{A}$ is an arbitrary nonsingular matrix within the constraint that S be completely controllable by admissible controls are:

1. The set $\Delta(\underline{x}_o, \underline{x}_t)$ may be empty for a given $\underline{x}_o$ and $\underline{x}_t$.
2. Even if $\Delta(\underline{x}_o, \underline{x}_t)$ is nonempty, the checking of ii to determine M may involve a large number of computations.

For the case where (1) takes the special form

$$\begin{bmatrix} x^1_{k+1} \\ x^2_{k+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x^1_t \\ x^2_k \end{bmatrix} + u_{k+1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad |u_{k+1}| \leq 1 \quad \text{for } k = 0, 1, 2 \ldots$$

the two difficulties are not encountered. Specifically, $\Delta(\underline{x}_o, \underline{x}_t)$ is nonempty for any $\underline{x}_o$ and $\underline{x}_t$ and the determination of M requires a fixed number of simple computations independent of the magnitude of M.

02-1-25 ENERGY OPTIMAL CONTROL FOR PULSE AMPLITUDE MODULATED LINEAR TIME-INVARIANT DISCRETE SYSTEMS. C. T. Lee (Prof. Desoer)

The problem under consideration is an idealization of the attitude control problem of a space vehicle. We assume that the control is exercised by gas jets that eject gas at a fixed velocity with respect to the satelite. The applied force is proportional to the rate of mass ejected. If u(t) is the rate at which mass is ejected, the cost of control over a time interval (t_o, t_1) is proportional to

$$\int_{t_o}^{t_1} |u(t)|\, dt .$$

We shall assume that the dynamics of the system are linear time-invariant and that the system is sampled periodically. Then the cost of the control over (0, NT) is proportional to

$$\sum_{i=1}^{N} |u_i|$$

where u_i is the control during the (i-1)th sampling period. In setting up the problem, we assume that a disturbance torque has changed the attitude of the vehicle and that the system considerations require that the attitude be brought back to the origin in a given time at a minimum cost.

The optimal strategies are mechanized for two cases: 1) the inputs u_i are unbounded; 2) the inputs u_i are bounded: $|u_i| \leq 1$.

A technical report describing the solution to this problem is in preparation.

02-1-21 ITERATIVE TECHNIQUES FOR GENERATING THE OPTIMAL CONTROL AS A FUNCTION OF THE STATE. (Prof. Desoer)

In the course of some preliminary work, some of the attributes of the pseudo-inverse of a matrix were necessary and resulted in a geometric way of looking at the pseudo-inverse in contrast to Penrose's purely algebraic method. This point of view resulted in ERL Report 426 entitled "A Geometric Interpretation of the Pseudo-Inverse of a Matrix".

02-1-30 A METHOD OF SOLVING LINEAR PROBLEMS BY USING THE ADJOINT SYSTEM. R. Sussman (Prof. Desoer)

A usual way of finding the response of a system B, at time t_1, to some input u, is the following: first, decompose the input into a set of δ-functions; next, find the response of B at t_1 to each individual δ-function, and finally add up all the contributions.

In this study an alternative method of finding the response of B at t_1 is proposed. By this method we use the adjoint system B^*, and the procedure is the following: apply a δ-function at time t_1 to B^* and find its response for all t. Then "weigh" this response according to the input u and thus get the response of the original system at time t_1.

A comparison between these two methods shows that while in the usual method a whole set of δ-functions has to be applied to the system B, only one δ-function has to be applied to B^*, namely at time t_1. However, in the usual method we have to find the response only at one point, while in the proposed method the response at all t is needed.

These concepts are further generalized.

Also some reciprocity relations between B and B^* are derived, relations which are a natural extension of the "reciprocity theorem" in various fields such as circuit theory, electromagnetics, etc.

The application of this adjoint method of solving linear problems is illustrated by several problems in circuit theory and in control system theory.

It is hoped that this study will point out the usefulness of the adjoint system, and show how this system can be applied, in a systematic way, to solve certain linear problems.

02-1-34 SIMULATION OF FINITE-STATE AUTOMATA. M. Silverstein (Prof. Gill)

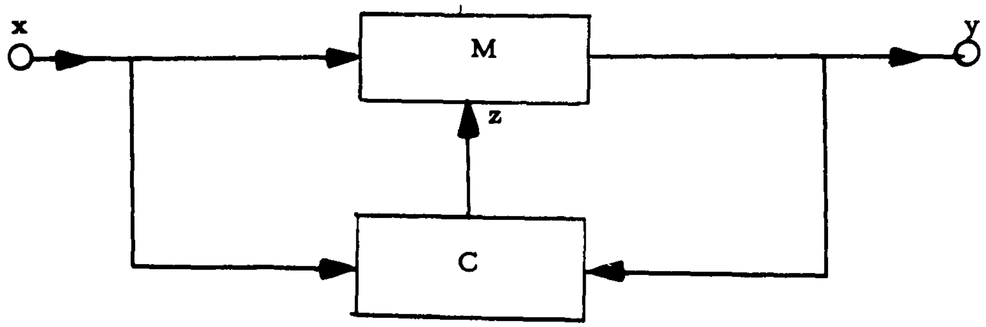
A general method for computer handling of Boolean algebra has been developed. It may be programmed in any language employing the notion of lists or associative memory; IPL-V is being used to obtain specific results. Commands exist for Boolean addition, multiplication and complementation of expressions. A small subroutine makes logical reductions. An algorithm has been found for the minimization of the automata, i. e., detection and elimination of equivalent states. Given a set of Boolean expressions representing a machine, the algorithm determines a new set of expressions representing an equivalent minimal machine. This algorithm has been programmed in the command structure developed for computer manipulation of Boolean expressions.

01-1-48 A STUDY OF THE PARALLEL ACCESS INFORMATION RETRIEVAL SYSTEM. D. G. Stone (Prof. Gill)

IR machines are described in terms of the complexities of the logical operations which they perform, and the type of code used to render the document description in machine language. For a large information store the most effective IR machine is a parallel access machine with a free field code. Such a machine has not yet been devised although its serial counterpart has existed for some time. Two fixed field parallel access IR machines have been outlined. Under study are parallel access machines of various complexities and coding schemes. The physical embodiment of the logical design and its influence on the design are noted.

01-1-65 ADAPTIVE FINITE STATE SYSTEMS. D. L. Moorehead (Prof. Gill)

One model of an adaptive finite state system might be as shown in the figure below:



M and C are finite state machines. The machine C receives information from the input and output of M and generates an output z which specifies a connection matrix for M.

Problems associated with this model are:
1. To what extent can it simulate adaptive behavior ?
2. Can conventional control systems be simulated by it?

As the first step towards answering these questions it was shown how the connection of M and C can be described by a single finite-state machine. Once this is done, the characteristics of the over-all system can be readily correlated with the characteristics of the individual machines.

43-1-17 CONSTRUCTION OF COMPACT SEQUENCES. S. G. Schaeffer (Prof. Gill)

The problem under investigation is the construction of compact sequences by a machine-like procedure. (A compact sequence is a sequence of $p^m + m - 1$ p-nary digits such that any subsequence of m digits is an m-digit p-nary number, with every m-digit p-nary number appearing exactly once.)

A. Gill and G. Machol have shown in an unpublished paper that all such sequences can be generated from certain binary matrices by simple permutation operations. The problem is to describe an algorithm using these operations and a set of selection rules which will result in the set containing all compact sequences for a given p and m, which contains no other sequences.

The most promising approach at present appears to be to draw on the theory of expansion of determinants to define the rules of operation on the matrix, and this line of attack is currently being undertaken. If it should prove to be unsuccessful, an attempt will be made to define the operations from a set of logical equations. It is felt that the determinant approach will be successful, and that it will lead directly to an algorithm which can easily be programmed for a computer.

17-1-02 NONLINEAR SEQUENTIAL CIRCUITS. A. Chang (Prof. Jury)

Investigations were begun on the cyclic behavior of nonlinear sequential circuits. Up to now, these investigations have been primarily limited to a literature survey.

02-1-24 STATISTICAL STUDY OF PULSE WIDTH MODULATED CONTROL SYSTEMS. S. C. Gupta (Prof. Jury)

The exact as well as the approximate model of the PWM control system was simulated on the IBM 704 to check the errors involved in approximations used in the Analysis and Synthesis of PWM Control Systems by analytical techniques as reported in two technical reports published by the Electronics Research Laboratory and also in a paper to be published by the Journal of Franklin Institute in May, 1962. It is found that the approximations are reasonable and some theoretical justification of these approximations and the errors involved has also been considered. All this work and some suggestions for future work in this area will be given in a published form soon.

02/17-1-03 TIME-VARYING SAMPLED-DATA SYSTEMS. C. A. Galtieri (Prof. Jury)

Some investigation on the present situation regarding the stability of time-varying sampled-data systems has been conducted. Explicit conditions for some particular uses have been derived. The results will be presented for publication in "Notes on System Theory".

17-1-04 ON THE ROOTS OF A REAL POLYNOMIAL INSIDE THE UNIT CIRCLE AND A STABILITY CRITERION FOR DISCRETE SYSTEMS. J. Blanchard (Prof. Jury)

Necessary and sufficient algebraic conditions for the roots of a real polynomial to lie inside the unit circle, in a table form, are given. In this form, the constraints are obtained only by evaluation of second order determinants. The connection and identity between the stability criterion established in the ERL Report, Series No. 60, Issue No. 425, and of a previously obtained criterion* are obtained. The table is very valuable if the coefficients of the real polynomial are given in numbers. It is similar to Routh's table obtained for the continuous case.

Conditions on the numbers of the roots inside, outside, or on the unit circle are also discussed under the cases when the determinants are zero or non-zero. Furthermore, necessary and sufficient conditions are formulated for all the roots to be inside a circle of radius σ less than unity, and in a ring between r_1 and r_2 inside the unit circle and also the conditions when all the roots are to be real and are to lie between plus and minus unity in the z-plane.

Various examples from discrete systems are presented which illustrate the applications of the new stability criterion and the other conditions formulated in this report.

In the conclusion, the various analytical stability criteria applied to linear discrete systems are enumerated and compared, with emphasis on the advantageous applications of each of the various criteria.

17-1-05 FORCED OSCILLATIONS IN NONLINEAR SAMPLED-DATA SYSTEMS. (Prof. Pai)

An analytic method to examine the forced oscillations in nonlinear sampled-data systems, where the ratio of sampling to signal frequency is not an integer, is being investigated and the results will be reported in due course.

Study is also being undertaken in the area of nonlinear systems with time lags.

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*E. I. Jury, "A Simplified Stability Criterion for Linear Discrete Systems," ERL Report, Series No. 60, Issue No. 373, University of California, Berkeley, June 1961.

43-1-16 ON AN OPTIMAL DISCRETE SYSTEM. K. Sakuda (Prof. Polak)

Work on a minimal energy control strategy for a PWM system with a double integrator plant is being continued. So far, it has been shown that an optimal control for a transient with an allowed time of N sampling periods, $N = 1, 2, \ldots,$ consists of N pulses, the first k of which are of one polarity whilst the last (N-k) are of the opposite polarity, $k = 1, 2, \ldots, N$. It has also been shown that the portion of the phase plane, over which transients of length N sampling periods are permitted, can be divided into disjoint regions indexed by (k, N-k), where k was defined above. It remains to find an algorithm for computing the lengths of the pulses of an optimal control.

43-1-18 ON THE EQUIVALENCE OF DISCRETE SYSTEMS IN TIME-OPTIMAL CONTROL. (Prof. Polak)

It was shown that under a transformation which maps the state space of a discrete regulator system onto a sequence space in a one-to-one manner, time optimal discrete regulator systems with different dynamics and modulators can be brought to equivalence and that this fact may be used to construct an optimal control strategy for one system from the known optimal strategy of another system to which it is equivalent. In particular, the optimal control strategy for a PWM discrete system with a plant described by a n-th order linear differential equation is derived in this manner from the optimal strategy of a PAM discrete system with the same plant.

01-1-21 DEAD TIME SYSTEM REGULATION. R. T. Lacoss (Prof. Smith)

A rational transfer function, $G_p(s)$, in cascade with a pure delay, e^{-sT}, is being used as a model for a plant. The plant is subjected to random disturbances which can be measured only at the output of the delay. The problem is to maintain the output as constant as possible by the use of a feedback system. If control is to be effective, certain restrictions are placed upon the statistics of the disturbance. These restrictions have been briefly investigated.

Professor Smith has suggested a regulating system which contains a odel of the plant with dead time. An analysis of such a topology has been completed for a particular plant transfer function to determine absolute and relative stability. The techniques used could be applied to a plant with an arbitrary $G_p(s)$.

It may be possible to adjust the plant model during normal operation of the regulating system. This is now being investigated. The sensitivity of various measures of performance to the adjustment of the model is now being obtained. When this is completed a very simple adaptive loop will be numerically simulated and tested. The application of such an adaptive loop will be limited by the rate of variation of the parameters in the actual plant.

01-1-22 TIME VARIABLE MAGNETIC DELAY LINE. K. E. Stoffers (Prof. Smith)

The analysis of the line has been done with piece wire linear core-functions. The results show that several assumptions underlying the design of the model, which was used to verify the applicability of the principle, were not true. An improved design is now being done.

01-1-61 OSCILLATION SUPPRESSOR FOR NON-REGULATED SUPPLY FOR PULSER. E. L. Harris (Prof. Smith)

An analysis has been completed of the use of a pentode with a delayed signal on the screen grid to prevent oscillations excited by a sudden change in load.

01-1-66 MINIMAL TIME CONTROL OF SYSTEMS WITH COMPLEX ROOTS. H. K. Knudsen (Prof. Smith)

The problem of obtaining minimal time control from a point $\underline{x}_o$ to the origin of the system state space is one of determining the control function u(t).

Given a linear, time-invariant, single input system, described by the vector differential equation:

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}u \qquad (1)$$

it has been shown by Pontriagin that a necessary condition for time optimal control on the forcing function u(t) is that:

$$u(t) = \mathrm{sgn}\langle \underline{g}(t), \underline{b} \rangle \qquad (2)$$

where $\underline{b}$ is a constant control vector, and $\underline{g}(t)$ is a solution of the system's adjoint differential equation.

Equation (2) would completely specify u(t) except for the fact that the vector solution $\underline{g}(t)$ of the system's adjoint differential equation is not completely defined, since the relationship between the initial conditions of the adjoint equations and the control function u(t) which drives the system output from any point $\underline{x}_o$ to the origin of the state space is not known.

An iterative procedure to find the initial conditions of the adjoint differential equations which will give time optimal control from an arbitrary (controllable) point $\underline{x}_o$ in the state space has been developed. This procedure uses a first order approximation of the systems behavior to changes in the initial conditions of the adjoint equation. At the present time, this iterative procedure is being programmed on the Bendix G-15 computer and DA-1 differential analyzer.

One complication in the method is that the set of equations generated during the iterative solution can become degenerate when switching hypersurfaces are intersected. Logic devices are being used to generate new sets of non-degenerate equations whenever degeneracy occurs.

01-0-24 HIGH RESOLUTION TV SYSTEM FOR STATIONARY PATTERNS. (Mr. Studer)

The purpose of this project is to establish criteria for correlating TV picture detail of stationary patterns and required minimum resolution of TV systems.

Tests have been carried out with a vidicon camera tube with exchangeable electronic circuitry to produce different degrees of horizontal versus vertical resolution. The aim was to test the sensitivity of various test patterns for measuring small differences in resolution. The results are presently under evaluation.

30-1-11 BINARY DECODING. T. J. Wagner (Prof. Thomasian)

A general model of binary coding and decoding is being formulated. Some recently proposed promising decoding procedures utilize encoding procedures (e. g., convolutional encoding) which are different from the extensively studied parity check block codes. Thus what is desired is a model that includes such procedures and reasonable generalizations of them. In the context of such a model it should be possible to consider alternative procedures and make comparative analyses.

02/30-1-12 IMPROVED HAMMING CODES. R. N. Miller (Prof. Thomasian)

This project is completed. The results have appeared in an ERL Report, Series No. 60, Issue No. 419, November 21, 1961.

02/30-1-13 OPTIMAL CONTROL WHEN THE STATE IS UNOBSERVABLE. G. K. Machol (Prof. Thomasian)

The problem presently being considered is that of finding optimal control policies for a finite-state probabilistic machine when only inputs and outputs are observed. This idealized problem corresponds roughly to physical problems where the device (rocket, chemical plant, etc.) being controlled is disturbed by random quantities and these random quantities are not observed. Also the complete condition of the device is not available at each instant of time; only the inputs and outputs are observed. Such a problem is considerably more involved than the case when the state is constantly observable. The additional complexity is due to the fact that an optimal policy may quite easily depend on many past outputs since these permit better guesses about the present state of the device.

01-1-54 ANALYSIS OF TRACKING CONTROL SYSTEM WITH NOISE DISTURBANCES. R. E. Brooks (Prof. Turin)

A study is being made of the probabilistic behavior of a class of (nonlinear) tracking systems with noise disturbances. The method of attack is through the Fokker-Planck diffusion equation, but instead of solving the equation exactly, attention has been confined to characterizing the transient and steady states modes probabilistic behavior. The transient mode depends on the initial state of the system and is associated with the tracking loss during acquisition, whereas the steady state mode is associated with tracking losses during the steady state. For the first order system with white noise disturbances explicit expressions have been obtained for the probability of the system remaining locked for t seconds.

12-1-06 INFORMATION THEORETIC STUDY OF SYNCHRONIZATION. H. Kaneko (Prof. Turin)

This project was completed and a report of the project is available in the form of a Master's Thesis, "A Statistical Analysis of the Synchronization of Digital Receivers," submitted to the Graduate Division of the University of California in January, 1962.

02-1-29 ANALYSIS OF ERRORS IN THE ESTIMATION OF THE IMPULSIVE RESPONSE. H. Kwakernaak (Prof. Turin)

The results of this study can be summarized as follows:

In order to be able to set up the problem of finding the impulsive response of a linear, time-invariant, single input-output system in terms of statistical estimation theory, it is necessary to make two essential assumptions on the nature of the impulsive response: 1) The impulsive response can be specified by its values at discrete intervals (sampling); 2) A finite number of such values suffices (truncation).

When these assumptions are not exactly met, errors are introduced. A third error stems from the noise-type disturbances of the observed signals. The effects of these three types of errors have been investigated more closely and some effects were pointed out, among which the rather disturbing phenomenon that in a number of cases the statistical errors may become very large as the sampling interval goes to zero. A condition for this to happen has been formulated.

An ERL Report, Series No. 60, Issue No. 434, is in preparation.

30-1-07 ADAPTIVE FILTERS. H. Scudder (Prof. Turin)

The period from November 15 to February 15 has been spent studying the results of previous work which is related to adaptive Wiener filters. Most of the time was spent studying optimum linear, time-invariant filters and learning the fundamentals of time-varying systems. Also being studied is the application of the Paley-Wiener criterion to time-varying systems.

43-1-11 EQUIVALENCE IN LINEAR SYSTEMS. G. Bacon (Prof. Zadeh)

The concept of k-equivalence of automata theory has been shown to have a particularly simple algebraic description in linear modular circuits. It was found that the set of states k-equivalent to the null state is a subspace depending upon k and that all other k-equivalent states are in cosets of this subspace. Further, it was seen that for any linear rational time-invariant system the states equivalent to the null state are those that are not observable. From this it can be shown that any such system has an observable equivalent.

41-1-05 A CANCELLATION TECHNIQUE FOR TIME-OPTIMAL CONTROL PROBLEM. J. H. Eaton (Prof. Zadeh)

Work is continuing on the problem of taking a given system from a specified initial state to a specified terminal state in minimum time. For systems with magnitude constraints on the input it is well known that the desired terminal state can be reached in minimum time using a bang-bang input, that is, an input which is always on the boundary of the space of constraints.

Using properties of the Laplace transformation, a set of transcendental equations has been obtained whose minimal time solution yields the required switching times. The convergence of a class of iterative procedures for obtaining the minimal time solution of the above set of equations has been established. At present, this investigation is concerned with determining the relative merits of various iterative procedures belonging to the above class.

43-1-20 DETERMINISTIC PROCESSES AND PREDICTION OPERATORS. J. F. A. Ormsby (Prof. Zadeh)

The work was, to a large extent, concerned with revising the report on the existence and relationship of perfect predictions. For the most part, this involved an enlargement based on making previous arguments more rigorous and adding certain new material. The finalized report together with its companion report on the analytic representation of a class of perfect predictions are scheduled for issue as ERL reports, numbers 421 and 422.

Preliminary work was started on the algebraic treatment of perfect prediction for discrete time series with later extensions to include time series over finite fields. Some initial relations were obtained. Future effort will be concentrated in this area.

43-1-03 IDENTIFICATION PROBLEMS IN FINITE-STATE MACHINES. J. Raviv (Prof. Zadeh)

Research is being continued on the problem of taking a deterministic finite-state machine to a specified state in the shortest (expected) time, when one is given an initial probability distribution on the states. At each stage the experimenter has to make a decision on what input to apply to the machine on the basis of an observed information pattern. After the input is applied and the output of the machine observed, the a posteriori probability distribution on the state space is calculated and the next decision is made on the basis of this distribution. The first problem which arises is of finding an optimal policy for taking the finite-state machine to a known terminal state without knowing the initial state. For purposes of decision making the probability distribution space is divided into connected regions by hypersurfaces such that a particular input is applied when the a posteriori probability distribution falls into a particular region. At present research is centered on the investigation of the forms of these hypersurfaces.

43-1-12 FUNDAMENTAL CHARACTERISTICS OF STOCHASTIC DISCRETE-STATE SYSTEMS. Y. Sekine (Prof. Zadeh)

The probabilistic characteristics of a complex system which consists of a finite number of elementary systems, has been investigated during this quarter.

The principal results are as follows:

(1). Though the probability density function of the duration of an elementary system is defined independently of the transition probability or in other words, independently of which state was occupied prior to the present one, the past history is more important for dealing with the complex system. The probabilistic characteristics of the complex system state depend on the previous complex system state, even though the present state is quite the same.

(2). The probability that the particular (recurrent) state of the complex system is originated by the transition of the μth elementary system is given by

$$a^{(\mu)} = \frac{T}{T_j^{(\mu)}}$$

where $T_j^{(\mu)}$ is the average duration of the jth state of the μth elementary system and T is the average duration of the particular state of the complex system under consideration.

(3). Using the probability $a^{(\mu)}$, the simple method of replacing the recurrent state of the complex system by an equivalent state has been considered. This method enables us to deal with the complex system very easily.

A report containing a detailed discussion on the probabilistic characteristics of elementary systems and complex systems is in preparation.

At present, investigations are being carried out on the optimum expansion policies of the multi-component stochastic system. The problem is stated below.

Given the m elementary systems and the elementary system functions $x_k(t)$'s $(k = 1, 2, \ldots m,\ t > 0)$ whose values are determined by the state the kth elementary system takes t hours after its start, we construct and expand the complex system by successively selecting elementary systems at $t = t_1, t_2, \ldots t_N$.

We wish to determine the optimal policy $\pi^o = \begin{bmatrix} k_1^o \\ k_2^o \\ k_N^o \end{bmatrix}$ of selecting the elementary systems to minimize the following objective function.

$$\Gamma_N = \sum_{t_n = t_1}^{t_N} f_{kn}(t_n) \tag{1}$$

under the constraint

$$\text{Prob}\{y(t_n \mid \pi) \geq d(t)\} \geq \gamma_n \qquad n = 1, 2, \ldots N \tag{2}$$

where $f_k(t)$ is the cost function needed to add the kth elementary system to the existing complex system at time t, and $y(t \mid \pi)$ is the complex system function defined by

$$y(t \mid \pi) = \sum_{t = t_1}^{t_N} x_{kn}(t - t_n) \tag{3}$$

π being the policy of selecting the elementary system. The function d(t) and the constant γ_n are prespecified.

Both of the dynamic programming and linear programming approaches have been examined. Though we can solve formally the above problem by the techniques of dynamic programming, the numerical computation is rather complicated, because the independent variables of the objective function in each stage are the vector quantity of N dimensions.

Other methods, such as integral linear programming and stochastic linear programming are currently being investigated.

As a matter of fact, the problem can be easily solved by the integer linear programming technique when the stochastic contraint (2) is replaced by the deterministic one setting $\gamma_n = 1$.

43-1-14 NOTATION FOR n-VARIABLE CALCULUS. B. Whalen (Prof. Zadeh)

Taylor's theorem, the Hamilton-Jacobi equation, some properties of the Jacobian, and some basic equations in the theory of control have been derived in the new notation. The notation tends to emphasize the geometrical aspects of the equations. These results will appear in volume two of the "Notes on System Theory".

43-1-19 LINEAR TECHNIQUES APPLIED TO DISCRETE-TIME OPTIMAL CONTROL PROBLEMS. B. Whalen (Prof. Zadeh)

The minimal time control law for unconstrained, not-completely controllable linear systems has been obtained. (The completely controllable system was treated by Kalman and Bertram.). The solution employs Schmidt orthogonalization.

Iterative solutions to the fixed time-minimum fuel problem and the minimum time control problem with inequality constraints have been devised. Each iteration involves the determination of the value of a linear program.

At present the technique is impractical because of the high dimensionality of the linear program. However, at least for the case of time invariant constraints, it may be possible to overcome this problem. This possibility is being investigated.

43-1-09 SYSTEM IDENTIFICATION. B. Whalen (Prof. Zadeh)

Work is continuing on this project. One result of this work is the use of differential equations for signal prediction. In many cases, orthogonal functions are used to predict signals which are obviously the solutions to differential equations. It seems more appropriate to estimate the differential equation and use this equation to predict the signal.

MISCELLANEOUS

MISCELLANEOUS

Bioelectronics

02-1-17 ELECTROPHYSIOLOGICAL PROPERTIES OF THE CEREBRAL CORTEX.[1] G. G. Furman (Prof. Susskind)

The objective of this project is the development of a mathematical description for electrical behavior in the olfactory cortex of the cat that is consistent with the experimental data. Toward this end, analysis of the data obtained during the past summer has continued.

The possibility that the waveforms mentioned in earlier quarterly reports have their origin in relaxation oscillations is under investigation. Some of the evidence pointing toward non-harmonic oscillation is:

1. The oscillations often cease abruptly with time in a discontinuous fashion.
2. Neurones are known to be capable of relaxation modes.
3. The waveforms appear very similar except for a change of scale of the time axis to those obtained by Kandel et al.[2] in a closely related cortex by the use of intracellular electrodes. Such waveforms are known to have their origins in a switching of state by the neurones under study even though they are smooth and near-sinusoidal.

33/14-9-02 CELLULAR AND LONGEVITY EFFECTS OF MICROWAVE RADIATION. P. O. Vogelhut (Prof. Susskind)

The results of this research are being written up in a technical report.

Nonlinear Magnetics

01-1-69 MAGNETICALLY CONTROLLED INVERTER USING SCRs. R. Shepard (Prof. Bourne)

Using special techniques, a Silicon Controlled Rectifier has been used to pulse a source of Direct Current. The main feature of the circuit is the use of a capacitor-saturable reactor combination to turn off the SCR at a predetermined time after it has been fired. Work is presently in progress to adapt the circuit to drive a transformer so that changes in voltage level may be accomplished. The final circuit configuration is expected to be quite similar to that of a power transistor D-C inverter.

01-1-70 FREQUENCY CONVERTER. A. Benchimol (Prof. Bourne)

Frequency conversion using magnetic amplifiers in a delta circuit fed by three-phase power shows promise in many low frequency applications. A system with a multiple winding power transformer and six magnetic amplifiers in a delta system was studied with the purpose of converting the power supply frequency to a lower frequency, continuously variable.

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[1] Prof. Freeman's laboratory, where the experimental work on this project is done, is supported by the Foundation's Fund for Research in Psychiatry and by the National Institute of Neurological Diseases and Blindness.

[2] Kandel, E. R., et al, J. Neurophysiol., vol. 24, no. 3 (May, 1961), pp. 225-271.

Experiments will be performed to confirm the theoretical studies undertaken.

Energy Conversion

01-1-62 **DEVELOPMENT OF VARIABLE SPEED INDUCTION MOTORS.** S. A. Nasar (Prof. Saunders)

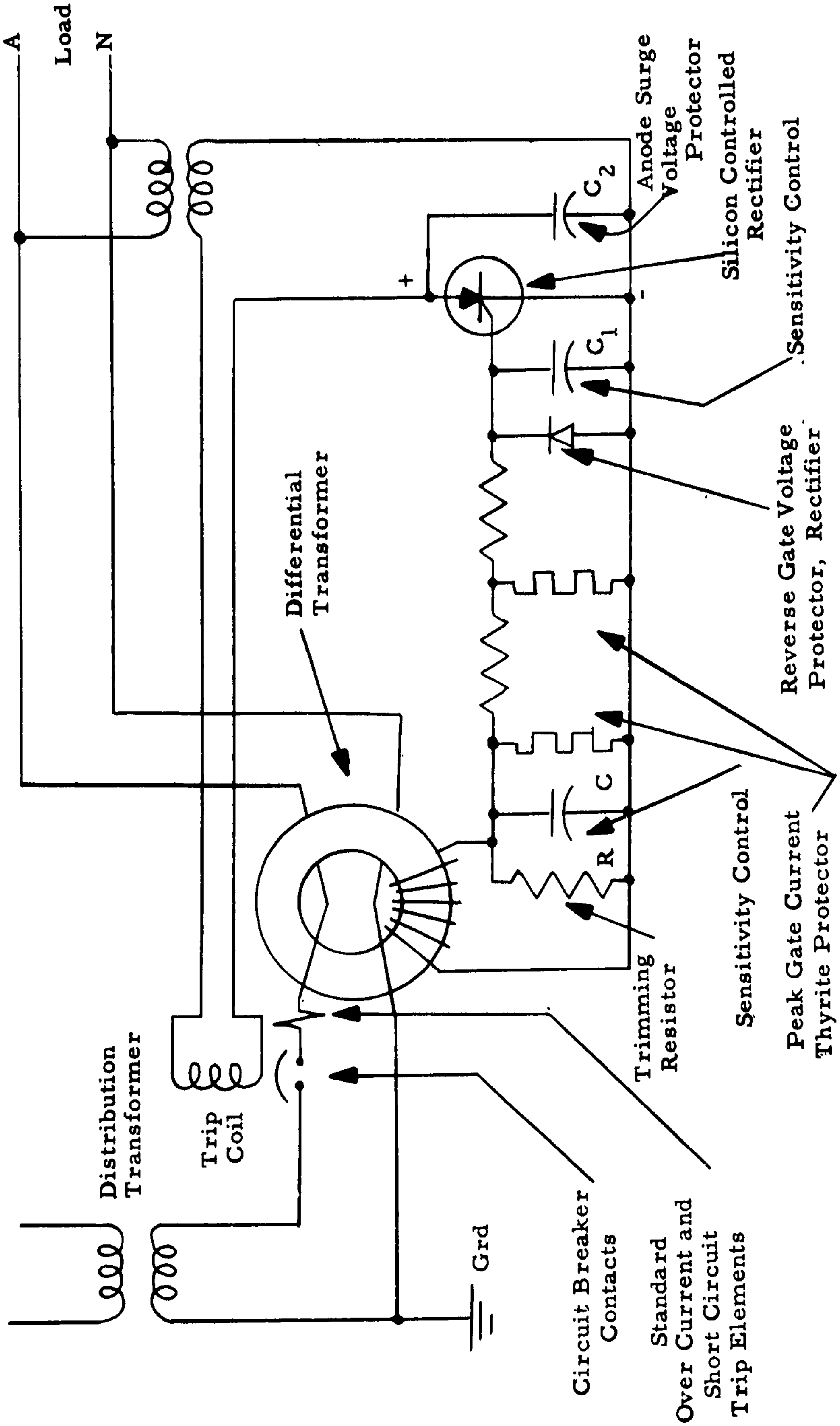
The phase of investigation related to testing and experimental work on the LOGMOTOR has been completed. The next step shall be to attempt to complete the formulation of a general theory for machines with magnetic structures of non-integral multiples of winding wavelengths. The LOG-MOTOR, being such a machine, is expected to serve as an example of application of the theory.

01-0-30 **MACHINE END-CONNECTION LEAKAGE.** S. A. Stone (Prof. Saunders)

Work is continuing on this project. Attention has been focused on the rotor end-connection leakage of squirrel-cage induction motors. In addition to image methods and direct solutions, some work is being done on conformal transformations.

01-0-30 **TRANSISTORIZED DIFFERENTIAL CIRCUIT BREAKERS FOR UTILIZATION CIRCUITS.** (Prof. Dalziel)

Experimental work during this period has been devoted to investigating transistor amplifying and switching circuits suitable for application to differential circuit breakers. Circuit breakers have been designed and perfected to permit protection of 2-wire single-phase 120 or 240 volt circuits and a typical circuit is shown herewith; however, other circuits have been developed for 3-wire single-phase 120/240 volt circuits, 4-wire three-phase 120/208 volt circuits, 3-wire three-phase 240 volt circuits, and the proposed compatible 2-wire single-phase 120/240 volt utilization circuit for residences and 4-wire three-phase 240/416 volt distribution systems for industry.



Two-Wire Single-Phase 120 or 240 Volt Circuit