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QUENCHING OF ADAPTIVE CONTROL SYSTEM RESPONSE TO TEST SIGNAL

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In adaptive control, a test signal may be used to identify the parameters of the system to be controlled. ~~A test signal disturbs the system~~. It is ~~therefore~~ desirable to eliminate the effects of this signal as soon as the identification has been completed. After identification a quenching signal may be introduced to eliminate the system response to the test signal. The systems treated here are described by linear differential equations with slowly varying coefficients. The nature of the quenching signal depends on whether or not it is bounded or unbounded. The unbounded quenching signal is a linear combination of a properly weighted impulse and derivatives of an impulse. The weights are determined by the initial conditions on the system at the instant of quenching and the system parameters. Unbounded quenching is considered to be optimum if the response to the test signal is eliminated with minimum integral squared error. The bounded quenching signal is obtained by scheduling the lengths of time its value is either at the upper or lower bound. The quenching signal is determined by the test signal and the system to be controlled. Therefore as soon as the system is identified quenching can be accomplished by scheduling regardless of the other disturbances to which the system is subject. The method applies to the quenching of system response whether or not adaptive control is involved.

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INTRODUCTION

State space for a system described by a differential equation of nth order is defined as a system of cartesian coordinates where the ordinates are the controlled variable and its first n-1 derivatives. The normal state of a system is the state due to the normal input. The normal input is the input to the system in the absence of the test or quenching signal.

Adaptive control is indicated where the parameters of the system to be controlled are time varying. It is important to know the value of the parameters to match the controller to the system. The determination of the parameters of the system is commonly referred to as the "identification problem". One method of identification is to use the normal input and the corresponding system response. The other is to introduce a test signal. The test signal disturbs the system. The response to the normal input is often difficult to distinguish from the noise present. In such cases a test signal for which the system response dominates the noise is preferred to the normal input.

Mishkin, Braun, Corbin, Merriam and others present techniques for solving the identification problem.[1]

~~We propose the~~ adaptive control ~~shown in Figure 1~~. The switches are closed during normal operation and open for the period of identification. The switches may be omitted, but then the computation becomes much more difficult. A test signal is used for identification to eliminate the effects of the test signal. The theory is restricted to controlled systems described by linear differential equations with constant coefficients. The form of the equation is known at the start but not the values of the coefficients which are parameters. It is assumed that the parameters of the system vary slowly enough so that they may be considered constant during identification. Immediately after identification the parameter values are available to calculate the quenching signal. The response to the test signal may then be quenched by scheduling determined only by the test signal and system characteristics. This signal is independent of the other disturbances to which the system is subject.

The nature of the quenching signal depends on whether or not it is bounded. It will be shown that the unbounded quenching signal is a linear combination of a properly weighted impulse and derivatives of an impulse. The order of the highest derivative is one less than the order of the system. The weights are determined by the state of the system at the instant of quenching and the system parameters. Unbounded quenching is considered to be optimum if the response to the test signal is eliminated with minimum integral squared error. The bounded quenching signal is obtained from a "bang-bang" controller. The signal is either positive or negative and its absolute value is less than or equal to a constant. Oldenburger, LaSalle, Bellman, Rozonoer and others present methods of obtaining the control functions necessary to determine when the signal is positive or negative.[2, 3, 4, 5]

The control time is defined as that time interval during which the bounded quenching signal is either positive or negative. The control functions for a bang-bang controller are used to obtain the control times. These time intervals are expressed as functions of the test signal, the magnitude of the quenching signal and the system to be controlled. The quenching signal is introduced by scheduling the control times. For some systems the control times are independent of the system parameters. A computer may be readily programmed to compute the control times if they are not independent of the system parameters.

Not all systems are treated; however the method is satisfactory for a large class of simple systems. For the general second order and higher order systems the expressions for the control times become complicated and impractical without approximations.

GENERAL THEORY FOR CASE OF UNBOUNDED QUENCHING SIGNAL

The general system is shown in Figure 2. It is assumed that this system is linear and that $g(t)$ is its impulse response. The Laplace transform $G(s)$ of $g(t)$ represents the transfer function of the system.

The signal $r(t)$ may be the normal input or an equivalent input due to disturbances entering the system at other points. The extra signal $m(t)$ is composed of two components $m_1(t)$ and $m_q(t)$. Here $m_1(t)$ is the test signal for identification and $m_q(t)$ is the quenching signal introduced to drive the system to the normal state. In general $r(t)$ is a function of time, either deterministic or random.

Since the system is assumed linear, superposition holds and the output $c(t)$ is the sum of the responses to the three inputs $r(t)$, $m_1(t)$ and $m_q(t)$. The three outputs are given by the convolution integrals

$$\begin{aligned} c_1(t) &= \int_{-\infty}^{\infty} g(t-\sigma)m_1(\sigma)d\sigma \\ c_q(t) &= \int_{-\infty}^{\infty} g(t-\tau)m_q(\tau)d\tau \\ c_r(t) &= \int_{-\infty}^{\infty} g(t-\lambda)r(\lambda)d\lambda \end{aligned} \tag{1}$$

where $c_r(t)$, $c_1(t)$ and $c_q(t)$ are the responses to $r(t)$, $m_1(t)$ and $m_q(t)$ respectively. Thus

$$c(t) = c_1(t) + c_q(t) + c_r(t) \ . \tag{2}$$

We define the system error e_q by

$$e_q = c_a - c_d \tag{3}$$

where c_a is the actual output of the system and c_d is the desired output.

When $m_1(t)$ is introduced the response to this signal is superimposed on the response to $r(t)$. If the identification requires time T and $m_1(t)$ is introduced at $t = 0$, it is desirable to return the system immediately to the normal state at $t = T$. Ideally we would like to return the system with zero error. Figure 3 shows the desired response.

For $0 < t \leq T$ the system response is the sum of c_r and c_1. For $t > T$ we then want the output c_r. Therefore we define the desired output c_d by

$$c_d = c_r \qquad t > T \ . \tag{4}$$

We have stated that ideally the system response should be returned to the normal state with zero error; this corresponds to moving from one point in state space to another in zero time. Due to limitations on the system and on the power available, a finite time will be required to return the system to the normal state. The actual response of the system is shown in Figure 4.

Since the system is not returned to the normal state instantaneously at $t = T$ the error e_q is given by

$$e_q = c_a - c_d = (c_1 + c_q + c_r) - c_r = c_1 + c_q \qquad t > T \ . \tag{5}$$

Since $c_q(t) = 0$ for $t < T$ it is convenient to introduce a new variable t' where

$$t' = t - T$$

and replace $c_q(t)$ by a new function $c_2(t')$ where

$$c_2(t') = c_q(t) \ .$$

Equation (5) may be written in terms of the new variable t' as

$$e(t') = c_1(t' + T) + c_2(t') \qquad t' \geq 0 \tag{6}$$

where

$$e(t') = e_q(t) \quad .$$

We introduce $m_2(t')$ where

$$m_2(t') = m_q(t) \quad .$$

The integral squared error I is given by

$$I = \int_0^{\infty} e^2(t')\, dt' \quad . \tag{7}$$

By Equations (6) and (7) we obtain

$$I = \int_0^{\infty} \left[c_1^2(t' + T) + 2c_1(t' + T)c_2(t') + c_2^2(t')\right] dt' \quad . \tag{8}$$

The error and hence the integral squared error is zero if

$$c_2(t') = - c_1(t' + T) \qquad t' \geq 0 \quad . \tag{9}$$

Now $c_2(t')$ is given by the convolution integral

$$c_2(t') = \int_{-\infty}^{\infty} g(t' - \tau)m_2(\tau)\, d\tau \quad . \tag{10}$$

By Equations (9) and (10)

$$\int_{-\infty}^{\infty} g(t' - \tau)m_2(\tau)\, d\tau = - c_1(t' + T) \quad . \tag{11}$$

Equation (11) may be solved for $m_2(t')$ by taking the Laplace transform of both sides to obtain

$$G(s)M_2(s) = - \mathcal{L}\left[c_1(t' + T)\right] \tag{12}$$

where $M_2(s)$ is the Laplace transform of $m_2(t')$ with respect to the time variable t', and $\mathcal{L}\left[c_1(t' + T)\right]$ is the Laplace transform of the output $c_1(t' + T)$ with respect to t'.

Since $m_1(t) = 0$ for $t \geq T$ the response of the system does not depend on $m_1(t)$ for $t \geq T$, but only on the initial conditions at $t = T$; that is, $t' = 0$.

In general the linear system under study is given by the differential equation

$$\frac{d^n c_1}{dt^n} + a_{n-1}\frac{d^{n-1}c_1}{dt^{n-1}} + \ldots + a_1\frac{dc_1}{dt} + a_o c_1 = b_q\frac{d^q m_1}{dt^q} + b_{q-1}\frac{d^{q-1}m_1}{dt^{q-1}} + \ldots + b_1\frac{dm_1}{dt} + b_o m_1 \tag{13}$$

for real coefficients $a_o, a_1, \ldots a_{n-1}$ and $b_o, \ldots b_q$. We assume that $q \leq n-1$.

For each i we have

$$\frac{d^i c_1}{dt^i} = \frac{d^i c_1}{d(t')^i} .$$

The initial conditions at $t = T$ are

$$c_1(T) = c_o$$

$$c_1'(T) = c_o'$$

$$\vdots \qquad \vdots$$

$$c_1^{(n-1)}(T) = c_o^{(n-1)}$$

where $c_o, c_o', \ldots c_o^{(n-1)}$ are the values of $c_1, c_1', \ldots c_1^{(n-1)}$ at $t' = 0$.

Let $C_1(s)$ denote $\mathcal{L}\left[c_1(t' + T)\right]$ for the time variable t'. By Equation (13)

$$\left[s^n C_1 - s^{n-1}c_o - s^{n-2}c_o' - \ldots - c_o^{n-1}\right] + a_{n-1}\left[s^{n-1}C_1 - s^{n-2}c_o - s^{n-3}c_o' - \ldots - c_o^{n-2}\right] + \ldots + a_1\left[sC_1 - c_o\right] + a_o C_1 = 0 \tag{14}$$

whence

$$C_1(s) = \frac{c_o\left[s^{n-1}+a_{n-1}s^{n-2}+\ldots+a_1\right]+c_o'\left[s^{n-2}+a_{n-1}s^{n-3}+\ldots+a_2\right]+\ldots+c_o^{n-1}}{s^n + a_{n-1}s^{n-1} + \ldots + a_1s + a_o} . \tag{15}$$

We note that $G(s)$ has the same denominator as $C_1(s)$ but that the numerator of $G(s)$ is

$$b_qs^q + b_{q-1}s^{q-1} + \ldots + b_o .$$

By Equation (12)

$$M_2(s) = \frac{-\left\{c_o\left[s^{n-1}+a_{n-1}s^{n-2}+\ldots+a_1\right]+c_o'\left[s^{n-2}+a_{n-1}s^{n-3}+\ldots+a_2\right]+\ldots+c_o^{n-1}\right\}}{b_qs^q + b_{q-1}s^{q-1} + \ldots + b_o} . \tag{16}$$

Now $c_2(t')$ is the solution of the differential equation with initial conditions

$$\begin{aligned} c_2(0) &= -c_1(T) \\ c_2'(0) &= -c_1'(T) \\ &\vdots \\ c_2^{(n-1)}(0) &= -c_1^{(n-1)}(T) . \end{aligned}$$

Since $c_2 = 0$ for $t' < 0$ the problem is to create the above initial values of c_2 and its derivatives.

If the inverse Laplace transform of both sides of Equation (16) is taken, then $m_2(t')$ will be that signal which produces the negative of the response $c_1(t)$ for $t > T$. Figure 5 shows the response $c_2(t')$.

If $c_1'(T) \neq 0$ the response $c_2(t')$ requires an instantaneous change in velocity and hence an infinite acceleration of the system output. For a physical system there is always mass or inertia associated with the output; therefore infinite acceleration requires infinite force.

SECOND AND THIRD ORDER CASES FOR UNBOUNDED QUENCHING SIGNAL

We consider a second order system with the transfer function

$$G(s) = \frac{k_1}{s^2 + a_1 s + a_0} \quad . \tag{17}$$

The differential equation describing the system is

$$\frac{d^2 c_1}{d(t')^2} + a_1 \frac{dc_1}{dt'} + a_0 c_1 = 0 \tag{18}$$

where the input is zero since we consider only the response for $t' \geq 0$ and $m_1(t) = 0$ for $t' \geq 0$.

Taking the Laplace transform of both sides of Equation (18) yields

$$(s^2 + a_1 s + a_0) C_1(s) - (sc_0 + c_0' + a_1 c_0) = 0 \tag{19}$$

where c_0 and c_0' are the initial conditions at $t' = 0$.

Solving Equation (19) for $C_1(s)$ we obtain

$$C_1(s) = \frac{c_0 s + a_1 c_0 + c_0'}{s^2 + a_1 s + a_0} \quad . \tag{20}$$

Substituting $C_1(s)$ from Eouation (20) and $G(s)$ from Equation (17) into Equation (12) and solving for $M_2(s)$ we get

$$M_2(s) = -\frac{1}{k_1}\left[c_0 s + (a_1 c_0 + c_0')\right] \quad . \tag{21}$$

The unit impulse $\delta(t')$ may be defined by

$$\delta(t') = \lim_{a \to 0} \frac{u(t') - u(t' - a)}{a} \tag{22}$$

where $u(t')$ and $u(t' - a)$ are unit step functions at $t' = 0$ and $t' = a$ respectively. The derivative of a unit impulse is taken as

$$\frac{d\,\delta(t')}{dt'} = \lim_{a \to 0} \frac{u(t') - 2u(t' - a) + u(t' - 2a)}{a^2} \tag{23}$$

in the sense that the limit of the Laplace transform of the fraction in Equation (23) as $a \to 0$ is s. The plot of this fraction versus t' is a double pulse, three of which are pictured in Figure 7c.

Similarly the second derivative is taken as

$$\frac{d^2\,\delta(t')}{dt'} = \lim_{a \to 0} \frac{u(t') - 3u(t' - a) + 3u(t' - 2a) - u(t' - 3a)}{a^3} \tag{24}$$

where the limit of the Laplace transform of the fraction in Eouation (24) is s^2. The plot of this fraction is a triple pulse.

We shall illustrate the use of the fraction on the right in relation (22). Consider a simple second order system described by the differential equation

$$c'' = k_1\, m_2\,(t') \tag{25}$$

where the primes denote differentiation with respect to t'.

Let $C(s)$ be the Laplace transform of $c(t')$ with respect to the time variable t' and $M_2(s)$ be the Laplace transform of $m_2(t')$. Taking the Laplace transform of both sides of Equation (25) and solving for $C(s)$ we obtain

$$C(s) = \frac{k_1 M_2(s)}{s^2} + \frac{sc_o + c_o'}{s^2} \tag{26}$$

where $c = c_o$ and $c' = c_o'$ at $t' = 0$.

The approximation to a unit impulse and its derivative is given by Equations (22) and (23) when $a \neq 0$. We let

$$m_2(t') = -W_1\left[\frac{u(t') - u(t' - a)}{a}\right] - W_2\left[\frac{u(t') - 2u(t' - a) + u(t' - 2a)}{a^2}\right] \quad (27)$$

where W_1 is the weight of the approximate impulse and W_2 is the weight of the approximate derivative.

Taking the Laplace transform of both sides of Equation (27) we get

$$M_2(s) = -\left(\frac{W_1}{a} + \frac{W_2}{a^2}\right)\frac{1}{s} + \left(\frac{W_1}{a} + \frac{2W_2}{a^2}\right)\frac{e^{-as}}{s} - \frac{W_2}{a^2}\frac{e^{-2as}}{s} \quad . \quad (28)$$

Substituting $M_2(s)$ from Equation (28) into Equation (26), taking the inverse Laplace transform and collecting t ms we obtain

$$\begin{aligned} c(t') &= \frac{k_1W_2a}{2} - k_1W_2 - k_1W_1t' + c_o + c_o't' \\ c'(t') &= c_o' - k_1W_1 \quad . \end{aligned} \quad (29)$$

We now let $a \to 0$ in Equations (29). The result is

$$\begin{aligned} c(t') &= c_o + c_o't' - k_1(W_2 + W_1t') \\ c'(t') &= c_o' - k_1W_1 \quad . \end{aligned} \quad (30)$$

If $c(t') = c'(t') = 0$ for $t' \to 0$ we must have

$$\begin{aligned} W_1 &= \frac{c_o'}{k_1} \\ W_2 &= \frac{c_o}{k_1} \quad . \end{aligned} \quad (31)$$

Letting $a \to 0$ in Equations (29) gives the same result as letting $a \to 0$ in Equations (22) and (23).

At $t' = 0^-$ we have $c = c_o$ and $c' = c_o'$. At $t' = 0^+$ we have $c = c' = 0$. Therefore c_o and c_o' are reduced to zero instantaneously at $t' = 0$ for the values of W_1 and W_2 given by Equations (31).

We now take the inverse Laplace transform of both sides of Equation (21) to obtain

$$m_2(t') = -\frac{1}{k_1}\left[c_o \mathcal{L}^{-1}(s) + (a_1c_o + c_o')\,\delta(t')\right] \tag{32}$$

where $\delta(t')$ is the unit impulse and $\mathcal{L}^{-1}(s)$ is the derivative of the unit impulse, referred to as the "doublet" 6 , 7. The initial condition c_o is the weight associated with the doublet and $(a_1c_o + c_o')$ is that of the impulse.

If the system is third order with the transfer function

$$G(s) = \frac{k_1}{s^3 + a_2s^2 + a_1s + a_o} \tag{33}$$

$M_2(s)$ is given by

$$M_2(s) = -\frac{1}{k_1}\left[c_os^2 + (c_o' + a_2c_o)s + (c_o'' + a_2c_o' + a_1c_o)\right] . \tag{34}$$

The inverse Laplace transform of both sides of Equation (34) now yields

$$m_2(t') = -\frac{1}{k_1}\left[c_o\mathcal{L}^{-1}(s^2) + (c_o' + a_2c_o)\mathcal{L}^{-1}(s) + (c_o'' + a_2c_o' + a_1c_o)\,\delta(t')\right] \tag{35}$$

where $\mathcal{L}^{-1}(s^2)$ is the second derivative of the unit impulse and is referred to as the "triplet". The weights of the triplet, doublet and impulse are c_o, $(c_o'+a_2c_o)$ and $(c_o''+a_2c_o'+a_1c_o)$ respectively.

It is noted that the weights of the impulse and its derivatives are determined from the initial conditions at $t' = 0$ and the system parameters. The order of the highest derivative is one less than the order of the system.

The signal $m_2(t')$ is not physically realizable due to the appearance of the impulse and its derivatives whence the error $e(t')$ will not be zero.

Truxal shows that the optimum manner for an ideal system to reach the origin from some point in the phase plane is as shown in Figure 6. 8. The path begins at some initial condition c_o, c_o' and goes to infinity then returns along the c' ordinate from infinity to the origin. Since the area under the reciprocal plot of $1/c'$ versus c is zero, and time in the phase plane is given by

$$t' = \int_{c_o}^{0} dc/c' ,$$

no time is required to reduce c to zero.

The unit impulse may be approximated by considering a pulse of finite height and short time duration such that the area under the pulse is unity.

Figure 7 a,b and c shows the family of paths in the phase plane, time domain and the respective quenching signals for a second order system. As the pulses become higher and of shorter time duration, the time to reach the origin from the initial condition at $t = T$ decreases.

To introduce the properly weighted pulses the computer must determine c_o, c_o', . . . at the instant $t = T$. In practice we cannot compute the third or higher order derivatives because of noise present in the system. We can however calculate the initial conditions from the known test signal and the system equation. Thus the control time may be computed in terms of the pulse height and the initial conditions at $t = T$.

SYSTEM WITH BOUNDED QUENCHING SIGNAL

The differential equation describing the linear system under study will be taken to be

$$\frac{d^n c}{d(t')^n} + a_{n-1}\frac{d^{n-1} c}{d(t')^{n-1}} + \ . \ . \ . + a_1 \frac{dc}{dt'} + a_o c = k_1 m_2 \tag{36}$$

where $|m_2| \leq k_2$. Then the quenching signal m_2 is bounded. Oldenburger has shown that under rather general conditions the best return to "equilibrium" is attained by operating "bang-bang".2. For the problem at hand equilibrium is the normal state of the system. In every reasonable sense the trajectory is optimum, i.e. least time to equilibrium, minimum overswing or underswing and minimum area between the trajectory and the t' axis, etc.

We shall now make use of the control functions for a bang-bang controller to obtain the control times. The quenching signal, scheduled according to the control times, will bring the system to equilibrium.

Example 1: Simple Second Order System

We consider the system described by the differential equation

$$c'' = k_1 m_2 \tag{37}$$

where the primes denote differentiation with respect to t'. The problem is to determine the control times in terms of the initial conditions at $t' = 0$.

The optimum control function Σ for the system of Equation (37) is given by .2

$$\sum = c + \frac{|c'| \ c'}{2k_1 k_2} \quad . \tag{38}$$

The schedule for optimum control is

$$m_2 = -(\operatorname{sgn} \Sigma)\, k_2 \tag{39}$$

where sgn Σ denotes the sign of Σ. Suppose that $\Sigma \neq 0$ at $t' = 0$. The transient from $t' = 0$ to the instant t_1' where $\Sigma = 0$ is the first phase of the solution. This is followed by a second phase terminating in equilibrium. The duration of this phase will be denoted by t_2'.

By Equation (37)

$$\begin{aligned} c' &= c_0' + k_1 m_2 t' \\ c &= c_0 + c_0' t' + \frac{k_1 m_2}{2} (t')^2 \end{aligned} \tag{40}$$

where $c = c_0$ and $c' = c_0'$ at $t' = 0$.

We substitute c' and c from Equations (40) into $\Sigma = 0$ to obtain

$$c_0 + c_0' t_1' + \frac{k_1 m_2}{2} (t_1')^2 + (\operatorname{sgn} c') \frac{(c_0' + k_1 m_2 t_1')^2}{2 k_1 k_2} = 0 \tag{41}$$

where sgn c' denotes the sign of c' when $\Sigma = 0$.

Solving for t_1' from Equation (41) we obtain

$$t_1' = \frac{(\operatorname{sgn} \Sigma) c_0' + \sqrt{\frac{1}{2}(c_0')^2 + (\operatorname{sgn} \Sigma) k_1 k_2 c_0}}{k_1\, k_2} \; . \tag{42}$$

Let c_1 and c_1' be the values of c and c' at $t' = t_1'$. By Equations (40) and (42) we have

$$c_1' = -(\operatorname{sgn} \Sigma) \sqrt{\frac{1}{2}(c_0')^2 + (\operatorname{sgn} \Sigma) k_1 k_2 c_0} \; . \tag{43}$$

The initial conditions for the last phase are c_1 and c_1'. Letting $t'' = t' - t_1'$ for the last phase we have

$$c' = c_1' + k_1 m_2 t'' \; . \tag{44}$$

When $t'' = t_2'$ the system is at equilibrium and $c' = 0$. Thus

$$t_2' = \frac{-\, c_1'}{k_1 m_2} \quad . \tag{45}$$

Substitution of c_1' from Equation (43) into Equation (45) yields

$$t_2' = \frac{\sqrt{\frac{1}{2}(c_0')^2 + (\operatorname{sgn} \Sigma) k_1 k_2 c_0}}{k_1\ k_2} \quad . \tag{46}$$

Equations (42) and (46) give t_1' and t_2' respectively in terms of the initial conditions c_0 and c_0'. It is unlikely that the case where $\Sigma = 0$ at $t = T$ will occur. This case is taken care of by having $t_1' = 0$. The times t_1' and t_2' approach zero as k_2 becomes arbitrarily large.

Equation (37) is the differential equation of the system after identification. We let $m_1(t)$ be the test signal, to be introduced at $t = 0$. The differential equation of the system for $t' < 0$ is

$$c'' = k_1\ m_1 \tag{47}$$

where the primes denote differentiation with respect to t.

We wish to express t_1' and t_2' in terms of the known test signal and the identification period T. We let the test signal be an impulse of weight W. Here m_1 is not bounded but it is understood that the quenching signal is. The solution to Equation (47) is

$$\begin{aligned} c(t) &= k_1 W t \\ c'(t) &= k_1 W \quad . \end{aligned} \tag{48}$$

At the end of the time interval T the identification is assumed to be complete and Equation (37) applies. Thus the initial conditions for Equation (37) are

$$c(T) = c_o = k_1WT$$
$$c'(T) = c_o' = k_1W \qquad (49)$$

Here c_o and c_o' are both positive. Hence $\Sigma > 0$. We substitute c_o and c_o' into Equations (42) and (46) to obtain t_1' and t_2' respectively. The control times are given by

$$t_1' = \frac{W + \sqrt{\frac{1}{2}W^2 + k_2WT}}{k_2}$$
$$t_2' = \frac{\sqrt{\frac{1}{2}W^2 + k_2WT}}{k_2} \qquad (50)$$

It is of interest to note that

$$t_2' = t_1' - W/k_2 \ .$$

Physically one cannot obtain an impulse but must approximate the impulse by a pulse of finite height and short time duration. Figure 8 shows an approximation to an impulse where H is the height and a is the duration. It is assumed that $a \ll T$.

If m_1 is the pulse shown in Figure 8, the initial conditions for Equation (37) are obtained by solving Equation (47). These initial conditions are

$$c_o = \tfrac{1}{2}k_1H(2aT - a^2)$$
$$c_o' = k_1Ha \qquad (51)$$

Substitution of the initial conditions from Equations (51) into Equations (42) and (46) yields

$$t_1' = \frac{Ha + \sqrt{\tfrac{1}{2}H^2a^2 + \tfrac{1}{2}k_2H(2aT - a^2)}}{k_2}$$

$$t_2' = \frac{\sqrt{\tfrac{1}{2}H^2a^2 + \tfrac{1}{2}k_2H(2aT - a^2)}}{k_2} \quad . \tag{52}$$

We define

$$H = W/a \ .$$

Letting $a \to 0$ while W remains constant Equations (52) reduce to Equations (50).

If m_1 is a step input of height H and duration T, the control times become

$$t_1' = \frac{HT + \sqrt{\tfrac{1}{2}H^2T^2 + \tfrac{1}{2}k_2HT^2}}{k_2}$$

$$t_2' = \frac{\sqrt{\tfrac{1}{2}H^2T^2 + \tfrac{1}{2}k_2HT^2}}{k_2} \quad . \tag{53}$$

We now have expressions for the control times in terms of the test signal m_1, T and k_2. We note that the control times for the simple second order system are independent of the system and are fixed by the test signal. Thus it is relatively easy to schedule the quenching signal when the test signal is known.

Example 2: Third Order System

We consider the system described by the differential equation

$$\tau c''' + c'' = k_1 m_2 \tag{54}$$

where $|m_2| \le k_2$, τ is the time constant of the system and the primes denote differentiation with respect to t'.

For the system of Equation (54) Oldenburger introduces the two control functions Σ_1 and Σ_2 where

$$\sum_1 = \psi + \frac{|\psi'|\psi'}{2k_1k_2} - (\operatorname{sgn}\psi')k_1k_2\tau^2 \ln^2\left\{1 + \sqrt{1 - \left[1 + (\operatorname{sgn}\psi')\frac{c''}{k_1k_2}\right]\exp\left(\frac{-|\psi'|}{k_1k_2\tau}\right)}\right\} \tag{55}$$

$$\sum_2 = \psi + \frac{|\psi'|\psi'}{2k_1k_2} \tag{56}$$

where

$$\psi = c + \tau c' \quad . \tag{57}$$

For disturbances normally encountered the log term of Σ_1 is small compared to the rest of the terms, and can therefore be dropped. 2. We therefore take $\Sigma = \Sigma_2$.

Optimum control is obtained by using Equation (39) and normally involves three phases, for each of which m_2 is k_2 or $-k_2$.

We begin with a set of initial conditions c_o, c_o' and c_o'' and let the system travel over the first phase to the first switch point where $\Sigma = 0$. During this phase Equation (54) becomes

$$\tau c''' + c'' = -(\operatorname{sgn}\Sigma)k_1k_2 \ . \tag{58}$$

The solution to Equation (58) is

$$c = \tau^2\left[c_o'' + (\operatorname{sgn}\Sigma)k_1k_2\right](e^{-\frac{1}{\tau}t'} - 1) + c_o + (c_o' + \tau c_o'')t' + (\operatorname{sgn}\Sigma)\tau k_1k_2t' - (\operatorname{sgn}\Sigma)\tfrac{1}{2}k_1k_2(t')^2$$

$$c' = (c_o' + \tau c_o'') + (\operatorname{sgn}\Sigma)\tau k_1k_2 - \tau\left[c_o'' + (\operatorname{sgn}\Sigma)k_1k_2\right]e^{-\frac{1}{\tau}t'} - (\operatorname{sgn}\Sigma)k_1k_2t'$$

$$c'' = \left[c_o'' + (\operatorname{sgn}\Sigma)k_1k_2\right]e^{-\frac{1}{\tau}t'} - (\operatorname{sgn}\Sigma)k_1k_2 \tag{59}$$

where $c = c_0$, $c' = c_0'$ and $c'' = c_0''$ at $t' = 0$. We let $c = c_1$, $c' = c_1'$ and $c'' = c_1''$ at $t' = t_1'$; these values of c, c' and c" are the initial conditions for the next phase. At $t' = t_1'$ we form the function ψ_1 where

$$\psi_1 = c_1 + \tau c_1' \quad . \tag{60}$$

Substitution of c_1, c_1' and c_1'' from Equations (59) into Equation (60) and the equation for ψ_1' yields

$$\begin{aligned} \psi_1 &= \psi_0'(t_1' + \tau) - (\operatorname{sgn}\Sigma)\tfrac{1}{2}k_1k_2(t_1')^2 + (c_0 - \tau^2 c_0'') \\ \psi_1' &= \psi_0' - (\operatorname{sgn}\Sigma)k_1k_2t_1' \end{aligned} \tag{61}$$

where ψ_0 and ψ_0' are the values of ψ and ψ' at $t' = 0$.

At $t' = t_1'$, ψ_1 and ψ_1' satisfy $\Sigma = 0$. Hence

$$\Sigma = \psi_0'(t_1' + \tau) - (\operatorname{sgn}\Sigma)\tfrac{1}{2}k_1k_2(t_1')^2 + (c_0 - \tau^2 c_0'') + (\operatorname{sgn}\psi_1')\frac{\left[\psi_0' - (\operatorname{sgn}\Sigma)k_1k_2t_1'\right]^2}{2k_1k_2} = 0 \ . \tag{62}$$

Solving for t_1' from Equation (62) we get

$$t_1' = \frac{(\operatorname{sgn}\Sigma)\psi_0' + \sqrt{\tfrac{1}{2}(\psi_0')^2 + (\operatorname{sgn}\Sigma)k_1k_2\psi_0}}{k_1k_2} \ . \tag{63}$$

For the second phase we have

$$\tau c''' + c'' = + (\operatorname{sgn}\Sigma)k_1k_2 \ . \tag{64}$$

The initial conditions are c_1, c_1' and c_1'' given by Equations (59) at $t' = t_1'$. Letting $t'' = t' - t_1'$ the solution of Equation (64) is

$$c = \tau^2\left[c_1'' - (\operatorname{sgn}\Sigma)k_1k_2\right](e^{-\frac{1}{\tau}t''} - 1) + c_1 + (c_1' + \tau c_1'')t''$$
$$- (\operatorname{sgn}\Sigma)\tau k_1k_2t'' + (\operatorname{sgn}\Sigma)\tfrac{1}{2}k_1k_2(t'')^2$$

$$c' = (c_1' + \tau c_1'') - (\operatorname{sgn}\Sigma)\tau k_1k_2 - \tau\left[c_1'' - (\operatorname{sgn}\Sigma)k_1k_2\right]e^{-\frac{1}{\tau}t''}$$
$$+ (\operatorname{sgn}\Sigma)k_1k_2t'' \tag{65}$$

$$c'' = \left[c_1'' - (\operatorname{sgn}\Sigma)k_1k_2\right]e^{-\frac{1}{\tau}t''} + (\operatorname{sgn}\Sigma)k_1k_2 \quad .$$

We let $c = c_2$, $c' = c_2'$, $c'' = c_2''$ at $t'' = t_2'$. Let Ψ_2 and Ψ_2' be the values of Ψ and Ψ' at $t'' = t_2'$. Substituting c_2, c_2' and c_2'' from Equations (65) into Ψ_2 and Ψ_2', and substituting the result into $\Sigma = 0$, we obtain

$$\Psi_1'(t_2' + \tau) + (\operatorname{sgn}\Sigma)\tfrac{1}{2}k_1k_2(t_2')^2 + (c_1 - \tau^2c_1'')$$
$$+ (\operatorname{sgn}\Psi_2')\frac{\left[\Psi_1' + (\operatorname{sgn}\Sigma)k_1k_2t_2'\right]^2}{2k_1k_2} = 0 \quad . \tag{66}$$

Solving for t_2' from Equation (66) we obtain

$$t_2' = \frac{-(\operatorname{sgn}\Sigma)\Psi_1' + \sqrt{\tfrac{1}{2}(\Psi_1')^2 - (\operatorname{sgn}\Sigma)k_1k_2\Psi_1}}{k_1k_2} \quad . \tag{67}$$

We may determine Ψ_1 and Ψ_1' in terms of c_o, c_o', c_o'' and t_1' by substituting c_1, c_1' and c_1'' from Equations (59) into Equation (60) and the equation for Ψ_1'.

For the last phase let $t''' = t' - (t_1' + t_2')$. For this phase we have Equation (58) valid where the initial conditions are now c_2, c_2' and c_2'', obtained from Equations (65) at $t'' = t_2'$. The form of the solution for this phase is given by Equations (59). At $t''' = t_3'$ we have $c_3 = c_3' = c_3'' = 0$. We may determine t_3' by setting any of the three quantities equal to zero. It is sufficient to write

$$c_3'' = \left[c_2'' + (\operatorname{sgn}\Sigma)k_1k_2\right]e^{-\frac{1}{\tau}t_3'} - (\operatorname{sgn}\Sigma)k_1k_2 = 0 \tag{68}$$

whence

$$t_3' = \tau \ln\left[1 + (\operatorname{sgn}\Sigma)\frac{c_2''}{k_1 k_2}\right] . \tag{69}$$

We may determine c_2'' in terms of c_o, c_o', c_o'', t_1' and t_2' by substituting c_1'' from the last of Equations (59) into the last of Equations (65).

Equation (54) is the differential equation of the system after identification. We let $m_1(t)$ be the test signal, to be introduced at $t = 0$. The differential equation of the system for $t' < 0$ is

$$\tau c''' + c'' = k_1 m_1 \tag{70}$$

where the primes denote differentiation with respect to t.

Let the test signal be an impulse of weight W. The quenching signal m_2 is still assumed bounded. The solution to Equation (70) is

$$\begin{aligned} c(t) &= k_1 W(\tau e^{-\frac{1}{\tau}t} + t - \tau) \\ c'(t) &= k_1 W(1 - e^{-\frac{1}{\tau}t}) \\ c''(t) &= \frac{k_1 W}{\tau} e^{-\frac{1}{\tau}t} \quad . \end{aligned} \tag{71}$$

At the end of the time interval T the identification is assumed to be complete and Equation (54) applies. The initial conditions for Equation (54) are

$$\begin{aligned} c(T) &= c_o = k_1 W(\tau e^{-\frac{1}{\tau}T} + T - \tau) \\ c'(T) &= c_o' = k_1 W(1 - e^{-\frac{1}{\tau}T}) \\ c''(T) &= c_o'' = \frac{k_1 W}{\tau} e^{-\frac{1}{\tau}T} \quad . \end{aligned} \tag{72}$$

From Equations (72), ψ_o and ψ_o' we see that $\Sigma > 0$ at $t' = 0$. Substitution of c_o, c_o' and c_o'' from Equations (72) into Equation (63) yields

$$t_1' = \frac{W + \sqrt{\frac{1}{2}W^2 + k_2 WT}}{k_2} \qquad (73)$$

We now substitute c_o, c_o' and c_o'' from Equations (72) into Equations (59) to obtain c_1, c_1' and c_1'' and substitute the result into Equation (60) to obtain ψ_1 and ψ_1'. Substituting the resulting expressions for ψ_1 and ψ_1' into Equation (67) we get

$$t_2' = \frac{1}{k_2}\left[(k_1 t_1' - W) + \sqrt{\tfrac{1}{2}(W - k_2 t_1')^2 + k_2\left(\tfrac{1}{2}k_2(t_1')^2 - W(t_1' + T)\right)}\right] . \qquad (74)$$

Substituting c_1'' from the last of Equations (59) into the last of Equations (65) and substituting the result into Equation (69) we obtain

$$t_3' = \tau \ln\left\{2 + \frac{1}{k_2}\left[\left(\frac{W}{\tau} e^{-\frac{1}{\tau}T} + k_2\right)e^{-\frac{1}{\tau}t_1'} - 2k_2\right] e^{-\frac{1}{\tau}t_2'}\right\} . \qquad (75)$$

If the impulse is approximated by a pulse of height H and duration a, the control times become

$$t_1' = \frac{Ha + \sqrt{\frac{1}{2}H^2a^2 + k_2 Ha(T - \frac{1}{2}a)}}{k_2}$$

$$t_2' = \frac{(k_2 t_1' - Ha)}{k_2} + \frac{1}{k_2}\sqrt{\tfrac{1}{2}(Ha - k_2 t_1')^2 + k_2\left(\tfrac{1}{2}k_2(t_1')^2 - Ha(t_1' + T - \tfrac{1}{2}a)\right)} \qquad (76)$$

$$t_3' = \tau \ln\left\{2 + \frac{1}{k_2}\left[\left(He^{-\frac{1}{\tau}T}(e^{\frac{1}{\tau}a} - 1) + k_2\right) e^{-\frac{1}{\tau}t_1'} - 2k_2\right] e^{-\frac{1}{\tau}t_2'}\right\} .$$

If the test signal is a step of height H and duration T, the control times are

$$t_1' = \frac{HT + \sqrt{\frac{1}{2}H^2T^2 + \frac{1}{2}k_2HT^2}}{k_2}$$

$$t_2' = \frac{(k_2t_1'-HT)}{k_2} + \frac{1}{k_2}\sqrt{\frac{1}{2}(HT-k_2t_1')^2 + k_2\left(\frac{1}{2}k_2(t_1')^2-HT(t_1'+\frac{1}{2}T)\right)} \qquad (77)$$

$$t_3' = \tau \ln\left\{2 + \frac{1}{k_2}\left[\left(H(1-e^{-\frac{1}{\tau}T})+k_2\right) e^{-\frac{1}{\tau}t_1'} -2k_2\right] e^{-\frac{1}{\tau}t_2'} \right\} .$$

We note that t_1' and t_2' <u>are independent of the system constants</u> k_1 <u>and</u> τ <u>for all three test signals</u>. The first two control times are then constant and only t_3' must be computed after τ is identified. The value of t_1' and t_2' can be stored in the computer memory and used to obtain t_3' immediately after identification.

Higher order systems were not considered, as the expressions for the control times become excessively complicated. Also, if the output is measured to determine the initial conditions, derivatives of order higher than the second are difficult to obtain because of noise present in the system.

For many systems one cannot solve for the control times explicitly. This is true for a general second order system.

<u>Experimental Results</u>

The results obtained for Example 1 were verified on the analog computer. Figure 9 shows the actual and theoretical system response to the bounded quenching signal. The initial conditions on the system at $t' = 0$ correspond to the conditions after identification. The control times were calculated from Equations (42) and (46). It is seen that the actual response is very close to the theoretical. A relay was used to obtain the quenching signal. The relay had a small amount of deadband, which accounts for some of the deviation from the theoretical response.

We have thus found that it is possible to determine the control times and obtain proper switching with sufficient accuracy to obtain a trajectory which is nearly the same as the theoretical trajectory.

LIST OF FIGURES

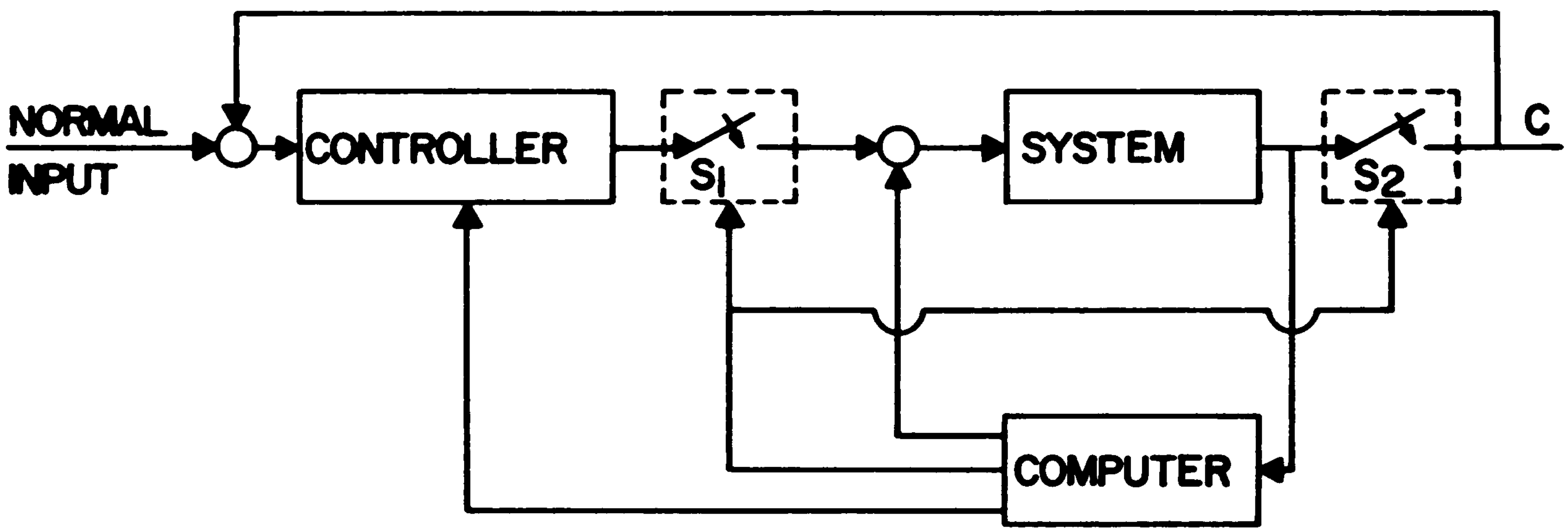


FIGURE 1

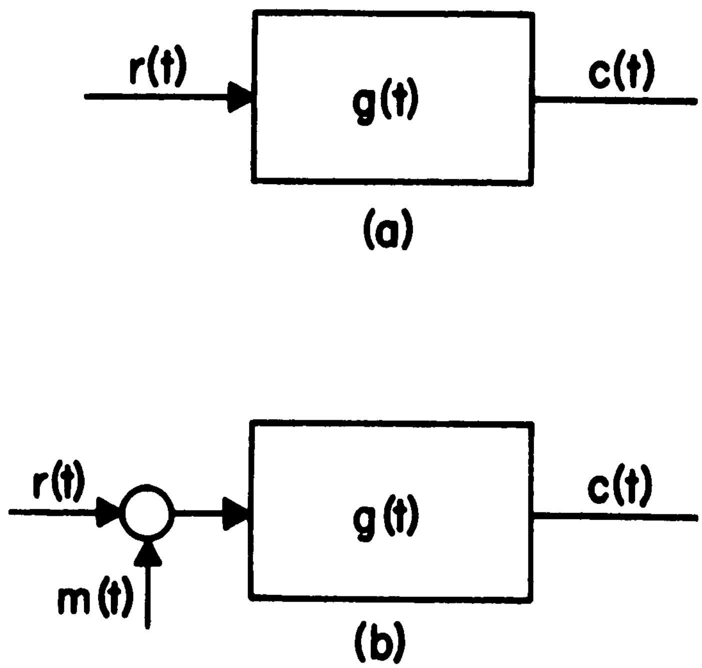


FIGURE 2

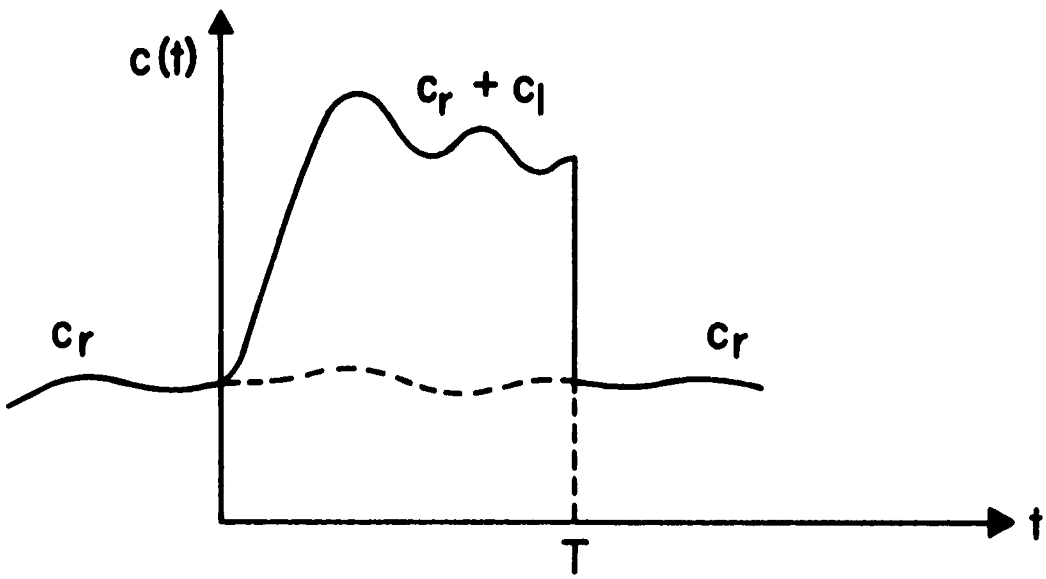


FIGURE 3

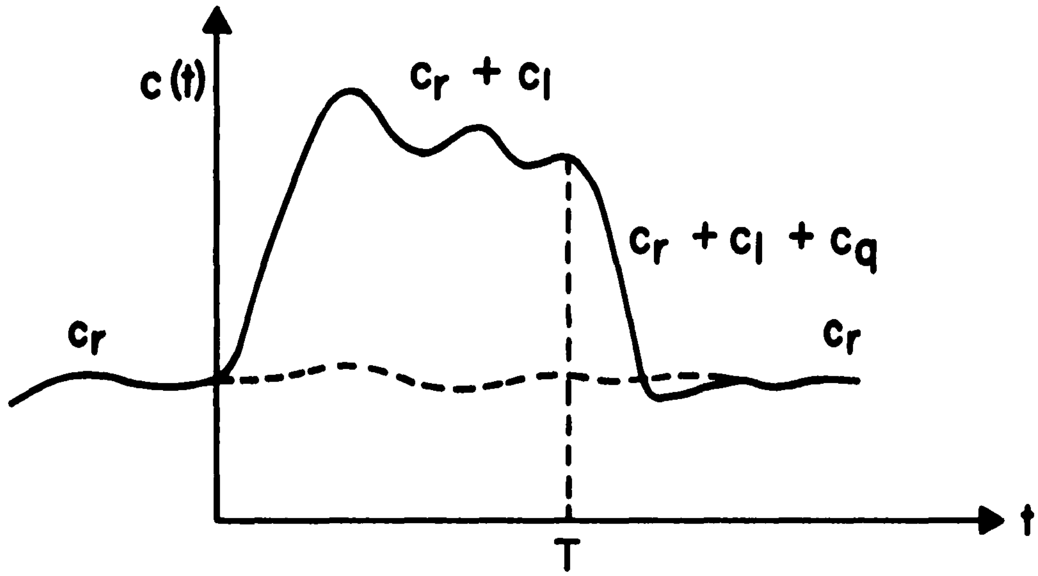


FIGURE 4

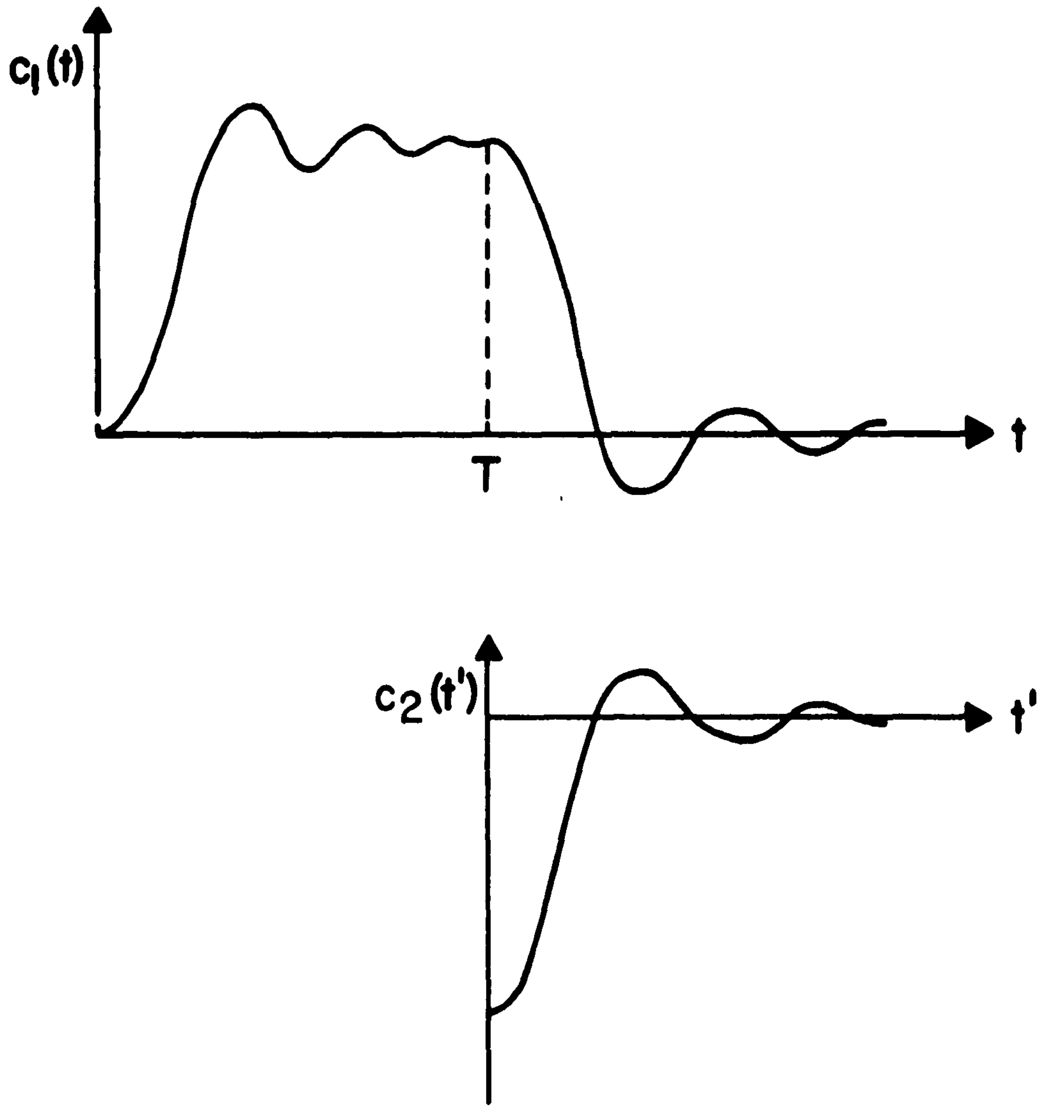
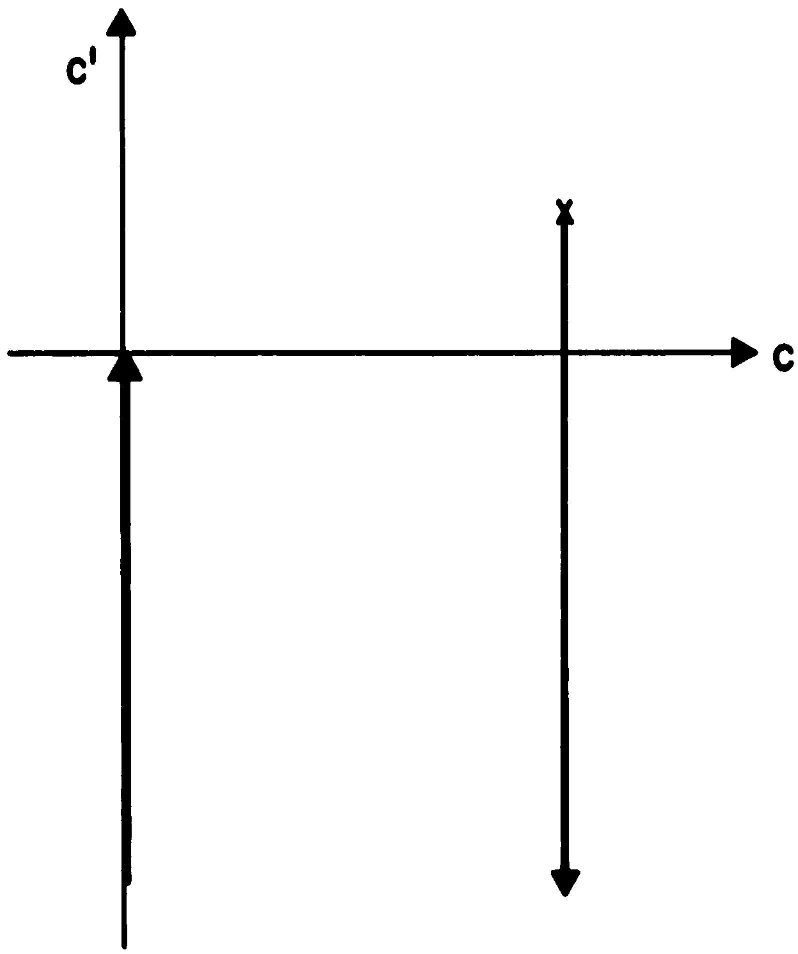


FIGURE 5



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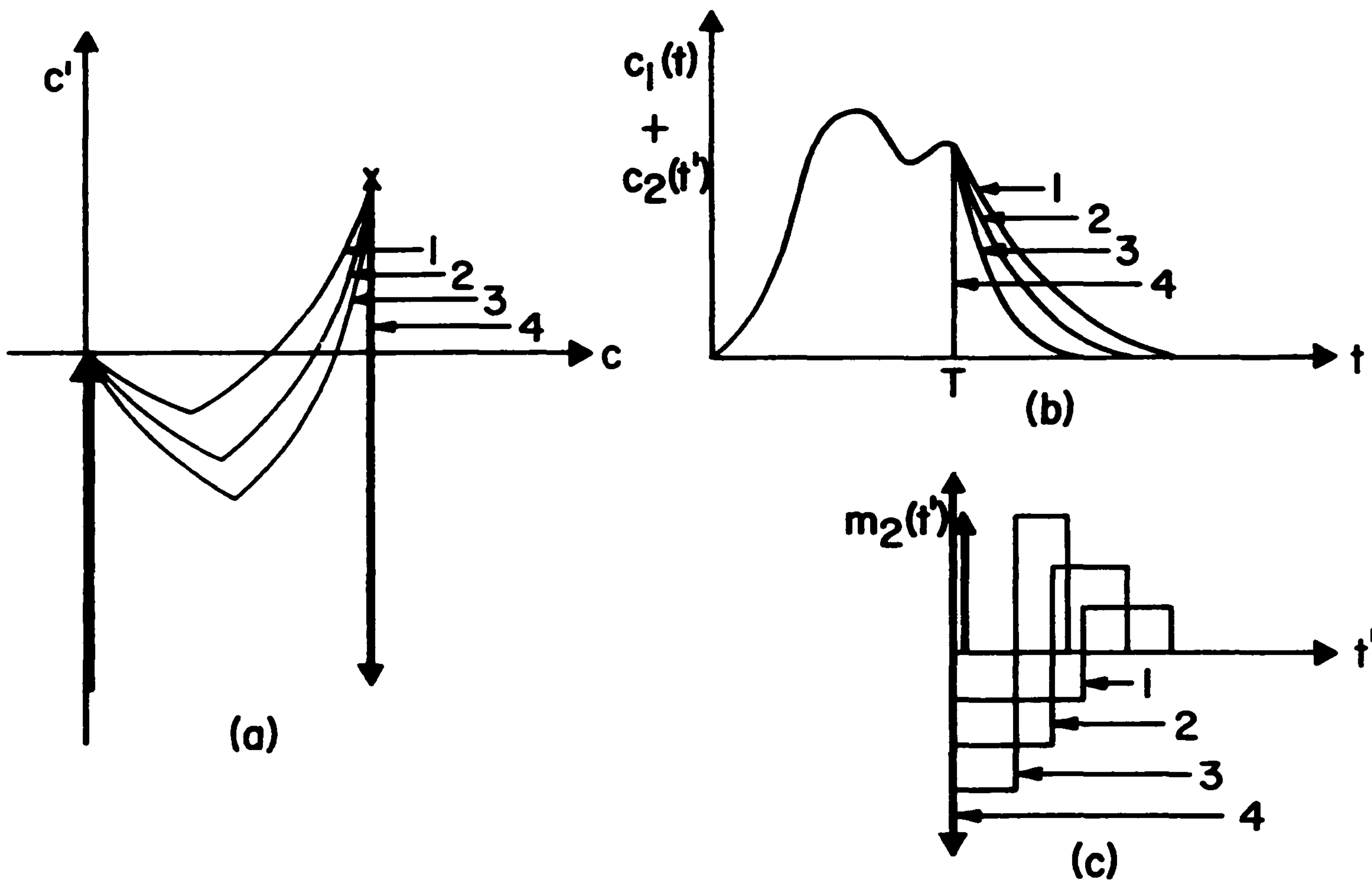

FIGURE 7

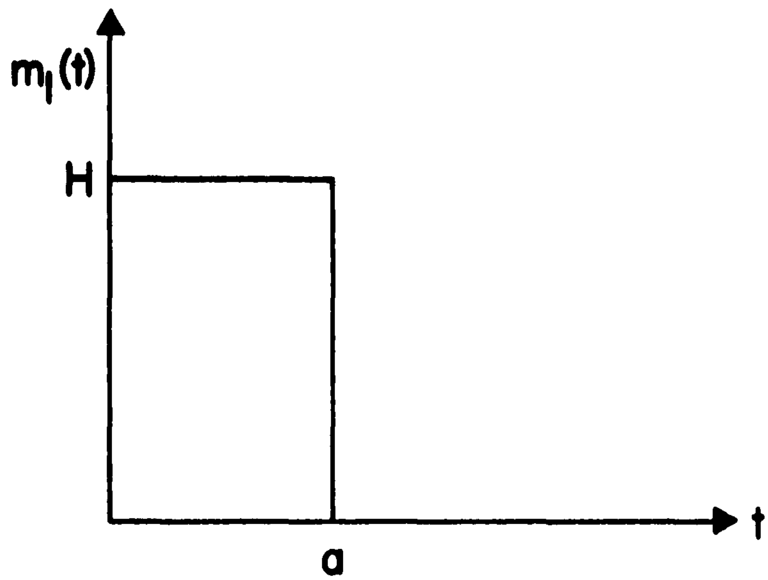

FIGURE 8

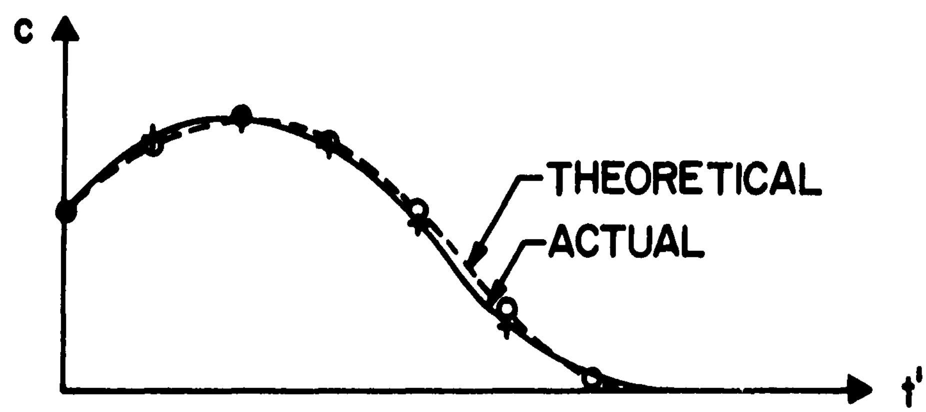

FIGURE 9

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