

UNCLASSIFIED

AD. 274 147

*Reproduced
by the*

**ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA**



UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

CATALOGED BY ASTIA
AS AD 147
274147

274147

TR-987

DIRECT-COUPLED WAVEGUIDE FILTERS

Robert F. Sullivan

12 March 1962

62-3-1



**DIAMOND ORDNANCE FUZE LABORATORIES
ORDNANCE CORPS ● DEPARTMENT OF THE ARMY
WASHINGTON 25, D. C.**

ORDNANCE CORPS
DIAMOND ORDNANCE FUZE LABORATORIES
WASHINGTON 25, D. C.

Robert W. McEvoy, Lt Col
COMMANDING

B. M. Horton
TECHNICAL DIRECTOR

The Diamond Ordnance Fuze Laboratories is a research, development, and engineering installation under the jurisdiction of the Chief of Ordnance.

The Diamond Ordnance Fuze Laboratories was established by the Ordnance Corps, Department of the Army, on 27 September 1953. The nucleus for these Laboratories was the personnel and facilities of the Ordnance Division of the National Bureau of Standards.

Typical fields of activity at the Diamond Ordnance Fuze Laboratories include electronics, physics, mechanics, chemistry, and applied mathematics. Examples of topics under these activities are radiation and field studies, circuit devices, chemical problems, and special electron tube design. The programs include all phases from basic research to product design.

The mission of the Laboratories is to:

1. Conduct research and development in the various physical science and engineering fields directed toward meeting the military characteristics for fuzes and related items.
2. Provide consulting and liaison services as required in connection with the development, production, and use of items developed in the laboratories, or of related items.
3. Fabricate models and prototypes of items under development at the laboratories.
4. Perform developmental testing, including destructive testing of prototypes.
5. Serve as principal Nuclear Radiation Effects Research Group to investigate and determine susceptibility of Ordnance electronic materiel to nuclear weapons radiation environment, mechanisms of those effects, and ways and means of developing less susceptible materiel.
6. Maintain and operate for OCO a special library of technical and progress reports, prepared by Army, Navy, Air Force, and their contractors.
7. Perform the Industrial Engineering Support Mission for all proximity fuze items.
8. Administer the Department of the Army Regional Training Center for the District of Columbia, Virginia, and Maryland region.

DIAMOND ORDNANCE FUZE LABORATORIES
ORDNANCE CORPS **WASHINGTON 25, D. C.**

DA-5N06-01-014
OMS Code 5530.11.62400
DOFL Proj No. 26300

TR-987

12 March 1962

DIRECT-COUPLED WAVEGUIDE FILTERS

Robert F. Sullivan

FOR THE COMMANDER:
APPROVED BY



H. Sommer

Chief, Laboratory 200



Qualified requesters may obtain copies of this report from ASTIA.

C O N T E N T S

	<u>Page</u>
ABSTRACT	5
1. INTRODUCTION	5
2. DESIGN DESCRIPTION	5
2.1 Mathematical Procedures and Construction Considerations	5
2.1.1 Insertion Loss	7
2.1.2 Waveguide Obstacles	9
2.1.3 Tuning	10
2.1.4 Construction Details	10
2.2 Summary of Design Analysis	11
3. SAMPLE DESIGN	23
3.1 Filter Requirements	23
3.2 Bandwidth Determination	23
 <u>APPENDICES:</u>	
A. WAVEGUIDE & FABRICATION TOLERANCES— MATHEMATICAL DERIVATIONS	35-38
B. SCALING	39
C. MEASUREMENTS	40-43
 Distribution	 45-46

ILLUSTRATIONS

Figure No.

- 1 Schematic filters.
- 2, 3 Two-post susceptances.
- 4, 5, 6 Two-post spacings.
- 7, 8 Three-post susceptances.
- 9, 10 Three-post spacings.
- 11 In-band filter characteristics.
- 12 Off-band filter characteristics.
- 13 Wavelength difference versus frequency.
- 14 Diagram of sample design.
- 15 Six-section detuned filter.
- 16 Insertion loss versus frequency of six-section filter.
- 17 In-band VSWR of six-section filter.
- 18 In-band insertion loss of six-section filter.
- C-1 Equivalent circuits used in measuring B and ϵ .
- C-2 Null-shift curve.

ABSTRACT

Design data are presented for rapid calculation in designing direct-coupled maximally flat bandpass filters in X-band using multiple-post obstacles. The data may be used for any TE_{10} mode waveguide by using appropriate scaling factors. A design example is given to illustrate the design procedure, and the effects of mechanical tolerances are discussed.

1. INTRODUCTION

The design of waveguide bandpass filters has been treated extensively from a theoretical viewpoint (ref 1, 2, 3). The two types of multisection waveguide filters are the quarter wavelength coupled filter and the direct-coupled filter. Design data in a form suitable for rapid calculation are available for quarter wavelength coupled filters (ref 4). Such data for direct-coupled maximally flat bandpass filters in X-band will be presented using the equations given by Cohn (ref 3).

Both types of filters are constructed of spaced susceptances. Because only normalized susceptance (the ratio of a susceptance to the characteristic admittance of the waveguide) may be measured, a susceptance as used in this report is assumed to be normalized.

The quarter wavelength coupled filter consists of a series of separate resonant waveguide sections coupled by waveguide sections an electrical quarter wavelength long. Each resonant section consists of two identical obstacles spaced about a half wavelength apart. The desired response of the filter is obtained by adjusting the Q's of the resonant sections; the Q's are a function of the susceptances of the obstacles. A direct-coupled filter consists of an array of obstacles spaced about a half wavelength apart. Each adjacent pair of obstacles forms a resonant section. The susceptances of the obstacles are adjusted to give the desired response. Both types of filters may be derived from the same low-pass prototype (ref 5) and may be designed to have identical characteristics. The direct-coupled filter has the advantage that the quarter wavelength sections are eliminated, eliminating the frequency sensitivity of these sections and reducing the number of obstacles required. The disadvantage of the direct-coupled filter is the large values of susceptance required. By choosing the proper type of obstacle, this disadvantage may be minimized without the need of specifying extremely close mechanical tolerances.

2. DESIGN DESCRIPTION

2.1 Mathematical Procedures and Construction Considerations

The equations given in references 1 and 2 were derived with the assumption that the susceptance of an obstacle is independent of

guide wavelength; and for bandpasses of 1 or 2 percent this assumption is justified. The susceptance of an obstacle in waveguide has been found to vary directly with guide wavelength. Cohn derived equations for direct-coupled maximally flat waveguide filters assuming this variation. These equations are given below in Cohn's notation and refer to the schematic filter in figure 1(a), resonant at λ_{g0} .

$$X_{i-1, i} = \frac{L / \sqrt{g_{i-1} g_i}}{1 - (L^2 / g_{i-1} g_i)}$$

where

$X_{i-1, i}$ = normalized reactance of element

$$L = \pi (\lambda_{g1} - \lambda_{g2}) / (\lambda_{g1} + \lambda_{g2})$$

λ_{g1} and λ_{g2} = the guide wavelengths at the 3-db response points.

$$g_0 = L$$

$$g_i = 2 \sin [(2i - 1)\pi / 2n] \quad i \geq 1$$

n = number of sections in the filter.

ϕ_i = electrical spacing between obstacles.

The response of the filter is given by

$$Db_{\lambda_g} = 10 \log_{10} [1 + \omega'^{2n}]$$

where

$$\omega' = 2 (\lambda_{g0} - \lambda_g) / (\lambda_{g1} - \lambda_{g2})$$

It is convenient for computational purposes to use the schematic filter in figure 1(b), with susceptances $B_r = -\frac{1}{X_{i-1,i}}$ (inductive susceptances) and to rewrite the resulting equation in terms of coefficients

$$A_r = \sqrt{g_{i-1} g_i}$$

This results in

$$B_1 = -\frac{A_1}{\sqrt{L}} + \frac{\sqrt{L}}{A_1}$$

$$B_r = -\frac{A_r}{L} + \frac{L}{A_r}; r > 1$$

Values of A_r are given in table 1 for n up to 12. Note that the filter is symmetrical:

$A_1 = A_n$, $A_2 = A_{n-1}$, etc. The spacing between obstacles depends upon the particular type of obstacle and its susceptance.

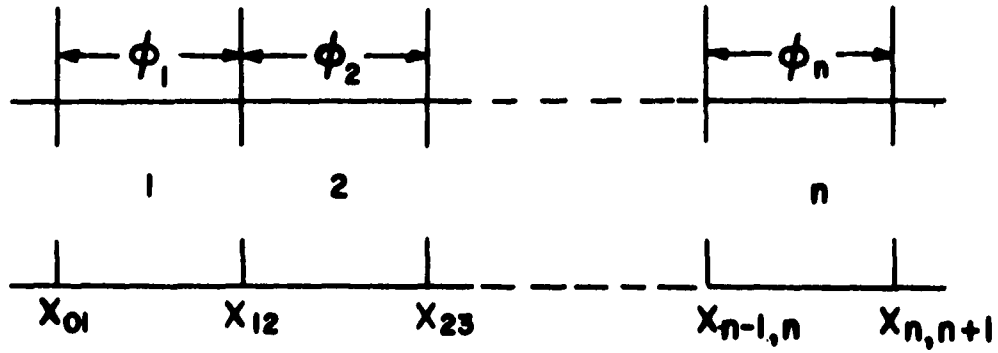
2.1.1 Insertion Loss

The equation for filter response assumes lossless waveguide. The loss effects are to decrease the off-band insertion loss of a filter and to increase the in-band insertion loss. Both effects have been treated by Cohn (ref 6). The insertion loss at resonance for a filter with loss is derived below, using a quarter wavelength coupled filter as a prototype rather than the low-pass prototype used by Cohn.

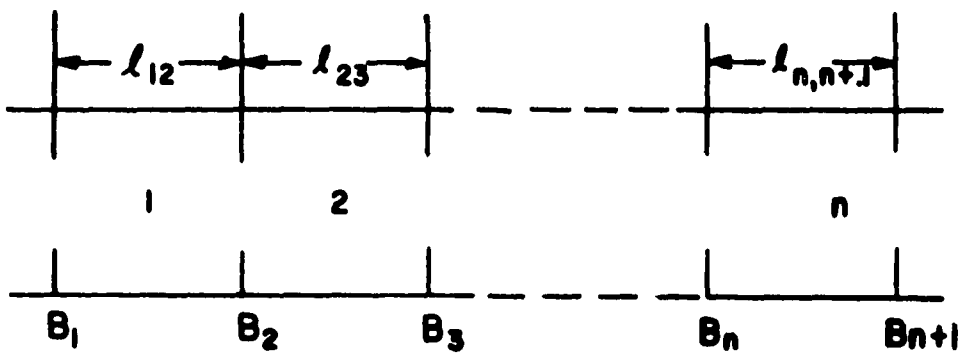
The insertion loss of a single resonant cavity is

$$D = 20 \log_{10} \frac{Q_0}{Q_0 - Q_L} \cong 8.68 \frac{Q_L}{Q_0},$$

where Q_0 is the unloaded cavity Q and Q_L the loaded Q . The unloaded



(Section a)



(Section b)

Figure 1. Schematic filters.

Q is a function of the energy dissipated in the interior of a cavity, and depends only upon the materials used in constructing the cavity and the type of obstacles employed. The loaded Q includes these losses as well as radiation to the connecting transmission lines; this is the Q that is measured and from which Q_o is calculated.

The Q_L 's of an n-section quarter wavelength coupled filter are distributed according to the equation (ref 1):

$$Q_r = \frac{f_o}{\Delta f} \sin \frac{2r-1}{2n} \pi$$

where Q_r is the Q_L of the r^{th} section,

f_o is the resonant frequency of the filter, and Δf is the frequency interval between half-power response points. The insertion loss of an n-section filter becomes

$$Db = \frac{8.68f_o}{Q_o \Delta f} \sum_{r=1}^n \sin \frac{2r-1}{2n} \pi = \frac{8.68}{Q_o} \frac{f_o}{\Delta f} K_n$$

The Q_o 's for full and half-size X-band waveguides and K_n and n up to 12 are given in tables 2 and 3. The tabulated values of Q_o are based on a measured Q_o of 3200 for full-size X-band 90 percent Cu and 10 percent Zn waveguide, using multiple post susceptances.

2.1.2 Waveguide Obstacles

The first consideration in choosing a type of obstacle is an adequate range of susceptances, since the maximum susceptances required for a direct-coupled filter may be as great as 90. The structure must be such that the filter may be fabricated without unduly stringent mechanical tolerances (appendix A). To construct filters from soft materials such as dip-brazed aluminum, an obstacle that adds rigidity to the structure is desirable, since for units that may be subject to severe shock and vibrational requirements, rigidity is necessary to avoid phase and amplitude-modulation effects. Multiple post susceptances meet these requirements. These obstacles are inductive susceptances and consist of two or three posts arranged symmetrically, transverse to the axis of the waveguide, and passing through both broad waveguide walls. Because the susceptances of post structures vary as λ_g/a , the susceptances are usually plotted as Ba/λ_g versus c, the

post displacement from the waveguide wall. The resonant spacing for a pair of identical obstacles can be expressed as $(\lambda_g/2)(1+\epsilon)$ where ϵ is a percentage correction to a half-wavelength spacing. Curves of Ba/λ_g and ϵ versus c are plotted in figures 2 through 10 for two- and three-post obstacles, with 3/32 in. diameters. These data are plotted for 1 in. x 1/2 in. O.D. waveguide and may be used for any TE_{10} mode waveguide by scaling as discussed in appendix B. The procedure for measuring susceptance and resonant spacing is given in appendix C.

2.1.3 Tuning

The cumulative effect of standard waveguide tolerances, fabrication tolerances, and measurement error in the susceptance data are discussed in appendix A. Because of the tolerances involved, a filter must be tuned after construction and designed such that the tuning method will bring the filter to the required resonant frequency. Tuning is accomplished either by inserting a screw in the center of the broad wall of each section of the filter, or by denting the broad wall. Both types of tuning add a capacitive susceptance at the center of each section and lower the resonant frequency. This requires that the spacing between obstacles be less than required for resonance at the desired frequency; 1/2- to 1-percent frequency tuning is usually provided.

Tuning a multiple section filter is done by the phase-shift method (ref 7). All sections are first detuned far off resonance; and the position of two adjacent nulls of the standing-wave pattern are found on a slotted line. The input and all odd sections are tuned so that the null falls exactly between the initial nulls. The even sections are all tuned to either original null and the final section is tuned for maximum transmission. The filter must be terminated in a matched load and the line must be well matched, looking from the slotted line back toward the generator. Because the filter response is a function of guide wavelength, the filter is tuned at the desired band center wavelength rather than at the center frequency.

2.1.4 Construction Details

Because of the tolerance in waveguide wall thickness, the posts should be dimensioned from the inside waveguide wall for best results. This may be done by one of three ways: (1) using a mandrel and selected waveguide, (2) accurately ground split-wedge parallel bars and unselected waveguide, or (3) by measuring wall thickness and skim cutting to a fixed wall thickness.

Solder fillets around the posts change the susceptances. These fillets can be minimized by using rivet-shaped posts and capping with a washer to prevent solder flow. For dip brazing, the retaining washer should be staked in place; the inner diameters of the brazing and retaining washers should be a close fit to the post diameter to insure that the retaining washer is well secured by staking and the brazing material flows without leaving gaps.

A tuning screw on a tunable filter is a probe and the screw and its bushing form a shorted coaxial line. Erratic tuning and increased insertion loss are caused by poor and variable thread contact. This can be avoided by using chokes or self-locking screws that thrust the screw against the thread. Where insertion loss is not critical, smooth tuning can be obtained by dielectric tuning such as a Teflon rod driven by a metal screw.

Provision for detuning must be made for detuned filters. A hole is drilled in the center of one broad wall of each filter section to receive a detuning probe; this hole also serves as a centering hole for a tuning clamp (ref 4). The clamp has two screws driven perpendicular to and in the center of the broad walls. One screw with a coarse thread locates the clamp and serves as a de-denting screw if the section has been over-tuned. The other screw with a small tip and a fine thread serves as the denting tool.

2.2 Summary of Design Analysis

In this section the necessary equations, graphs and tables for designing an n-section maximally flat waveguide filter are consolidated for convenience.

$$g = \frac{\lambda}{\sqrt{1 - (\lambda/2a)^2}} \quad (1)$$

$$L = \pi(\lambda_{g1} - \lambda_{g2}) / (\lambda_{g1} + \lambda_{g2}) \quad (2)$$

$$B_1 = -\frac{A_1}{\sqrt{L}} + \frac{\sqrt{L}}{A_1} \quad (3)$$

$$B_r = -\frac{A_r}{L} + \frac{L}{A_r}; \quad r > 1$$

$$B_r = B_{n-r+1}$$

$$l_{r, r+1} = (\lambda_g / 2)(1 + \epsilon_r / 2 + \epsilon_{r+1} / 2) \quad (4)$$

$$Db_{\lambda_{g0}} = \frac{8.68}{Q_0} \frac{f_0}{\Delta f} K_n \quad (5)$$

$$Db_{\lambda_g} = 10 \log_{10} [1 + \omega'^{2n}] \quad (6)$$

$$\omega' = (\lambda_{g0} - \lambda_g) / (\lambda_{g1} + \lambda_{g2})$$

where

λ_g = guide wavelength

λ = free-space wavelength = $\frac{1.1802 \times 10^{10}}{f}$ in.

a = largest waveguide internal dimension

$\lambda_{g1}, \lambda_{g2}$ = guide wavelengths corresponding to 3-db

response points.

B_r = the susceptance of the r^{th} obstacle.

A_r = a coefficient from table 1.

$l_{r, r+1}$ = the length of the resonant section between

B_r and B_{r+1} .

Q_0 = the unloaded Q of a resonant section (table 3).

K_n = a coefficient from table 2, computed for X-band waveguide at 9300 Mc.

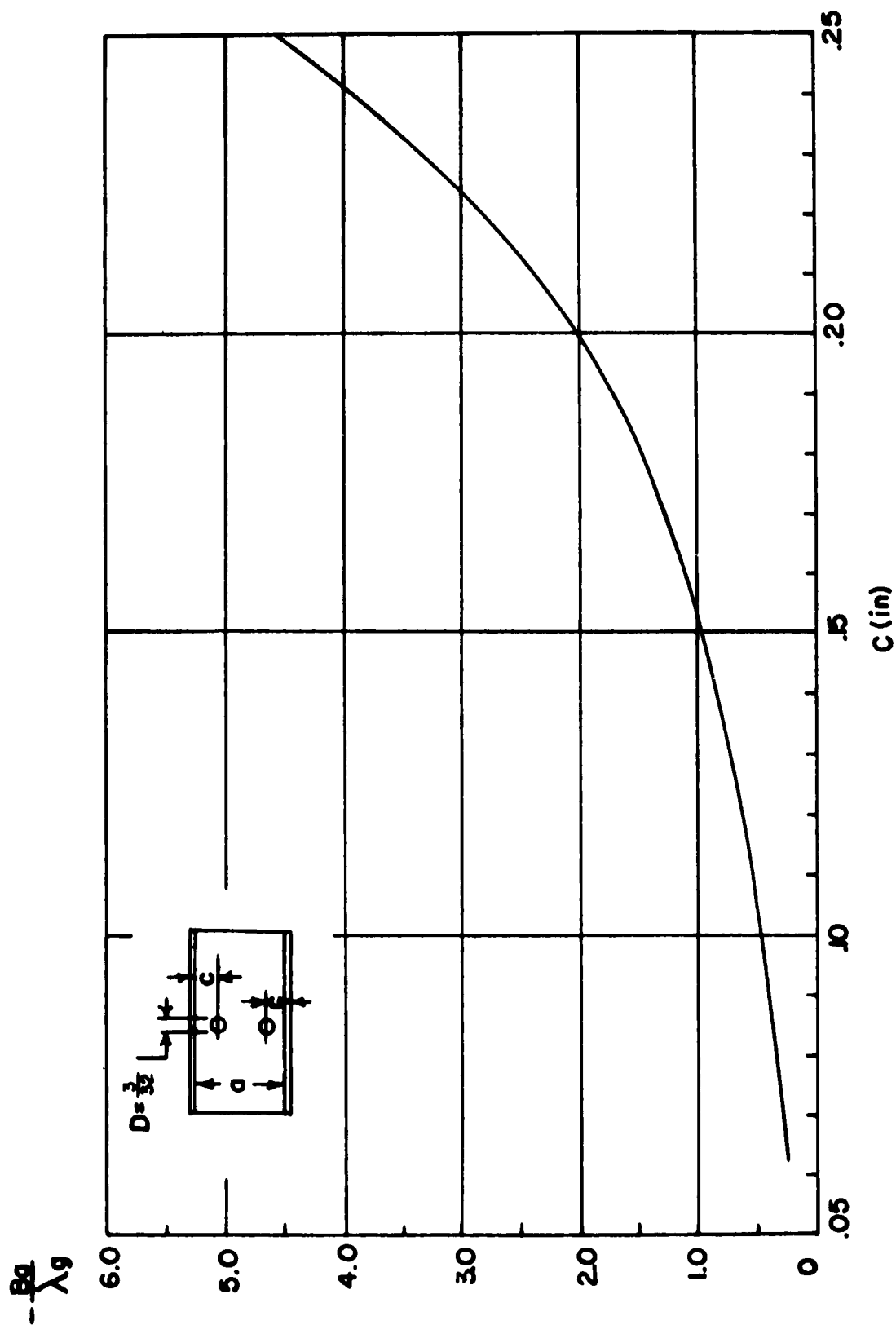


Figure 2. Two-post susceptances.

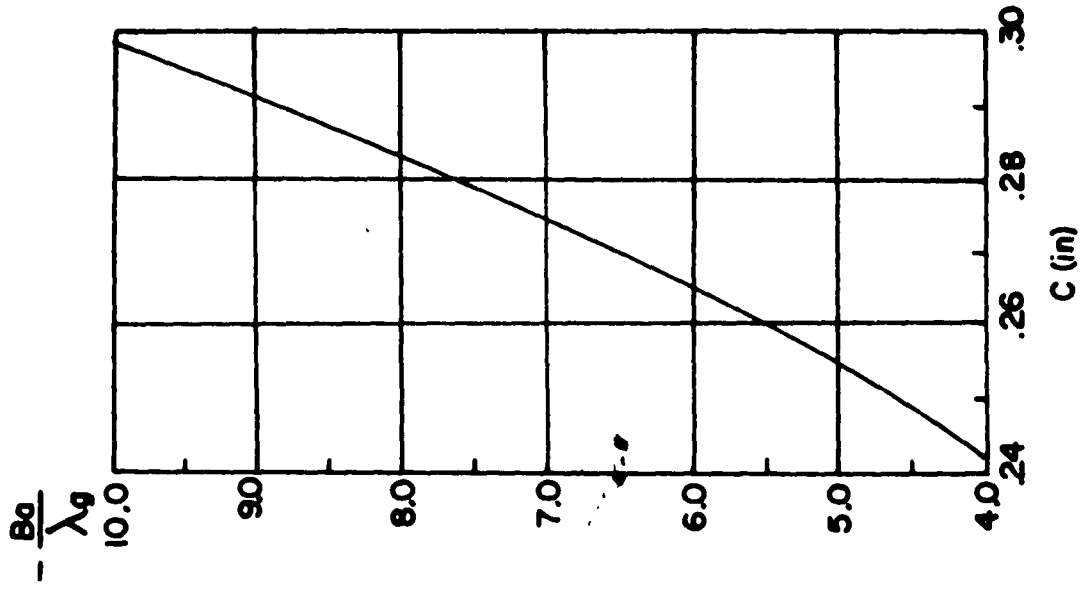


Figure 3. Two-post susceptances.

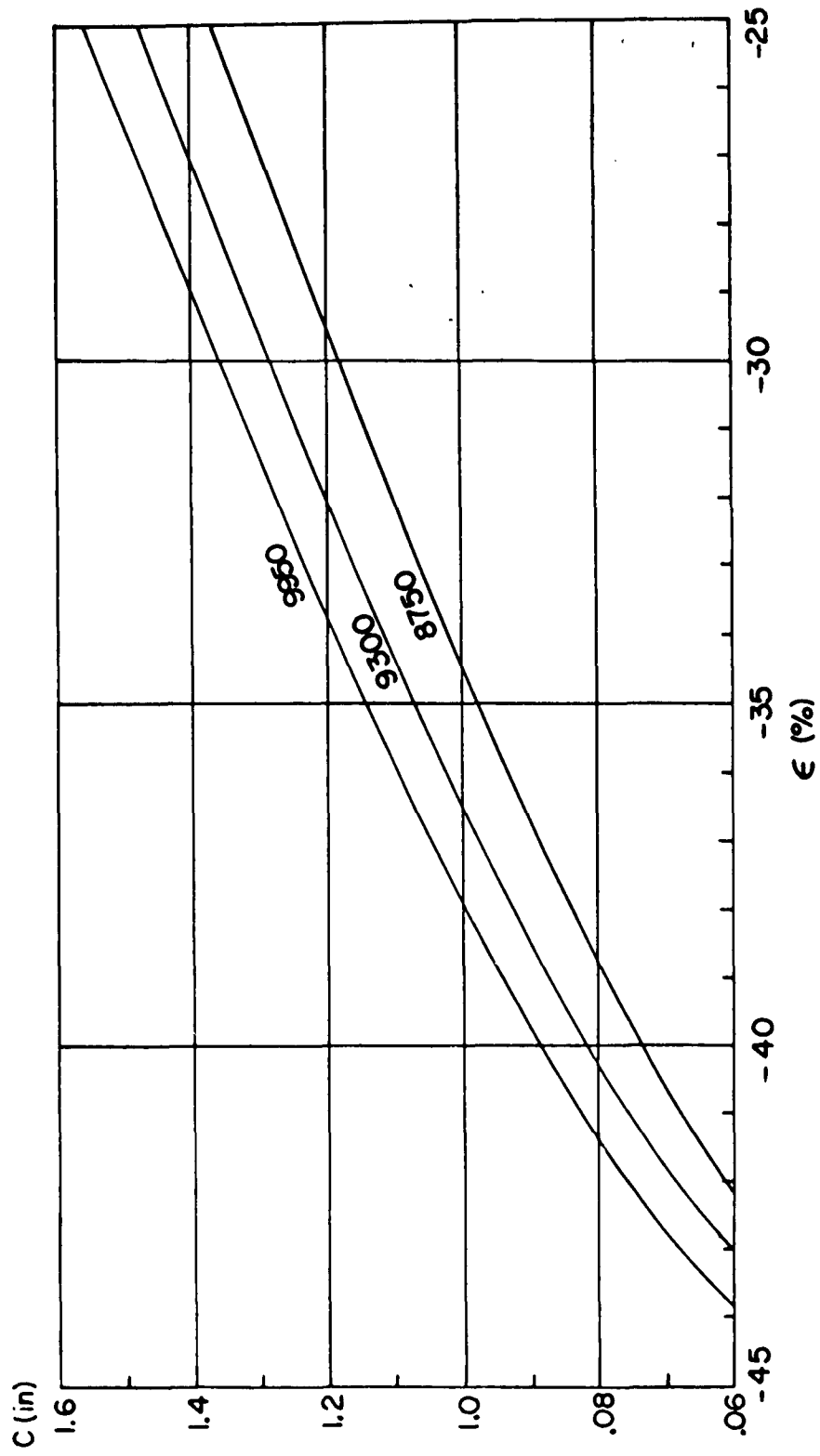


Figure 4. Two-post spacings.

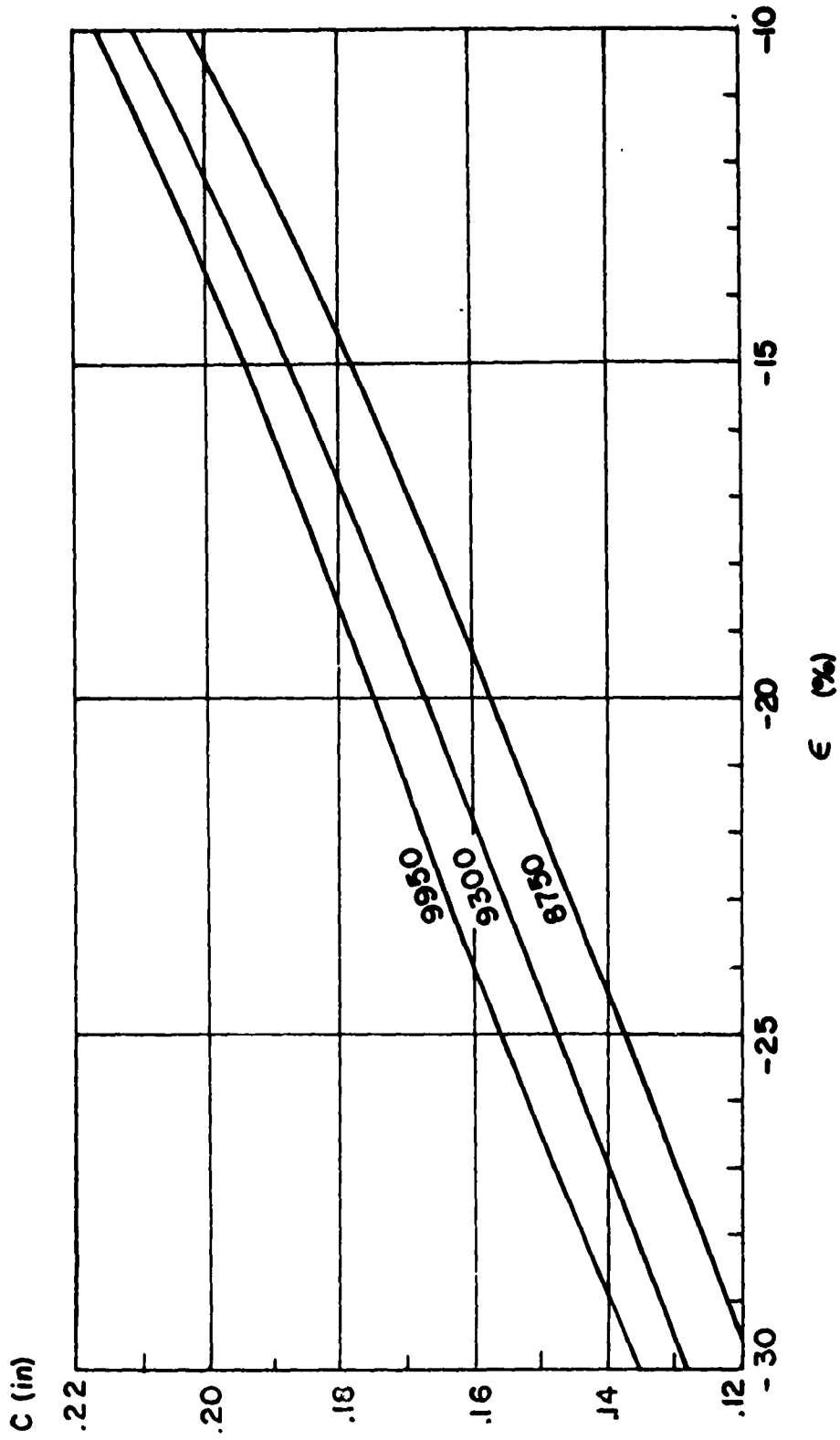


Figure 5. Two-post spacings.

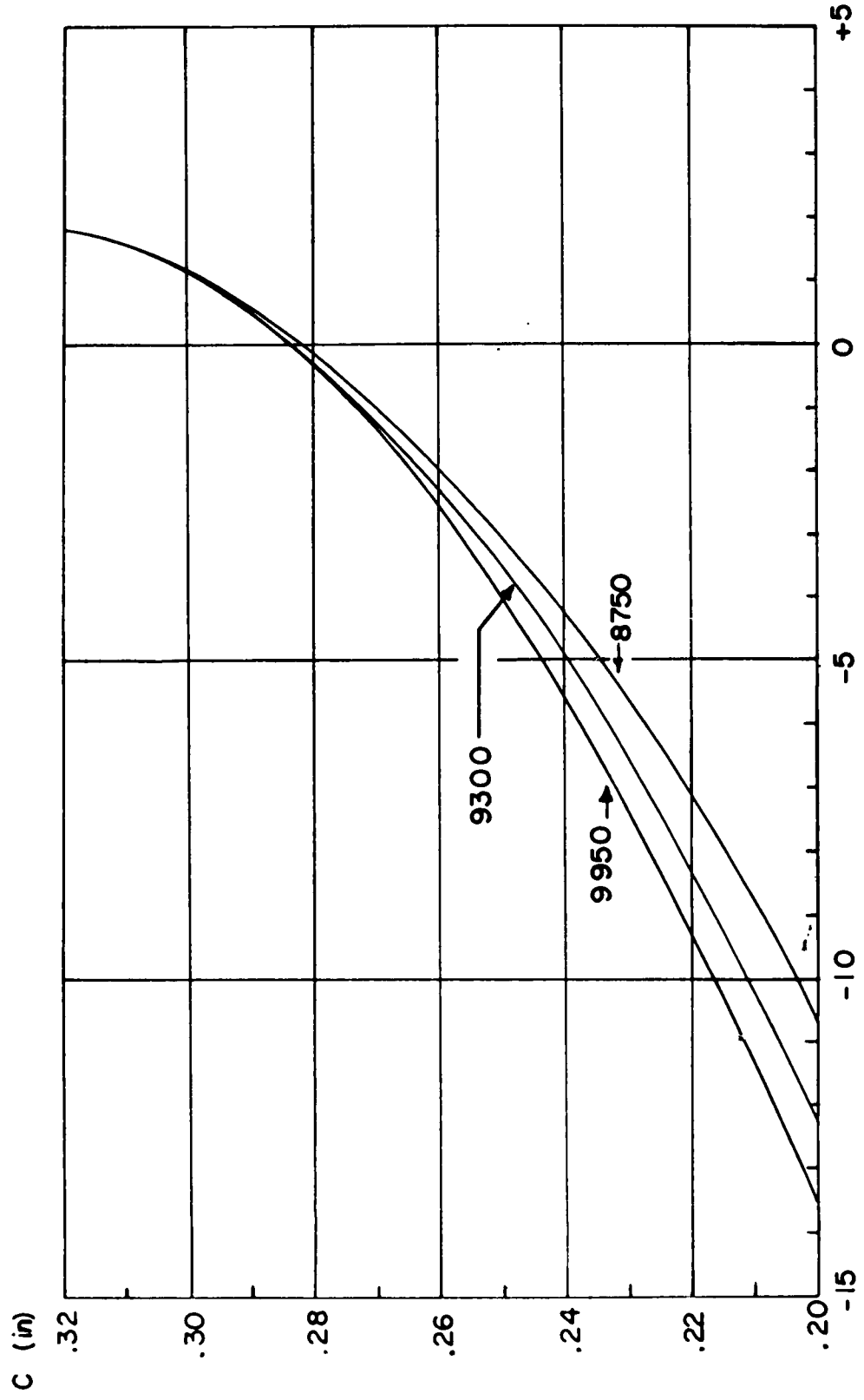


Figure 6. Two-post spacings.

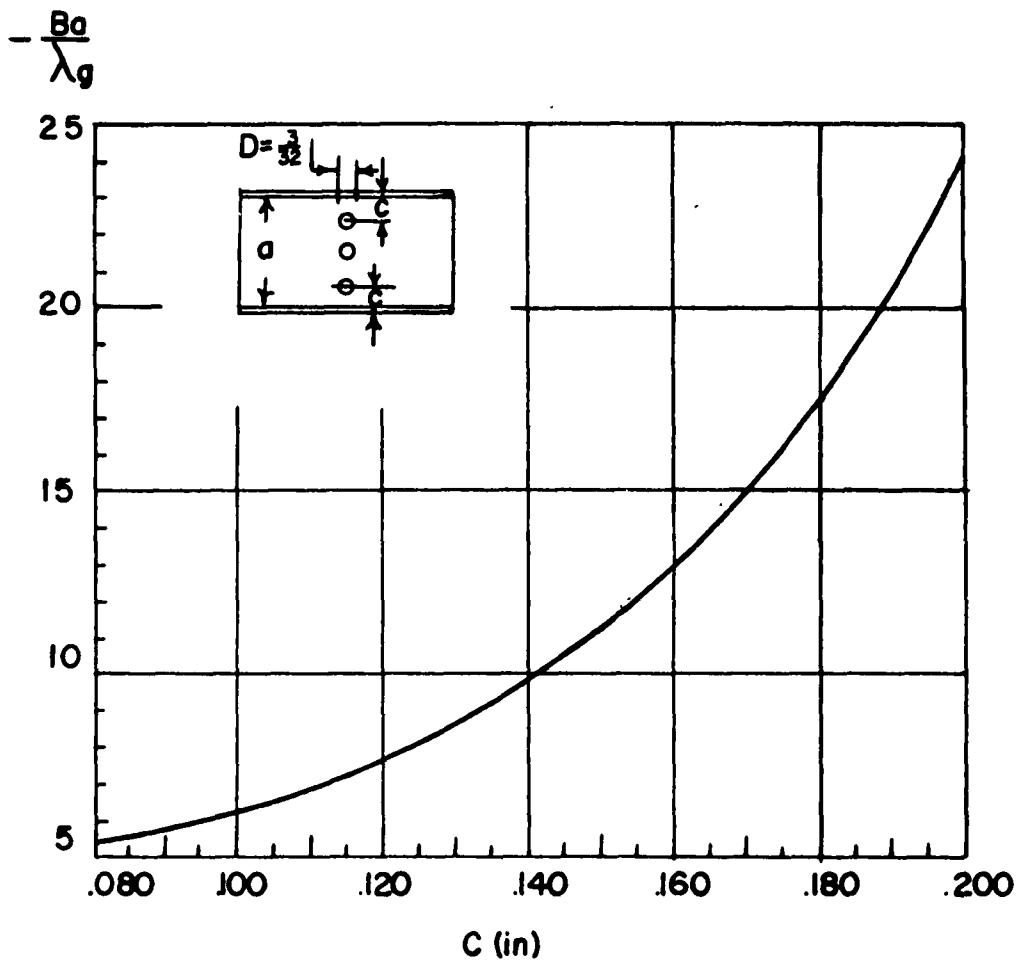


Figure 7. Three-post susceptances.

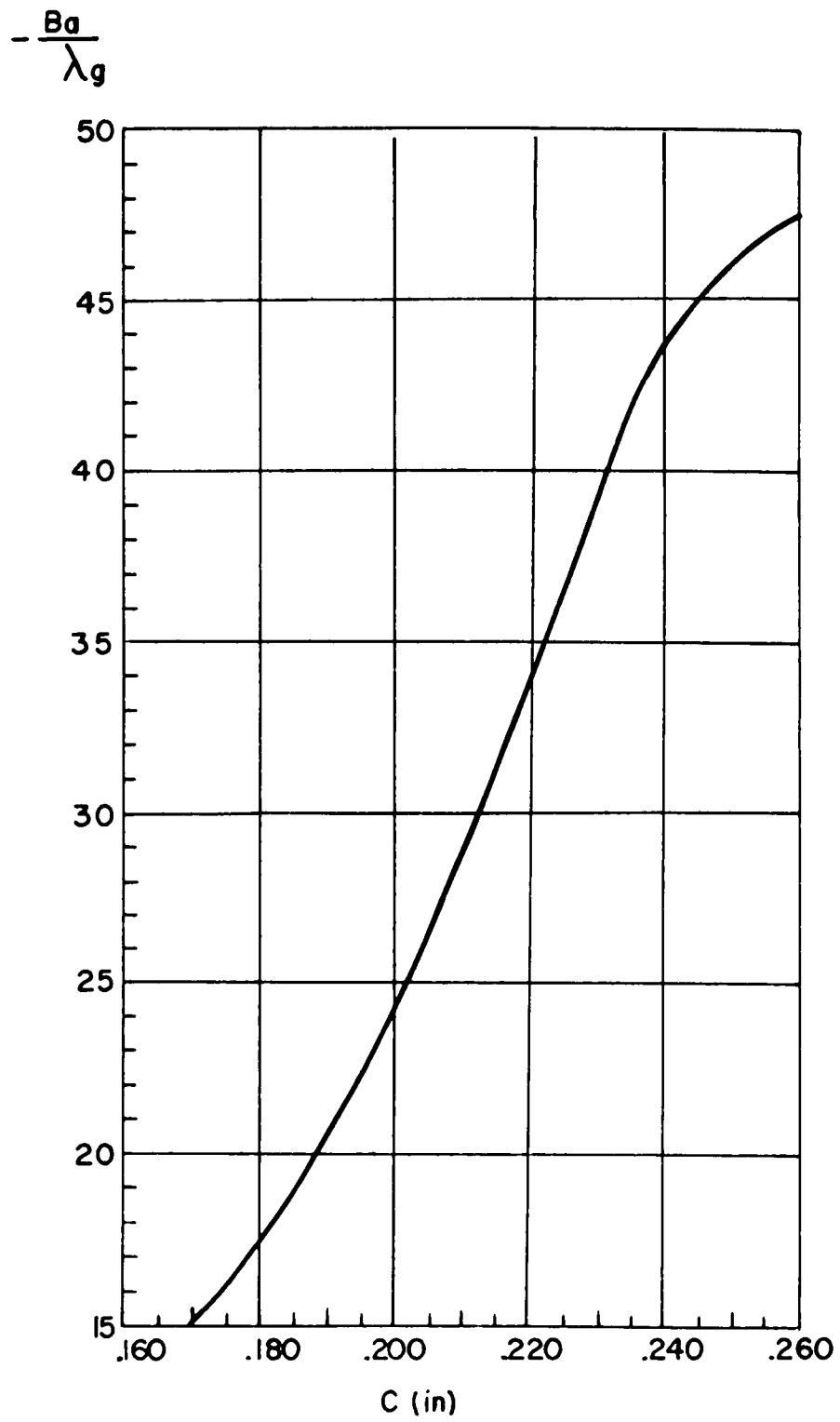


Figure 8. Three-post susceptances.

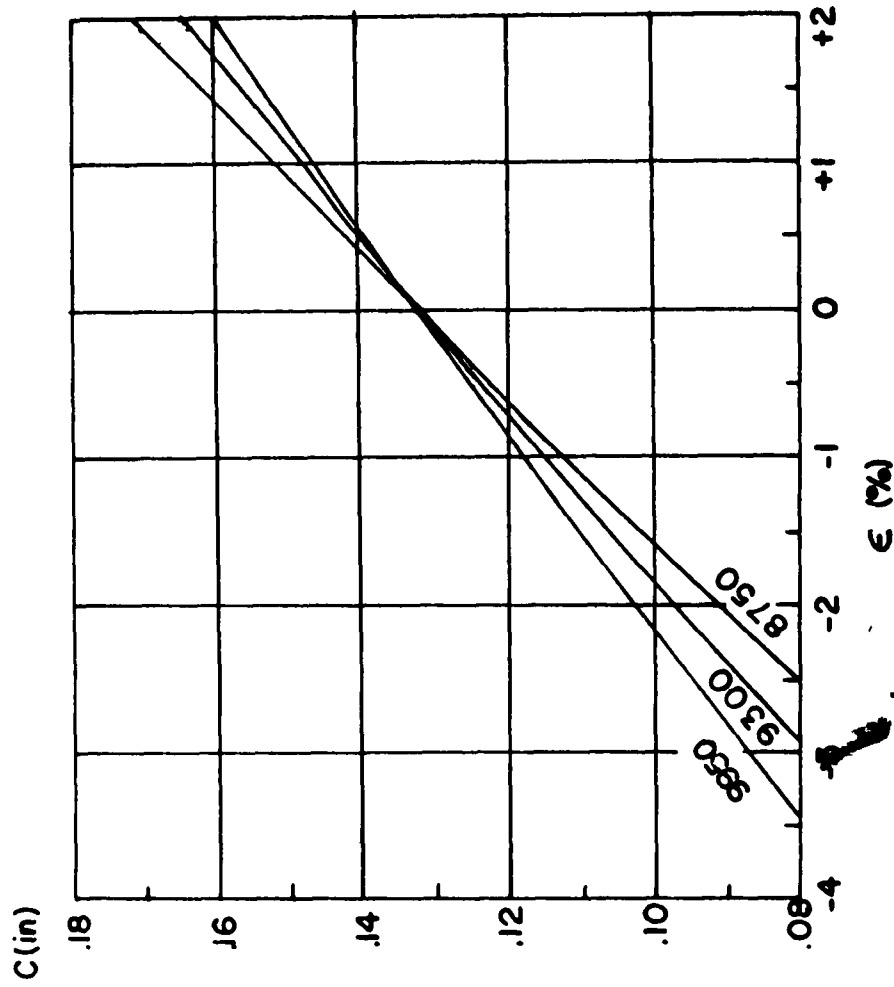


Figure 9. Three-post spacings.

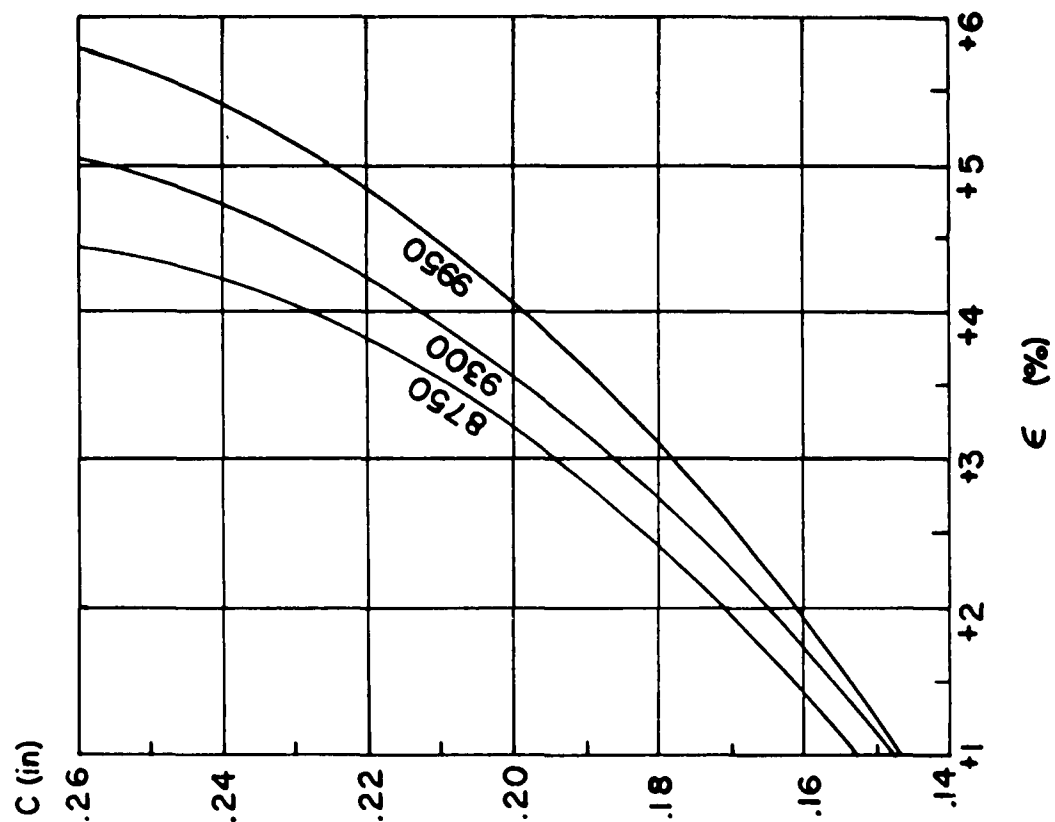


Figure 10. Three-post spacings.

Table 1. Coefficients used in susceptance calculations.

n	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇
2	1.189	1.414					
3	1.000	1.414					
4	.8749	1.189	1.848				
5	.7861	1.000	1.799				
6	.7195	.8556	1.653	1.931			
7	.6671	.7449	1.499	1.898			
8	.6247	.6584	1.362	1.806	1.962		
9	.5894	.5893	1.238	1.697	1.939		
10	.5594	.5330	1.133	1.588	1.876	1.975	
11	.5335	.4863	1.043	1.484	1.797	1.959	
12	.5110	.4470	.9653	1.390	1.712	1.914	1.983

Table 2. Coefficients used in computing insertion loss

n	2	3	4	5	6	7	8	9	10	11	12
K _n	1.41	2.00	2.61	3.24	3.86	4.49	5.13	5.76	6.39	7.02	7.66

Table 3. Unloaded Q's for 1x½" waveguide

Material	Q _o (Full size)	Q _o (Half size)
Copper	5000	3100
Brass (90% Cu, 10% Zn)	3200	2000
Brass - Yellow	2500	1600
Alumium	4000	2400

3. SAMPLE DESIGN

3.1 Filter Requirements:

- (a) Center frequency = 9300 Mc
- (b) Insertion loss at CF < 1 db
- (c) Insertion loss at 9250 and 9350 Mc < 0.5 db greater than CF insertion loss
- (d) 36 db minimum insertion loss at 9300 ± 150 Mc
- (e) Waveguide size: 1 in. x 1/2 in. OD.

Before proceeding with the filter design, the minimum number of filter sections and the bandwidth required to meet the specifications must be determined. In-band insertion loss above center frequency insertion loss is plotted in figure 11 as a function

of $\left| \frac{\lambda_{g0} - \lambda_g}{\lambda_{g1} - \lambda_{g2}} \right|$ with the number of sections n as a parameter.

The out-of-band insertion loss per filter section is plotted in figure 12 as a single curve; the curve is accurate for points lying above those indicated. Because the filter response is a function

of guide wavelength, $|\lambda'_{g0} - \lambda_g|$ has been plotted as function of frequency in figure 13 where λ'_{g0} is the guide wavelength at the center frequency. λ'_{g0} is used instead of λ_{g0} since λ_{g0} cannot be determined until the required bandwidth has been found.

3.2 Bandwidth Determination

To determine the bandwidth and the number of sections, a bandwidth is assumed and, using figures 12 and 13, the minimum number of sections required to meet the out-of-band specifications is determined. This process is repeated for several assumed bandwidths. From the filter requirement c, the bandwidth must be greater than 100 Mc. The minimum number of sections required for bandwidths from 110 to 140 Mc are tabulated below with insertion losses at 9450 Mc, since the insertion loss will be less at 9450 Mc than at 9150 Mc. To allow for deviations from theoretical response due to tolerance, tuning error, and the effect of dissipation on the response curve, a 3-db margin is added to the 36-db off-band response desired. The insertion loss at resonance is calculated from equation 5 as tabulated on page 27.

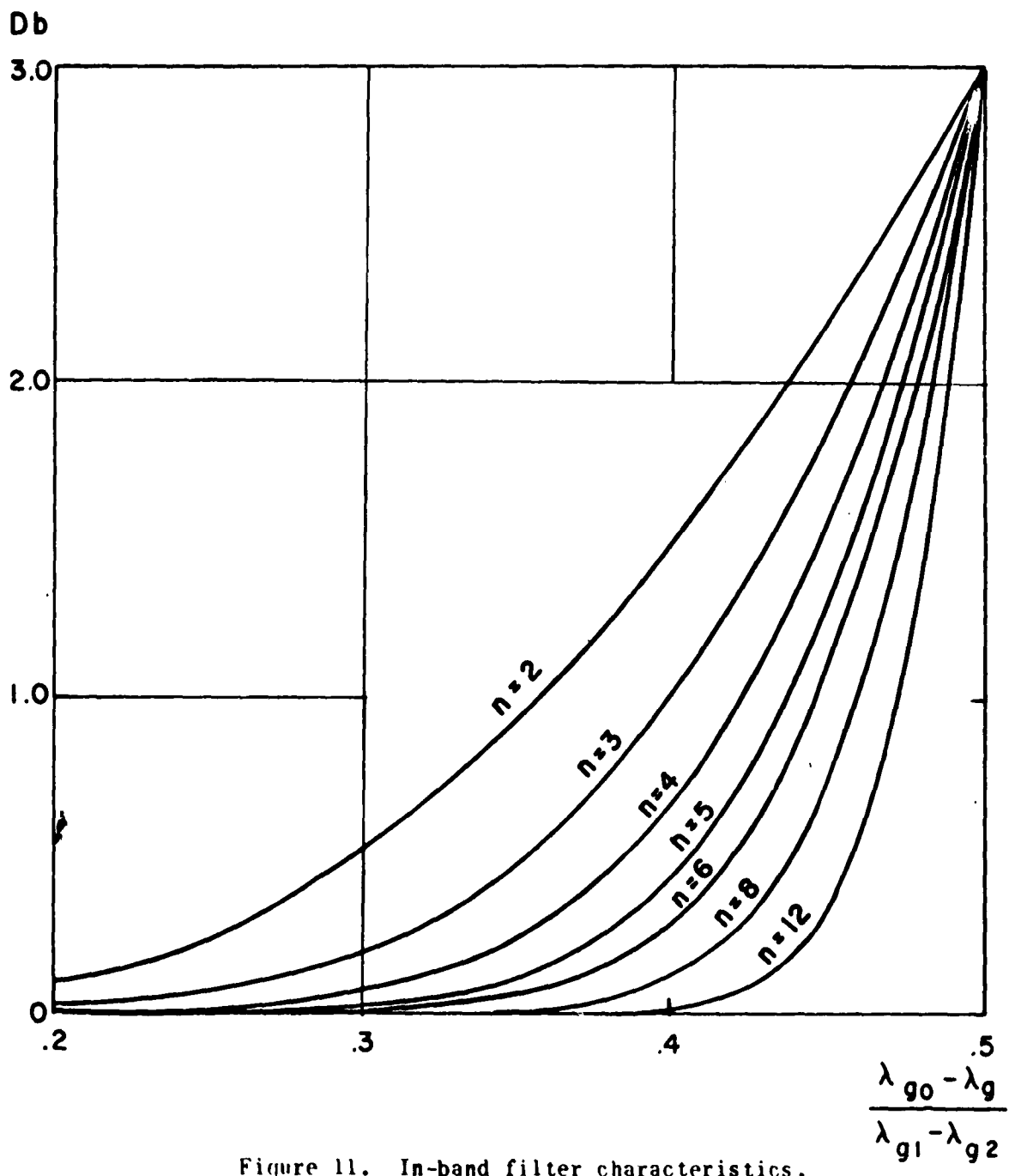


Figure 11. In-band filter characteristics.

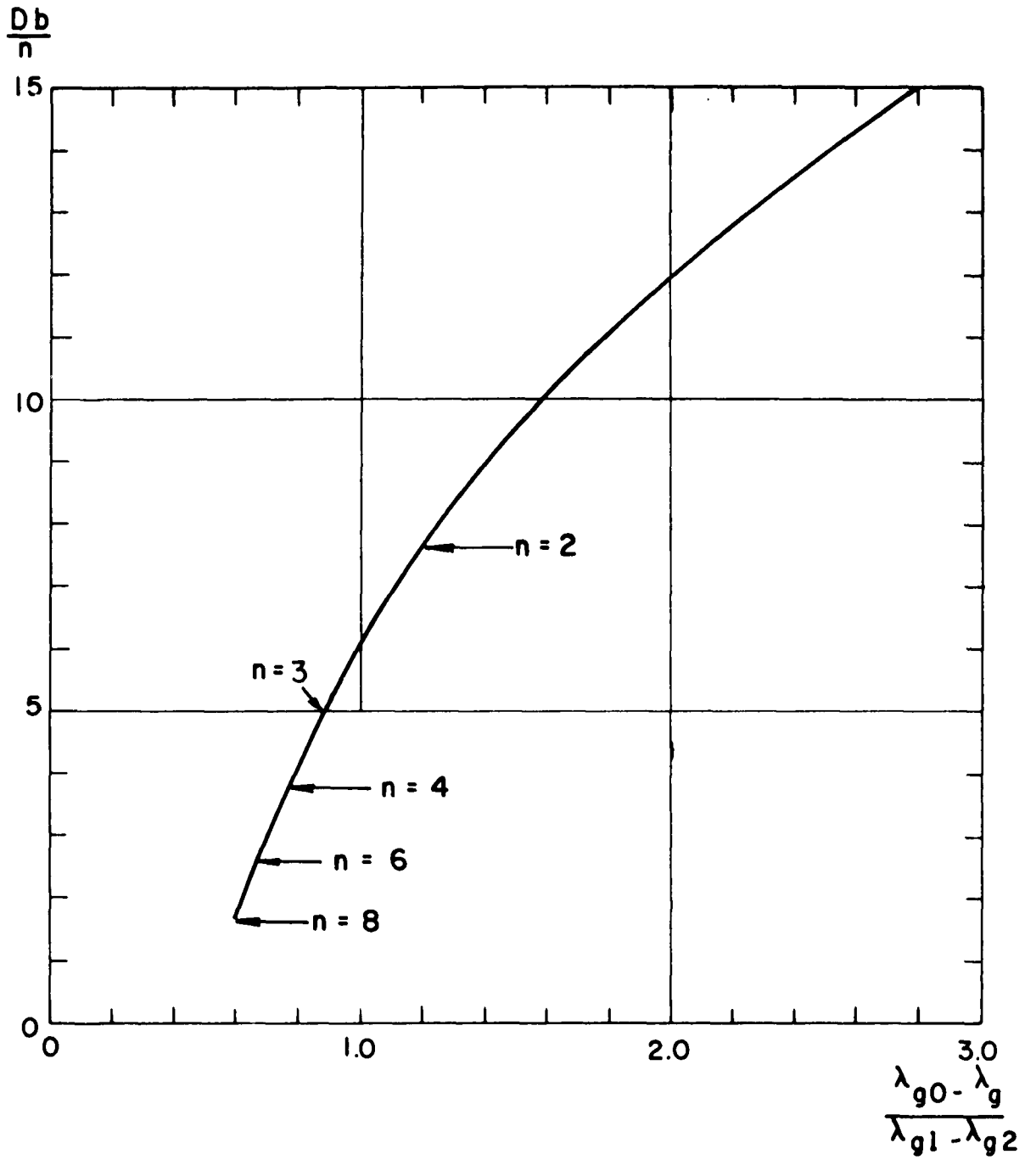


Figure 12. Off-band filter characteristics.

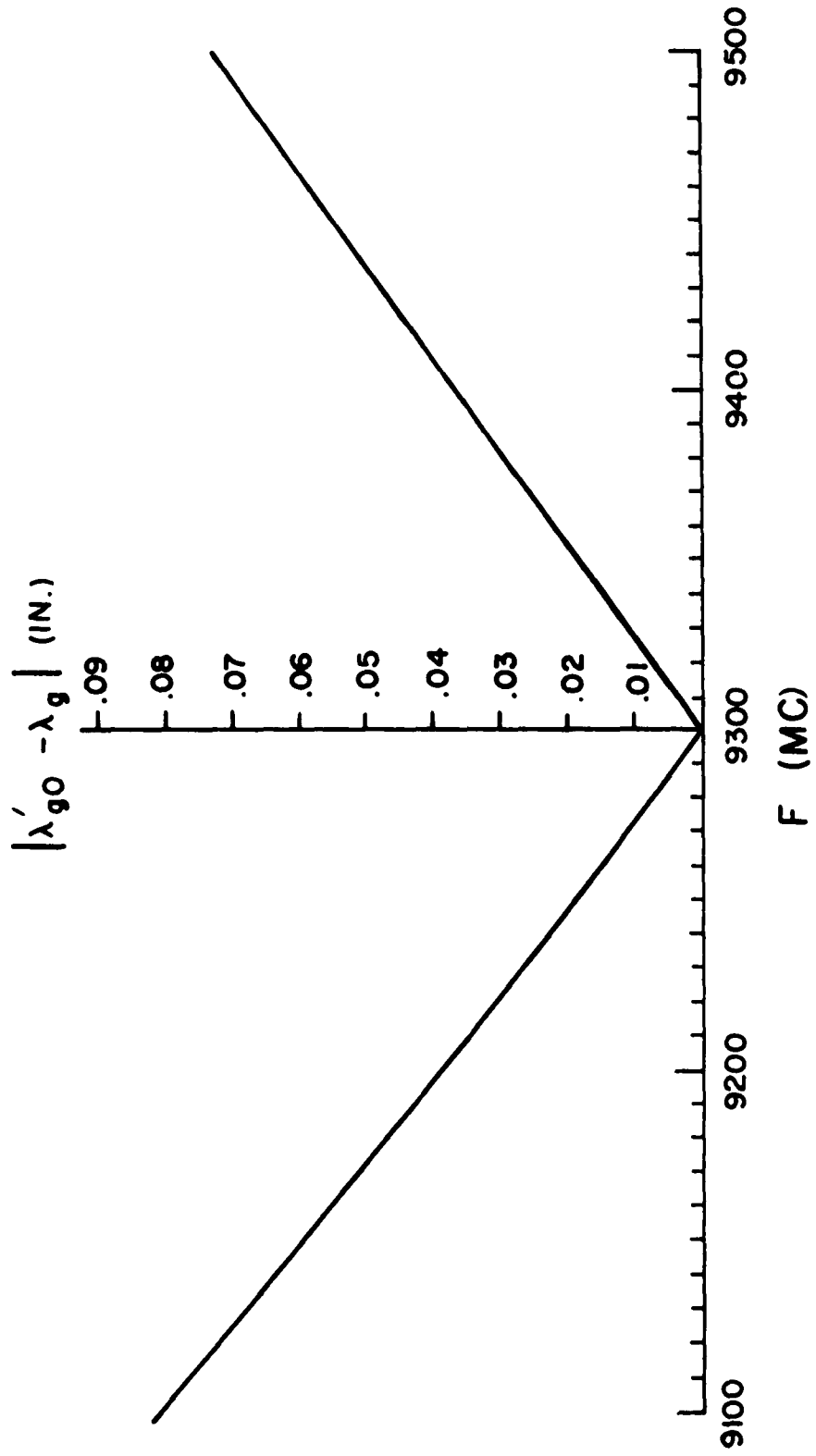


Figure 13. Wavelength difference versus frequency.

BW	$\left \frac{\lambda_{g0} - \lambda_g}{\lambda_{g1} - \lambda_{g2}} \right $	N	DB 9450	DB 9300
110	1.31	5	42	0.74
120	1.20	6	46	0.81
130	1.10	6	41	0.75
140	1.04	7	45	0.81

Two of the three specifications (out-of-band responses and center frequency insertion loss) are met by all four cases considered above. The insertion loss above the center frequency insertion loss for each case at ± 50 Mc from the center frequency is calculated using figures 11 and 13 and tabulated below. Calculations are made at 9250 Mc, since the insertion loss will be higher at the lower frequency.

BW	$\left \frac{\lambda_{g0} - \lambda_g}{\lambda_{g1} - \lambda_{g2}} \right $	N	Db(9350)
110	0.452	5	1.34
120	0.415	6	0.45
130	0.384	6	0.21
140	0.358	7	0.05

The minimum number of sections that meet all three of the filter response specifications is 6. To allow margin for tolerances, a 130-Mc bandwidth is chosen instead of 120 Mc. The next step is to calculate susceptances and spacings for a six-section filter with 3-db response points at 9235 and 9365 Mc.

$$\lambda_{g1} = \frac{\lambda_1}{\sqrt{1 - (\lambda_1 / 2a)^2}} = 1.8148 \text{ in. for } 9235 \text{ Mc}$$

$$\lambda_{g2} = 1.7649 \text{ in. for } 9365 \text{ Mc}$$

$$\lambda_{g0} = \frac{\lambda_{g1} + \lambda_{g2}}{2} = 1.7898 \text{ in.}$$

$$L = \pi \frac{\lambda_{g1} - \lambda_{g2}}{\lambda_{g1} + \lambda_{g2}} = 0.0437 \text{ in.}$$

Using equation (3) and table 1,

$$B_1 = B_7 = -3.15$$

$$B_2 = B_6 = -19.56$$

$$B_3 = B_5 = -37.81$$

$$B_4 = -44.18$$

Since the susceptance curves are given in Ba/λ_g , the B's are multiplied by a/λ_{g0} .

$$-B_1 a / \lambda_g = 1.58$$

$$-B_2 a / \lambda_g = 9.83$$

$$-B_3 a / \lambda_g = 19.01$$

$$-B_4 a / \lambda_g = 22.21$$

For a two-post structure for B_1 and three-post structures for the remaining susceptances, the spacings c are found from figures 2, 3, 7, and 8, and ϵ_i from figures 4, 5, 6, 9, and 10.

$$C_1 = C_7 = 0.184 \text{ in.} \quad \epsilon_1 = \epsilon_7 = - 0.160$$

$$C_2 = C_6 = 0.140 \text{ in.} \quad \epsilon_2 = \epsilon_6 = + 0.005$$

$$C_3 = C_5 = 0.186 \text{ in.} \quad \epsilon_3 = \epsilon_5 = + 0.032$$

$$C_4 = 0.195 \text{ in.} \quad \epsilon_4 = + 0.034$$

Allowing 50 Mc for the effects of mechanical tolerances, the spacings l_i, l_{i+1} are computed at 9350 Mc; $\lambda_g/2 = 0.885$ in.

$$l_{i, i+1} = (\lambda_g/2)(1 + \epsilon_i/2 + \epsilon_{i+1}/2)$$

$$l_{12} = l_{67} = 0.817 \text{ in.}$$

$$l_{23} = l_{56} = 0.901 \text{ in.}$$

$$l_{34} = l_{45} = 0.914 \text{ in.}$$

This completes the design procedure for the six-section filter, a sketch of which (showing spacings and tolerances) is shown in figure 14. Figure 15 is a photograph of the completed dent-tuned filter showing the holes used in the dent-tuning procedure. The measured characteristics of the filter are shown in figures 16, 17 and 18.

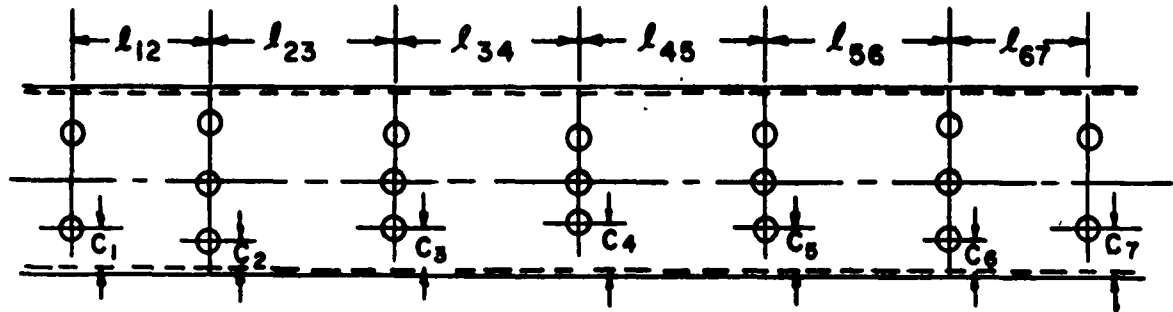


Figure 14. Diagram of sample design.

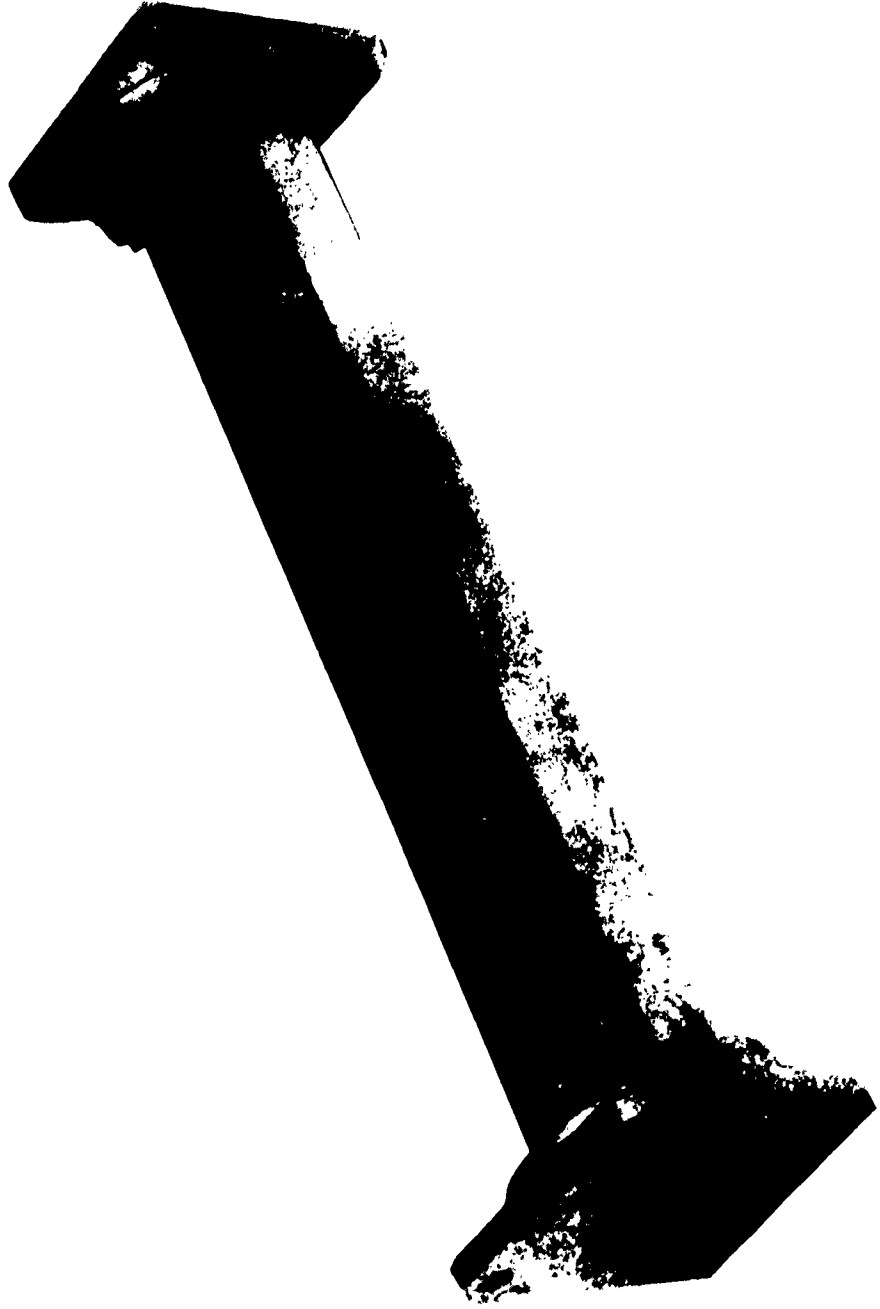


Figure 15. Six-section dent-tuned filter

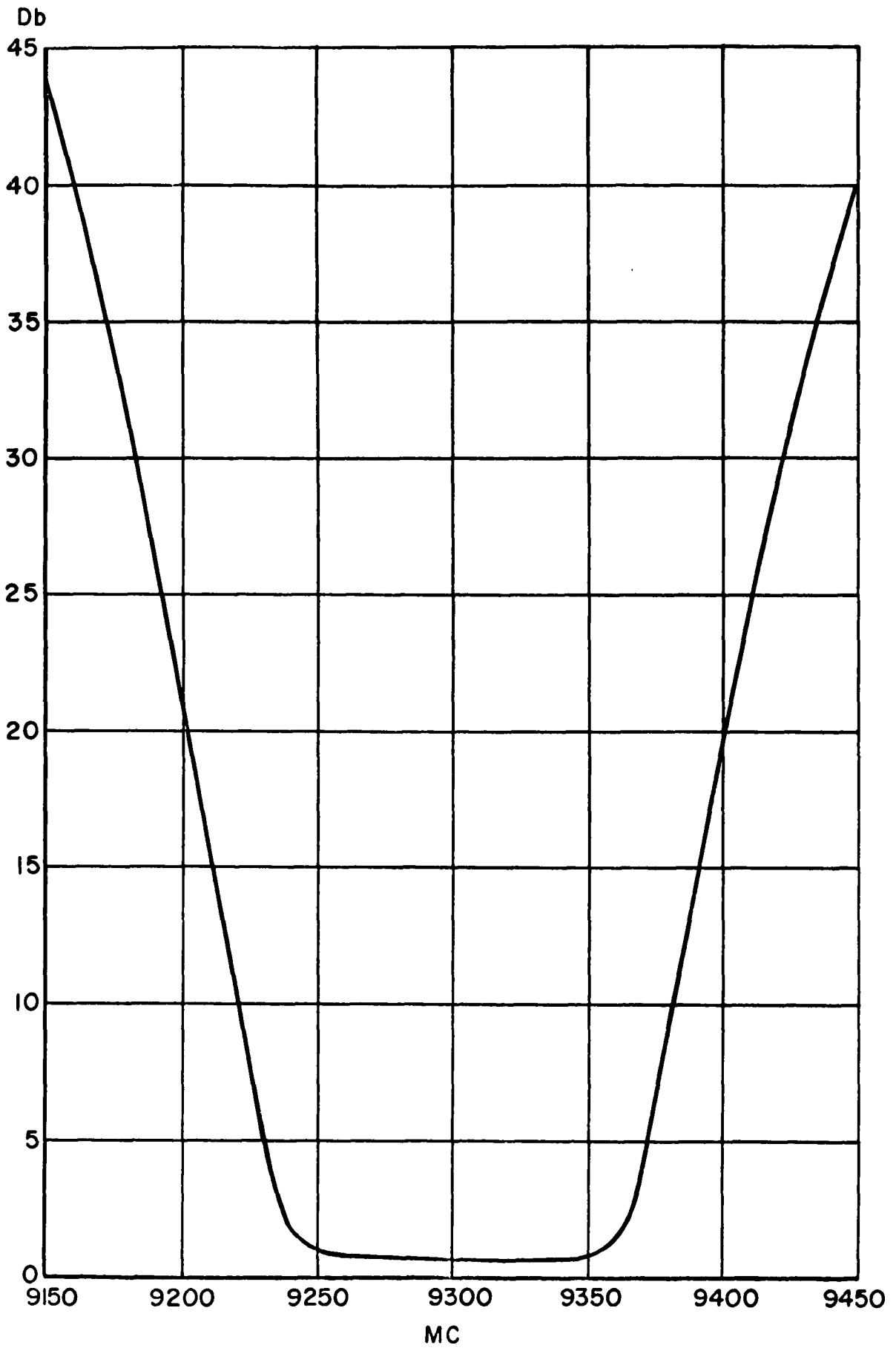


Figure 16. Insertion loss versus frequency of six-section filter.

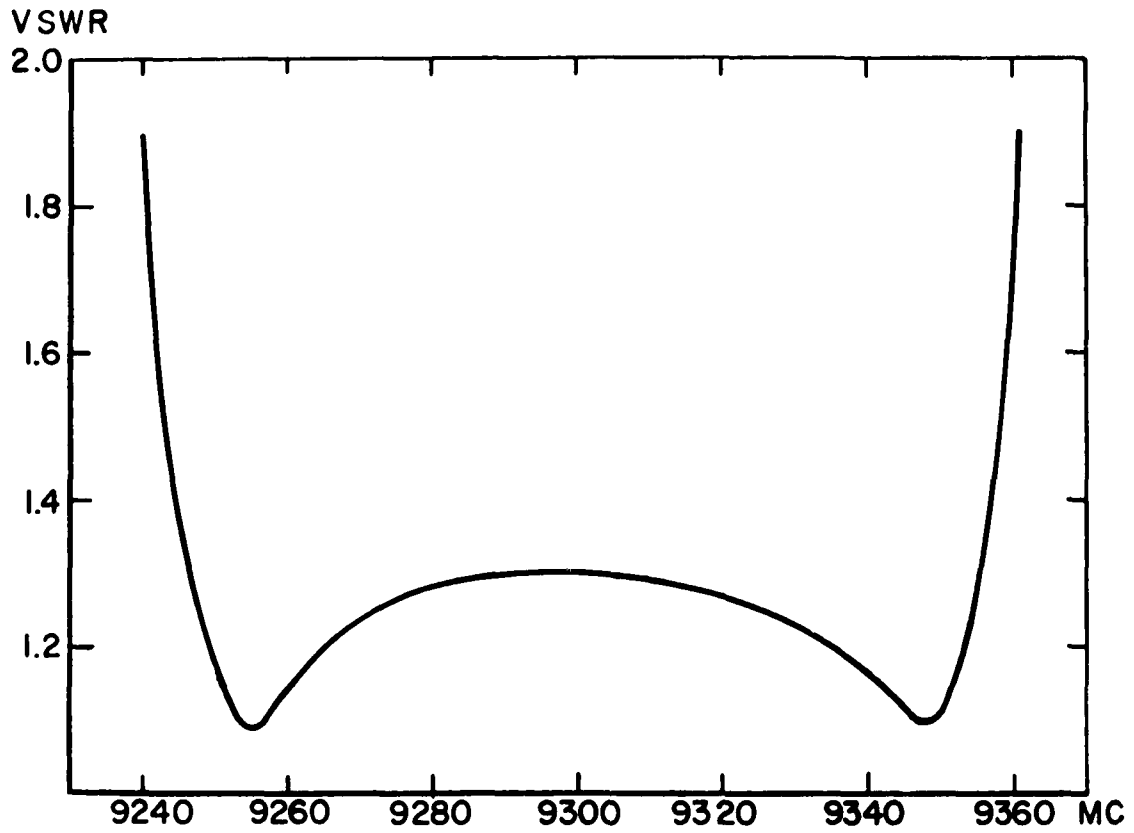


Figure 17. In-band VSWR of six-section filter.

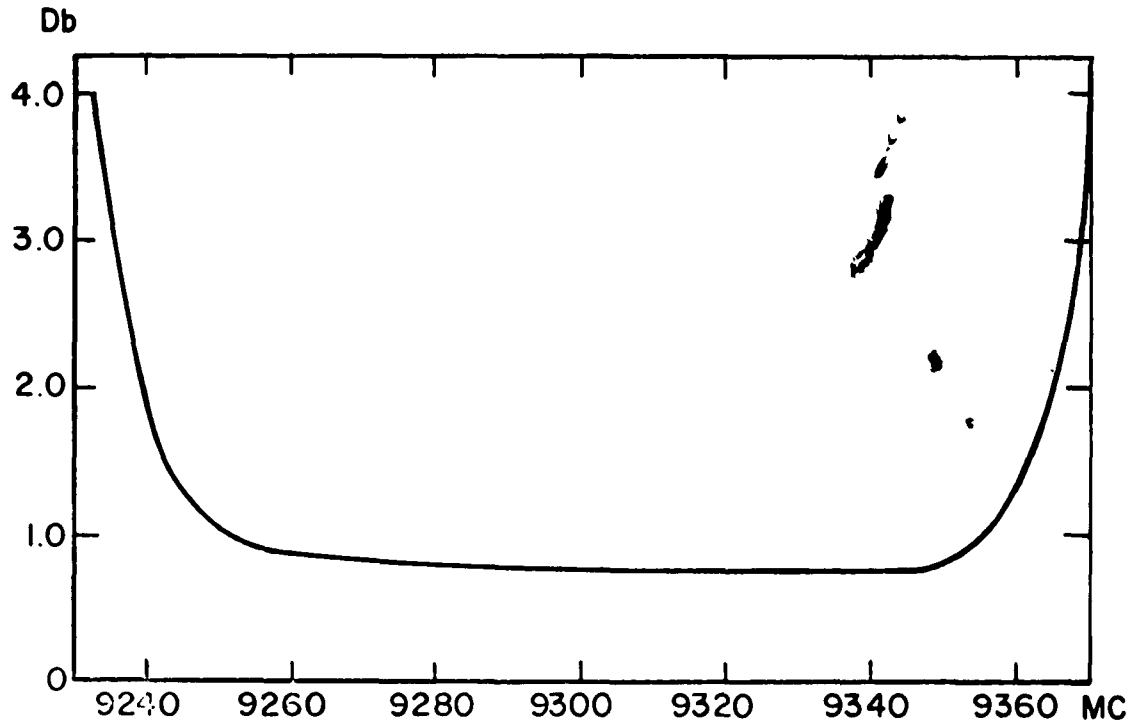


Figure 18. In-band insertion loss of six-section filter.

APPENDIX A

Waveguide and Fabrication Tolerances—Mathematical Derivations

Waveguide tolerances and fabrication tolerances make it difficult, except for extremely broad-band filters, to construct a filter without providing for tuning. The allowance to be made for tuning may be determined by computing the effects of each tolerance on λ_{go} .

The equation for guide wavelength is

$$\lambda_{go} = \frac{\lambda}{\sqrt{1 - (\lambda/2a)^2}}$$

where a is the broad internal dimension of the guide, and for this discussion is the nominal width to which must be added the tolerance δ .

$$\begin{aligned} \lambda'_{go} &= \frac{\lambda}{\sqrt{1 - (\lambda/2(a+\delta))^2}} \\ &\cong \frac{\lambda}{\sqrt{1 - \lambda^2/(4a^2 + 8a\delta)}} \\ &= \frac{\lambda}{\sqrt{1 - \lambda^2/4a^2(1+2\delta/a)}} \\ &\cong \frac{\lambda}{\sqrt{1 - \lambda^2/4a^2 + 2\delta\lambda^2/4a^3}} \\ &= \frac{\lambda_{go}}{\sqrt{1 + 2\lambda^2\delta/a(4a^2 - \lambda^2)}} \end{aligned}$$

$$\lambda'_{g0} = \lambda_{g0} \left(\frac{1}{1 + 2\lambda_{g0}^2 \delta / 4a^3} \right)^{1/2}$$

$$\approx \lambda_{g0} \left(1 - \lambda_{g0}^2 \delta / 4a^3 \right)$$

$$\Delta \lambda_{g0} = \lambda_{g0} - \lambda'_{g0} \approx \lambda_{g0}^3 \delta / 4a^3.$$

For the sample design $\delta = 0.003$ in, $\lambda_{g0} = 1.7895$ in, $a = 0.900$ in,
 $\Delta \lambda_{g0} = 0.006$ in.

The effect of the other tolerances are calculated using the equation

$$l_{i, i+1} = (\lambda_{g0} / 2) \left(1 + \epsilon_i / 2 + \epsilon_{i+1} / 2 \right)$$

$$\lambda_{g0} = \frac{2 l_i, l_{i+1}}{\left(1 + \epsilon_i / 2 + \epsilon_{i+1} / 2 \right)}$$

The effect of mechanical tolerances given on the post spacings c is greatest where ϵ varies most rapidly with c ; in the sample design this occurs for B_1 . The effect of mechanical tolerances on the post spacings l is greatest where ϵ is greatest, which occurs in the first section of the sample design filter. The usual way of specifying the spacings l on a machine drawing is to reference each obstacle to an index line, giving a tolerance Δl on the distance of each obstacle from this line. This results in two independent errors in the spacing between obstacles.

$$\Delta \lambda_{g2} = \Delta \lambda_{g3} = \frac{2 \Delta l}{1 + \epsilon_i / 2 + \epsilon_{i+1} / 2}$$

For the first section of the sample design $\Delta l = 0.002$ in.,

$\epsilon_1 = -0.160$, and $\epsilon_2 = +0.005$ and

$$\Delta \lambda_{g2} = \Delta \lambda_{g3} = 0.004 \text{ in.}$$

The tolerances on the spacings c result in two independent errors for each susceptance since each post is indexed separately from the sidewall. It is assumed that the effect of displacing the center post in a three-post structure by the amount given as a tolerance is negligible, and that the effect of displacing a post nearest the wall has half the effect of displacing both posts an equal distance. The error in λ_{g0} for displacing a post results from the change in ϵ .

$$\Delta \lambda_{g4} = \Delta \lambda_{g5} = \frac{\lambda_{g0} \Delta \epsilon_1}{2 + \epsilon_1 + \epsilon_2}$$

$$\Delta \lambda_{g6} = \Delta \lambda_{g7} = \frac{\lambda_{g0} \Delta \epsilon_2}{2 + \epsilon_1 + \epsilon_2}$$

For the sample design: $C_1 = 1/2 \Delta > C_1 = 0.184 + 0.001 \text{ in.} = 0.185 \text{ in.}$
(two-post structure)

$C_2 + 1/2 \Delta > C_2 = 0.140 + 0.001 \text{ in.} = 0.141 \text{ in.}$ (three-post structure)

$$\Delta \epsilon_1 = 0.002 \text{ in.}, \Delta \epsilon_2 = 0.001 \text{ in.}$$

$$\Delta \lambda_{g4} = \Delta \lambda_{g5} = .002 \text{ in.}$$

$$\Delta \lambda_{g6} = \Delta \lambda_{g7} = .001 \text{ in.}$$

The error in the spacing data is $\Delta \lambda_{g8} = 0.002 \text{ in.}$ The rms error is $\sqrt{\sum_{i=1}^8 (\Delta \lambda_{gi})^2} = \pm 0.009 \text{ in.}$

or approximately 25 Mc. An allowance of 50 Mc for tuning should be adequate.

The bandwidth error is relatively independent of the waveguide dimensional tolerance since the $\Delta \lambda_g$'s in the numerator of the equation

$$L = \pi \frac{\lambda_{g1} - \lambda_{g2}}{\lambda_{g1} + \lambda_{g2}} \text{ tend to cancel, making } \Delta L$$

quite small. The bandwidth of the filter is determined primarily

by the highest Q sections, those with the largest values of susceptance. An approximation of the bandwidth error can be obtained by finding the effect of the tolerances on the largest susceptance and computing the corresponding ΔL 's. This is done for B_4 in the design sample.

The error in the susceptance data is approximately 2 percent.

$$B_4 = -\frac{A_4}{L} + \frac{L}{A_4}$$

$$B_4 \approx -\frac{A_4}{L} \quad \text{since} \quad \frac{L}{A_4} \ll \frac{A_4}{L}$$

$$L_1 = L \frac{\Delta B_4}{B_4}$$

$$= 0.020 L.$$

The error in B_4 due to the 0.002-in. tolerance on c is from figure 8

$$\frac{\Delta B \lambda}{a} \approx 1.6$$

$$\Delta L_2 = L \frac{1.6}{44.2} = 0.036L$$

The maximum error in bandwidth is $\frac{\Delta L_1 + \Delta L_2}{L} = 5.6$ percent.

The maximum error is computed rather than the probable error since no means exist for correcting the effect of tolerances on bandwidth. The choice of bandwidth and number of sections should be made to insure meeting filter specifications, taking into account this bandwidth error.

APPENDIX B

Scaling

The susceptance and spacing curves in this report may be used for any size rectangular waveguide operating in the TE_{10} mode by appropriate scale changes. The principle is one of normalizing to obtain universal curves. This normalization must be such that the geometry and field distribution are held constant for any given point on a curve. The diameter of the posts is a geometric parameter which is normalized keeping d/a constant. If the post spacings c are written in terms of c/a for the d/a as shown, the half wavelength correction curve is normalized. The susceptance curves are normalized by transforming c to c/a .

The scaled post diameter may not be a stock (or standard) rod size. However, if a large number of filters are required, the use of other than a standard diameter may not be economically desirable. Susceptance and spacing data for stock rod sizes must be obtained by measurement.

APPENDIX C

Measurements

Susceptance may be found by VSWR and insertion loss measurements (ref 8). The spacing required for resonance may be measured either by varying the short circuit in figure C-1(a) locating the short position for minimum input VSWR, or by using the 90-deg phase shift in the VSWR null, which occurs when the circuit is brought from anti-resonance to resonance. When B is small, neither method yields accurate values for the spacing since both the VSWR and the phase of the reflection change slowly with θ_1 .

The null-shift method (ref 9), for which the equations are derived below, yields accurate results for susceptance and resonant spacing of small susceptances. The normalized input impedance Z_i of the circuit, shown in figure C-1(a), is found by using the lossless transmission line transformation:

$$Z_i = \frac{Z + j \tan \theta}{1 + jZ \tan \theta}.$$

To obtain

$$Z_i = j \frac{\tan \theta_1 + \tan \theta_2 - B \tan \theta_1 \tan \theta_2}{1 - B \tan \theta_1 - \tan \theta_1 \tan \theta_2}$$

At a null in the standing wave pattern, $Z_i = 0$ since the input impedance at this point is real.

$$\therefore \tan \theta_1 + \tan \theta_2 = B \tan \theta_1 \tan \theta_2$$

$$\sin (\theta_1 + \theta_2) = B \sin \theta_1 \sin \theta_2$$

$$\cot \theta_1 + \cot \theta_2 = B$$

from which

$$\tan (\theta_1 + \theta_2) = \frac{B(1 - \cos 2\theta_1)}{B \sin 2\theta_1 - 2}.$$

For a given value of B the plot of $(\theta_1 + \theta_2)$ versus θ_1 is a curve of the form shown in figure C-2.

The maximum and minimum of θ_1 and θ_2 versus θ_1 are found by setting $d(\theta_1 + \theta_2)/d\theta_1 = 0$. Since $\theta_1 + \theta_2$ repeats every π radians, only the variation of θ_1 over the region $0 < \theta_1 + \theta_2 < \pi$ be considered. For negative B, $\theta_{11} = \pi$, $\theta_{12} = \tan^{-1}(B/2)$ and $\theta_{22} = \tan^{-1}(B/2)$. The susceptance can be found from the peak-to-peak excursion of the curve since

$$\tan(\Delta\theta/2) = \tan[(\theta_{12} + \theta_{22} - \pi)/2] = -\tan(\theta_{12}/2) = -B/2$$

The equivalent circuit shown in figure C-1(b) is the same as that shown in figure C-1(a) with θ_1 and θ_2 replaced by $x + l$ and $y + l$, where x and y are measured distances. At the minimum of the curve, $\theta_1 + \theta_2 = \pi$. Substituting $x_m + l$ for θ_1 and $y_m + l$ for θ_2 , $x_m + y_m + 2l = \lambda_g/2$.

The resonant spacing for identical susceptances is given by

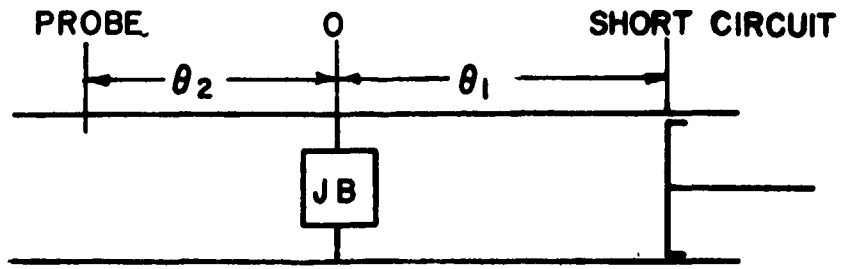
$\tan \theta = 2/B = \tan(\pi l_o/\lambda_g)$. Using the relation $\tan(\Delta\theta/2) = \tan(\pi\Delta l/\lambda_g) = -B/2$ the resonant distance between identical susceptances is found to be

$$l_o - 2l = x_m + y_m + (\Delta l/2) - (\lambda_g/4)$$

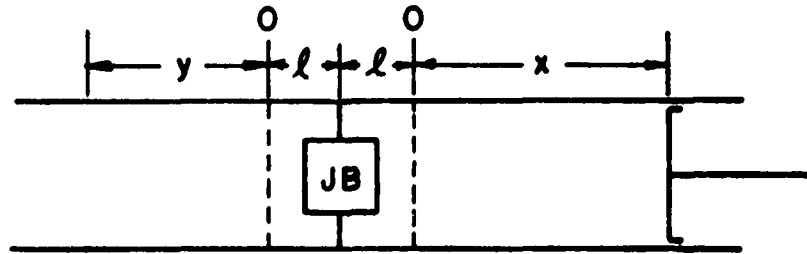
Measurements were made using a micrometer-driven short and a dial gauge to measure the probe position. The guide wavelength was measured in the slotted line and in the line containing the short circuit; these values were used in computing B. The distances x and y can be measured to ± 0.0005 in. resulting in a maximum error of 0.002 in. in Δl . The error in the measured values of B is 2 percent or less for $B < 10$. The resonant spacings for $B < 10$ were measured to 0.002 in. with this method.

Normalized susceptances greater than 10 were measured by the insertion-loss method. The susceptance is given by

$$|B| = 2 \sqrt{\log_{10}^{-1} |l/10 - 1|}$$



(Section a)



(Section b)

Figure C-1. Equivalent circuits used in measuring b and ϵ .

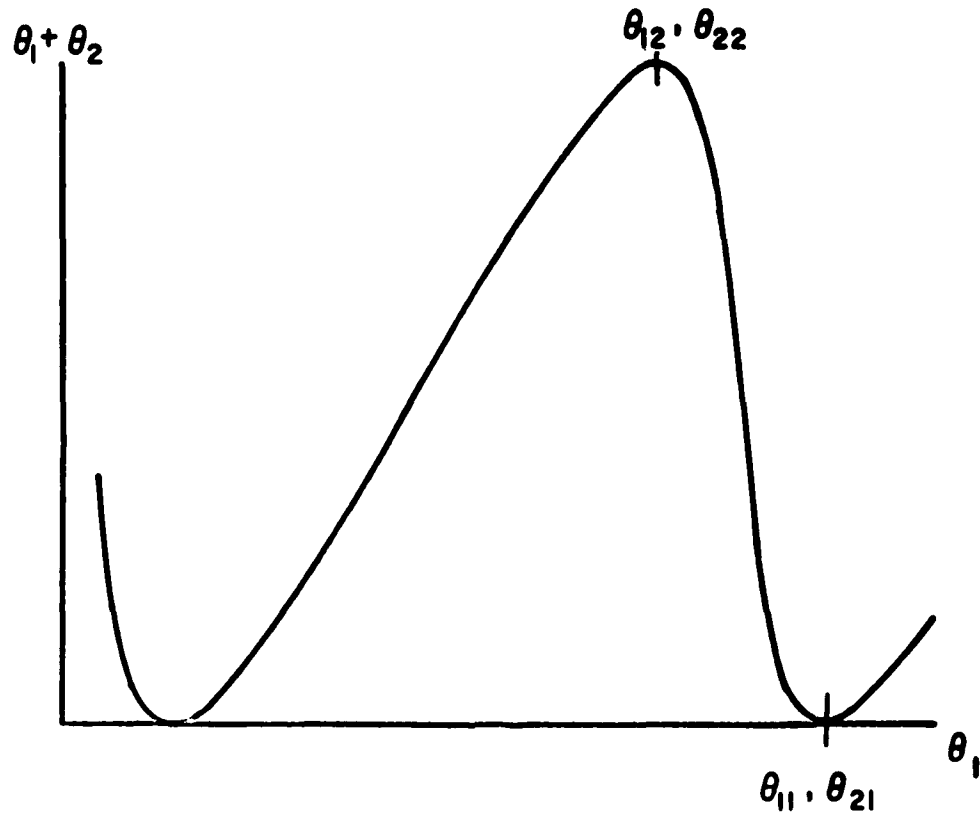


Figure C-2. Null-shift curve.

where M is the insertion loss in db. The attenuator used was accurate to within ± 0.1 db, provided the line was well matched looking from the attenuator toward both the generator and the load. A well matched isolator was used between the generator and the attenuator, and a bilaterally matched 20-db pad was used between the attenuator and the load. For normalized susceptances of 10 or greater, the error is less than 2 percent.

The spacing for resonance can be determined by the short position required for minimum VSWR. As B gets large, the minimum VSWR is high and the measurement becomes inaccurate. The 90-deg shift in the null position, which occurs when tuning from anti-resonance to resonance, was used to locate the short position at resonance. At resonance the spacing between the short and the susceptance is given by

$$L_r = \frac{\lambda_g}{2} \left(1 + \frac{\epsilon}{2} \right)$$

where ϵ is the half-wavelength correction for a resonant section made up of identical susceptances.

B and ϵ were measured at frequencies of 8750, 9300, and 9950 Mc. The curves of Ba/λ_g for the three frequencies differed by less than the maximum experimental error, hence, only the curve for 9300 Mc is plotted (fig. 2, 3, 7, and 8).

REFERENCES

1. W. W. Mumford, "Maximally Flat Filters in Waveguide," *BSTJ*, Vol 27 (Oct 1948), pp 648-714.
2. W. D. Lewis and W. W. Mumford, Patent No. 2,585,563, 12 Feb 1952.
3. S. B. Cohn, "Direct Coupled Resonator Filters," *Proc Inst Radio Engrs*, Vol 39, (Nov. 1951) pp 187-196.
4. R. O. Stone, and Gustave Shapiro. "Miniaturization Techniques For Passive Detection Receivers," NBS Report No. 6416, 1 May 1959, pp 29-91.
5. E. S. Hensperger, "A Simplified Approach to the Design of Bandpass Filters in Waveguide," *The Microwave Journal*, Vol 2 (Nov 1959), pp 39-44.
6. S. Cohn, "Dissipation Loss in Multiple-Coupled Resonator Filters," *Proc Inst Radio Engrs*, Vol 47 (Aug 1959), pp 1342-1348.
7. M. Dishall, "Alignment and Adjustment of Synchronously Tuned Multiple Resonant Circuit Filters," *Proc Inst Radio Engrs*, Vol 39 (Nov 1951), pp 1448-1455.
8. G. C. Southworth, "Principles and Applications of Waveguide Transmission," New York, D. Van Nostrand Co., Inc. (1950), pp 214-220.
9. M.H.N. Potok, "Inductive Post Type Filters," *Journal Brit. IRE*, Vol 18 (May 1958).

DISTRIBUTION

Department of the Army
Office of the Chief of Ordnance
The Pentagon, Washington 25, D. C.
Attn: ORDTN (Nuclear & Special Components Br)
Attn: ORDTU (GM Systems Br)

Commanding General
U. S. Army Ordnance Missile Command
Redstone Arsenal, Alabama
Attn: ORDXM-R (Ass't Chief of Staff for R & D)

Commanding General
Army Ballistic Missile Agency
Redstone Arsenal, Alabama
Attn: Technical Documents Library

Commanding Officer
Picatinny Arsenal
Dover, New Jersey
Attn: Library

Commander
U. S. Naval Ordnance Laboratory
Corona, California
Attn: Documents Librarian

Commander
U. S. Naval Ordnance Laboratory
White Oak, Silver Spring 19, Maryland
Attn: Tech Library

Department of the Navy
Bureau of Naval Weapons
Washington 25, D. C.
Attn: DLI-3, Tech Library

Commander
Naval Research Laboratory
Washington 25, D. C.
Attn: Tech Library

U. S. Army Liaison Officer
U. S. Naval School
Explosive Ordnance Disposal
Indian Head, Maryland

DISTRIBUTION (Continued)

Commander
Aerospace Services Technical Information Agency
Arlington Hall Station
Arlington 12, Virginia
Attn: TIPDR (10 copies)

INTERNAL DISTRIBUTION

Horton, B. M./McEvoy, R. W., Lt Col
Apstein, M./Gerwin, H. L./Guarino, P. A./Kalmus, H. P.
Schwenk, C. C.
Hardin, C. D., Lab 100
Sommer H., Lab 200
Hatcher, Robert D., Lab 300
Landis, P. E., Lab 400
Fong, Louis B., Lab 500
Flyer, I. N., Lab 600
Campagna, J. H./Apolenis, C. J., Div 700
DeMasi, R., Div 800
Franklin, P. J./Horsey, E. F., Lab 900
Seaton, J. W., 260
Witte, J. J., 250
Inas, P., 230
Tazza, L. M., 210
Griffin, P. W., 240
Jones, H. S., 250
Carson, A. A., 240
Paradis, R. J., 640
Howard, W. G., 250
Kasser, J. A., 250
Brune, H. B., 250
Prytulak, M. D., 250
Reggio, F., 250
Carver, R. V., 250
Marshburn, C. E., 630
Sullivan, R. F., 250 (15 copies)
Technical Reports Unit/800
Technical Information Office, 010 (10 copies)
DOFL Library (5 copies)

(Two pages of abstract cards follow)

<p>AD _____</p> <p>Diamond Ordnance Fuse Laboratories, Washington 25, D. C.</p> <p>DIRECT-COUPLED WAVEGUIDE FILTERS - Robert F. Sullivan</p> <p>TR-987, 12 March 1962, 22 pp text, 20 illus. DA-5806-01-014, CJS Code 5530.11.62400, DDFL Proj No 26300 - UNCLASSIFIED Report</p> <p>Design data are presented for rapid calculation in designing direct-coupled maximally flat bandpass filters in X-band using multiple-post obstacles. The data may be used for any TE₁₀ mode waveguide by using appropriate scaling factors. A design example is given to illustrate the design procedure, and the effects of mechanical tolerances is discussed.</p> <p style="text-align: center;">UNCLASSIFIED</p>	<p style="text-align: center;">Accession No. _____</p> <p>1. Multisection waveguide filters--Design data</p> <p>2. Direct-coupled pass filters--Calculations--Design</p> <p>3. Waveguide band-pass filters--Calculations--Design</p> <p>4. Waveguide filters--Electrical characteristics</p>	<p>AD _____</p> <p>Diamond Ordnance Fuse Laboratories, Washington 25, D. C.</p> <p>DIRECT-COUPLED WAVEGUIDE FILTERS - Robert F. Sullivan</p> <p>TR-987, 12 March 1962, 22 pp text, 20 illus. DA-5806-01-014, CJS Code 5530.11.62400, DDFL Proj No 26300 - UNCLASSIFIED Report</p> <p>Design data are presented for rapid calculation in designing direct-coupled maximally flat bandpass filters in X-band using multiple-post obstacles. The data may be used for any TE₁₀ mode waveguide by using appropriate scaling factors. A design example is given to illustrate the design procedure, and the effects of mechanical tolerances is discussed.</p> <p style="text-align: center;">UNCLASSIFIED</p>	<p style="text-align: center;">Accession No. _____</p> <p>1. Multisection waveguide filters--Design data</p> <p>2. Direct-coupled pass filters--Calculations--Design</p> <p>3. Waveguide band-pass filters--Calculations--Design</p> <p>4. Waveguide filters--Electrical characteristics</p>	<p>AD _____</p> <p>Diamond Ordnance Fuse Laboratories, Washington 25, D. C.</p> <p>DIRECT-COUPLED WAVEGUIDE FILTERS - Robert F. Sullivan</p> <p>TR-987, 12 March 1962, 22 pp text, 20 illus. DA-5806-01-014, CJS Code 5530.11.62400, DDFL Proj No 26300 - UNCLASSIFIED Report</p> <p>Design data are presented for rapid calculation in designing direct-coupled maximally flat bandpass filters in X-band using multiple-post obstacles. The data may be used for any TE₁₀ mode waveguide by using appropriate scaling factors. A design example is given to illustrate the design procedure, and the effects of mechanical tolerances is discussed.</p> <p style="text-align: center;">UNCLASSIFIED</p>	<p style="text-align: center;">Accession No. _____</p> <p>1. Multisection waveguide filters--Design data</p> <p>2. Direct-coupled pass filters--Calculations--Design</p> <p>3. Waveguide band-pass filters--Calculations--Design</p> <p>4. Waveguide filters--Electrical characteristics</p>
--	--	--	--	--	--

REMOVAL OF EACH CARD WILL BE NOTED ON INSIDE BACK COVER, AND REMOVED CARDS WILL BE TREATED AS REQUIRED BY THEIR SECURITY CLASSIFICATION.