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A HEURISTIC EXAMINATION OF THE KINEMATICS OF SIMPLIFIED ROCKET TRAILS

J. PRESSMAN R. CASEY

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UNITED STATES AIR FORCE, BEDFORD, MASSACHUSETTS

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A HEURISTIC EXAMINATION OF THE KINEMATICS OF SIMPLIFIED ROCKET TRAILS

I. INTRODUCTION

This report is the second of an initial series of reports whose purpose it is to clarify some of the individual aspects of the complex integrated behavior of missile trails. This initial series which, in effect, attempts to clear the ground is preliminary to more advanced forthcoming studies currently in progress and, frankly, is preparatory to them. The first report in this present series was "A Survey of Chemiluminescent Reactions Important to Missile Trails", which was performed prior to an experimental chemiluminescence program. The second report as titled above is an initial examination of missile trail shapes under simplified assumptions to give an intuitive cast to later more detailed studies. This second preliminary report will be followed by a third report entitled "The Penetrating Power of Particulate Matter in an Exponential Atmosphere".

By the title of this second report is meant that this study in itself does not attack the rocket trail shape problem completely directly or realistically but that the development will be of interest and value to later studies. The rationale here was to excise out one major aspect - in this case the effect of the rocket movement itself on the phenomenon and simplify the other aspects so as to sharpen this first aspect. Thus, the present sutdy will only have a reasonable degree of verisimilitude in that portion of the trajectory whose characteristics most closely matches the assumptions of this study. The assumptions made are given below and their virtue lies in the fact that simple analytical expressions are reached from which the effects of an accelerating source may be judged. The assumptions are:

(1) No significant interaction between the exhaust matter (either molecules or particulate matter) and the ambient atmosphere occurs.

(2) The acceleration of the rocket is assumed constant.

(3) A simple cosine law of velocities is assumed for the emitted particles.

Hence, it can be seen that the model above is most appropriate to the particulate matter component at high altitudes. This is so because of the small effect of the attenuated atmosphere on the relatively high momentum particulate matter. Also, at high altitudes where a reasonably large amount of the fuel has been consumed, the acceleration in the time interval before cut-off becomes more constant (see Equation 3) and, hence, is in closer agreement with assumption number two.

In the following there will be presented first an elementary statement of the elements of exterior ballistics appropriate to the development. This will be followed by an analysis of particle trajectories (under the assumptions stated above) together with the density resulting from these trajectories. Then, an analysis in particular is made of the trail behavior after burnout. This includes some attention to the inverse problem in which it is shown that from photographs of missile trails there can be deduced some of the properties of the missile itself.

It is again remarked that this is on the basis of the chosen model, but it might be very interesting to pursue this avenue in a more sophisticated development. Then some analysis is devoted to trajectory segments which are not vertical but horizontal or slanted. Finally, there are some concluding remarks of a general nature.

It is noted that in general this analysis is not valid in the lower region of the atmosphere where interaction through drag occurs. Hence, although in some cases, particle trajections are drawn on the ancillary graphs in this lower region, they are only valid in the zone of non-interaction indicated schematically on these graphs.

II. KINEMATICS OF SIMPLIFIED ROCKET TRAILS

A. SOME ELEMENTS OF EXTERIOR BALLISTICS

First we shall consider the rocket flight itself. Consider a rocket that is exhausting it's mass at the rate r, with a velocity c with respect to the rocket. We see that

$$d(mv) = Fdt - (c - v)\ dm \tag{1}$$

where m is the mass of the rocket, F is the external force acting on it (here -mg) v is the instantaneous velocity of the rocket with respect to the grounds, and dm is the mass of the exhaust in the time dt. Then

$$m = m_o - rt \tag{2}$$

where for m_o the initial total mass of the rocket we have

$$m_o = m_r + m_f$$

and (2a)

$$m_f = \text{total mass of fuel},\ m_r = \text{mass of rocket structure.}$$

Use of Equation (1) and (2) gives

$$[m_o - rt] \left(\frac{dv}{dt}\right) = rc + F$$

or

$$a = \frac{rc}{m_o - rt} - g$$

$$= \frac{rc}{m_r + m_f - rt} - g \tag{3}$$

$$\text{up to } t = \frac{m_f}{r}$$

or

$$a = \frac{rc}{m} - g$$

where a is the acceleration of the rocket. Separating and integrating to find v - the instantaneous rocket velocity.

$$v = c \ \ell n \left[\frac{m_o}{m_o - rt}\right] - gt \tag{4}$$

and y the distance the rocket has traveled upward,

$$y = c\left[t + \{t - \frac{m_o}{r}\} \ \ell n \left[\frac{m_o}{m_o - rt}\right]\right] - \frac{1}{2} gt^2 \tag{5}$$

B. PARTICLE TRAJECTORY AND ROCKET TRAIL DENSITY BEFORE BURNOUT

For the purpose of the following analysis the assumptions listed in the introduction have been made. These assumptions as previously stated are aimed at sharpening up the kinematic effect on the rocket trail caused by <u>effusion from an accelerating source</u>. The equations of motion for an exhaust particle leaving the rocket are:

$$x = [v_g \sin \varphi] [T - \tau] \tag{6}$$

$$y = -\frac{1}{2} g[T-\tau]^2 + [a\tau - v_g \cos \varphi] [T-\tau] + \frac{1}{2} a\tau^2 \tag{7}$$

where: v_g is the velocity of the particle with respect to the rocket

φ is the cone angle of the rocket motor

T is the total time since firing

τ is the time (after firing) at which the particle of interest left the rocket

a is the assured constant acceleration of the rocket

These equations give the envelope of the trail, if φ is the maximum cone angle. For a particle leaving the rocket at φ = 0 the downward component of the initial velocity is always greater than that of a particle with φ > 0. Hence, we can expect to find the particle fired in any short time interval dt to be in the form of a disk with its lowest point at the center. Figure 1 shows some of these disks. The constants chosen for this analysis were

$$a = 1.67 \times 10^{-2} \text{ km/sec}^2$$

$$g = 9.8 \times 10^{-3} \text{ km/sec}^2$$

T & τ are marked on the graph

Figures 2 and 3 show some of the individual trajectories of the particle, fired at different times as well as the overall envelope of the trail. Figure 3 is presented to illustrate the behavior of the visible trail at the critical region where the exhaust velocity equals the rocket velocity. Note that these graphs are valid only above roughly 100 km, the actual height depending upon the particle size since at lower altitude atmospheric interaction occurs invalidating our assumption.

Next, we wish to examine the density of the rocket trail. Equation (7) may be written

$$y = -\frac{1}{2}(g+a)\,\tau^2 + \left[(g+a)\,T + v_g \cos\varphi\right]\tau - (v_g \cos\varphi)T - \frac{1}{2}gT^2 \quad (8)$$

Taking the derivative with respect to the firing time of the particles, τ, we obtain

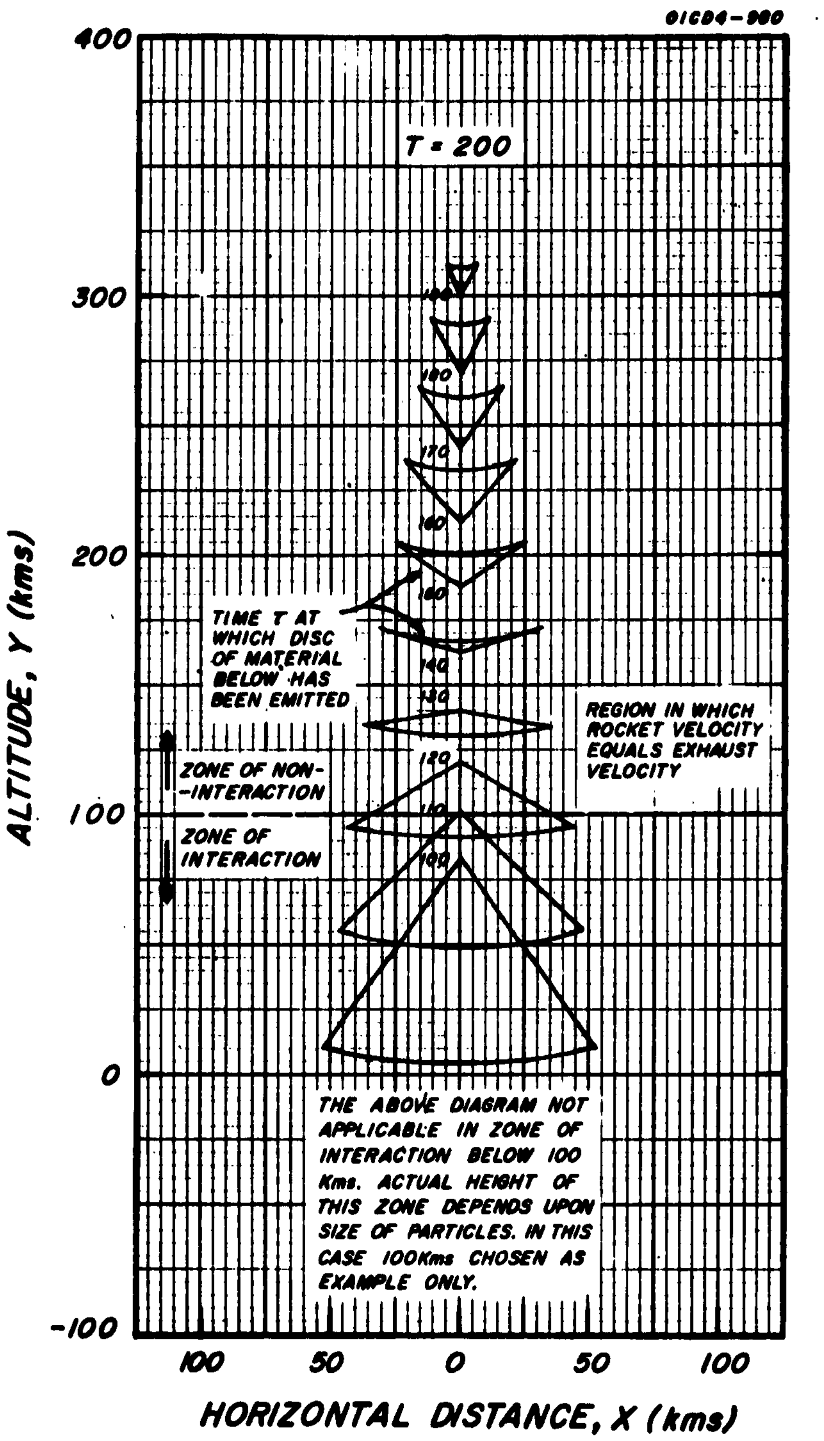

Figure 1. Position of Particles at T = 200 Seconds which have been Emitted at τ = 100 to τ = 200 seconds.

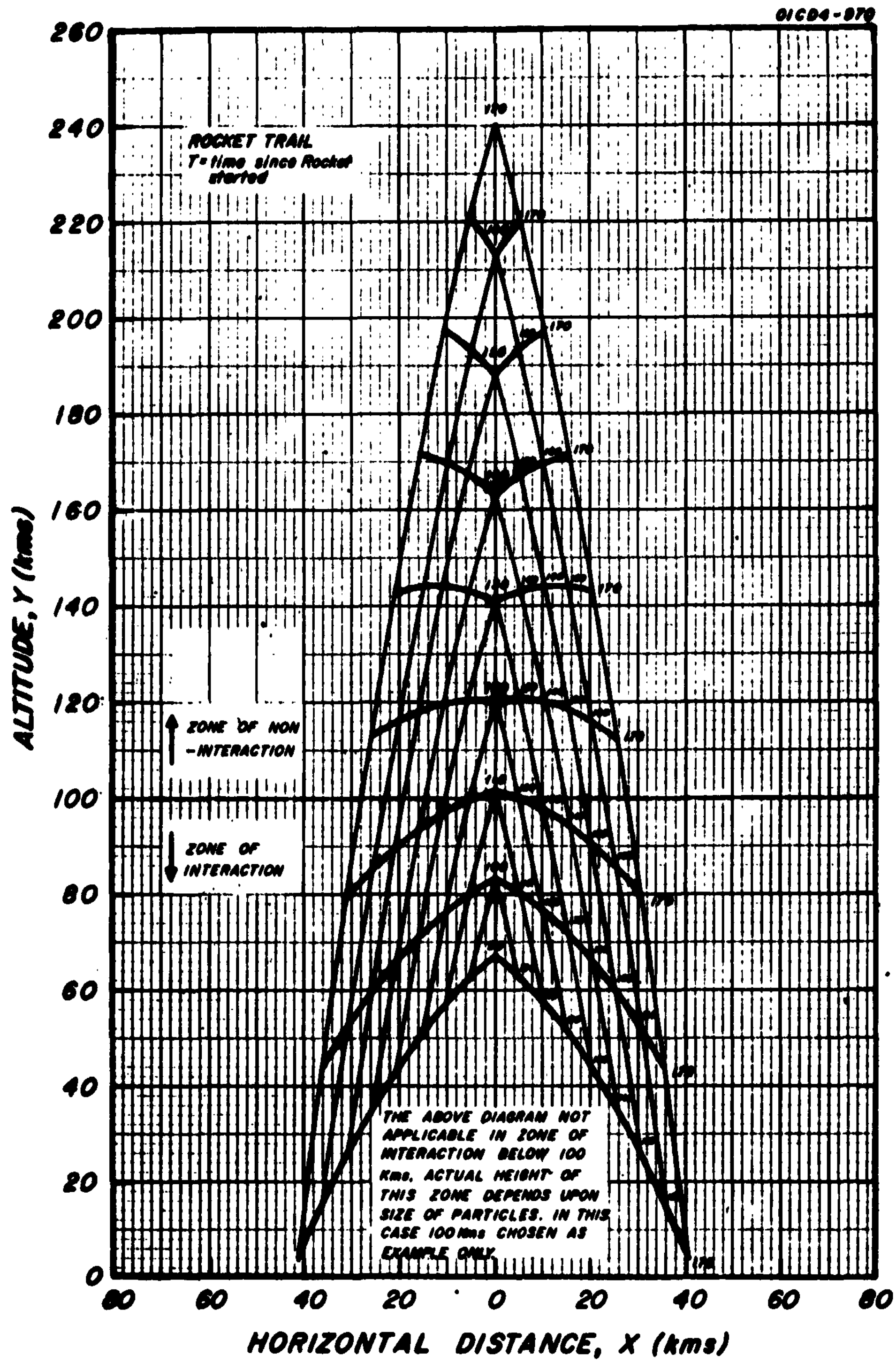

Figure 2. Rocket Trail Shape and Individual Particle Trajectories for T = 90-170 Sec.

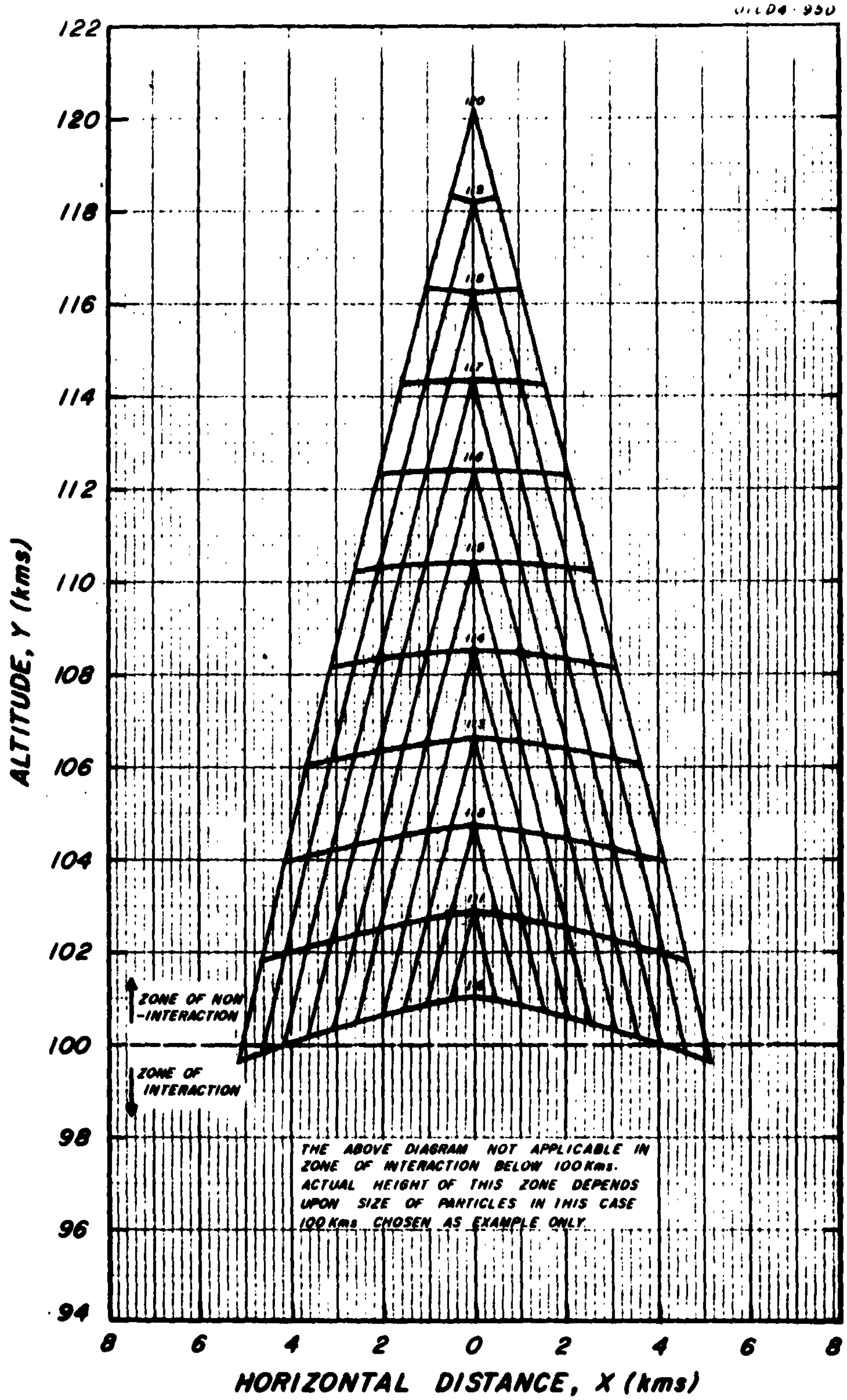

Figure 3. Rocket Trail Shape and Particle Trajectories for Z = 110 to 120 Sectonds and T = 120 to Demonstrate Behavior when Particle Velocity Equals Rocket Velocity.

$$dy = \{-(g+a)\ \tau + [(g+a)\ T + v_g \cos\varphi]\}\ d\tau$$

or

$$dy = [(g+a)\ (T-\tau) + (v_g \cos\varphi)]\ d\tau$$

The volume element (assuming symmetry about the y axis) is:

$$V = \pi\ X^2\ dy \qquad (9)$$

Making use of Equation (6) we find

$$V = \pi\ (v_g \sin\varphi)^2\ [(g+a)(T-\tau)^3 + v_g \cos\varphi\ (T-\tau)^2]\ d\tau \qquad (10)$$

This is the volume occupied by the particles fired in the time element $d\tau$. Since the density ρ is given by:

$$\rho = \frac{M}{V} \qquad (11)$$

and we know that a mass dm was expended during the time element $d\tau$

$$\rho = \frac{dm}{dt}\left[\frac{1}{\pi\ v_g^2 \sin^2\varphi}\left\{\frac{1}{(g+a)(T-\tau)^3 + v_g \cos\varphi\ (T-\tau)^2}\right\}\right] \qquad (12)$$

Since $\frac{dm}{dt}$ = constant = r then

$$\rho = r\left[\frac{1}{\pi\ v_g^2 \sin^2\varphi}\right]\left[\frac{1}{(g+a)(T-\tau)^3 + v_g \cos\varphi\ (T-\tau)^2}\right] \qquad (13)$$

Equation 13 indicates that as τ increases for fixed T, ρ always increases since $\tau < T$. Hence an examination of the trail from bottom to top shows that the density of the trail is always increasing for fixed T.

C. ROCKET TRAIL AFTER BURNOUT (VERTICAL FLIGHT) AND DETERMINATION OF ROCKET CHARACTERISTICS FROM MISSILE TRAIL PHOTOGRAPHS

Let us assume that photographs are taken of a rocket trail after the burnout of the motor. We assume that we have no data prior to the actual burnout. As an example consider Figure 4, when data are only available after burnout from 800 km upward. The equations of motion of a particle which has left the rocket at the moment of burnout are given at time τ after burnout by:

$$X = (v_g \sin \phi)\, \sigma \tag{14}$$

$$Y = -\frac{1}{2} g \sigma^2 + (v_r - v_g \cos \varphi)\, \sigma + y_o \tag{15}$$

where:

v_g is the velocity of the gas with respect to the rocket.

σ is the time of motion of the particle I.E. time after burnout.

φ is the maximum cone angle of the motor.

v_r is the velocity of the rocket at burnout.

From Equation 14 and Figure 5 we have

$$\frac{dx}{d\sigma} = v_g \sin \varphi = c \tag{16}$$

where c is a constant, which may be determined from any two photographs since

$$\frac{dx}{d\sigma} = \frac{x_2 - x_1}{\sigma_2 - \sigma_1} = \frac{\Delta x}{\Delta \sigma} \quad \text{or} \quad x = c\sigma \tag{17}$$

So knowing c and x it is possible to find σ.

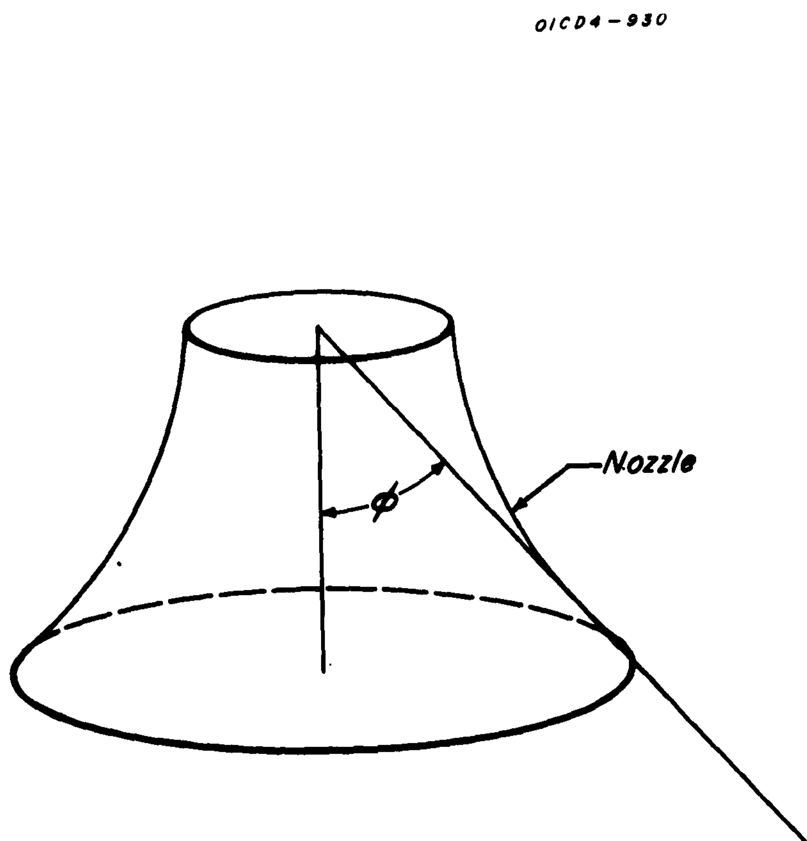

Figure 5. Schematic of Nozzle Angle Designation.

From Equation 15 we see that:

$$Y\sigma_2 = \frac{1}{2} g \sigma_2^2 + (v_r - v_g \cos \varphi) \sigma_2 + Y_o$$

$$Y\sigma_1 = - \frac{1}{2} \sigma_1^2 + (v_r - v_g \cos \varphi) \sigma_1 + Y_o$$

or

$$\Delta Y = Y\sigma_2 - Y\sigma_1 = - \frac{1}{2} g (\sigma_2^2 - \sigma_1^2) + (v_r - v_g \cos \varphi)(\sigma_2 - \sigma_1) \quad (18)$$

Since σ_2 and σ_1 are known by the use of Equation 18, since ΔY may be determined from any two photographs, and since g is known, we find that

$$\frac{\Delta Y + \frac{1}{2} g (\sigma_2^2 - \sigma_1^2)}{\sigma_2 - \sigma_1} = v_r - v_g \cos \varphi = D \quad (19)$$

where D is a constant that is determined from the above measurements.

We shall now obtain some information about the cone angle of the rocket motor. To do this we will examine the angle between the exhaust envelope and the vertical. (See Figure 6.)

Let us assume that the jth particle is released exactly at burnout and the ith particle is released sometime before burnout. We have

$$X_j = (v_g \sin \varphi) t_j$$

$$X_i = (v_g \sin \varphi) t_i$$

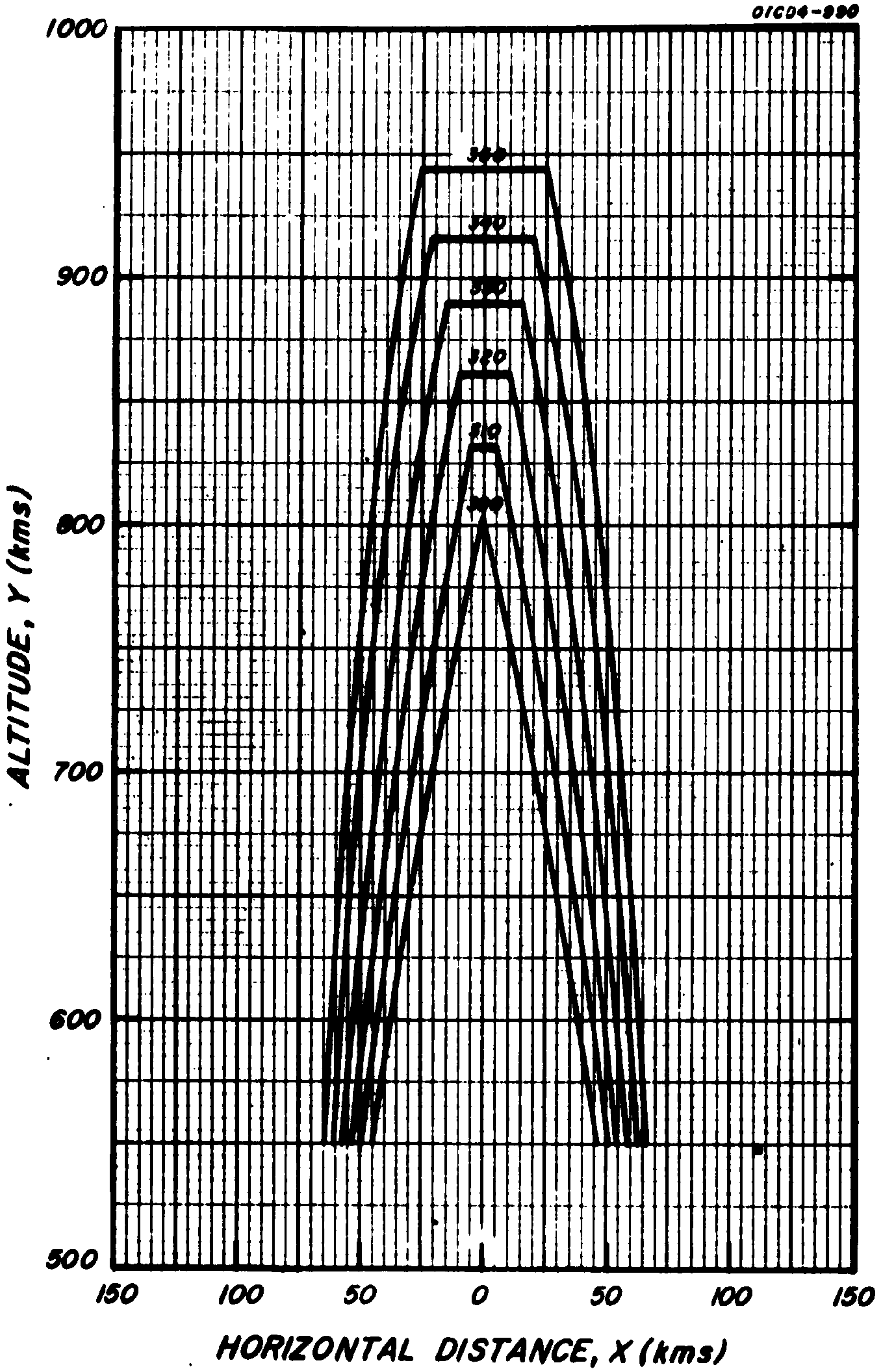

Figure 4. Shape of Rocket Exhausts After Burnout (300 Seconds).

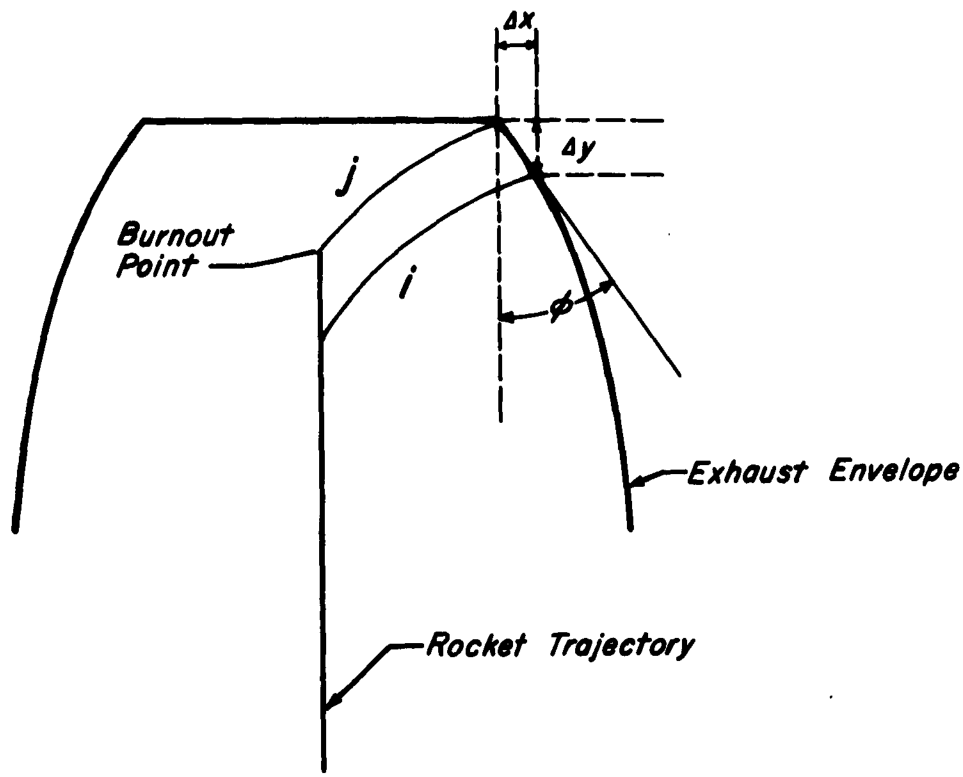

Figure 6. Schematic of Post-Burnout Rocket Exhaust.

and

$$\Delta X = (v_g \sin \varphi)\ (t_i - t_j)$$

where

$$t_j = \sigma \qquad (20)$$

$$t_i = T - \tau + \sigma \qquad (21)$$

From Equation 15

$$Y_j = \frac{1}{2}\, aT^2 - \frac{1}{2}\, g t_i^{\ 2} + [aT - v_g \cos \varphi]\ t_j$$

and

$$Y_i = \frac{1}{2}\, aT^2 - \frac{1}{2}\, g\ t_i^{\ 2} + [a\,\tau - v_g \cos \varphi]\ t_i$$

or

$$Y_j = \frac{1}{2}\, aT^2 - \frac{1}{2}\, g\sigma^2 + aT\sigma - v_g \cos \varphi\, \sigma$$

and

$$Y_i = \frac{1}{2}\, a\tau^2 - \frac{1}{2}\, g\ [(T - \tau)^2 + 2\sigma\ (T - \tau) + \sigma^2]$$

$$+ a\tau[T - \tau] + a\tau\sigma - v_g \cos \varphi\ [T-\tau] - v_g \cos \varphi\ \sigma$$

Since $\Delta Y = Y_j - Y_i$

$$\Delta Y = \frac{1}{2}\, a[T-\tau]^2 + \frac{1}{2}\, g\ [T-\tau]\ [(T-\tau) + 2\sigma]$$

$$+ v_g \cos \varphi\ (T-\tau) + a\sigma[T-\tau] \qquad (22)$$

Since by definition in the coordinate system selected we have

$$\tan\theta = \frac{\Delta X}{\Delta Y} \qquad \text{then} \tag{23}$$

$$\tan\theta = \frac{[v_g \sin\varphi]\,[T-\tau]}{\frac{1}{2}a\,[T-\tau]^2 + \frac{1}{2}g\,[T-\tau]\,\{(T-\tau) + 2\sigma\} + v_g \cos\varphi\,(T-\tau) + a\sigma(T-\tau)}$$

or

$$\tan\theta = \frac{v_g \sin\varphi}{\frac{1}{2}\,a(T-\tau) + \frac{1}{2}\,g[T-\tau] + g\sigma + v_g \cos\varphi + g\sigma}$$

or

$$\tan\theta = \frac{v_g \sin\varphi}{\frac{1}{2}(a+g)\,[T-\tau] + (g+a)\,\sigma + v_g \cos\varphi} \tag{24}$$

Let us consider two particles close to one another, so that $[T-\tau]\to 0$.

Then

$$\tan\theta = \frac{v_g \sin\varphi}{(a+g)\sigma + v_g \cos\varphi} \tag{25}$$

Now $\tan\theta$ may be determined from photographs, and making use of Equation 16 and Equation 19, while setting $\tan\theta = \zeta$ we obtain:

$$(a+g)\sigma - D + v_r = \frac{c}{\zeta} \tag{26}$$

So knowing ζ from two photos

$$(a+g)\sigma_2 - D + v_r = \frac{c}{\zeta_2}$$

$$(a + g)\sigma_1 - D + v_r = \frac{c}{\zeta_1}$$

We obtain $(a + g)(\sigma_2 - \sigma_1) = c\left(\frac{1}{\zeta_2} - \frac{1}{\zeta_1}\right)$

Since σ_1, σ_2, and c are known and ζ_1 and ζ_2 may be measured and

$$a = - g + \frac{c}{(\sigma_2 - \sigma_1)}\left(\frac{\zeta_1 - \zeta_2}{\zeta_1 \zeta_2}\right)$$

so the acceleration of the rocket may be determined. Then Equation 25 becomes

$$\tan \theta = \frac{c}{(g + a)\sigma + v_g \cos \varphi} \qquad (27)$$

Also
$$\frac{v_g \sin \varphi}{v_g \cos \varphi} = \frac{c}{v_g \cos d} = \tan \varphi \qquad (28)$$

Since $v_g \cos \varphi$ may be found from Equation 27, the cone angle φ may be found from Equation 28. Also since c is known and φ is known, the velocity of the gas with respect to the rocket, v_g, may be found from Equation 16 since

$$v_g = \frac{c}{\sin \varphi}$$

From Equation 14 the velocity of the rocket at burnout may be determined. Since, at burnout

$$aT = v_r \qquad (29)$$

It is possible to determine T the total burning time and also since

$$1/2\ aT^2 = Y_o \qquad (30)$$

the height of the rocket at burnout can be determined. The determination of the total burning time and the height of the rocket at burnout may be in doubt as the acceleration of the rocket is really not constant.

Our conclusion then is that for this highly idealized case one can measure many of the significant performance characteristics of a rocket such as, exhaust velocity, cone angle, time of burnout and height of burnout from pictures taken after burnout. It may very well be that for more realistic models similar deductions may be made from post-burnout photographs.

D. ANALYSIS OF HORIZONTAL AND SLANTED PATHS

We are now concerned with the rocket trail when the rocket is traveling parallel to the ground. See Fig. 7.. It can be seen that the equations of motion now become more complicated. If the rocket started from the point X = 0, Y = 0 then, the x coordinate and velocity of the released particles are given by

$$X = X_o + v_{gX} t$$

$v_{gX} = (v_r - v_g \cos \varphi)$ and in this case

$$v_r = a\tau$$

as before v_g is the velocity of the gas with respect to the rocket and v_r is the velocity of the rocket (for many rockets $v_g \simeq 2$ km/sec) where τ = time from firing of the rocket at which the particles are released, and t = time of flight of the released particle.

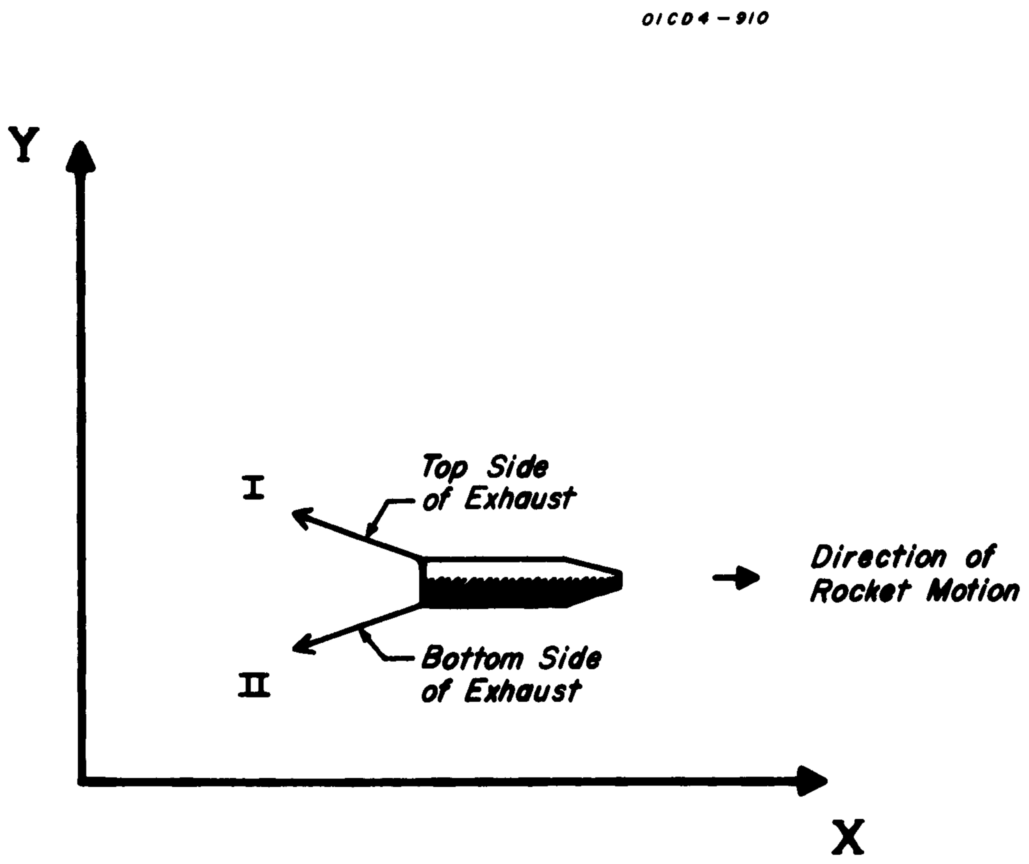

Figure 7. Schematic of Horizontal Flight.

For Y we obtain

$$Y_I = +(v_g \sin \varphi)t - \frac{1}{2} gt^2$$

$$Y_{II} = -(v_g \sin \varphi)t - \frac{1}{2} gt^2$$

for the top and bottom component.

So that the equations of motion for the particle fired upward are:

$$X_I = \frac{1}{2} a\tau^2 + (a\tau - v_g \cos \varphi)t \quad (31)$$

$$Y_I = (v_g \sin \varphi) t - \frac{1}{2} gt^2 \quad (32)$$

and for the particles fired downward:

$$X_{II} = \frac{1}{2} a\tau^2 + (a\tau - v_g \cos \varphi) t \quad (33)$$

$$Y_{II} = - (v_g \sin \varphi) t - \frac{1}{2} gt^2 \quad (34)$$

One of the trails for a horizontal flight has been computed using the same parameters as in Section A. It is interesting to note that there appears to be no separation in the trail, when $v_g = v_r$. It would also seem (for these parameters) that a possible small separation between the particles would fall too low in the atmosphere for the separation to be noticed. That is the particles would be in the lower, more dense region of the atmosphere before the separation would become observable. See Figure 8.

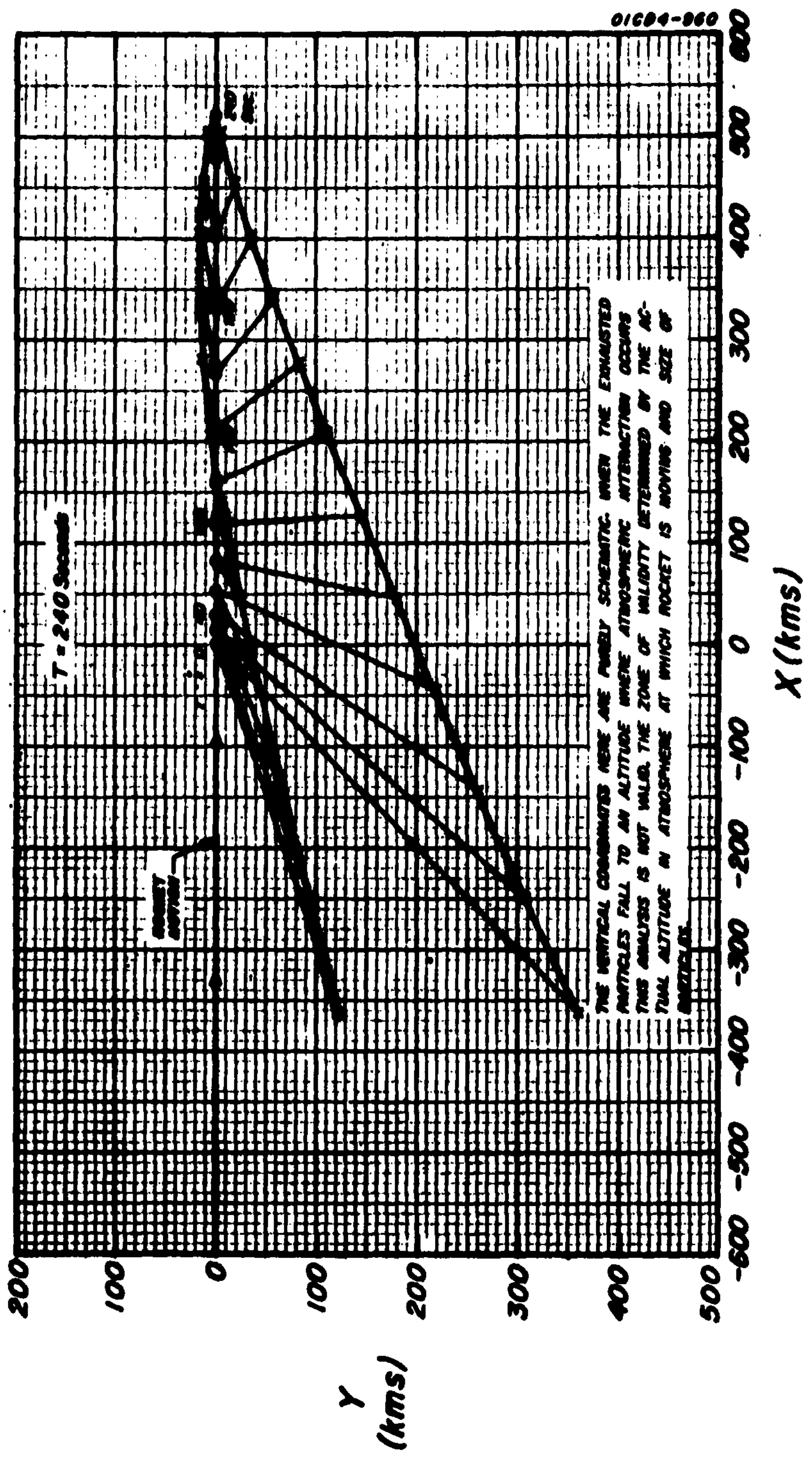

Figure 8. Shape of Missile Trail at 240 Seconds for Horizontal Flight ($a = 1.67 \times 10^{-2}$ km/sec^2). Points Calculated at Z = 20 sec. Intervals.

Next let us consider a rocket moving at some angle, θ, with the vertical. We expect equations of motion of the form:

$$X_1 = X_o + v_{X1}t$$

$$Y_1 = Y_o - \frac{1}{2} gt^2 + v_{Y1}t$$

$$X_{II} = X_o + v_{XII}t$$

$$Y_{II} = Y_o + v_{YII}t - \frac{1}{2} gt^2$$

We write v_{X1}, v_{XII}, v_{YI} and v_{YII}

$$v_{Xr} = - v_r \sin \theta$$

$$v_{Yr} = v_r \cos \theta$$

$$v_{XIg} = v_g \sin (\theta + \varphi)$$

$$v_{XIIg} = v_g \sin (\theta - \varphi)$$

$$v_{YIg} = -v_g \cos (\theta + \varphi)$$

$$v_{YIIg} = - v_g \cos (\theta - \varphi) .$$

From the diagram on the preceeding page, as

$$v_{XI} = v_{Xr} - v_{XIg}$$

etc.

we obtain;

$$v_{X1} = - v_r \sin \theta + v_g \sin (\theta + \Phi) \tag{35}$$

$$v_{XII} = - v_r \sin \theta + v_g \sin (\theta - \varphi) \tag{36}$$

$$v_{Y1} = v_r \cos \theta - v_g \cos (\theta + \varphi) \tag{37}$$

$$v_{Y11} = v_r \cos \theta - v_g \cos (\theta - \varphi) \tag{38}$$

It is noted that the signs of the terms depend upon the particular values of θ and φ selected. For our purposes we will deal only with the geometrical configuration indicated in Figure 9. The remaining configurations may be analyzed in a similar manner.

Let us examine these velocities under various conditions, i.e., when there are restrictions on the velocity of the exhaust gas and the velocity of the rocket.

If; $v_g \sin (\theta - \varphi),\ v_g \sin (\theta + \varphi) < v_r \sin \theta$

then; v_{XII} neg v_{XI} neg

if; $v_g \sin (\theta - \varphi) < v_r \sin \theta;\ v_g \sin (\theta + \varphi) > v_r \sin \theta$

then; v_{XII} neg v_{X1} pos.

if; $v_g \sin (\theta - \varphi),\ v_g \sin (\theta + \varphi) > v_r \sin \theta$

then; v_{XII} pos v_{X1} pos

From the above discussion we see that the top side of the trail may be drifting in a specified direction, while the bottom of the trail is drifting in the opposite direction.

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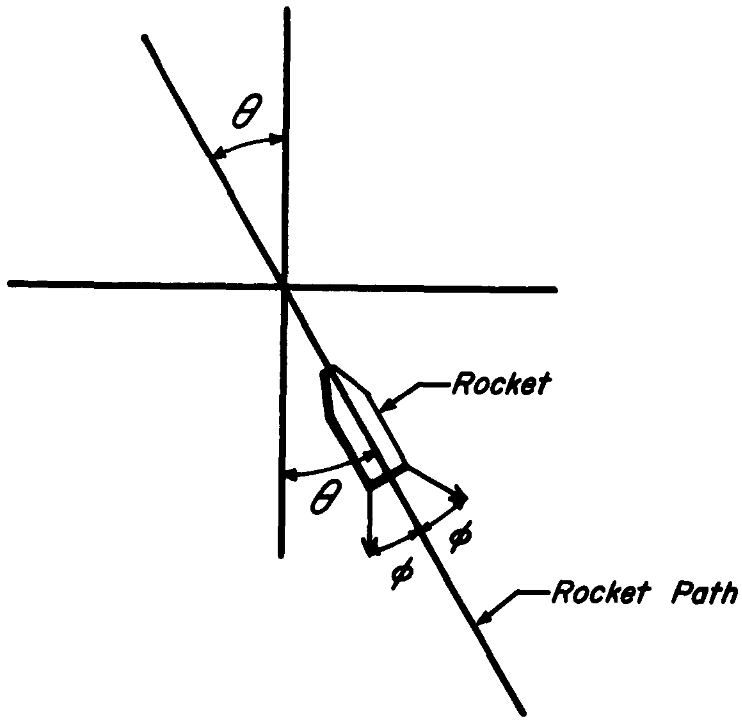

Figure 9. Schematic for Rocket Oblique Path.

Let us look for the conditions under which we can expect a transition to occur.

If $\theta < \varphi$

then we may expect a transition where

$$v_r \sin\theta = v_g \sin(\theta + \varphi) \quad \text{or}$$

$$\tan\theta = \frac{\sin\varphi}{\frac{v_r}{v_g} - \cos\varphi}$$

As an example, if the velocity of the rocket is 5 km/sec, the velocity of the gas is 2 km/sec, and the cone angle $\varphi = 15^\circ$ then

$$2.5 = \frac{\sin(\theta + 15^\circ)}{\sin\theta} \quad \text{and } \theta \sim 4.6^\circ$$

For various times a "photograph" may be constructed of what the trail looks like. For instance if we choose a photograph time $T = 400$ seconds then for times of release $t = 100, 200, 300$ seconds we may compute the ballistic trajectories and final position of the particles for $\tau = 300$, 200, and 100 seconds respectively.

No figures will be presented since the patterns are similar to the horizontal case.

E. INFORMATION FROM ROCKET TRAILS

As stated in Section B, we feel that useful data concerning the rocket and rocket flight may be determined from an examination of the trail after burnout. At burnout we can determine within the assumptions made:

1. The cone angle of the motor
2. The velocity of the rocket at burnout
3. The time since burnout
4. The velocity of the gas with respect to the rocket
5. The acceleration of the rocket at burnout (assumed constant)
6. The total time of rocket burning (assuming constant acceleration
7. The height of the rocket at burnout

Note that this analysis does not require that we see the trail at the time of burnout.

It is felt that if there were some method of "marking" the trail - such as a bump in the smooth envelope - it would be possible to obtain the same results as the burnout case. The difficulty is that the particles are indistinguishable in the envelope. It is also possible that the time that the particles have been in motion may be determined from the radiation pattern emitted by the trail.

In the previous sections we have analyzed the trails of rockets in vertical flight, horizontal flight, flight at an angle with the vertical. For a vertical flight since the envelope is uniform (no distinguishing markings) the only useful information obtainable before burnout is the cone angle.

For the horizontal flight it is interesting to notice the difference in path for the particle whose initial vertical velocity is upward and the particle whose initial velocity is downward. This and the cone angle could possible give us the three equations necessary to find:

1. The cone angle

2. The velocity of the rocket at burnout

3. The velocity of the gas with respect to the rocket

This is for the case before burnout. As can be seen from Section C, this has not been worked out since this study is not meant to be exhaustive.

For the flight at an angle with the vertical we again see the different paths for particles fired with an upward and downward component. Analysis not presented here also indicates that one of the trails may cross the flight path of the rocket twice. Thus we might be able to find;

1. The cone angle

2. The velocity of the rocket at burnout

3. The velocity of the gas with respect to the rocket.

4. The angle of flight with the vertical.

It is possible that the same analysis, that was applied to the vertical flight, would be applicable to the horizontal and angular flight for the post-burnout case.

Conclusions:

First it seems that further studies are warranted by the information obtainable after burnout. If an experimental study of the trails was made it would then be possible to examine the trail for irregularities before burnout

It seems that future work should be divided into two parts, immediate and long range.

Immediate Work

1. Application of burnout analysis to the case of horizontal and angular flight.

2. Determination of the significance of the envelope of horizontal and angular flight.

Long range Work

1. Reanalysis of the three modes of flight with variable acceleration

2. For the case of angular flight allow θ to become a variable.

3. Work out the density function for horizontal and angular flight.

F. CONCLUSIONS

The assumptions made have been drastic enough to perhaps make the applicability of the results open to serious question. However, we do obtain at this price a simple tractable analysis. For additional more realistic assumptions, the resulting set of equations are sufficiently complicated so that we may well gain little insight into the nature of the effect of the missile movement on the trail shape and so that involved numerical analysis is necessary.

However, it is felt that the preceding analysis indicates, in a simple fashion, some of the complex morphology and phenomena due to the kinematics of missile trails and hence serves as a rough guide to future work.