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STUDY OF THE USE OF MODELS TO SIMULATE DYNAMICALLY LOADED UNDERGROUND STRUCTURES

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Final Report

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Research Directorate
AIR FORCE SPECIAL WEAPONS CENTER
Air Force Systems Command
Kirtland Air Force Base
New Mexico

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(Prepared under contract AF 29(601)-4374
by American Machine & Foundry Company,
Mechanics Research Division, Niles, Illinois)

HEADQUARTERS
AIR FORCE SPECIAL WEAPONS CENTER
Air Force Systems Command
Kirtland Air Force Base
New Mexico

FOREWORD

Mechanics Research Division of American Machine & Foundry Company takes pleasure in presenting this final report on the Study of the Use of Models to Simulate Dynamically Loaded Underground Structures in compliance with the provisions of Contract No. AF 29(601)-4374.

This study was carried out under the supervision of A.A. Arentz, Jr., Project Engineer. Acknowledgement is made to Messrs. D.H. Duel, J.B. Harkin, L.E. Fugelso, and M.F. Darienzo of Mechanics Research Division for their contributions to this report. Other members of Mechanics Research Division's staff too numerous to mention, have also contributed to this program through their suggestions and criticisms.

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ABSTRACT

This report presents the results of an analytical investigation of the use of models to simulate dynamically loaded structures buried under the ground.

Scaling laws were proposed relating the model structure and soil medium to the corresponding prototype components. Physical properties of some materials were presented as a guide in choosing possible model simulants and a method presented for simulating soils. The report represents the initial effort towards a solution of the dynamic similitude problem.

PUBLICATION REVIEW

This report has been reviewed and is approved.

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NOTATION

$\overline{AS}_1$ =	Slant distance that the C_1 wave propagates (See Figure 3.4)
$\overline{BS}_2$ =	Slant distance that the C_2 wave propagates (See Figure 3.4)
C_1 (Wave)	Dilatational (Wave)
C_2 (Wave)	Shear (Wave)
c_1	Velocity of C_1 wave
c_2	Velocity of C_2 wave
c_o	Velocity of sound in air
c (Subscript)	...of the shell
D	diameter of the shell
D (Subscript)	...of the medium (soil)
dyn (Subscript)	Dynamic value of ...
E =	Modulus of elasticity
h =	Thickness of shell
$k = \frac{6\ c_o^2}{7\ P_o}$	"Ambient" constant
L =	Length of the cylindrical shell
ℓ =	Structural deflection
m (Subscript)	...in the model (shock tube)
p =	Overpressure
P_o	Ambient air pressure
p (Subscript)	...in the prototype
t =	any time

t_A = Time when C_1 wave leaves point A (See Figure 3.4)

t_B = Time when C_2 wave leaves point B (See Figure 3.4)

t_{S1} = Time when C_1 wave arrives at S_1 (See Figure 3.4)

t_{S2} = Time when C_2 wave arrives at S_2 (See Figure 3.4)

t_w = Pulse duration

$\tilde{t} = t_{S2} - t_{S1}$ = Time lapse between arrival at the structure of C_1 and C_2 waves

$\hat{t}$ = Time lapse between the two parts of a wave split by a non-linear material

U = Air shock velocity

u = Displacement of the medium (soil)

$v = \dot{u}$ = Particle velocity

x = Any distance coördinate

z = Depth of burial

$\beta_{dyn} = \frac{c_1}{c_2} = \sqrt{\frac{2(1-\nu_{D, dyn})}{1 - 2\nu_{D, dyn}}}$

γ = Weight density

$\Delta = \frac{E}{1-\nu_c^2}$ = Flexural rigidity

δ = Logarithmic decrement for percyclic damping

ϵ = Strain

θ_1 = Mach angle of C_1 wave

θ_2 = Mach angle of C_2 wave

NOTATION (Continued)

$\lambda_{...}$ =	Scale factor of the quantity indicated by its subscript (See Table 4.1)
Λ =	Wave length
Λ_R =	Characteristic length of response wave
ν =	Poisson's Ratio
ρ =	Mass density
σ =	Stress
τ =	Any natural period of the structure

Note: The notation appearing in the appendices is defined where used.

1.0 INTRODUCTION

The study program pursued under the present contract has been directed toward the objective of initiating a logically directed approach to the prediction of soil and structure response to air-blast effects of thermonuclear weapons by means of experiments with small models subjected to simulated blast loading. Specifically, it is intended to lay a theoretically based foundation for the future direction of shock-tube requirements.

It is important to recognize the philosophy that must underlie the approach to the present study. Generally, a wide gap exists between theory and practice in the use of models to study physical phenomena. This is mainly due to the difference in interests of the theoretician and the experimenter. Because of this, model experiments have not always been as fruitful as could otherwise be expected. The problems of model experimentation cannot be successfully solved exclusively by theory or experiment, but by a compatible combination of both.

While there has been effort expended in both theory and experiment in previous model studies in the area of underground structure response to airblast loading, no unifying approach has been advanced. The present study is directed toward gathering the information produced in previous model studies and in theoretical work and welding it into a framework upon which future model experiments may be based. Such future experiments then will be both directed and purposeful.

Thus, the present study is the first link in a balanced overall program of theory and experiment. The results of this first stage will not be a panacea for model test procedures, but will be the basis for the initial phase of meaningful experimentation.

The philosophy of the present study then is one of perspective. It gathers experience and theory from the past and applies it to the production of more meaningful results in the future. It will mainly determine the questions to be answered by future work and the proposed methods for obtaining these answers.

1.1 Objectives

The fundamental problem with which this study program is concerned is the development of scaling laws which will enable the structural response of underground structures subjected to air-blast loading to be predicted by means of model experiments.

This problem involves the application of dimensional analysis to formulate a means for qualitative (and, where possible, quantitative) prediction of full-scale behavior of dynamically loaded structures through interpretation of model experiments.

The problem also involves the use of distorted scaling and model simulants where such are necessary to achieve the desired end result. Determination of the type and usefulness of the information that may be gathered from model experiments is another facet of the problem. The results of the study program consist of recommended methods, procedures, and scaling laws for future shock-tube experiments.

1.2 Approach

The research program under the present contract has been divided into three phases: (1) Preliminary Studies, (2) Development of Scaling Law, and (3) Model Material Investigations.

Preliminary studies include a literature search and ascertainment of the characteristics of the Air Force shock tube.

During the conduct of the study program the approach to the second phase was materially altered. Initially, as reported in the interim report, this phase was subdivided into a well ordered system of catagories of distinct phenomena. These phenomena (loading, soil, structure, action of loading on soil, and interaction between soil and structure) and their attending parameters were first considered more or less in vacuo. Then it was planned to use the relationships developed for these distinct catagories and the classical methods of dimensional analysis and theory of models to formulate scaling relationships. The conflicts encountered between the scaling relationships thus developed were to be adjusted and resolved into a meaningful scaling law.

As this phase of the work continued it became apparent that the approach was leading to a complexly intertwined and conflicting scaling system. Fundamental phenomena and peripheral trivia were so inextricably combined that the basic questions to be answered by experimental processes were obscured.

At this juncture, a re-evaluation of the problem led to a revision of the method of approach. It was decided to concentrate on the aforementioned basic questions and to determine how they could best be answered in as unfettered a manner as possible.

In accordance with the revised approach, maximum effort was directed toward determining which questions concerning the general problem of dynamically loaded underground structures were most in need of answers. Extant theories were reviewed to determine upon what assumptions their major premises were based. Conflicting theories were compared to determine wherein they differed and how they could be resolved. These considerations led to the formulation of several propositions which were used as the basis for the scaling relationships developed.

This approach, when implemented by shock tube experimentation, will serve to either confirm or disprove current leading theories in the field of dynamically loaded underground structures. Such confirmation or denial should lead to a more rapid advance in theory and thus to a new set of shock tube experiments. This then is in conformance with the philosophy of this study as presented in Section 1.0.

Phase 3 of the program consisted of developing an outline of the method to be used for determining the properties of model simulants necessary to properly scale the desired effects. A list of possible simulants and their properties was also prepared.

In the derivation of the scaling laws, the shape of the structure used was a cylinder in all cases. This allowed the derivation of the scaling law to be as uncluttered as possible and will make experimentation easier. Cylinders are relatively easy to fabricate (as opposed to spheres, domes, and the like). Since the phenomena of interest are general rather than directed to given structures, a single geometry will serve the purpose of indicating the effects of the phenomena instead of the peculiarities of different shapes.

2.0 STATEMENT OF THE PROBLEM

In order to retain perspective, the statement of the problem will be in three parts corresponding with the three phases of the program as outlined in Section 1.2. This is not to imply that the aforementioned 3 phases are co-equal in importance or that they represent equal amounts of effort. The important phase and that upon which the most effort was expended was the second. The great bulk of this report will deal with phase 2.

2.1 Preliminary Studies

The first step in conducting an investigation of the use of models to simulate dynamically loaded underground structures was to determine the results of previous investigations pertinent to the problem. To accomplish this, the problem was divided into three sections and a literature search was conducted to establish the present state of the art in each field. The areas selected for the literature search were:

a. Blast simulation techniques (shock tube)

b. Dynamic behavior of soils

c. Theoretical investigations involving the response of buried shell-type structures to a shock wave.

The literature survey accomplished its intended purpose of indicating present approaches to the problem. A bibliography of the portion of the literature surveyed which was most helpful to the present problem is presented in the Appendix of this report.

A visit to the Air Force Shock Tube Facility and discussions with the operating personnel thereof served to acquaint AMF personnel with the physical realities of the model experiment program which will follow the present study.

2.2 Development of Scaling Law

In order to develop a scaling law that was not just another rehash of the familiar applications of the theory of models as developed for problems in hydraulics and aerodynamics, it was necessary to formulate a fresh approach to the specific problem at hand-how to make use of models to determine the phenomenology of dynamically loaded underground structures. Unfortunately, the use of models was conceived in the field of hydraulics and developed in the field of aerodynamics. This genesis has led to stereotyped applications of the theory of models to other problem areas and provided a road block to those endeavoring to use models in other fields. Thus, it was decided to forego tradition in the present study in order to develop a scaling law which would be meaningful to the solution of important problems in the field being investigated.

Essentially, the procedure involved the formulation of basic conditions that reflected the important problems. Then the scaling law was developed.

2.2.1 Formulation of Basic Conditions

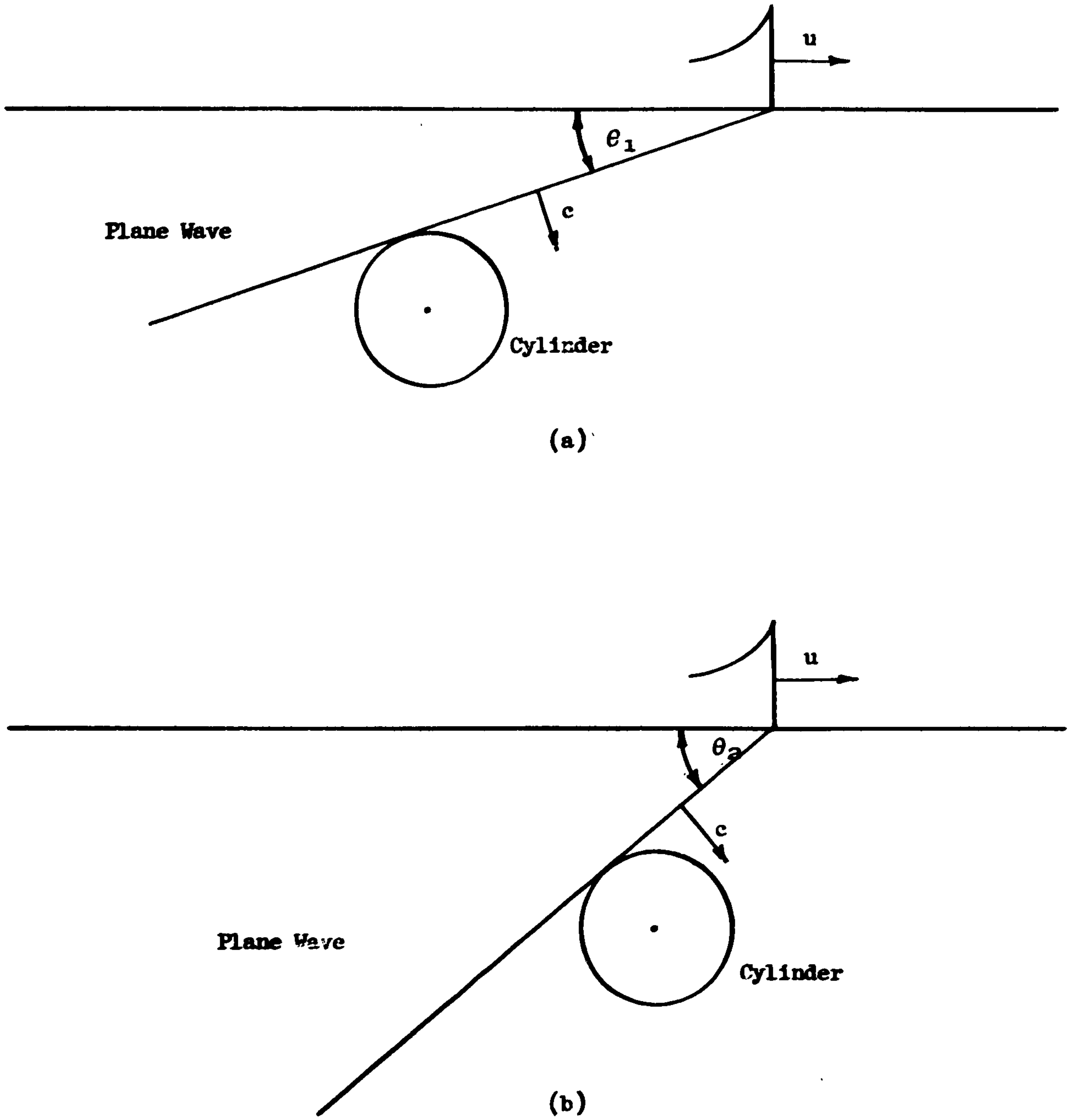
The experience of MRD personnel in the field of underground structures and blast loading and the results of the literature search have led to the formulation of a list of basic conditions or questions which bear heavily on the ultimate solution of the problem of buried structures. The entries on this list are not necessarily co-equal in significance or importance.

Two objectives were used to guide the formulation of this list. The first was to determine the relevance of certain phenomena whose significance to the problem was of a debatable nature. The second was to develop the capability of obtaining at least qualitative information as to certain phenomena which are felt to be important but have not yet yielded to theoretical investigations.

The list, as formulated, begins with the importance of the role of structural response in the total problem. Under what circumstances does the response of the structure, when combined with the nature of the forces exerted upon the structure, lead to one history of deformation and under what set of circumstances does it lead to another? The first family of conditions deals with the relation of the natural period (or periods) of the structure and the pulse width of the incoming wave. In postulating this family, the ratio of the pulse width to a natural period of the structure was investigated. The range of values which this ratio could run was divided into 3 regimes. Regime I includes those values of the ratio where the pulse width is extremely short in comparison with the natural period of the structure. Regime II contains pulses whose durations are roughly one-half as large as that of the natural period of the structure.
Regime III includes the remaining higher values of the ratio. The boundaries of these regimes have been worked out and this analysis is included in later portions of this report.

The second condition developed in the consideration of a realistic scaling law also involved the natural periods of the structure. This condition arose from consideration of the question: Which modes of the structure that are excited by the dilatational wave are of such frequency and mode shape that they are reinforced by the shear wave by virtue of its time of arrival? If such relationships do exist, it was felt that they must be preserved.

Consideration of the influence of Mach angles led to the third condition imposed on the scaling system. If the Mach angles must be preserved, the ratio of the shock velocity in the air to the seismic velocity must be preserved since it is this ratio which determines the Mach angle. However, it was determined



Plane Waves Striking a Cylinder

Figure 2.1

that the requirement to preserve Mach angles is not a strict one. Consider, for example, a structure of the form of a horizontal cylinder whose longitudinal axis is parallel with the wave front. This example is illustrated in Figure 2.1. Geometrically, since the cylinder is symmetric about its axis, the result is basically the same in Figures 2.1(a) & 2.1(b). The limiting case is $\theta = 90^\circ$, for beyond this, the superseismic case is not preserved. Thus, on purely geometric considerations, Mach angles do not have to be precisely preserved. The only consideration would be to preserve superseismicity. It should be pointed out, however, that the degree of non-preservation of the Mach angle, even within the broad limitation imposed, does have an effect on the ratio of stress impinging on the structure to overpressure at the surface. As the Mach angle increases, this ratio decreases. These considerations then form another condition for the scaling law developed.

It is interesting to note here the relative insensitivity of the Mach angle to changes in air-shock velocity under certain conditions. This may be seen in Figure 3.5 which shows Mach angle as a function of shock velocity for different values of seismic velocity. Consider, for example, the curve for c = 1500 ft/sec. Between values of 10,000 and 20,000 ft/sec for shock velocity, the Mach angle varies only from about 9° to 4°. Such behavior can be useful in the practical solution of difficult scaling problems.

Non-linearities of the soil stress-strain curve formed the basis of the next condition. Presuming that soil exhibits a stress-strain curve like that shown in Figure 2.2, the questions that arise concern the need for preserving the shape of this curve in the model.

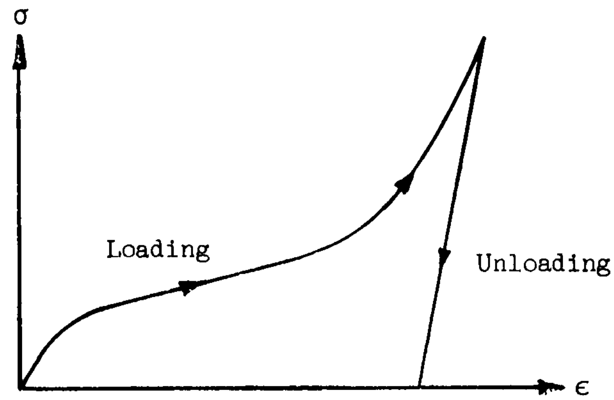


Figure 2.2 Typical Stress-Strain Curve for Soil

For example, assuming that the scaling law developed would be non-linear and that σ and ε would be scaled independently of each other, the question of the relationship between the loading and unloading portions of the curve could be significant, since the slope of the unloading portion of the curve affects recovery displacement and thus energy propagation away from the structure.

Another condition imposed was that involving the relationship between soil displacement and structural deflection. Since soil-structure interaction depends in part on the relative motions of both soil and structure and the maintenance or non-maintenance of contact between the two, it becomes necessary to scale soil displacement and structural deflection the same.

In addition to the above major basic conditions the following factors also influence the derivation of a scaling law.

Depth of burial of the structure affects the problem in several ways. It may, for example, affect the amount of attenuation of the stress wave if the attenuation is due to plastic or percyclic dissipation. It is also involved in determining whether, in the case of shallow burial, any significance may be assigned to the reflection of stress waves from the ground surface. Depth may also be significant if, again in the case of shallow burial, the mass of soil above the structure is significant.

When a structure has a high acoustic impedance compared with that of the soil, the effect of the response of the structure may become significant. (Structure responses can cause attenuation of local soil pressure which propagates as what is termed the "response wave.") If these phenomena are significant (this is probably dependent on the soil recovery capacity) the characteristic length of the response wave must be scaled as the incoming wave length.

The analysis of the above conditions and other limiting factors will be found in Section 3.

2.2.2 Derivation of Scaling Relationships

The end product of the aforementioned analyses was expressed in terms of scaling relationships. Certain scale factors were deemed fixed or given (such as overpressure and therefore shock velocity) and the remaining scale factors were expressed in terms of these. Each desired condition dictated a number of such relationships. These were tabulated in a form such that the scaling law for any particular combination of desired conditions may be readily ascertained.

From the scaling relationships and associated tables, the phenomena which formed the bases for the desired conditions can be incorporated into an experimental program and these phenomena can be imposed on model structures. By varying the values of the scale factors, the effect of the phenomena may be studied both separately and in concert. In this way, more qualitative (and quantitative) information may be obtained to enable further theoretical studies to be made. The existence of other effects than those described herein may also be tested. By verifying the existence or non-existence of certain phenomena, a clearer picture of the total problem can be obtained.

Suggestions as to the implementation of the above approach are contained in Section 6.0. The suggested tests given there are by no means all inclusive, but are intended to point up basic problems which might be solved via the experimental approach. Other problems which may be of present interest to AFSWC personnel may be amenable to this approach. It is also expected that the results of the testing program will uncover new problem areas which in turn can be made the subject of new tests.

2.3 Model Investigation

After the scaling laws have been developed, the successful application of these laws will depend on the ability to find model materials whose physical properties are compatible with the development of usable scale factors. The material parameters of a large group of available and inexpensive materials must be obtained either through published data or, when the required data is unavailable from this source, by tests conducted to furnish these parameters.

As an aid in choosing model material simulants several important criteria are outlined and illustrated by an example. In addition a tabulated list is included which presents some possible model simulant materials and their material properties which are pertinent to the development of scaling factors. While not all inclusive either in extent or pertinent parameters, it is intended to furnish a guide for the initial experimental program which, it is expected, will follow the theoretical work presented in this report.

3.0 ANALYSIS OF CONDITIONS

In order to develop a set of scaling relationships based on the aforementioned approach of setting given requirements which are directed toward immediate problems, it is necessary to analyse the conditions postulated. This section is concerned with the analyses of these conditions.

The conditions are divided into two groups, those which must hold for those scaling relationships which are intended to explore a given phenomenon and those which must hold for all the phenomena considered. These two groups will be considered separately. Definitional statements which serve to implement the analysis and are the basis for developing scaling relationships will be presented where needed. All definitions, definitional statements, and conditions used are listed in Section 4. The structure chosen for all cases is a cylindrical shell.

3.1 Conditions Imposed on All Scaling Relationships

One of the basic prescribed parameters of the shock tube is the overpressure. By means of the Rankine-Hugoniot equation, the velocity of the shock front may be determined.

$$\frac{U^2}{c_o^2} = \left(1 + \frac{6}{7}\frac{p}{P_o}\right) \qquad (3.1)$$

A valid approximation of this equation for higher overpressures may be given as

$$U^2 = kp \qquad \text{where} \qquad k = \frac{6}{7}\cdot\frac{c_o^2}{P_o} \qquad (3.2)$$

Figure 3.1 shows the valid range of the approximation. For overpressures over 40 psi, the value of U given by equation 3.2 is within 10% or less of that value given by equation 3.1. If overpressures of over 25 psi are maintained, the approximation should give adequate results.

Because of the boundary condition existing between the air and soil, the

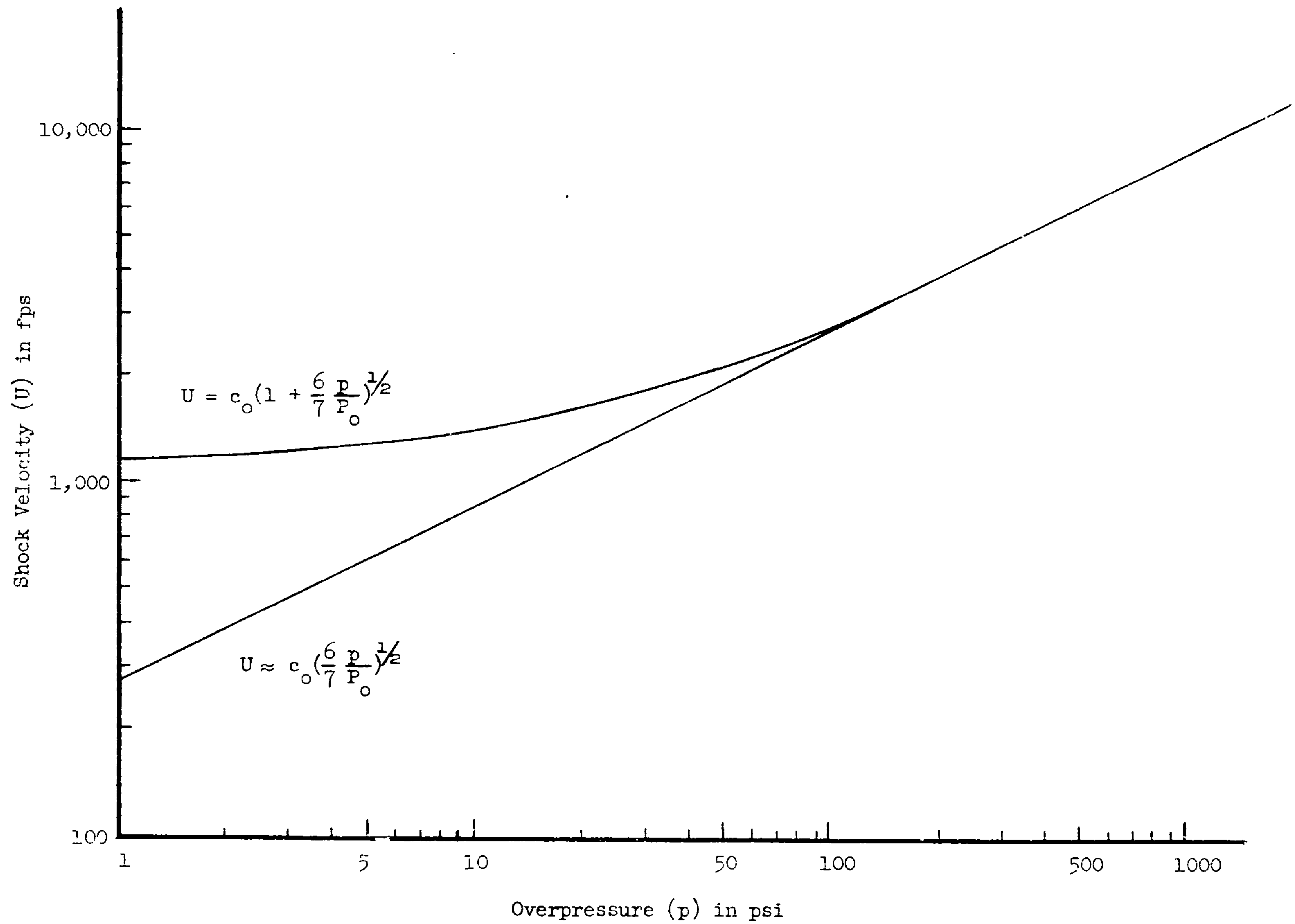


Figure 3.1 Shock Velocity vs. Overpressure - Comparison of Actual Values with Approximate Values

value of stress in the soil at the surface must be the same as the overpressure. For this reason, the condition is imposed that the scale factor for stress in the soil is the same as the scale factor for overpressure. ($\lambda_{\sigma_D} = \lambda_p$)

The boundary condition between the soil and the structure imposes another condition. The stress in the soil must equal the pressure on the structure at any particular point. This does not mean that the stress field in the simulated soil is required to be the scaled stress field of the prototype.

This same boundary condition imposes the condition that the displacement of the structure must, for any given point, be equal to the soil displacement at that point.

3.2 Conditions Desired for Useful Scaling

The different phenomena on which it has been deemed desirable to base the development of scaling laws are hereinafter analyzed. Basic formulae upon which the relationships have been developed are contained in Tables 4.2 and 4.3.

3.2.1 Pulse Width and Natural Period of the Structure

One of the many factors which determine the manner in which a structure responds to dynamic loading is the relation between the pulse width of the loading stress and the natural period of the structure. Thus, in the case where the pulse is extremely short in comparison with a natural period of the structure, it may be regarded simply as an impulse. On the other hand, in the case where the pulse is long and has a long rise time compared with a natural period, structural response is affected by maximum pressure behind the wave front rather than duration of the load.

In the transitional case, both the peak pressure and the duration time affect structure response. Rise time also has an independent effect on response. For the applications which are of future interest, however, very short (near zero) rise times dominate consideration. Therefore, variable rise time has not been considered in this study.

Consideration of a simple mass-spring system with various time-dependent forcing functions has led to a delineation of the three cases mentioned above. Figure 3.2 gives graphs of the results of this computation. From these graphs, it has been concluded that if the total pulse duration $\leq$ 0.1 of the natural period under consideration, the pulse may be treated as an impulse. If the pulse is over 0.8 of the natural period, peak pressure is the dominating factor. Graph (b) of Figure 3.2 shows the effect of rise time, but, as previously mentioned, this has not been taken into account in the derivation of the scaling law. The following table gives the limits of the regimes and the scaling conditions pertinent to each.

TABLE 3.1

Regime	Ratio of Pulse Width to Natural Period	Scaling Condition
I	0 to 0.1	Scale structural deflection in accordance with impulse criteria
II	0.1 to 0.8	Preserve ratio of pulse width to natural period, scale structure deflection in accordance with peak pressure criteria
III	0.8 or more	Scale structure deflection in accordance with peak pressure criteria

In regime I, the table shows that structural deflection must be scaled in accordance with impulse criteria. Thus

$$\lambda_{\ell} = \lambda_u = \frac{\lambda_{\sigma_D} \lambda_t}{\lambda_{\rho_D} \lambda_{c_1}} \qquad (3.3)$$

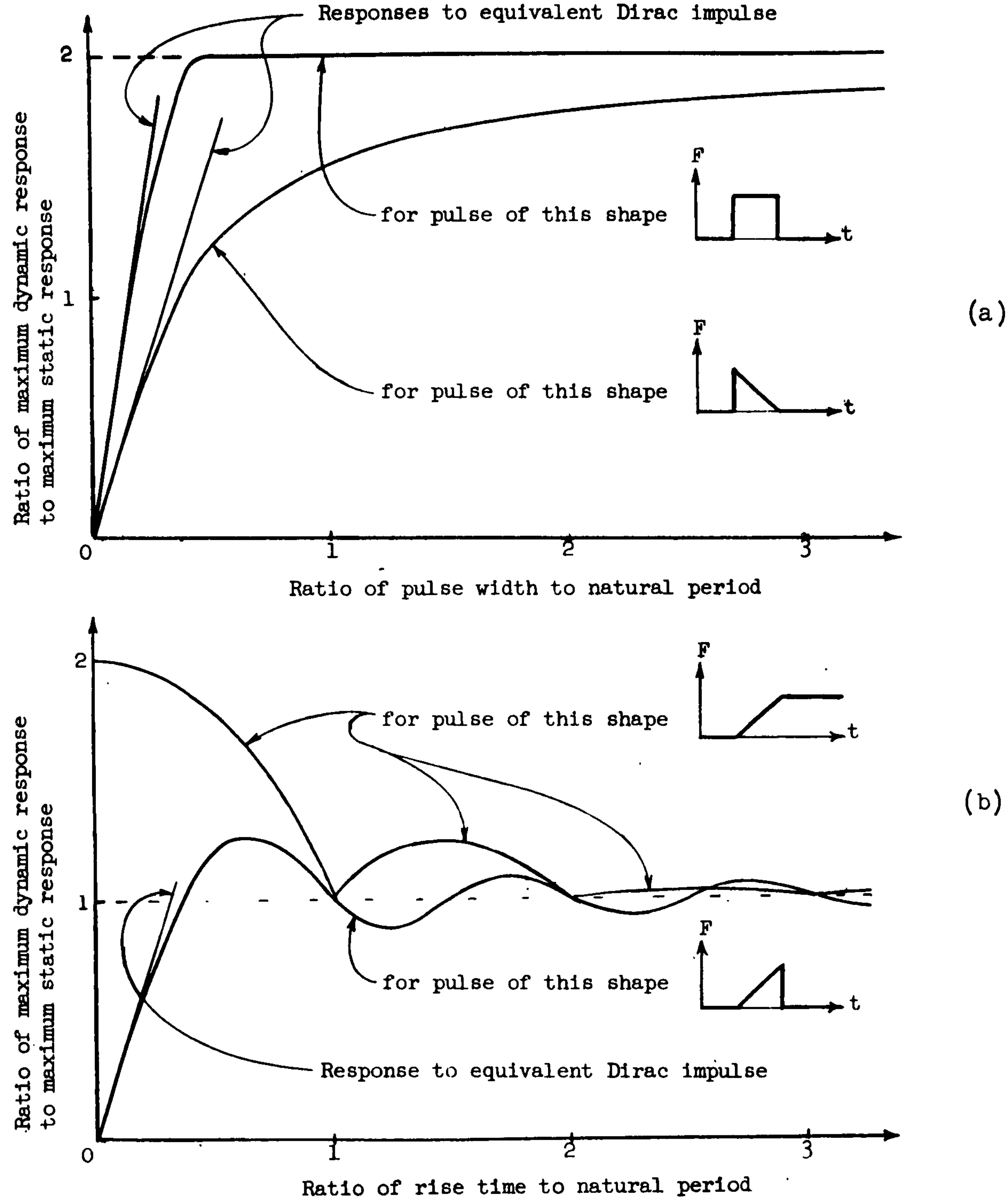


Figure 3.2 - Dynamic Response of Mass-spring System to Various Loading Functions

Regime II has two conditions which are pertinent. The first of these states that the ratio of pulse width to natural period must be preserved from model to prototype. Thus

$$\lambda_{t_w} = \lambda_\tau \qquad (3.4)$$

The second qualification is that structure deflection must be scaled according to peak pressure. This dictates that

$$\lambda_\ell = \frac{\lambda_{\sigma_D} \lambda_{t_w} \lambda_\tau}{\lambda_{\rho_c} \lambda_h} \qquad (3.5)$$

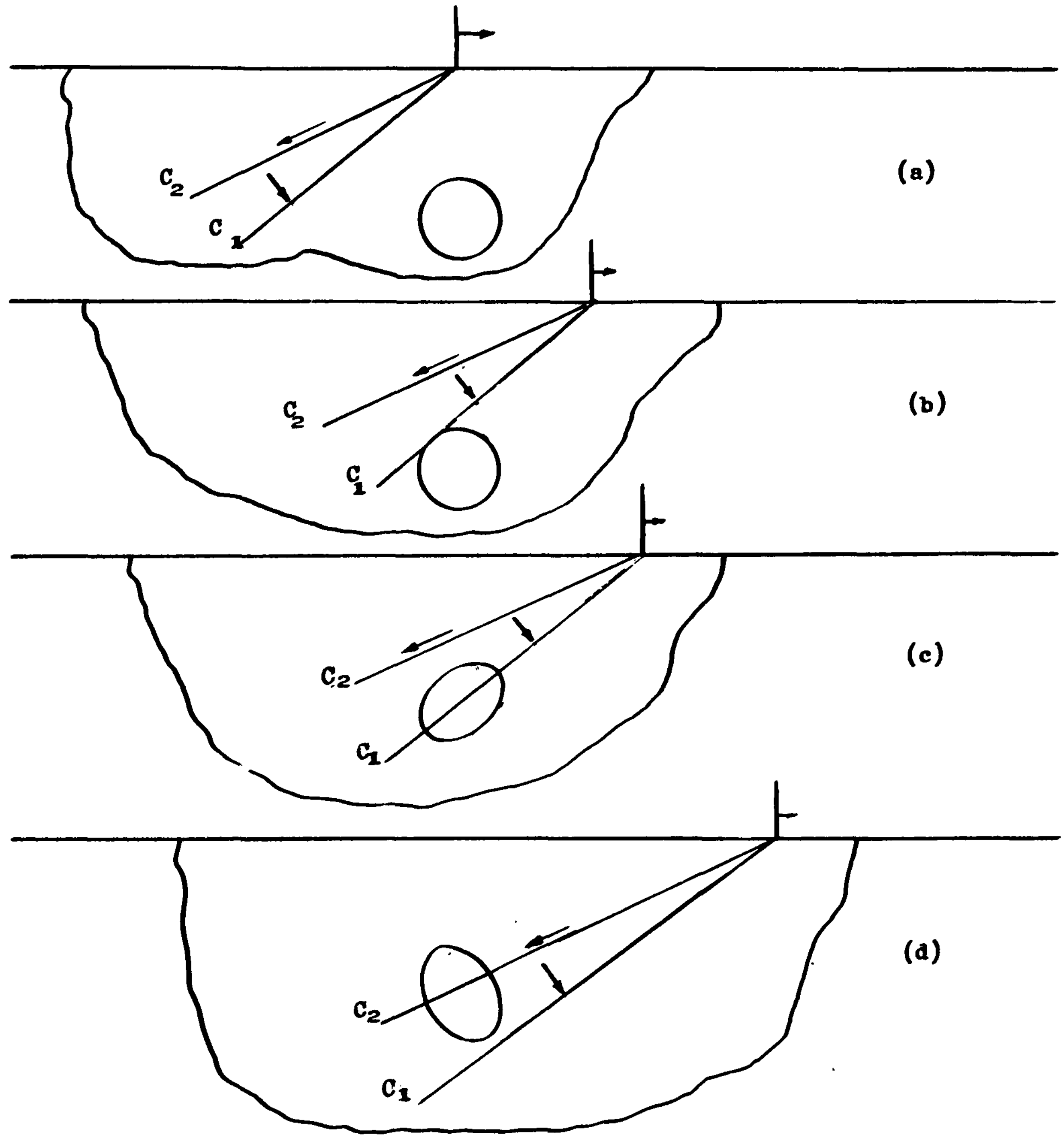
In the third regime, deflection of the structure must be scaled according to peak pressure only. Thus

$$\lambda_\ell = \frac{\lambda_{\delta_D} \lambda_L \lambda^3_{D/h}}{\lambda_\Delta} \qquad (3.6)$$

3.2.2 Arrival Times of C_1 and C_2 Waves

Considering the fact that there are two wave fronts being propagated at different velocities through the medium and that both these fronts will impinge on the structure, the question arises as to what influence their combined effect will have on the total problem. More specifically, which modes of vibration of the structure that are excited by the dilatational wave are of such frequency and shape that they are reinforced by the shear wave by virtue of its time of arrival? If such a phenomenon is important, it should be preserved.

The velocity of the dilatational wave is of course greater than that of the shear wave, and it produces a different particle motion within the medium. Figure 3.3 illustrates the subject phenomenon. For purposes of simplification, the waves have been shown as plane waves with no loss in generality. The direction of propagation of the waves in each case is perpendicular to the front.



Structure Subjected to C_1 & C_2 Waves

Figure 3.3

The particle motion contributed by each wave is shown by arrows on the illustration. In (a), the waves are approaching the structure. A short time later, the dilatational wave impinges on the cylinder (b). The waves continue and the structure deflects (c). If a vibrational mode of the structure causes it to have a cross-sectional shape as shown in (d) at the same time as the shear wave contacts it, the result will be a reinforcement of the vibrational mode.

For the purposes of this analysis, the velocity of the shock front in air (U) is assumed to be constant. This assumption is very reasonable for present purposes since the length of the soil bin over which the wave passes is small. If, for example, the overpressure in the shock tube is 100 psi, the shock velocity will be roughly 2800 feet/second. Thus, the wave will pass over the soil bin in around .003 seconds. There will be very little change in shock front velocity in this short time. The assumption goes a long way to simplify the analysis.

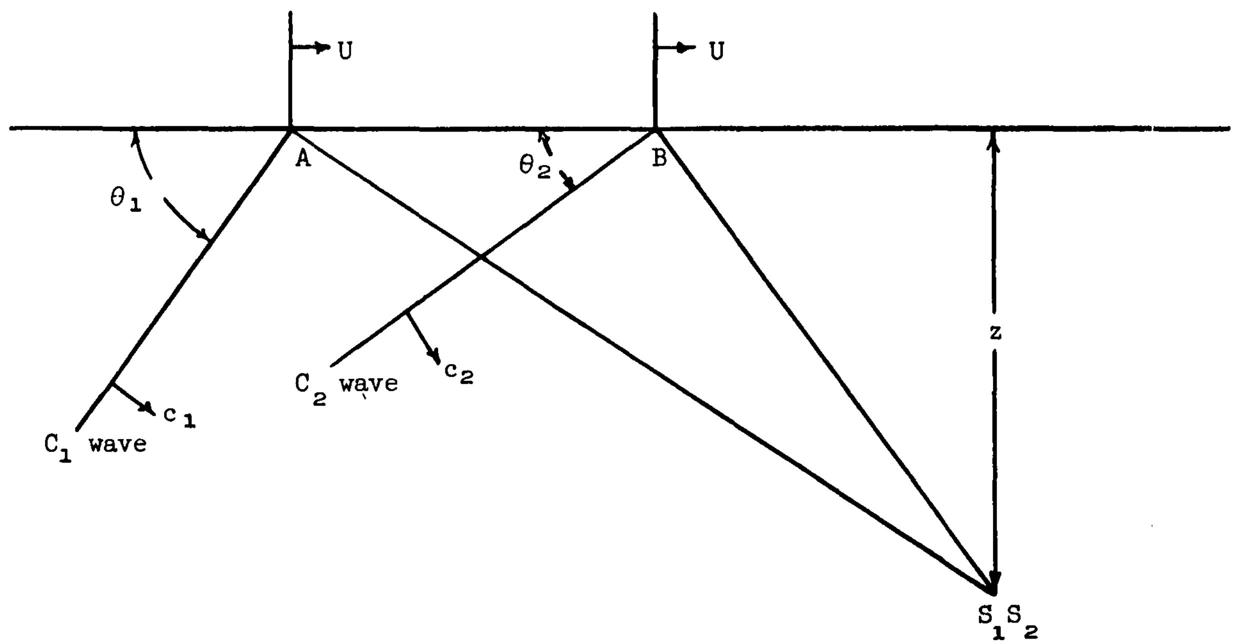
Considering now a point in the medium, the problem is to determine the difference in arrival time between the two waves (See Figure 3.4). The point in the medium has been given two designations (S_1 and S_2) to distinguish between the arrival times of the two waves. When considering the arrival of the first wave, the point will be referred to as S_1; when considering the second wave, it will be called S_2.

From the figure, we see that

$$\sin (90^\circ - \theta_1) = z/\overline{AS}_1$$

and

$$\sin (90^\circ - \theta_2) = z/\overline{BS}_2$$



Arrival Times of C_1 & C_2 Waves

Figure 3.4

Since

$$\sin(90° - \theta_1) = \frac{(U^2 - c_1{}^2)^{1/2}}{U}$$

and

$$\sin(90° - \theta_2) = \frac{(U^2 - c_2{}^2)^{1/2}}{U}$$

$$\frac{(U^2 - c_1{}^2)^{1/2}}{U} = \frac{z}{c_1(t_{S_1} - t_A)} \tag{3.7}$$

and

$$\frac{(U^2 - c_2{}^2)^{1/2}}{U} = \frac{z}{c_2(t_{S_2} - t_B)} \tag{3.8}$$

(It may be seen from the figure that $\overline{AS}_1$ and $\overline{BS}_2$ are equal to $c_1(t_{S_1} - t_A)$ and $c_2(t_{S_2} - t_B)$ respectively.)

One more relationship is needed to establish the required time interval $(t_{S_2} - t_{S_1})$. The geometry of the figure establishes the necessary equation as

$$(t_B - t_A) = \frac{z}{U}\left[\frac{1}{\left(\frac{U^2}{c_1^2} - 1\right)^{1/2}} - \frac{1}{\left(\frac{U^2}{c_2^2} - 1\right)^{1/2}}\right] \qquad (3.9)$$

By algebraic manipulation we may obtain

$$\tilde{t} = t_{S_2} - t_{S_1} = \frac{z}{U}\left[\left(\frac{U^2}{c_2^2} - 1\right)^{1/2} - \left(\frac{U^2}{c_1^2} - 1\right)^{1/2}\right] \qquad (3.10)$$

which is the delay time we are seeking.

From this equation we may determine that, for a given shock velocity and soil (or soil simulant) the time between arrival of the waves is purely a function of the depth of burial.

A simplifying approximation that is useful in ranges when U >> c is

$$\tilde{t} = \frac{z}{U}\left(\frac{U}{c_2} - \frac{U}{c_1}\right) = z\left(\frac{1}{c_2} - \frac{1}{c_1}\right) \qquad (3.11)$$

By using the definition

$$\frac{c_1}{c_2} = \sqrt{\frac{2(1-\nu_{D,dyn})}{1-2\nu_{D,dyn}}} = \beta \qquad (3.12)$$

the following is obtained

$$\tilde{t} = \frac{z}{c_1}\sqrt{\frac{2(1-\nu_{D,dyn})}{1-2\nu_{D,dyn}}} \qquad (3.13)$$

3.2.3 Mach Angle Preservation

As was stated in Section 2.2.1 (and illustrated in Figure 2.1) the requirement to preserve Mach angle is not necessarily strict. What is necessary is that the superseismic case be preserved.

If the Mach angles were to be precisely preserved, the sines of these angles would have to remain invariant between the model and the prototype. Since the sines of the Mach angles θ_1 and θ_2 are equal to the velocity ratios c_1/U and c_2/U respectively, the resulting relation between shock velocity and Mach angle is as shown in Figure 3.5 for selected values of seismic velocity. As long as these ratios remain < 1, the waves will retain their required superseismic character.

More precisely, since the dilatational wave has the greater velocity, if we restrict the ratio of it to the shock velocity to values less than one, the shear wave is insured to be superseismic. Therefore, our scaling law may be flexible in that the ratio (c_1/U) in the model may vary from something more than zero to almost 1. As this ratio in the model varies from that in the prototype, however, the ratio of stress at a given point in the medium to the overpressure at the surface also varies.

3.2.4 <u>Non-Linear Stress-Strain Curve</u>

Perhaps the greatest difficulty in the theoretical solutions of underground structures subjected to nuclear blast effects lies in the non-linear character of the soil-stress-strain curve. Figure 2.2 gives a typical example of a stress-strain curve for soil.

A great deal of useful information can be derived from using a soil simulant which will produce a similar curve. The scale factor for strain will most probably be different from that for stress. By obtaining a close similarity of the curves for soil and simulant, the scale factor for slope of the stress-strain curve would then be the ratio of the scale factor for stress to the scale factor for strain.

Perhaps the most important aspect of the non-linear stress-strain curve in regard to scaling is its irreversible character. This should be p[illegible]rved in the soil simulant. Such a property can probably be simulated best with foamed materials.

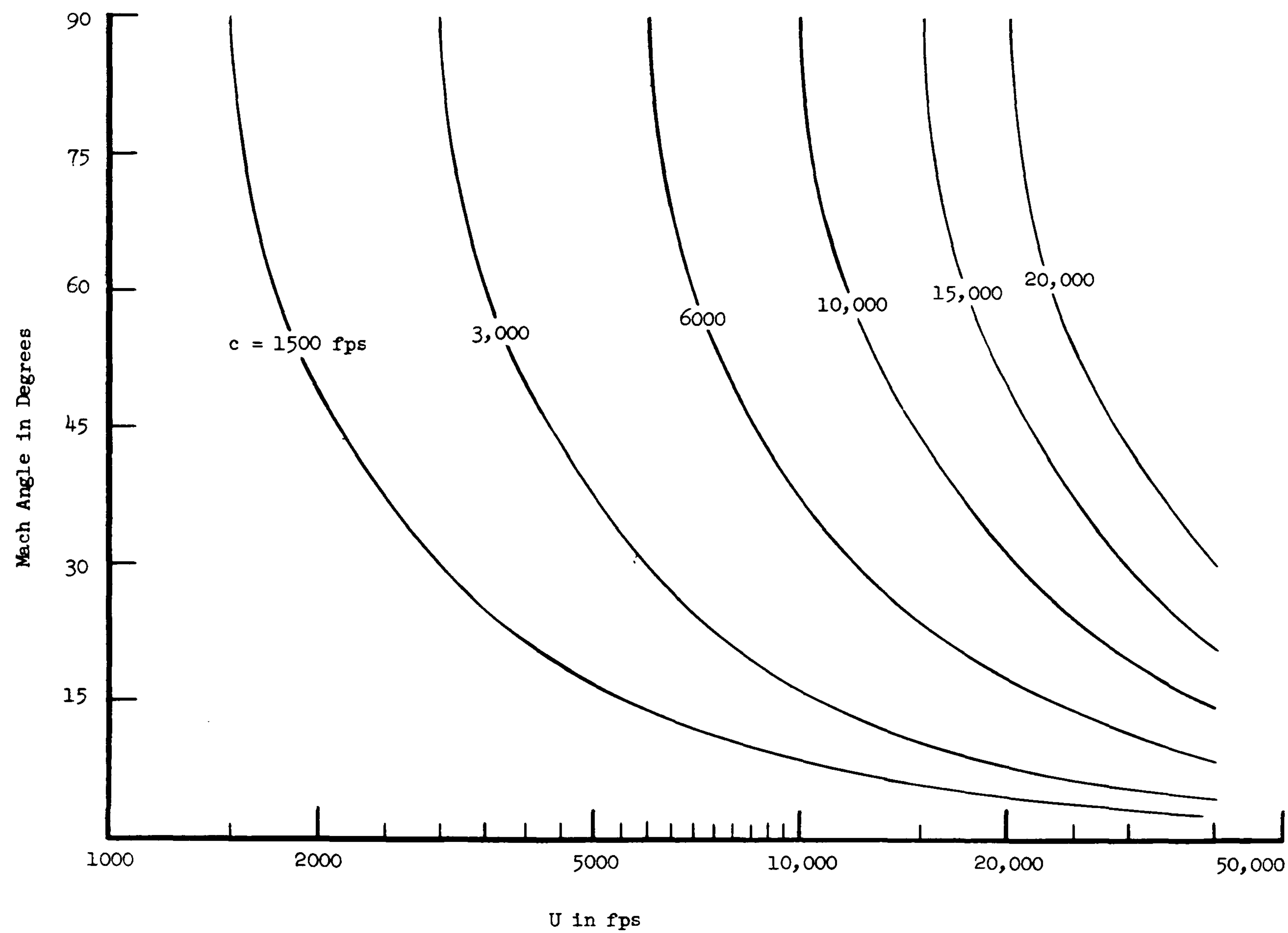


Figure 3.5 - Mach Angle versus Shock Velocity

Since the slope of the unloading portion of the stress-strain curve affects recovery displacement, and hence energy propagation away from the structure, it would be advantageous to be able to scale this the same as the loading portion. This puts another qualification on the soil simulant. It also requires more knowledge of the actual soils which are to be simulated. However, as more of these conditions are met, more qualitative information can be obtained which will lead to greater advances in theory.

If the non-linear stress-strain curve can be simulated as indicated above, another phenomenon can be scaled. Consider the case where the shape of the nonlinearity is such that it splits the pulse in two. The analysis of these two waves and the difference in their times of arrival at a given point is not unlike the case of the time difference between the C_1 and C_2 waves as given in Section 3.2.2. Here we may say that

$$\hat{t} = z\left(\frac{1}{c_B} - \frac{1}{c_A}\right) \qquad (3.14)$$

By substituting the equivalents of c_B and c_A we see that

$$\lambda_{\hat{t}} = \frac{\lambda_z \, \lambda_{\rho_D}^{1/2} \, \lambda_{\epsilon_{D,dyn}}^{1/2}}{\lambda_{\sigma_D}^{1/2}} \qquad (3.15)$$

By requiring the scaling of the period of the structure to conform with the scaling of $\hat{t}$, the effect of such a split pulse can be determined.

3.2.5 Structural Deflection and Soil Displacement

The stress wave in passing through the soil medium generates a localized displacement of the soil particles along the path of the wave front. When the wave front contacts the surface of the buried structure, the displaced soil adjacent to this surface exerts a force on the structure wall. This force results in a displacement (ℓ) of the structure surface the magnitude of which is dependent on the magnitude of the exerting force and on the specific structural parameters which are pertinent to the phenomenon. Since the displacement of the buried structure wall must be compatible with the corresponding displacement of the soil adjacent to the surface of the structure at any given point, it is necessary that the displacements be scaled the same in both media.

Assuming that the buried structure is a thin walled cylindrical shell, it is shown in the theory of cylindrical shells [49] that the inextensional deformation of a circular cylindrical shell will involve an expression for the deflection, ℓ, of the form

$$\ell = \frac{\sigma_D L D^3(1-\nu_c^2)}{E_c h^3} = \frac{\sigma_D L(\frac{D}{h})^3}{\Delta} \qquad (3.16)$$

It is seen from the above expression that the structural parameters pertinent to the structural deflection are E_c, L_c, $\left(\frac{D_c}{h_c}\right)^3$, and ν_c. Soil displace- is given by

$$u = \frac{\sigma_D t}{\rho_D c_1} \qquad (3.17)$$

so that for scaling purposes the relationship governing soil displacement is given by

$$\frac{\lambda_{\sigma_D} \lambda_t}{\lambda_{\rho_D} \lambda_{c_1}} \qquad (3.18)$$

The desired condition is that both stresses and displacements (and hence strains) in the structure and adjacent soil have not only point to point equality, but that the scale factors for these remain invariant throughout the model.

3.2.6 Attenuation

Attenuation of stress waves through dissipative mechanisms of the medium is an important consideration in the problem of buried structures subjected to blast effects. One of the possible mechanisms by which such attenuation occurs is through percyclic damping.

Percyclic damping by definition obeys the law such that each sinusoidal component of a propagating wave is damped by a constant reduction factor in propagating a distance equal to its own wave length. Thus, a waveform would change with time according to the equation

$$f(x,t) = \int_{-\infty}^{\infty} e^{-\frac{\delta c t}{\Lambda}} g(\xi)\, e^{i\xi x}\, d\xi \qquad (3.19)$$

where $f(x,o) = \int_{-\infty}^{\infty} g(\xi)\, e^{i\xi x}\, d\xi$ is the initial shape of the waveform and $g(\xi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x,o)\, e^{-i\xi x}\, dx$ is the spectrum.

Thus assuming percyclic damping as the attenuation mechanism, the ratio of the initial wave amplitude to the amplitude at some distance ct is given by $e^{-\frac{\delta c t}{\Lambda}}$

Since $e^{-\frac{\delta c t}{\Lambda}}$ is a transcendental function, the argument $-\frac{\delta c t}{\Lambda}$ must be dimensionless. Thus to preserve similitude in the damping ratios, $\frac{\delta c t}{\Lambda}$ must remain invariant between model and prototype. In addition, the wave length of the propagating wave must be scaled as the depth. This requirement need only be met where percyclic damping is to be scaled.

3.2.7 Response Wave

The responses of a structure always cause an attenuation of local soil pressure. When a structure has a high acoustic impedance in comparison with that of the soil, this attenuation comes gradually and only after the sharp rise upon initial reflection. This attenuation propagates away as what is termed the response wave. Any characteristic length of the response wave is proportional to the ratio of the structure mass per area in the median of the shell to the soil mass per volume. In order to preserve the effect of this phenomenon in the model, it will be necessary to scale the response wave the same as the incoming wave.

The relative importance of preserving this ratio is dependent on the recovery capacity of the soil in question.

3.2.8 Engulfment Time

The time it takes for one wave length of the stress wave to pass over the structure is of some importance. When the ratio $\left(\frac{D}{c_1 \tau}\right)_p$ is in the neighborhood of 1 (e.g. 1/3, 1/2, 1 1/2 etc.), the scale condition should be

$$\frac{\lambda_D}{\lambda_{c_1} \lambda_\tau} = 1 \tag{3.20}$$

If the ratio in the prototype is much different than 1, this ratio need not be scaled closely.

3.2.9 Scaling of Weight

It is inescapable that weight density must be scaled the same as mass density, at least as long as we are earthbound. Thus,

$$\lambda_{\gamma_D} = \lambda_{\rho_D} \tag{3.21}$$

$$\lambda_{\gamma_c} = \lambda_{\rho_c} \tag{3.22}$$

Since weight is a force, it appears at first that it should be scaled as other forces. This scaling would actually relate strèsses, in the form

$$\lambda_{\gamma_D} \lambda_z = \lambda_{\sigma_D} \tag{3.23}$$

$$\lambda_{\gamma_c} \lambda_D = \lambda_{\sigma_c} \tag{3.24}$$

Bu , scaling of weight as force might conflict with other scaling relations. If such conflict arises, there are several possible remedies.

It is likely that in many cases the stresses due to structure weight, and possibly soil weight also, will be only a fraction of the stresses due to blast overpressure. In such cases 3.24, and possibly 3.23, can be ignored.

If weight is responsible for a major portion of total stress, an attempt should be made to deal only with overpressure-induced stress, which would be the excess above weight-induced stress. This amounts simply to a translation of the stress scale, since weight remains constant, and should not interfere with the dynamic phenomena being studied. If this can be done, then 3.23 and 3.24 can again be ignored. Difficulty might be encountered in translating the stress scale of the soil due to nonlinearity of its stress-strain curve. The stress-strain curves of the prototype and of the model must now be similar after translation. But chances are 50-50 that this should present no more difficulty than if they had to be similar before translation.

As a last resort, to account for soil weight, λ_p should be replaced by $\lambda_{p + z\gamma_D}$ in all primary scaling relations except the one relating overpressure to air-shock velocity. If the effect of structure weight cannot be neglected, it can always be eliminated by a translation of the scale of structure stress because the stress-strain curve of the structure is linear. Again both 3.23 and 3.24 can be ignored.

4.0 DERIVED SCALING RELATIONSHIPS

Table 4.5 gives the derived scaling relationships. The scale factors are given in terms of known or prescribed scale factors for the given desired conditions set forth in Section 3.2 and listed in Table 4.4. The imposed conditions (given in Section 3.1 and listed in Table 4.3) are included in the derivations when applicable.

Two of the desired conditions have not been included in the derived scaling relationships - the condition for preserving superseismicity and the scaling of weight. As was pointed out in Section 3.2.3, the preservation of the superseismic case is not an exceedingly strict requirement and it may be checked after the rest of the scaling requirement has been developed (i.e. check to see that $(\frac{c_1}{U})_m < 1.$) If weight is required to be scaled, the procedure given in Section 3.2.9 should be followed.

In the column for each desired requirement in Table 4.5, certain parameters are listed as given. Most often, the given parameters include the physical parameters of the structure. These are either easily controllable or dictated by the physical limitations of the shock tube. Other given parameters must be ascertained experimentally or just chosen. An example of the former category would be the determination of λ_{c_1}, $\lambda_{\beta,dyn}$ and $\lambda_{\epsilon_{D,dyn}}$. Since these refer to dynamic properties of the soil and the soil simulant, they cannot be computed from published static values. Known field test data must provide the values for the soil and "free-field" experiments in the shock tube must provide the values for the soil simulant. In cases where the scale factors for properties of the structure are considered, the model material will have to be chosen before the other scale factors are determined. This may involve several attempts at arriving at the

TABLE 4.1

SCALE FACTOR IDENTIFICATION

Scale Factor	Parameter
$\lambda_p = \frac{p_m}{p_p}$	overpressure
$\lambda_{\sigma_D} = \frac{\sigma_{D,m}}{\sigma_{D,p}}$	stress in soil
$\lambda_{\epsilon_{D,dyn}} = \frac{\epsilon_{D,dyn,m}}{\epsilon_{D,dyn,p}}$	strain in soil
$\lambda_\ell = \frac{\ell_m}{\ell_p}$	deflection in structure
$\lambda_u = \frac{u_m}{u_p}$	displacement in soil
$\lambda_D = \frac{D_m}{D_p}$	diameter of structure
$\lambda_z = \frac{z_m}{z_p}$	depth in soil
$\lambda_L = \frac{L_m}{L_p}$	length of structure
$\lambda_{\left(\frac{d\sigma}{d\epsilon}\right),dyn} = \frac{\left(\frac{d\sigma}{d\epsilon}\right),dyn,m}{\left(\frac{d\sigma}{d\epsilon}\right),dyn,p}$	Dynamic soil modulus
$\lambda_t = \frac{t_m}{t_p}$	time

TABLE 4.1 (Continued)

SCALE FACTOR IDENTIFICATION

Scale Factor	Parameter
$\lambda_{t_w} = \frac{t_{w,m}}{t_{w,p}}$	pulse width (in units of time)
$\lambda_{\gamma_c} = \frac{\gamma_{c,m}}{\gamma_{c,p}}$	weight density of structure
$\lambda_{\gamma_D} = \frac{\gamma_{D,m}}{\gamma_{D,p}}$	weight density of soil
$\lambda_{(\beta-1),dyn} = \frac{(\beta-1)_{dyn,m}}{(\beta-1)_{dyn,p}}$	ratio involving seismic velocities
$\lambda_{\sigma_c} = \frac{\sigma_{c,m}}{\sigma_{c,p}}$	bending stress in structure
$\lambda_{c_1} = \frac{c_{1m}}{c_{1p}}$	velocity of dilatational wave in medium
$\lambda_{\rho_D} = \frac{\rho_{D,m}}{\rho_{D,p}}$	density of medium
$\lambda_U = \frac{U_m}{U_p}$	velocity of shock front in air
$\lambda_\tau = \frac{\tau_m}{\tau_p}$	any natural period of structure
$\lambda_h = \frac{h_m}{h_p}$	thickness of structure shell
$\lambda_{\rho_c} = \frac{\rho_{c,m}}{\rho_{c,p}}$	density of shell material

TABLE 4.1 (Continued)

SCALE FACTOR IDENTIFICATION

Scale Factor	Parameter
$\lambda_k = \frac{k_m}{k_p}$	"ambient" constant for air
$\lambda_\Delta = \frac{\Delta_m}{\Delta_p}$	flexural rigidity modulus $= \frac{E_c}{1-\nu_c^2}$
$\lambda_{\frac{D}{h}} = \frac{\lambda_D}{\lambda_h}$	ratio of Dia. of structure to thickness of structure
$\lambda_{\Lambda_R} = \frac{\Lambda_{R,m}}{\Lambda_{R,p}}$	characteristic length of response wave

simulant which will give usable scale factors.

Each column in Table 4.5 gives the scaling relationship imposed by a desired condition. Columns A(1), A(2), and A(3) are mutually exclusive in ascertaining a scaling law for a given experiment. Column E need only be included when it is desired to scale percyclic damping and column G need be included only when tne conditions as cited in Section 3.2.8 warrant it.

To ascertain the scaling relationship for a given test, the conditions to be met must be decided upon. These conditions dictate the scaling factors. Where two or more relationships for one quantity are encountered (one dictated by each separate condition which involves that quantity), these must be made compatible. For example, in order to have conditions B through G hold with condition A(1), $\lambda_{D/h}$ must be scaled as dictated by condition A(1) and can no longer be chosen freely. By the same token, the requirement for scaling ℓ dictates that

$$\frac{\lambda_p \lambda_t}{\lambda_{\rho_D} \lambda_{c_1}} = \frac{\lambda_p \lambda_L \lambda_{D/h}^3}{\lambda_\Delta}$$

TABLE 4.2

DEFINITIONAL STATEMENTS

$$\frac{c_1}{c_2} = \beta_{\text{dyn}}$$

$$\beta = \sqrt{\frac{2-2\nu_{D,\text{dyn}}}{1-2\nu_{D,\text{dyn}}}}$$

$$\tau = \frac{D^2 \cdot \rho_c{}^{1/2}}{h\ \Delta^{1/2}}$$

$$\Delta = \frac{E_c}{1-\nu_c{}^2}$$

$$\tilde{t} = \frac{z}{c_1}(\beta-1)_{\text{dyn}}$$

$$\hat{t} = z\ \rho_D{}^{1/2}\left[\left(\frac{d\sigma}{d\epsilon}\right)_{B,\text{dyn}}^{1/2} - \left(\frac{d\sigma}{d\epsilon}\right)_{A,\text{dyn}}^{1/2}\right]$$

---

thus restricting the choice of the scale factors involved.

The choice of A(1), A(2), or A(3) depends on the relation of the pulse width to the natural period as explained in Section 3.2.1. The time scale factor in A(1) is given by $\lambda_t = \frac{t_m}{t_p}$ where t_m and t_p represent measured time intervals of the loading wave in shock tube and prototype respectively. Figure 4.1 shows an example of this.

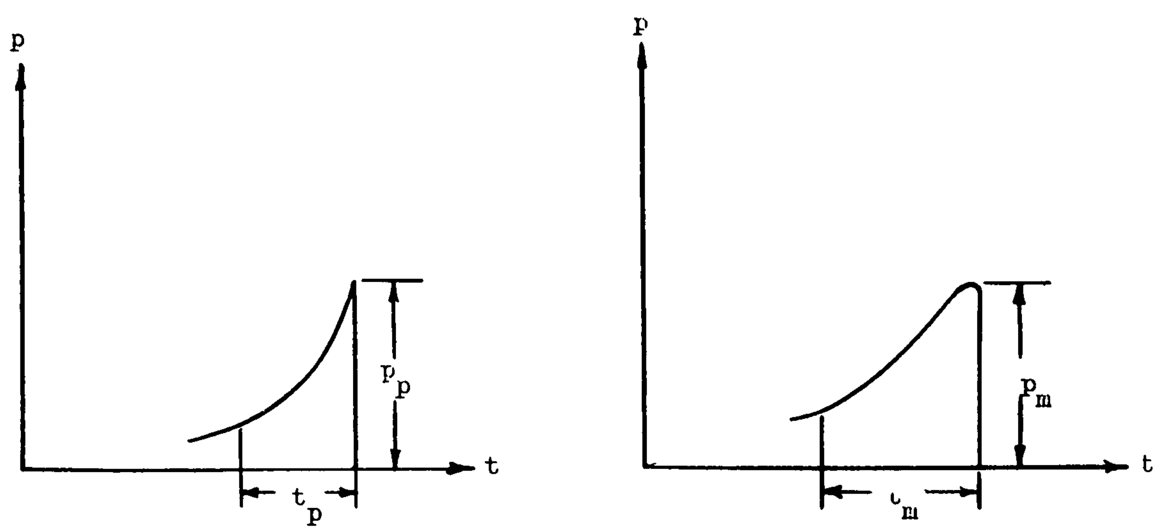
TABLE 4.3

PRIMARY RELATIONSHIPS IMPOSED FOR ALL CASES

$$\lambda_{\dot{u}}^{2} = \lambda_k \lambda_p$$

$$\lambda_p \doteq \lambda_{\sigma_D}$$

$$\lambda_u = \lambda_\ell \qquad \text{(point wise)}$$



(a)
Prototype Shock Wave

(b)
Model Shock Wave

Figure 4.1

TABLE 4.4

PRIMARY RELATIONSHIPS DESIRED FOR USEFUL SCALING

Identification	Requirement	Conditions Imposed Thereby	Equations Used
A(1) (Regime I in Section 3.2.1)	Scale structural deflection according to impulse	$\lambda_{\ell} = \frac{\lambda_{\sigma_D} \lambda_t}{\lambda_{\rho_D} \lambda_{c_1}}$	$\sigma_D = \frac{\rho_D c_1 u}{t}$; $\ell = u$
A(2) (Regime II in Section 3.2.1)	Preserve $\frac{\text{Pulse Width}}{\text{Natural Period}}$ & scale deflection according to rise time & peak pressure	$\lambda_{t_w} = \lambda_{\tau}$ $\lambda_{\ell} = \frac{\lambda_{\sigma_D} \lambda_{t_w} \lambda_{\tau}}{\lambda_{\rho_D} \lambda_h}$	$\ell = \frac{v_{max}}{\omega}$; $\left(\begin{matrix} v = u \\ \omega = \frac{1}{\tau} \end{matrix}\right)$ $v_{max} = \frac{\sigma_D t_w}{\rho_c h}$
A(3) (Regime III in Section 3.2.1)	Scale deflection according to peak pressure	$\lambda_{\ell} = \frac{\lambda_{\sigma_D} \lambda^3_{\frac{D}{h}} \lambda_L}{\lambda_{\Delta}}$	$\ell = \frac{\sigma_D L D^3}{\Delta h^3}$
B (Sec. 3.2.2)	Scale Structural Period as delay time between C_1 & C_2 waves	$\lambda_{\tau} = \lambda_{\bar{t}}$	See Table 4.2
C (Sec. 3.2.4)	Scale non-linear stress-strain curve & scale structural period by split pulse time interval	$\lambda_{\left(\frac{d\sigma}{d\epsilon}\right)_{dyn}} = \frac{\lambda_{\sigma_{D,dyn}}}{\lambda_{\epsilon_{D,dyn}}}$ $\lambda_{\hat{t}} = \frac{\lambda_z \lambda^{1/2}_{\rho_D} \lambda^{1/2}_{\epsilon_{D,dyn}}}{\lambda^{1/2}_{\sigma_{D,dyn}}}$	See Table 4.2

TABLE 4.4 (Continued)

PRIMARY RELATIONSHIPS DESIRED FOR USEFUL SCALING

Identification	Requirement	Conditions Imposed Thereby	Equations Used
D (Sec. 3.2.5)	Scale structural deflection as soil displacement	$\lambda_{\ell} = \lambda_u$	$u = \frac{\sigma_D t}{\rho_D c_1}$; also see Table 4.2
E (Sec. 3.2.6)	Scale attenuation due to percyclic damping as seismic wavelength	$\lambda_{\Lambda} = \lambda_z$ $\lambda_{\Lambda} = \lambda_{\delta} \lambda_{c_1} \lambda_t$	Amplitude Ratio = $e^{\frac{-\delta c t}{\Lambda}}$
F (Sec. 3.2.7)	Scale characteristic length of response wave as incoming pulse wave length	$\lambda_{\Lambda_R} = \lambda_z$	$\Lambda_R = \frac{h\rho_c}{\rho_D}$
G (Sec. 3.2.8)	Scale engulfment time	$\lambda_D = \lambda_{c_1} \lambda_{\tau}$	See Table 4.2

TABLE 4.5

DERIVED SCALING RELATIONSHIPS

Scale Factor	Desired Condition: A (1)	A (2)	A (3)	B	C	D	E	F	G
λ_Δ	*	*	*	*	*	*			*
λ_L	*		*			*			
λ_{ρ_c}		*		*	*			*	*
$\lambda_{(D/h)}$	$\frac{\lambda_\Delta^{1/3}\lambda_t^{1/3}}{\lambda_L^{1/3}\lambda_{\rho_D}^{1/3}\lambda_{c_1}^{1/3}}$	*	*			*			*
λ_D	§	§	§		*	*			
λ_h	§	§	§		*	*		*	
λ_{c_1}	*			*		*	*		$\frac{\lambda_\Delta^{1/2}}{\lambda_{\rho_c}^{1/2}\lambda_{(D/h)}}$
λ_{t_w}		*							
λ_t	*					$\frac{\lambda_L\lambda_{\rho_D}\lambda_{c_1}\lambda_{(D/h)}^3}{\lambda_\Delta}$			
λ_ℓ	$\frac{\lambda_p\lambda_t}{\lambda_{\rho_D}\lambda_{c_1}}$	$\frac{\lambda_p\lambda_{t_w}\lambda_{(D/h)}^2}{\lambda_{\rho_c}^{1/2}\lambda_\Delta^{1/2}}$	$\frac{\lambda_p\lambda_L\lambda_{(D/h)}^3}{\lambda_\Delta}$			$\frac{\lambda_p\lambda_L\lambda_{(D/h)}^3}{\lambda_\Delta}$			
λ_z				$\frac{\lambda_D^2\lambda_{\rho_c}^{1/2}\lambda_{c_1}}{\lambda_h\lambda_\Delta^{1/2}\lambda_{(\beta-1),dyn}}$	$\frac{\lambda_D^2\lambda_{\rho_c}^{1/2}\lambda_p^{1/2}}{\lambda_h\lambda_\Delta^{1/2}\lambda_{\rho_D}^{1/2}\lambda_{\epsilon_{D,dyn}}}$		$\lambda_\delta\lambda_{c_1}\lambda_t$	$\frac{\lambda_h\lambda_{\rho_c}}{\lambda_{\rho_D}}$	
λ_δ							*		

* Denote "given" or "chosen" scale factor for that case

§ D & h are here given only by ratio D/h

Blank spaces indicate no requirement for this factor in the particular case

For all cases: λ_k & λ_p are given or chosen

$$\lambda_u = \lambda_k^{1/2}\lambda_p^{1/2} \qquad \lambda_{\delta_D} = \lambda_p \qquad \lambda_{\delta_c} = \lambda_p\lambda_{(D/h)}^3$$

The pulse width (t_w) of A(2) may be scaled in like manner.

The scale factors for the material properties of both the structure and the soil have a great bearing on the achievement of usable scale factors for the other quantities. The ability to find a variety of simulants to provide the combinations of scale factors necessary to bring the remaining factors within usable limits will increase the contributions that can be made by the shock tube.

5.0 MODEL MATERIALS

In order to properly implement the scaling laws developed it is necessary to have a variety of model simulants available so that usable scale factors may be obtained. As an impetus to the development of a list of simulants, a variety of some materials have been investigated. These include prototype materials and possible simulants for both structure and soil.

The choice of the proper model simulant will depend on the total scaling requirements for the particular experiment.

A rather obvious requirement for choosing a model simulant is its availability which is generally, although not necessarily, directly related to a reasonable cost value. Materials not easily available or of an expensive nature would not be desirable unless their use is unavoidable and the results justify the extra effort and expenditures. Avoiding these choices if possible, the material must be chosen whose parameters will produce usable scale factors for the particular case.

One important fact to also keep in mind when choosing simulant materials which have passed the criteria mentioned above, is the ratios between model and prototype yield or ultimate stresses. These ratios must not be less than the scale factor for stress being used. Whether either yield or ultimate stress values are employed in the choice will be determined by the particular phenomena being investigated. For example it may be desired to predict stress values in the prototype due to the shock wave impact and a scale factor for stress of 1:10 is indicated in projecting from model to prototype. If the prototype is constructed with 10,000 psi ultimate strength concrete the model simulant must have an ultimate strength equal to or exceeding 1000 psi or the ratio of ultimate strengths between model and prototype must exceed $\frac{1}{10}$. In this example case the yield strength need not follow this rule since deflections are not being measured and only the breaking point is of importance.

As a guide in formulating a testing program based on the theoretical background presented in this report, the following tabulated list is provided. Some "free field" experiments in the shock tube should produce the properties not available from regular known sources. Such properties as were available from the literature investigated have been included. Blank spaces indicate no information available.

Material	Tensile Ultimate Stress (psi) X 10^{-3}	Compressive Ultimate Stress (psi) X 10^{-3}	Tensile Yield Stress (psi) X 10^{-3}	Compressive Yield Stress (psi) X 10^{-3}	Poisson's Ratio	Density $\frac{lb\,sec^2}{in^4}$ X 10^{-5}	Modulus of Elasticity (psi)	Remarks
Metals:								
Aluminum 17ST	55	---	32	32	0.33	26.4	10.4 X 10^6	Ref. 55
Aluminum 51ST	44	---	42	37	0.33	26.4	10.1 X 10^6	Ref. 55
Brass 70-30	40-120	---	8-80	--	0.36	80	16 X 10^6	Ref. 54
Bronze (Ph)	40-130	---	20-80	--	0.35	83	16 X 10^6	Ref. 54
Cast Iron	18-60	---	8-40	--	0.265	67	15 X 10^6	Ref. 54
Cast Steel, heat treated	60-125	---	30-90	--	0.265	74	30 X 10^6	Ref. 54
Copper (Cu), hard drawn	68	---	60	--	0.355	83	16 X 10^6	Ref. 54
Lead	1.8-3	---	--	--	---	106	2.25 X 10^6	Ref. 52
Titanium (Ti)	60	---	50	50	---	43	15.5 X 10^6	Ref. 55
Zinc	--	---	13	--	---	67	11.5 X 10^6	Ref. 54
Rocks:								
Concrete	.3-.5	2-7.5	--	--	0.12	21	3 X 10^6	Ref. 36
Granite	--	---	--	--	0.06-0.13	25	6 X 10^6	Ref. 36
Soils:								
Loess	--	---	--	--	0.44	--	4.6 X 10^6	Ref. 36
Overburden	--	---	--	--	0.45	--	4 X 10^6	Ref. 36
Alloys:								
Al Bronze (cast)	65-110	---	25-60	--	---	70	15 X 10^6	Ref. 55
6Al-4V(Ti)	130	---	120	126	---	41.6	16 X 10^6	Ref. 55
A-286 (Steel)	130-140	---	85-95	88-98	---	74.5	29 X 10^6	Ref. 55
Beryllium Cu	160-185	---	120-130	120-130	---	104	---	Ref. 55
Inconel X	155	---	100	105	---	79	31 X 10^6	Ref. 55
8Mn(Ti)	120	---	110	110	---	44.5	15.5 X 10^6	Ref. 55
Mn Bronze	60-110	---	20-60	--	---	77	15 X 10^6	Ref. 55

Material	Tensile Ultimate Stress (psi) X 10^{-3}	Compressive Ultimate Stress (psi) X 10^{-3}	Tensile Yield Stress (psi) X 10^{-3}	Compressive Yield Stress (psi) X 10^{-3}	Poisson's Ratio	Density $\frac{lb\ sec^2}{in^4}$ X 10^{-5}	Modulus of Elasticity (psi)	Remarks
<u>Plastics:</u>								
ABS Foam	1.3-1.6	---	--	--	---	5	76.8 X 10^3	Ref. 37
ABS Resin	4-8.5	---	--	--	---	9.7	2.5 X 10^3	Ref. 53
Cellular Cellulose Acetate	.170	---	--	--	---	1.0	59 X 10^2	Ref. 37
Cellulose Acetate	1.9-8.5	---	--	--	---	12.0	4 X 10^5	Ref. 53
CR-39	--	---	--	--	.34	11.3	7.8 X 10^5	Ref. 36 *
Epoxy Foam	.3-.6	.5-1.4	--	--	---	1.9-2.2	---	Ref. 37
Epoxy (general purpose)	2-12	20-40	--	--	---	10.5-22	---	Ref. 53
Hysol-8705	--	---	--	--	.46	10.0	4.4 X 10^2	Ref. 36 **
Hysol-8705	--	---	--	--	.46	10.0	8 X 10^2	Ref. 36 ***
Lucite	--	---	--	--	.34	11.3	7.8 X 10^5	Ref. 36
Phenolic Foam	.004-.75	.009-.130	--	--	---	.3-1.5	---	Ref. 53
Phenolic (molded)	5-10	22-35	--	--	---	12.5-17.5	8-33 X 10^5	Ref. 53
Polystyrene (expanded)	.035-.185	.010-.140	--	--	---	.23-.68	9-53 X 10^2	Ref. 37
Polystyrene (general purpose)	5-8	11.5-16	--	--	---	9.7	4.5 X 10^5	Ref. 53
Polyurethane Foam	20-660	10-100	--	--	---	.45-2.7	5-36 X 10^3	Ref. 37
Polyvinyl Chloride	5.5-9	---	--	10-11	---	12-13	3.5-4 X 10^5	Ref. 53
Polyvinyl Formal	9-11	---	--	--	---	11	5-7 X 10^5	Ref. 53
Vinyl Foams	.01-.2	---	--	--	---	.6	---	Ref. 53
Vulkollan	--	---	--	--	.45	10.5	5.2 X 10^2	Ref. 36 ****
Vulkollan	--	---	--	--	.45	10.5	8.1 X 10^2	Ref. 36 *****

* Dynamic measurements at 53,600 cps
** Static values
*** Dynamic impact velocity 63.6 in/sec.
**** Static values
***** Dynamic impact velocity 4.31 in/sec.

6.0 RECOMMENDATIONS

In conjunction with the shock tube facility, the scaling law derived is a tool that can be used to fill voids in existing knowledge. The particular field of underground structures subjected to nuclear weapons effects has many such voids. No particular approach can be used to fill all of these in one fell swoop.

The scaling law developed will provide insight into some of the major voids. Other deficiencies in present knowledge can also be overcome by experimental work based on this law. Some of the areas which are now subject to question or conflicting views can be filled in with simple yes or no answers that can be obtained by shock tube experiments. For example, the importance of damping in the structure itself has been debated. The primary question here concerns whether damping within the structure is important or not. Secondarily, if it is important, how important is it? Using the scale law (imposing those conditions which bear on structural deflection and vibration) an experimental program can be devised to answer the primary question. If two model cylinders which are the same except for their damping characteristics are tested under the same conditions, the results should provide the information sought. In order to change the damping characteristics, one cylinder could be coated on the inside with a damping material (such as Vibrodamp). If damping does have an influence, a quantitative estimate of the degree of this effect can be obtained by a series of tests with varying amounts of damping in the cylinder.

Another area of question involves the importance of the ratio of soil mass to structure mass in the case of shallow burial. Here again, by varying the simulated soil and structure materials, different values of the ratio can be obtained. A wide variance in two tests can establish whether the ratio is important. If it is important, more subtle variations can be made to ascertain more quantitative information as to the effect. Then, the depth of burial can be varied to determine its effect on the problem.

The extent to which some questions can be answered depends on the degree of success achieved in finding suitable model simulants. Simulating the soil is the greatest problem here.

One of the important problems that can be answered if a suitable set of soil simulants can be found is, "What is the effect of the unloading slope of the soil stress-strain curve?" If two simulants can be found whose loading characteristics are similar (can both be scaled to same prototype) but whose unloading characteristics vary to a significant degree, a series of experiments can be devised to determine the effect in question. By the same token, the importance of the dependence of the effect of the response wave on soil recovery capacity can also be determined. Also, the effect of dissipation in the soil can be checked in the same manner (if percyclic damping is the criterion).

By slight variations of some of the scaled dimensions, the significance of other effects can be observed. The effect of the relation between the arrival times of the C_1 and C_2 wave and the natural period of the structure can be ascertained by a series of experiments where everything is held constant except the depth of burial.

While not included in the scaling law as derived, the asserted existence of viscoelastic properties of the soil is the basis of a problem area. The existence of a soil simulant which exhibited viscoelastic properties that varied rather widely over fairly short temperature ranges could provide qualitative information that would shed light on these areas. Such a material could be used in a series of experiments run at different temperatures and the results compared.

There are of course many other tests that may and should be performed in the shock tube. The results of these tests should then be made available to theoreticians so that further advances may be made. With new and revised theories, a revision of the scaling law and a repetition of the process will lead to more refined and exact theories.

APPENDIX A

A METHOD OF SIMULATING SOILS

APPENDIX A

A Method of Simulating Soils

As was pointed out in the body of this report, one of the most difficult aspects of a shock tube experimental program will lie in the simulation of soils. The complicating factor here is the non-linearity of the stress-strain curve. In conjunction with an internally sponsored AMF project, a report was prepared on the subject of test methods for underground shock problems. One part of this report dealt with a method of simulating soils. This portion of the report has been extracted and edited and is included in this appendix as a possible approach to the problem of soil simulation.

Since this report covered a number of subjects and approaches to the problem of testing, it cannot be applied directly to the scaling relationships given in the body of the present report. The soil study dealt with pressure-specific volume relationships rather than stress-strain relationships. However, the method given here will provide an insight into the general problems of soil simulation and one approach to their solution. The report was prepared for AMF by Mr. Vincent J. Cushing of the Engineering Physics Company.

A.1 Introduction

If one is going to employ less than full-scale pressures in connection with soils it is evident that:

1. A technique must be found for manufacturing a soil simulant which will provide realistic P-v behavior at scaled pressures;
2. The P-v behavior of real soils must be reasonably well understood in a quantitative theoretical manner in order that one might develop a cogent process for manufacturing the simulant indicated in 1 above.

In our discussion here, then, we will concentrate on such non-linear stress-strain soil characteristics; discuss a theoretical model which describes real P-v behavior; and thus exhibit the parameters which must be controlled if one is to fabricate a soil simulant.

A.2 Full-Scale Soil Behavior

If a continuously increasing pressure is applied to soil-like materials, a pressure-volume (i.e., specific volume) relationship is generally that which is indicated in Fig. A.1. Starting with the initial specific volume v_b, the P-v curve is initially concave downward as indicated in region A. In this region the (local) sound speed is a decreasing function of pressure, and gives rise to the well-known dispersion of pressure waves in soils.

As the pressure approaches a so-called crushing pressure, P_c, the P-v curve flattens out, becoming nearly horizontal, and there is a rather sudden decrease in specific volume. In this region, indicated as region B in Fig. A.1 the soil rapidly loses its structural integrity, becoming essentially fully compacted near a specific volume, v_s', as indicated in Fig. A.1. As the pressure is further increased this now fully compacted soil has a P-v curve as indicated in region C; here the soil behavior is described by a Tait or bulk modulus type of equation of state.[1]

If now the pressure is unloaded, the soil's P-v relationship will return along the same Tait type of curve; and in fact will follow such a curve all the way down to zero pressure, finally ending up with a specific volume of v_s, as indicated in Fig. A.1. In this overall process the soil has become fully compacted, and in

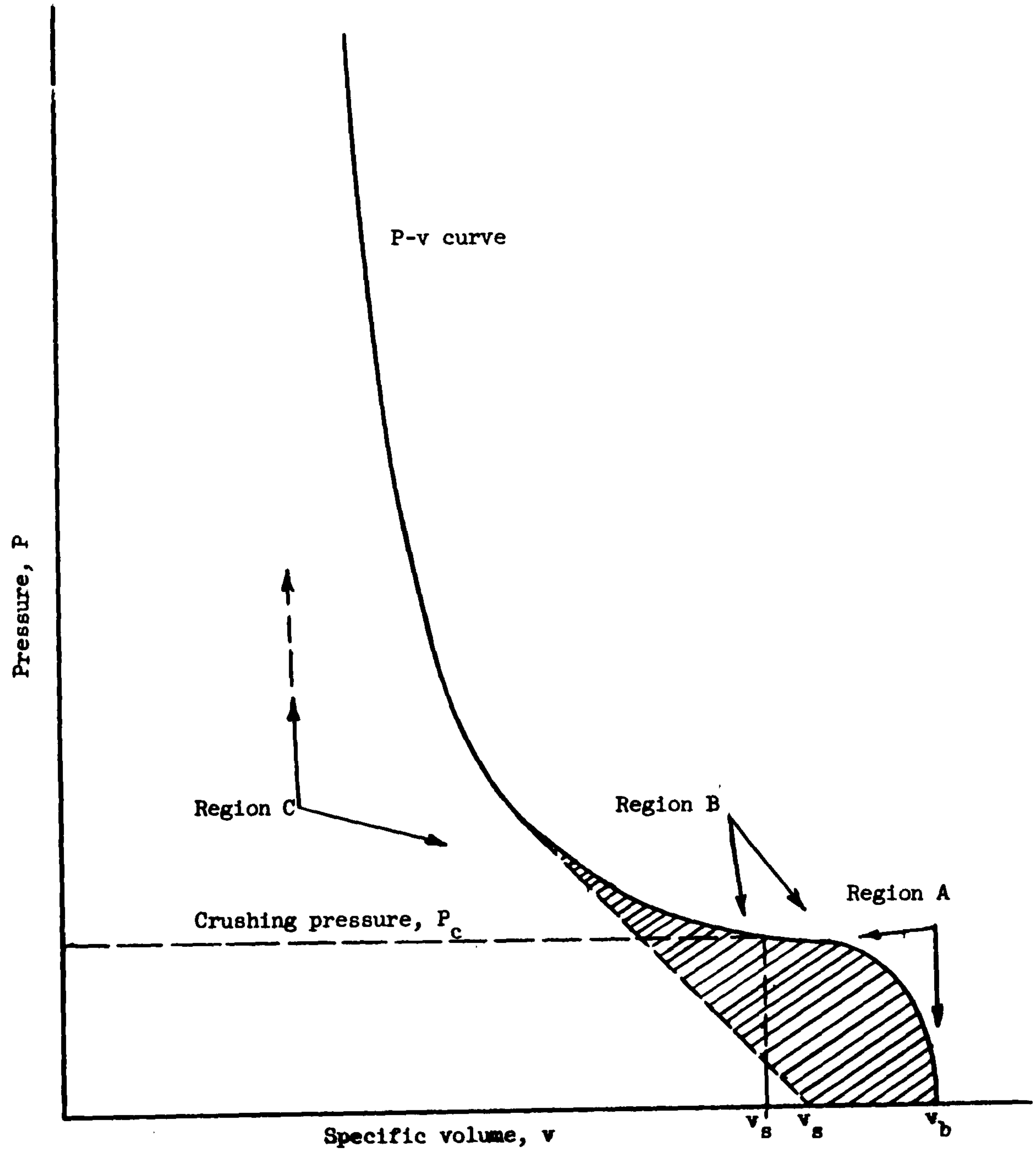


Figure A.1 Typical pressure-volume relationship for Nevada soil.

process of compaction has absorbed from the pressure source an amount of specific energy as indicated by the hatched area in Fig. A.1.

Terzaghi and Peck[2] indicate that for certain sands and clay the aforementioned crushing pressure, P_c, is in the neighborhood of 60-80 psi. On the other hand, however, the Lawrence Radiation Laboratory[3] indicates that, for Nevada tuff, the crushing pressure, P_c, is in the neighborhood of 20,000-30,000 psi. It seems clear, therefore, that we must be analytically and experimentally prepared for very extreme variations in soil characteristics.

A.3 Empirical P-v Relationships

In a material such as soil, the fine details of a P-v relationship depend on a manifold of factors such as strain-rate, thermoviscosity,[4] porosity, moisture content,[5] etc. However, at this early stage of our analysis, as reported here, we will limit our considerations to porosity--and we shall see later that the characteristic shape of a soil's P-v behavior can be described by consideration of this parameter alone. Since we are not going to consider the manifold of other factors, we shall accordingly limit our discussion now of empirical P-v relationships to those obtained by quasi-static compression wherein strain-rate and thermoviscosity can be neglected.

Region C of Fig. A.1 is the simplest to describe. It is generally called a Tait or bulk modulus type of equation of state, and has been employed extensively in the study of underwater explosions. The most notable characteristic of such a Tait form is that the P-v relationship is concave upward. Such a curvature obtains if the isentropic bulk modulus, b, is a linear function of pressure

$$b \equiv - v(\partial P/\partial v)_S$$
$$= A(1 + P/P_1) \qquad (A.1)$$

where A is, strictly speaking, a function of the entropy S, but for practical purposes may be considered as a constant. From Eq. (A.1) we see that the bulk modulus increases with pressure. Such a bulk modulus which increases linearly with pressure is known to describe quite well the behavior of liquids and (already compacted, i.e., without porosity) solids at very high pressures.

If the isentropic bulk modulus, b, is a constant (i.e., does not vary with pressure), then it can be shown that for all pressures we have

$$c^2 = \partial P/\partial \rho$$
$$= Pv = P_o v_o \qquad (A.2)$$

where $\rho = 1/v$, and (P_o, v_o) is any point on the P-v curve. Equation (A.2) indicates that the product Pv is equal to a constant; and also that the sound speed, c, is constant for all pressures. This, of course, would be the case for a constant bulk modulus or constant elastic modulus--and if soil materials truly were typified by such a characteristic, our problem of scaling would be relatively simple.

Equation (A.1) indicates a bulk modulus relationship for which the sound speed increases with increasing pressure and the P-v curve is concave upward. Equation (A.2), corresponding to the constant value of bulk modulus, corresponds to the situation where the sound speed is constant. If now we desire a relationship where the P-v relationship is concave downward, evidently we require a bulk modulus relationship which is a decreasing function of pressure.

Empirically, the overall curve of Fig. A.1 can be approximately described by the following kind of a bulk modulus relationship

$$b = A \left[1 + (P/P_2) \frac{1 + P/P_1}{1 + P/P_2} \right] \qquad (A.3)$$

where A, P_1, and P_2 are empirical constants.

Clearly for P small,

$$b \approx A \left[1 + (P/P_2) \right]$$

If P_2 is a negative number our bulk modulus is the desired decreasing function of pressure, and we would therefore have the P-v curve concave downward as indicated in Fig. A.1 for low pressures. On the other hand, for pressures sufficiently large Eq. (A.3) clearly approaches the relationship of Eq. (A.1); and therefore for high pressures the empirical relationship of Eq. (A.3) will simulate the Tait regime as indicated by region C of Fig. A.1.

Equation (A.3) is a three-parameter (A, P_1, P_2) fit to an empirical curve typified by Fig. A.1. Clearly expressions with a larger number of parameters could be devised if one requires a better fit.

A.4 Compression and Decompression--Irreversibility

We indicated earlier that in soils the compression-decompression cycle is irreversible in that the soil is compacted during the compression phase, but does not "decompact" during the decompression phase; rather the decompression portion of the P-v curve tends to follow down an extrapolated portion of a Tait form typified by region C.

Today, interest in underground shock is centered in the 3,000-10,000 psi region, i.e., ground shock induced by air pressures in the 3,000-10,000 psi region. Now, if the crushing pressure of the soil, P_c, is large compared to our region of interest, then in actuality a typical compression-decompression cycle is as indicated in Fig. A.2. The material begins with a specific volume of v_b, and is compressed through the concave-downward portion of the P-v curve until it reaches the maximum pressure, P_m. During this compression portion of the cycle the material has been partially compacted to the specific volume v_m'. If now pressure is relieved, the decompression phase of the P-v curve will be a Tait form which passes through P_m; when the pressure, P, has returned to ambient the specific volume of the material does not of course return to its initial specific volume, v_b, but rather returns to its partially compacted specific volume, v_m.

A.5 Theory of Compressibility of Porous Media

As perhaps suggested in the previous portion of this Appendix, our requirement is to synthesize a material wherein the bulk modulus varies with pressure in a prescribed manner. And furthermore since we may be interested in scaling down our pressures by a large ratio (10-100 or more), we would like this variation of bulk modulus with pressure to be effected in the scaled-down pressure regime.

For the purposes of our analysis we will assume that we are given a desired P-v relationship such as that indicated in Fig. A.1. We shall now develop a theory of material compressibility which closely duplicates real behavior; the significant parameters in this theory will then suggest techniques for simulating a medium with real P-v characteristics. (We will not attempt to describe in detail how one could fabricate such a soil simulant--for indeed that would be an extensive undertaking and could only be expected within the framework of a much more extensive

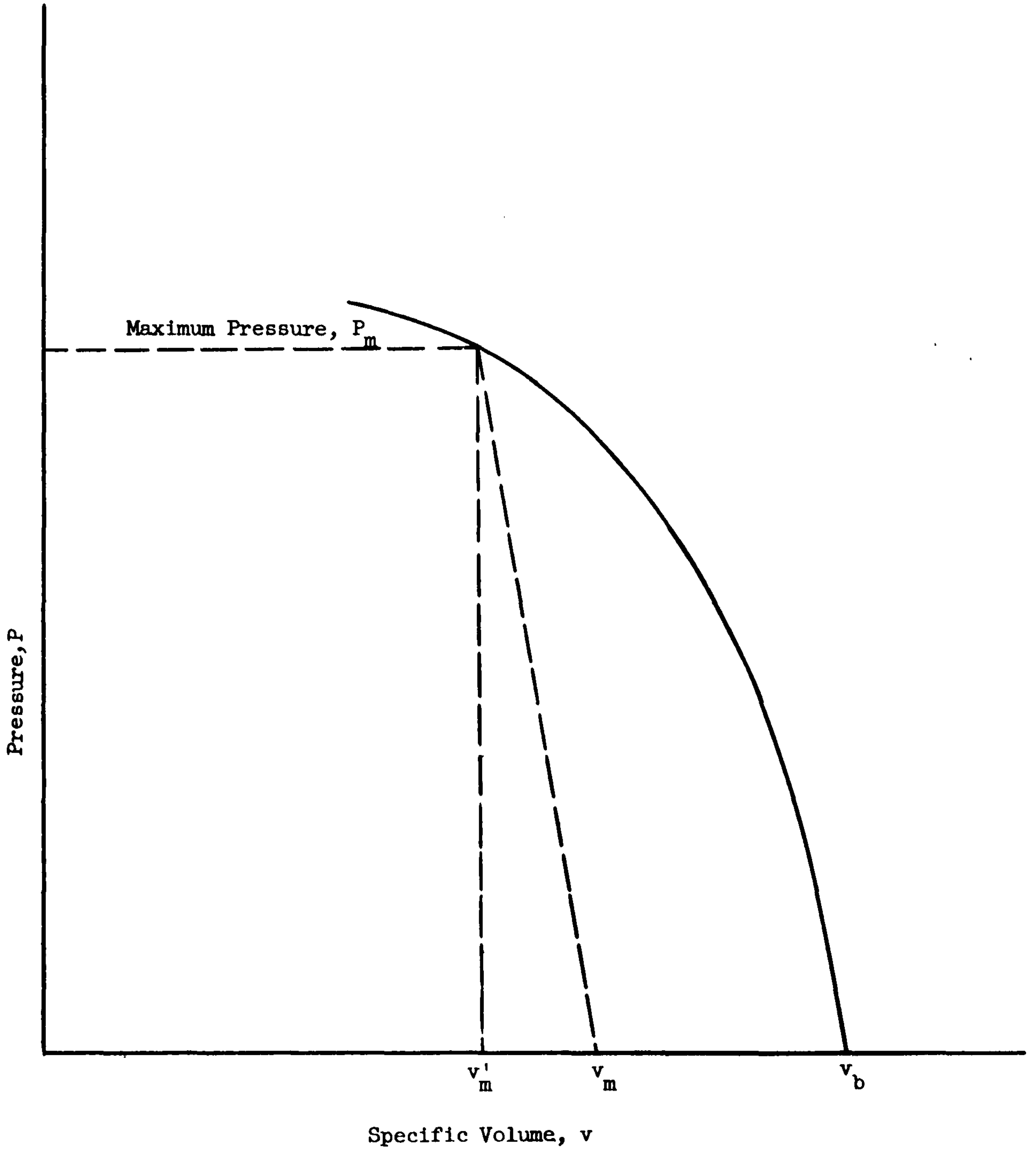


Figure A.2 Pressure cycle where maximum pressure is less than soil's crushing pressure.

endeavor--but we will make some suggestions as to the general approach which might be followed.)

A.5.1 General Picture--The physical picture we have of a compactable porous material is quite similar to that of a foam. Generally a foam is made up of a liquid phase with an inter-mixed gaseous phase, and perhaps the whole system stabilized by suitable emulsifying agents. However, for a solid compactable material we think of the foam made up of a solid material which is interspersed with voids. In real soil-like materials these voids are most often filled up with air and/or water; however, to keep out picture simple at this stage of the theoretical development we will assume that no matter is in the voids, and hence we will neglect any compressibility effects of the matter which may be in the voids. Later, if desired, the theory presented here could be embellished in a straightforward manner to account for the effect of matter in the voids.

If the system of voids were perfectly regular (i.e., if the voids were located with something like crystalline regularity, and each void had the same volume and shape), we would have, as the writer sees it, a very unrealistic situation--and the apparent mechanical properties of such a system would be quite different (as we shall see in the forthcoming analysis) from the P-v behavior of observed soil-like materials. Rather, we believe a better picture is one wherein the voids are randomly distributed with respect to size, and we assume that this distribution is uniform throughout the volume of the "solid foam"--hence for analytical purposes we assume that the distribution function describing these voids is invariant from point to point throughout the foam. And for simplicity in analysis we will assume that this distribution is Gaussian with the mean value of the porosity equal to $\bar{a}$, and the standard deviation in porosity equal to a constant, σ_a. Let us now look into the apparent consequences of this assumed physical picture.

A.5.2 Theory of Compaction--Let the specific volume of our foam or bulk material be v_b, and let the specific volume of the solid component or constituent be equal to v_s; then the porosity, a, is generally defined as

$$a = 1 - v_s/v_b \tag{A.5}$$

Although the porosity, a, is a most often used parameter in discussions of compactable material, it will be easier for our analysis to employ what we shall call the pore ratio, R, which we define as

$$R = v_p/v_s \tag{A.6}$$

where v_s is the specific volume of the solid component of the foam;

v_p is the volume of pores per unit mass of the solid component.

Now the specific volume of the bulk material is equal to

$$v_b = v_s + v_p \tag{A.7}$$

so that, using Eqs. (A.5) and (A.6), the relationship between our pore ratio, R, and the normally employed porosity, a, is given by

$$R = a/(1 - a) \tag{A.8}$$

Let us now consider a sphere of our foam of unit mass immersed in a fluid so that a hydrostatic pressure, P, can be applied. If we take a section through a diameter of our sphere, the area of this section is given by

$$A_b = \pi^{1/3}(3v_b/4)^{2/3} \tag{A.9}$$

and, under the hydrostatic pressure, P, the total force acting across this section is given by

$$F = \pi^{1/3}(3v_b/4)^{2/3}P \tag{A.10}$$

Now, the force, F, indicated in Eq. (A.10) must be borne by the load-bearing solid component of our foam. Because of the foam's porosity the effective cross section of load-bearing material is, of course, less than the A_b of Eq. (A.9); and in fact the effective area, A_s, of load-bearing material is given by

$$A_s = \pi^{1/3}(3v_s/4)^{2/3} \tag{A.11}$$

Let P_s be the stress level in the load-bearing, solid component of our foam. Then, if equilibrium maintains we must have

$$F = \pi^{1/3}(3v_s/4)^{2/3}P_s$$

where F is the force, as indicated in Eq. (A.10) which is acting across the diametric section of our sphere of foam.

Before going farther we would like to make one embellishment to the relationships that have been derived so far. We should like to assume that, at each point in the spherical volume which we are considering, the solid component is indeed load-bearing as long as the stress level, P_s, is less than the ultimate stress level, P_u, of the solid material of our foam; and we shall assume that the material is no longer capable of carrying any load whenever the local stress, P_s, exceeds P_u. At the instant that P_s exceeds P_u we shall say that the material has failed at that point; and we shall assume that the stresses in the solid material are continuously redistributed so that the load is always borne by the material which has not yet been crushed. Taking this into consideration, then, Eq. (A.12) would be modified to read

$$F = \pi^{1/3}[3v_s(1 - f)/4]^{2/3}P_s \tag{A.13}$$

where f is the fraction of the solid component of our foam which has been crushed.

Furthermore, we shall assume that the porosity disappears at those points where the load-bearing material has failed; that is to say, when the load-bearing material fails locally, it collapses and fills the voids in the locale. Taking this into consideration, then, Eq. (A.7) and Eq. (A 10) must be modified to read

$$\begin{aligned} v_b &= v_s + v_p(1 - f) \\ &= v_s\,[1 + R(1 - f)] \end{aligned} \tag{A.14}$$

and

$$F = \pi^{1/3}\,[3v_s\,\{1 + R(1 - f)\}\,/4\,]^{2/3}P \tag{A.15}$$

From Eqs. (A.13) and (A.15) we obtain

$$P_s = P/G^{2/3} \tag{A.16}$$

where

$$G \equiv [1 - f\,]/\,[1 + R(1 - f)\,] \tag{A.17}$$

Equations (A.16) and (A.17) show that, at (each neighborhood of) each point, the stress level, P_s, in the load-bearing component of our foam is a function of the local values of:

a. the pore ratio, R;

b. the failure fraction, f.

We shall not attempt to specify explicitly the value of R throughout our foam; rather we shall be satisfied to make some assumptions about the statistical distribution of pore ratio, R. First of all, we shall assume that in our sphere of foam the mean value of the pore ratio is equal to $\bar{R}$. Accordingly, the average value of the load-bearing material's stress level is given by

$$\bar{P}_s = P/(\bar{G})^{2/3} \tag{A.18}$$

where

$$\bar{G} = [1 - f\,]\,/\,[1 + \bar{R}(1 - f)\,] \tag{A.19}$$

Next we shall assume that the pore ratio, R, has a Gaussian distribution in respect to magnitude and that the standard deviation, σ_R, is a constant.

Since R and hence G and P_s are distributed variables, we should like to find a relationship between σ_R and P_s's standard deviation, σ_{P_s}. Before we can find such a relationship we must first express P_s as a linear function of R. To do this we shall assume that throughout our foam R does not differ too much from $\bar{R}$; and therefore that P_s does not differ too much from $\bar{P}_s$. Under this assumption we can linearize Eqs. (A.16) and (A.17), i.e., simply form differentials so as to express ΔP_s as a linear function of ΔR,

$$\Delta P_s = (2P/3)(\bar{G})^{-1/3} \Delta R \tag{A.20}$$

To a first order approximation Eq. (A.20) expresses variations in P_s as a linear function of the variations in R; hence [6]

$$\sigma_{P_s} = (2P/3)(\bar{G})^{-1/3} \sigma_R \tag{A.21}$$

The distribution function for a Gaussian distribution with mean value described by Eq. (A.18) and standard deviation described by Eq. (A.21) is

$$dX = \frac{dP_s}{\sqrt{2\pi}\,\sigma_{P_s}} \exp - \left[(P_s - \bar{P}_s)^2 / 2\sigma_{P_s}^2 \right] \tag{A.22}$$

Equation (A.22) is, of course, normalized so that

$$\int_{-\infty}^{\infty} dX = 1 \tag{A.23}$$

Using Eq. (A.22) we are now able to set up an expression which ascertains the fraction, f, of the load-bearing material which has failed; that is, by hypothesis, the fraction of material wherein the local stress level, P_s, exceeds the load-bearing material's yield stress, P_u. The expression for f is

$$f = \int_{P_u}^{\infty} dX \tag{A.24}$$

Equation (A.24) can be expressed in terms of readily available tabulated functions as

$$f = [\, 1 - \mathrm{erf}(x_u) \,] /2 \tag{A.25}$$

where

$$x_u = [(P_u/P)\bar{G}^{2/3} - 1\,] / [2\sqrt{2}\,\bar{G}\,\sigma_R] \tag{A.26}$$

where we have made use of Eq. (A.21), and where erf(x) is the error function [7] or error integral.[8] Equations (A.25) and (A.26), then, (together, of course, with Eq. (A.17)) express the failure fraction, f, as a function of the applied hydrostatic pressure, P--and in terms of the three parameters, P_u, $\bar{R}$, and σ_R. Control of these three parameters will provide a control over the manner in which failure takes place in the load-bearing component of our foam. Figure A.3 is a plot of f versus P/P_u for the following two sets of conditions:

a. $\bar{R} = 1$, and $\sigma_R = 0.5$;

b. $\bar{R} = 0.5$, and $\sigma_R = 0.25$.

Once the failure fraction, f, is known as a function of the hydrostatic pressure, P, we can utilize Eq. (A.14) to ascertain the specific volume of our foam as a function of the hydrostatic pressure, P--that is to say, we have now determined the P-v relationship for our theoretical model of a porous, compactable material. In Fig. A.4 we make use of the data in Fig. A.3 and plot P/P_u as a function of v_b. The solid curves in Fig. A.4 indicate the behavior of our compactable material if the stresses in the material are always in equilibrium with the hydrostatic pressure. In practice, of course, when such a material is subjected to a hydrodynamic shock

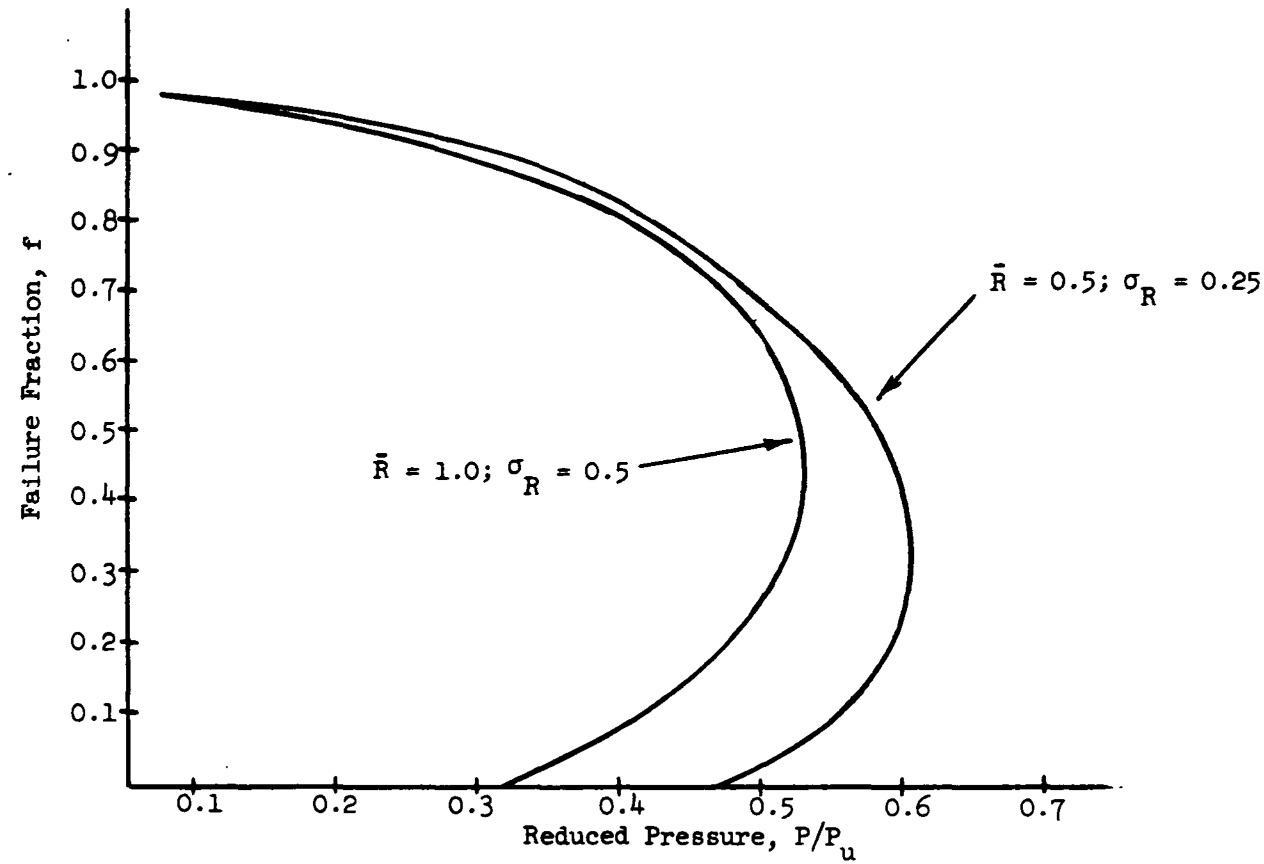


Figure A 3 Fraction of failed load-bearing material as a function of hydrostatic pressure P and material's yield stress, P_u; and with average pore ratio, $\bar{R}$, and standard deviation in pore ratio, σ_R, as parameters.

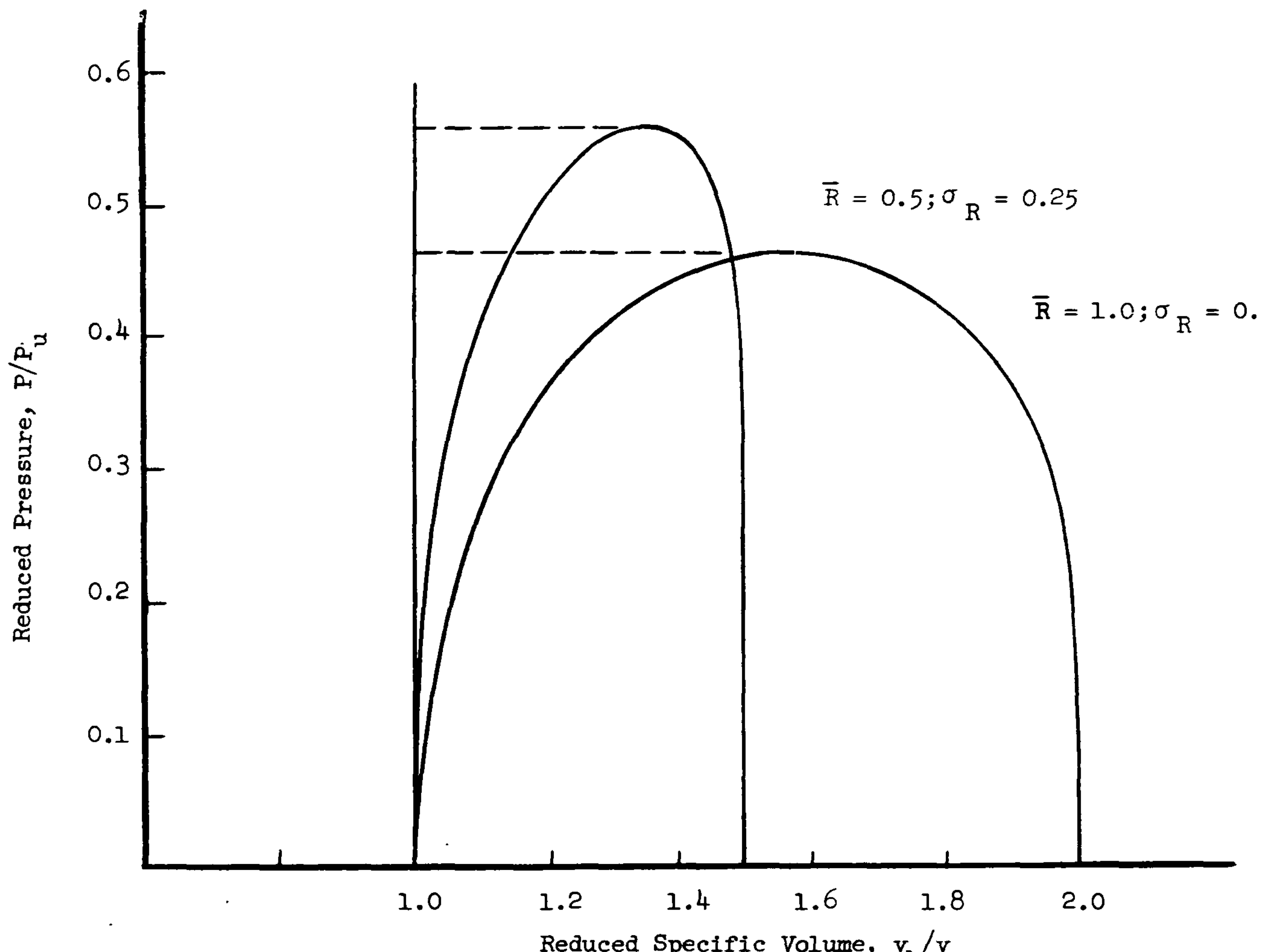


Figure A.4 Theoretical P-v relationship, where P is hydrostatic pressure, P_u is load-bearing material's yield strength, v_b is specific volume of bulk (porous) material, v_s is specific volume of solid component; and with average pore ratio, $\overline{R}$, and standard deviation in pore ratio, σ_R, as parameters.

wave, the compactable material compresses in the manner of the solid portion of Fig. A-4 until the collapsed pressure, P_c, is reached—and at that instant the material suddenly collapses and fully compacts right down to the specific volume of the solid constituent. Hence, under hydrodynamic shock the observed P-v curve follows the dashed portion as indicated in Fig. A.4.

A.5.3 Conclusion--In subsection A.5.2 we have outlined a theory of compaction for soil-like materials. Within the framework of this elementary theory we have seen that the significant parameters are:

a. P_u, the ultimate allowed stress in the solid, load-bearing component of the porous material;

b. the average pore ratio, $\overline{R}$, throughout the compactable material;

c. the standard deviation, σ_R, in the distribution of pores throughout the compactable material. These three parameters control the P-v behavior of our soil-like material, and, in particular, control the hydrostatic pressure level, P_c, at which crushing takes place.

In this first approach to a theoretical model we have assumed a Gaussian distribution of pore ratio; however, further consideration would perhaps indicate whether other statistical distributions might be better descriptive of the real situation.

But whatever distribution function is employed, the fundamental physical picture which we portray here is that, with increasing hydrostatic pressure, P, there is a probabilistic progression of local failure in the load-bearing component of the soil-like material; and this progression of failure takes out of play a fraction, f, of this load-bearing material. Consequently, as the hydrostatic pressure, P, is increased the effective cross sectional area of the load-bearing material is progressively decreased. As this process is continued there will be reached a state where instability sets in: the fraction, f, rises rapidly, and the soil-like material loses its identity and collapses (i.e., fully compacts) to the specific volume, v_s, of the solid component of the soil-like material. Once the porous material has collapsed or compacted, then further increase in hydrostatic pressure will find a P-v variation corresponding to the bulk modulus equation of state which is proper to the solid constituent.

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