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DYNAMIC HEAVING MOTION
OF GROUND EFFECT MACHINES

by

W. C. Webster and J. D. Lin

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ABSTRACT

The dynamics of simple ground effect machines undergoing heaving motion over a wavy surface is formulated here. The equation of motion is a third order ordinary differential equation with the coefficients depending essentially on the mass flow into or out of the cavity under the machine and on the geometric characteristics of the machine. The response of machines traveling over a sinusoidal surface is derived as a function of the encounter frequency and mass flow coefficients. Then, mass flow coefficients are obtained for different types of machines. Peripheral jet machines without an exceptionally large volume of concave bottom are found to be stable in the heave mode. The responses of both a peripheral jet machine and a plenum chamber machine are computed. The peak response of plenum chamber machines is generally higher than that of peripheral jet machines.

NOTATION

A	-	horizontal projection of the base area of machine
A_o	-	effective nozzle area of peripheral jet
A_i	-	effective nozzle area of interior jet
a	-	speed of sound
C	-	coefficient of the distribution of base pressure
g	-	the acceleration of gravity
H	-	distance from the base of machine to the ground
H_m	-	distance from the base of machine to the mean level of wavy surface
H_e	-	equilibrium height of machine relative to the base of machine
H_o	-	distance from the exit of peripheral nozzle to the ground
H_1	-	distance from the edge of the labyrinth of plenum chamber machine to the ground
H_a	-	unsteady part of H or $H = H_e + H_a$
H_s	-	half-amplitude of the response of machine
h_a	-	distance from the wavy surface to its mean level
h_s	-	half-amplitude of the wavy surface
L	-	lift
M	-	mass flow into or out of the cavity under machine
M_i	-	mass flow of interior jet nozzle or exhaust
m	-	mass of the machine
p_u	-	base pressure
$\bar{p}_u$	-	p_u/p_o
p_{uo}	-	base pressure at the equilibrium height

NOTATION (Continued)

p_a - unsteady part of p_u or $p_u = p_{uo} + p_a$

p_o - total pressure in the plenum chamber for peripheral jet

p_i - total pressure in the plenum chamber for interior jet

S - length of peripheral jet nozzle

T - thrust of jet

t_o - effective thickness of peripheral jet nozzle

t_u - thickness of mass flow out of the cavity of machine under underfed peripheral jet

U - speed of the machine

U_w - speed of the wavy surface

V_o - fluid velocity at the exit of peripheral jet nozzle

V_i - fluid velocity at the exit of interior jet nozzle or exhaust

$\forall$ - total volume of the cavity under the machine

α - mass flow factor of overfed jet

β - stability index

γ - the ratio of the specific heats of air

ϵ - phase angle

η_o - nozzle efficiency

θ - angle of the nozzle axis of peripheral jet at its exit

λ_w - wave length of the wavy surface

μ - percentage of the weight of machine supported by its base pressure

ρ - fluid density

ω - angular frequency, rad./sec.

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INTRODUCTION

Ground effect machines are a class of vehicles supported by an air cushion and utilizing peripheral jet curtains or labyrinths as a sealing device. Due to the compressibility of the air cushion and the flexibility of the peripheral jet, the machine-air cushion system is susceptible to oscillatory motions. The base pressure under the machine depends strongly on the variation of the altitude during forward flight. The efficiency of ground effect machines is generally derived by using a low base loading and close proximity to the ground. These pertinent characteristics make the problem of dynamic stability exceptionally important when a machine is designed to be operated on a rough terrain or seaway.

In free flight, a machine is subject to six degrees of freedom in motion. Many of these degrees may be coupled with one another. There are, however, three motions which are essential to the stability of ground effect machines: heaving, pitching and rolling. Tulin (1) first treated heaving motion of an edge-jet machine and brought out the idea of underfed and overfed jets for unbalanced operation of a machine. His work on heaving stability provides the foundation for further theoretical analysis of dynamic motion in heaving and pitching. It is the purpose of this report to examine Tulin's original work and to extend the analysis to the dynamic heaving response for other types of ground effect machines.

FUNDAMENTALS

Ground effect machines can be divided into two basic categories depending on the method by which the air cushion is maintained. They are:

(i) Machines that maintain the air cushion by means of a high momentum jet curtain around the periphery of the machine. It is common for machines of this type to have the air cushion compartmented by interior jets for the purpose of stability. We shall call this type of machine a peripheral-jet machine. Figure 1 shows a two-dimensional slice of such a machine.

(ii) Machines that maintain the air cushion by a physical barrier or labyrinth under which the fluid is discharged from the air cushion. We shall call this type of machine a plenum chamber machine as shown in Figure 2.

An analysis of dynamic motion of ground effect machines involves the unsteady motion of both the machine itself and the flow field of peripheral jet and air cushion. The aerodynamic force due to forward motion is generally ignored at the present stage. As the machine is disturbed from its equilibrium condition, unsteadiness is induced into the air cushion due to its compressibility and due to mass flux into and out of the cavity; the latter is controlled by the behavior of the jet curtain. The dynamic process of the compression of the air cushion is assumed to be instantaneous as compared to the disturbance (the ratio of speed of sound to the product of the disturbance frequency and largest dimension in the cavity is assumed to be large). The instantaneous change of jet momentum is of the order of $\frac{A_o}{A}(1+\frac{A_1}{A_o})$ which can be ignored for practical machines. Therefore, the dynamic analysis has only to be concerned with the mass flow into or out of the cavity due to instantaneous changes of base pressure and the volume of cavity. The mass flow coefficients depend essentially on the behavior of peripheral jet under unbalanced operation. As we shall see later, further assumptions have to be made for the derivation of mass flow coefficients so that the problem can be treated analytically.

FORMULATION OF THE PROBLEM

Consider the machine shown in Figure 3 travelling over a surface with sinusoidal undulations. If the ratios of wave amplitude to wave length and machine width to wave length are small, then one can assume that the motion of machine is essentially pure heaving. For a planar motion, the equation of motion of the mass center of machine is expressible as

$$L - mg = m\ddot{H}_m \qquad [1]$$

The instantaneous lifting force consists of the pressure force due to the ground effect and the thrust of jets, and thus

$$L = p_u A + T = C(p_{uo}+p_a)A + T \qquad [2]$$

The unsteady part of jet thrust results from the change of ambient pressure due to unsteady base pressure. If the analysis is based on the thin jet theory, which assumes an ambient pressure equal to the atmospheric pressure, then T is independent of time. In the thick jet theory, the contributions is of the order of

$\frac{A_o}{A}\left(1+\frac{A_i}{A_o}\right)$, which may be neglected for practical machines; however, this effect can also be absorbed in the coefficient, C, for the pressure distribution over the base of the machine. Decomposing H_m into

$$H_m = H_e + H_a + h_a \qquad [3]$$

where H_e is the equilibrium height and independent of time, Equation [1] becomes

$$m(\ddot{H}_a + \ddot{h}_a) = Cp_a A \qquad [4]$$

by use of Equations [2] and [3] and

$$Cp_{uo}A + T = m_g$$

which is the condition at equilibrium. Equation [4] implies that the vertical oscillation of machine is forced by a wavy surface and the variation of base pressure due to the mass flow into or out of the cavity. The former is a given function while the latter remains to be derived from the mass flow under the machine and the configuration of peripheral jet with or without an interior jet. The instantaneous mass flow into or out of the cavity under machine satisfies

$$\frac{d}{dt}(\rho V) = \left.\frac{\partial M}{\partial H}\right|_{Eq.} H_a + \left.\frac{\partial M}{\partial p_u}\right|_{Eq.} p_a \qquad [5]$$

where the linearized mass flow coefficients, $\left.\frac{\partial M}{\partial H}\right|_{Eq.}$ and $\left.\frac{\partial M}{\partial p_u}\right|_{Eq.}$ will be evaluated for different **types** of machines as mentioned in the last section. Carrying out the differentiation of the left-hand-side of Equation [5], one has

$$\left.\frac{\partial M}{\partial H}\right|_{Eq.} H_a + \left.\frac{\partial M}{\partial p_u}\right|_{Eq.} p_a = \frac{V}{a^2}\dot{p}_a + \rho A\dot{H}_a \qquad [6]$$

Equations [4] and [6] can be combined as the equation of heaving motion,

$$\dddot{H}_a + a_2\ddot{H}_a + a_1\dot{H}_a + a_o H_a = -(\dddot{h}_a + a_2\ddot{h}_a) \qquad [7]$$

with the mass flow coefficients,

$$a_o = - \frac{CAa^2}{m\forall} \left.\frac{\partial M}{\partial H}\right|_{Eq.}$$

$$a_1 = \left.\frac{C\rho\mu g A a^2}{p_u \forall}\right|_{Eq.} \qquad [8]$$

and

$$a_2 = - \frac{a^2}{\forall} \left.\frac{\partial M}{\partial p_u}\right|_{Eq.}$$

Immediately from Equation [7] and the Routh-Hurwitz criterion, it is obvious that the machine is stable (not capable of self-excited heaving motions) if and only if

$$\beta = a_1 a_2 / a_o \geqslant 1 \qquad [9]$$

RESPONSE CHARACTERISTICS

It is now possible to investigate the response of machine over a given wavy surface by Equation [7]. Let the surface be a train of sinusoidal waves of the following form:

$$h_a = h_s \sin(\omega t + \epsilon) \qquad [10]$$

where h_s is the half-amplitude of waves, ϵ is the phase angle and the encounter frequency

$$\omega = 2\pi \left(\frac{U-U_w}{\lambda w}\right),$$

then the machine must have a periodic motion as

$$H_a = H_s \sin\omega t \qquad [11]$$

with a frequency identical with the forcing frequency. Substituting Equations [10] and [11] into Equation [7], one finds the response of the machine as

$$\left|\frac{H_s}{h_s}\right| = \frac{\omega^2(a_2^2+\omega^2)}{\left\{[a_2(a_0-a_2\omega^2)+\omega^2(a_1-\omega^2)]^2+\omega^2(a_1a_2-a_0)^2\right\}^{\frac{1}{2}}}$$

and [12]

$$\epsilon = \text{arc tan}\left[\frac{\omega(a_1a_2-a_0)}{a_0a_2+(a_1-a_2^2)\omega^2-\omega^4}\right]$$

Applying the stability criterion, Equation [9], for the neutral stability, Equation [12] is reduced to

$$\left|\frac{H_s}{h_s}\right| = \frac{\omega^2}{\left|a_1-\omega^2\right|} \qquad [13]$$

This response function has a resonance frequency,

$$\omega_n=\sqrt{a_1}$$

or by Equation [8] with C = 1.0,

$$\omega_n=\sqrt{\frac{g}{H_e}}$$

which corresponds to the undamped oscillation with a sealed air cushion. In general, the critical resonance frequency and damping coefficient can be found directly from the secular equation of Equation [7] after the mass flow coefficients are found for a specific configuration.

MASS FLOW COEFFICIENTS

To predict the response of a machine by Equation [12], the mass flow coefficients a_Q and a_2 are derived on the basis of the configuration of a specific machine. The principle of momentum balance is applied to the unbalanced operation of peripheral jets with or without interior jets. In the case of the plenum-chamber machine, the flow under the labyrinth is unique. It can be shown that the contribution of the unsteady part of jet momentum is of the order of ${}^{H\omega}/V_j$, where V_j is the velocity of peripheral jet or the discharge velocity for plenum chamber machines. For the present analysis the unsteady part of jet momentum is neglected because both the height and disturbance frequency are small in comparison with the velocity in the peripheral jet. Furthermore, the local undulation near the jet is also neglected. Then, the mass flow coefficients can be derived for each type of machine discussed in the following sections.

Peripheral Jet Machine

A typical section of this type of machine is shown in Figure 1. The thin jet theory is modified by taking the average value of base pressure and atmospheric pressure as the mean static pressure at the nozzle exit and extended to the condition of unbalanced jets. For balanced operation, the base pressure satisfies

$$\tfrac{1}{2}\bar{p}_u H_o = \eta_o t_o (1 + \sin\theta)(1 - \tfrac{1}{2}\bar{p}_u) \qquad [14]$$

where the unsteady part of momentum is neglected. When the machine is disturbed from its equilibrium height, the peripheral jet may be operated under one of these three possible conditions:

(a) The instantaneous base pressure becomes higher than the pressure required by the jet to operate under balanced condition at the instantaneous height. Hence, there is mass flow out of the cavity and it is defined as an "underfed jet".

(b) The lower instantaneous base pressure cannot support balanced operation and causes the peripheral jet to split.

This is an "overfed jet".

(c) The instantaneous base pressure and the height of machine remain under balanced operation.

It is known that the configuration of the peripheral jet depends on the predominance of the so called "jet effect" or "compression effect". When a machine becomes lower than its equilibrium height, both effects tend to increase the base pressure. The predominance of the compression effect tends to make the jet underfed, but the predominance of the jet effect tends to make the jet overfed. The situation is just opposite when the machine is higher than its equilibrium height. Tulin (1) showed that for practical machines without exceptionally large volume due to the concave bottom of machine, the compression effect always predominates; therefore, one can assume the configuration a priori, i.e. peripheral jet underfed for the machine above the equilibrium height and overfed below the equilibrium height. Thus a discontinuity in the jet configuration may exist for peripheral jet machines during heaving. It should be noted that the case (c) above corresponds to the condition,

$$M = 0 \quad \text{or} \quad a_0 = a_2 = 0$$

Then, from Equation [8],

$$\left|\frac{H_s}{h_s}\right| = \frac{\omega^2}{|a_1 - \omega^2|}$$

This is exactly the same result as Equation [13], which expresses the condition of neutral stability. With the perfect gas law, air trapped in the cushion acts like a spring with no damping and thus the system is similar to a one-dimensional mass spring system.

For the cases of (a) and (b), Equation [14] does not apply to unbalanced operation. The instantaneous base pressure now satisfies the following equations:

Overfed: $\frac{1}{2}\overline{p}_u H_o = \eta_o t_o (1+\sin\theta - 2\alpha)(1-\frac{1}{2}\overline{p}_u)$ [15]

Underfed: $\frac{1}{2}\overline{p}_u (H_o - 2t_u) = \eta_o t_o (1+\sin\theta)(1-\frac{1}{2}\overline{p}_u)$ [16]

where α is the ratio of mass flow into the cavity due to overfed jet and t_u is the thickness of discharge jet from the cavity due to the underfed jet. Hence, the mass flow can be expressed as

Overfed: $M = \alpha\eta_o \rho V_o t_o S$ [17]

Underfed: $M = \rho V_u t_u S$ [18]

By differentiating Equations [17] and [18] partially with respect to H and p_u, and by use of Equations [14] to [16], the mass flow coefficients are derived and tabulated as follows.

	Overfed Jet	Underfed Jet
a_o	$\left.\frac{C\mu g a^2 S}{2\forall \sqrt{(2p_o - p_u)/\rho}}\right\vert_{Eq.}$	$\left.\frac{C\mu g a^2 S}{\forall \sqrt{2p_u/\rho}}\right\vert_{Eq.}$
a_2	$\left.\frac{a^2 S[H_o + \eta_o t_o(1+\sin\theta)]}{2\forall \sqrt{(2p_o - p_u)/\rho}}\right\vert_{Eq.}$	$\left.\frac{a^2 S[H_o + \eta_o t_o(1+\sin\theta)]}{\forall \sqrt{2p_u/\rho}}\right\vert_{Eq.}$

All quantities in the table are linearized at the equilibrium height. The stability index β can now be computed from the above table

$$\beta = \frac{a_1 a_2}{a_o} = \left.\frac{\gamma A[H_o + \eta_o t_o(1+\sin\theta)](p_u + p_{atm})}{\mu \forall p_u}\right\vert_{Eq.} \quad [19]$$

where p_{atm} is the atmospheric pressure. It should be noted the β is identical for both overfed and underfed jets.

A cursory inspection of Equation [19] reveals that it would indeed be a rare machine for which $\beta \leq 1$, since

$$\left.\frac{\gamma(p_u+p_{atm})}{p_u}\right|_{Eq.} >> 1$$

When the machine is oscillated about its equilibrium height, as shown in the table, there is a discontinuity in the mass flow coefficients.

The configuration of the peripheral jet has to change from underfed to overfed as it heaves upward and passes the equilibrium height or visa versa for a downward motion. This non-linearity cannot be taken into account by the present analysis of the linearized version of dynamic motion. For each cycle of heaving, the net mass flow into and out of the cavity should be zero; therefore, the total amplitude of heaving should be between the amplitudes based on these two sets of mass flow coefficients.

As an example, the computation is carried out for the following machine:

Diameter = 50 feet (circular planform)

$H_e = H_o = 5$ feet (flat base)

$t_o = 1$ foot

$p_{uo} = 20$ psf

$p_o = 35$ psf

$C = 1.0$

$\rho = 0.0025$ slug/ft^3

$g = 32.2$ ft./sec.^{2}

then

"Overfed"	"Underfed"
$a_o = 2090$	$a_o = 4650$
$a_2 = 425$	$a_2 = 940$

and $a_1 = 975$.

The response curves based on the mass flow coefficients for underfed and overfed jets are computed and shown in Figure 5.

Peripheral Jet Machines with a Central Jet

Interior jets are generally required to divide the cavity under a machine into compartments for the purpose of pitch and roll stability (2). Most practical designs of interior jets may simply be approximated by a central jet of equivalent characteristics. A section of this type of machine is shown in Figure 6. With sufficient mass flow from the central jet, the peripheral jet can remain underfed for a whole cycle of motion and thus the mass flow coefficients are continuous. For an underfed peripheral jet, the equation of momentum balance is

$$\tfrac{1}{2}\bar{p}_u(H_o-2t_u) = \eta_o t_o(1+\sin\theta)(1-\tfrac{1}{2}\bar{p}_u) \qquad [20]$$

and the mass flow into or out of the cavity is

$$M = \rho V_u t_u S + M_i \qquad [21]$$

The mass flow of a central jet for a constant total head in the interior nozzle may be computed by

$$M_i = \eta_o A_i \sqrt{2\rho(p_i-p_u)} \qquad [22]$$

The condition to maintain the peripheral jet underfed can be expressed as

$$M_i > M_{max} \qquad [23]$$

where M_{max} is the mass flow required due to compression and change of volume in the cavity at the maximum response expected during heaving motion by Equation [5]; hence this condition has to be specified after the solution is found. The mass flow coefficients are derived as the following:

$$a_o = \left. \frac{C\mu g a^2 S}{\forall \sqrt{2p_u/\rho}} \right|_{Eq.}$$

and

$$a_2 = \frac{a^2}{\forall} \left\{ \frac{S[H_o + \eta_o t_o (1+\sin\theta)]}{\sqrt{2p_u/\rho}} - \frac{\partial M_1}{\partial p_u} \right\} \Bigg|_{Eq.}$$

For a neglegible value of $\left. \frac{\partial M_1}{\partial p_u} \right|_{Eq.}$ the response curve is similar to that of the underfed jet shown in Figure 5. The heaving response of this type of machine is expected to be larger than that of a machine with the same peripheral jet but without the central jet.

Peripheral Jet with a Central Exhaust

In contrast to the idea of a central jet, a central exhaust can be installed in ground effect machines. By changing the operation of peripheral jet from the underfed mode to the overfed mode, the heaving response is significantly reduced as indicated in Figure 5. The central exhause is not as effective as a central jet as a compartmentation device for the purpose of pitch stability; however, it does reduce the cross flow under the machine to a certain degree. The following equations are derived for the balance of momentum of the peripheral jet and of the mass flow under overfed operation:

$$\tfrac{1}{2}\bar{p}_u H_o = \eta_o t_o (1+\sin\theta - 2\alpha)(1-\tfrac{1}{2}\bar{p}_u) \qquad [24]$$

and

$$M = \alpha\eta_o \rho V_o t_o S - M_i \qquad [25]$$

where $M_i = \eta_o A_i \sqrt{2\rho p_u}$ for the exhaust with an atmospheric ambient pressure. The mass flow coefficients are then derived as

$$a_o = \frac{C\mu g a^2 S}{2\forall \sqrt{(2p_o - p_u)/\rho}}\Bigg|_{Eq.}$$

and

$$a_2 = \frac{a^2}{\forall}\left\{\frac{S[H_o + \eta_o t_o(1+\sin\theta)]}{2\sqrt{(2p_o-p_u)/\rho}} + \frac{\partial M_i}{\partial p_u}\right\}\Bigg|_{Eq.}$$

Double Peripheral Jet Machines

A typical section of double peripheral jet machines is shown in Figure 7, which represents a better approximation to a practical machine than a peripheral jet machine with a central jet. The equation of motion, Equation [7], can be applied here with some modification as follows. The base pressure may no longer be uniform during heaving motion; then p_a is the unsteady part of base pressure relative to either the inner or outer compartment. This discontinuity of base pressure across the interior jet may be taken into account simply by a modification of the coefficient, C. Also, the mass flow coefficient, $\frac{\partial M}{\partial p_u}\Big|_{Eq.}$, consists of two parts:

$$\frac{\partial M}{\partial p_u}\Bigg|_{Eq.} = \frac{p_{uo_c}}{p_{uo}}\frac{\partial M_c}{\partial p_u}\Bigg|_{Eq.} + \frac{\partial M}{\partial p_u}\Bigg|_{Eq.} \qquad [26]$$

where the subscript, c, denotes the quantities of inner compartment and the non-subscript for the outer. Similar to a central jet machine, a sufficiently strong interior jet can keep the peripheral jet underfed throughout the whole cycle of motion. From Equations [20] and [21], the mass flow coefficients are derived as

$$a_o = \left. \frac{C\mu g a^2 S}{\forall \sqrt{2p_u/\rho}} \right|_{Eq.}$$

and

$$a_2 = \left. \frac{a^2}{\forall}\left\{ \frac{\sqrt{2\rho p_u}}{p_{u_c}} S[H_1 + \eta_o t_o(1+\sin\theta)] - \frac{\partial M_i}{\partial p_u} \right\} \right|_{Eq.}$$

where $M_i = \eta_o A_i \sqrt{2\rho[p_i - \frac{1}{2}(p_u + p_{u_c})]}$. The behavior of interior jets is very complicated during heaving; however, for the linearized mass flow coefficients derived above, the jet only serves as a compartmentation device and provides sufficient mass flow to maintain the peripheral jet underfed.

Plenum Chamber Machines

This type of machine is quite different from those just discussed due to the absence of both peripheral and interior jets. The base pressure is maintained by the air input to the plenum chamber and the gap between the labyrinth and the ground. The mass flow for any height can then be expressed as

$$M_o = C_d S H_1 \sqrt{2\rho p_u} \qquad [27]$$

which is also equal to the mass flow delivered by the fan at the equilibrium height only. At any height the mass flow of the fan is a function of the base pressure only. For a small change of base pressure, this mass flow can be expanded into a Taylor's series in the neighborhood of its equilibrium height, i.e.

$$M_f(p_u)=M_f\Big|_{Eq.}+M_f'(p_{uo})p_a+\frac{1}{2!}M_f''(p_{uo})p_a^2+\ldots$$

or

$$=M_f\Big|_{Eq.}+M_f'(p_{uo})p_a \qquad [28]$$

by neglecting the higher order terms and M_f' is assumed to be a known characteristic of the fan. Then, the difference in mass flow of the fan and through the gap under the labyrinth is derived as

$$M=M_f\Big|_{Eq.}+M_f'(p_{uo})p_a-C_dSH_1\sqrt{2\rho p_u} \qquad [29]$$

Hence, the mass flow coefficients are derived as the following:

$$a_o=\frac{C\mu gC_d a^2S}{\text{¥}\sqrt{p_u/2\rho}}\Bigg|_{Eq.}$$

and

$$a_2=\frac{a^2}{\text{¥}}\left(\frac{C_dSH_1}{2\sqrt{p_u/2\rho}}-M_f'\right)\Bigg|_{Eq.}$$

The stability of this type of machines is crucial due to the fact that β may become smaller than unity if $M_f' > 0$. For a_o and a_2 derived above, one has

$$\beta=\left[\frac{\gamma(p_u+p_a)AH_1}{2p_u\text{¥}}\left(1-\frac{M_f'\sqrt{2p_u/\rho}}{C_dS}\right)\right]\Bigg|_{Eq.} \qquad [30]$$

Even if $M_f' > 0$, the stability is very weak, for H_1 of the machine is usually small in comparison with the value of ¥/A. In general, β given in Equation [30] is smaller than that given by Equation [19] of a peripheral jet machine with underfed jet. The response of this type of machine is worked out for the following example:

Diameter = 50 feet (circular planform)

$H_1 = 2$ feet

$\mu = 0.98$

$\forall/A = 10$ feet

$p_{uo} = 20$ psf

$M_f' = 0$

$C_d = 0.8$

which give $a_o = 38{,}000$, $a_1 = 487$ and $a_2 = 1{,}230$. The response of this machine shown in Figure 8 is much larger than those of peripheral jet machines.

SUMMARY OF RESULTS

1. Equation of heaving motion, Equation [7], is here derived for the following types of ground effect machines:

   (a) Peripheral jet machines

   (b) Peripheral and central jet machines

   (c) Peripheral jet and central exhaust machines

   (d) Double jet machines

   (e) Plenum-chamber type machines

2. The response function for a machine traveling over a sinusoidal surface is expressed in terms of mass flow coefficients as given in Equation [12].

3. The mass flow coefficients and the response for each type of the machines are discussed in the following:

(a) Peripheral jet machines have two modes of operation, overfed and underfed peripheral jets. Thus, the mass flow coefficients are discontinuous as the machine passes its equilibrium height in each cycle of heaving. The response of machine over a sinusoidal surface can be obtained uniquely only by taking this non-linearity into account. However, the limit of response was obtained here for both underfed and overfed jets. It is shown in Figure 5 that the response for an underfed jet is higher than that for an overfed jet.

(b) Peripheral jet machines with a central jet can be designed in such a way that the central jet is strong enough to maintain the peripheral jet underfed during the whole cycle of motion. Then, the mass flow coefficients are continuous and unique. In this case, the heaving response becomes relatively larger than that of a machine without a central jet. However, the most important role of the central jet is to serve as a compartmentation device for the purpose of providing pitch stability.

(c) On the basis of the behavior of peripheral jet machines with or without a central jet, central exhaust machines may have continuous mass flow coefficients and a small response; however, the effectiveness of compartmentation, as compared with a central jet, is greatly reduced.

(d) The configuration of double jet machines represents a better approximation to realistic machines. In general, the mass flow coefficients and response are similar to a central jet machine.

REFERENCES

(1) Tulin, M. P., "On the Vertical Motions of Edge Jet Vehicles", Symposium on Ground Effect Phenomena, Princeton Univeristy, p.p. 119-134, October 1959.

(2) Lin, J. D., "Dynamic Motions of Ground Effect Machines", HYDRONAUTICS, Incorporated Technical Report 011-4, in preparation.

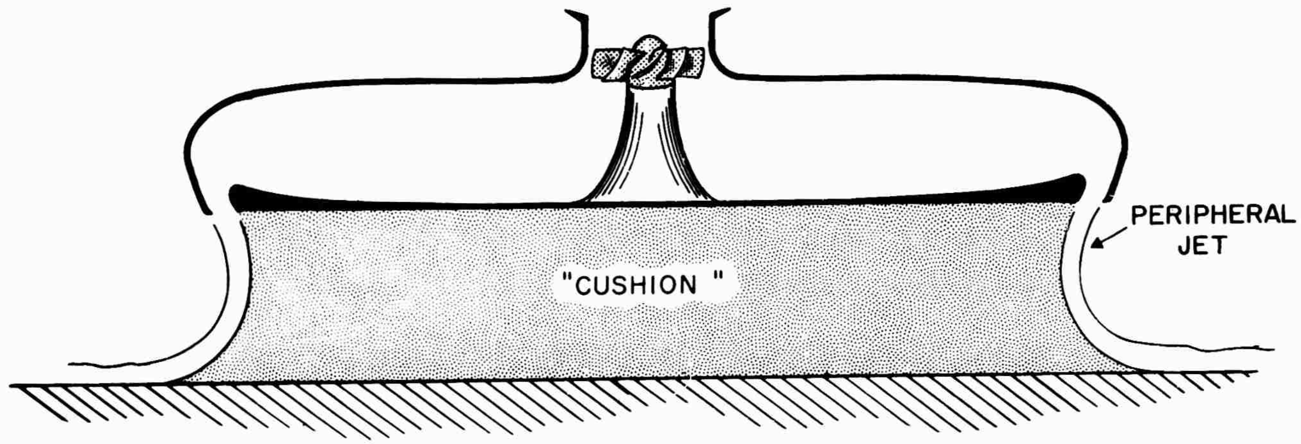


Figure 1- Section of a Typical Peripheral Jet Machine

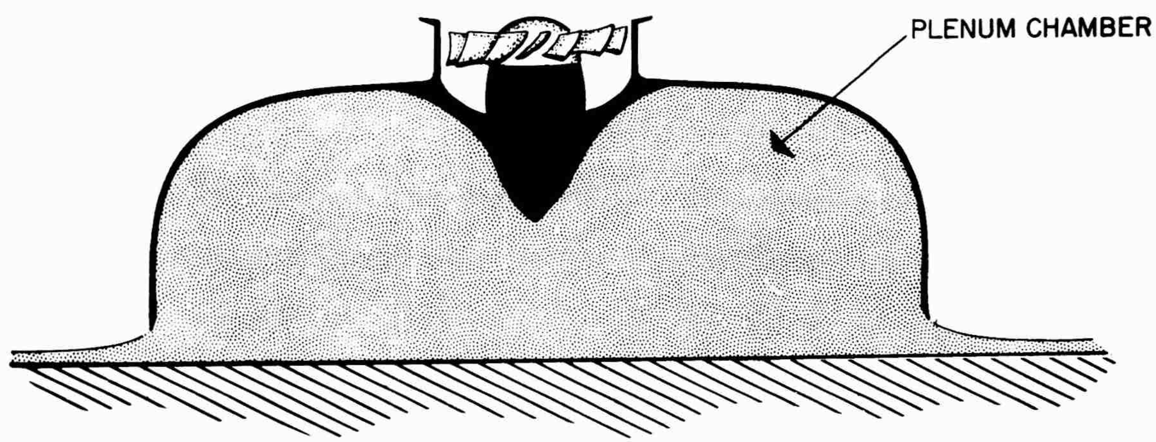


Figure 2- Section of a Plenum Chamber Machine

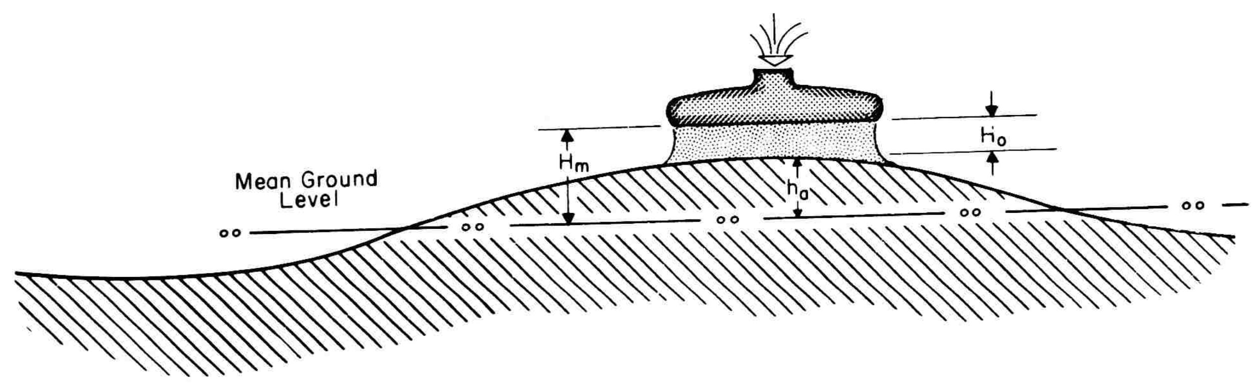
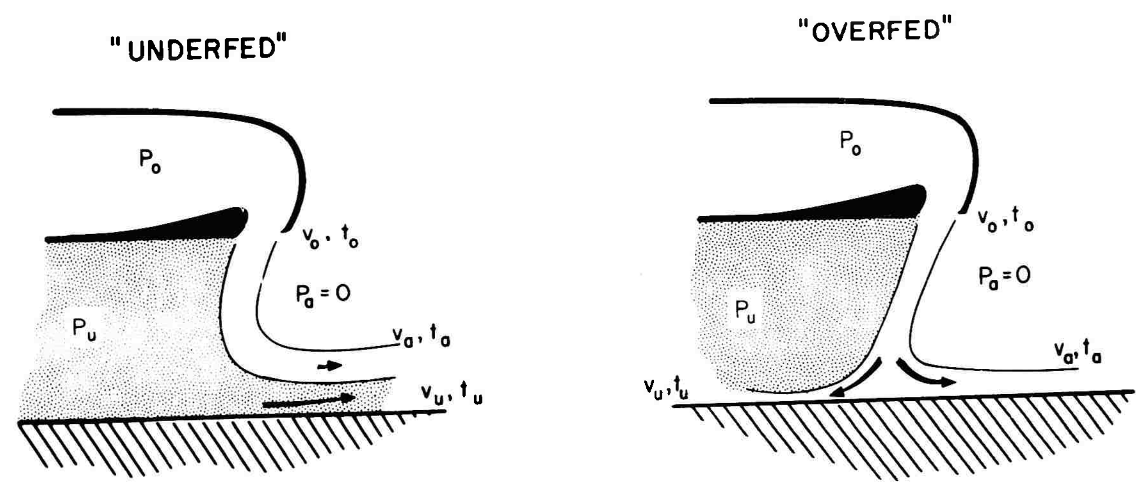


Figure 3- Heaving Motion Over a Sinusoidal Surface



(a)-Underfed Jet

(b)- Overfed Jet

Figure 4- Unbalanced Operations of Peripheral Jet

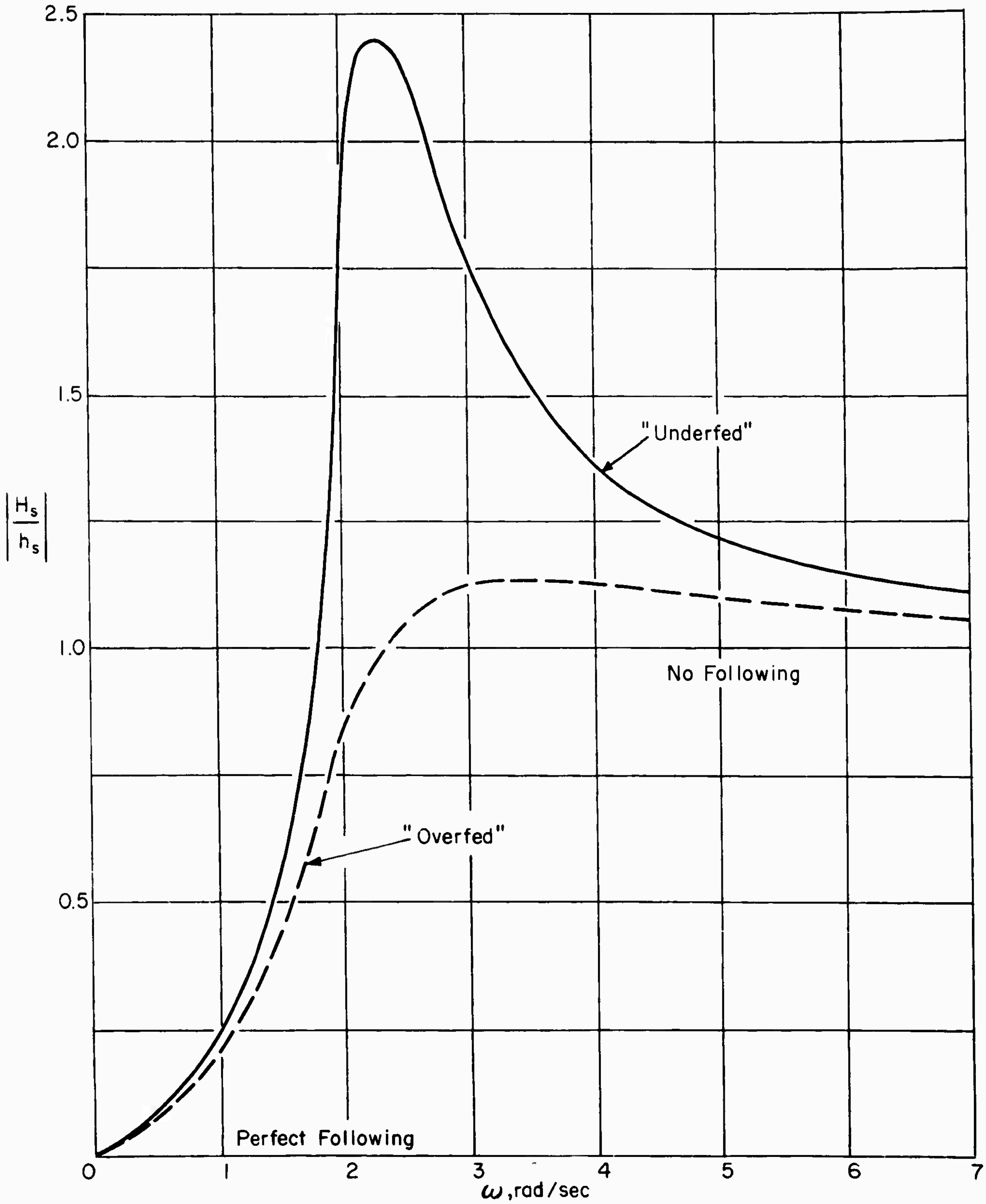


Figure 5- Response Curves of a Peripheral Jet Machine

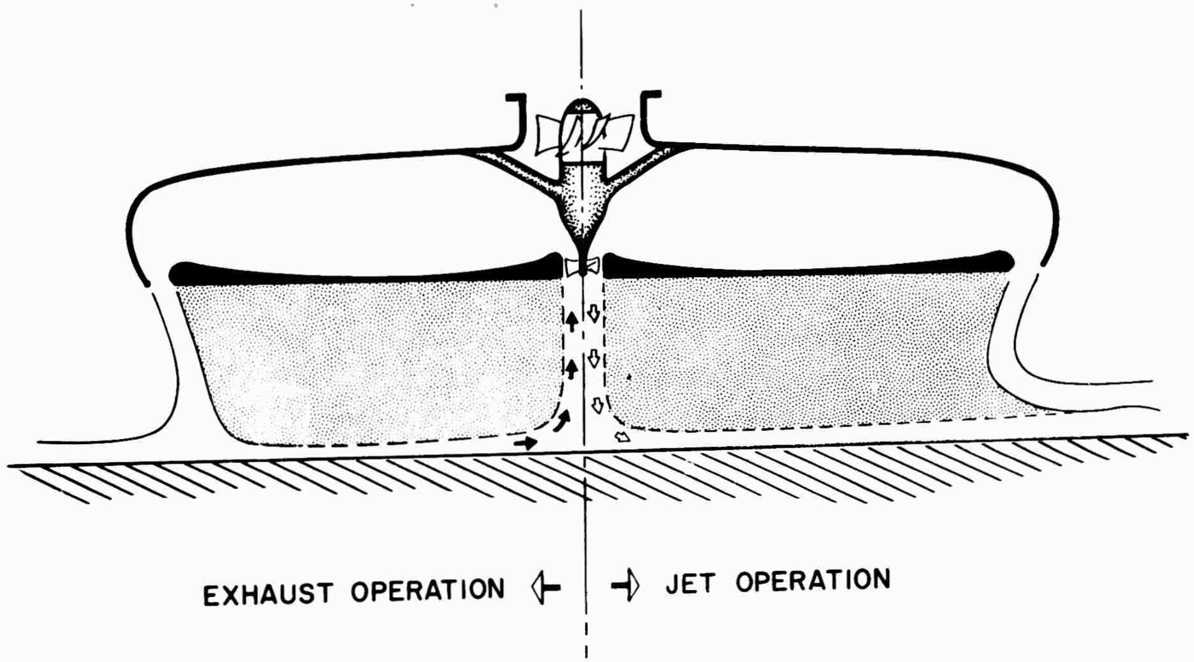


Figure 6- Section of a Typical Central Jet and a Central Exhaust Machines

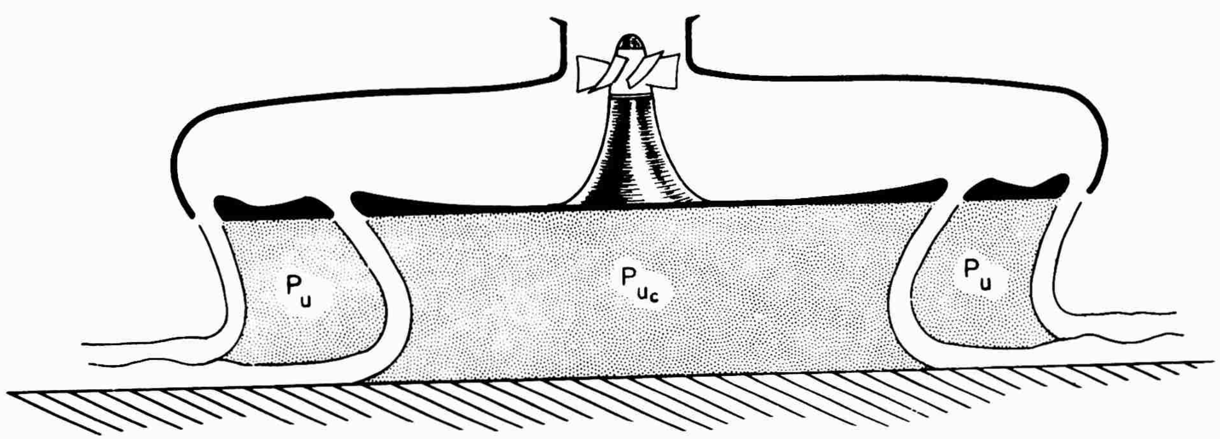


Figure 7- Section of a Typical Double Peripheral Jet Machine

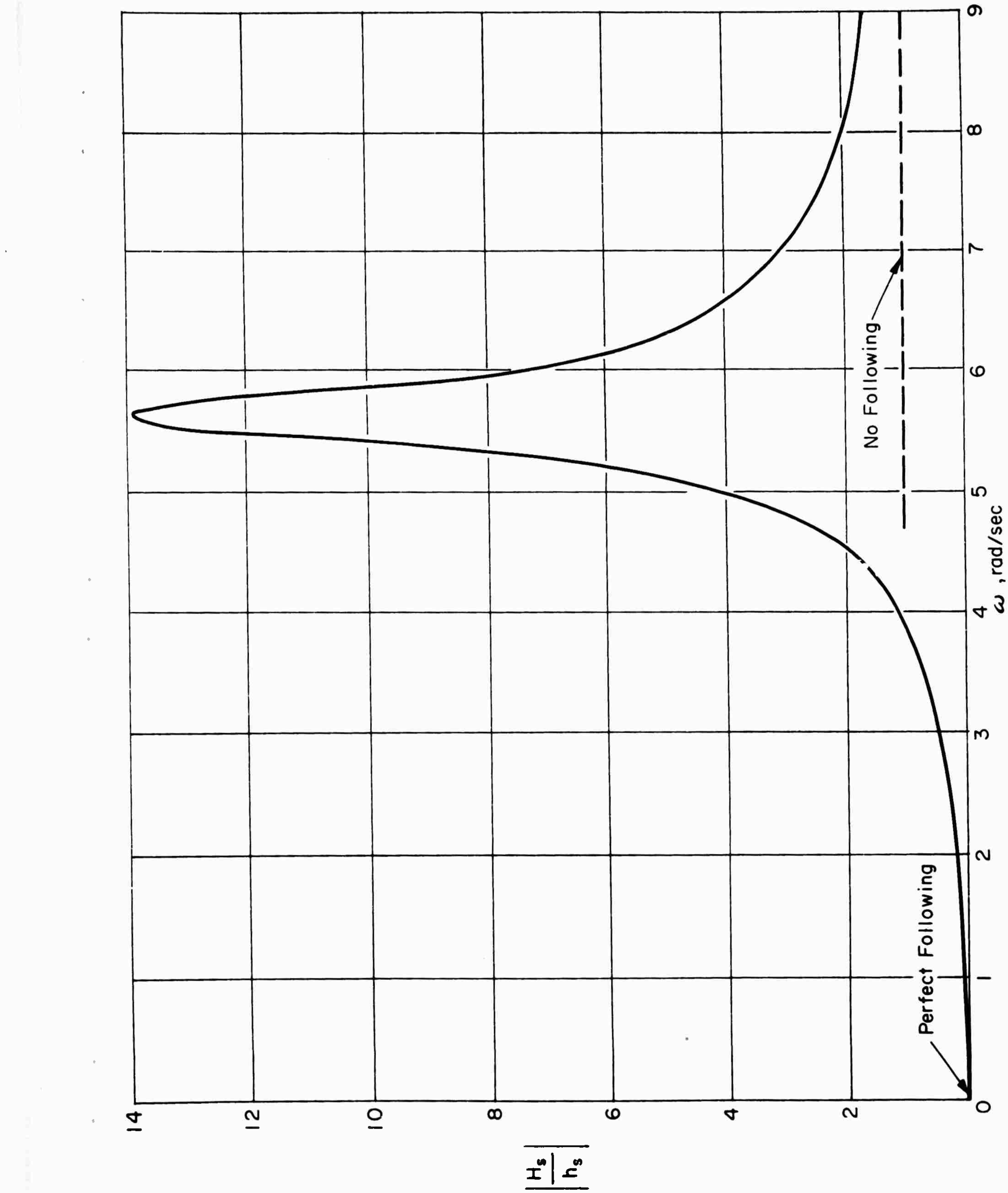


Figure 8 - Response Curve of a Plenum Chamber Machine

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