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METEOROLOGICAL EFFECTS ON AN ACOUSTIC WAVE WITHIN A SOUND RANGING ARRAY

PREPARED BY

Missile Meteorology Division

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NEW MEXICO

MISSILE METEOROLOGY DIVISION

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ABSTRACT

Equations are derived for determining the direction cosines of a plane or spherical wave front by assuming the arrival time at each microphone in an array to be an independent observation and requiring that the sum of the squares of the corrections to the individual recordings be a minimum.

In addition, the effects on the direction cosines and/or the time arrival errors resulting from considering speed of sound variations (profile) with height are also considered.

In particular, five examples were considered for different profiles, none of which were extreme, and resulted in direction angle errors as large as two degrees and apparent time errors on the order of .04 second.

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I N T R O D U C T I O N

Various publications appear in the literature [1, 2, 3] in which methods are presented for determining the direction (azimuth and elevation) of propagation of a sound wave. Only a few attempts are made to correct for timing errors and, in general, no attempt is made to correct for wind and temperature variations within the array. This is probably because of the assumption that the variations due to timing inaccuracies are greater than the meteorological variations. Dean [4] recently presented a least squares method for determining the direction cosines of the wave by minimizing the effects of timing errors.

The purpose of this report is twofold: (1) to present a general derivation of equations that can be used for determining the direction cosines of a plane or spherical wave front when considering the vertical meteorological profile over the array and minimizing the time errors; and (2) to present the results of several examples in which wind and temperature profiles over the array are considered.

D I S C U S S I O N

A least squares solution and a linear solution are derived for determination of the direction cosines of a plane or spherical wave traveling within a sound ranging array where the meteorological parameters vary as a function of height. The solution is based on but not restricted to a square array of four microphones, one at each corner of the array. All computations of examples presented are referred to one of the microphones of the array as a base.

Vertical profiles of meteorological data (in some cases hypothetical and in others actual) were introduced into the equations and the time of arrival of a plane wave front at each microphone was obtained. Using these times and restricted meteorological data (surface data only), an inverse set of direction angles was obtained. The difference between the two sets is considered the error due to neglecting the total vertical profiles. (Examples are presented which reflect the order of magnitude of these errors, but unfortunately no data were available which would reflect extreme errors.)

The observation equations used in the following type of adjustment are based on the assumptions of the so-called "direction" method of geodesy [5]. Each time recording is treated as an independent observation, and the sum of the squares of the corrections to the individual recordings is to be made a minimum. A single time, however, taken by itself determines nothing, for if each of the recorded times be changed by a constant amount, the new set has the same significance as before. The effect is simply a change in zero time, which is purely an arbitrary matter. If a set of time corrections for an array has been determined in any manner and the mean of these corrections is not zero, the sum of the squares of these corrections can always be diminished by subtracting from each correction the mean of all the corrections so that the algebraic sum of the reduced corrections is zero. Hence in any least squares adjustment of the time recordings of an array, the algebraic sum of the time corrections must be zero. To allow for this change of time base or for a constant correction to all recorded times in an array, an unknown constant correction, w, enters into all equations expressing the results of observations at an array.

As is usual in this type of derivation [6], the observation equation will be written

$$t_i + dt_i - t_i^o + w - v_i = 0 \qquad (1)$$

where: t_i^o is the actual (observed) time of recording at the i^{th} station,

t_i is the computed time of arrival at the i^{th} station,

dt_i is the change in recording time at the i^{th} station due to a small change in the direction cosines l, m, n of the wave-front (subject to the condition that $l^2 + m^2 + n^2 = 1$),

w is a constant time correction for all stations of the array,

and v_i is the residual at the i^{th} station. It is the sum of the squares of the v_i's which is to be made a minimum.

Since the origin of time is arbitrary, time can be measured conveniently from any recording, and for this station the arrival time and the calculated arrival time will both be zero. Also for this station, as can be seen later, v = w.

The basic equation connecting time and direction cosines will first be developed for the simplest of all possible cases, that of a plane wave propagated at a constant velocity in a still atmosphere, and later modified to more complicated cases.

If a plane wave (unit normal having direction cosines l, m, n) passes Station F at time $t_f^o = t_f = 0$ and is propagated with a speed c, its calculated time of arrival at Station I will be

$$t_i = \frac{1}{c} (X_i - X_f)l + (Y_i - Y_f)m + (Z_i - Z_f)n + t_f^o \; (t_f^o = 0). \qquad (2)$$

Here X, Y, Z are station coordinates. Since n is a function of l and m ($n = \sqrt{1 - l^2 - m^2}$), substituting in the above equation leads to:

$$t_i = \frac{1}{c} (X_i - X_f)l + (Y_i - Y_f)m \pm (Z_i - Z_f)(1 - l^2 - m^2)^{\frac{1}{2}}. \qquad (3)$$

This is not linear in l and m. Expanding in a Taylor's series about an assumed set of direction cosines (l_o, m_o, n_o), noting that

$$n = n_o - (l_o/n_o)dl - (m_o/n_o)dm,$$

and (just to reduce the number of terms) setting

$$\left.\begin{aligned} (D_{if})_o &= (X_i - X_f)l_o + (Y_i - Y_f)m_o + (Z_i - Z_f)n_o \\ (E_{if})_o &= (X_i - X_f) - (Z_i - Z_f)l_o/n_o \\ (F_{if})_o &= (Y_i - Y_f) - (Z_i - Z_f)m_o/n_o \end{aligned}\right\} \qquad (4),(4a)*$$

gives

$$t_i + dt_i = \frac{1}{c} (D_{if})_o + \frac{1}{c} (E_{if})_o dl + \frac{1}{c} (F_{if})_o dm. \qquad (5)$$

Note that for any given least-squares adjustment $(D_{if})_o$, $(E_{if})_o$, and $(F_{if})_o$ will be numbers depending only on the station coordinates and the assumed direction cosines, but must be recomputed for each iteration. Substituting back into equation (1) gives

$$v_i = w + \frac{1}{c} (E_{if})_o dl + \frac{1}{c} (F_{if})_o dm + \frac{1}{c} (D_{if})_o - t_i^o. \qquad (6)$$

---

* For the set of "a" equations which begin on page 5 , equation (4a) is the same as equation (4).

These equations contain w which is of no importance in itself, being simply a constant correction to all times. Since the sum of the v_i's is to equal zero (N = number of stations)

$$\sum v_i = Nw + \frac{1}{c}\sum (E_{if})_o dl + \frac{1}{c}\sum (F_{if})_o dm + \frac{1}{c}\sum (D_{if})_o - \sum (t_i^o) = 0 \quad (7)$$

$$w = -\frac{1}{Nc}\left[\sum (E_{if})_o dl + \sum (F_{if})_o dm + \sum (D_{if})_o\right] + \frac{1}{N}\sum t_i^o . \quad (8)$$

Substituting (8) into (6) gives the reduced observation equation:

$$v_i = L_i dl + M_i dm + K_i \quad (9)$$

where

$$L_i = \frac{1}{c}[(E_{if})_o - \frac{1}{N}\sum (E_{if})_o]$$

$$M_i = \frac{1}{c}[(F_{if})_o - \frac{1}{N}\sum (F_{if})_o]$$

$$K_i = \frac{1}{c}[(D_{if})_o - \frac{1}{N}\sum (D_{if})_o] - (t_i^o - \frac{1}{N}\sum t_i^o). \quad (10)$$

As before, K_i, L_i, M_i are sets of numbers, one triad for each station, which depend only on the initial conditions and must be recomputed for each iteration. From here on the usual solution scheme is used. In Gaussian notation the normal equations are

$$[LL]dl + [LM]dm = -[LK]$$

$$[LM]dl + [MM]dm = -[MK] \quad (11)$$

and by Cramer's Rule:

$$dl = \frac{[LM]\,[MK] - [LK]\,[MM]}{[LL]\,[MM] - [LM]\,[LM]}$$

$$dm = \frac{[LK]\,[LM] - [MK]\,[LL]}{[LL]\,[MM] - [LM]\,[LM]} . \quad (12)$$

A new set of direction cosines is calculated from

$$l_o' = l_o + dl$$

$$m_o' = m_o + dm$$

$$n_o' = \pm [1 - (l_o')^2 - (m_o')^2]^{\frac{1}{2}}. \quad (13)$$

Equations (4) and (10) are used to set up new observation equations (9) and the process repeated until dl and dm are negligible. Then w is calculated from equation (8) and the v_i's from equation (6).

To derive the equations to take into account a wind of constant velocity, $\bar{W}$, parallel to the earth's surface, assume $W = S_x i + S_y j$. This will reduce the difference in arrival times at Stations I and F by

$$\frac{t_i}{c}(S_x l + S_y m) \quad \text{since } t_f = 0. \quad (1a)$$

Then equation (2) becomes

$$t_i = \frac{1}{c}[(X_i - X_f)l + (Y_i - Y_f)m + (Z_i - Z_f)n] - (\frac{t_i}{c})(S_x l + S_y m). (2a)$$

Solving for t_i and substituting for n, gives

$$t_i = \frac{1}{c + S_x l + S_y m}[(X_i - X_f)l + (Y_i - Y_f)m \pm (Z_i - Z_f)(1 - l^2 - m^2)^{\frac{1}{2}}], (3a)$$

This again must be linearized. Letting

$$C_o = c + S_x l_o + S_y m_o \quad (14),(14a)^*$$

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* For the set "a" equations which begin on this page, equation (14a) is the same as equation (14).

then

$$\left.\frac{\partial t_i}{\partial l}\right)_{l_o,m_o,n_o} = \frac{(E_{if})_o}{C_o} - \frac{(D_{if})_o}{C_o^{\,2}} S_x$$

$$\left.\frac{\partial t_i}{\partial m}\right)_{l_o,m_o,n_o} = \frac{(F_{if})}{C_o} - \frac{(D_{if})_o}{C_o^{\,2}} S_y$$

$$t_i + dt_i = \left[\frac{(E_{if})_o}{C_o} - \frac{(D_{if})_o}{C_o^{\,2}} S_x\right]dl +$$

$$\left[\frac{(F_{if})_o}{C_o} - \frac{(D_{if})_o}{C_o^{\,2}} S_y\right]dm + \left[\frac{(D_{if})_o}{C_o}\right] . \qquad (5a)$$

Letting

$$L_i = \left[\frac{(E_{if})_o}{C_o} - \frac{(D_{if})_o}{C_o^{\,2}} S_x\right] - \frac{1}{N}\sum\left[\frac{(E_{if})_o}{C_o} - \frac{(D_{if})_o}{C_o^{\,2}} S_x\right]$$

$$M_i = \left[\frac{(F_{if})_o}{C_o} - \frac{(D_{if})_o}{C_o^{\,2}} S_y\right] - \frac{1}{N}\sum\left[\frac{(F_{if})_o}{C_o} - \frac{(D_{if})_o}{C_o^{\,2}} S_y\right]$$

$$K_i = \left[\frac{(D_{if})_o}{C_o} - \frac{1}{N}\sum\frac{(D_{if})_o}{C_o}\right] - \left[t_i^o - \frac{1}{N}\sum t_i^o\right] , \qquad (10a)$$

substituting (5a) in equation (1), and eliminating v, we again get equation (9)

$$v_i = L_i dl + M_i dm + K_i .$$

From here on equations (11), (12), and (13) carry on the solution.

The assumption of a constant wind velocity is, in general, unrealistic since it is a function of both space and time. However, whatever the functions S_x and S_y were of time and space, there must exist a $(W^*)_{if} = (S_x^*)_{if}i + (S_y^*)_{if}j$, that constant wind velocity which would have the same effect as the variable wind during the test. Parentheses and subscripts have been added to indicate that these are to be the best estimates of the "average" wind velocity components during the time interval (t_f^o, t_i^o) and over the path the sound ray travelled to Station I. If S_x and S_y are considered to be functions of height (h) only, then, to a first approximation

$$S_x^* = \frac{1}{h} \int_0^h S_x(h)\, dh$$

$$S_y^* = \frac{1}{h} \int_0^h S_y(h)\, dh \,. \tag{15}$$

If it is desired to modify these equations with corrections for wave-front curvature, then information must be obtained regarding the change in direction cosines over the array. The simplest assumption would be that the wave front spreads out from a point in space (X,Y,Z). Then the direction cosines to Station I would be

$$\begin{aligned} \cos\alpha_i &= l_i = (X_i - X)/R_i \\ \cos\beta_i &= m_i = (Y_i - Y)/R_i \\ \cos\gamma_i &= n_i = (Z_i - Z)/R_i \end{aligned} \tag{16}$$

where

$$R_i = [(X - X_i)^2 + (Y - Y_i)^2 + (Z - Z_i)^2]^{\frac{1}{2}} \,. \tag{17}$$

Setting

$$\begin{aligned} l_i &= l_f + \Delta l_{if} \\ m_i &= m_f + \Delta m_{if} \\ n_i &= n_f + \Delta n_{if} \end{aligned} \tag{18}$$

then

$$\begin{aligned} \Delta l_{if} &= \frac{X_i - X}{R_i} - \frac{X_f - X}{R_f} \\ \Delta m_{if} &= \frac{Y_i - Y}{R_i} - \frac{Y_f - Y}{R_f} \\ \Delta n_{if} &= \frac{Z_i - Z}{R_i} - \frac{Z_f - Z}{R_f} \,. \end{aligned} \tag{19}$$

To apply this model it will be necessary to determine either an R or a point X,Y,Z and calculate the $\Delta l_{if}'s$, $\Delta m_{if}'s$, and $\Delta n_{if}'s$. Then equation (4) will be evaluated for each station with respect to the chosen base station. This will result in

$$(D_{if})_{oi} = (X_i - X_f)(l_o + \Delta l_{if}) + (Y_i - Y_f)(m_o + \Delta m_{if}) + (Z_i - Z_f)(n_o + \Delta n_{if})$$

$$(E_{if})_{oi} = (X_i - X_f) - (Z_i - Z_f)(l_o + \Delta l_{if})/n_o + \Delta n_{if}) \quad (4b)$$

$$(F_{if})_{oi} = (Y_i - Y_f) - (Z_i - Z_f)(m_o + \Delta m_{if})/n_o + \Delta n_{if})$$

where l_o, m_o, n_o are now the assumed direction cosines at the base station (Station F).

Then equation (5a) will be changed to

$$t_i + dt_i = \left[\frac{(E_{if})_{oi}}{C_{oi}} - \frac{(D_{if})_{oi}}{C_{oi}^2}(S_x^*)_{if}\right]dl + \left[\frac{(F_{if})_{oi}}{C_{oi}} - \frac{(D_{if})_{oi}}{C_{oi}^2}(S_y^*)_{if}\right]dm + \left[\frac{(D_{if})_{oi}}{C_{oi}}\right] \quad (5b)$$

where

$$C_{oi} = C + (S_x^*)_{if}\,(l_o + \Delta l_{if}) + (S_y^*)_{if}\,(m_o + \Delta m_{if}). \quad (14b)$$

Equations (10a) become

$$L_i = \left[\frac{(E_{if})_{oi}}{C_{oi}} - \frac{(D_{if})_{oi}}{C_{oi}^2}(S_x^*)_{if}\right] - \frac{1}{N}\sum\left[\frac{(E_{if})_{oi}}{C_{oi}} - \frac{(D_{if})_{oi}}{C_{oi}^2}(S_x^*)_{if}\right]$$

$$M_i = \left[\frac{(F_{if})_{oi}}{C_{oi}} - \frac{(D_{if})_{oi}}{C_{oi}^2}(S_y^*)_{if}\right] - \frac{1}{N}\sum\left[\frac{(F_{if})_{oi}}{C_{oi}} - \frac{(D_{if})_{oi}}{C_{oi}^2}(S_x^*)_{if}\right] \quad (10b)$$

$$K_i = \left[\frac{(D_{if})_{oi}}{C_{oi}} - \frac{1}{N}\sum\frac{(D_{if})_{oi}}{C_{oi}}\right] - \left[t_i^o - \frac{1}{N}\sum t_i^o\right],$$

and again substituting (5b) and (10b) into equation (1) we get

$$v_i = L_i dl + M_i dm + K_i . \qquad (9)$$

After solving for dl and dm using equations (11) and (12) we now have

$$l_i = l_o + \Delta l_{if} + dl$$

$$m_i = m_o + \Delta m_{if} + dm$$

$$n_i = \sqrt{1 - l_i^2 - m_i^2} .$$

For i = f this gives

$$l_f = l_o + dl$$

$$m_f = m_o + dm$$

$$n_f = \sqrt{1 - l_f^2 - m_f^2} .$$

These will be the new assumed direction cosines to be put back into equations (4b).

When the iteration has reduced dl and dm to a negligible value, l_f, m_f, and n_f will be the direction cosines of the wave-front at Station F. If the direction cosines at another station are desired, they can be computed using equations (18).

Other assumptions regarding the curvature of the wave-front can be put into these equations by redefining Δl_{if}, Δm_{if}, and Δn_{if} in equations (18) and (19). The form of the equations following will not be changed.

E X A M P L E S

In the presentation that follows it has been assumed that the wave front when at the first microphone is plane, but changes orientation as it passes over the array. This results because the speed of sound over the array is not constant with respect to elevation. Using a modification of equation (2a), t_i is determined for three microphones relative to the first. In turn, these values of t_i along with a "local" speed of sound are used in equation (2a)' below to calculate the direction cosines which are then compared with those initially specified.

Equation (2a) is rewritten as

$$(X_i - X_f)l + (Y_i - Y_f)m + (Z_i - Z_f)n - (C + S_x l + S_y m)t_i = 0 \quad (2a)'$$

where C is the speed of sound due to temperature only as a function of height; Sx and Sy are wind components in the X and Y directions respectively, as a function of height.

Letting

$$C(h) = c_1 + c_2 h$$

$$S_x(h) = a_1 + a_2 h$$

$$S_y(h) = b_1 + b_2 h,$$

(2a)' can be written as

$$lX_i + mY_i + nZ_i - \int_{t_o}^{t_i} \left[c_1 + c_2 h + l(a_1 + a_2 h) + m(b_1 + b_2 h) \right] dt = 0 \quad (2C)$$

where

$$t_o = 0, \text{ and } X_f = Y_f = Z_f = 0.$$

This can be simplified by using the average values of $C(h)$, $S_x(h)$, and $S_y(h)$, i.e.,

$$\tilde{C}(h) = \frac{1}{h}\int_o^h (c_1 + c_2 h)dh = c_1 + \frac{c_2 h}{2}$$

$$\tilde{S}_x(h) = \frac{1}{h}\int_o^h (a_1 + a_2 h)dh = a_1 + \frac{a_2 h}{2}$$

$$\tilde{S}_y(h) = \frac{1}{h}\int_o^h (b_1 + b_2 h)dh = b_1 + \frac{b_2 h}{2},$$

Then equation (20) reduces to

$$lX_i + mY_i + nZ_i - \left[c_i + \frac{\frac{c\,h_i}{2}}{2} + l(a_i + \frac{\frac{a\,h_i}{2}}{2}) + m(b_i + \frac{\frac{b\,h_i}{2}}{2})\right] t_i = 0.$$

Combining like terms, the above reduces to

$$l\left[X_i - t_i(a_i + \frac{\frac{a\,h_i}{2}}{2})\right] + m\left[Y_i - t_i(b_i + \frac{\frac{b\,h_i}{2}}{2})\right] + nZ_i - t_i(c_i + \frac{\frac{c\,h_i}{2}}{2}) = 0. \quad (21)$$

Equation (21) contains four unknowns, l, m, n, and h_i. For a square array of microphones as shown in Figures 1 and 2, h_i can be determined and written as

$$h_i = \frac{d_i}{\operatorname{Ctn}(\gamma - \frac{\pi}{2}) + \operatorname{Ctn}(\pi - \gamma)} \quad (22)$$

where

$d_1 = L \operatorname{Sin} B$ (L is the length of the side of the square)

$d_2 = L \operatorname{Cos} B$

$d_3 = L\sqrt{2} \operatorname{Sin} (\frac{\pi}{4} + B) = L(\operatorname{Cos} B + \operatorname{Sin} B) = d_1 + d_2$

$B = \tan^{-1} (\frac{\alpha}{\beta})$ (Figure 3).

If

angle B = angle A = $\frac{\pi}{4}$, then $d_1 = d_2 = \frac{\sqrt{2}}{2} L$ and $d_3 = L\sqrt{2}$.

Substituting the value of h_i (where i = 1, 2, and 3) into equation (21) reduces it to three unknowns, l, m, and n. Having three microphones to work with will give three equations in three unknowns, i.e.,

$$\begin{aligned} lA_1 + mB_1 + nZ_1 - C_1 &= 0 \\ lA_2 + mB_2 + nZ_2 - C_2 &= 0 \\ lA_3 + mB_3 + nZ_3 - C_3 &= 0 \end{aligned} \quad (23)$$

where

$$A_i = X_i - (a_i + \frac{\frac{a\,h_i}{2}}{2})\, t_i$$

$$B_i = Y_i - (b_i + \frac{\frac{b\,h_i}{2}}{2})\, t_i$$

$$C_i = (c_i + \frac{\frac{c\,h_i}{2}}{2})\, t_i$$

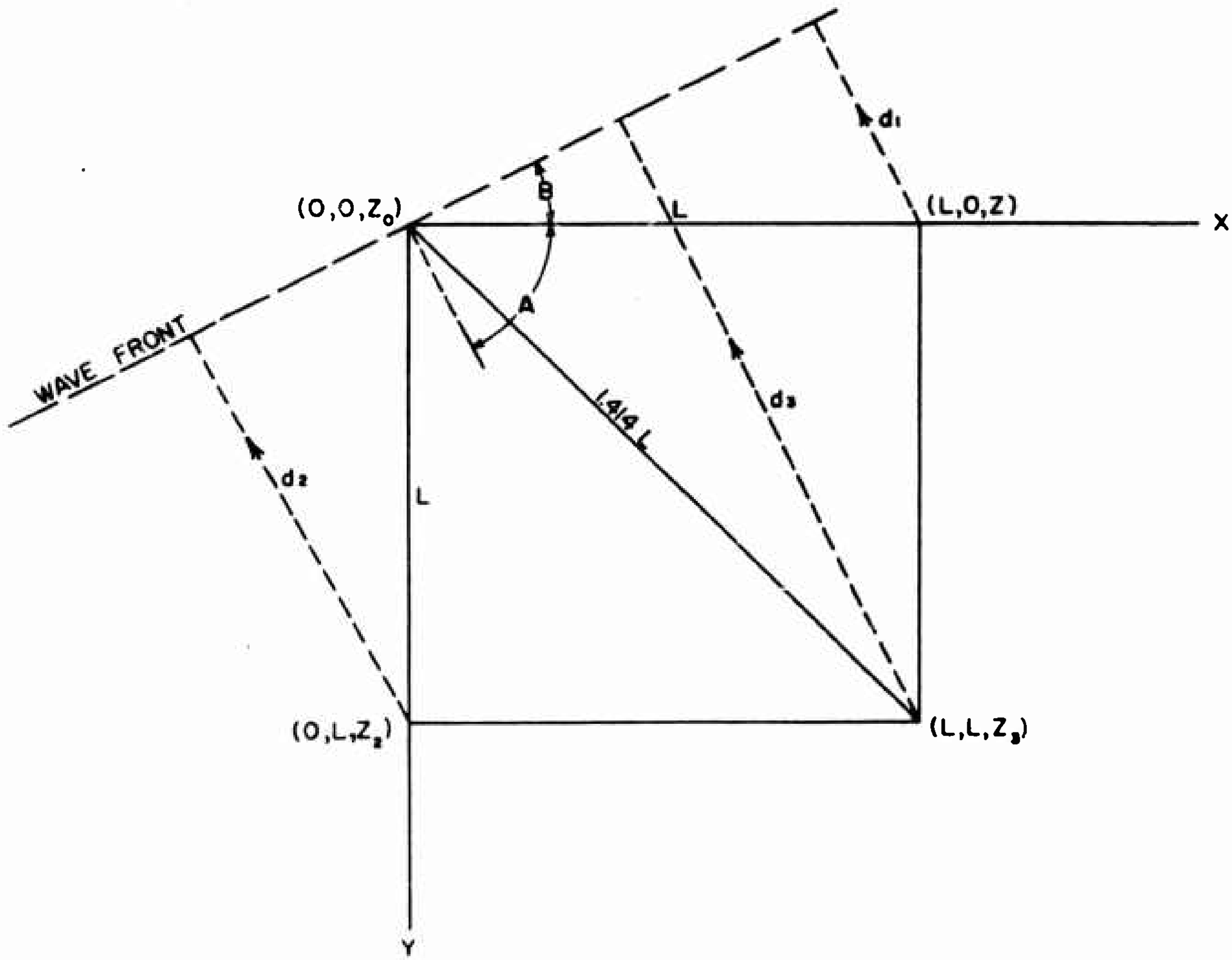

FIG. 1

VIEW FROM ABOVE

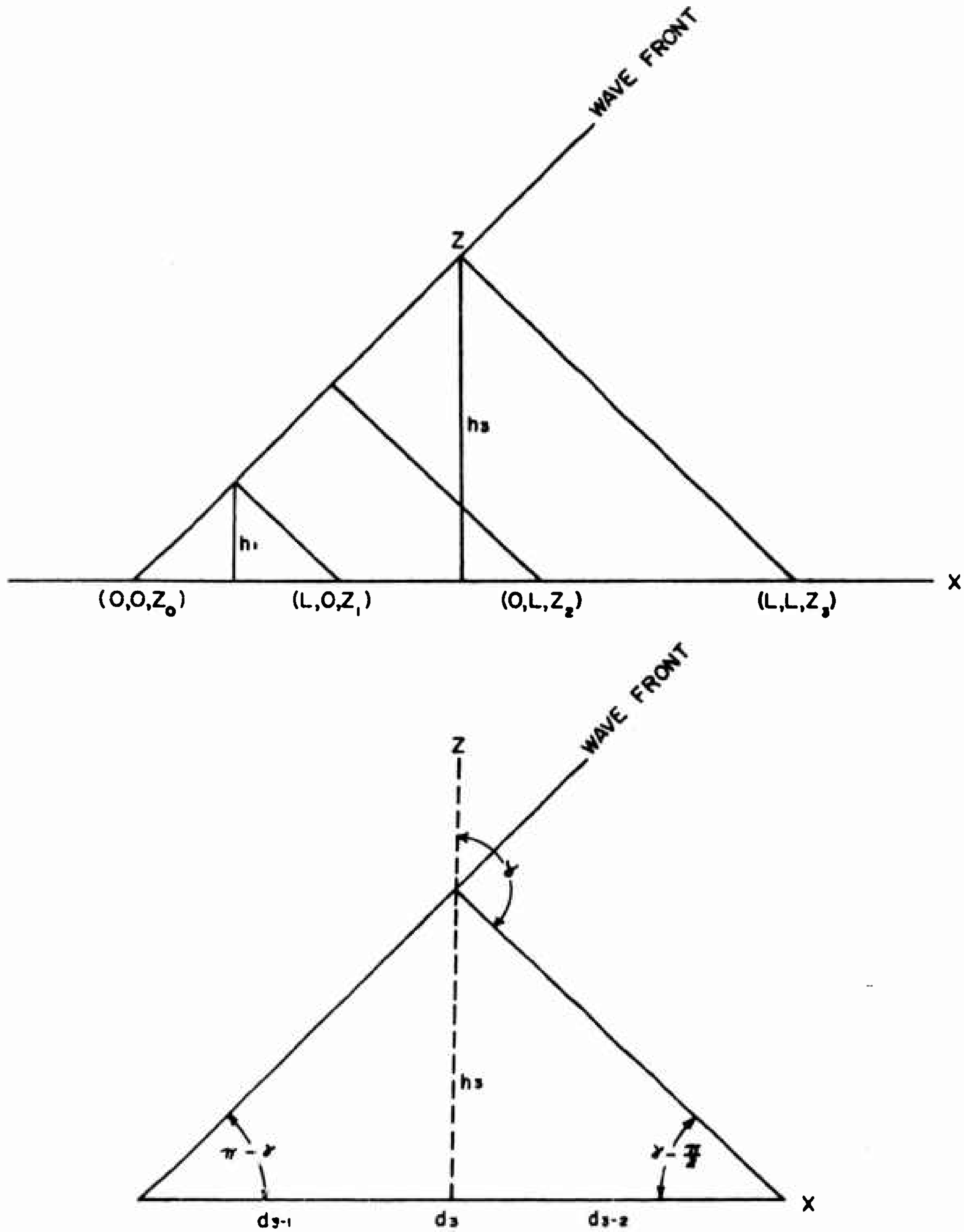
WAVE FRONT
Z
hs
h1
(O,O,Z0)
(L,O,Z1)
(O,L,Z2)
(L,L,Z3)
X
WAVE FRONT
Z
hs
X
ds-1
ds
ds-2

FIG. 2

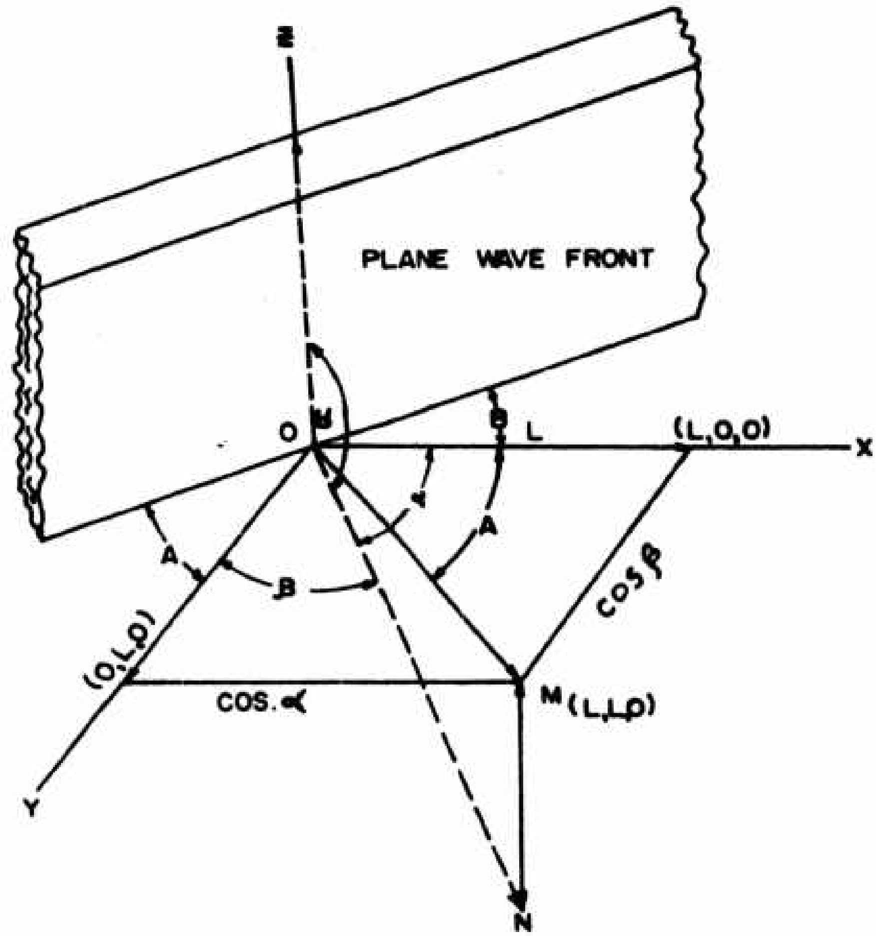

FIG. 3

A is the angle the wave front makes with the y-axis.

B is the angle the wave front makes with the x-axis.

$\text{TAN } A = \frac{\cos\beta}{\cos\alpha}$ or $A = \frac{\pi}{2} - B$

ON is normal to the wave front and is unity.

$\text{TAN } B = \frac{\cos\alpha}{\cos\beta}$ or $B = \frac{\pi}{2} - A$

OM is a projection of the normal onto the xy-plane.

and h_i is equal to equation (22).

Values for l, m, and n may be obtained by a simultaneous solution of equations (23).

One should note that instead of using the linear form, one could use a higher order polynomial, i.e.,

$$C(h) = c_1 + c_2 h + c_3 h^2 + \cdot \cdot \cdot + c_n h^{n-1}$$

$$S_x(h) = a_1 + a_2 h + a_3 h^2 + \cdot \cdot \cdot + a_n h^{n-1}$$

$$S_y(h) = b_1 + b_2 h + b_3 h^2 + \cdot \cdot \cdot + b_n h^{n-1}$$

The use of an n order polynomial may be necessary to represent the meteorological data. A_i, B_i, and C_i above could then be written as

$$A_i = X_i - \left(a_1 + \frac{a_2 h_i}{2} + \frac{a_3 h_i^2}{3} + \cdot \cdot \cdot + \frac{a_n h_i^{n-1}}{n} \right) t_i$$

$$B_i = Y_i - \left(b_1 + \frac{b_2 h_i}{2} + \frac{b_3 h_i^2}{3} + \cdot \cdot \cdot + \frac{b_n h_i^{n-1}}{n} \right) t_i$$

$$C_i = \left(c_1 + \frac{c_2 h_i}{2} + \frac{c_3 h_i^2}{3} + \cdot \cdot \cdot + \frac{c_n h_i^{n-1}}{n} \right) t_i \ .$$

Equation (21) can be written as

$$l[X_i - \left(a_1 + \frac{a_2 h_i}{2} + \cdot \cdot \cdot + \frac{a_n h_i^{n-1}}{n} \right) t_i] +$$

$$m[Y_i - \left(b_1 + \frac{b_2 h_i}{2} + \cdot \cdot \cdot + \frac{b_n h_i^{n-1}}{n} \right) t_i] +$$

$$nZ_i - \left(c_1 + \frac{c_2 h_i}{2} + \cdot \cdot \cdot + \frac{c_n h_i^{n-1}}{n} \right) t_i = 0. \qquad (21.a)$$

CONCLUSIONS

The equations presented in this report are readily adaptable for studies of both plane and spherical waves for different meteorological profiles, but are necessarily more complicated than most conventional methods [3].

Results of five selected examples (see Appendix A) show that errors as large as two degrees occur in the direction angles of a plane wave if one ignores the total vertical meteorological (wind components and temperature) profile, i.e., to a height h_3 as shown in Figure 2, within the array. Possibly even more significant is the finding that when assuming a plane wave and neglecting most of the total vertical profile (only the lowest level meteorological data were used), there were apparent time arrival errors as large as .04 second.

Even more important is the fact that these results do not reflect the extreme errors, since extreme profiles were not considered.

R E F E R E N C E S

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APPENDIX A

Results of five examples are presented in table form.

All columns will be denoted by the number of the column in parenthesis and interpreted as follows:

1. Original selected direction cosines and azimuth of a plane wave.

2. See Figure 1.

3. Heights at which meteorological data are available.

4. East-west components of winds for heights in (3); (East is plus).

5. North-south components of wind for heights in (3); (North is plus).

6. Speed of sound as a function of temperature only for heights in (3).

7. Arrival times calculated by using values in columns (4), (5), and (6) in equation (21.a).

8. Angles: Using t_i of (7) and S_{xo}, S_{yo}, and C_o of (4), (5) and (6) in equation 21.a for obtaining angles.

9. Times: Using α, β, γ of column (1) and S_{xo}, S_{yo}, C_o, from columns (4), (5), (6) in equation 21.a.

10. Time arrival errors: (9) - (7).

11. Direction cosine errors: (8) - (1).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
EXAMPLE I	$\alpha = 45^\circ$ $\beta = 75^\circ$ $\gamma = 131.149^\circ$ $B = 69.896^\circ$	$d_1 = 1{,}408.61'$ $d_2 = 515.69'$ $d_3 = 1{,}924.14'$	$l_0 = 0$ $l_1 = 699.95'$ $l_2 = 255.47'$ $l_3 = 953.42'$	$S_{x_0} = 4.264$ $S_{x_1} = 22.960$ $S_{x_2} = 14.760$ $S_{x_3} = 31.488$	$S_{y_0} = 9.184$ $S_{y_1} = 79.693$ $S_{y_2} = 54.776$ $S_{y_3} = 60.680$	$C_0 = 1124.91$ $C_1 = 1132.86$ $C_2 = 1128.32$ $C_3 = 1131.60$	$t_1 = 0.9093$ $t_2 = 0.3367$ $t_3 = 1.3439$	$\alpha = 46.186^\circ$ $\beta = 75.307^\circ$ $\gamma = 134.268^\circ$ $B = 69.640^\circ$	$t_1 = 0.9386$ $t_2 = 0.3435$ $t_3 = 1.2821$	$\Delta t_1 = 0.0313$ $\Delta t_2 = 0.0068$ $\Delta t_3 = 0.0382$	$\Delta\alpha = 1.086^\circ$ $\Delta\beta = 0.307^\circ$ $\Delta\gamma = 2.087^\circ$ $\Delta B = -0.256^\circ$
EXAMPLE II	$\alpha = 46^\circ$ $\beta = 75^\circ$ $\gamma = 131.149^\circ$ $B = 69.896^\circ$	$d_1 = 1408.61'$ $d_2 = 515.69'$ $d_3 = 1924.19'$	$l_0 = 0$ $l_1 = 699.96'$ $l_2 = 255.47'$ $l_3 = 953.42'$	$S_{x_0} = -0.933$ $S_{x_1} = -9.633$ $S_{x_2} = -13.640$ $S_{x_3} = -11.440$	$S_{y_0} = 2.786$ $S_{y_1} = 22.933$ $S_{y_2} = 13.933$ $S_{y_3} = 34.467$	$C_0 = 1134.92$ $C_1 = 1128.16$ $C_2 = 1131.54$ $C_3 = 1128.16$	$t_1 = 0.9409$ $t_2 = 0.3449$ $t_3 = 1.2721$	$\alpha = 44.600^\circ$ $\beta = 74.872^\circ$ $\gamma = 130.681^\circ$ $B = 69.869^\circ$	$t_1 = 0.9344$ $t_2 = 0.3420$ $t_3 = 1.2764$	$\Delta t_1 = 0.0065$ $\Delta t_2 = 0.0029$ $\Delta t_3 = -0.0045$	$\Delta\alpha = -0.400^\circ$ $\Delta\beta = -0.128^\circ$ $\Delta\gamma = -0.465^\circ$ $\Delta B = -0.025^\circ$
EXAMPLE III	$\alpha = 60^\circ$ $\beta = 60^\circ$ $\gamma = 135^\circ$ $B = 45^\circ$	$d_1 = 1060.66'$ $d_2 = 1060.66'$ $d_3 = 2121.32'$	$l_0 = 0$ $l_1 = 530.33'$ $l_2 = 530.33'$ $l_3 = 606.16'$	$S_{x_0} = -0.733$ $S_{x_1} = -13.640$ $S_{x_2} = -13.640$ $S_{x_3} = -11.440$	$S_{y_0} = 2.989$ $S_{y_1} = 13.933$ $S_{y_2} = 13.933$ $S_{y_3} = 34.467$	$C_0 = 1134.92$ $C_1 = 1131.54$ $C_2 = 1131.54$ $C_3 = 1128.16$	$t_1 = 0.6627$ $t_2 = 0.6627$ $t_3 = 1.3030$	$\alpha = 60.025^\circ$ $\beta = 61.032^\circ$ $\gamma = 135.898^\circ$ $B = 45.898^\circ$	$t_1 = 0.6602$ $t_2 = 0.6602$ $t_3 = 1.3205$	$\Delta t_1 = 0.0025$ $\Delta t_2 = 0.0025$ $\Delta t_3 = -0.0175$	$\Delta\alpha = 0.025^\circ$ $\Delta\beta = 0.025^\circ$ $\Delta\gamma = 0.898^\circ$ $\Delta B = 0.898^\circ$
EXAMPLE IV	$\alpha = 75^\circ$ $\beta = 15.849^\circ$ $\gamma = 95^\circ$ $B = 15.059^\circ$	$d_1 = 389.71'$ $d_2 = 1448.49'$ $d_3 = 1838.20'$	$l_0 = 0$ $l_1 = 33.84'$ $l_2 = 125.76'$ $l_3 = 157.60'$	$S_{x_0} = 0$ $S_{x_1} = 0$ $S_{x_2} = 0$ $S_{x_3} = 0$	$S_{y_0} = 0$ $S_{y_1} = 0$ $S_{y_2} = 0$ $S_{y_3} = 0$	$C_0 = 1116.57$ $C_1 = 1126.96$ $C_2 = 1139.39$ $C_3 = 1142.55$	$t_1 = 0.3446$ $t_2 = 1.2664$ $t_3 = 1.6098$	$\alpha = 74.998$ $\beta = 15.856$ $\gamma = 95.015$ $B = 15.061$	$t_1 = 0.3480$ $t_2 = 1.2985$ $t_3 = 1.6415$	$\Delta t_1 = -0.0034$ $\Delta t_2 = -0.0321$ $\Delta t_3 = -0.0317$	$\Delta\alpha = -0.002^\circ$ $\Delta\beta = 0.007^\circ$ $\Delta\gamma = 0.016^\circ$ $\Delta B = 0.002^\circ$
EXAMPLE V	$\alpha = 46^\circ$ $\beta = 75^\circ$ $\gamma = 131.149^\circ$ $B = 69.896^\circ$	$d_1 = 1408.61'$ $d_2 = 515.69'$ $d_3 = 1924.19'$	$l_0 = 0$ $l_1 = 697.95'$ $l_2 = 255.47'$ $l_3 = 953.42'$	$S_{x_0} = 8.20$ $S_{x_1} = 18.60$ $S_{x_2} = 12.40$ $S_{x_3} = 16.90$	$S_{y_0} = 4.40$ $S_{y_1} = 19.60$ $S_{y_2} = 15.40$ $S_{y_3} = 18.40$	$C_0 = 1129.13$ $C_1 = 1129.79$ $C_2 = 1132.47$ $C_3 = 1129.79$	$t_1 = 0.9239$ $t_2 = 0.3390$ $t_3 = 1.2715$	$\alpha = 44.991^\circ$ $\beta = 75.003^\circ$ $\gamma = 131.140^\circ$ $B = 69.901^\circ$	$t_1 = 0.9360$ $t_2 = 0.3426$ $t_3 = 1.2985$	$\Delta t_1 = -0.0121$ $\Delta t_2 = -0.0036$ $\Delta t_3 = -0.0070$	$\Delta\alpha = -0.009^\circ$ $\Delta\beta = 0.003^\circ$ $\Delta\gamma = 0.009^\circ$ $\Delta B = 0.005^\circ$

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