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Supersonic Linear and Nonlinear Normal Force Characteristics of Wing-Dominated Slender Configurations

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MISSILE & SPACE SYSTEMS DIVISION
DOUGLAS AIRCRAFT COMPANY, INC.
SANTA MONICA/CALIFORNIA

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MISSILE & SPACE SYSTEMS DIVISION
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SANTA MONICA, CALIFORNIA

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MISSILE & SPACE SYSTEMS DIVISION
Douglas Aircraft Company, Inc.

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KX:mma
Enclosure as noted

ABSTRACT

Previous work is refined and extended to fairly arbitrary symmetric configurations with subsonic leading edges. (Ref. 1). ~~In the present report~~ linear and nonlinear normal force characteristics of slender wing body configurations with subsonic leading edges are empirically shown to be functions only of the similarity parameter $\beta A\!\!\!R$, even for rather arbitrary wing geometries. This choice of parameter is discussed in terms of certain limiting cases. By redefining the functional form of the nonlinear normal force in terms of appropriate trigonometric functions of the angle of attack, increased precision of correlation is obtained.

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NOMENCLATURE

Symbol	
$A\!R$	Aspect ratio
b	Gross span
C_z	Normal force coefficient $\frac{N}{q_\infty S}$ where N = Normal force q_∞ = Free stream dynamic pressure
D	Maximum body diameter
ℓ	Characteristic length
M	Mach number
S	Gross wing area
α	Angle of attack
β	$\sqrt{M^2-1}$
τ	Characteristic slope
ω	Delta wing semi-apex angle

Subscripts

∞	Free stream
x	Axial component

1. INTRODUCTION

Correlations of normal force characteristics of delta winged configurations based on the appropriate choice of similarity parameters, in part through the consideration of "limiting" or "analogous" cases, were presented in Ref. 1. These correlations divided into two regimes: Regime I, Subsonic Leading Edges; Regime II, Supersonic Leading Edges. In the present report, Regime I is reconsidered and extended to fairly arbitrary (i.e., nondelta) wing geometries.

In view of the small angle assumptions generally indigenous[*] to hypersonic (Ref. 2) and "generalized" (Ref. 3) similitude, a certain latitude of choice of particular transcendental functions is implied (Ref. 3). Previously, it was shown that the compatibility of limiting cases (shock-expansion and Newtonian theories) dictated a particular choice for the description of the linear and nonlinear normal force behavior of the class of delta winged configurations with supersonic leading edges (Ref. 1). Specifically it was found that the total normal force coefficient should be expressed as

$$C_z = C_{z_\alpha} \frac{\sin 2\alpha}{2} + C_{z_{\alpha/|\alpha|}} \sin^2 \alpha \qquad (1\text{-}1)$$ [**]

The first term on the right hand side in Eq. (1-1) is the conventional "linear" term. The second, or nonlinear, term is, effectively, the "cross drag" term consistent with Allen's analogy (Ref. 4) and Sychev's analysis (Ref. 5).

---

[*] Except cf. Ref. 5

[**] For supersonic leading edges $C_{z_{\alpha/|\alpha|}}$ is a function of α or, more explicitly, of $\beta \tan \alpha$ (Ref. 1). The term $\alpha/|\alpha|$ accounts for the direction of the cross drag force.

The choice of form in Eq. (1-1) is also appropriate to subsonic leading edges. A consistent functional form best characterizes both regimes. This fact was overlooked in Ref. 1.

All of the available[*] experimental data are least squares fit to obtain the coefficients of Eq. (1-1). Configurations and assigned symbols (consistent with Ref. 1) are given in Table I.

2. CHOICE OF THE FUNCTIONAL FORM

From linear theory for delta wings (Ref. 6),

$$\frac{\beta C_z}{\alpha} = \frac{2\pi\beta\omega}{E'(\beta\omega)} \quad , \qquad (2\text{-}1)^{**}$$

or, alternatively,

$$\frac{C_z}{\alpha^2} = \frac{2\pi\beta\omega}{\beta\alpha E'(\beta\omega)} \qquad (2\text{-}2)$$

If, consistent with the Newtonian limits (Refs. 1 and 3), the pressures are associated with $\sin^2\tau$ and the similarity parameters are appropriately $\beta\tan\tau$, Eq. (2-2) becomes

$$\frac{C_z}{\sin^2\alpha} = \frac{2\pi\beta\tan\omega}{\beta\tan\alpha\, E'(\beta\tan\omega)} \qquad (2\text{-}3)$$

which gives

$$\frac{\beta C_z}{4} = \frac{\pi}{2}\,\frac{\sin 2\alpha}{2}\,\frac{\beta\tan\omega}{E'(\beta\tan\omega)} \qquad (2\text{-}4)$$

---

[*] All of the appropriate data of Ref. 1 which were readily available plus additional (nondelta) data were employed.

[**] $E'(\beta\omega)$ is the complete elliptic integral of the second kind with modulus $\sqrt{1-\beta^2\omega^2}$

Eq. (2-4) demonstrates the functional appropriateness of the form of the "linear" term in Eq. (1-1) to the case of subsonic leading edges (Regime I).

In consequence, Eq. (1-1) may be rewritten:

$$C_z = C_{z_\alpha}(\beta \tan \omega)\frac{\sin 2\alpha}{2} + C_{z_{\alpha/\alpha/}}(\beta \tan \omega, \beta \tan \alpha)\sin^2 \alpha \qquad (2\text{-}5)$$

For subsonic leading edges it is empirically demonstrated that $C_{z_{\alpha/\alpha/}}$ is a function of $\beta \tan \omega$ only (Ref. 1).

Using the slender body theory form of the similarity parameters, $\tan \omega$ is replaced by $A\!\!R/4$. Then Eq. (2-5) becomes

$$C_z = C_{z_\alpha}\left(\frac{\beta A\!\!R}{4}\right)\frac{\sin 2\alpha}{2} + C_{z_{\alpha/\alpha/}}\left(\frac{\beta A\!\!R}{4}\right)\sin^2 \alpha \qquad (2\text{-}6)$$

In Fig. 1 and 2 the empirically determined coefficients C_{z_α} and $C_{z_{\alpha/\alpha/}}$ are presented as functions of the similarity parameter $\beta A\!\!R$. The overall correlation is seen to be superior (owing to the functional redefinition) to the correlation previously restricted to delta wings for the nonlinear term, and equivalent for the linear (C_{z_α}) coefficient.

A comparison of the experimental values of the normal-force coefficient and those values predicted by Eq. (1-1) is shown in Fig. 3 for delta wing configurations of aspect ratio 3/8, 2/3, and 1, with a Mach number range of 1.96 to 3.30.

In this case coefficients were not obtained by a least squares fit owing to the lack of "raw" normal force data and the sensitivity of the coefficients to C_z accuracy at small α . Alternatively we could have used Lampert's initial slopes (Ref. 7) and fit $C_{z_{\alpha/\alpha/}}$; the quality of the agreement is, however, self-evident.

3. CHOICE OF THE SIMILARITY PARAMETER $\beta A\!\!R$

From slender body theory,

$$C_{z_\alpha} \cong 2 \quad , \tag{3-1}$$

[*]

based on $\frac{\pi b^2}{4}$ as reference area.

If gross wing area is used as reference area,

$$C_{z_\alpha} = \frac{2}{S} \cdot \frac{\pi b^2}{4} = 2\pi \frac{A\!\!R}{4} \quad , \tag{3-2}$$

so that

$$\frac{\beta C_{z_\alpha}}{4} = \frac{\pi}{2} \frac{\beta A\!\!R}{4} \tag{3-3}$$

for arbitrary configurations according to slender body theory.

From linear theory for delta wings the variation of C_{z_α} with Mach number is obtained:

$$\frac{\beta C_{z_\alpha}}{4} = \frac{\pi}{2} \frac{\beta \tan\omega}{E'(\beta \tan\omega)} = \frac{\pi}{2} \frac{\beta A\!\!R}{4} \frac{1}{E'(\beta A\!\!R/4)} \tag{3-4}$$

Note the consistency of the functional form with slender body theory.

Therefore in the present case the appropriate similarity parameters are obtained to define the "linear" normal force characteristics of arbitrary slender configurations on the basis of slender body theory. Note the

---

[*] From Ref. 8, $C_{z_\alpha} \cong 2$ (to within ~ 5 percent) independent of b/D ; this is the basis for considering normal force characteristics of slender wing body configuration to be geometrically functions of the gross wing alone.

correspondence, in the limit, to linear theory for delta wings. It is demonstrated empirically that the Mach number dependence predicted by linear theory for delta wings applies,* as well, to the "corresponding" arbitrary configurations. Data were taken from Ref. 8.

4. COMPARISON WITH SYCHEV'S FORM

Another limiting case, that of Sychev, (Ref. 5) comes from hypersonic small disturbance theory:

$$
\begin{aligned}
C_z &= \sin^2\alpha \, C_z^*(k_1, k_2) \\
k_1 &= \frac{b}{\ell}\cot\alpha \; ; \quad k_2 = M_\infty \sin\alpha
\end{aligned}
\tag{4-1}
$$

for

$$\frac{M_\infty b}{\ell} \overset{\sim}{>} 1$$

Whereas for the present correlations we have the restriction $\frac{M_\infty b}{\ell} \overset{\sim}{<} 1$, if the axial component of M_∞, $M_x = M_\infty \cos\alpha$ is considered, Sychev's result may be expressed as:

$$C_z = \sin^2\alpha \, C_z^*(k_1^*, k_2^*) \tag{4-2}$$

where

$$k_1^* = \frac{AR}{4}\cot\alpha \; ; \quad k_2^* = M_x \tan\alpha \tag{4-3}$$

Using $k_1^* k_2^*$ and k_2^* as similarity parameters, we obtain

$$
\begin{aligned}
C_z &= \sin^2\alpha \, C_z^*\left(\frac{M_x AR}{4}, \; M_x \tan\alpha\right) \\
&\cong \sin^2\alpha \, C_z^*\left(\frac{\beta_x AR}{4}, \; \beta_x \tan\alpha\right)
\end{aligned}
\tag{4-4}
$$

---

*Actually Lampert's correlation (Ref. 7) is used.

Therefore, if we base the similarity parameters on the axial component of the free stream, Sychev's result is parallel to the present result (the same arguments apply also for the functional form of the linear term, i.e., $C_{z_{lin}} = C_{z_\alpha} \frac{\sin 2\alpha}{2}$).

TABLE I

RANGE OF CONFIGURATIONS CORRELATED

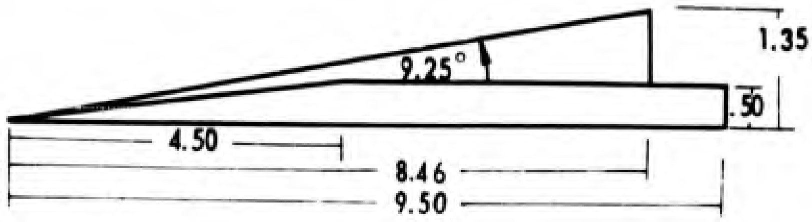
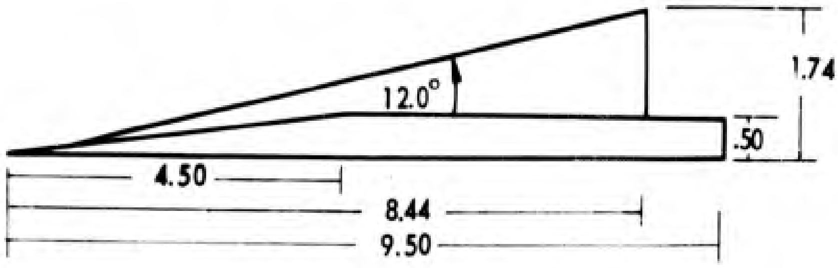
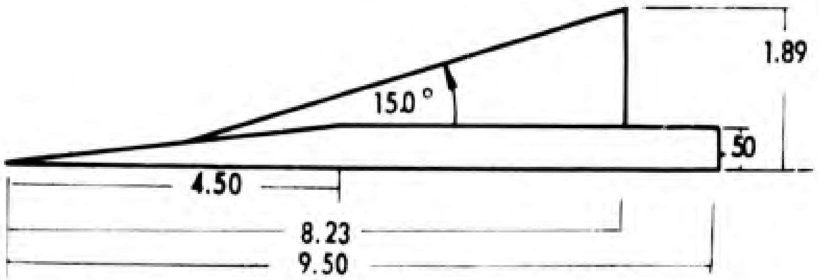
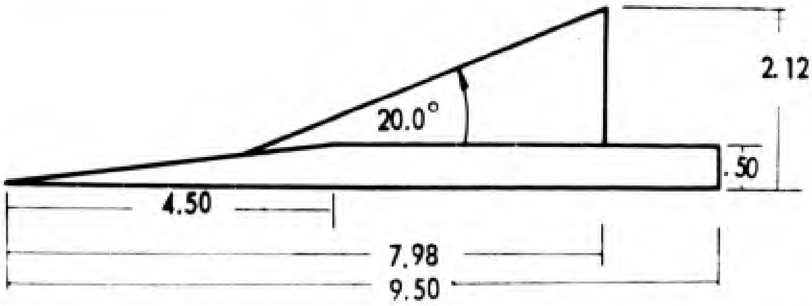
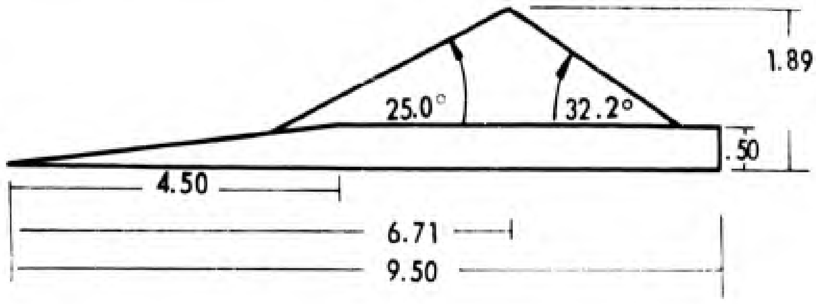
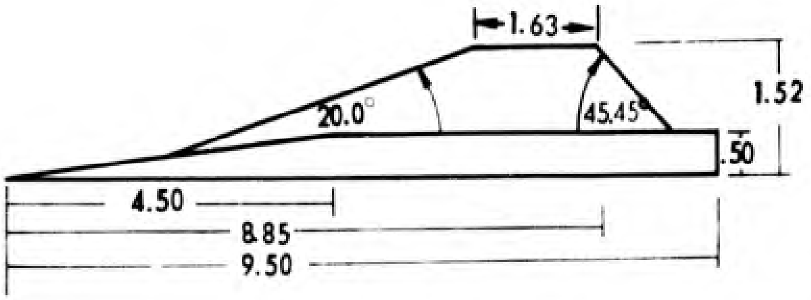
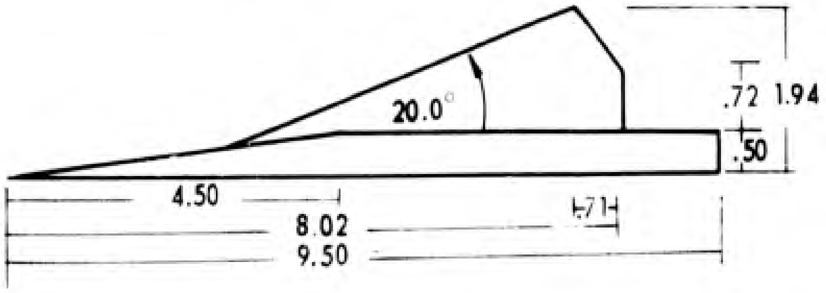
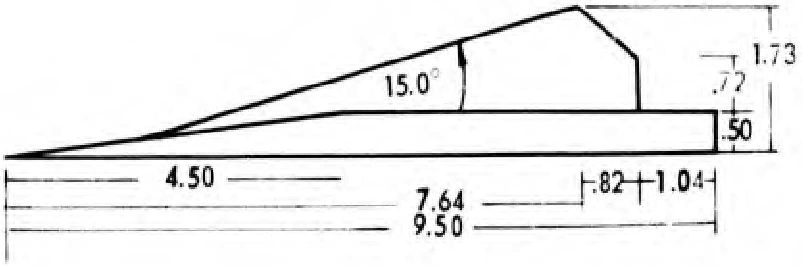
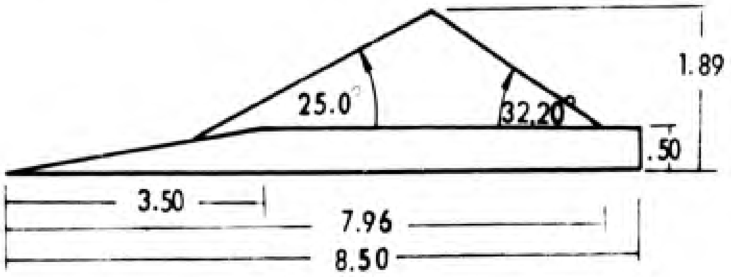
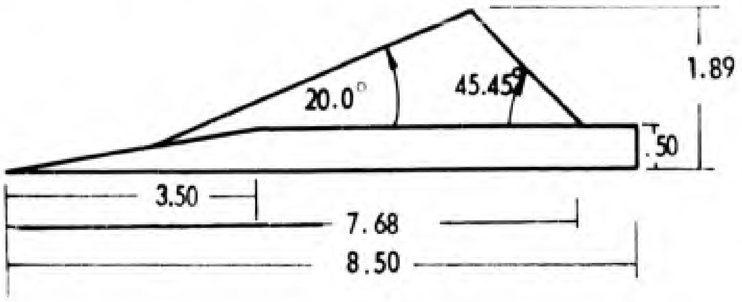
SYMBOL	CONFIGURATION (All dimensions expressed in inches)	ASPECT RATIO
○		0.652
△		0.850
◇		1.072
□		1.456
▽		1.072

TABLE I (CONTINUED)

RANGE OF CONFIGURATIONS CORRELATED

SYMBOL	CONFIGURATION	ASPECT RATIO
▽		0.680
◊		1.197
⌂		0.886
▢		1.072
D		1.067

INITIAL SLOPE NORMAL FORCE CORRELATION FOR SUBSONIC LEADING EDGES

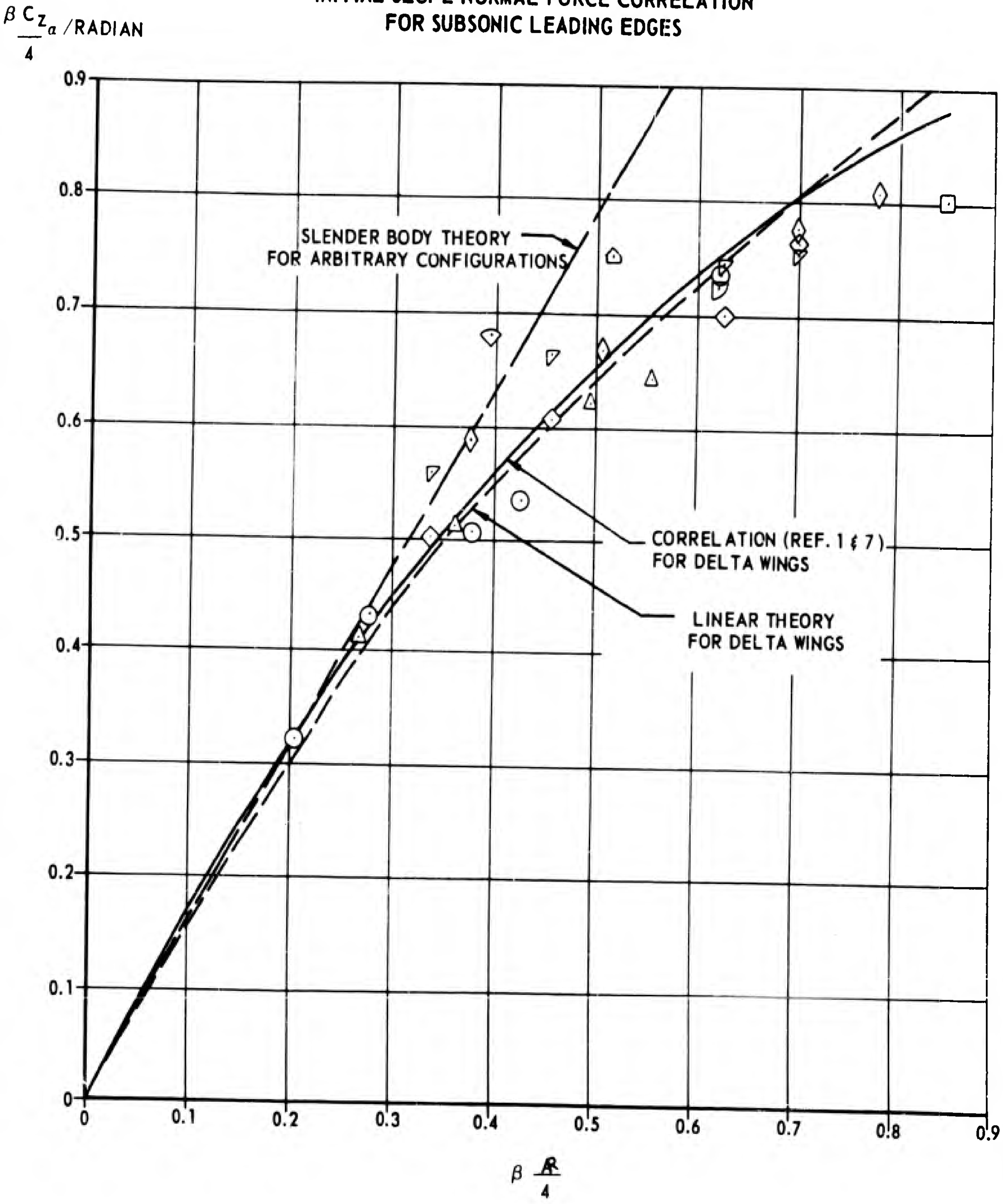


FIGURE 1

NONLINEAR NORMAL FORCE CORRELATION FOR SUBSONIC LEADING EDGES

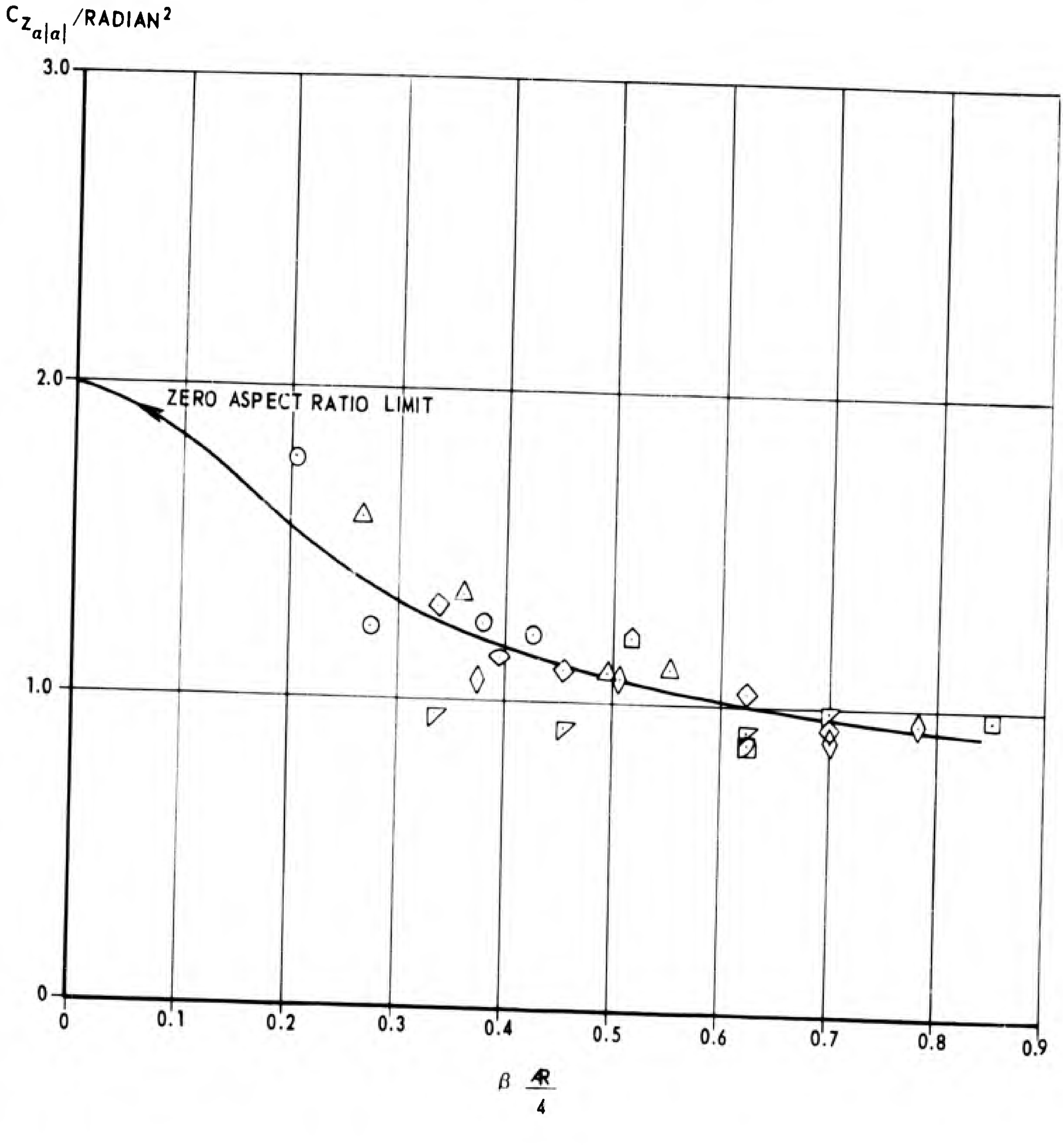


FIGURE 2

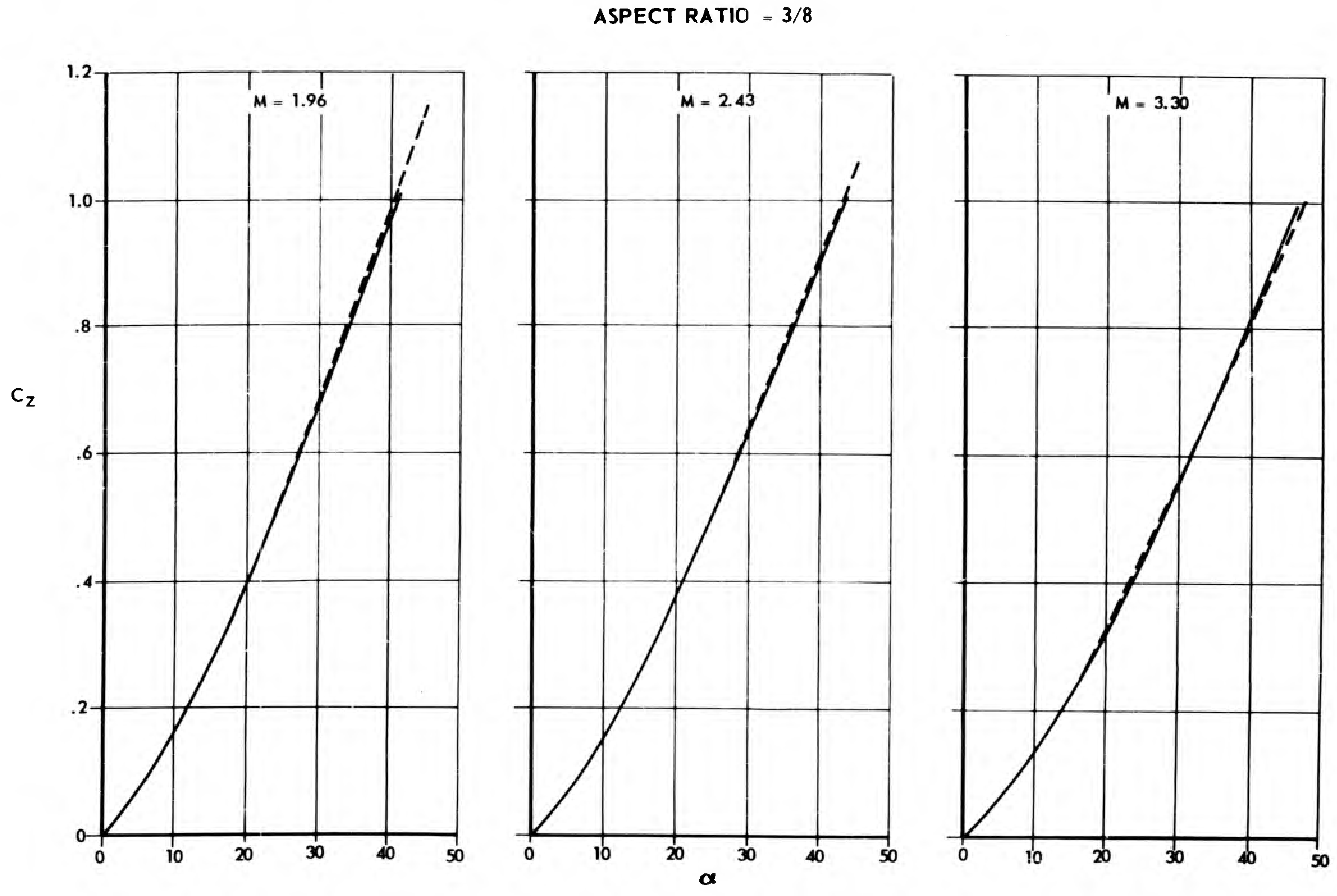

ASPECT RATIO = 3/8
M = 1.96
M = 2.43
M = 3.30
C_Z
1.2
1.0
.8
.6
.4
.2
0
0
10
20
30
40
50
α
---- PRESENT CORRELATION
—— EXPERIMENT (REF.10)

FIGURE 3A

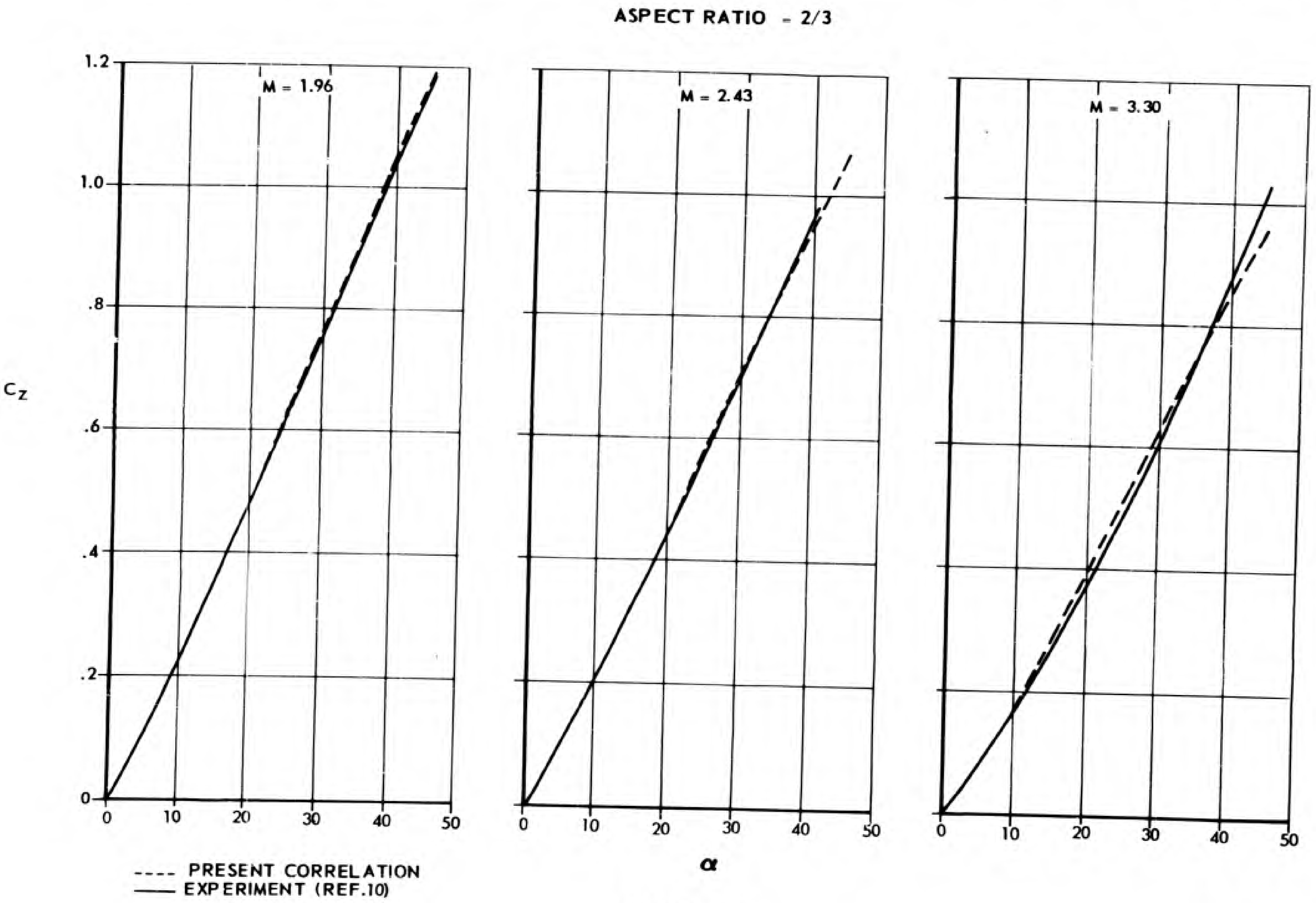

ASPECT RATIO - 2/3
M = 1.96
M = 2.43
M = 3.30
C_Z
1.2
1.0
.8
.6
.4
.2
0
0
10
20
30
40
50
α
---- PRESENT CORRELATION
—— EXPERIMENT (REF.10)

FIGURE 3B

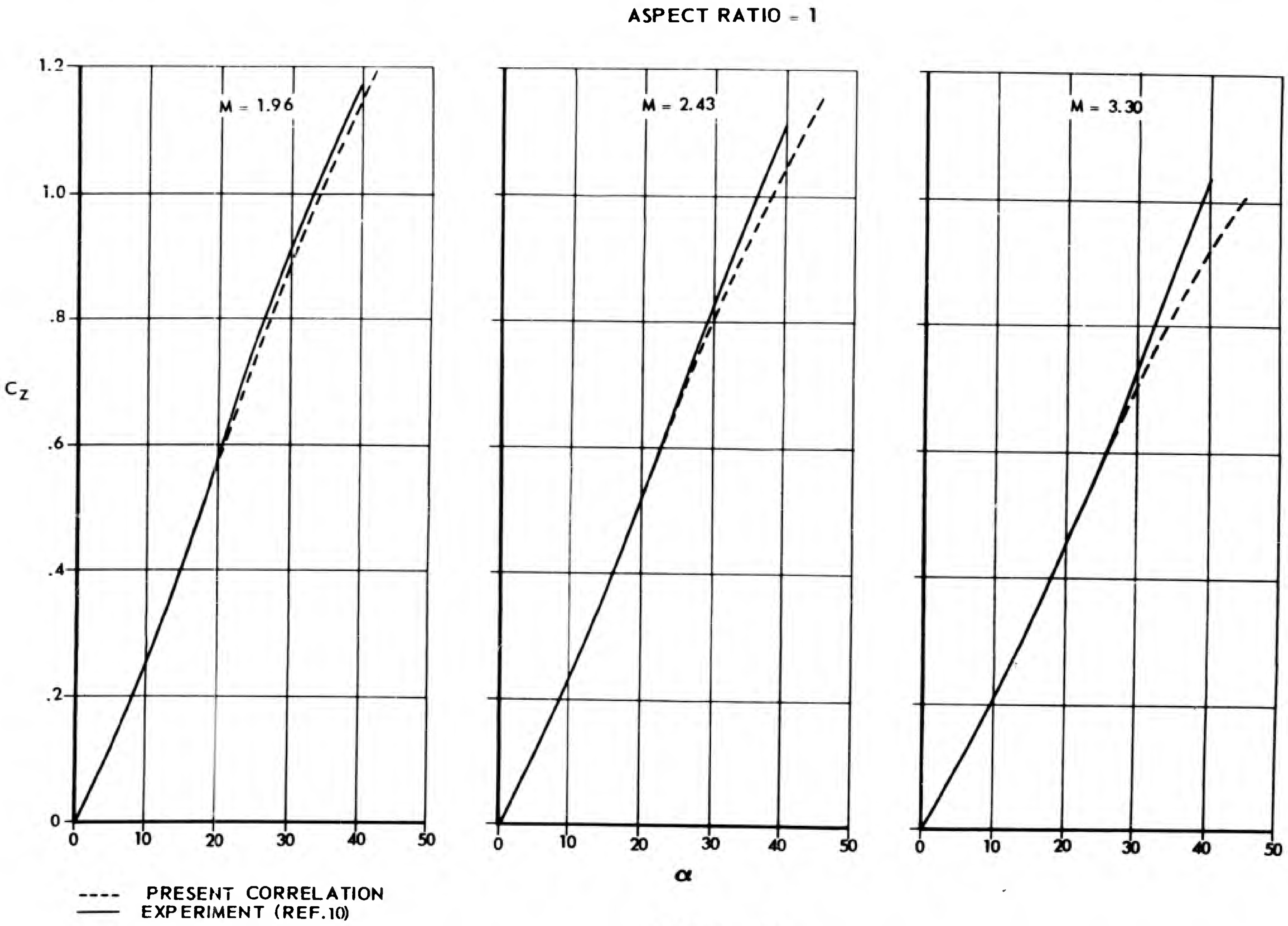

ASPECT RATIO = 1
M = 1.96
M = 2.43
M = 3.30
C_z
1.2
1.0
.8
.6
.4
.2
0
0
10
20
30
40
50
α
---- PRESENT CORRELATION
—— EXPERIMENT (REF.10)

FIGURE 3C

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