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THERMODYNAMIC CALCULATIONS FOR ROCKET ENGINES

By

B. A. Nikolayev

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UNEDITED ROUGH DRAFT TRANSLATION

THERMODYNAMIC CALCULATIONS FOR ROCKET ENGINES

BY: B. A. Nikolayev

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This book presents an approximate method for the calculation of thermodynamic parameters of the products of combustion in a rocket engine, as well as a method for the calculation of the specific thrust, without a determination of the composition of the products of combustion.

The proposed method will make it possible to calculate these parameters with relatively little effort but with a sufficient degree of accuracy from the practical standpoint. This method is valid for the products of combustion of fuels composed of carbon, hydrogen, nitrogen, and oxygen, with some reservations with regard to the method for the latter. Examples of the calculations are presented. In addition, there is a description of a method for the determination of the composition of the products of combustion from the indicated fuels, and this method is simpler with respect to calculation than in the case of the methods generally employed.

The book may be useful for specialists engaged in the area of rocket engineering.

PREFACE

In the design of a flying craft (airplane, rockets) it is necessary to devise the optimum power-plant version. One of the most important problems here is the selection of the fuel on which the engine is to operate, and also the designation of the operating regime for the engine.

The selection of the propellant (fuel, oxidizer, and the ratio between them) as well as the operating regime for the engine (the pressure of the products of combustion within the combustion chamber and at the outlet section of the nozzle) is based on a variety of thermodynamic calculations. The basic purpose of these calculations is to determine the magnitude of specific thrust - the most important characteristic for a rocket engine.

Existing methods of thermodynamic calculations for rocket engines, based on the preliminary calculation of the chemical composition of the products of combustion, are complicated and difficult. It is therefore expedient to use approximate calculation methods which make it possible to arrive at sufficiently accurate results.

This book proposes an approximate method of thermodynamic calculations for rocket engines, and this method requires no preliminary calculation of the composition of the products of combustion, and the present book further considers the application of this method in the cases that are most frequently encountered in practice. The proposed method is valid for the calculation of the operating characteristics for rocket engines employing fuels consisting of carbon, hydrogen, nitrogen, and oxygen. The composition and the characteristics of

such fuels are taken from data published in foreign literature. The accuracy of this data has been checked by the most precise calculation methods for instances in which the pressure of the products of combustion in the combustion chamber ranges from 20 to 100 atm abs, in which the pressure at the outlet section from the nozzle dropped to 0.4-0.5 atm abs, and in which the temperature of the products of combustion attains values of 3800-4000° abs. In this case, the above-mentioned chemical elements are contained in the following quantities:

carbon	from	4 to 40%
hydrogen	from	0.5 to 13%
nitrogen	from	0 to 80%
oxygen	from	15 to 85%

The coefficient of excess oxygen α ranges from 0.55 to 0.95*. This factor is associated with a more conventional parameter - the coefficient of excess oxidizer α_0 in the following function:

$$\alpha_0 = \frac{1}{\nu_0} \frac{\frac{8}{3}C_r + 8H_r - \frac{O_r}{\alpha}}{\frac{O_0}{\alpha} - \frac{8}{3}C_0 - 8H_0}$$

In the calculations that are carried out within the above-recommended limits, the changes in the chemical composition of the fuel, and in the pressure and temperature of the products of combustion, the error in the value of specific thrust does not, in the overwhelming majority of cases, exceed 0.5 to 1.0%.

The error in the value of the temperature for the products of combustion does not exceed 30 to 50°. This statement follows from a series of comparative control calculations performed for instances in which the products of the fuel combustion exhibited sharply divergent elementary chemical composition.

This approximate calculation method for rocket engines is ex-

pediently employed in those cases in which the known methods of calculation require a tremendous amount of calculation work. We have reference here, primarily, to the calculations that must be carried out in order to undertake a comparative analysis of the operating characteristics for engines employing different fuels and regimes. These calculations become necessary in the selection of the most efficient fuels for the new engine, for the designation of the operating regime of the engine, as well as in the development of new types of fuel. With a comparatively small volume of calculations, using a 50-centimeter slide rule, it is possible to obtain answers to all of the questions above with a sufficient degree of accuracy.

If extremely precise values for the thermodynamic parameters of the products of combustion for the indicated rocket fuels are required, and if extremely precise values for specific thrusts are also needed, the proposed method may yet be useful. In this case, it is recommended that the approximate value of these parameters be determined and then conventional methods of successive approximations should be used in order to find the exact solution.

This book also presents a new method of calculating the chemical composition of the products of rocket-fuel combustion. This method is valid for the products of combustion of the same fuels for which it is possible to use the approximate method of thermodynamic calculations for rocket engines which we developed. The proposed method of calculating the chemical composition of the products of combustion from rocket engines is simpler, from the calculation standpoint, than the methods conventionally employed. To facilitate the calculation operations involved in this method, auxiliary nomograms are presented.

[FOOTNOTES]

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- 2 * The ratio of the actual oxygen content in the propellant to the quantity of oxygen required for the complete combustion of the fuel element contained in the propellant is referred to as the coefficient of excess oxygen:

$$a = \frac{A_o}{2A_c + 0.5A_H}$$

TRANSLITERATED SYMBOL

- 2 $C_r, H_r, O_r = C_g, H_g, O_g = C, H, O_{\text{goryucheye}} = C, H, O_{\text{fuel}}$

INTRODUCTION

Specific thrust is, first of all, determined by the propellant used; however, it also depends substantially on the design of the rocket engine as well as the operating regime of the engine. We know that the processes involved in the combustion of the propellant and discharge of the gases - the products of combustion from the combustion chambers of rocket engines - are complex and have not been fully studied. In this connection, we do not yet have rigorous mathematical facilities to calculate the specific thrust in a manner which will enable us to take into consideration the losses which occur in the operations of actual engines.

Thermodynamic calculations are, as a rule, performed without taking into consideration any energy losses with the exception of the losses that are attributable to the dissociation processes taking place within the products of combustion (the decomposition of the products of combustion as a result of the effect that the high temperatures have on the simpler monatomic and biatomic components). The rated (calculated) value of specific thrust, determined without any calculation of losses, will be greater than the true value obtained during hot-firing tests of the engine. However, when the engine is properly designed, the divergence between the rated and actual specific-thrust values is not greater than several percentage points. With some experience, the true specific-thrust value can be found from its rated value by means of a simple recalculation - the multiplication of the rated specific-thrust value by a certain coefficient

less than 1. The magnitude of this coefficient is determined through practical experience.

Thermodynamic calculations for rocket engines are generally carried out with the following assumptions. The mixing of the propellant components prior to combustion is assumed to be ideal, and the combustion of the propellant is assumed to be complete. The heat content of the products of combustion within the combustion chamber of the engine is assumed to be equal to the heat content of the propellant. For the given pressure for the products of combustion within the combustion chamber of the engine, this condition makes it possible to calculate the temperature of the products of combustion.

The process of the expansion of the products of combustion and their discharge from the engine is assumed to take place without friction, without a supply of energy from external sources, and without losses to the surrounding space. Here it is assumed that the components of the products of combustion are in a state of chemical and energy equilibrium. This is equivalent to a statement that the process involved in the expansion of the products of combustion and the discharge of the products of combustion from the rocket engine are subject to the law of isentropy. The entropy of the products of combustion at the outlet of the nozzle is assumed to be equal to the entropy of the products of combustion within the combustion chamber. At the selected pressure for the products of combustion at the outlet section of the nozzle, this latter condition makes it possible to calculate the temperature of the products of combustion in this section and, consequently, to calculate the heat content of the products of combustion.

The kinetic energy of the stream of exhaust gases - the products of combustion - is formed as a result of a reduction in the

heat content of these products. Given ideal combustion and discharge processes, the ideal exhaust velocity is determined from the following equation:

$$w_{s,c} = 91,53 \sqrt{i_t - i_{s,c}}$$

where $i_{v,s}$ is the heat content of the products of combustion at the outlet section of the nozzle, given an isentropic discharge process; i_t is the heat content of the propellant.

The ideal specific thrust in the rated operating regime for the nozzle, in which the pressure of the products of combustion at the outlet section of the nozzle is equal to atmospheric pressure, will be

$$P_{ya} = \frac{w_{s,c}}{g}$$

Further on, we will refer to ideal specific thrust simply as specific thrust.

We have developed approximate mathematical relationships between the heat content and the entropy of the products of combustion, between their pressure and temperature, as well as for the initial elementary chemical composition of the propellant, and these relationships serve as the basis for the proposed engine-design method. These relationships were established in two stages: we first obtained the relationship for undissociated products of combustion, and subsequently for the dissociated products of combustion. By undissociated products of combustion for the propellant under consideration we refer to those products of combustion consisting only of carbon monoxide, carbon dioxide, water vapors, biatomic hydrogen, and biatomic nitrogen. The numerical coefficients in the approximate mathematical expressions have been determined on the basis of the data presented in Table 1 (in the Appendix) for the heat content of the products of combustion.

The heat-content origin may be chosen quite arbitrarily; however, in establishing the heat content of individual components of the products of combustion it is necessary to take into consideration the possibility of chemical reactions between these products. Therefore, for all of the substances participating in the process (the propellant components and the products of combustion), a uniform reference system must be adopted. A special feature of the heat-content reference system used in this book is found in the fact that the heat content of carbon dioxide, water vapors, biatomic hydrogen, and biatomic nitrogen at 0° abs, is assumed to be equal to zero. Since the thermal energy of the remaining components of the products of combustion at 0° abs is also equal to zero, the chemical energy of these components at 0° abs is defined as their heat of formation from CO_2 , H_2O , O_2 , and N_2 , referred to a temperature of absolute zero.

In using the proposed calculation method for rocket engines, we must present the heat content of the propellant in the same heat-content reference system which was adopted for the heat content of the products of combustion. If we have data on the heat content of the propellant components, said data calculated on the basis of another reference system, these data must then be referred to the reference system adopted in this book. The method used in the calculation of the heat content of the propellant components and the propellant as a whole is presented at the end of the book, in Section 20.

[TRANSLITERATED SYMBOLS]

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$i_{B.C} = i_{v.s} = i_{vykhodnoye\ secheniye} = i_{outlet\ section}$

7

$i_T = i_t = i_{toplivo} = i_{propellant}$

7

$P_{yD} = P_{ud} = P_{udel'naya\ (tyaga)} = P_{specific\ (thrust)}$

ADOPTED DENOTATIONS

- α_0 - coefficient of excess oxidizer;
- α - coefficient of excess oxygen;
- C_g, H_g, N_g, O_g - content of chemical elements, by weight, in 1 kg of fuel, expressed in g;
- C_o, H_o, N_o, O_o - content of chemical elements, by weight, in 1 kg of oxidizer, expressed in g;
- q_g, q_o - mass fraction of fuel and oxidizer in 1 kg of propellant;
- v_o - theoretical relationship between components (quantity of mass units of oxidizer necessary for complete oxidation of one mass unit of fuel);
- v - actual relationship between components (quantity of mass units of oxidizer referred to one mass unit of fuel, for a given α_0);
- A_C, A_H, A_N, A_O - number of gram-atoms for carbon, hydrogen, nitrogen, and oxygen, respectively, in 1 kg of propellant;
- M^0, M - number of gram-molecules in 1 kg of undissociated and dissociated products of combustion, respectively;
- μ_k^0, μ_k - average ("apparent") molecular weight of undissociated and dissociated products of combustion, respectively;

$$x_C = x_C^0; x_H = x_H^0;$$

$$x_N = x_N^0; x_O = x_O^0$$

- parameters characterizing the composition of the propellant;
- Q_{sg} - heat of combustion of substance (determined from handbooks);
- Q_{obr} - heat of formation of substance (determined from handbooks);
- p^0, p - pressure of undissociated and dissociated products of combustion, respectively in atm abs;
- T_0 - temperature of propellant combustion, with no dissociation of products of combustion, in $^{\circ}\text{abs}$;
- i_t, i_g, i_o - heat content of 1 kg of propellant, fuel (combustible), and oxidizer, respectively, in kcal/kg;
- $T_{k.s}, T_{v.s}$ - temperature of products of combustion in combustion chamber of the engine and at outlet section of nozzle, respectively in $^{\circ}\text{abs}$;
- $S_{k.s}, S_{v.s}$ - entropy of products of combustion in combustion chamber of engine and outlet section of nozzle, respectively, in kcal/kg deg;
- i_B^0, i_B - heat content of undissociated and dissociated products of combustion, respectively, in temperature zone "B" ($2800-3800^{\circ}\text{abs}$) in kcal/kg;

- $i_{H'}^0, i_H^0$ - heat content of undissociated and dissociated products of combustion, respectively, in temperature zone "H" (1400-2800° abs) in kcal/kg;
- $c_{p,B}^0, c_{p,H}^0$ - heat capacity of undissociated products of combustion in temperature zones "B" and "H," respectively, in kcal/kg deg;
- $p_{H_2O}^0$ - partial water-vapor pressure in dissociated products of combustion, at a temperature of 2800° abs, in atm abs;
- i_{2800}^0 - heat content of 1 kg of undissociated products of combustion at a temperature of 2800° abs, in kcal/kg;
- i_v^0 - the parameter characterizing the heat content of undissociated products of combustion at a temperature of 2800° abs;
- $i_{T_{k.s.}}, i_{v.s.}$ - heat content of 1 kg of products of combustion in combustion chamber of engine (at temperature $T_{k.s.}$) and at the outlet section of the nozzle, respectively, in kcal/kg;
- v^0, v - specific volume of undissociated and dissociated products of combustion, respectively, in m³/kg;
- Δi - increment in heat content of 1 kg of products of combustion, as a result of their dissociation, in kcal/kg;
- $\Delta i_B^{(40)}$ - increment in heat content for 1 kg of prod-

- ucts of combustion, as a result of their dissociation in the temperature zone "B" at a pressure of 40 atm abs, in kcal/kg;
- $\Delta i_H^{(1)}$ - increment in heat content for 1 kg of products of combustion as a result of their dissociation in temperature zone "H" at a pressure of 1 atm abs, in kcal/kg;
- $\Delta i_{B,v}^{(40)}$ - parameter characterizing increment in heat content as a result of the dissociation of the products of combustion in the temperature zone "B" at a pressure $p = 40$ atm abs;
- $\Delta i_{H,v}^{(1)}$ - parameter characterizing increment in heat content as a result of the dissociation of the products of combustion in the temperature zone of "H" at a pressure $p = 1$ atm abs;
- $\Delta i_{B,v}^{(3400)}$ - parameter characterizing increment in heat content as a result of the dissociation of products of combustion in temperature zone "B" at a temperature of 3400° abs;
- $\Delta i_{H,v}^{(2800)}$ - parameter characterizing increment in heat content as a result of the dissociation of products of combustion in temperature zone "H" at a temperature of 2800° abs;
- Δi_B - increment in heat content as a result of the dissociation of 1 kg of products of combustion in the temperature zone "B" at temperature T and pressure p , in kcal/kg;

Δi_H - increment in heat content as a result of dissociation of 1 kg of combustion products in temperature zone "H" at temperature T and pressure p, in kcal/kg;

$\Delta i_{B,v,f}$ - parameter characterizing the increment in heat content of the products of combustion as a result of dissociation at fixed temperatures $T_{B,f} = 3400^\circ$ abs, and a pressure $p_{B,f} = 40$ atm abs;

$\Delta i_{H,v,f}$ - parameter characterizing increment in heat content for products of combustion as a result of dissociation at fixed temperature $T_{B,f} = 2800^\circ$ abs, and pressure $p_{H,f} = 1$ atm abs;

m_B, m_H - exponents in formulas describing parameters associated with dissociation as functions of the temperature of the products of combustion (in zones "B" and "H" respectively);

n_{ef} - effective exponent in formulas describing parameters associated with dissociation as functions of the pressure of the products of combustion;

q - ratio between increment in heat content for products of combustion as a result of dissociation to increments in number of gram-molecules;

ω - relative increment in pressure of products

of combustion as a result of dissociation taking place in a constant volume of products of combustion;

S_B, S_H - entropy of products of combustion in temperature zones "B" and "H," respectively, in kcal/kg.deg.;

$w_{v.s.}$ - ideal exhaust velocity for products of combustion from engine nozzle, in m/sec;

P_{ud} - specific engine thrust, in kg sec/kg;

k - adiabatic exponent for discharge;

B_1, B_2 - parameters characterizing intensity of dissociation reaction;

n_0, n - ratio of partial pressure of water vapors to partial pressure of biatomic hydrogen in undissociated and dissociated products of combustion, respectively.

[LIST OF TRANSLITERATED SYMBOLS]

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11	$Q_{cr} = Q_{sg} = Q_{sgoraniye} = Q_{combustion}$
11	$Q_{obp} = Q_{obr} = Q_{obrazovaniye} = Q_{formation}$
11	$T_{k.c} = T_{k.s} = T_{kamera\ sgoraniya} = T_{combustion\ chamber}$
14	$\Delta i_{B,v,\varnothing} = \Delta i_{B,v,f} = \Delta i_{B,v,fiksirovannaya} = \Delta i_{B,v,fixed}$
14	$n_{e\varnothing} = n_{ef} = n_{effektivnyy} = n_{effective}$

Chapter 1

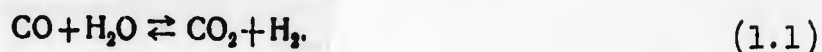
THERMODYNAMIC PARAMETERS OF UNDISSOCIATED PRODUCTS OF COMBUSTION

1. Undissociated Products of Combustion

In the majority of cases, products of combustion in rocket engines are dissociated in view of the high temperatures. The composition of the dissociated products of combustion depends not only on the initial chemical composition of the propellant and the temperature of the products of combustion, but it depends on pressure as well. If we assume that the pressure of the products of combustion increases without limit, the composition of the products of combustion will tend toward a fully defined limit. The products of combustion for the propellants under consideration, in this case, consist of water vapors (H_2O), biatomic hydrogen (H_2), carbon dioxide (CO_2), carbon monoxide (CO), and biatomic nitrogen (N_2), (if we do not take into consideration the oxidation reaction of the nitrogen).

The presence of one or another gas in a unit weight of products of combustion is determined only by the initial chemical composition of the propellant and the temperature of the products of combustion. Under conditions of unlimited great pressure, all reactions involving an increase in the number of moles (i. e., volume) are suppressed. In this case, chemical reactions involving no change in the number of moles are possible, and as a result the magnitude of pressure has no effect. The most characteristic reaction of this type, taking place in the products of combustion of the propellants under consideration, is

the reaction resulting in the formation of water gas



The relative content of the components of the products of combustion will, in the case of a change in temperature, change in accordance with this reaction. The equilibrium equation of the reaction (1.1), expressed in terms of the partial pressure of the products of combustion, takes the following form:

$$K = \frac{p_{\text{CO}_2}^0 p_{\text{H}_2}^0}{p_{\text{CO}}^0 p_{\text{H}_2\text{O}}^0} = f(T) \quad (1.2)$$

The numerical value of the equilibrium constant depends only on temperature and for the given reaction diminishes with an increase in the latter. In this case, the content of water vapors and carbon monoxide increases within the products of combustion as a result of the reduced content of carbon dioxide and biatomic hydrogen. The biatomic-nitrogen content remains constant with a change in temperature. The superscript "0" indicates that the given parameter refers to the products of combustion whose chemical composition corresponds to an infinitely great pressure.

While in calculating the composition of the products of combustion at any finite pressure we may conventionally assume that they consist only of H_2O , H_2 , CO_2 , CO , and N_2 , the quantity of the components of the products of combustion and their relative content at a given temperature, will be independent of the magnitude of pressure and will be precisely the same as in the case of infinitely great pressure.

The products of combustion, whose calculation was carried out under the above-indicated conditions, will, in the future, be referred to as undissociated products of combustion. For uniformity in handling the data of this book, the content of the components of undissociated products of combustion in Eq. (1.2) and later on will be written in

terms of their partial pressure. The relationship between the number of gram-molecules of a certain i th component M_i^0 of the products of combustion and its partial pressure p_i^0 can be found from the known relationship:

$$\frac{p_i^0}{p^0} = \frac{M_i^0}{M^0} \quad (1.3)$$

Since

$$M^0 = \frac{1000}{\mu_k^0}, \quad (1.4)$$

where μ_k^0 is the "apparent" (average) molecular rate of the mixture of the products of combustion; therefore, denoting

$$\mu^0 = \frac{\mu_k^0}{1000}, \quad (1.5)$$

we will finally obtain

$$p_i^0 = p^0 \mu^0 M_i^0. \quad (1.6)$$

The value of the parameter μ^0 is calculated in terms of the elementary chemical composition of the propellant in the following manner. In burning 1 gram-atom of carbon, 1 gram-molecule of carbon dioxide and carbon monoxide are formed (total); in the combustion of 1 gram-atom of hydrogen, 0.5 gram-molecules (total) of water vapor and biatomic hydrogen are formed; the combustion of 1 gram-atom of nitrogen forms 0.5 gram-molecules of biatomic nitrogen. Consequently, in the combustion of 1 kg of propellant in which there are, respectively, $A_C, A_H,$ and A_N gram atoms of carbon, hydrogen, and nitrogen, a gram-molecule of the products of combustion is formed;

$$M^0 = A_C + 0.5(A_H + A_N). \quad (1.7)$$

If we examine Expressions (1.4), (1.5), and (1.7) simultaneously, we will obtain

$$\mu^0 = \frac{1}{A_C + 0.5(A_H + A_N)}. \quad (1.8)$$

The parameter μ^0 is fully determined by the composition of the propellant and remains constant with changes in the pressure and temperature of the products of combustion.

To calculate the temperature of the undissociated products of

combustion it is necessary to devise a system of algebraic equations, and here the number of equations must be equal to the number of unknowns. There are a total of six unknowns, of which five are the partial pressures of the products of combustion making up the gaseous mixture, and the temperature of combustion. Consequently, the system must consist of no less than six equations.

The equation of the equilibrium constants for the reaction resulting in the formation of water gas is used as one of the equations. Further on, four material-balance equations are employed in the solution (the number of material-balance equations is equal to the number of chemical elements entering into the composition of the propellant).

The principle involved in the derivation of the material-balance equations can be seen with the example of the chemical element oxygen. There are A_0 gram-atoms of oxygen in one kilogram of propellant. The products of combustion - carbon dioxide, carbon monoxide, and water vapors - also contain oxygen. There are two gram-atoms of oxygen in one gram-molecule of CO_2 ; there is one gram-atom of oxygen in one gram-molecule of CO ; there is one gram-atom of oxygen in one gram-molecule of H_2O . On the basis of the law of the conservation of matter, the number of gram-atoms of oxygen is identical both for one kilogram of propellant as well as for one kilogram of the products of combustion; consequently,

$$A_0 = 2M_{\text{CO}_2}^0 + M_{\text{CO}}^0 + M_{\text{H}_2\text{O}}^0. \quad (1.9)$$

Let us transform Expression (1.9). We will multiply the right- and left-hand parts by $p^0 \mu^0$ and we will designate with X_0^0 the product of the parameters of the left-hand part:

$$X_0^0 = p^0 \mu^0 A_0. \quad (1.10)$$

Taking into consideration (1.6), we finally obtain

$$X_O^0 = 2p_{CO_2}^0 + p_{CO}^0 + p_{H_2O}^0 \quad (1.11)$$

For the three remaining chemical elements, the material-balance equations are obtained in similar fashion:

$$X_C^0 = p_{CO_2}^0 + p_{CO}^0 \quad (1.12)$$

$$X_H^0 = 2p_{H_2O}^0 + 2p_{H_2}^0 \quad (1.13)$$

$$X_N^0 = 2p_{N_2}^0 \quad (1.14)$$

Here, in analogy with the parameter X_O^0 , the symbols X_C^0 , X_H^0 , and X_N^0 denote:

$$X_C^0 = p^0 \mu^0 A_C \quad (1.15)$$

$$X_H^0 = p^0 \mu^0 A_H \quad (1.16)$$

$$X_N^0 = p^0 \mu^0 A_N \quad (1.17)$$

The last, the sixth, equation of the system under consideration will be the equation expressing the equality between the heat content of a weight unit of propellant and the heat content of the products of combustion. If we assume that there are no heat losses in the combustion of the fuel, we may write

$$i_T = i_T^0$$

Here $i_{T_0}^0$ is the heat content of 1 kg of undissociated products of combustion, at a combustion temperature T_0 .

In this case; if we know the composition of the products of combustion, the heat content $i_{T_0}^0$ may be determined in accordance with the following equation:

$$i_T^0 = \frac{1}{p^0 \mu^0} (I_{CO_2} p_{CO_2}^0 + I_{CO} p_{CO}^0 + I_{H_2} p_{H_2}^0 + I_{H_2O} p_{H_2O}^0 + I_{N_2} p_{N_2}^0) \quad (1.18a)$$

where I_{CO_2} , I_{CO} , and similar denotations, here and further on, indicate the heat-content values for the components at a given temperature, in kcal/g-mole, in accordance with Table 1 of the Appendix.

We will assemble the derived equations into a uniform system:

$$K = \frac{p_{CO_2}^0 p_{H_2}^0}{p_{CO}^0 p_{H_2O}^0} = f(T); \quad (1.19)$$

$$X_O^0 = 2p_{CO_2}^0 + p_{CO}^0 + p_{H_2O}^0;$$

$$\begin{aligned}
 X_C^0 &= p_{CO}^0 + p_{CO_2}^0; \\
 X_H^0 &= 2p_{H_2O}^0 + 2p_{H_2}^0; \\
 X_N^0 &= 2p_{N_2}^0; \\
 i_T &= i_T^0.
 \end{aligned}
 \tag{1.19}$$

The most efficient method of solving this system of equations consists in the following. We must assume a series of values for the temperatures, close to the anticipated combustion temperature T_0 , and for each chosen temperature value we must find the partial pressure for the products of combustion, as well as the heat content of their mixture (in terms of Eq. (1.18a)).

To check on the correctness of the temperatures selected, we use Eq. (1.18), which will be satisfied only if the selected temperature equals T_0 . To speed up the calculations, an auxiliary curve representing the heat content of the products of combustion as a function of temperature is generally constructed, and the sought temperature T_0 is determined from this curve.

The solution of the system of equations (1.19) for a fixed temperature makes it possible to reduce this system to a quadratic equation for the partial pressure of any of the products of combustion (with the exception of biatomic nitrogen). The partial pressure of biatomic nitrogen is determined directly from Eq. (1.14). For purposes of better coordination with the data discussed in subsequent chapters, we will solve the quadratic equation for the parameter $n_0 = p_{H_2O}^0 / p_{H_2}^0$:

$$a_1 n_0^2 + b_1 n_0 + c_1 = 0, \tag{1.20}$$

where

$$\begin{aligned}
 a_1 &= K(2X_C^0 - X_O^0 + 0.5X_H^0); \\
 b_1 &= K(2X_C^0 - X_O^0) + X_C^0 - X_O^0 + 0.5X_H^0; \\
 c_1 &= X_C^0 - X_O^0.
 \end{aligned}$$

The partial pressures of the components of the products of combustion will be equal to:

$$\begin{aligned}
 p_{H_2}^0 &= \frac{0,5X_H^0}{n_0 + 1}; \\
 p_{H_2O}^0 &= 0,5X_H^0 - p_{H_2}^0; \\
 p_{CO}^0 &= \frac{X_C^0}{1 + Kn_0}; \\
 p_{CO_2}^0 &= X_C^0 - p_{CO}^0.
 \end{aligned}$$

The numerical values of the equilibrium constants K , at the chosen temperatures, are determined in accordance with Table 2 of the Appendix.

The relative content of the components of undissociated products of combustion is independent of the magnitude of pressure. With a change in pressure p^0 , the partial pressures of the gases - the products of combustion - change in proportion to this change, so that their total remains equal to p^0 . The relative partial pressures of the products of combustion, or their molar fractions [see Formula (1.3)], do not change in this case. Dividing the right- and left-hand parts of Eq. (1.19) by p^0 and denoting the relative partial pressure of the i th component by $p_i^0 = p_i^0/p^0$, we will obtain

$$\left. \begin{aligned}
 K &= \frac{\bar{p}_{CO_2}^0 \bar{p}_{H_2}^0}{\bar{p}_{CO}^0 \bar{p}_{H_2O}^0}; \\
 \mu^0 A_O &= 2\bar{p}_{CO}^0 + \bar{p}_{CO}^0 + \bar{p}_{H_2O}^0; \\
 \mu^0 A_C &= \bar{p}_{CO}^0 + \bar{p}_{CO}^0; \\
 \mu^0 A_H &= 2\bar{p}_{H_2O}^0 + 2\bar{p}_{H_2}^0; \\
 \mu^0 A_N &= 2\bar{p}_N^0.
 \end{aligned} \right\} \quad (1.21)$$

From an examination of the system of equations (1.21) we may come to the conclusion that the relative partial pressures of undissociated products of combustion are independent of the magnitude of the total pressure of p^0 of these products. The relative partial pressure of biatomic nitrogen is independent both of the pressure and the temperature of the products of combustion.

At comparatively low temperatures for the products of combustion (below 1800° abs), the processes of dissociation are, for all intents

and purposes, nonexistent. The composition of the products of combustion changes with a change in temperature only in accordance with the laws of equilibrium governing the reaction resulting in the formation of water gas. Consequently, Eq. (1.20) is fully applicable for the calculation of the actual composition of the process of combustion, at temperatures not exceeding 1800° abs.

2. Heat Content of Undissociated Products of Combustion as a Function of their Temperature

The heat content of 1 kg of undissociated products of combustion from the propellants under consideration, in the general case, is determined according to Eq. (1.18a). We will add to and subtract from the right-hand portion of Eq. (1.18,a) the following terms

$$\frac{I_{CO} p_{CO}^0}{p^0 \mu^0} \quad \text{and} \quad \frac{I_{H_2O} p_{H_2O}^0}{p^0 \mu^0}.$$

Further, after the simultaneous examination of Eqs. (1.11), (1.12), (1.10), and (1.15), we will obtain

$$p_{CO}^0 = p^0 \mu^0 (A_O - A_C) - p_{H_2O}^0. \quad (1.22)$$

And, finally, using (1.10), (1.12) - (1.16), (1.18,a), and (1.22), we will find .

$$\begin{aligned} i^0 = & -(I_{CO} - I_{CO_2})(A_O - A_C) + \frac{1}{\mu^0} (I_{CO} - I_{CO_2} - I_{H_2} + \\ & + I_{H_2O}) \frac{p_{H_2O}^0}{p^0} + I_{CO} A_C + 0,5 I_{H_2} A_H + 0,5 I_{N_2} A_N. \end{aligned} \quad (1.23)$$

It is the purpose of the present section to find an approximate, but sufficiently accurate, relationship between the heat content and temperature of the undissociated products of combustion in the case of a constant elementary chemical composition of the fuel (at constant values of A_O , A_C , A_H , A_N , and μ^0).

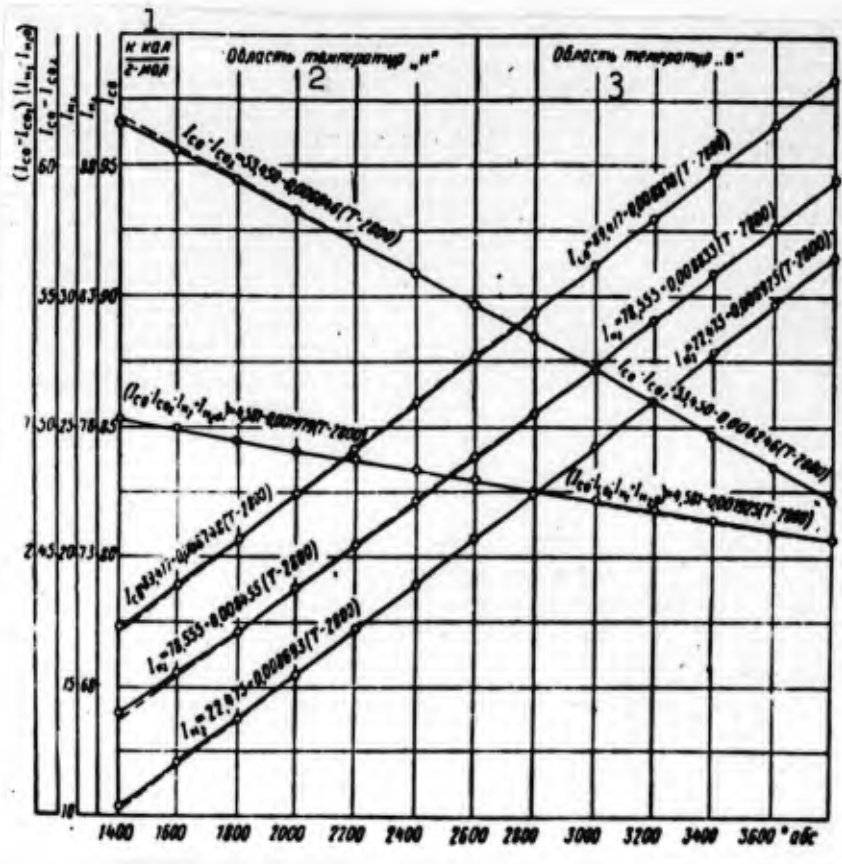


Fig. 1. 1) kcal/g-mole; 2) temperature re-
 tion 'H'; 3) temperature region "B".

This relationship is defined as the relationship between temperature and heat contents or the algebraic sums of the heat contents of the following components of the products of combustion:

$$\left. \begin{aligned}
 &I_{CO} - I_{CO,i} \\
 &I_{CO} - I_{CO,i} - I_{H_2} + I_{H_2O} \\
 &I_{CO,i} \\
 &I_{H_2} \\
 &I_{N_2}
 \end{aligned} \right\} \quad (1.24)$$

as well as the quantity $p_{H_2O}^0/p^0$.

The temperature region for the products of combustion in the case of rocket propellants is bounded by the temperatures of the products of combustion within the combustion chamber and at the outlet section of the nozzle. At a pressure of 20 to 100 atm abs for the products of combustion within the combustion chamber, the

temperature is generally equal to 2500 to 4000° abs., and lies primarily within the limits of from 2800 to 3800° abs.* The temperature of the products of combustion at the outlet section of the nozzle, at a pressure of 0.5 to 1.5 atm abs, in the overwhelming majority of cases lies within the limits of from 1400 to 2800° abs. Consequently, the sought law governing changes in the heat content of the products of combustion must be valid, first of all, for the temperature region bounded by the values 1400 and 3800° abs.

Figure 1 presents graphically the relationship between the values of the parameters (1.24) and the absolute temperature (solid lines). The values for the heat content of the components in the products of combustion (on the curve, denoted by circles) are taken from Table 1 of the Appendix. As we can see, the relationship between the parameters (1.24) and the temperature in the temperature region between 1400 and 3800° abs., proved to be impossible. Therefore, the temperature region under consideration was divided into two zones:

- 1400 to 2800° abs (zone "H"),
- 2800 to 3800° abs (zone "B").

The temperature of 2800° abs was selected as the dividing line for the following reasons; this line is situated approximately in the middle of the region under consideration and, in the overwhelming majority of cases, divides the temperatures of the products of combustion within the combustion chamber of a rocket engine from the temperatures prevailing at the outlet section of the nozzle.

For each of the parameters (1.24) in the two temperature zones, we selected a certain approximate linear function (the

dashed lines in Fig. 1) which has the form

$$y = y_{2800} + b(T - 2800) \quad (1.25)$$

Here y_{2800} represents the values of the parameters (1.24) at a temperature of 2800° abs. The values of the coefficients "b" for parameters (1.24) were determined graphically.

The expanded functions (1.25) have the following form (heat content in kcal/g-mole).

The temperature zone "B":

$$\left. \begin{aligned} I_{CO} &= 89,417 + 0,008878 (T - 2800); \\ I_{H_2} &= 78,555 + 0,008833 (T - 2800); \\ I_{N_2} &= 22,475 + 0,008925 (T - 2800); \\ I_{CO} - I_{CO_2} &= 53,450 - 0,006246 (T - 2800); \\ I_{CO} - I_{CO_2} - I_{H_2} + I_{H_2O} &= 4,581 - 0,001925 (T - 2800). \end{aligned} \right\} \quad (1.26)$$

The temperature zone "H":

$$\left. \begin{aligned} I_{CO} &= 89,417 + 0,008748 (T - 2800); \\ I_{H_2} &= 78,555 + 0,008455 (T - 2800); \\ I_{N_2} &= 22,475 + 0,008693 (T - 2800); \\ I_{CO} - I_{CO_2} &= 53,450 - 0,006046 (T - 2800); \\ I_{CO} - I_{CO_2} - I_{H_2} + I_{H_2O} &= 4,581 - 0,001979 (T - 2800). \end{aligned} \right\} \quad (1.27)$$

The maximum errors in the calculation of the values of I_{CO} , I_{H_2} , I_{N_2} , and $I_{CO} - I_{CO_2}$ in accordance with Eqs. (1.26) and (1.27) do not exceed 0.5%. The maximum magnitude of the error for the parameter $I_{CO} - I_{CO_2} - I_{H_2} + I_{H_2O}$ comes to 1.5%. Since, however, for the products of combustion of all of the fuels under consideration, the fraction of the term

$$\frac{1}{\mu^0} (I_{CO} - I_{CO_2} - I_{H_2} + I_{H_2O}) \frac{p_{H_2O}^0}{p^0}$$

in the Sum (1.23) is not great, the indicated error for this parameter will not result in any substantial error in the determination of the heat content i^0 .

Solving Eqs. (1.23) and (1.26) simultaneously, we find a

relationship between the heat content i_B^0 and the temperature of the products of combustion for the temperature zone "B" (2800-3800° abs):

$$\begin{aligned}
 i_B^0 = & [142,867 + 0,002632(T-2800)] A_C + \\
 & + [39,278 + 0,00417(T-2800)] A_H + \\
 & + [11,238 + 0,004463(T-2800)] A_N - \\
 & - [53,450 - 0,006246(T-2800)] A_O + \\
 & + \frac{1}{\mu^0} [4,581 - 0,001925(T-2800)] \frac{p_{H_2O}^0}{p^0}.
 \end{aligned} \tag{1.28}$$

Here the dimensionalities of the numerical coefficients are expressed in kcal/g-mole; kcal/g-mole·deg, and °abs. The units of i_B^0 are kcal/kg.

It is impossible to make direct use of Eq. (1.28) for the determination of i_B^0 , since the relationship between the parameter $p_{H_2O}^0/p^0$ and the temperature is not known. To simplify the form of the equation, we will adopt the method discussed below.

We will find the specific heat of the products of combustion at constant pressure, the specific heat equal to the derivative of the heat content with respect to temperature (with a constant elementary composition of the products of combustion):

$$\begin{aligned}
 c_{p,s}^0 = \frac{di_B^0}{dT} = & 0,002632A_C + 0,00417A_H + \\
 & + 0,004463A_N + 0,006246A_O + \\
 & + \frac{1}{\mu^0} [4,581 - 0,001925(T-2800)] \frac{d\left(\frac{p_{H_2O}^0}{p^0}\right)}{dT} - 0,001925 \frac{p_{H_2O}^0}{\mu^0 p^0}.
 \end{aligned} \tag{1.29}$$

An evaluation of the importance of the two last terms in the sum indicate the following. The fraction of the term

$$\frac{1}{\mu^0} [4,581 - 0,001925(T-2800)] \frac{d\left(\frac{p_{H_2O}^0}{p^0}\right)}{dT}$$

does not exceed 0.1% of the quantity $c_{p,B}^0$; consequently, this term may be neglected. The fraction of the term $0.001925 p_{H_2O}^0 / p^0 \mu^0$ attains 10 to 15% of the quantity $c_{p,B}^0$. Therefore, this term cannot be neglected. There is a possibility of simplifying this expression considerably. The partial pressure of the water vapors in the undissociated products of combustion undergo little change as temperature changes. Even under the most unfavorable of circumstances (in the case of products of combustion from fuels with limited content of hydrogen and oxygen) a change in the partial pressure of the water vapors comes to approximately 5% of its magnitude, as temperature changes from 3800 to 2800° abs.

On the basis of these considerations, if we take into account that the fraction of the term $0.001925 p_{H_2O}^0 / p^0 \mu^0$ is nevertheless small, we made the assumption that regardless of the temperature assumed in the calculation of $c_{p,B}^0$ it is possible to use the parameter $p_{H_2O,2800}^0$ - the partial pressure of the water vapors at a temperature of 2800° abs., instead of the partial pressure $p_{H_2O}^0$ of water:

$$p_{H_2O,2800}^0 \cong p_{H_2O}^0 \quad (1.30)$$

With this assumption, as calculations for specific fuels indicate, the error in the determination of the quantity $c_{p,B}^0$ does not exceed 0.1%. For engineering calculations this is permissible.

As a result of the term $(1/\mu^0) \cdot (4.581 - 0.001925 [T - 2800]) \cdot (d(p_{H_2O}^0/p^0)/dT)$ having been neglected, we obtain a somewhat reduced value for $c_{p,B}^0$. Using $p_{H_2O,2800}^0/p^0 \mu^0$ instead of $p_{H_2O}^0/p^0 \mu^0$ results in a slight increase in the value of $c_{p,B}^0$. As a result, we find that there is partial and reciprocal compensation of the two errors.

The relative partial pressure of the water vapors in the

undissociated products of combustion for the fuels under consideration, at a temperature of 2800° abs., can be calculated according to the following formula with sufficient accuracy:

$$\frac{p_{H_2O, 2800}^0}{p_{H_2O}^0} = [0,612 + 0,415 \lg(\alpha - 0,42)] A_H. \quad (1.31)$$

Formula (1.31) was found as a result of the processing of direct calculations of the chemical composition of the undissociated products of combustion. This formula yields a sufficiently accurate relationship between the relative partial pressure of the water vapors at 2800° abs and the elementary chemical composition of the propellant.

The adopted simplifications result in an insignificant increase in the magnitude of $c_{p,B}^0$; this magnitude was determined according to the approximate formula, and the increase is in comparison to its value obtained by direct calculation. This demonstrated the comparability of the results obtained in corresponding calculations for a series of propellants. For purposes of compensating the errors, the right-hand portion of Expression (1.29) must be multiplied by a correction factor less than unity. In accordance with our comparative calculations, the correction factor equals 0.997. Finally, we will obtain

$$c_{p,B}^0 = \{2,624A_C + 4,450A_H + 6,227A_O + [3,229 - 0,797 \lg(\alpha - 0,42)] A_H\} 10^{-3}. \quad (1.32)$$

For an evaluation of the accuracy of Formula (1.32) we carried out a comparison of the values of $c_{p,B}^0$ obtained by direct calculation and according to the given formula for markedly divergent elementary chemical compositions. The results of the comparative

* $\lg = \log$

TABLE 1

1 Условный номер хими- ческого со- става	$\frac{A_C}{A_H}$	$\frac{A_N}{A_H}$	A_H	$\alpha=0,90$		
				2 $c_{p,B}^0$, оп- ределенное прямым расчетом	3 $c_{p,B}^0$, вы- численное по формуле (1.32)	4 Погреш- ность зна- чения $c_{p,B}^0$ %
1	0,5	8	7,116	0,3468	0,3471	0,086
2	0,1333	1,4	30,10	0,4323	0,4321	0,023
3	0,077	0,385	59,83	0,5440	0,5422	0,331
4	0,05556	0	95,60	0,6780	0,6741	0,575
5	1,75	5,5	6,382	0,3506	0,3508	0,057
9	3,125	2,75	5,707	0,3525	0,3525	0
11	0,8333	0	23,68	0,4224	0,4218	0,142
14	1,5	0	14,41	0,3902	0,3896	0,154
15	4,5	0	5,215	0,3580	0,3573	0,195

1) Conventional number for chemical composition; 2) $c_{p,B}^0$, determined through direct calculation; 3) $c_{p,B}^0$, calculated according to Formula (1.32); 4) error in value of $c_{p,B}^0$, %.

CONTINUATION

1 Условный номер хими- ческого со- става	$\frac{A_C}{A_H}$	$\frac{A_N}{A_H}$	A_H	$\alpha=0,60$		
				2 $c_{p,B}^0$, оп- ределенное прямым расчетом	3 $c_{p,B}^0$, вы- численное по формуле (1.32)	4 Погреш- ность зна- чения $c_{p,B}^0$ %
1 ₃	0,5	8	7,500	0,3468	0,3475	0,202
2 ₃	0,1333	1,4	33,83	0,4485	0,4489	0,089
3 ₃	0,077	0,385	73,63	0,6022	0,6021	0,016
4 ₃	0,05556	0	132,8	0,8300	0,8302	0,024
5 ₃	1,75	5,5	7,280	0,3483	0,3483	0
9 ₃	3,125	2,75	7,004	0,3480	0,3458	0,632
11 ₃	0,8333	0	31,47	0,4454	0,4438	0,359

1) Conventional number for chemical composition; 2) $c_{p,B}^0$, determined through direct calculation; 3) $c_{p,B}^0$, calculated according to Formula (1.32); 4) error in value of $c_{p,B}^0$, %.

calculations indicate that the error in the magnitude of $c_{p,B}^0$, calculated according to Formula (1.32), does not exceed 0.6% (Table 1).

Let us establish an analogous relationship for the temperature zone "H" (1400 - 2800° abs) as was established in (1.32). Through the simultaneous solution of Eqs. (1.23) and (1.27) we will find the expression which will make it possible to calculate the value of heat content for the undissociated products of combustion in the temperature zone "H":

$$\begin{aligned}
 i_n^0 = & [142,867 + 0,002702(T-2800)] A_C + \\
 & + [39,278 + 0,004228(T-2800)] A_H + \\
 & + [11,238 + 0,004347(T-2800)] A_N - \\
 & - [53,450 - 0,006046(T-2800)] A_O + \\
 & + \frac{1}{\mu^0} [4,581 - 0,001979(T-2800)] \frac{p_{H_2O}^0}{p^0}.
 \end{aligned}
 \tag{1.33}$$

The direct utilization of Eq. (1.33) for the determination of i_H^0 is impossible, since the relationship between $p_{H_2O}^0/p^0$ and temperature is unknown.

Specific heat $c_{p,H}^0$, at constant pressure for the products of combustion, is equal to the derivative of heat content with respect to temperature (given a constant elementary chemical composition of the products of combustion):

$$\begin{aligned}
 c_{p,H}^0 = \frac{di_n^0}{dT} = & 0,002702A_C + 0,004228A_H + 0,004347A_N + 0,006046A_O + \\
 & + \frac{1}{\mu^0} [4,581 - 0,001979(T-2800)] \frac{d\left(\frac{p_{H_2O}^0}{p^0}\right)}{dT} - 0,001979 \frac{p_{H_2O}^0}{\mu^0}.
 \end{aligned}
 \tag{1.34}$$

Let us evaluate the importance of the two last terms of the right-hand part of Expression (1.34). An analysis of the results obtained from the calculations carried out for this purpose indicate that the fraction of the term

TABLE 2

$\alpha=0,90$

1 Условный номер химического состава	$\frac{A_C}{A_H}$	$\frac{A_N}{A_H}$	A_H	2 $c_{p,H}^0$, определенное прямым методом	3 $c_{p,H}^0$, вычисленное по формуле (1.35)	4 Погрешность значения $c_{p,H}^0$, %
1	0,5	8	7,116	0,3421	0,3400	0,61
2	0,1333	1,4	30,10	0,4258	0,4203	1,29
5	1,75	5,5	6,382	0,3464	0,3444	0,57
6	0,7	1,6	16,91	0,3816	0,3817	0,75
9	3,125	2,75	5,707	0,3182	0,3165	0,48
10	1,25	0,5	15,11	0,3844	0,3818	0,67
11	0,8333	0	23,68	0,4161	0,4122	0,93
12	3,75	1,5	5,490	0,3512	0,3501	0,31
13	2,14	0,428	9,840	0,3683	0,3657	0,70
14	1,5	0	14,41	0,3844	0,3820	0,57
15	4,5	0	5,215	0,3538	0,3518	0,56

1) Conventional number of chemical composition; 2) $c_{p,H}^0$, determined by direct method; 3) $c_{p,H}^0$, calculated according to Formula (1.35); 4) error in value of $c_{p,H}^0$, %.

$$\frac{1}{\mu^0} \left[4,581 - 0,001979 (T - 2800) \frac{d \left(\frac{p_{H_2O}^0}{p^0} \right)}{dT} \right]$$

does not exceed 0.6% of the magnitude of $c_{p,H}^0$; therefore, this term is neglected. The fraction of the term $0.001979 p_{H_2O}^0 / p^0 \mu^0$ attains 10% of the magnitude of $c_{p,H}^0$; apparently, this term cannot be neglected in the calculation of $c_{p,H}^0$. However, it is possible to simplify the determination of this quantity without any substantial reduction in the accuracy of the results obtained in the calculation. For this purpose, the value of the parameter $p_{H_2O}^0 / p^0 \mu^0$, taken for any temperature, will be replaced (as in zone "B^{II}") by its value for a temperature of 2800° abs. - $p_{H_2O,2800}^0 / p^0 \mu^0$. The parameter $p_{H_2O,2800}^0 / p^0 \mu^0$ is determined according to

Formula (1.31).

The above-mentioned assumptions result in a certain reduction in the value of $c_{p,H}^0$; therefore, for the determination of $c_{p,H}^0$, according to the approximate method, the right-hand part of Expression (1.34) must be multiplied by a constant correction factor, numerically greater than unity. Comparative calculations, which we carried out, result in a value of 1.005 for this correction factor.

Consequently, having carried out the above-mentioned simplifications for Eq. (1.34), we find

$$c_{p,n}^0 = (2,716A_C + 4,369A_N + 6,076A_O + \dots + [3,032 - 0,825 \lg(\alpha - 0,42)] A_H) 10^{-3}. \quad (1.35)$$

The error in the value of $c_{p,H}^0$, obtained according to the approximate Expression (1.35), for the products of combustion of a series of elementary chemical compositions, markedly different from one another, does not exceed 1%. The results of the comparative calculations are cited in Table 2.

Expressions (1.32) and (1.35) determine the derivative of the heat content of 1 kg of the products of combustion with respect to temperature in the corresponding temperature zones. Integrating these equations, we find the heat content of 1 kg of undissociated products of combustion in the corresponding temperature zones.

For the temperature zone "B"

$$i_B^0 - i_{2800}^0 = \int_{2800}^T c_{p,n}^0 dT.$$

Since the parameter $c_{p,B}^0$ is independent of temperature, we finally obtain

$$i_B^0 = i_{2800}^0 + c_{p,n}^0 (T - 2800). \quad (1.36)$$

Similarly, for the temperature zone "H"

$$i_n^0 = i_{2800}^0 + c_{p,n}^0 (T - 2800). \quad (1.37)$$

The parameter i_{2800}^0 , entering into Eqs. (1.36) and (1.37), represents the heat content of the undissociated products of combustion of the propellants, at a temperature of 2800° abs. For the calculation of i_B^0 and i_H^0 according to Expressions (1.36) and (1.37) we must know the value of the heat content i_{2800}^0 .

The method for the calculation of the value of the heat content of the undissociated products of combustion is presented in the following section.

3. The Heat Content of Undissociated Products of Combustion

Equations (1.32) and (1.35) make it possible to calculate the values of parameters $c_{p,B}^0$ and $c_{p,H}^0$ without any preliminary calculation of the chemical composition of the undissociated products of combustion. For this, it is sufficient to know only the initial elementary chemical composition of the propellant, defined by the quantities A_C , A_H , A_N , and A_O . Consequently, in addition to the term i_{2800}^0 , there are no other terms requiring preliminary calculation of the chemical composition of the products of combustion for the calculation of their values, which enter into the right-hand parts of Eqs. (1.36) and (1.37).

For a determination of i_{2800}^0 we require, in the general case, a calculation of the chemical composition of the products of combustion at a temperature of 2800° abs. Thus, for the calculation of the values of heat content at other temperatures, said calculation performed according to Eqs. (1.36) and (1.37), we must first carry out a calculation of the chemical composition of the products of combustion at a temperature of 2800° abs. The latter

circumstance makes markedly more difficult the utilization of Eqs. (1.36) and (1.37) for the calculation of the heat content of undissociated products of combustion. It is desirable to simplify the calculation method for the parameter i_{2800}^0 to the maximum.

The simplification of the calculation method for the parameter i_{2800}^0 , and at the same time of the parameters i_B^0 and i_H^0 , is precisely the matter with which the present section deals.

The heat content of the undissociated products of combustion, at a temperature of 2800° abs., as well as at any other temperature, is calculated according to Eq. (1.18a). In the general case, the heat content is a function of the following parameters:

$$i^0 = f(A_C, A_H, A_N, A_O, T).$$

In an examination of the heat content i^0 , with constant temperature for the products of combustion ($T = \text{const}$), the number of parameters decreases to 1. Thus, for example, at $T = 2800^\circ$ abs., we obtain

$$i_{2800}^0 = f(A_C, A_H, A_N, A_O).$$

In this case, the heat content of the undissociated products of combustion depends only on the initial elementary chemical composition of the propellant. The number of gram-atoms of one of the chemical elements, given known numbers of gram-atoms of the three remaining elements (and known atomic weights for all four elements), can be found, as a supplement to the weight of up to 1 kg of propellant. Thus, the number of parameters on which i_{2800}^0 depends, can be reduced to three as a result of the elimination of, for example, the parameter A_N .

The number of gram-atoms A_O of oxygen can be expressed through

parameters A_H , A_C , and α in accordance with the formula

$$\alpha = \frac{A_0}{2A_C + 0,5A_H}.$$

Finally, we will obtain

$$i_{2800}^0 = f(A_C, A_H, \alpha). \quad (1.38)$$

In the case of propellants for rocket engines, the carbon and hydrogen content ranges within wide limits. The situation is somewhat different with α , the coefficient of excess oxygen. The operating range for excess oxygen, the range in which rocket engines operate, is comparatively small. Let us examine the operation of engines with coefficients of excess oxygen ranging in value from 0.5 to 1.0. Within this range of values for the coefficient of excess oxygen, however, we generally undertake a theoretical investigation of the characteristics for the combustion of the propellant and the operation of the engine. For this purpose it is sufficient to carry out thermal calculations at several values of α and subsequently by methods involving the graphical interpolation for the determination of the required characteristics at all intermediate values of α .

If we take into consideration all that has been mentioned above, the problem of determining the parameter i_{2800}^0 becomes substantially clearer. We will assume the values of α , the coefficient of excess oxygen to be equal to 0.95, 0.90, 0.80, 0.70, 0.60, and 0.55. At each of these values, the heat content i_{2800}^0 proves to be a function of only two parameters: A_C and A_H .

At each fixed value of α , it is felt to be expedient to carry out direct calculations for the determination of heat content i_{2800}^0 as a function of the parameters A_C and A_H and to construct the corresponding nomograms. For a number of considerations,

TABLE 3

Условный номер химического состава 1	x_C^0	x_H^0	x_N^0	α					
				0,95	0,90	0,80	0,70	0,60	0,55
				x_O^0	x_O^0	x_O^0	x_O^0	x_O^0	x_O^0
1	4	8	61	11,4	10,8	9,6	8,4	7,2	6,6
2	4	30	42	21,85	20,7	18,4	16,1	13,8	12,65
3	4	52	20	32,3	30,6	27,2	23,8	20,4	18,7
4	4	72	0	41,8	39,6	35,2	30,8	26,4	24,2
5	14	8	44	39,1	28,8	25,6	22,4	19,2	17,6
6	14	20	32	36,1	34,2	30,4	26,6	22,8	20,9
7	14	40	12	45,6	43,2	38,4	33,6	28,8	26,4
8	14	52	0	51,3	48,6	43,2	37,8	32,4	29,7
9	25	8	22	51,3	48,6	43,2	37,8	32,4	29,7
10	25	20	10	57,0	51,0	48,0	42,0	36,0	33,0
11	25	30	0	61,75	58,5	52,0	45,5	39,0	35,75
12	30	8	12	60,8	57,6	51,2	44,8	38,4	35,2
13	30	14	6	63,65	60,3	53,6	46,9	40,2	36,85
14	30	20	0	66,5	63,0	56,0	49,0	42,0	38,5
15	35	8	0	72,2	68,4	60,8	53,2	45,0	41,2

Note. The conventional number for the composition, noted without index (column 1), corresponds to $\alpha = 0.90$ (column 6). For other values of α , the coefficient of excess oxygen, the conventional composition number takes subscripts 0, 1, 2, 3, or 4. For example, for composition No. 4:

α	Conventional Composition Number
0.95	4 ₀
0.90	4
0.80	4 ₁
0.70	4 ₂
0.60	4 ₃
0.55	4 ₄

Other compositions are given analogous denotations.

1) Conventional composition number.

the reasons for which will become clearer after having read the subsequent chapters of this book, we have selected not only the parameters A_C and A_H as the independent variables for the construction of the nomograms, but we have also chosen the parameters x_C^0 and x_H^0 , associated with A_C and A_H in the following manner:

$$X_C^0 = 40\mu^0 A_C; \quad (1.39)$$

$$X_H^0 = 40\mu^0 A_H. \quad (1.40)$$

The sought-after function in these nomograms is not the quantity i_{2800}^0 - the heat content of the undissociated products of combustion at a temperature of 2800° abs - but the quantity i_V^0 , associated with i_{2800}^0 by the expression

$$i_V^0 = 40\mu^0 i_{2800}^0. \quad (1.41)$$

The nomograms are constructed for the above-mentioned coefficients of excess oxygen, and are shown in Figs. 1 to 6 of the Appendix.

In order to plot the support points in the construction of the nomograms, we made use of the results obtained in the direct calculation of the values of the parameter i_V^0 for the products of combustion of the chemical compositions which are cited in Table 3.

The limits for changes in the values of the parameters X_C^0 , X_H^0 , and α in Table 3 correspond to the limits of changes in the percentage content of the chemical elements in the propellant, indicated in the preface to this book. The compositions denoted in Tables 1, 2, and 3 by one and the same conventional number, regardless of through which parameters they are expressed (through A_C , A_H , A_N , A_O or X_C^0 , X_H^0 , X_N^0 , X_O^0). The calculation of the parameter i_V^0 was carried out in accordance with the conventional method, involving the utilization of the table for the heat contents for the products of combustion, presented in the Appendix.

The sought value of i_{2800}^0 is determined from the nomograms in the following manner. Let us assume that it is necessary to

determine the value of i_{2800}^0 at a certain fixed value for the coefficient of excess oxygen ($\alpha = \text{const}$) (coinciding with one of the values of α , at which the nomograms were constructed) for the propellant whose chemical composition is known. For this purpose, the values of the independent variables X_C^0 and X_H^0 are calculated according to Formulas (1.39) and (1.40).

Then, a straight line is drawn from a point on the axis of abscissas, said point corresponding to the value of X_H^0 , and the straight line is to be parallel to the axis of ordinates, to the point at which it intersects with the line corresponding to the value of X_C^0 . The ordinate of the point of intersection determines the value of i_v^0 . The sought heat content i_{2800}^0 is found according to Expression (1.41):

$$i_{2800}^0 = \frac{i_v^0}{40\mu^0}.$$

If the given value of α , the coefficient of excess oxygen, does not coincide with any one of the values for this coefficient at which the nomograms were constructed, and if it does not exceed the limits $0.55 \leq \alpha \leq 0.95$, for the determination of the value of the parameter i_{2800}^0 at a given α it is necessary to determine successively the value of this parameter by means of the nomograms constructed at values close to the given value of α and, having constructed the auxiliary curve with coordinates (i_{2800}^0, α) , to find the value of i_{2800}^0 .

Thus the nomograms presented in Figs. 1 to 6 of the Appendix make it possible to determine the parameter i_{2800}^0 without any preliminary calculation of the chemical composition of the products of combustion.

Consequently, the heat content of the undissociated products of combustion for the two temperature ("B" and "H"), unlike the methods presented earlier, can be determined according to Eqs.

(1.36) and (1.37), in terms of the elementary chemical composition of the propellant, without any preliminary calculation of the chemical composition of the products of combustion.

Since the independent parameters X_C^0 and X_H^0 are used for the determination of the value of i_{2800}^0 , it is more convenient to carry out a calculation of the values of $c_{p,B}^0$ and $c_{p,H}^0$ in terms of these parameters rather than in terms of A_C and A_H .

With a modification of Expressions (1.32) and (1.35), we find

$$c_{p,B}^0 = \frac{1}{40\mu^0} [356,0 + (12,454x - 6,276) X_C^0 + (3,114x - 1,221) X_H^0 - 0,797 \lg(x - 0,42) X_H^0] 10^{-3}; \quad (1.42)$$

$$c_{p,H}^0 = \frac{1}{40\mu^0} [349,6 + (12,152x - 6,022) X_C^0 + (3,038x - 1,336) X_H^0 - 0,825 \lg(x - 0,42) X_H^0] 10^{-3}. \quad (1.43)$$

For an evaluation of the error in the values of i_B^0 and i_H^0 we carried out comparative calculations of the values of these parameters for the products of combustion for a series of markedly divergent chemical compositions. The values of i_B^0 and i_H^0 were calculated by the direct method and according to Eqs. (1.36) and (1.37).

The results of the calculations are presented in Table 4.

From an analysis of this table it follows that for the products of composition of the compositions under consideration the error in the magnitude of i_B^0 and i_H^0 , calculated according to Eqs. (1.36) and (1.37), does not exceed 0.4%.

TABLE 4

1 Условный номер химического состава	2 Температура продуктов сгорания T° абс.	3 Значение i_B^0 или i_H^0 по уравнениям (1.36) или (1.37) ккал/кг	4 Значение i_B^0 или i_H^0 при расчете прямым методом ккал/кг	5 Погрешность определения i_B^0 или i_H^0 ккал/кг	6 Погрешность определения i_B^0 или i_H^0 в процентах от i_B^0 или i_H^0
1	3800	1279,0	1283,0	4,0	0,31
1	3400	1140,2	1136,0	4,2	0,37
1	3000	1001,4	998,0	3,4	0,34
4	3800	2579,0	2584,0	5,0	0,19
4	3400	2309,4	2310,0	0,6	0,03
4	3000	2039,8	2038,0	1,8	0,09
5	3400	1236,6	1238,0	1,4	0,11
5	2400	888,4	888,5	0,1	0,01
8	3400	1872,2	1876,0	3,8	0,20
8	2400	1356,6	1356,0	0,6	0,04
9	3400	1321,8	1327,0	5,2	0,39
9	2400	971,6	975,0	3,4	0,35
11	3400	1584,2	1586,0	1,8	0,11
11	2400	1166,2	1167,0	0,8	0,07
15	3400	1400,2	1400,0	0,2	0,01
15	3000	1257,4	1256,0	1,4	0,11
15	2400	1045,2	1044,0	1,2	0,11

1) Conventional chemical composition number; 2) temperature of the products of combustion, T° abs; 3) value of i_B^0 or i_H^0 according to Eqs. (1.36) or (1.37), kcal/kg; 4) value of i_B^0 or i_H^0 , in calculation by direct method, kcal/kg; 5) error in determination of i_B^0 or i_H^0 , kcal/kg; 6) error in determination of i_B^0 or i_H^0 , in per cent of i_B^0 or i_H^0 .

FOOTNOTES

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*We have reference here to the propellants of the class under consideration.

Chapter 2

THERMODYNAMIC PARAMETERS OF DISSOCIATED PRODUCTS OF COMBUSTION

4. Dissociated Products of Combustion

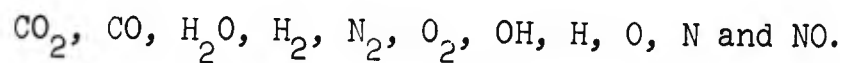
The high temperature encountered in the combustion of propellants in rocket engines leads to the dissociation of individual components of the products of combustion, and in this dissociation the more complex molecules decompose into simpler molecules and monatomic particles. The dissociation reaction is accompanied by an expenditure of heat and an increase in chemical energy for the products of combustion. The expenditure of heat in the dissociation of the product of combustion is a factor which tends to limit the magnitude of the maximum temperature of combustion for the given propellant. The degree of dissociation is determined, primarily, by the temperature.

The formation of lighter monatomic and biatomic gases - the products of the dissociation - as a result of the dissociation results in a decrease in the average ("apparent") molecular weight of the gaseous products of combustion. If we examine the dissociation taking place at constant pressure, it is accompanied by an increase in the volume of the products of combustion. Dissociation taking place in a constant volume results in an increase in the pressure of the products of combustion.

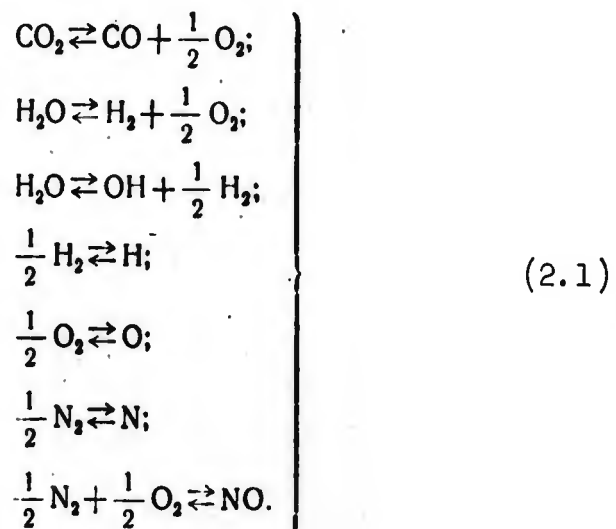
With a substantial increase in the pressure of the dissociated products of combustion (theoretically, at $p \rightarrow \infty$) it is possible to suppress all reactions taking place with an increase in the number

of moles, and to cause a reaction resulting in the combination of the molecules and atoms that were dissociated earlier. These reactions are referred to as reactions of recombination. The recombination phenomenon will take place with a drop in the temperature of the products of combustion.

In the general case, the composition of the products of combustion depends on the initial chemical composition of the propellant, the pressures, and on temperature. Calculations indicate that in the products of combustion of the rocket propellants under consideration, there are 11 components present in considerable quantities:



The formation of these components is a consequence of six dissociation reactions and the reaction of the oxidation of biatomic nitrogen:



For the calculation of the true temperature of the combustion of the propellant, i.e., the temperature prevailing during the dissociation of the products of combustion, as well as for the calculation of the partial pressures of these products, it is necessary to derive a system of equations in which the total

number of equations must be equal to the number of unknowns.

If we consider the process of dissociation as the process taking place without any change in the volume of the products of combustion, pressure will increase as a result of the increase in the number of moles, this latter increase due to dissociation, and the pressure will become greater than the pressure of the products of combustion, calculated without dissociation having been taken into consideration.

With such an approach to the solution of the problem, the determination of the characteristics of the products of combustion under conditions of dissociation is carried out in two stages. During the first stage, we resolve the simpler problem of determining the characteristics of the products of combustion without taking into consideration dissociation and the obtained results are regarded as the first approximation to the actual case. In the second stage, we determine the correction factors for the resultant parameters of the first approximation, and these will make it possible to obtain an exact solution. The transition from the results of the first approximation to the exact values of the parameters is carried out at constant volume and temperature for the products of combustion.

For the solution of the problem it is necessary to determine 13 unknowns, and we have reference here to the 11 partial pressures of the products of combustion, as well as the temperature and volume of the mixture of these products. Consequently, the system must consist of no less than 13 equations. The first seven equations are the equations of the equilibrium constants of the reaction (2.1):

$$\frac{p_{\text{CO}} \sqrt{p_{\text{O}_2}}}{p_{\text{CO}_2}} = K_1 = f_1(T);$$

$$\frac{p_{H_2} \sqrt{p_{O_2}}}{p_{H_2O}} = K_2 = f_2(T);$$

$$\frac{p_{OH} \sqrt{p_{H_2}}}{p_{H_2O}} = K_3 = f_3(T);$$

$$\frac{p_{H}}{\sqrt{p_{H_2}}} = K_4 = f_4(T);$$

$$\frac{p_{O}}{\sqrt{p_{O_2}}} = K_5 = f_5(T);$$

$$\frac{p_{N}}{\sqrt{p_{N_2}}} = K_6 = f_6(T);$$

$$\frac{p_{NO}}{\sqrt{p_{N_2} p_{O_2}}} = K_7 = f_7(T).$$

Generally, the magnitude of the equilibrium constants for the products of combustion of the propellants under consideration are presented in a table, and here the temperature values are taken to the nearest hundred degrees, with an interval of 100 to 200°. In the present book, the equilibrium constants used in the calculations are those presented in the Appendix. It should be pointed out that the equilibrium constants are always being refined, and therefore we may encounter tables in other works in which there are somewhat different values for these same equilibrium constants.

Further on, for the solution of the problem we make use of the equations of material balance.

The equations of material balance for the case of propellant combustion, with dissociation of the products of combustion having been taken into consideration, are obtained in similar fashion by Eqs. (1.11) - (1.14), valid for undissociated products of combustion. Let us, for example, derive a balanced equation in terms of a chemical element - oxygen. There are A_0 gram-atoms of oxygen in a single kilogram of propellant. A number of com-

ponents of the products of combustion also contain oxygen, and namely: in each gram-molecule of CO_2 and O_2 there are two gram-atoms of oxygen, and in each gram-molecule of CO , H_2O , O , OH , and NO , there is a single gram-atom of oxygen. In accordance with the law of the conservation of matter, the number of gram-atoms of oxygen in the products of combustion must be equal to their quantity in the propellant. Consequently,

$$A_0 = 2M_{\text{CO}_2} + 2M_{\text{O}_2} + M_{\text{CO}} + M_{\text{H}_2\text{O}} + M_{\text{O}} + M_{\text{OH}} + M_{\text{NO}}.$$

Let us modify the last expression. We will multiply the right- and left-hand parts by $p\mu$ and the product of the parameters of the left-hand part will be denoted by the symbol X_0 :

$$X_0 = p\mu A_0. \quad (2.2)$$

Here p is the pressure of the dissociated products of combustion; μ is the parameter which is a function of μ_k , the "apparent" molecular weight of the dissociated products of combustion - determined according to the following formula:

$$\mu = \frac{\mu_k}{1000}. \quad (2.3)$$

In analogy with (1.6), the terms of the sum in the right-hand part are written in terms of partial pressures, since for each component of the products of combustion the following equality proves valid

$$p_i = p\mu M_i. \quad (2.4)$$

The balance expression with respect to oxygen takes the form

$$X_0 = 2p_{\text{CO}_2} + 2p_{\text{O}_2} + p_{\text{CO}} + p_{\text{H}_2\text{O}} + p_{\text{O}} + p_{\text{OH}} + p_{\text{NO}}. \quad (2.5)$$

Similarly, for the three remaining chemical elements

$$X_C = p_{\text{CO}_2} + p_{\text{CO}}; \quad (2.6)$$

$$X_H = 2p_{H_2O} + 2p_{H_2} + p_{OH} + p_{H} \quad (2.7)$$

$$X_N = 2p_{N_2} + p_{NO} + p_N \quad (2.8)$$

Here, in analogy with the parameter X_O , the symbols X_C , X_H , and X_N denote

$$X_C = p^* A_C \quad (2.9)$$

$$X_H = p^* A_H \quad (2.10)$$

$$X_N = p^* A_N \quad (2.11)$$

For the solution of the problem presented, it is necessary to derive an additional two equations, since the number of unknowns is 13, and at the moment we have only 11 equations.

In the absence of heat losses, the heat content of the propellant is equal to the heat content of the products of combustion, whose composition corresponds to the combustion temperature $T_{k.s}$:

$$i_T = i_{T_{k.s}} \quad (2.12)$$

For the solution of Eq. (2.12) it is necessary to be able to determine the heat content of the products of combustion in terms of the composition of the products of combustion for any given temperature T . This heat content may be calculated according to a formula similar to Formula (1.18,a):

$$i = \frac{1}{p^*} (I_{CO_2} p_{CO_2} + I_{CO} p_{CO} + I_{H_2O} p_{H_2O} + I_{H_2} p_{H_2} + I_{OH} p_{OH} + I_O p_O + I_{NO} p_{NO} + I_O p_O + I_H p_H + I_N p_N) \quad (2.13)$$

Here I_{CO_2} , I_{CO} , etc., are the heat-content values for the components in the products of combustion, at temperature T .

The last missing equation is determined from the method which we have adopted for achieving the transition from the parameters of the undissociated products of combustion to the parameters of the

dissociated products, without any change in volume or temperature. Consequently,

$$v = v^0 \text{ (at } T = \text{const).} \quad (2.14)$$

The mixture of the gaseous products of combustion, whose individual components may enter into chemical reactions with one another, may be regarded as an ideal gas governed by the equation of state. Let us break down this equation for the cases of undissociated and dissociated products of combustion:

$$v^0 = \frac{\bar{R}}{p^0 \mu_x^0} T; \quad (2.15)$$

$$v = \frac{\bar{R}}{p \mu_x} T. \quad (2.16)$$

Since $T = \text{const}$, and $v = v^0$, if we take into consideration Expressions (1.5) and (2.3) we will see that for the given propellant the following relationship is valid, given the conditions indicated above:

$$p \mu_x = p^0 \mu_x^0 = \text{const.} \quad (2.17)$$

The last equation is extremely noteworthy. From a comparison of Expressions (1.10), (1.15)-(1.17), and (2.2), (2.9)-(2.11), and taking into consideration Expression (2.17) it follows directly that parameters X_O , X_C , X_H , and X_N , found in the left-hand parts of the equations of material balance, said equations written for dissociated products of combustion [(2.5)-(2.8)], are equal to the corresponding parameters of Eqs. (1.11)-(1.14), written for the products of combustion, dissociation not having been taken into consideration. Consequently,

$$\left. \begin{aligned} X_O &= X_O^0; \\ X_C &= X_C^0; \\ X_H &= X_H^0; \\ X_N &= X_N^0. \end{aligned} \right\} \quad (2.18)$$

The parameters X_O^0 , X_C^0 , X_H^0 , and X_N^0 are easily determined accord-

ing to Formulas (1.10) and (1.15)-(1.17) at given p^0 and a known elementary chemical composition of the propellant (A_O , A_C , A_H , and A_N). Employing the Equation (2.17) makes it easier to apply Formula (2.13) to calculate the heat content of the product of combustion under conditions of dissociation.

Let us combine the derived system of equations, taking into consideration the fact that Eq. (2.14) has already been used:

$$\begin{aligned}
 1) \quad & \frac{p_{CO} \sqrt{p_{O_2}}}{p_{CO_2}} = K_1 = f_1(T); \\
 2) \quad & \frac{p_H \sqrt{p_{O_2}}}{p_{H_2O}} = K_2 = f_2(T); \\
 3) \quad & \frac{p_{OH} \sqrt{p_{H_2}}}{p_{H_2O}} = K_3 = f_3(T); \\
 4) \quad & \frac{p_H}{\sqrt{p_{H_2}}} = K_4 = f_4(T); \\
 5) \quad & \frac{p_O}{\sqrt{p_{O_2}}} = K_5 = f_5(T); \\
 6) \quad & \frac{p_N}{\sqrt{p_{N_2}}} = K_6 = f_6(T); \\
 7) \quad & \frac{p_{NO}}{\sqrt{p_N} \sqrt{p_{O_2}}} = K_7 = f_7(T); \\
 8) \quad & X_O^0 = 2p_{CO_2} + 2p_{O_2} + p_{CO} + p_{H_2O} + p_O + p_{OH} + p_{NO}; \\
 9) \quad & X_C^0 = p_{CO_2} + p_{CO}; \\
 10) \quad & X_H^0 = 2p_{H_2O} + 2p_H + p_{OH} + p_{H_2}; \\
 11) \quad & X_N^0 = 2p_{N_2} + p_{NO} + p_N; \\
 12) \quad & i_T = i_{T_{k.c.}}
 \end{aligned} \tag{2.19}$$

In the presence of dissociation, the composition of the mixture of the gaseous products of combustion is, in the general case, a function of the following variables: A_O , A_C , A_H , A_N , p and T . This follows from the condition that if the temperature, pressure, and elementary chemical composition of the propellant are given, the coefficient of the system of equations (2.19) are fully given, said system of equations determining the only solution to the problem. After using additional functions as independent variables, we can

also use the parameters A_C , A_H , α , \underline{p} , T or X_C , X_H , α , \underline{p} , and T . Evidently, such a substitution is completely permissible.

5. Heat Content of Dissociated Products of Combustion as a Function of the Temperature of the Products of Combustion

The chemical composition of the products of combustion, given dissociation, differs from the composition of the products of combustion determined for the case in which the dissociation reaction has not been taken into consideration. The appearance of such additional products of combustion as OH, NO, H, N, etc., changes (increases) the number of moles for the products of combustion per unit weight of mixture. Together with a change in the chemical composition of the products of combustion there is, doubtlessly, a change in the thermodynamic parameters of these products of combustion, particularly insofar as this pertains to heat content.

Following the method that we have adopted for the examination of the parameters of the dissociated products of combustion in two stages (see Part 4), we will present the heat content \underline{i} of 1 kg of dissociated products of combustion in the form of the sum

$$i = i^0 + \Delta i. \quad (2.20)$$

Here i^0 is the heat content of the undissociated products of combustion; Δi is the increment in heat content to the mixture of the products of combustion, as a result of dissociation.

Let us investigate the parameter \underline{i} as a function of temperature, for the products of combustion of the propellants under consideration. It is evident that this function will be determined by the extent to which the parameters i^0 and Δi are functions of temperature.

Earlier we derived Eqs. (1.36) and (1.37) which make it possible to calculate the heat content i^0 of the products of combustion in

the absence of dissociation, at temperatures falling within the temperature zones "B" and "H."

Let us examine the parameter Δi . Just as other thermodynamic parameters, determined on the basis of the preliminarily calculated chemical composition of the products of combustion under conditions of dissociation, the parameter Δi depends on the independent variables X_C , X_H , α , p , and T :

$$\Delta i = f(X_C, X_H, \alpha, p, \text{ and } T). \quad (2.21)$$

Let us temporarily exclude the independent variable p from our examination, assuming that the change Δi due to temperature and other independent variables takes place at constant pressure for the products of combustion. For this particular case, we will denote the parameter Δi as $\Delta i^{(p)}$.

It is clear that under these conditions $\Delta i^{(p)}$ is a function of the following independent variables:

$$\Delta i^{(p)} = f(X_C, X_H, \alpha, T). \quad (2.22)$$

Let us investigate Function (2.22) in order to derive simple but sufficiently precise analytical expressions that relate $\Delta i^{(p)}$ with the elementary chemical composition of the propellant and the temperature of the products of combustion. The limits for changes in the variables X_C , X_H , and α will be assumed to be the same as for the case of undissociated products of combustion (see Table 3), and the temperature region under investigation will be divided, as before, into temperature zones "B" and "H." The pressure for the dissociated products of combustion, at which the increment $\Delta i^{(p)}$ in heat content is determined, must be selected in advance.

The pressure of the dissociated products of combustion must be sufficiently characteristic for the conditions prevailing for the products of combustion in rocket engines. In the temperature zone

"B" (2800-3800° abs.), for the most part coincident with the temperatures of the products of combustion in the combustion chambers of the engines, the calculations of the parameter $\Delta i^{(p)}$ are carried out at a pressure of $p_{B, f} = 40$ atm abs. In the temperature zone "H" (1400-2800° abs.), for the most part coincident with the temperature of the products of combustion at the outlet section of the nozzle of the engine, the calculations are carried out at a pressure of $p_{H, f} = 1$ atm abs.

Let us investigate the nature of the parameter $\Delta i^{(p)}$ as a function of the temperature in the temperature zone "B" (denoting this parameter in this zone as $\Delta i_B^{(40)}$). We will employ the investigational method that is similar to the processing methods used for the results of the physical measurements, said method involving the assumption of a series of values for the independent variable (in this case, T) in the region of its change and to determine the value of the sought function at each of these values, after which the resultant function is analyzed. Only in our case will the calculated values of this function serve in the role of measured values for the function. We will carry out the investigation, first of all, for the chemical compositions cited in Table 3 under the conventional numbers 1, 1₁, 4, 4₂, 4₃, 15, 15₁, and 15₂, since, as it follows from an examination of their elementary chemical composition, these compositions seem to be "border-line cases" with respect to the chemical elements which they contain - nitrogen, hydrogen, and carbon, respectively.

Calculations of the parameter $\Delta i_B^{(40)}$, as well as of other parameters relating to dissociated products of combustion, are carried out directly, according to the known chemical composition of the products of combustion, determined through the solution of

the system of equations (2.19) at a pressure of 40 atm abs for the dissociated products of combustion.

The results of the calculations are presented graphically in Fig. 2. The curve was constructed in logarithmic coordinates. The lines on this graph were drawn as close as possible to the support points, corresponding to the results of the calculations that were carried out (denoted by circles on the curve). The values of $\log(T-2200)$ have been plotted along the axis of abscissas, and the values of the common logarithm of the parameter $\Delta i_{B,v}^{(40)}$, associated with $\Delta i_B^{(40)}$ by the following expression, are plotted along the axis of ordinates:

$$\Delta i_{B,v}^{(40)} = 40 \mu^0 \Delta i_B^{(40)}. \quad (2.23)$$

From an examination of the curve in Fig. 2 it follows that for the given chemical compositions, the relationship between $\Delta i_{B,v}^{(40)}$ and T is virtually linear in logarithmic coordinates.

The linear function is preserved at values of α (the coefficient of excess oxygen) ranging from 0.60 to 0.90.

The slope with respect to the axis of abscissas for the lines relating to the compositions with identical quantities X_C and X_H , but with various values of α , remains the same for all values of α , assumed for the purpose of the calculation. The magnitude of this slope is in strict accord with the initial elementary chemical composition (X_C, X_H) and increases in conjunction with an increase in the carbon content. The smallest slope with respect to the axis of abscissas for the line is noted in the calculation cases involving propellants with high nitrogen content.

The nature of the relationship between $\Delta i_{B,v}^{(40)}$ and the temperature for the intermediate chemical compositions (denoted in Table 3 by the conventional numbers 2, 3, 5, 6, 8, 9, 10, 11, 12, and 14),

is investigated only for a single value of the coefficient of excess oxygen ($\alpha = 0.90$) assuming that in view of the smoothness with which the properties of the products of combustion change with a change in the chemical composition, the nature of the relationships between the parameter $\Delta i_{B, v}^{(40)}$ and the temperature, at other values of α , will be the same as in the case of the "border-line" propellants.

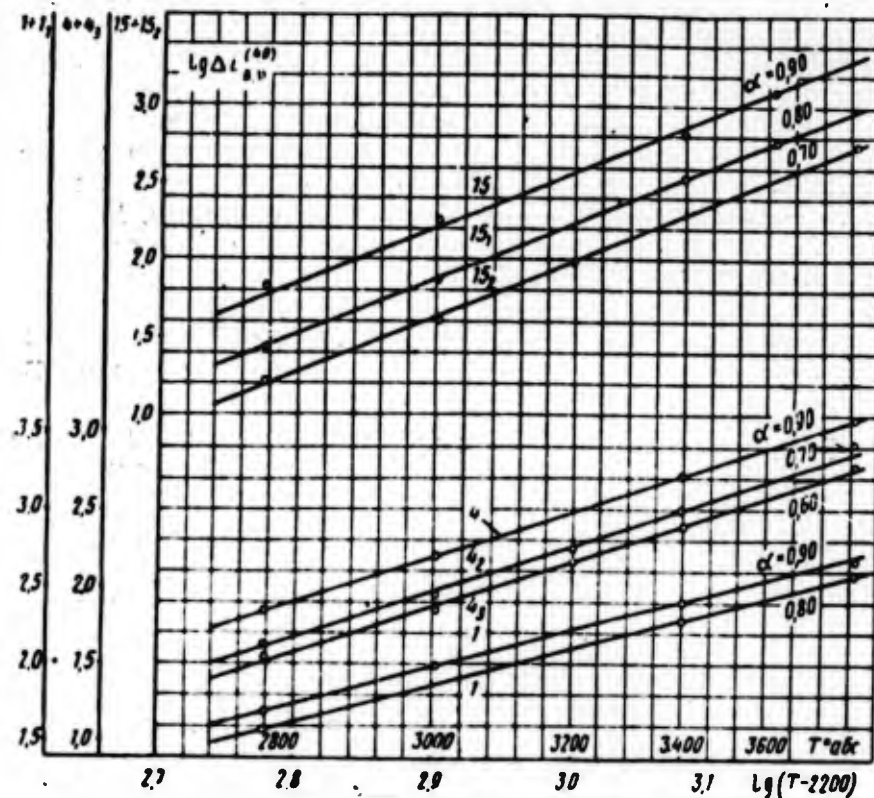


Fig. 2. 1) T° abs; 2) $\log (T-2200)$.

The results obtained in the calculations of these compositions are presented graphically in Fig. 3. From an examination of this graph we find that for the intermediate chemical compositions the established quantitative relationship is preserved: the parameter $\Delta i_{B, v}^{(40)}$ as a function of temperature, in its graphic representation in the above-indicated logarithmic coordinates, remains virtually linear, while the angle formed by the straight lines and the positive segment of the axis of abscissas increases in direct proportion to the carbon content in the propellant.

Consequently, it may be maintained with sufficient reliability

that for all of the compositions under consideration and throughout the entire temperature zone "B" there exists an approximate proportionality of the type

$$\lg \Delta i_{B,v}^{(40)} \sim m_B \lg (T - 2200)$$

or, changing from the logarithmic record to a conventional system,

$$\Delta i_{B,v}^{(40)} \sim (T - 2200)^{m_B} \quad (2.24)$$

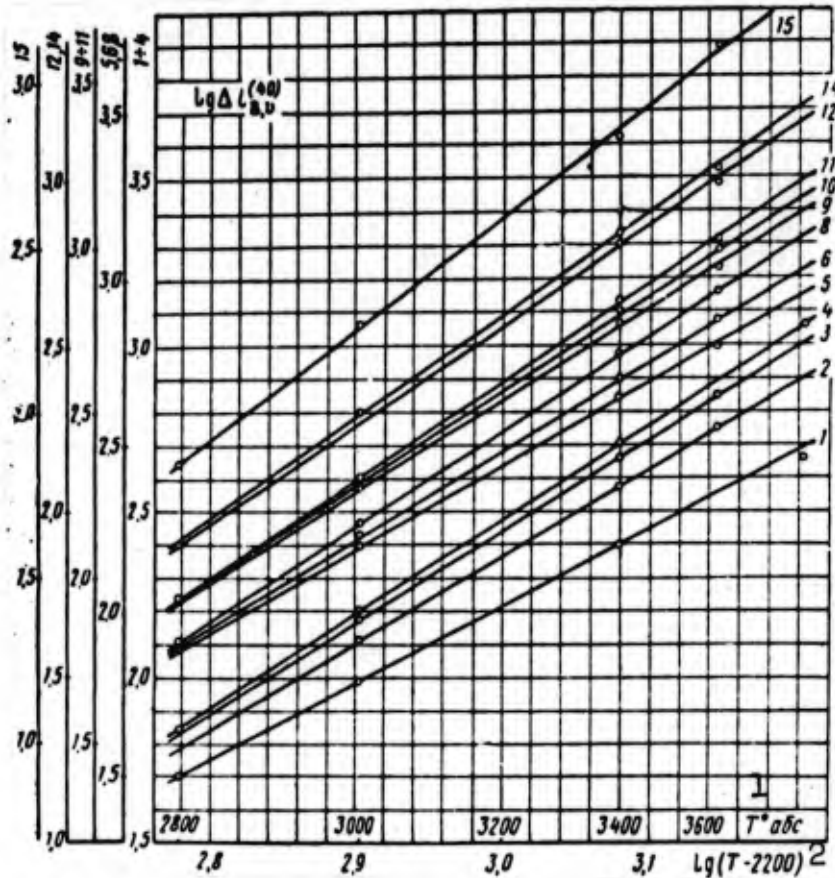


Fig. 3. 1) T° abs; 2) $\lg (T-2200)$.

The exponent m_B is a function only of the parameters X_C and X_H , which characterize the initial chemical composition of the propellant, but it is independent of the quantity α . Numerically, the parameter m_B is equal to the tangent of the angle formed by the temperature axis and the straight lines which represent $\Delta i_{B,v}^{(40)}$ as a function of temperature on the graphs constructed in the above-mentioned coordinates. The magnitude of the parameter m_B for the propellants under consideration in the given work can easily be determined according to a specially constructed nomogram, presented in Fig. 7 of the Appendix.

For the construction of the nomogram, we made use of the results from the same thermodynamic calculations that had been carried out to establish the type of relationship existing between the parameter $\Delta i_{B, v}^{(40)}$ and temperature. The values of X_H have been plotted along the axis of abscissas on the nomogram. The lines of the graph correspond to the fixed values of X_C . No clarification is required for the sequence involved in the determination of the parameter m_B according to the nomogram, for known X_C and X_H .

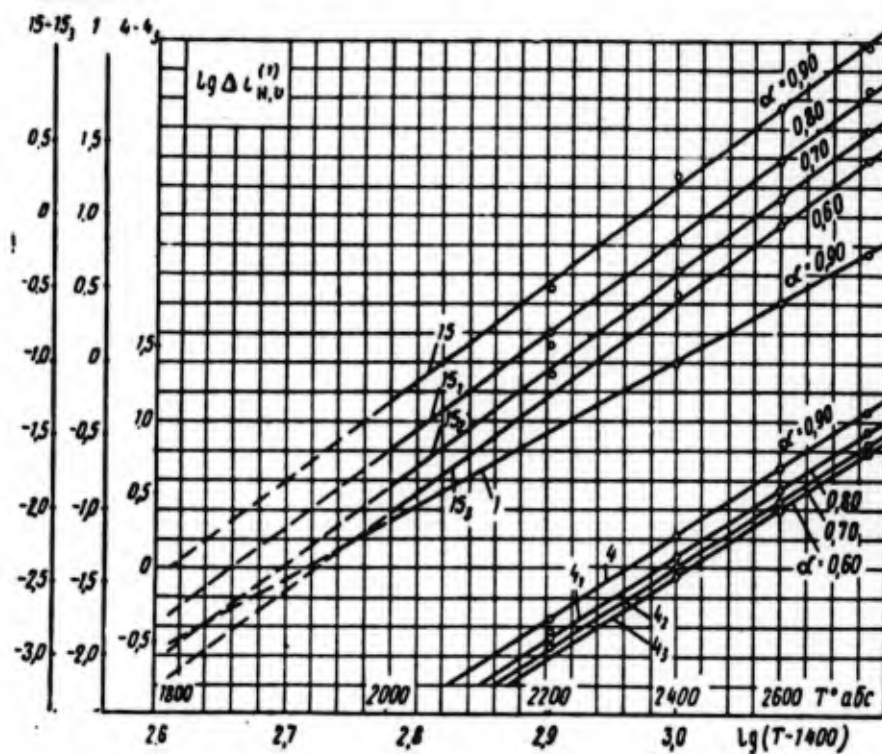


Fig. 4. 1) T° abs; 2) $\log (T-1400)$.

Let us now investigate the parameter Δi as a function of temperature T in the temperature zone "H." We will denote this parameter by $\Delta i_H^{(1)}$. We will employ the same investigational method which was used for the temperature zone "B." We will perform direct calculations for the determination of the parameter $\Delta i_{H, v}^{(1)}$ (at a pressure $p_{H, f} = 1$ atm abs for the dissociated products of combustion), associated with the parameter $\Delta i_H^{(1)}$ by the expression

$$\Delta i_{H, v}^{(1)} = \mu^0 \Delta i_H^{(1)}, \quad (2.25)$$

at various temperatures for the compositions denoted in Table 3 by the conventional numbers 1, 4, 4₁, 4₂, 4₃, 15, 15₁, 15₂, 15₃. The results of the calculations are presented graphically (Fig. 4). The graph has been constructed in logarithmic coordinates: the values of $\log (T-1400)$ have been plotted along the axis of abscissas, and the values of $\log \Delta i_{H, v}^{(1)}$ have been plotted along the axis of ordinates. The support points according to which the curve was constructed are denoted by circles. From an examination of the graph in Fig. 4 it follows that in a graphical representation of the relationship between the parameters $\Delta i_{H, v}^{(1)}$ and T in coordinates $[\log \Delta i_{H, v}^{(1)}, \log (T-1400)]$ this relationship for the propellants under consideration becomes virtually linear. The linear function is preserved at values for the coefficient of excess oxygen α between 0.60 to 0.90. The angle formed by the positive segment of the axis of abscissas and the lines of the graph for one and the same composition (X_C, X_H) but for various values of α , remains the same at all values of α assumed for purposes of the calculation.

We should take note of the fact that linearity of the function is impaired at temperatures below 2200° abs. The reason for this may be the actual disruption of the quantitative relationship at comparatively low temperatures, as well as the circumstance that the parameter $\Delta i_{H, v}^{(1)}$ becomes so small that in its calculation noticeable errors might arise. However, in view of the absolute smallness of the magnitude of $\Delta i_{H, v}^{(1)}$ at these temperatures we can assume that the established linear function is not violated, even at temperatures below 2200° abs.

If we take into consideration all that has been said above, we can maintain with sufficient reliability that for all compositions under consideration and throughout the entire temperature zone "H"

there exists an approximate proportionality of the form

$$\lg \Delta i_{n,v}^{(1)} \sim m_n \lg (T - 1400)$$

or, converting to conventional terms,

$$\Delta i_{n,v}^{(1)} \sim (T - 1400)^{m_n}. \quad (2.26)$$

The exponent m_H is a function only of the parameters X_C and X_H which characterize the initial chemical composition of the propellant, but it is independent of the quantity α . Numerically, the parameter m_H for the propellants under consideration may be determined according to the nomogram presented in Fig. 8 of the Appendix. This nomogram is constructed in similar fashion to the nomogram for the determination of the parameter m_B . The sequence of operations with this parameter requires no clarification.

Let us now return to the temperature zone "B." We will assume that for the products of combustion of any of the propellants under consideration, direct calculation was used to determine the value of the parameter $\Delta i_{B,v}^{(40)}$ at a certain temperature $T_{B,f}$ (and at a pressure $p_{B,f} = 40$ atm abs). We will denote this parameter as $\Delta i_{B,v,f}$. In this case, in accordance with Expression (2.24) there arises the possibility of calculating the value of the parameter $\Delta i_{B,v}^{(40)}$ for the given propellant at any temperature T (but at the same pressure $p_{B,f} = 40$ atm abs), without resorting to direct calculations for the determination of $\Delta i_{B,v}^{(40)}$ at this temperature. In actual fact, it follows from Expression (2.24) that

$$\Delta i_{n,v}^{(40)} = \Delta i_{n,v,\phi} \left(\frac{T - 2200}{T_{n,\phi} - 2200} \right)^{m_n}. \quad (2.27)$$

The utilization of Expression (2.27) in the thermodynamic calculation for rocket engines is possible only if we will have found a sufficiently simple method of determining the parameter $\Delta i_{B,v,f}$ at the temperature $T_{B,f}$ and pressure $p_{B,f} = 40$ atm abs for the

products of combustion of the propellant under consideration.

There are possibilities for the establishment of such a method. The parameter $\Delta i_{B, v, f}$, determined at the selected values of $T_{B, f}$ and $p_{B, f}$, is a function only of the initial chemical composition, i.e., of the independent variables X_C , X_H and α . It would be expedient to assume a series of values for the coefficient of excess oxygen α and for each of these values to construct a nomogram for the determination of the parameter $\Delta i_{B, v, f}$ for various values of X_C and X_H .

At which temperature $T_{B, f}$ should the calculations of the parameter $\Delta i_{B, v, f}$ and the subsequent construction of the nomograms be carried out? We have pointed out earlier that the sought functions in the temperature zone "B" will be used primarily for the calculation of the thermodynamic characteristics of the products of combustion in the combustion chambers of rocket engines. On the basis of these considerations, we will select $T_{B, f} = 3400^\circ$ abs.

Figures 9 to 14 of the Appendix show the nomograms for the determination of the parameter $\Delta i_{B, v, f}$ at a pressure $p_{B, f} = 40$ atm abs and a temperature $T_{B, f} = 3400^\circ$ abs., constructed for values of α equal to 0.95, 0.90, 0.80, 0.70, 0.60, and 0.55. The nomograms have been constructed on the basis of direct calculations of the parameter $\Delta i_{B, v, f}$ for the propellants presented in Table 3. The values of the parameter X_H are plotted on the axis of abscissas, and the lines on the nomograms correspond to the fixed values of the parameter X_C . The sought quantity $\Delta i_{B, v, f}$ is determined according to the ordinate of the point at which the perpendicular dropped to the axis of abscissas from the point on the axis, corresponding to the value of X_H , intersects with the line which corresponds to the value of X_C .

If we take into consideration Expressions (2.23) and (2.27),

we will find an equation which associates the quantity $\Delta i_B^{(40)}$ in the temperature zone "B" with the temperature T of these products (at a pressure $p_{B,f} = 40$ atm abs):

$$\Delta i_B^{(40)} = \frac{\Delta i_{B,v,\phi}}{40\mu^0} \left(\frac{T-2200}{T_{B,\phi}-2200} \right)^{m_B}. \quad (2.28)$$

The simultaneous examination of Eqs. (1.36), (2.20), and (2.28) will make it possible to derive a formula for the determination of the heat content in 1 kg of dissociated products of combustion in the temperature zone "B" (at a pressure $p_{B,f} = 40$ atm abs):

$$i_B^{(40)} = i_{2800}^0 + c_{p,B}^0 (T-2800) + \frac{\Delta i_{B,v,\phi}}{40\mu^0} \left(\frac{T-2200}{T_{B,\phi}-2200} \right)^{m_B}. \quad (2.29)$$

Let us return to the temperature zone "H." We will assume that for the products of combustion of any of the propellants under consideration, the value of the parameter $\Delta i_{H,v}^{(1)}$ was determined directly at a certain temperature $T_{H,f}$ (and at a pressure $p_{H,f} = 1$ atm abs). We will denote this parameter by $\Delta i_{H,v,f}$. In accordance with Expression (2.26) in this case there arises the possibility of calculating the value of the parameter $\Delta i_{H,v}^{(1)}$ for the given propellant at any temperature T (but at the same pressure $p_{H,f} = 1$ atm abs.), without undertaking any direct calculations for the determination of $\Delta i_{H,v}^{(1)}$ at these temperatures. Indeed, it follows from (2.26) that

$$\Delta i_{H,v}^{(1)} = \Delta i_{H,v,\phi} \left(\frac{T-1400}{T_{H,\phi}-1400} \right)^{m_H}. \quad (2.30)$$

The parameter $\Delta i_{H,v,f}$, determined at selected values of $p_{H,f}$ and $T_{H,f}$ is a function only of the independent variables X_C , X_H , and α . It is expedient to assume a series of values for α and to construct a nomogram for each of these values to determine the parameter $\Delta i_{H,v,f}$ at various values of X_C and X_H .

The value of $T_{H,f}$ must be close to the temperatures for the products of combustion at the outlet section of the nozzle. However, it must simultaneously be sufficiently high in order for the dis-

sociation of the products of combustion to be perceptible, and for the numerical value of the parameter $\Delta i_{H,v,f}$ to be sufficiently great. Only in this case will the error of the determination of the parameter $\Delta i_{H,v,f}$ in its calculation not exceed the permissible. On the basis of these considerations we select $T_{H,f} = 2800^\circ$ abs.

Figures 15 to 20 of the Appendix show the nomograms for the determination of the parameter $\Delta i_{H,v,f}$ at a pressure $p_{H,f} = 1$ atm abs and a temperature $T_{H,f} = 2800^\circ$ abs., constructed at the same values of α at which the nomograms for the determination of the parameter $\Delta i_{B,v,f}$ were constructed for the temperature zone "B."

The nomograms were constructed on the basis of direct calculations of the parameter $\Delta i_{H,v,f}$ for the compositions presented in Table 3. The determination of the parameter $\Delta i_{H,v,f}$ according to the nomograms is carried out in a manner similar to the determination of the parameter $\Delta i_{B,v,f}$.

Taking into consideration Expressions (2.25) and (2.30), we will find

$$\Delta i_n^{(1)} = \frac{\Delta i_{n,v,\phi}}{\mu^0} \left(\frac{T-1400}{T_{n,\phi}-1400} \right)^{m_n}. \quad (2.31)$$

Solving Eqs. (1.37), (2.20), and (2.31) simultaneously, we will derive a formula for the determination of the heat content in 1 kg of dissociated products of combustion in the temperature zone "H" (at a pressure $p_{H,f} = 1$ atm abs):

$$i_n^{(1)} = i_{2800}^0 + c_{p,n}^0 (T - 2800) + \frac{\Delta i_{n,v,\phi}}{\mu^0} \left(\frac{T-1400}{T_{n,\phi}-1400} \right)^{m_n}. \quad (2.32)$$

6. The Relationship Between the Increment in Heat Content and the Increment in the Number of Gram-Molecules for the Products of Combustion, on Their Dissociation

Let us investigate the ratio

$$q = \frac{i - i^0}{M - M^0} = \frac{\Delta i}{\Delta M}. \quad (2.33)$$

From an examination of Expression (2.33) we can come to the

conclusion that the parameter q is a ratio between parameters closely related to one another. As a result of the dissociation of the products of combustion of any of the propellants under consideration both the number of gram molecules of the products of combustion as well as their heat content increase simultaneously. The intensity

TABLE 5

1 Условный номер химического состава	X_C	X_H	X_N	2 $p=40 \text{ атм}$				$p=1 \text{ атм}$			
				3 $T=3400^\circ \text{ абс.}$		$T=3000^\circ \text{ абс.}$		$T=2800^\circ \text{ абс.}$		$T=2400^\circ \text{ абс.}$	
				α	q	α	q	α	q	α	q
1	4	8	64	0,90	132,4	0,90	140,7	0,90	132,2	0,60	128,0
2	4	30	42	0,90	131,0	0,90	139,0	0,94	129,3	0,90	122,4
3	4	52	20	0,76	129,0	0,89	125,3	0,89	141,1	—	—
4	4	72	0	0,62	128,7	0,90	127,9	0,90	125,3	0,90	124,6
5	14	8	44	0,90	127,0	0,90	127,9	0,90	125,3	0,90	124,6
6	14	8	44	0,90	135,4	0,89	130,8	0,92	133,4	0,90	129,9
7	14	20	32	0,63	122,6	0,88	135,3	0,94	131,9	—	—
8	14	20	32	0,90	135,0	0,88	135,3	0,94	131,9	—	—
9	14	40	12	0,66	128,7	—	—	0,90	129,6	—	—
10	14	40	12	0,90	131,5	—	—	0,90	129,6	—	—
11	14	52	0	0,71	127,4	—	—	0,90	129,6	—	—
12	14	52	0	0,90	128,4	0,89	129,8	0,92	128,1	0,90	131,9
13	25	8	22	0,70	124,7	0,89	129,8	0,92	128,1	0,90	131,9
14	25	8	22	0,90	132,5	0,90	131,5	0,88	132,4	0,86	133,8
15	25	8	22	0,66	127,6	0,90	131,5	0,88	132,4	0,86	133,8
16	25	20	10	0,90	133,0	0,89	133,6	0,93	131,2	0,93	133,0
17	25	20	10	0,67	128,6	0,89	133,6	0,93	131,2	0,87	134,6
18	25	30	0	0,90	132,0	0,90	130,7	0,92	129,5	0,93	131,7
19	25	30	0	0,68	126,0	0,90	130,7	0,92	129,5	0,86	130,0
20	25	30	0	0,90	132,5	0,91	134,6	0,94	130,4	0,99	135,5
21	30	8	12	0,68	131,2	0,91	134,6	0,94	130,4	0,99	135,5
22	30	8	12	0,90	131,3	—	—	0,80	122,3	—	—
23	30	14	6	0,70	130,0	—	—	0,80	122,3	—	—
24	30	14	6	0,90	131,3	—	—	0,80	122,3	—	—
25	30	20	0	0,90	131,3	0,90	130,8	0,92	129,8	0,99	131,9
26	30	20	0	0,70	127,1	0,90	130,8	0,92	129,8	0,99	131,9
27	36	8	0	0,90	129,0	0,90	127,0	0,90	130,0	—	—
28	36	8	0	0,90	129,0	0,90	127,0	0,80	129,2	—	—
29	36	8	0	0,90	129,0	0,90	127,0	0,70	127,3	—	—

1) Conventional number for chemical composition; 2) $p = 40 \text{ atm abs}$;
3) $T = 3400^\circ \text{ abs}$.

with which these two parameter increase is determined primarily by the temperature of the products of combustion and is, from all that can be seen, approximately equal for each.

Let us investigate the effect of such independent variables as the elementary chemical composition of the propellant (X_C , X_H , α), the pressure p and the temperature T of the products of combustion on the magnitude of q for the class of propellants under consideration.

There does not exist an exact analytical relationship between the quantities Δi and ΔM , which would not require a preliminary calculation of the composition of the products of combustion. An investigation of the parameter \underline{q} may be carried out only in accordance with the results obtained through direct calculations of the quantities Δi and ΔM for the products of combustion from propellants of various compositions, at various temperatures and pressures. For this purpose, calculations were carried out for the quantities Δi and ΔM , and at the same time for the parameter \underline{q} , for the compositions presented in Table 3, denoted by the conventional numbers 1 through 15, with the coefficients of excess oxygen (α) ranging from 0.60 to 0.99, at temperatures ranging from 2400 to 3400° abs, and for pressures of 1 and 40 atm abs for the dissociated products of combustion.

The results of the calculations are presented in Table 5.

An analysis of the table indicates that for the adopted compositions, given the temperature and pressure of the products of combustion, the parameter \underline{q} is a virtually constant quantity. The averaged value of the parameter \underline{q} , obtained according to the results of 69 separate calculations, is close to 130:

$$q = \frac{\Delta i}{\Delta M} \cong 130. \quad (2.34)$$

The scattering of the values for the parameters \underline{q} for individual propellants does not exceed, as can be seen from Table 5, 8.5% of its average value. This figure includes, of course, errors which might have been permitted in the calculations of \underline{q} .

Table 5 covers such a wide class of chemical compositions, as well as such a wide range of temperatures and pressures for the products of combustion that it is difficult to assume that this relative constancy of \underline{q} will not be maintained under slightly altered

conditions.

It is well known that with a gradual change in the chemical composition of a propellant, as well as with a gradual change in the pressure and temperature of the products of combustion, the thermodynamic parameters of the products of combustion change smoothly, monotonically. As confirmation of this, we have the experience gained in the thermodynamic calculations for rocket engines and the data obtained in hot-firing tests.

7. The Relationship Between the Increment in the Heat Content of the Products of Combustion as a Result of Dissociation and the Pressure of the Products of Combustion

In the present section we are investigating the relationship between the parameters Δi and \underline{p} for the products of combustion of the propellants under consideration.

It is well known that there exists no analytical relationship between these parameters that does not require the preliminary calculation of the composition of the products of combustion. Earlier, other authors established only that the relative increment in the pressure of the products of combustion ($\omega = \Delta p/p^0 = (p-p^0)/p^0$) under the conditions of dissociation, taking place at constant value and temperature, is a function of the absolute magnitude of pressure, and here this function can be described by the approximate interpolation formula

$$\omega = Bp^{-n_1}. \quad (2.35)$$

Here B is some coefficient, and n_1 is an exponent, in absolute terms less than 1.

Formula (2.35) establishes that an increase in the pressure of the products of combustion during dissociation is proportional to the negative fractional exponent of the pressure. In principle, this formula makes it possible to calculate the value of the param-

eter ω at any pressure \underline{p} , if we know the value of ω_x at any pressure p_x :

$$\omega = \omega_x \left(\frac{p}{p_x} \right)^{-n_1}. \quad (2.36)$$

A similar formula may be written for the parameter Δi — the increment in heat content for the products of combustion as a result of dissociation:

$$\Delta i = \Delta i_x \left(\frac{p}{p_x} \right)^{-n_2}. \quad (2.37)$$

It was demonstrated through a series of examples that the values of the exponents n_1 and n_2 are close to one another, in which case for the products of combustion of the propellants under considerations, in the case of $\alpha < 1$, we can assume in approximate terms that

$$n_1 = n_2 = 0.5.$$

A satisfactory degree of accuracy for the determination of the parameters ω and Δi in Formulas (2.36) and (2.37), given the condition $n_1 = n_2 = 0.5$, may be obtained only if the values of \underline{p} and p_x are close. At values of \underline{p} and p_x , sufficiently far apart, we cannot assume that the exponent is equal to 0.5; this quantity must be determined by special calculation. For a calculation of the value of n_1 or n_2 , we can use the same Formulas (2.36) or (2.37), having determined preliminarily, by direct calculation, the parameters ω or Δi for no less than two values of the pressure for the products of combustion. To increase the accuracy in the subsequent calculations, the parameter n_1 (or n_2) must be determined at values of pressure for the products of combustion, comparable to \underline{p} and p_x , respectively.

In addition to pressure, temperature also has an effect on the quantities n_1 and n_2 . In the general case, the parameters n_1 and n_2 will be functions of the chemical composition of the propellant, the

pressure, and the temperature. The thermodynamic calculations for rocket engines that we carried out indicate, however, that it is possible, without great error in the final calculation results; to operate with a single value for the exponent for the given propellant. We will refer to this exponent as the effective exponent n_{ef} .

The magnitude of n_{ef} must be determined at values of the pressure and temperature of the products of combustion that are characteristic of rocket engines. We will assume, as before, that a pressure of 40 atm abs in the combustion chamber and 1 atm abs at the outlet section of the nozzle are characteristic. The temperature of the products of combustion at which dissociation becomes noticeable is limited, in the region under consideration, approximately by the values 1800 and 3800° abs. The actual relationship between n_1 (or n_2) and temperature is not known. We know only that with an increase in the temperature of the products of combustion, a temperature at which the calculation is carried out, the numerical values of the exponent, in the majority of cases, diminish smoothly. We will assume that there exists a linear function between the parameters n_1 (or n_2) and temperature in the range of from 1800 to 3800° abs. In this case, as the value of the parameter n_{ef} for temperatures from 1800 to 3800° abs., we can accept the value of n_1 (or n_2), calculated for 2800° abs. Consequently,

$$n_{ef} \cong n_{2800}. \quad (2.38)$$

The determination of n_{2800} at pressure values of 40 and 1 atm abs and a temperature of 2800° abs., can be carried out in terms of parameter ω according to Formula (2.36) or in terms of Δi according to Formula (2.37).

Using the interrelationship between these quantities that we established earlier (see Chapter II, Section 6) we will demonstrate

that the values of n_1 and n_2 are close to one another.

In fact, since

$$\omega = \frac{\Delta p}{p} = \frac{\Delta M}{M^0},$$

in accordance with (2.34)

$$\frac{\Delta I}{\omega} = 130 M^0 = \text{const}, \quad (2.39)$$

whence for the given propellant

$$n_1 = n_2 = \frac{\lg \frac{\omega}{\omega_x}}{\lg \frac{p_x}{p}} = \frac{\lg \frac{\Delta I}{\Delta I_x}}{\lg \frac{p_x}{p}}. \quad (2.40)$$

Consequently, the equality of the values of n_1 and n_2 is maintained to the extent to which the constancy of \underline{q} is maintained for the propellants under consideration.

For the determination of $n_{ef} = n_{2800}$ we will find the increment in the heat content of the products of combustion at a temperature of 2800° abs., and a pressure of 40 atm abs according to Eq. (2.28):

$$\Delta i_{\phi}^{(40)} = \frac{\Delta i_{\phi, v, \phi}}{40 \mu^0} \left(\frac{2800 - 2200}{T_{\phi, \phi} - 2200} \right)^{m_{\phi}} = \frac{0,5^{m_{\phi}} \Delta i_{\phi, v, \phi}}{40 \mu^0}$$

(since $T_{B, f} = 3400^\circ$ abs.).

For this same purpose we will use Formula (2.25) to find the magnitude of $\Delta i_H^{(1)}$ at the same temperature of 2800° abs., (since $T_{H, f} = 2800^\circ$ abs.), but at a pressure of 1 atm abs (or $p_{H, f} = 1$ atm abs):

$$\Delta i_H^{(1)} = \frac{\Delta i_{H, v, \phi}}{\mu^0}.$$

Then, in accordance with (2.37)

$$\frac{\Delta i_H^{(1)}}{\Delta i_{\phi}^{(40)}} = \frac{40 \Delta i_{H, v, \phi}}{0,5^{m_{\phi}} \Delta i_{\phi, v, \phi}} = \left(\frac{40}{1} \right)^{n_{\phi}},$$

whence

$$n_{\phi} = 0,6242 \lg \frac{40 \cdot 2^{m_{\phi}} \Delta i_{H, v, \phi}}{\Delta i_{\phi, v, \phi}}. \quad (2.41)$$

The parameters m_B , $\Delta i_{H,v,f}$, and $\Delta i_{B,v,f}$ are determined in the calculation carried out in accordance with the nomograms constructed earlier.

It becomes possible, with the determination of the parameter n_{ef} to calculate the increment in heat content as a result of the dissociation of the products of combustion at any pressure p , if we know the increment in heat content for these products at a certain selected pressure (and at the same temperature). In the temperature zone "B" the nomograms shown in Figs. 9-14 of the Appendix make it possible to determine the increment $\Delta i_{B,v,f}$ in heat content at a pressure of $p_{B,f} = 40$ atmospheres absolute and a temperature $T_{B,f} = 3400^\circ$ abs. At any other pressure in this zone, but at the same temperature of 3400° abs, the increment in heat content will be equal to

$$\Delta i_{B,v}^{(3400)} = \Delta i_{B,v,\phi} \left(\frac{p_{B,\phi}}{p} \right)^{n_{\phi}}. \quad (2.42)$$

In the joint consideration of Eqs. (2.28) and (2.42) we derive the equation which describes the increment Δi_B in heat content for 1 kg of products of combustion as a function of the chemical composition of the propellant, of pressure, and of the temperature in the temperature zone "B":

$$\Delta i_B = \frac{\Delta i_{B,v,\phi}}{40^{n_{\phi}}} \left(\frac{p_{B,\phi}}{p} \right)^{n_{\phi}} \left(\frac{T-2200}{T_{B,\phi}-2200} \right)^{m_B}. \quad (2.43)$$

Denoting

$$c'_B = \frac{\Delta i_{B,v,\phi}}{40^{n_{\phi}}} \left(\frac{p_{B,\phi}}{p} \right)^{n_{\phi}}, \quad (2.44)$$

we will obtain

$$\Delta i_B = c'_B \left(\frac{T-2200}{T_{B,\phi}-2200} \right)^{m_B}. \quad (2.45)$$

The equation for the heat content of 1 kg of dissociated products of combustion in the temperature zone "B" is determined through the joint consideration of Eqs. (1.36), (2.20), and (2.45):

$$i_p = i_{2800}^0 + c_{p,n}^0 (T - 2800) + c'_n \left(\frac{T - 2200}{T_{n,\phi} - 2200} \right)^{m_n}. \quad (2.46)$$

In the temperature zone "H" the nomograms in Figs. 15-20 of the Appendix make it possible to determine the increment $\Delta i_{H,v,f}$ in heat content at a pressure of $p_{H,f} = 1$ atmospheres absolute and a temperature $T_{H,f} = 2800^\circ$ abs. Consequently, at any other pressure in this zone, but at the same temperature, the increment in heat content for the products of combustion is equal to

$$\Delta i_{n,v}^{(2800)} = \Delta i_{n,v,\phi} \left(\frac{p_{n,\phi}}{p} \right)^{n_{\phi}}. \quad (2.47)$$

Through the simultaneous examination of Eqs. (2.31) and (2.47) we will derive the equation which describes the increment Δi_H in the heat content of 1 kg of products of combustion as a function of the chemical composition of the propellant, of pressure, and of the temperature in the temperature zone "H":

$$\Delta i_n = \frac{\Delta i_{n,v,\phi}}{\mu^0} \left(\frac{p_{n,\phi}}{p} \right)^{n_{\phi}} \left(\frac{T - 1400}{T_{n,\phi} - 1400} \right)^{m_n}. \quad (2.48)$$

Denoting

$$c'_n = \frac{\Delta i_{n,v,\phi}}{\mu^0} \left(\frac{p_{n,\phi}}{p} \right)^{n_{\phi}}, \quad (2.49)$$

we will obtain

$$\Delta i_n = c'_n \left(\frac{T - 1400}{T_{n,\phi} - 1400} \right)^{m_n}. \quad (2.50)$$

The equation for the heat content of 1 kg of dissociated products of combustion in the temperature zone "H" is determined through the simultaneous examination of Eqs. (1.37), (2.20), and (2.50):

$$i_n = i_{2800}^0 + c_{p,n}^0 (T - 2800) + c'_n \left(\frac{T - 1400}{T_{n,\phi} - 1400} \right)^{m_n}. \quad (2.51)$$

8. The Entropy of the Products of Combustion

It is the purpose of this section to derive the equations which will make it possible to calculate the entropy of the dissociated products of combustion at a temperature T and a pressure p without any

preliminary calculation of their chemical composition.

We will select a temperature $T_1 = 1400^\circ$ abs , and a pressure $p_1 = 1$ atmospheres absolute as the parameters for the entropy origin. The temperature $T_1 = 1400^\circ$ abs is the lower boundary of the temperature region under consideration and its selection as the temperature origin therefore requires no additional clarification. It should be pointed out that the selection of any other temperature as the entropy origin would have no effect on the results of the thermodynamic calculations for engines, since the absolute entropy value has no effect on these results. We will assume that under these initial conditions, the entropy for the products of combustion is equal to S_0 , the products of combustion consisting of carbon dioxide, carbon monoxide, water vapors, and biatomic nitrogen and hydrogen, each in a state of chemical equilibrium with respect to the others.

The transition from the initial parameters T_1 and p_1 for the products of combustion to the given T and p may be accomplished through the following three reversible processes, in which a change in entropy can be determined quite simply. The determination of entropy in all of these processes is carried out by the integration of the following differential equation:

$$dS = \frac{di}{T} - v \frac{dp}{T}. \quad (2.52)$$

The first process. The products of combustion are reversibly compressed from a pressure of $p_1 = 1$ atmospheres absolute to a high pressure of p^0 at a constant temperature $T_1 = 1400^\circ$ abs. During this process, the chemical composition of the products of combustion undergoes no change; therefore, the change in the heat content of the products of combustion is $di = 0$ (the products of combustion, by definition, represent an ideal gas).

A change in entropy in this process is found through the integration of Expression (2.52) at $T = \text{const}$ and $\mu^0 = \text{const}$.

It is equal to

$$\Delta S_1 = -\frac{\bar{R}}{\mu^0} \ln p^0. \quad (2.53)$$

The second process. The products of combustion are reversibly heated at a constant pressure p^0 (without dissociation) from a temperature $T_1 = 1400^\circ \text{ abs}$ to a temperature T . The heat content for the products of combustion up to a temperature of 2800° abs in this case is determined by Expression (2.51), and above 2800° abs by Expression (2.46). Having entered these values into Expression (2.52) and integrating this expression at a constant pressure ($p^0 = \text{const}$) we find the change in entropy in the second process if the value of the temperature T falls within the temperature zone "B" ($2800\text{-}3800^\circ \text{ abs}$):

$$\begin{aligned} \Delta S_{2, n} &= \int_{1400}^{2800} \frac{dl_n^0}{T} + \int_{2800}^T \frac{dl_n^0}{T} = \\ &= c_{p, n}^0 \ln \frac{2800}{1400} + c_{p, n}^0 \ln \frac{T}{2800}. \end{aligned} \quad (2.54)$$

If the value of the temperature T lies within the temperature zone "H" ($1400\text{-}2800^\circ \text{ abs}$), then

$$\Delta S_{2, n} = \int_{1400}^T \frac{dl_n^0}{T} = c_{p, n}^0 \ln \frac{T}{2800}. \quad (2.55)$$

The third process. The products of combustion expand reversibly from a pressure p^0 to the values of pressure p under consideration at a constant value for the final temperature T . The change in entropy in the third process is equal to

$$\Delta S_3 = \frac{1}{T} \int_{p^0}^p (di - v dp) = \frac{i - i^0}{T} - \int_{p^0}^p \frac{\bar{R}(1 + \epsilon)}{\mu^0 p} dp =$$

$$= \frac{\Delta i}{T} - \frac{\bar{R}}{\mu^0} \ln \frac{p}{p^0} - \frac{\bar{R}}{\mu^0} \int_{p^0}^p \frac{\omega dp}{p}, \quad (2.56)$$

since

$$\mu = \frac{\mu^0}{1 + \omega}. \quad (2.57)$$

The relative fraction of the last term of Expression (2.56) amounts to 4 to 8% of the total change in entropy. Therefore, the integral $\int_{p^0}^p (\omega dp/p)$ can be calculated with limited accuracy. From Expression (2.39) we obtain

$$\omega = \frac{\mu^0 \Delta i}{130}.$$

Consequently,

$$\int_{p^0}^p \frac{\omega dp}{p} = \frac{\mu^0}{130} \int_{p^0}^p \frac{\Delta i dp}{p}.$$

Substituting the value of Δi_B , as given by Expression (2.43), we find the value of this integral for the case of temperature zone "B":

$$\left(\int_{p^0}^p \frac{\omega dp}{p} \right)_B = \frac{\Delta i_{B,v,f}}{40 \cdot 130} \left(\frac{T - 2200}{T_{B,\phi} - 2200} \right)^{n_B} p_{B,\phi}^{n_B} \int_{p^0}^p \frac{dp}{p^{n_B+1}},$$

since the parameters $\Delta i_{B,v,f}$, n_{ef} , $T_{B,f}$, $p_{B,f}$, and m_B are independent of pressure.

Since at a pressure p^0 there are no dissociation reactions,

$$\int_{p^0}^p \frac{dp}{p^{n_B+1}} = - \frac{1}{n_B p^{n_B}}.$$

Consequently, for the case of temperature zone "B"

$$\left(\int_{p^0}^p \frac{\omega dp}{p} \right)_B = - \frac{c'_B \mu^0}{130 n_B} \left(\frac{T - 2200}{T_{B,\phi} - 2200} \right)^{n_B}, \quad (2.58)$$

where c'_B is determined from Expression (2.44).

We find the value of the integral for the case of the temperature zone "H" in similar fashion:

$$\left(\int_{p_0}^p \frac{u dp}{p} \right)_n = - \frac{c'_{n, \mu^0}}{130 n_{\phi}} \left(\frac{T-1400}{T_{n, \phi}-1400} \right)^{m_n} \quad (2.59)$$

where c'_{H} is determined from Expression (2.49).

The entropy of the products of combustion having temperature T and pressure p is equal to

$$S = S_0 + \Delta S_1 + \Delta S_2 + \Delta S_3.$$

There is no need to know the absolute value of the entropy of the products of combustion in these calculations, but we must know only the entropy difference that is found in the processes being calculated. Therefore, the value of the initial entropy S_0 may be selected arbitrarily. For the sake of simplicity in the calculation formulas, we will select the following initial value of entropy

$$S_0 = -c'_{p, n} \ln \frac{2800}{1400}. \quad (2.60)$$

The equation for the entropy of the products of combustion for the case of the temperature zone "B" is derived through the joint consideration of Expressions (2.53), (2.54), (2.56), (2.58), and (2.60):

$$S_p = 2,303 c'_{p, s} \lg \frac{T}{2800} - 2,303 \frac{\bar{R}}{\mu^0} \lg p + c'_s \left(\frac{T-2200}{T_{s, \phi}-2200} \right)^{m_s} \left(\frac{1}{T} + \frac{\bar{R}}{130 n_{\phi}} \right). \quad (2.61)$$

Similarly, for the case of the temperature zone "H" we will obtain

$$S_n = 2,303 c'_{p, n} \lg \frac{T}{2800} - 2,303 \frac{\bar{R}}{\mu^0} \lg p + c'_n \left(\frac{T-1400}{T_{n, \phi}-1400} \right)^{m_n} \left(\frac{1}{T} + \frac{\bar{R}}{130 n_{\phi}} \right). \quad (2.62)$$

Equations (2.61) and (2.62) make it possible to calculate the entropy of the products of combustion at their pressure p and temperature T without any preliminary calculation of their chemical composition. These equations, used in conjunction with the equations established

earlier for the heat content of the products of combustion, and presented in this work in the form of nomograms, make it possible to develop a new method of thermodynamic calculation for rocket engines operating on the propellants under consideration. The new method of calculation for engines is examined in the following chapter.

Chapter 3
THERMODYNAMIC CALCULATION METHOD FOR
LIQUID-FUEL ROCKET ENGINES

9. The Temperature of the Products of Combustion in the Combustion Chamber of the Engine

Let us assume that we are given certain components of a propellant — the fuel and the oxidizer — and that we know both their chemical composition and the value of the coefficient of excess oxidizer α'_0 at which these components burn up within the combustion chamber of the engine, and let us also assume that we know the pressure $p_{k.s}$ of the products of combustion. We must find the temperature of the products of combustion of the propellant under given conditions.

In carrying out the calculation with the same premises and assumptions which were enumerated in the "Introduction," the theoretical temperature of the products of combustion $T_{k.s}$ in the combustion chamber of the engine can be determined from the conditions of equality between the heat content of a unit weight (1 kg) of the products of combustion and the heat content of the same unit weight of propellant

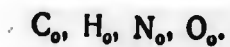
$$i_r = i_{r,c}. \quad (3.1)$$

The heat content of the unit weight (1 kg) of propellant may be known or can be found through separate calculation. The method used to determine this heat content is demonstrated in Section 20 of this book. Here we will assume that the value of the parameter i_t is known.

To calculate the value of temperature $T_{k.s}$ we will assume the

following calculation sequence.

1. We will determine, in grams, the mass content of the chemical elements in 1 kg of oxidizer:



2. We will determine, in grams, the mass content of the chemical elements in 1 kg of fuel (combustible):



3. We will determine the quantity of oxidizer required for the oxidation of 1 kg of fuel:

$$v_o = \frac{\frac{8}{3} C_r + 8H_r - O_r}{O_o - \frac{8}{3} C_o - 8H_o} \quad (3.2)$$

4. We will assume several values for the coefficient of excess oxygen (α), i.e., precisely those values for which the nomograms presented in Figs. 1 to 6 and 9 to 20 of the Appendix were constructed (0.95, 0.90, 0.80, 0.70, 0.60, and 0.55), and we will determine the corresponding values for the coefficient of excess oxidizer α_o according to the following formula:

$$\alpha_o = \frac{1}{v_o} \frac{\frac{8}{3} C_r + 8H_r - \frac{O_r}{\alpha}}{\frac{O_o}{\alpha} - \frac{8}{3} C_o - 8H_o} \quad (3.3)$$

We will determine the value of α' , corresponding to the given α'_o :

$$\alpha' = \frac{O_r + \alpha'_o v_o}{\frac{8}{3} (C_r + \alpha'_o v_o C_o) + 8(H_r + \alpha'_o v_o H_o)} \quad (3.3a)$$

5. We will calculate the weight ratio v for the propellant components at the selected values of α and the obtained value of α' according to the following formula:

$$v = \alpha_o v_o \quad (3.4)$$

6. We will find the number of gram-atoms of the chemical elements carbon, hydrogen, and nitrogen in 1 kg of propellant at the selected

values of α and the obtained value of α' according to the following formulas:

$$\left. \begin{aligned} A_C &= \frac{C_r + vC_o}{12(1+v)}; \\ A_H &= \frac{H_r + vH_o}{1+v}; \\ A_N &= \frac{N_r + vN_o}{14(1+v)}. \end{aligned} \right\} \quad (3.5)$$

7. At the selected values of α and the obtained value of α' , we will determine the parameter μ^0 according to the following formula:

$$\mu^0 = \frac{1}{M^0} = \frac{1}{A_C + 0,5(A_H + A_N)}. \quad (3.6)$$

Formula (3.6) is derived as a result of the joint consideration of Expressions (1.4), (1.5), and (1.7).

8. According to Formulas (1.39) and (1.40) we will find the values for the parameters $X_C = X_C^0$ and $X_H = X_H^0$ at the selected values of α and the obtained value of α' :

$$X_C = 40\mu^0 A_C; \quad (1.39)$$

$$X_H = 40\mu^0 A_H. \quad (1.40)$$

9. According to the nomograms in Figs. 1 to 6 and 9 to 20 of the Appendix, we will determine the values of the parameters i_v^0 , $\Delta i_{B,v,f}$, and $\Delta i_{H,v,f}$ for the selected values of α .

10. We will construct an auxiliary curve which indicates the change in the parameters i_v^0 , $\Delta i_{B,v,f}$, and $\Delta i_{H,v,f}$ as functions of the change in α .

11. We will employ the methods of graphical interpolation to determine the values of the parameters i_v^0 , $\Delta i_{B,v,f}$, and $\Delta i_{H,v,f}$, which correspond to the given value of the coefficient of excess oxidizer (α'_0) (i.e., correspond to the obtained value of α').

A further calculation of the temperature $T_{k.s}$ of the products of combustion within the combustion chamber of the engine, in accordance with the conditions of the problem, will be carried out only for the

obtained value of α' . The values of $T_{k.s}$ generally fall into the temperature "B," i.e., above 2800° abs. Most frequently, it will be necessary to determine the temperature according to Eq. (2.46). We will demonstrate the method of calculating the temperature $T_{k.s}$ according to this equation.

12. We will use the nomogram in Fig. 7 of the Appendix to find the value of the parameter m_B .

13. We will calculate the value of the parameter $c_{p,B}^0$ according to Formula (1.42).

14. We will calculate the value of the parameter n_{ef} according to Formula (2.41).

15. We will calculate the value of the parameter i_{2800}^0 according to Formula (1.41).

16. We will calculate the value of the parameter c'_B according to Formula (2.44), assuming that $p = p_{k.s}$. In this case, $p_{B,f} = 40$ atmospheres absolute.

17. We will write Eq. (2.46) to conform to our case:

$$i_r = i_{2800}^0 + c_{p,s}^0 (T_{k,c} - 2800) + c'_s \left(\frac{T_{k,c} - 2200}{T_{s,\phi} - 2200} \right)^{m_s}.$$

This equation will be written in a form convenient for the calculation of $T_{k.s}$: those terms independent of temperature will be collected in the left-hand part of the equation and the algebraic sum of the terms in the left-hand part will be denoted by i_k :

$$i_k = i_r - i_{2800}^0 + 2800c_{p,s}^0. \quad (3.7)$$

Then, if we take into consideration that $T_{B,f} = 3400^\circ$ abs., we will obtain

$$i_k = c_{p,s}^0 T_{k,c} + c'_s \left(\frac{T_{k,c} - 2200}{1200} \right)^{m_s}. \quad (3.8)$$

It is recommended that the value of the temperature $T_{k.s}$ be determined, in the following manner, according to the equation: three or

four values of the temperature $T_{k\ i}$ are assumed, close to the anticipated temperature $T_{k.s}$ of the products of combustion in the combustion chamber of the engine and to employ Eq. (3.8) to find the heat-content ($i_{k\ i}$) values which correspond to the assumed temperature values. Further, from the condition $i_{k\ i} = i_k$, to find the sought value of $T_{k.s}$ by some method.

When the anticipated value of the temperature $T_{k.s}$ falls below 2800° abs (in the temperature zone "H"), Eq. (2.51) is used to determine this temperature value.

10. Entropy of Products of Combustion in the Combustion Chamber of an Engine

The value of the entropy of the products of combustion in the combustion chamber of an engine, at temperatures of 2800° abs and higher for the products of combustion can be found from Eq. (2.61). For the given case, the equation takes the form

$$S_{k.c} = 2,303c_{p,B}^0 \lg \frac{T_{k.c}}{2800} - \frac{0,004574}{\mu^0} \lg p_{k.c} + \\ + c'_B \left(\frac{T_{k.c} - 2200}{1200} \right)^{m_B} \left(\frac{1}{T_{k.c}} + \frac{1,527 \cdot 10^{-8}}{n_{ef}} \right). \quad (3.9)$$

Here it is assumed that $S_{k.s} = S_B$, $T = T_{k.s}$, $p = p_{k.s}$,

$$\bar{R} = 0.001986 \text{ kcal/deg.}$$

The values of the parameters $c_{p,B}^0$, μ^0 , c'_B , m_B , and n_{ef} were determined earlier.

At temperatures below 2800° abs, for the products of combustion in the combustion chamber of an engine, the calculation of the entropy of the products of combustion is carried out in accordance with Eq. (2.62).

11. Temperature and Heat Content of the Products of Combustion at the Outlet Section of the Engine Nozzle

The temperature of the products of combustion at the outlet section of the nozzle is determined on the basis of the premises adopted

in this work, according to which the expansion of the products of combustion and their discharge through the engine are reversible processes, involving neither loss nor addition of energy.

In this case, the entropy $S_{k.s}$ of the products of combustion is equal to the entropy $S_{v.s}$ at the outlet section of the nozzle:

$$S_{k.c} = S_{v.c} \quad (3.10)$$

We will assume, in this case, that the value of the entropy $S_{k.s}$ which we found in accordance with the method described in the previous section and the pressure $p_{v.s}$ of the products of combustion at the outlet section of the nozzle to have been given.

The temperature $T_{v.s}$ for the products of combustion at the outlet section of the nozzle is, in the absolute majority of cases, in the temperature zone "H," i.e., below 2800° abs. To determine the temperature it is most frequently necessary to employ Eq. (2.62). We will first determine the values of certain of the parameters that are contained in Eq. (2.62).

1. We will calculate the value of the parameter $c_{p,H}^0$ in accordance with Formula (1.43).

2. We will determine the value of the parameter m_H in accordance with the nomogram presented in Fig. 8 of the Appendix.

3. We will calculate the value of the parameter c'_H , assuming that $p = p_{v.s}$, according to Formula (2.49). In this case, $p_{H,f} = 1$ atm abs.

We will write Eq. (2.62) in a form convenient for the calculation of the value of $T_{v.s}$. With this goal in mind, we will collect all of the terms in the equation that are independent of temperature on the left-hand side, and the algebraic sum of the terms in the left-hand side will be denoted by S_a . Assuming $\bar{R} = 0.001986$ kcal/deg, we will obtain

$$S_a = S_{u.c} + 2,303c_{p,u}^0 \lg 2800 + \frac{0,004574}{\mu^0} \lg p_{u.c}. \quad (3.11)$$

In this case,

$$S_a = 2,303c_{p,u}^0 \lg T_s + c_u' \left(\frac{T_{u.c} - 1400}{1400} \right)^{m_u} \left(\frac{1}{T_{u.c}} + \frac{1,527 \cdot 10^{-8}}{n_{u,\phi}} \right) \quad (3.12)$$

(since $T_{H,f} = 2800^\circ$ abs).

The value of temperature $T_{v.s}$ is most expediently determined from Eq. (3.12) in the following manner: we assume several values for the temperature $T_{v.s}$, close to the anticipated value of $T_{v.s}$, and at each of these values we find the value of the entropy S_{a1} from Eq. (3.12). Further, from the condition $S_{a1} = S_a$ we determine the true value of the temperature $T_{v.s}$.

At $S_a/c_{p,H}^0 \leq 7.50$, the calculation of $T_{v.s}$ can be carried out according to the following formula:

$$\lg T_{u.c} = \frac{S_a}{2,303c_{p,u}^0}. \quad (3.13)$$

In this case $T_{v.s} \leq 1800^\circ$ abs.

The value of the heat content $i_{v.s}$ for the products of combustion at the outlet section of the nozzle can be found from Eq. (2.51). In the solution of the equation it is necessary to assume that $i_H = i_{v.s}$, and $T = T_{v.s}$,

$$p = p_{v.s}, \quad T_{H,f} = 2800^\circ \text{ abs}; \quad p_{H,f} = 1 \text{ atm abs.}$$

12. Exhaust Velocity for the Products of Combustion and Specific Thrust During Rated Operating Regime of Engine

The ideal exhaust velocity for the products of combustion is determined according to the following equation:

$$w_{u.c} = 91,53 \sqrt{i_t - i_{u.c}} \quad (3.14)$$

Specific thrust for the rated operating regime of the engine is determined according to the following equation:

$$P_{ud} = w_{v.s}/g \text{ [kg/(kg/sec)]}. \quad (3.15)$$

13. Specific Volume of Products of Combustion

When the thrust of the engine is given, the thermodynamic calculation may be continued in order to determine the basic dimensions of the combustion chamber. In this case, it becomes necessary to determine the specific volumes of the products of combustion at various sections of the chamber.

Let us demonstrate a method for the calculation of the specific volumes of the products of combustion by using the approximate mathematical expressions established in this work. In accordance with (2.34) the number of gram-molecules of dissociated products of combustion in 1 kg is equal to

$$M = M^0 + \frac{\Delta i}{130}.$$

Here Δi is expressed in kcal/kg.

We also know that

$$M = \frac{1000}{\mu_k} = \frac{1000}{848} R,$$

where R is the gas constant of the dissociated products of combustion, in kg-m/kg·deg. Consequently,

$$R = 0,848 M^0 + 0,00652 \Delta i. \quad (3.16)$$

For the case of the combustion chamber of an engine, the gas constant of the products of combustion will be equal to

$$R_{k.c} = 0,848 M^0 + 0,00652 \Delta i_{k.c},$$

where

$$\Delta i_{k.c} = c'_s \left(\frac{T_{k.c} - 2200}{1200} \right)^{m_s}.$$

For the conditions prevailing at the outlet section of the nozzle, the gas constant is equal to

$$R_{n.c} = 0,848 M^0 + 0,00652 \Delta i_{n.c},$$

where

$$\Delta i_{n.c} = c'_n \left(\frac{T_{n.c} - 1400}{1400} \right)^{m_n}.$$

The specific volumes of the products of combustion in the combustion chamber of the engine at the outlet section of the nozzle are determined on the basis of the following equations:

$$v_{k.c} = \frac{R_{k.c} T_{k.c}}{p_{k.c}}; \quad (3.17)$$

$$v_{b.c} = \frac{R_{b.c} T_{b.c}}{p_{b.c}}. \quad (3.18)$$

The adiabatic exponent for the exhaust outflow is determined according to the well-known expression

$$k = \frac{\lg\left(\frac{p_{k.c}}{p_{b.c}}\right)}{\lg\left(\frac{p_{k.c} T_{b.c}}{p_{b.c} T_{k.c}}\right)}. \quad (3.19)$$

Thus the proposed calculation method will make it possible to establish the basic dimensions of the combustion chamber in an engine, without having to resort to the calculation of the chemical composition of the products of combustion.

14. Methods Employed in Thermodynamic Calculations in Certain Special Cases

A. Effect of coefficient of excess oxidizer in the propellant on the operating characteristics of the engine

In carrying out thermodynamic calculations for rocket engines it frequently becomes necessary to establish the relationship between the magnitudes of the most important parameters characterizing the operation of the engine: the temperature of the products of combustion in the engine chamber and at the outlet section of the nozzle, the exhaust velocity, specific thrust, etc., and the coefficient of excess oxidizer α_0 .

We will demonstrate the most suitable sequence of carrying out the thermodynamic calculations in this case. We will assume the chemical composition of the fuel (combustible) and oxidizer, as well as

the pressure $p_{k.s}$ of the products of combustion within the combustion chamber and the pressure $p_{v.s}$ at the outlet section of the nozzle to have been given. We must conduct the investigation at values for the coefficient of excess oxidizer α_0 within the approximate limits of 0.95 to 0.55.

For the solution of the given problem, we will assume a series of values for the coefficient of excess oxygen α from those for which nomograms have been constructed, and with each of these we will successively determine the values of the parameters α_0 , v , A_C , A_H , A_N , μ^0 , X_C , X_H , and according to the nomograms presented in Figs. 1 to 20 of the Appendix, the values of the parameters i_v^0 , m_B , m_H , $\Delta i_{B,v,f}$, and $\Delta i_{H,v,f}$. Further, at each fixed value of α we will continue the calculation to its end, successively determining the values of the parameters i_t , $c_{p,B}^0$, i_{2800}^0 , n_{ef} , c'_B , $T_{k.s}$, as well as $S_{k.s}$, S_a , $c_{p,H}^0$, c'_H , $T_{v.s}$, $i_{v.s}$, $w_{v.s}$, and P_{ud} .

Having determined the values of the sought parameters at all the selected values of α , we will construct the curve for these parameters as a function of the coefficient of excess oxidizer and we will use this curve to determine the values of the parameters at any intermediate values of α_0 .

B. Effect of the pressure of the products of combustion within the combustion chamber of an engine on the operating characteristics of the engine

We will assume that it is necessary to establish the effect that a change in the pressure of the products of combustion within the combustion chamber of an engine has on the operating characteristics of the engine. Here, we must bear in mind that because of the approximate nature of the mathematical expressions established earlier the investigated region of change for pressure of the products of combustion within the engine combustion chamber will be limited approximately to a

range of from 20 to 100 atm abs. The least error in the values of the sought parameters is attained if the calculation is carried out at a pressure of 40 atm abs for the products of combustion within the combustion chamber of the engine. (The nomograms presented in Figs. 9 to 14 of the Appendix have been constructed for a pressure of 40 atm abs.) At greater pressures, the effect of dissociation diminishes and the utilization of the earlier established approximate expressions also results in no pronounced errors in the values of the sought parameters. At pressures below 40 atm abs for the products of combustion within the combustion chamber of the engine, the degree of calculation accuracy diminishes, and this is all the more pronounced the lower the pressure. At these pressures, the dissociation reactions are fully developed and the approximate nature of the derived expressions begins to have an effect on the calculation results.

We will assume the chemical composition of the fuel (combustible) and oxidizer, the value of α'_0 , and this means the value of α' (we will assume for simplicity's sake that α' coincides with one of the values of α for which the nomograms in Figs. 1 to 6 and 9 to 20 of the Appendix have been constructed), and the magnitude of the pressure $p_{v.s}$ of the products of combustion at the outlet section of the nozzle to have been given. A feature of the calculation procedure in this case is the fact that the values of all parameters independent of the pressure of the products of combustion can be determined in advance and then used at any values of $p_{k.s}$.

Consequently, for a given α' we first determine the values of the parameters v , A_C , A_H , A_N , μ^0 , X_C , X_H , and according to the nomograms presented in Figs. 1 to 20 of the Appendix, the values of the parameters i_V^0 , m_B , m_H , $\Delta i_{B,v,f}$, and $\Delta i_{H,v,f}$.

Subsequently, we will assume a series of values for the pressure

$p_{k.s}$ of the products of combustion within the combustion chamber of the engine and, at each of these values, we will carry the calculation to its conclusion, successively determining the parameters

$$c'_B, S_{k.s}, S_a, c'_H, T_{v.s}, w_{v.s}, \text{ and } P_{ud}.$$

Having determined the values of the sought parameters at all the selected values of $p_{k.s}$, we construct the curve representing these parameters as a function of the pressure of the products of combustion within the combustion chamber and we use this curve to determine the values of the parameters at any intermediate values of $p_{k.s}$.

C. Effect of pressure of products of combustion at outlet section of nozzle on operating characteristics of the engine

Let us assume that it is necessary to establish the effect of a change in the pressure in the products of combustion at the outlet section of the nozzle on the above-mentioned operating characteristics of the engine. In this case it is assumed that the chemical composition of the fuel (combustible) and oxidizer as well as the pressure $p_{k.s}$ of the products of combustion within the combustion chamber of the engine are known.

A feature of the calculation procedure in the given case is the fact that the characteristics of the products of combustion are constant (primarily, temperature $T_{k.s}$) at all values of the pressure of the products of combustion at the outlet section of the nozzle.

We will adopt the following calculation sequence.

1. At given values of α'_0 (α') and $p_{k.s}$ we will carry out the calculation of the parameters whose values are independent of the quantity $p_{v.s}$:

$$\begin{aligned} & \nu, A_C, A_H, A_N, \mu^0, X_C, X_H, i_v^0, m_s, m_u, \\ & \Delta i_{s.v.\phi}, \Delta i_{n.v.\phi}, c_{p,s}^0, c_{p,u}^0, n_{s\phi}, i_{2800}^0, \\ & i_T, T_{k.c}, c'_s, S_{k.c}. \end{aligned}$$

2. We will assume a series of values for $p_{v.s}$ in the chosen region of pressure change and at each of these values we will carry out the calculation to its conclusion, determining the values of the parameters S_a , c'_H , $T_{v.s}$, $w_{v.s}$, and P_{ud} .

3. We will construct the curve representing the parameters $T_{v.s}$, $w_{v.s}$, and P_{ud} as a function of the parameter $p_{v.s}$ according to the calculation data.

All other conditions being equal, the calculation accuracy for the parameters $T_{v.s}$, $w_{v.s}$, and P_{ud} increases as the value of $p_{v.s}$ approaches 1 atm abs, since the nomograms for the determination of the parameter $\Delta i_{H,v,f}$ have been constructed precisely for this pressure.

We have subdivided the calculation sequence into several of the most frequently encountered practical cases. In the subsequent section, we present calculation examples.

In Example 1 we show the thermodynamic calculation for a liquid-fuel rocket engine operating on a propellant consisting of four chemical elements: carbon, hydrogen, nitrogen, and oxygen.

In Example 2 we present the calculation for an engine operating on a propellant that is free of nitrogen and at a pressure of $p_{k.s} = 50$ atm abs for the products of combustion within the combustion chamber of the engine.

In Example 3 we demonstrate the method used to carry out the calculations for established relationships between the operating characteristics of the engine and the coefficient of excess oxidizer in the propellant.

In Examples 4 and 5 we demonstrate the method used in the calculations for the establishment of relationships between the operating characteristics of an engine and the pressure of the products of combustion within the combustion chamber and at the outlet section of the

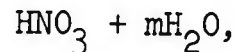
engine nozzle, respectively.

15. Examples of Thermodynamic Calculations

Example 1. Determine the exhaust velocity and specific thrust during the rated operating regime of a liquid-fuel rocket engine operating on technically pure (96%, by weight) nitric acid (admixture, water) and kerosene ($C_g = 86.8\%$, $H_g = 13.2\%$) at $\alpha'_o = 0.8$. The pressure of the products of combustion within the combustion chamber was assumed to be $p_{k.s} = 40$ atm abs, and the pressure at the outlet section of the nozzle was assumed to be $p_{v.s} = 1$ atm abs.

1. We will determine the weight content of the chemical elements in 1 kg of oxidizer.

The conventional chemical formula for the oxidizer is



where m is the number of gram-molecules of water per single gram-molecule of nitric acid. In the given case, there is $\xi = 0.04$ (by weight) of the admixture, water, in the nitric acid.

Consequently,

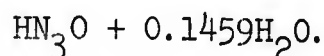
$$\frac{\xi}{1-\xi} = \frac{m\mu_{H_2O}}{\mu_{HNO_3}}.$$

Here μ_{H_2O} and μ_{HNO_3} are the molecular weights of the water and the nitric acid.

Evidently,

$$m = \frac{\xi\mu_{HNO_3}}{(1-\xi)\mu_{H_2O}} = \frac{0.04 \cdot 63}{0.96 \cdot 18} = 0.1459.$$

The conventional formula for the oxidizer is



The conventional molecular weight of the oxidizer is

$$\begin{aligned}\mu_o &= \mu_{HNO_3} + 0.1459\mu_{H_2O}; \\ \mu_o &= 63 + 0.1459 \cdot 18 = 65.627.\end{aligned}$$

The weight content of the chemical elements in 1 kg of oxidizer,

$$C_o = 0:$$

$$H_o = \frac{1 + 2 \cdot 0,1459}{65,627} 1000 = 19,69 \text{ g};$$

$$N_o = \frac{14}{63,627} 1000 = 213,2 \text{ g};$$

$$O_o = \frac{(3 + 0,1459) 16}{65,627} 1000 = 766,5 \text{ g}.$$

2. We will determine the weight content of the chemical elements in 1 kg of fuel (combustible):

$$C_g = 868 \text{ g}; H_g = 132 \text{ g}; N_g = 0; O_g = 0.$$

3. The theoretically required quantity of oxidizer for the oxidation of 1 kg of fuel (combustible) is determined in accordance with Formula (3.2):

$$v_o = \frac{\frac{8}{3} \cdot 868 + 8 \cdot 132 - 0}{766,5 - \frac{8}{3} \cdot 0 - 8 \cdot 19,69} = 5,54.$$

We will find the value of the coefficient of excess oxygen α' , which corresponds to the given α'_o in accordance with Formula (3.3a):

$$\alpha' = \frac{0,8 \cdot 5,54}{\frac{8}{3} \cdot 868 + 8(132 + 0,8 \cdot 5,54 \cdot 19,69)};$$

$$\alpha' = 0,834.$$

4. We will assume a series of values for α : 0.95, 0.90, 0.80, 0.70, and for these we will calculate the values of α_o in accordance with Formula (3.3).

α	0,95	0,90	0,834	0,80	0,70
α_o	0,939	0,877	0,800	0,761	0,649

An example of the calculation for $\alpha = 0.95$ is:

$$\alpha_o = \frac{1}{5,54} \frac{\frac{8}{3} \cdot 868 + 8 \cdot 132 - 0}{\frac{766,5}{0,95} - 0 - 8 \cdot 19,69};$$

$$\alpha_o = 0,939.$$

5. The actual weight ratio v of oxidizer to fuel (combustible) in the propellant is determined in accordance with Formula (3.4):

α	0,95	0,90	0,834	0,80	0,70
v	5,200	4,860	4,432	4,216	3,595

An example of the calculation for $\alpha = 0.95$ is:

$$v = 0,939 \cdot 5,54; v = 5,200.$$

6. The quantity of gram-atoms of carbon, hydrogen, and nitrogen in 1 kg of propellant is determined in accordance with Formula (3.5).

α	A_C	A_H	A_N
0,95	11,69	37,82	12,78
0,90	12,35	38,82	12,63
0,834	13,32	40,39	12,46
0,80	13,87	41,22	12,31
0,70	15,77	44,14	11,94

7. The value of the parameter μ^0 is determined in accordance with Formula (3.6).

α	0,95	0,90	0,834	0,80	0,70
μ^0	0,02703	0,02625	0,02517	0,02458	0,02283

An example of the calculation for $\alpha = 0.95$ is:

$$\mu^0 = \frac{1}{11,69 + 0,5(37,82 + 12,78)};$$

$$\mu^0 = 0,02703.$$

8. The independent variables X_C and X_H are determined in accordance with Formulas (1.39) and (1.40).

α	X_C	X_H
0,95	12,65	40,90
0,90	12,98	40,78
0,834	13,42	40,65
0,80	13,66	40,60
0,70	14,40	40,32

An example of the calculation for $\alpha = 0.95$ is:

$$X_C = 40 \cdot 0,02703 \cdot 11,69 = 12,65;$$

$$X_H = 40 \cdot 0,02703 \cdot 37,82 = 40,90.$$

9. We determine the following parameters at selected values of α according to the nomograms presented in Figs. 1 to 4, 9 to 12, and 15 to 18 of the Appendix.

α	i_v^0	$\Delta i_{B,v,\phi}$	$\Delta i_{H,v,\phi}$
0,95	1335	666	14,52
0,90	1462	560	11,87
0,80	1721	393	8,20
0,70	1998	274	5,82

10. We will construct the curve representing the parameters i_v^0 , $\Delta i_{B,v,\phi}$, and $\Delta i_{H,v,\phi}$ as functions of the coefficient of excess oxygen α (Fig. 5).

11. We will read the values of the parameters for the found coefficient of excess oxygen ($\alpha' = 0.834$) from the curve:

$$i_v^0 = 1632;$$

$$\Delta i_{B,v,\phi} = 438;$$

$$\Delta i_{H,v,\phi} = 9,15.$$

We will carry out our further calculations with a coefficient of excess oxygen of $\alpha' = 0.834$.

12. We will determine m_B in accordance with the nomogram presented

in Fig. 7 of the Appendix:

$$m_B = 2.881.$$

13. We will determine $c_{p,B}^0$ according to Formula (1.42):

$$c_{p,B}^0 = \frac{1}{40 \cdot 0,02517} [356,0 + (12,454 \cdot 0,834 - 6,276) \cdot 13,42 + \\ + (3,114 \cdot 0,834 - 1,221) \cdot 40,65 - \\ - 0,797 \lg(0,834 - 0,420) \cdot 40,65] \cdot 10^{-3}; c_{p,B}^0 = 0,4760.$$

14. We will calculate n_{ef} according to Formula (2.41):

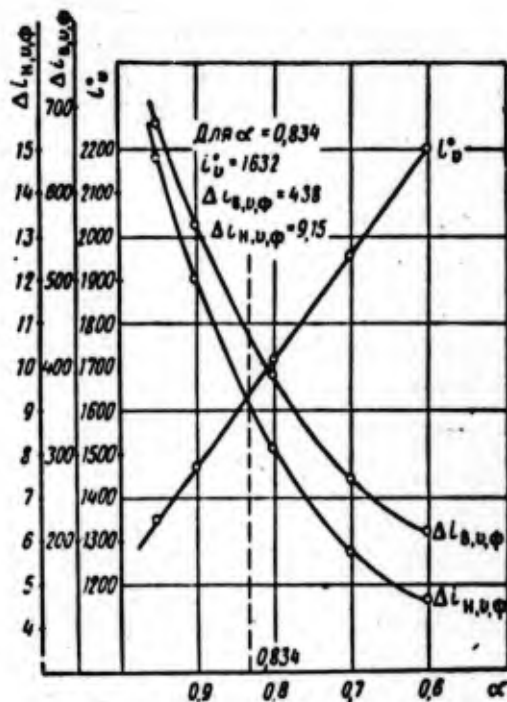


Fig. 5

$$n_{\phi} = 0,6242 \lg \frac{40 \cdot 2^{2,881} \cdot 9,15}{438}; \\ n_{\phi} = 0,4925.$$

15. We will determine i_{2800}^0 according to Formula (1.41):

$$i_{2800}^0 = 1632 \frac{1}{40 \cdot 0,02517}; \\ i_{2800}^0 = 1621,7.$$

16. We will determine c'_B , assuming $p = p_{k.s} = 40$ atm abs, according to Formula (2.44):

$$c'_B = \frac{\Delta i_{B,v,\phi}}{40 \mu^0} \left(\frac{p_{B,\phi}}{p_{k.s}} \right)^{n_{\phi}}.$$

Since $p_{k.s} = 40$ atm abs and $p_{B,f} = 40$ atm abs, $c'_B = \Delta i_{B,v,f} / 40 \mu^0 =$

$$= 438/(40 \cdot 0.02517),$$

$$c'_B = 435.2.$$

17. We will determine the heat content i_t of 1 kg of propellant according to the method presented in Section 20 of the given book.

For $\alpha' = 0.834$, the conventional formula for the propellant is



The conventional molecular weight of the propellant is

$$100 + 424.62 + 17.71 = 542.33.$$

The weight fractions of the components in the propellant are:

$$\text{kerosene } q_{\text{ker}} = 100/542.33 = 0.1843 \text{ kg};$$

$$\text{water } q_{H_2O} = (0.984 \cdot 18)/542.33 = 0.0326 \text{ kg};$$

$$\text{nitric acid (100\%)} q_{HNO_3} = 424.62/542.33 = 0.7828 \text{ kg}.$$

The heat content of the propellant components has been determined by us in the solution of the examples in Section 20:

$$i_{\text{ker}} = 10,399 \text{ kcal/kg};$$

$$i_{H_2O(\text{liquid})} = -459 \text{ kcal/kg};$$

$$i_{HNO_3} = -128 \text{ kcal/kg}.$$

We should also take into consideration (with negative sign) the heat of solution for water in nitric acid, approximately equal to 12.6 kcal per 1 kg of nitric acid. If we take into consideration the heat of solution, the heat content of nitric acid will be equal to:

$$i_{HNO_3} = -128 - 12.6 = -140.6 \text{ kcal/kg}.$$

The heat content of 1 kg of propellant is:

$$i_r = i_{\text{ker}} q_{\text{ker}} + i_{HNO_3} q_{HNO_3} + i_{H_2O} q_{H_2O};$$

$$i_r = 10399 \cdot 0.1843 - 140.6 \cdot 0.7828 - 459 \cdot 0.0326;$$

$$i_r = 1791.9 \text{ kcal/kg}.$$

18. We determine i_k according to Formula (3.7):

$$i_k = 1791.9 - 1621.7 + 0.4760 \cdot 2800; \quad i_k = 1504.2.$$

19. The temperature $T_{k.s}$ of the products of combustion in the

combustion chamber of the engine is determined in accordance with Eq. (3.8).

We will assume a series of values for $T_{k\ i}$, close to the anticipated $T_{k\ s}$, and from the condition $i_{k\ i} = i_{k\ s}$ we will determine $T_{k\ s}$. We carry out the calculations with the aid of the following type of table (only a single conclusive calculation line is presented in the table).

$T_{k\ i}$	$\frac{T_{k\ i} - 2200}{1200}$	$\lg [2]$	$m_s [3]$	$\lg c'_s$	$\lg \frac{\Delta i_{k\ i}}{=}$ $=[4] + [5]$	$\Delta i_{k\ i}$	$c_{p, s}^0 T_{k\ i}$	$\frac{i_{k\ i} =}{= [7] + [8]}$
2940	0,6164	$\overline{1,7900}$ -0,2100	-0,6048	2,6388	2,034	108,1	1399,0	1507,1

$$T_{k\ s} = 2940^\circ \text{ abs, since } i_{k\ i} \approx i_{k\ s}.$$

20. We determine the entropy $S_{k\ s}$ of the products of combustion within the combustion chamber of the engine according to Eq. (3.9).

Since $c'_B ((T_{k\ s} - 2200)/1200)^{m_B} = \Delta i_{k\ s} = 108.1$ (see column seven of the table),

$$S_{k\ s} = 2,303 \cdot 0,4760 \lg \frac{2940}{2800} - \frac{0,004574}{0,02517} \lg 40 + 103,1 \left(\frac{1}{2940} + \frac{1,527 \cdot 10^{-5}}{0,4925} \right);$$

$$S_{k\ s} = -0,2261.$$

21. We determine the parameter $c_{p, s}^0$ according to Formula (1.43):

$$c_{p, s}^0 = \frac{1}{40 \cdot 0,02517} [349,6 + (12,152 \cdot 0,834 - 6,022) 13,42 +$$

$$+ (3,038 \cdot 0,834 - 1,336) 40,65 - 0,825 \lg (0,834 - 0,42) 40,65] \cdot 10^{-2};$$

$$c_{p, s}^0 = 0,4630.$$

22. We determine the parameter m_H according to the nomogram presented in Fig. 8 of the Appendix:

$$m_H = 5.727.$$

23. We determine S_a according to Formula (3.11) taking into consideration that $p_{v, s} = 1 \text{ atm abs}$:

$$S_a = -0,2261 - 2,303 \cdot 0,4630 \cdot \lg 2800;$$

$$S_a = 3,4489.$$

24. We determine the temperature $T_{v.s}$ of the products of combustion at the outlet section of the nozzle. Since, in this case,

$$\frac{S_g}{c_{p,n}^0} = \frac{3,4489}{0,4630} = 7,44 < 750,$$

for the determination of the temperature we need not employ Eq. (3.12), we can carry out the calculation in accordance with Formula (3.13);

$$\lg T_{v.s} = \frac{3,4489}{2,303 \cdot 0,4630} = 3,239;$$

$$T_{v.s} = 1707^\circ \text{ abs.}$$

25. The heat content of the products of combustion at the outlet section of the nozzle are determined in accordance with Eq. (2.51).

Here

$$c_n \left(\frac{T_{v.s} - 1400}{1400} \right)^{m_n} = \Delta i_{v.s} = 0;$$

$$i_{v.s} = 1621,7 + 0,4630 (1707 - 2800) + 0;$$

$$i_{v.s} = 1115,7 \text{ kcal/kg.}$$

26. The theoretical ideal exhaust velocity for the products of combustion is determined according to Eq. (3.14):

$$w_{v.s} = 91,53 \sqrt{1791,9 - 1115,7};$$

$$w_{v.s} = 2382 \text{ m/sec.}$$

27. The specific thrust in the case of a rated engine operating regime is equal to

$$P_{ya} = \frac{2382}{9,81};$$

$$P_{ud} = 242,8 \text{ kg} \cdot \text{sec/kg.}$$

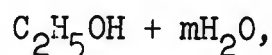
28. The calculation results are presented in the curve in Fig. 7 (see the points corresponding to $\alpha = 0,834$).

Example 2. Calculate the exhaust velocity and specific thrust in the case of a rated engine operating regime, said engine operating on ethyl alcohol (95%, by weight) (admixture, water) with a coefficient of excess oxidizer $\alpha'_0 = 0,7$ ($\alpha' = 0,748$). The pressure of the products of combustion in the combustion chamber of the engine is $p_{k.s} = 50$

atm abs, and the pressure of the products of combustion at the nozzle outlet is $p_{v.s} = 1$ atm abs.

There is an entropy diagram for the given propellant, and this diagram makes it possible to compare the calculation results produced in accordance with the proposed method and from the diagram.

1. We will find the weight content of the chemical elements in 1 kg of fuel (combustible). The conventional chemical formula of the fuel (combustible) is



where m is the number of gram-molecules of water per single gram-molecule of ethyl alcohol.

In this case there is $\xi = 0.05$ (by weight) of the admixture, water, in the ethyl alcohol. Consequently,

$$\frac{\xi}{1-\xi} = \frac{m\mu_{H_2O}}{\mu_{C_2H_5OH}}.$$

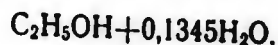
Here, μ_{H_2O} and $\mu_{C_2H_5OH}$ are the molecular weights of the water and the ethyl alcohol, respectively.

Evidently,

$$m = \frac{\xi\mu_{C_2H_5OH}}{(1-\xi)\mu_{H_2O}} = \frac{0,05 \cdot 46}{0,95 \cdot 18};$$

$$m = 0.1345.$$

The conventional formula for the fuel (combustible) is



The conventional molecular weight of the fuel (combustible) is

$$\begin{aligned} \mu_r &= \mu_{C_2H_5OH} + 0,1345\mu_{H_2O}; \\ \mu_r &= 46 + 0,1345 \cdot 18 = 48,422. \end{aligned}$$

The weight content of the chemical elements in 1 kg of fuel (combustible) is

$$C_r = \frac{2 \cdot 12}{48,422} \cdot 1000 = 495,7 \text{ g};$$

$$H_r = \frac{6 + 0,1345 \cdot 2}{48,422} \cdot 1000 = 129,5 \text{ g};$$

$$N_r = 0;$$

$$O_r = \frac{(1 + 0,1345) 16}{48,422} \cdot 1000 = 375,0 \text{ g}.$$

2. The weight content of the chemical elements in 1 kg of oxidizer is:

$$C_o = 0;$$

$$H_o = 0;$$

$$O_o = 1000 \text{ g};$$

$$N_o = 0.$$

3. Let us assume a series of values for α : 0.95, 0.90, 0.80, 0.70, 0.60, and 0.55; for these, as well as for the given $\alpha' = 0.748$, we will carry out the calculation of the independent parameters X_C and X_H .

In the given example, for the calculation of the values of X_C and X_H we do not require any preliminary determination of the parameters v_o , v , A_C , A_H , and μ^0 , since for X_C and X_H we can find simpler expressions. Indeed, since in the given case

$$C_o = H_o = N_r = N_o = 0,$$

then

$$A_C = \frac{C_r}{12(1+v)}; \quad A_H = \frac{H_r}{1+v};$$

$$\mu^0 = \frac{1}{A_C + 0,5A_H} = \frac{12(1+v)}{C_r + 6H_r}.$$

Consequently, the parameters X_C and X_H are determined by the following expressions, regardless of the selected value of α :

$$X_C = 40\mu^0 A_C = \frac{40C_r}{C_r + 6H_r} = \frac{40 \cdot 495,7}{495,7 + 6 \cdot 129,5} = 15,59;$$

$$X_H = 40\mu^0 A_H = \frac{40 \cdot 12H_r}{C_r + 6H_r} = \frac{40 \cdot 129,5 \cdot 12}{495,7 + 6 \cdot 129,5} = 48,83.$$

4. We determine the parameters i_V^0 , $\Delta i_{B,v,f}$, and $\Delta i_{H,v,f}$ at the selected values of α in accordance with the nomograms presented in Figs. 1 to 6 and 9 to 20 of the Appendix.

α	i_v^0	$\Delta i_{B,v,f}$	$\Delta i_{H,v,f}$
0,95	1430	744	16,12
0,90	1575	607	13,02
0,80	1861	415	8,70
0,70	2148	287	6,15
0,60	2429	227	5,10
0,55	2578	214	4,77

5. We will construct the curve representing the parameters i_v^0 , $\Delta i_{B,v,f}$, and $\Delta i_{H,v,f}$ as functions of the coefficient of excess oxygen α (Fig. 6).

6. The values of the parameters for a given coefficient of excess oxygen $\alpha' = 0.748$ are taken from the curve:

$$\begin{aligned} i_v^0 &= 2010; \\ \Delta i_{B,v,f} &= 342; \\ \Delta i_{H,v,f} &= 7,13. \end{aligned}$$

We will continue the calculation only with a coefficient of excess oxygen $\alpha' = 0.748$.

7. We will use Formula (3.2) to calculate the quantity of oxidizer theoretically required for the oxidation of 1 kg of fuel (combustible):

$$\begin{aligned} v_o &= \frac{\frac{8}{3} 495,7 + 8 \cdot 129,5 - 375,0}{1000}; \\ v_o &= 1,983. \end{aligned}$$

8. Indeed, the weight ratio of the propellant components is determined according to Formula (3.4):

$$\begin{aligned} v &= 0,70 \cdot 1,983; \\ v &= 1,390. \end{aligned}$$

9. The number of gram-atoms of carbon and hydrogen in 1 kg of propellant is determined according to the formulas in (3.5):

$$A_c = \frac{495,7}{12(1 + 1,39)} = 17,30;$$

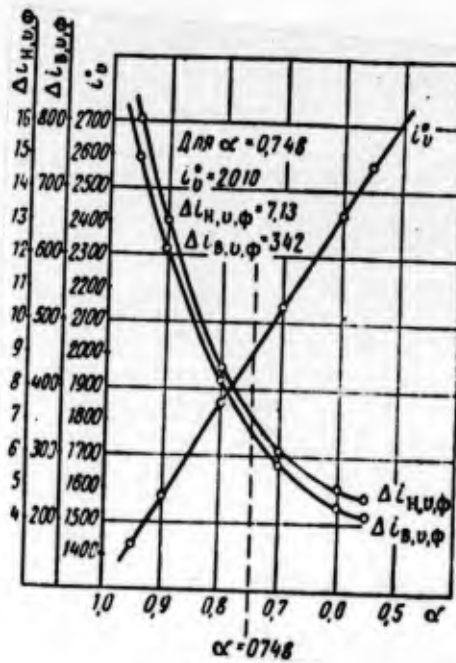


Fig. 6

$$A_H = \frac{129,5}{1 + 1,39} = 54,24.$$

10. The value of the parameter μ^0 is determined according to Formula (3.6):

$$\mu^0 = \frac{1}{17,30 + 0,5 \cdot 54,24};$$

$$\mu^0 = 0,02252.$$

11. We will determine m_B in accordance with the nomogram presented in Fig. 7 of the Appendix:

$$m_B = 2.96.$$

12. We will find $c_{p,B}^0$ according to Formula (1.42):

$$c_{p,B}^0 = \frac{1}{40 \cdot 0,02252} [356,0 +$$

$$+ (12,454 \cdot 0,748 - 6,276) \times$$

$$\times 15,59 + (3,114 \cdot 0,743 -$$

$$- 1,221) 48,83 - 0,797 \lg \times$$

$$\times (0,748 - 0,420) 48,83] \cdot 10^{-3};$$

$$c_{p,B}^0 = 0,5297.$$

13. We will calculate n_{ef} in accordance with Formula (2.41):

$$n_{\phi} = 0,6242 \lg \frac{40 \cdot 2^{2,96} \cdot 7,13}{342};$$

$$n_{ef} = 0.507.$$

14. We will determine i_{2800}^0 according to Formula (1.41):

$$i_{2800}^0 = \frac{2010}{40 \cdot 0.02252};$$

$$i_{2800}^0 = 2232 \text{ kcal/kg.}$$

15. We will determine c'_B according to Formula (2.44), assuming

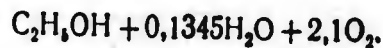
$$p = p_{k.s} = 50 \text{ atm abs;}$$

$$c'_s = \frac{342}{40 \cdot 0.02252} \left(\frac{40}{50} \right)^{0.507};$$

$$c'_s = 339.2.$$

16. In accordance with the method outlined in Section 20, we will find the heat content i_t of 1 kg of propellant.

For $\alpha' = 0.748$, the conventional formula for the propellant is



The conventional molecular weight of the propellant is

$$46 + 0.1345 \cdot 18 + 2.1 \cdot 32 = 115.62.$$

The weight fractions of the components of the propellant are:
alcohol (100%)

$$q_{C_2H_5OH} = \frac{46}{115.62} = 0.3975 \text{ kg;}$$

water

$$q_{H_2O} = \frac{0.1345 \cdot 18}{115.62} = 0.0209 \text{ kg;}$$

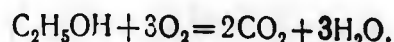
oxygen

$$q_{O_2} = \frac{2.1 \cdot 32}{115.62} = 0.5818 \text{ kg.}$$

We will determine the heat content of the ethyl alcohol through special calculation in terms of the known magnitude of the heat of combustion:

$$Q_{sg} = 326 \text{ kcal/g-mole.}$$

At $\alpha_0 = 1$, the equation for the burning of alcohol in oxygen is



According to Formula (2) in Section 20

$$i_{C_2H_5OH} = Q_{sg} - 3I_{O_2} + 2I_{CO_2} + 3I_{H_2O} \text{ (liquid)}.$$

According to Table 1 of the Appendix, under normal conditions (+20°C)

$$I_{O_2} = 2.03 \text{ kcal/g-mole};$$
$$I_{CO_2} = 2.18 \text{ kcal/g-mole}.$$

The heat content of the liquid water has been determined in Section 20:

$$I_{H_2O} \text{ (liquid)} = - 8.26 \text{ kcal/g-mole} = - 459 \text{ kcal/kg}.$$

Consequently, the heat content of the alcohol is equal to:

$$i_{c,H,OH} = 326 - 3 \cdot 2.03 + 2 \cdot 2.18 + 3(-8.26);$$

$$i_{c,H,OH} = 291.6 \text{ kcal/g-mole} = 6510 \text{ kcal/kg}.$$

The heat content of the liquid oxygen, at the boiling point, at atmospheric pressure, can be found by subtracting the heats of vaporization from the heat content of the oxygen gas (at -183°C).

The heat content of the oxygen gas at -183°C is equal to 0.63 kcal/g-mole.

The heat of vaporization for oxygen is

$$Q_{isp} = 1.63 \text{ kcal/g-mole}.$$

The heat content of liquid oxygen is

$$I_{O_2} \text{ (liquid)} = 0.63 - 1.63 = - 1.00 \text{ kcal/g-mole};$$

$$i_{O_2} \text{ (liquid)} = - 31.25 \text{ kcal/kg}.$$

We will now determine the heat content of 1 kg of propellant:

$$i_r = 0.3975 \cdot 6510 + 0.0209(-459) + 0.5818(-31.25);$$

$$i_r = 2559 \text{ kcal/kg}.$$

17. We will determine i_k according to Formula (3.7):

$$i_k = 2559 - 2232 + 0.5297 \cdot 2800;$$

$$i_k = 1810.$$

18. The temperature of the products of combustion in the combustion chamber of the engine is determined according to Eq. (3.8) in accordance with the method described in Section 9. The final result of the calculations on the determination of the value of temperature $T_{k.s}$ is presented in the table. The value of the temperature $T_{k.s}$ has been determined from the condition $i_{k1} = i_k$

$$T_{k.s} = 3120^\circ \text{ abs.}$$

T_{k1}	$\frac{T_{k1}-2200}{1200}$	$\lg [2]$	$m_s [3]$	$\lg c'_s$	$\lg \frac{\Delta i_{k1}}{= [4] + [5]}$	Δi_{k1}	$c_{p,s}^0 T_{k1}$	$\frac{i_{k1}}{= [7] + [8]}$
3120	0,7670	1,8848	-0,341	2,5305	2,1895	154,7	1652	1806,7

19. The heat content in the products of combustion in the combustion chamber of the engine is determined according to Eq. (3.9).

Since

$$c'_s \left(\frac{T_{k.c} - 2200}{1200} \right)^{m_s} = \Delta i_{k.c} = 154,7$$

(see column seven of the table),

$$S_{k.c} = 2,303 \cdot 0,5297 \lg \frac{3120}{2800} - \frac{0,004574}{0,02252} \lg 50 + 154,7 \left(\frac{1}{3120} + \frac{1,527 \cdot 10^{-8}}{0,507} \right);$$

$$S_{k.c} = -0,2295.$$

20. We determine the parameter $c_{p,H}^0$ according to Formula (1.43):

$$c_{p,H}^0 = \frac{1}{40 \cdot 0,02252} [349,6 + (12,152 \cdot 0,748 - 6,022) 15,59 + (3,038 \cdot 0,748 - 1,336) 48,83 - 0,825 \lg (0,748 - 0,42) 48,83] \cdot 10^{-8};$$

$$c_{p,H}^0 = 0,5142.$$

21. We determine the parameter m_H according to the nomogram presented in Fig. 8 of the Appendix:

$$m_H = 5,78.$$

22. We find S_a according to Formula (3.11).

Since $p_{v.s} = 1 \text{ atm abs,}$

$$S_a = -0,2295 + 2,303 \cdot 0,5142 \cdot \lg 2800;$$

$$S_a = 3,8525.$$

23. Since in this case

$$\frac{S_a}{c_{p,a}^0} = \frac{3,8525}{0,5142} < 7,50,$$

the calculation of the temperature $T_{v.s}$ of the products of the combustion can be carried out in a simplified manner, using Formula (3.13):

$$\lg T_{v.s} = \frac{3,8525}{2,303 \cdot 0,5142} = 3,251;$$

$$T_{v.s} = 1780^{\circ} \text{ abs.}$$

24. The heat content of the products of combustion at the outlet section of the nozzle can be determined according to Eq. (2.51). In this case,

$$c_a' \left(\frac{T_{v.s} - 1400}{1400} \right)^{m_a} = \Delta i_{v.s} = 0;$$

$$i_{v.s} = 2232 + 0,5142 (1780 - 2800) + 0;$$

$$i_{v.s} = 1707 \text{ kcal/kg.}$$

25. We determine the theoretically ideal exhaust velocity for the products of combustion from the engine according to Eq. (3.14):

$$w_{v.s} = 91,53 \sqrt{2559 - 1707};$$

$$w_{v.s} = 2673 \text{ m/sec.}$$

26. The specific thrust for the rated operating regime of the engine is equal to:

$$P_{ud} = 2673/9.81;$$

$$P_{ud} = 272.5 \text{ kg/kg/sec.}$$

Let us compare the calculation results against the data of the entropy diagram (see [7]).

The calculation was carried out by means of a 50-centimeter slide rule, by one operator during 3 hours of working time.

Example 3. For the propellant components cited in Example 1, establish the specific thrust and temperature of the products of combustion in the combustion chamber and at the outlet section of the engine

	1 $T_{k.s}$ ° abs.	2 $T_{v.s}$ ° abs.	3 $w_{v.s}$ в м/сек	4 $P_{уд}$ в $\frac{кг \cdot сек}{кг}$	Погрешность значения удельной тяги в процентах
6 Расчет	3120	1780	2673	272,5	0,4
7 Диаграмма	3125	1780	2683	273,7	

- 1) $T_{k.s}$, ° abs; 2) $T_{v.s}$, ° abs; 3) $w_{v.s}$, in m/sec; 4) $P_{уд}$, in kg·sec/kg; 5) error in the value of specific thrust, in percent; 6) calculation; 7) diagram.

α	m_o	m_d
0,95	2,870	5,71
0,90	2,875	5,72
0,80	2,884	5,73
0,70	2,892	5,74

α	1 q_{ker} кг/кг	2 q_{HNO_3} кг/кг	3 q_{H_2O} кг/кг	4 i_t $\frac{ккал}{кг}$
0,95	0,1615	0,8045	0,0340	1550,3
0,90	0,1710	0,7960	0,0332	1650,9
0,80	0,1921	0,7760	0,0323	1873,1
0,70	0,2180	0,7510	0,0313	2148,1

- 1) q_{ker} , kg/kg; 2) q_{HNO_3} , kg/kg; 3) q_{H_2O} , kg/kg; 4) i_t , kcal/kg.

α_o	0,95	0,90	0,80	0,70
$c_{p,s}^o$	0,4683	0,4712	0,4796	0,4899

nozzle as functions of the coefficient of excess oxidizer (in the α -value region from 0.95 to 0.70). For the products of combustion within the combustion chamber, the pressure $p_{k.s} = 40$ atm abs, and at the outlet section of the nozzle, $p_{v.s} = 1$ atm abs.

We will carry out the calculation for values of the coefficient of excess oxygen α , equal to 0.95, 0.90, 0.80, and 0.70. The initial portion of the calculation (to item 9) is in complete agreement with the calculations carried out in Example 1. Farther on, we will carry out the calculation in accordance with the procedure described in Section 14A.

1. We find the parameters m_B and m_H according to the nomogram shown in Figs. 7 and 8 of the Appendix.

2. The heat content of the fuel (combustible) and the heat content of the oxidizer were determined in item 17 of Example 1. Having calculated the weight fractions of the fuel (combustible) and oxidizer (q_g) and (q_o), respectively, in 1 kg of propellant, we find the heat content of 1 kg of propellant according to Formula (1) in Section 20 at the selected values of α . The results of the calculations are presented in the table (see third table, page 104).

3. We determine $c_{p,B}^0$ according to Formula (1.42) (see fourth table, page 104).

An example of calculation for $\alpha = 0.95$:

$$c_{p,n}^0 = \frac{1}{40 \cdot 0,02703} [356,0 + (12,454 \cdot 0,95 - 6,276) 12,65 + (3,114 \cdot 0,95 - 1,221) 40,90 - 0,797 \lg (0,95 - 0,42) \cdot 40,90] 10^{-3};$$

$$c_{p,n}^0 = 0,4683.$$

4. We determine $c_{p,H}^0$ according to Formula (1.43).

α	0,95	0,90	0,80	0,70
$c_{p,n}^0$	0,4552	0,4582	0,4663	0,4767

An example of calculation for $\alpha = 0.95$:

$$c_{p,n}^0 = \frac{1}{40 \cdot 0,02703} [349,6 + (12,152 \cdot 0,95 - 6,022) 12,65 + (3,038 \cdot 0,95 - 1,336) 40,90 - 0,825 \lg (0,95 - 0,42) \cdot 40,90] 10^{-3};$$

$$c_{p,H}^0 = 0.4552.$$

5. We will find i_{2800}^0 according to Formula (1.41).

α	0,95	0,90	0,80	0,70
i_{2800}^0	1234,9	1392,8	1749,2	2188,2

An example of calculation for $\alpha = 0.95$:

$$i_{2800}^0 = 1335 \frac{1}{40 \cdot 0,02703};$$

$$i_{2800}^0 = 1234,9.$$

6. We will determine i_k according to Formula (3.7).

α	0,95	0,90	0,80	0,70
i_k	1625,9	1577,4	1466,8	1331,6

An example of calculation for $\alpha = 0.95$:

$$i_k = 1550,3 - 1234,9 + 2800 \cdot 0,4683;$$

$$i_k = 1625,9.$$

7. We will find n_{ef} according to Formula (2.41).

α	0,95	0,90	0,80	0,70
n_{ef}	0,5022	0,4947	0,4927	0,4994

An example of calculation for $\alpha = 0.95$:

$$n_{ef} = 0,6242 \lg \frac{40 \cdot 2^{2,87} \cdot 14,52}{666};$$

$$n_{ef} = 0,5022.$$

8. We will determine c'_B according to Formula (2.44).

α_0	0,95	0,90	0,80	0,70
c'_B	616,0	533,5	400,0	300,0

An example of calculation for $\alpha = 0.95$:

$$c'_s = \frac{\Delta i_{s, v, \phi} (p_{s, \phi})^{n_{s, \phi}}}{40 p_{k, c}}$$

or, since $p_{k, s} = 40$ atm abs and $p_{B, f} = 40$ atm abs,

$$c'_s = \frac{666}{40 \cdot 0,02703} = 616,0$$

The temperature of the products of combustion within the combustion chamber of the engine can be determined according to Eq. (3.8), following the method described in Section 9.

The final calculation of results at the selected values of α are presented in the following table.

$T_{k, i}$	$\frac{T_{k, i} - 2200}{1200}$	$\lg 2 $	$m_s 3 $	$\lg c'_s$	$\lg \Delta i_{k, i} = \frac{ 4 }{ 4 + 5 }$	$\Delta i_{k, i}$	$c'_{p, s} T_{k, i}$	$i_{k, i} = \frac{ 7 }{ 7 + 8 }$
3023	0,6860	$\bar{1},8363$ -0,1637	-0,4700	2,7896	2,3196	208,8	1416	1625
2998	0,6650	$\bar{1},8230$ -0,1770	-0,5088	2,7272	2,2184	165,4	1413	1578
2890	0,5747	$\bar{1},7596$ -0,2404	-0,6934	2,6020	1,9086	81,0	1386	1467
2670	0,3915	$\bar{1},5930$ -0,4070	-1,177	2,4770	1,3000	20,0	1308	1328

Consequently, at $i_{k, i} = i_{k'}$, the temperature $T_{k, s}$ is equal to

α	0,95	0,90	0,80	0,70
1) $T_{k, c}$ ° abs.	3023	2998	2890	2670

1) $T_{k, s}$, ° abs.

9. We will calculate $S_{k, s}$ according to Eq. (3.9).

α	0,95	0,90	0,80	0,70
$S_{k, c}$	-0,1602	-0,1869	-0,2524	-0,3354

An example of calculation for $\alpha = 0.95$:

$$S_{x,c} = 2,303 \cdot 0,4683 \lg \frac{3023}{2800} - \frac{0,004574}{0,02703} \lg 40 +$$

$$+ 208,8 \left(\frac{1}{3023} + \frac{1,527 \cdot 10^{-8}}{0,5022} \right);$$

$$S_{x,c} = -0,1602.$$

10. We will determine S_a according to Formula (3.11), taking into consideration that $p_{v,s} = 1 \text{ atm abs.}$

α	0,95	0,90	0,80	0,70
S_a	3,4528	3,4501	3,4498	3,4496

An example of calculation for $\alpha = 0.95$:

$$S_a = -0,1602 + 0,4552 \cdot 2,303 \lg 2800;$$

$$S_a = 3,4528.$$

11. We will determine c'_H according to Formula (2.49), taking into consideration that

$$p_{H,f} = 1 \text{ atm abs and } p_{v,s} = 1 \text{ atm abs.}$$

α	0,95	0,90	0,80	0,70
c'_H	537,2	439,0	303,5	215,0

An example of calculation for $\alpha = 0.95$:

$$c'_H = \frac{14,52}{0,02703};$$

$$c'_H = 537,2.$$

12. According to Eq. (3.12) we find the temperature of the products of combustion at the outlet section of the nozzle, at a coefficient of excess oxygen $\alpha = 0.95$.

The final calculation results are presented in the following table.

$T_{b.c.t}$	$\frac{T_{b.c.t}-1400}{1400}$	$\lg [2]$	$m_H [3]$	$\lg c'_x$	$\lg \Delta i_{b.c.t} = [4] + [5]$
1964	0,403	$\begin{matrix} 1,6054 \\ -0,3946 \end{matrix}$	-2,2540	2,730	0,476

Continuation

$\Delta i_{b.c.t}$	$\frac{1}{T_{b.c.t}} + \frac{1,527 \cdot 10^{-8}}{n_{\text{ox}}}$	$[7] \cdot [8]$	$2,303 c_{p,H}^0 \lg T_{b.c.t}$	$S_{ai} = [9] + [10]$
2,99	0,000539	0,0016	3,4513	3,4529

Consequently, since $S_{ai} \approx S_a$, the temperature of the products of combustion at the outlet section of the nozzle, at $\alpha = 0.95$, is equal to 1964° abs. For the remaining values of the coefficient of excess oxygen, the calculation of $T_{v.s}$ is conducted according to Formula (3.13). Finally, we obtain the following data.

α	0,95	0,90	0,80	0,70
$T_{b.c.t}^{\text{acc}}$	1964	1861	1631	1387

13. The heat content of the products of combustion at the outlet section of the nozzle is determined according to Eq. (2.51).

α	0,95	0,90	0,80	0,70
$i_{b.c}$	857,4	962,6	1204,2	1514,2

An example of calculation for $\alpha = 0.95$:

$$i_{b.c} = 1234,9 + 0,4552(1964 - 2800) + 3,0;$$

$$i_{b.c} = 857,4 \text{ kcal/kg.}$$

14. Specific thrust.

α	0,95	0,90	0,80	0,70
P_{ya}	245,6	245,0	241,7	235,6

An example of calculation for $\alpha = 0.95$:

$$P_{yA} = 9,33 \sqrt{1550,3 - 857,4};$$

$$P_{yA} = 245,6 \text{ kg/kg/sec.}$$

The results of the calculations are presented in the curve shown in Fig. 7 in the form of the sought quantities as functions of the coefficient of excess oxygen as well as of the coefficient of excess oxidizer.

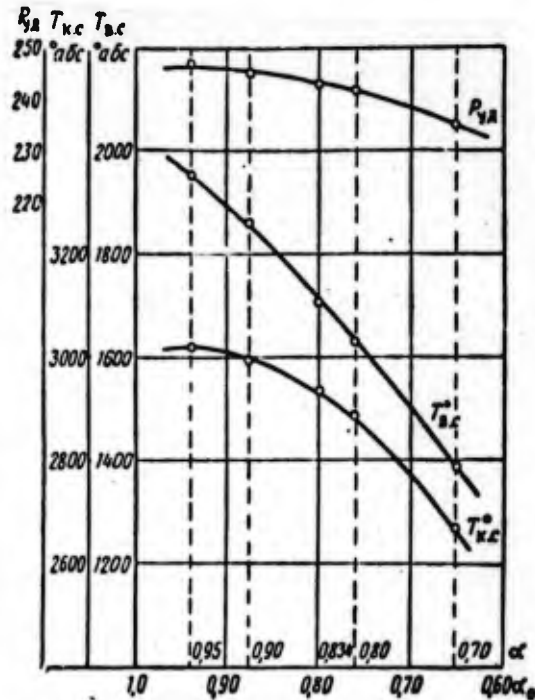


Fig. 7

Example 4. Establish specific thrust and the temperature of the products of combustion within the combustion chamber and at the outlet section of the nozzle of a liquid-fuel rocket engine, operating on technically pure (96%, by weight) nitric acid (admixture, water) and kerosene ($C_g = 86.8\%$, $H_g = 13.2\%$) as a function of the pressure $p_{k.s}$ of the products of combustion within the combustion chamber, as $p_{k.s}$ changes within the limits of 20 to 100 atm abs.

The coefficient of excess oxidizer $\alpha'_0 = 0.8$ ($\alpha' = 0.834$), and the pressure of the products of combustion at the outlet section of

the nozzle is $p_{v.s} = 1$ atm abs.

As we can see from the conditions of the problem, the composition of the propellant components and the coefficient of excess oxidizer in this example coincide fully with the corresponding parameters in Example 1. The initial part of the calculation (up to item 15) coincides with the calculations presented in Example 1. This calculation will, farther on, be carried out in accordance with the sequence described in Section 14B.

We will assume a series of values for $p_{k.s}$ (20, 40, 60, 80, and 100 atm abs) and at each of these values we will carry out the following calculations.

1. We will determine c'_B according to Formula (2.44).

$p_{k.s}$	20	40	60	80	100
c'_B	612,3	435,2	352,5	309,4	277,2

An example of calculation for $p_{k.s} = 20$ atm abs:

$$c'_B = \frac{438}{40 \cdot 0,02517} \left(\frac{40}{20} \right)^{0,4925};$$

$$c'_B = 612,3.$$

2. According to Eq. (3.8) we will determine the temperature $T_{k.s}$ of the products of combustion within the combustion chamber. Let us remember that $i_k = 1504,2$.

To calculate the temperature $T_{k.s}$ at each of the selected values of the pressure $p_{k.s}$ we will assume a series of values for the temperature $T_{k.i}$, close to the anticipated $T_{k.s}$, and from the condition $i_{k.i} = i_k$ we will determine $T_{k.s}$.

The final calculation results are presented in the following table.

$1) T_{k,i}$ ° abs.	$\frac{T_{k,i}-2200}{1200}$	$\lg 2 $	$m_s 3 $	$\lg c'_s$	$\lg \frac{\Delta i_{k,i}}{= [7] + [8]}$	$\Delta i_{k,i}$	$c_{p,s}^0 T_{k,i}$	$\frac{i_{k,i}}{= [7] + [8]}$
2895	0,5790	$\bar{1},7630$ -0,2370	-0,6825	2,7870	2,1045	127,2	1377	1504,2
2940	0,6164	$\bar{1},7900$ -0,2100	-0,6048	2,6388	2,0340	108,1	1399	1507,1
2987	0,6360	$\bar{1},8035$ -0,1965	-0,5660	2,5472	1,9812	95,7	1410	1505,7
2977	0,6474	$\bar{1},8120$ -0,1880	-0,5413	2,4905	1,9492	89,0	1417	1506,0
2987	0,6560	$\bar{1},8170$ -0,1830	-0,5272	2,4430	1,9158	82,3	1421	1503,3

1) $T_{k,i}$, ° abs.

Thus,

$p_{k,s}$	20	40	60	80	100
$1) T_{k,s}^{\circ}$ abs.	2895	2940	2963	2977	2987

1) $T_{k,s}$, ° abs.

3. We will determine $S_{k,s}$ according to Eq. (3.9).

$p_{k,c}$	20	40	60	80	100
$S_{k,c}$	-0,1688	-0,2261	-0,2560	-0,2790	-0,2986

An example of calculation for $p_{k,s} = 20$ atm abs:

$$S_{k,c} = 2,303 \cdot 0,4760 \lg \frac{2895}{2800} - \frac{0,004574}{0,02517} \lg 20 + 127,2 \left(\frac{1}{2895} + \frac{1,527 \cdot 10^{-5}}{0,4925} \right);$$

$$S_{k,c} = -0,1688.$$

4. We will determine S_a according to Formula (3.11).

$p_{k,c}$	20	40	60	80	100
S_a	3,5062	3,4489	3,4190	3,3960	3,3764

An example of calculation at $p_{k,s} = 20$ atm abs.

Since $p_{v.s} = 1 \text{ atm abs}$,

$$S_a = -0,1688 + 2,303 \cdot 0,4630 \cdot \lg 2800;$$

$$S_a = 3,5062.$$

5. We will determine the temperature $T_{v.s}$ of the products of combustion at the outlet section of the nozzle according to Formula (3.13).

$p_{k.c}$	20	40	60	80	100
$T_{s.c}^{\circ}$	1932	1707	1599	1517	1452

1) $T_{v.s}, ^{\circ}\text{abs}$.

An example of calculation for $p_{k.s} = 20 \text{ atm abs}$:

$$\lg T_{s.c} = \frac{3,5062}{2,303 \cdot 0,4630} = 3,286;$$

$$T_{v.s} = 1932^{\circ} \text{ abs}.$$

6. We will determine $i_{v.s}$ according to Eq. (2.51).

$p_{k.c}$	20	40	60	80	100
$i_{s.c}$	1222,1	1115,7	1065,7	1027,7	997,7

An example of calculation at $p_{k.s} = 20 \text{ atm abs}$:

$$i_{s.c} = 1621,7 + 0,4630 (1932 - 2800) + 0;$$

$$i_{s.c} = 1222,1.$$

7. We will determine P_{ud} .

$p_{k.c}$	20	40	60	80	100
P_{ya}	222,9	242,8	251,7	258,1	263,0

An example of calculation at $p_{k.s} = 20 \text{ atm abs}$:

$$P_{ya} = 9,33 \sqrt{1791,9 - 1222,1};$$

$$P_{ya} = 222,9 \text{ kg} \cdot \text{sec}/\text{kg}.$$

The results of the calculation are presented on the curve shown

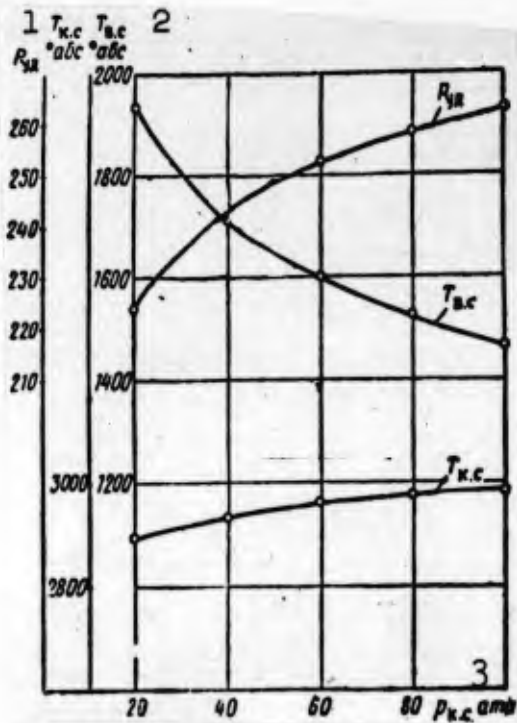


Fig. 8. 1) $T_{k.s}$, $^{\circ}abs$; 2) $T_{v.s}$, $^{\circ}abs$; 3) $p_{k.s}$, atm abs.

in Fig. 8.

Example 5. Establish specific thrust and the temperature of the products of combustion at the outlet section of the nozzle of a liquid-fuel rocket engine, operating on ethyl alcohol (95%, by weight) (admixture, water) and liquid oxygen as a function of the pressure $p_{v.s}$ of the products of combustion at the outlet section of the nozzle, with $p_{v.s}$ ranging from 0.2 to 1.0 atm abs.

The coefficient of excess oxidizer $\alpha'_o = 0.7$ ($\alpha' = 0.748$), and the pressure of the products of combustion within the combustion chamber of the engine is $p_{k.s} = 50$ atm abs.

As can be seen from the condition, the composition of the propellant components, the coefficient of excess oxidizer, and the pressure of the products of combustion within the combustion chamber in the given example are the same as in Example 2 of this section. The solution of the problem (to the actual determination of the entropy $S_{k.s}$

of the products of combustion within the combustion chamber) is carried out in the manner shown in Example 2. Later on, we will handle the calculation in accordance with the procedure described in Section 14C. We will assume the following values for $p_{v.s}$: 1.0, 0.8, 0.5, and 0.2 atm abs, and we will continue the calculation at selected values of $p_{v.s}$.

1. We will find S_a according to Formula (3.11).

$p_{b.c}$	1,0	0,8	0,5	0,2
S_a	3,8525	3,8368	3,7935	3,7143

An example of calculation for $p_{v.s} = 0.8$ atm abs.

$$S_a = -0,2295 + 2,303 \cdot 0,5142 \cdot \lg 2800 + 2,303 \cdot 0,001986 \lg 0,8;$$

$$S_a = 3,8525.$$

2. Since in all cases here $S_a/c_{p,H}^0 < 7.50$, we will carry out the calculation of $T_{v.s}$ according to Formula (3.13).

$p_{b.c}$	1,0	0,8	0,5	0,2
$T_{b.c}^{o_{ab.c}}$	1780	1734	1585	1356

An example of calculation for $p_{v.s} = 0.8$ atm abs:

$$\lg T_{b.c} = \frac{3,8368}{2,303 \cdot 0,5142} = 3,239;$$

$$T_{v.s} = 1734^{\circ} \text{ abs.}$$

3. We will determine the heat content $i_{v.s}$ of the products of combustion at the outlet section of the nozzle according to Eq. (2.51).

$p_{b.c}$	1,0	0,8	0,5	0,2
$i_{b.c}^{\circ}$	1707,0	1683,8	1607,2	1489,2

An example of calculation for $p_{v,s} = 0.8$ atm abs:

$$i_{s,c} = 2232 + 0,5142(1734 - 2800) + 0;$$

$$i_{s,c} = 1683,8 \text{ kcal/kg.}$$

4. We will determine P_{ud} .

$p_{s,c}$	1,0	0,8	0,5	0,2
P_{ya}	272,5	276,0	288,0	305,2

An example of calculation for $p_{v,s} = 0.8$ atm abs:

$$P_{ya} = 9,33 \sqrt{2559 - 1683,8};$$

$$P_{ya} = 276,0 \text{ kg/kg/sec.}$$

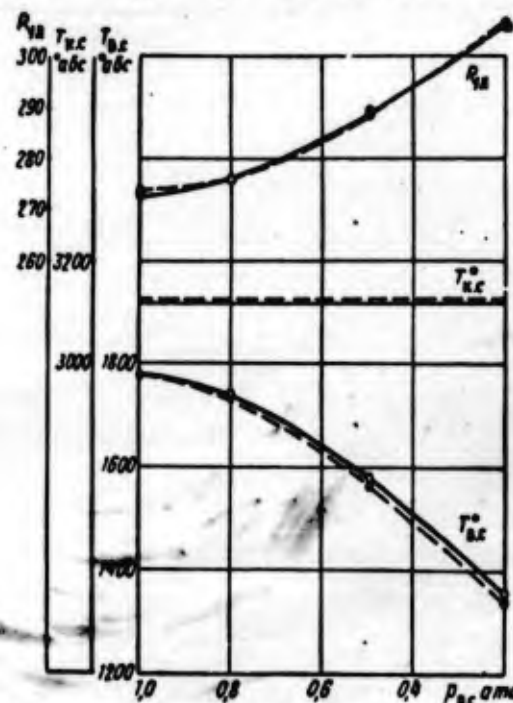


Fig. 9

G.B. Sinyarev and M.V. Dobrovol'skiy [9] have constructed an entropy diagram for the given case. The result of our calculations can be compared against the data of the diagram.

1 $p_{v.s}$ атм	2 $T_{v.s}$ в °абс.		5 $P_{уд}$ в $\frac{кг\ сек}{кг}$		8 Погрешность удельной тяги %
	3 по расчету	4 по диаграмме	6 по расчету	7 по диаграмме	
1,0	1780	1780	272,5	273,7	0,4
0,8	1734	нет данных	276,0	нет данных	—
0,5	1585	1565	288,0	289,0	0,3
0,2	1356	1340	305,2	306,0	0,3

1) $p_{v.s}$, atm abs; 2) $T_{v.s}$, in °abs;
 3) according to calculation; 4) ac-
 cording to diagram; 5) $P_{уд}$, in
 kg·sec/kg; 6) according to calcula-
 tion; 7) according to diagram; 8)
 error in specific thrust, %.

Figure 9 presents a curve constructed according to the results obtained in the calculations that were carried out. The dashed lines on the diagram refer to data from the entropy diagram.

16. Calculation Accuracy

Let us examine the calculation accuracy for the proposed method. We assume that certain methods of thermodynamic calculation for rocket engines are based on the preliminary determination of the composition of the products of combustion, and these methods are exact. The degree of calculation accuracy for the basic operating characteristics of the engine in an approximate method can be evaluated by a comparison of the values of these characteristics, during their determination, by an exact method and by the approximate method proposed in this book.

It was indicated earlier that with respect to the approximate expressions, established in this book, the heat content i^0 of undissociated products of combustion can be calculated with an error not exceeding 0.4% of its magnitude, and in the absolute majority of cases under consideration, the error did not exceed 0.1 to 0.2%. The heat content i^0 makes up the basic fraction of the heat content of the dis-

sociated products of combustion. The parameter Δi - the increment in heat content for the products of combustion as a result of dissociation - in all cases functions as a correction for the parameter i^0 , and it is greater to the extent that temperature is higher and pressure lower.

In this work we have adopted a system for the construction of nomograms which eliminates great errors in the calculation of Δi , and we have reference here to:

at a temperature below 1800 to 1900° abs, for the products of combustion, the magnitude of the parameter Δi may be neglected because of its smallness;

at a temperature ranging approximately between 1900 to 2400° abs, for the products of combustion, Δi is small in absolute magnitude and the possible error in its determination, therefore, will have virtually no effect on the calculation results;

at a somewhat higher temperature for the products of combustion at the outlet section of the engine nozzle, a temperature approaching 2600 to 2800° abs, the possible error in the determination of Δi is smallest, since the nomograms for the determination of the parameter $\Delta i_{H,v,f}$ in the given temperature zone "H" were constructed for a temperature of 2800° abs;

at a temperature of 3200 to 3600° abs, for the products of combustion within the combustion chamber of the engine, the error in the determination of Δi will be smallest, since the nomograms for the parameter $\Delta i_{B,v,f}$ were constructed in the temperature zone "B" for a temperature of 3400° abs.

It was established earlier that the interpolation formulas of the type (2.35) are approximate, and the method, recommended in this book, for calculation of the exponent n_{ef} is not rigorous (see Chapter 2,

Section 7).

It is for this reason that the calculation accuracy in the case of the operating characteristics of the engine increases if the thermodynamic calculation for the engine is carried out at a pressure of $p_{k.s} = 40$ atm abs for the products of combustion in the combustion chamber, and at $p_{v.s} = 1$ atm abs for the products of combustion at the outlet section of the nozzle. Since the nomograms for the determination of the parameters $\Delta i_{B,v,f}$ and $\Delta i_{H,v,f}$ were constructed for precisely these values of pressure, this circumstance eliminates the need to interpolate for pressure and makes it possible to avoid the errors that are associated with this interpolation.

The calculation accuracy in the case of the operating characteristics of the engine, as was demonstrated in the calculations that we carried out, remains completely satisfactory if the calculation is carried out at a pressure in excess of 20 atm abs for the products of combustion within the combustion chamber.

Speaking of the calculation accuracy of the proposed method, we should stress the circumstance that the entire calculation, on the basis of which the approximate mathematical expressions have been established and on the basis of which the nomograms were constructed, were carried out by conventional methods in which we have taken into consideration the products of combustion for six dissociation reactions and the reactions of the oxidation of nitrogen. On the basis of the considerations referred to above, we need not anticipate any major errors in the final calculation results. A sufficiently large quantity of thermodynamic calculations, carried out according to the proposed method, makes it possible to assert that the maximum error in the determination of specific thrust, when calculated according to the proposed method, in the absolute majority of cases, does not exceed 1% of

its magnitude. The error in the determination of the temperature of the products of combustion is not greater than 40 to 50°.

Such an error in technical calculations is completely permissible, all the more so since the law governing variations in the operating characteristics of the engine from the most important parameters (X_c , X_H , α , $p_{k.s}$, $p_{v.s}$) remains completely intact.

[LIST OF TRANSLITERATED SYMBOLS]

Manu-
script
Page
No.

93 $i_{кер} = i_{ker} = i_{kerosin} = i_{kerosene}$

93 $i_{H_2O} (жидкой) = i_{H_2O} (zhidkoy) = i_{H_2O} (liquid)$

Chapter 4

CALCULATION OF THE CHEMICAL COMPOSITION OF THE PRODUCTS OF COMBUSTION AT HIGH TEMPERATURES

As was pointed out earlier, we require no preliminary calculation of the chemical composition of the products of combustion in carrying out the thermodynamic calculations for rocket engines according to the method recommended in this book. However, in certain special cases, particularly in the design of the cooling system, as well as in carrying out particularly exact calculations of operating characteristics for engines, it becomes necessary to calculate the chemical composition of the products of combustion.

The present chapter presents an outline of the method used to calculate the composition of the products of combustion taking into consideration their dissociation. This method will make it possible to calculate the composition of the products of combustion for the propellants in the pressure region ranging approximately from 0.5 to 100 atm abs and at temperatures up to 3600° abs. The amount of work involved in the calculations according to the proposed method is several times less than would be required in calculations for the products of combustion on the basis of certain methods of successive approximations.

The essence of the proposed calculation method consists in the following: the system of equilibrium and material-balance equations

(2.19) is reduced to a quadratic equation for the quantity $n = p_{H_2O}/p_{H_2}$, which is solved by a method of successive approximations with use of the nomograms presented in Figs. 21 to 42 of the Appendix. In deriving the calculation equation, no substitutions were undertaken nor were any of the existing relationships replaced; there may be a drop in the accuracy of the determination of the content in individual components in the products of combustion only as a result of errors occurring in work done with the above-mentioned nomograms.

A feature of the proposed method is that we should assume the conventional pressure p^0 of the undissociated products rather than the actual pressure p of the products of combustion. The quantities p^0 and p are interrelated in the following manner:

$$p = p^0 + \Delta p,$$

where Δp is the change in the pressure of the products of combustion as a result of dissociation in a constant volume.

17. Derivation of Calculation Equations

For the derivation of the calculation equation we will carry out a series of transformations of existing equations. We will derive an algebraic sum of the type

$$A'_i = X_C + X_H - p. \quad (4.1)$$

In a consideration of Eqs. (2.6) and (2.7), we will obtain

$$A'_1 = p_{H_2O} + p_{H_2} - p_{O_2} - p_O - p_{NO} - p_{N_2} - p_N \quad (4.2)$$

Expression (4.2), in consideration of (2.8), may be written in the form

$$A'_1 = p_{H_2O} + p_H - p_O - p_O - 0,5p_{NO} - 0,5p_N - 0,5X_N.$$

Denoting

$$A'_i = A_i + 0,5X_N. \quad (4.3)$$

we will obtain

$$A'_i = p_{H_2O} + p_H - p_O - p_O - 0,5p_{NO} - 0,5p_N. \quad (4.4)$$

Let us now derive the algebraic sum of the form

$$A_2' = X_C + X_O + X_H + X_N - 2p. \quad (4.5)$$

Taking into consideration Eqs. (2.5)-(2.8), we will obtain

$$A_2' = p_{CO} + p_{H_2O} - p_H - p_O - p_N. \quad (4.6)$$

And, finally, we will derive the algebraic sum

$$A_3' = X_C - X_O - X_N + p. \quad (4.7)$$

Substituting the values of the terms in the right-hand part in accordance with Eqs. (2.5), (2.6), and (2.8), we will be able to write

$$A_3' = p_{CO} + p_{H_2} + p_H - p_O - p_{NO} - p_N. \quad (4.8)$$

Let us introduce yet another parameter into the examination

$$A_5' = A_3' + 0,5X_N. \quad (4.9)$$

It is not difficult to prove that

$$A_5' = p_{CO} + p_{H_2} + p_H - p_O + 0,5p_N - 0,5p_{NO}. \quad (4.10)$$

From Expression (4.2) we will find the relationship between p_{H_2} and p_O :

$$p_{H_2O} + p_{H_2} - p_O = A_5' + E, \quad (4.11)$$

where

$$E = p_O + p_{NO} + p_N + p_N = p_O + 0,5p_{NO} + 0,5p_N + 0,5X_N. \quad (4.11a)$$

The subsequent derivations can be simplified if we introduce arbitrary equilibrium constants into the examination, said constants associated with the basic constants in the following fashion:

$$K_8 = K_4 \sqrt{K_3} = \frac{p_H \sqrt{p_O}}{p_{H_2O}}; \quad K_9 = K_5 K_2 = \frac{p_O p_{H_2}}{p_{H_2O}}$$

$$K_{10} = K_7 K_2 = \frac{p_{NO} p_{H_2}}{p_{H_2O} \sqrt{p_N}}; \quad K_{11} = \frac{K_3}{\sqrt{K_2}} = \frac{p_{OH}}{\sqrt{p_{H_2O}} \sqrt{p_O}}$$

$$K_{12} = K_2^2 = \frac{p_O p_{H_2}^2}{p_{H_2O}^2}.$$

Substituting into (4.11) the value of the parameter p_{H_2O} , denoted in terms of the equilibrium constant K_9 , we will find

$$p_{H_2} = \frac{A'_1 + E + p_0}{n + 1} \quad (4.12)$$

where

$$n = \frac{p_{H_2O}}{p_{H_2}} = \frac{p_0}{K_9}$$

It follows from (4.8) that

$$p_{CO} = A'_3 + E - p_{H_2} - p_H - p_N \quad (4.13)$$

Substituting into (4.13) the value of p_{H_2} from (4.12), we will obtain

$$\frac{x_C}{1 + Kn} = \frac{A'_3 n + A'_3 - A'_1 + En - p_0 - (p_H + p_N)(1 + n)}{1 + n} \quad (4.14)$$

Let us break down the right-hand part of Eq. (4.14). The numerical values of the parameters A'_1 , A'_2 , and A'_3 cannot be determined prior to the calculation of the chemical composition of the products of combustion, since the quantity p - the pressure of the dissociated products - is not known.

However, we have every possibility of calculating the values of the parameters A_1 , A_2 , and A_3 , determined by the following expressions:

$$A_1 = x_C + x_H - p^0; \quad (4.15)$$

$$A_2 = x_C + x_H + x_O + x_N - 2p^0; \quad (4.16)$$

$$A_3 = x_C - x_O - x_N + p^0, \quad (4.17)$$

since in view of (2.18)

$$x_C = x_C^0; \quad x_H = x_H^0; \quad x_O = x_O^0; \quad x_N = x_N^0.$$

The pressure p^0 of the undissociated products of combustion we assume, as was pointed out above, to be known. Taking into consideration that

$$p^0 = p - \Delta p,$$

in the simultaneous examination of Expressions (4.1), (4.5), (4.7), (4.15), (4.16), and (4.17), we will obtain

$$A'_1 = A_1 - \Delta p; \quad (4.18)$$

$$A_2' = A_2 - 2\Delta p; \quad (4.19)$$

$$A_3' = A_3 + \Delta p, \quad (4.20)$$

and subsequently

$$A_4' = A_1 - \Delta p + 0,5X_N = A_4 - \Delta p, \quad (4.21)$$

$$A_5' = A_3 + \Delta p + 0,5X_N = A_5 + \Delta p. \quad (4.22)$$

Taking into consideration the derived expressions, we will transform (4.14):

$$\frac{X_C}{1 + Kn} = \frac{A_3n + A_3 - A_1 + n\Delta p + 2\Delta p + En - p_O - (p_H + p_N)(1 + n)}{1 + n}. \quad (4.23)$$

The numerical value of the parameter Δp , evidently, is equal to the total pressure of the products of dissociation (at $\alpha < 1$, these are O_2 , OH , NO , O , H , N) after the subtraction of the reduced pressure of the "basic" gases, present in the undissociated products of combustion. The total pressure of the products of dissociation is

$$p_{p.d} = p_O + p_{OH} + p_{NO} + p_O + p_H + p_N.$$

The drop in the partial pressure in the "basic" products of combustion during dissociation may be found through a comparison of Expressions (1.11)-(1.14) and (2.5)-(2.8). Thus to balance the chemical element - hydrogen - the following expressions are valid:

$$X_H^0 = 2p_{H,O}^0 + 2p_{H,i}^0;$$

$$X_H = 2p_{H,O} + 2p_{H,i} + p_{OH} + p_H.$$

In accordance with (2.18)

$$X_H = X_H^0.$$

Denoting

$$\Delta p_{H,O} = p_{H,O}^0 - p_{H,O};$$

$$\Delta p_{H,i} = p_{H,i}^0 - p_{H,i};$$

we will find that the total drop in the partial pressures of the water vapor and the biatomic hydrogen, as a result of dissociation, is equal to:

$$\Delta p_{H,O} + \Delta p_{H,i} = 0,5p_{OH} + 0,5p_H.$$

Analogously, for the other chemical elements, we will obtain

$$\begin{aligned}\Delta p_{CO} + \Delta p_{CO} &= 0; \\ \Delta p_{N_2} &= 0,5p_{NO} + 0,5p_N.\end{aligned}$$

Then, by definition

$$\Delta p = p_{n,2} - (\Delta p_{H_2O} + \Delta p_{H_2} + \Delta p_{CO} + \Delta p_{CO} + \Delta p_{N_2}).$$

or finally

$$\Delta p = 0,5(p_H + p_{OH} + p_N + p_{NO}) + p_O + p_{O_2}. \quad (4.23a)$$

Taking into consideration Formulas (4.23a) and (4.11a), we will obtain

$$\begin{aligned}\frac{X_C}{1 + \lambda n} &= \\ = \frac{A_5 n + A_3 - A_1 + \left(0,5p_{OH} + \frac{1}{n} p_{OH} - 0,5p_H\right)n + (1+n)(p_{NO} + p_O + 2p_{O_2})}{1+n}.\end{aligned} \quad (4.24)$$

Here

$$A_5 = 2X_C - X_O + 0,5X_H, \quad (4.25)$$

$$A_3 - A_1 = 2X_C - X_O. \quad (4.26)$$

It also follows from a comparison of (4.25) against (4.26) that

$$A_3 - A_1 = A_5 - 0,5X_H. \quad (4.27)$$

We will denote

$$B_1 = 0,5p_{OH} + \frac{1}{n} p_{OH} - 0,5p_H, \quad (4.28)$$

$$B_2 = p_{NO} + p_O + 2p_{O_2}. \quad (4.29)$$

Equation (4.24) takes the following form, if we take into consideration (4.25)-(4.29):

$$\frac{X_C}{1 + \lambda n} = \frac{A_5 n + A_5 - 0,5X_H + B_1 n + B_2(n+1)}{1+n}. \quad (4.30)$$

We will transform the expression for B_1 :

$$\begin{aligned}B_1 = 0,5p_{OH} + \frac{1}{n} p_{OH} - 0,5p_H &= 0,5K_3 \frac{p_{OH}}{\sqrt{p_{H_2}}} + \frac{1}{n} K_3 \frac{p_{H_2O}}{\sqrt{p_{H_2}}} - \\ &- 0,5K_4 \sqrt{p_{H_2}} = \sqrt{p_{H_2}} [(0,5n+1)K_3 - 0,5K_4].\end{aligned} \quad (4.31)$$

We will express $\sqrt{p_{H_2}}$ in terms of the ratio $n = p_{H_2O}/p_{H_2}$.

At one time, we obtained the following expression for p_{H_2} :

$$p_{H_2} = \frac{A_1 + E + p_0}{1+n} = \frac{A_1 - \Delta p + E + p_0}{1+n} \quad (4.12)$$

Examining (4.12), (4.23a), and (4.11a) jointly, we will find

or

$$p_{H_2} = \frac{A_4 - 0,5(p_{OH} + p_{H_2})}{1+n}$$

$$p_{H_2} = \frac{A_4 - 0,5\sqrt{p_{H_2}}(K_3n + K_4)}{n+1},$$

since

$$p_{OH} = K_3 \frac{p_{H_2O}}{\sqrt{p_{H_2}}} \text{ и } p_{H_2} = K_4 \sqrt{p_{H_2}},$$

and further

$$\sqrt{p_{H_2}} = \frac{-b' + \sqrt{b'^2 - 4a'c'}}{2a'}, \quad (4.32)$$

where

$$b' = 0,5(K_3n + K_4);$$

$$a' = n + 1;$$

$$c' = -A_4.$$

Thus parameter B_1 is a function of temperature, of the elementary composition of the propellant, and of the ratio $n = p_{H_2O} / p_{H_2}$.

Now let us examine parameter B_2 :

$$B_2 = p_{NO} + p_0 + 2p_{O_2}.$$

We know that

$$p_{NO} = K_{10}n \sqrt{p_{N_2}};$$

$$p_0 = K_9n;$$

$$p_{O_2} = K_{12}n^2.$$

We will determine p_{N_2} under conditions of dissociation:

$$p_{N_2} = 0,5X_N - 0,5K_{10}n \sqrt{p_{N_2}} - 0,5K_9 \sqrt{p_{N_2}}.$$

The quantity p_{N_2} of this equation can be found by solving the equation for $\sqrt{p_{N_2}}$. However, in actual practice, all cases of calculation can be simplified by effecting the following substitution in the right-hand part of the equation:

$$\sqrt{p_{N_2}} \approx \sqrt{0,5X_N}.$$

Thus,

$$p_N \approx 0,5X_N - 0,5(K_{10}n + K_4)\sqrt{0,5X_N}.$$

Finally, for B_2 , we obtain

$$B_2 = (K_{10}\sqrt{p_N} + K_4 + K_{12}n)n. \quad (4.33)$$

We will rewrite Eq. (4.30) in the following form:

$$\frac{X_C}{1 + Kn} = \frac{(A_5 + B_1 + B_2)n + A_5 - 0,5X_H + B_2}{1 + n}.$$

Solving (4.30) for \underline{n} , we will obtain a quadratic equation of the type

$$an^2 + bn + c = 0, \quad (4.34)$$

where

$$\left. \begin{aligned} a &= K(A_5 + B_1 + B_2); \\ b &= (A_5 + B_1 + B_2) + K(A_5 - 0,5X_H + B_2) - X_C; \\ c &= (A_5 - 0,5X_H + B_2) - X_C. \end{aligned} \right\} \quad (4.35)$$

If we assume that $B_1 = B_2 = 0$, Eq. (4.34) can be transformed into a conventional quadratic equation for $n_0 = p_{H_2}^0 / p_{H_2}^0$ for the undissociated products of combustion at the same temperature.

In the case of dissociated products of combustion, the parameters B_1 and B_2 are not equal to zero. As can be seen from (4.31), the parameter B_1 is a function of temperature, and of the ratios \underline{n} and A_4 .

Since

$$\begin{aligned} A_4 &= A_1 + 0,5X_N = X_C + X_H - p^0 + 0,5X_N = X_C + X_H - X_C - 0,5X_H - \\ &\quad - 0,5X_N + 0,5X_N = 0,5X_H, \end{aligned}$$

in the final calculation B_1 will be a function of temperature, \underline{n} , and X_H . The parameter B_2 depends upon temperature, \underline{n} , and X_N .

18. Method for the Calculation of the Chemical Composition of the Products of Combustion

Let us maintain that it is necessary to calculate the chemical composition of the dissociated products of combustion for a given elementary chemical composition of the propellants and a temperature T .

Let us examine the course of the solution for this problem when using

the proposed calculation method.

We know that a calculation of the partial pressures of individual components of the products of combustion is quite simple if we know the partial pressure of one of the components or the ratio of any two components, such as, for example $n = p_{H_2O} / p_{H_2}$.

In calculations employing the proposed method, the primary goal is to seek out the magnitude of \underline{n} . The calculation of the parameter \underline{n} is carried out by the method of successive approximations on the basis of the method discussed below. Having assumed certain values for n_1 (in the first approximation), we determine parameters B_1 and B_2 and, solving Eq. (4.34), we will find the value of n_2 . The true magnitude of \underline{n} will be found between n_1 and n_2 , and here, as has been demonstrated in calculation practice, the magnitude of \underline{n} is found closer to n_2 . For this reason, in the second approximation, it is expedient to assume a value of n_3 , determined by the following formula:

$$n_3 = \frac{2}{3} n_2 + \frac{1}{3} n_1. \quad (4.36)$$

Once we have the value of n_3 we again determine the parameters B_1 and B_2 , and as a result of solving Eq. (4.34), we find the quantity n_4 , which is then compared against n_3 .

The calculation is continued until the value of the parameter \underline{n} , selected for the calculation, coincides with the value of this parameter, as derived in accordance with Eq. (4.34), or until we obtain values that are close to one another. Generally, the problem is resolved after two or three attempts. The found value of \underline{n} is then used for the determination of the partial pressures of the components of the products of combustion.

Which magnitude of n_1 should we take first?

For an approximate determination of n_1 we should assume $B_1 = B_2 =$

= 0 and according to Eq. (4.34) to determine n_0 - the ratio of the partial pressures of the same gases, in the absence of dissociation. We know that as a result of dissociation the partial pressure of water vapor diminishes, and the partial pressure of biatomic hydrogen increases. Thus in all cases, $n < n_0$. To reduce the number of attempts, an initial value of n_1 that is somewhat lower than n_0 should be selected.

As yet nothing has been said as to the manner in which we should calculate the quantities B_1 and B_2 . The determination of these quantities on the basis of Eqs. (4.31) and (4.33) is not desirable. This operation can be avoided by calculating B_1 and B_2 beforehand for a series of temperatures, and by constructing the corresponding nomograms. For B_1 and B_2 two types of nomograms have been constructed:

for the region of sufficiently high pressures and temperatures:

$$p = 3-100 \text{ atm abs, } T = 2600-3600^\circ \text{ abs}$$

(see Figs. 21-32 of the Appendix);

for the region of relatively low pressures and temperatures:

$$p = 0.5-3 \text{ atm abs, } T = 2000-2800^\circ \text{ abs,}$$

(see Figs. 33-42 of the Appendix).

The nomograms shown in Figs. 21-32 of the Appendix are intended primarily for the determination of the chemical composition of the dissociated products of combustion within the combustion chamber of a liquid-fuel rocket engine. The nomograms shown in Figs. 33-42 of the Appendix make it possible to determine the chemical composition of the products of combustion at the outlet section of the nozzle. A sufficiently wide range of variation in the quantity n makes it possible to employ these nomograms, in practice, at all values for the coefficient of excess oxygen α of interest to us.

Having determined the ratio $n = p_{H_2O}/p_{H_2}$, it is easy to calculate

the chemical composition of the products of combustion according to the known relationships

$$b' = 0,5(K_3n + K_4);$$

$$a' = n + 1; \quad c' = -0,5X_H;$$

$$\sqrt{p_{H_2}} = \frac{-b' + \sqrt{b'^2 - 4a'c'}}{2a'};$$

$$p_{H_2O} = n p_{H_2}; \quad p_{OH} = \frac{p_{H_2O}}{\sqrt{p_{H_2}}} K_3;$$

$$p_H = K_4 \sqrt{p_{H_2}}; \quad p_O = K_5 n; \quad \sqrt{p_{O_2}} = K_2 n;$$

$$p_{CO} = \frac{X_C}{1 + K_6}; \quad p_{CO_2} = X_C - p_{CO};$$

$$\sqrt{p_{N_2}} = -0,25K_{10}n + \sqrt{(0,25K_{10}n)^2 + 0,5X_N};$$

$$p_{NO} = K_{10}n \sqrt{p_{N_2}}; \quad p_N = K_6 \sqrt{p_{N_2}};$$

$$p_{N_2} = 0,5X_N - 0,5(p_{NO} + p_N).$$

If it becomes necessary repeatedly to undertake any special calculations in the region of change for the parameters n , X_H , X_N , and T , said region not encompassed by the nomograms that have been constructed, these can easily be constructed in accordance with the known Relationships (4.31) and (4.33). When the number of special calculations is small and the construction of additional nomograms pointless, we can carry out calculations without nomograms, using Eqs. (4.31) and (4.33) for the determination of the parameters B_1 and B_2 .

The principle involved in the calculation of the chemical composition of the products of combustion, the method proposed in this book, is maintained at any values for the equilibrium constants. However, nomograms have been constructed for specific values of the equilibrium constants, and these are presented in Table 2 of the Appendix. Since the equilibrium constants are refined with time, should it be necessary to determine B_1 and B_2 at new values for these constants, new nomograms may be constructed or Eqs. (4.31) and (4.33) can be used directly.

To calculate the heat content and the entropy of the products of combustion, we may make use of the tabular heat-content and entropy values for individual gases — the products of dissociation — regardless of the point which has been selected as the origin of the system for these parameters in the determination of their numerical values. In particular, Tables 1 and 3 of the Appendix may be used.

The calculation of the chemical composition of the products of combustion is simplest with those temperature values for which the nomograms shown in Figs. 21-42 of the Appendix were constructed. The chemical composition is best determined for the intermediate temperature values by the method of graphical interpolation for the known partial pressures of the components of the products of combustion at three or four adjacent temperatures. If desired, the calculation of the chemical composition of the products of combustion may be carried out at any selected temperature. For this, it is necessary beforehand to construct curves representing the equilibrium constants as functions of temperature and to determine the parameters B_1 and B_2 directly from Eqs. (4.31) and (4.33) rather than according to the nomograms.

To calculate the composition of the products of combustion and, first of all, to determine the values of the initial parameters X_C , X_H , X_N , and X_O according to Formulas (1.10), (1.15)-(1.17), and (2.18), it is necessary to assume the quantity p^0 — the pressure of the products of combustion in the absence of dissociation. The actual pressure p of the dissociated products of combustion, whose composition is determined by calculation, exceeds p^0 by Δp .

Since the quantity Δp is a function of the temperature of the products of combustion, it is clear that with a constant p^0 the pressure p of the dissociated products of combustion will take on various values at various temperatures.

In the design of rocket engines this method for the calculation of the products of combustion is inconvenient, since the composition of the products of combustion in rocket engines is determined at a given pressure p .

In this case, the quantity Δp should be calculated in advance, and then the quantity p^0 , this quantity to correspond to the pressure p .

In the temperature zone "B" (in excess of 2800° abs) to calculate the parameter Δp at given pressure and temperature for the dissociated products of combustion, we can use the equation

$$\Delta p = \frac{p}{130.40} \Delta i_{s, v, f} \left(\frac{p_{s, \phi}}{p} \right)^{n_{ef}} \left(\frac{T - 2200}{T_{s, \phi} - 2200} \right)^{m_B} \quad (4.37)$$

Equation (4.37) was derived through the joint consideration of Eqs. (2.43) and (2.34), wherein we took into consideration the circumstance that in the general case p is not equal to 40 atm abs. The parameters $\Delta i_{B, v, f}$, n_{ef} , and m_B are determined, as before, in accordance with the nomograms and the formulas presented in Chapter 2 of the present book, with the value of α at which the calculation of the composition of the products of combustion is carried out (the value of m_B is independent of α). The numerical values of the parameters $X_C = X_C^0$ and $X_H = X_H^0$, by means of which we determine the values of $\Delta i_{B, v, f}$, n_{ef} , and m_B according to the nomograms and the formulas in Chapter 2, are found by Formulas (1.39) and (1.40) for the given propellant at one time and they are independent of the temperature T and the pressure p of the dissociated products of combustion that were assumed for the purposes of the calculation.

The parameter Δp is a function of the temperature and the pressure of the dissociated products of combustion and changes with a change in the pressure and the temperature, in accordance with Eq. (4.37).

If the magnitude of the pressure p remains constant, and only temperature changes, Δp will change in accordance with the change in temperature. The sought pressure of the undissociated products of combustion (we will refer to this as p_T^0) will change simultaneously with a change in Δp , since

$$p_T^0 = p - \Delta p. \quad (4.38)$$

In order for the pressure of the dissociated products of combustion to be equal to the selected pressure p , it is necessary for parameters X_C and X_H , used in the nomograms presented in Figs. 21-42 of the Appendix for the determination of B_1 and B_2 , to be calculated according to the following formulas:

$$X_{C_T} = p_T^0 \mu^0 A_C; \quad (4.39)$$

$$X_{H_T} = p_T^0 \mu^0 A_H. \quad (4.40)$$

The values of the parameters X_{C_T} and X_{H_T} may be used for the determination of the parameters B_1 and B_2 from the nomograms only at the temperature T at which the calculation of the chemical composition of the products of combustion is carried out. If the calculations are carried out at another temperature for the products of combustion it is necessary to determine a new value of Δp according to Eq. (4.37) and to use this new value to find new values for p_T^0 , X_{C_T} , and X_{H_T} .

Experience has demonstrated, however, that for a variety of reasons the pressure of the dissociated products of combustion is not always obtained precisely equal to the given pressure p , but is somewhat different, either one way or another. If the difference is small, we can, respectively, reduce or increase the partial pressures of all of the components of the products of combustion, assuming a linear law for the change in the partial pressures relative to the total pressure. In the case of a more or less substantial difference, the calculation should be repeated with corrected values of X_{C_T} and X_{H_T} (multiplied by

the ratio between the given pressure of the dissociated products of combustion and the obtained value for this pressure).

At temperatures lying within the temperature zone "H" (below 2800° abs), we use the following equation for the determination of the parameter Δp at a pressure p :

$$\Delta p = \frac{p}{130} \Delta i_{n, v, \phi} \left(\frac{1}{p} \right)^{n_{\phi}} \left(\frac{T - 1400}{T_{n, \phi} - 1400} \right)^{m_{\phi}}. \quad (4.41)$$

19. Examples of the Calculation of the Chemical Composition of the Dissociated Products of Combustion

Below we present four examples of the calculation of chemical composition in accordance with the proposed method. The first two of these examples pertain to propellants consisting primarily of hydrogen and containing no nitrogen. The two following calculation examples pertain to propellants consisting primarily of carbon and nitrogen.

In Examples 1 and 3 we show the sequence employed in the calculation of the chemical composition of the products of combustion at a given pressure p^0 .

In Examples 2 and 4 we present the calculation of the chemical composition of the products of combustion at a given pressure p of dissociated products of combustion.

Example 1. Calculate the composition of the products of combustion of a propellant characterized by the following parameters: $X_C = 14$, $X_H = 52$, $X_N = 0$, and $\alpha = 0.8945$, at $p^0 = 40$ atm abs and $T = 3000^\circ$ abs.

1. We will determine the ratio $n_0 = p_{H_2O}^0 / p_{H_2}^0$ (in the absence of dissociation). It is with this purpose in mind that we will carry out the following calculations:

a) we will find the independent oxygen parameter:

$$X_0 = (2X_C + 0.5X_H)\alpha;$$

$$X_0 = (2 \cdot 14 + 0.5 \cdot 52) 0.8945;$$

$$X_0 = 48.34;$$

b) we will determine the coefficient A_5 :

$$A_5 = 2X_c - X_0 + 0.5X_H;$$

$$A_5 = 2 \cdot 14 - 48.34 + 0.5 \cdot 52;$$

$$A_5 = 5.66;$$

c) we will use Formula (4.35) to establish the magnitude of the coefficients in the quadratic equation, assuming

$$B_1 = B_2 = 0;$$

$$a = K \cdot A_5;$$

$$a = 0.1445 \cdot 5.66;$$

$$a = 0.8182;$$

$$b = A_5 + K(A_5 - 0.5X_H) - X_c;$$

$$b = 5.66 + 0.1445(5.66 - 0.5 \cdot 52) - 14;$$

$$b = -11.282;$$

$$c = (A_5 - 0.5X_H) - X_c;$$

$$c = (5.66 - 26) - 14;$$

$$c = -34.34;$$

d) we will find n_0

$$n_0 = \frac{11.282 + \sqrt{11.282^2 + 4 \cdot 0.8182 \cdot 34.34}}{2 \cdot 0.8182};$$

$$n_0 = 16.35.$$

2. Under given conditions we will determine the ratio \underline{n} .

a) in the first approximation we will select

$$n_1 < n_0 = 14.$$

We will find $B_1 = 0.925$ according to the nomogram presented in Fig. 27 of the Appendix.

We will find $B_2 = 1.02$ according to the nomogram presented in Fig. 28 of the Appendix.

We will use the formulas in (4.35) to determine the coefficients of the quadratic equation (4.34):

$$a = 1.099; b = -9.183; c = -33.32.$$

Having solved Eq. (4.34), we will find

$$n_2=11,09.$$

The difference between the values of n_1 and n_2 turned out to be rather substantial;

b) in the second approximation we will select n_3 , found according to Formula (4.36):

$$n_3 = \frac{2}{3} \cdot 11,09 + \frac{1}{3} \cdot 14 = 12,07.$$

After rounding off, $n_3 = 12$.

We will find $B_1 = 0.858$ according to the nomogram presented in Fig. 27 of the Appendix.

We will find $B_2 = 0.76$ according to the nomogram presented in Fig. 28 of the Appendix.

We will find the coefficients of Eq. (4.34):

$$a=1,051; b=-9,552; c=-33,58.$$

We will solve Eq. (4.34), and we will find

$$n_4=11,79;$$

c) since the difference between the values of n_3 and n_4 is not great, the quantity n_5 for the subsequent, third approximation need not be calculated according to Formula (4.36), but we may assume that $n_5 = n_4$. With minor rounding off, $n_5 = 11.80$.

According to the nomograms indicated above, we will find $B_1 = 0.850$ and $B_2 = 0.74$.

We will determine the coefficients of Eq. (4.34):

$$a=1,048; b=-9,582; c=-33,60.$$

Having solved Eq. (4.34), we obtain $n_6 = 11.84$;

d) for the true value of \underline{n} , we assume

$$n = \frac{n_5 + n_6}{2} = \frac{11,80 + 11,84}{2} = 11,82.$$

3. Solving the chain of equations presented in Section 18, we

will find the partial pressures of the components of the products of combustion under conditions of dissociation. The values of the equilibrium constants are taken from Table 2 of the Appendix.

1 Последовательность вычислений	2 Значения парциальных давлений газов <i>атм</i>	3 Значения парциальных давлений газов при расчете по точному методу <i>атм</i>	1 Последовательность вычислений	2 Значения парциальных давлений газов <i>атм</i>	3 Значения парциальных давлений газов при расчете по точному методу <i>атм</i>
$b' = 0,5(K_3n + K_4)$	0,6766		P_0	0,0682	0,068
$a' = n + 1$	12,82		$\sqrt{P_0}$	0,579	
$c' = -0,5X_H$	-26		P_{O_2}	0,335	0,3331
$\sqrt{P_{H_2}}$	1,397		P_{CO}	5,170	5,1801
P_{H_2}	1,952	1,960	P_{CO_2}	8,830	8,8199
P_{H_2O}	23,09	23,095	4 Полное давление продуктов сгорания	41,336	41,3354
P_{OH}	1,656	1,6545			
P_H	0,235	0,2356			

1) Sequence of calculations; 2) values of the partial pressures of the gases, atm abs; 3) values of the partial pressures of the gases, in calculation according to the exact method, atm abs; 4) total pressure of the products of combustion.

The entire calculation in this and in the following examples is carried out by means of a 50-centimeter slide rule. For purposes of comparison, the right-hand column shows the partial pressures of the products of combustion for the same propellant, said partial pressures determined by an exact method involving the use of a computer. The calculation data produced by the two methods are so close to one another that the feasibility of using the proposed method in this case cannot be subject to any doubt.

As can be seen from Example 1, the pressure of the products of combustion increased by 1.336 atm abs as a result of dissociation. If

the calculation of the chemical composition had been carried out at some other temperature, the pressure increment for the products of combustion and the total pressure of the dissociated products of combustion would correspond to this new temperature and would deviate from the values presented in the Table. In the following example, we will demonstrate the sequence employed in the calculation of the chemical composition of the products of combustion at a given pressure p for dissociated products of combustion.

Example 2. Calculate the chemical composition of products of combustion for the same propellant as used in Example 1, at a pressure of $p = 40$ atm abs for the dissociated mixture of the products of combustion and at a temperature of $T = 3000^\circ$ abs.

1. We will determine the pressure increment Δp for the products of combustion, due to dissociation, said increment corresponding to the pressure $p = 40$ atm abs of the dissociated products of combustion.

With this purpose in mind:

a) we will find the parameter $\Delta i_{B,v,f}$ for $\alpha = 0.8945$ by a method of graphical interpolation according to the data of the nomograms in Figs. 10 and 11 of the Appendix. In this case, we need not construct an auxiliary curve, since $\alpha = 0.8945$ is quite close to $\alpha = 0.90$. Assuming that $\Delta i_{B,v,f}$ is a linear function of α in the sector ranging from $\alpha = 0.80$ to $\alpha = 0.90$, we will find its value at $\alpha = 0.8945$ from the following relationship

$$\frac{0,90 - 0,80}{0,90 - 0,8945} = \frac{(\Delta i_{B,v,\phi})_{0,90} - (\Delta i_{B,v,\phi})_{0,80}}{(\Delta i_{B,v,\phi})_{0,90} - (\Delta i_{B,v,\phi})_{0,8945}}$$

For $\alpha = 0.90$ we will find $\Delta i_{B,v,f} = 596$ according to the nomogram presented in Fig. 10 of the Appendix.

For $\alpha = 0.80$ we will find $\Delta i_{H,v,f} = 412$ according to the nomogram presented in Fig. 11 of the Appendix.

Replacing the relationship written above with these values, we will find that at $\alpha = 0.8945$, the parameter $\Delta i_{B,v,f} = 586$;

b) we will find $m_B = 2.95$ according to the nomogram presented in Fig. 7 of the Appendix (the parameter m_B is independent of the coefficient of excess oxygen);

c) we will determine $\Delta i_{H,v,f}$ for $\alpha = 0.8945$ and we will use Formula (2.41) to find n_{ef} . The method used for the determination of $\Delta i_{H,v,f}$ for the given α is similar to the method used for the determination of the parameter $\Delta i_{B,v,f}$ (in the given example, n_{ef} need not be determined, since the pressure $p = 40$ atm abs of the dissociated mixture is equal to the pressure at which the nomograms presented in Figs. 9 to 14 of the Appendix have been constructed, and there is no longer any need to interpolate the magnitude of Δp for pressure);

d) we will find Δp for $\alpha = 0.8945$ according to Eq. (4.37):

$$\Delta p = \frac{p}{130.40} \Delta i_{B,v,f} \left(\frac{p_{B,f}}{p} \right)^{m_B} \left(\frac{T - 2200}{T_{B,f} - 2220} \right)^{m_B}$$

In our case

$$p = 40 \text{ atm abs}; \Delta i_{B,v,f} = 586; p_{B,f} = 40 \text{ atm abs};$$

$$T_{B,f} = 3400^\circ \text{ abs}; T = 3000^\circ \text{ abs}; m_B = 2.95.$$

Substituting the values of these parameters into Eq. (4.37), we will obtain

$$\Delta p = 1.362 \text{ atm abs.}$$

2. We will determine the pressure of the undissociated products of combustion:

$$p_T^0 = p - \Delta p;$$

$$p_T^0 = 40 - 1.362 = 38.638 \text{ atm abs.}$$

3. We will find the independent parameters:

$$X_{C_T} = 14 \frac{38.638}{40} = 13.52;$$

$$X_{H_T} = 52 \frac{38.638}{40} = 50.23;$$

$$X_{O_2} = 48,34 \frac{38,638}{40} = 46,70;$$

$$A_5 = 2 \cdot 13,52 + 0,5 \cdot 50,23 - 46,70 = 5,462.$$

4. The problem is then solved according to the procedure outlined in Example 1. In this case, the finding of the ratio n is facilitated by the circumstance that we know the value of $n = 11,82$ for the same propellant, at $p = 41,336$ (see Example 1). Since here $p = 40$ atm abs, we may expect that the ratio of the partial pressure of the water vapor to the partial pressure of the biatomic hydrogen will be somewhat less than 11,82.

a) as the first approximation, we will take

$$n_1 = 11,50.$$

For $0,5 X_{H_T} = 25,12$ we will find $B_1 = 0,825$ according to the nomogram presented in Fig. 27 of the Appendix.

For $0,5 X_{N_T} = 0$ we will find $B_2 = 0,700$ according to the nomogram presented in Fig. 28 of the Appendix.

Using conventional formulas, we will determine the coefficients:

$$a = 1,010; b = -9,273; c = -32,478.$$

We will use Eq. (4.34) to find

$$n_2 = 11,88.$$

We will use Formula (4.36) to find n_3 for the second approximation:

$$n_3 = \frac{2}{3} \cdot 11,88 + \frac{1}{3} \cdot 11,50;$$

$$n_3 = 11,72 \approx 11,70.$$

According to the nomogram presented in Fig. 27 of the Appendix, we will determine

$$B_1 = 0,832.$$

According to the nomogram presented in Fig. 28 of the Appendix, we will find $B_2 = 0,730$.

We will determine the coefficients:

$$a=1,01; b=-9,232; c=-32,448.$$

Having solved Eq. (4.34), we will find

$$n_4=11,78;$$

c) we will finally establish, according to Formula (4.36), that

$$n = \frac{2}{3} \cdot 11,78 + \frac{1}{3} \cdot 11,70;$$

$$n = 11,75.$$

5. At $n = 11.75$ we will determine the partial pressures of the products of combustion at $T = 3000^\circ$ abs and at $p = 40$ atm abs.

Последовательность вычислений 1	Значения парциальных давлений газов в <i>атм</i> 2	Последовательность вычислений 1	Значения парциальных давлений газов в <i>атм</i> 2
$b' = 0,5(K_3n + K_4)$	0,6731	P_0	0,0678
$a' = n + 1$	12,75	$\sqrt{P_0}$	0,575
$c' = -0,5X_H$	-25,12	P_{O_2}	0,331
$\sqrt{P_H}$	1,378	P_{CO}	5,015
P_H	1,900	P_{CO_2}	8,505
$\Gamma_{H,0}$	22,32		
P_{OH}	1,625		
P_H	0,232	3) Давление диссоциированных продуктов сгорания	39,996

1) Calculation sequence; 2) values of the partial pressures of the gases, in atm abs; 3) pressure of the dissociated products of combustion.

The results are so close to $p = 40$ atm abs, that it is absolutely unnecessary to undertake any recalculations of the partial pressures of the products of combustion to bring them into line with a pressure of 40 atm abs.

Example 3. Calculate the composition of the products of combustion of a propellant characterized by the following parameters: $X_C = 25$, $X_H = 8$, and $X_N = 22$ at $\alpha = 0.9018$, $p^0 = 40$ atm abs, and $T =$

= 3000° abs.

1. We will determine the ratio n_0 (in the absence of dissociation).

With this purpose in mind, we will carry out the following sequence of preliminary calculations:

a) we will find the parameters

$$\begin{aligned}X_0 &= (2X_C + 0,5X_H) \alpha; \\X_0 &= (2 \cdot 25 + 0,5 \cdot 8) \cdot 0,9018; \\X_0 &= 48,69 \\A_s &= 2X_C - X_0 + 0,5X_H; \\A_s &= 2 \cdot 25 - 48,69 + 0,5 \cdot 8; \\A_s &= 5,31\end{aligned}$$

b) we will determine the coefficients of the quadratic equation (4.34), assuming that $B_1 = B_2 = 0$, and we will find n_0 :

$$\begin{aligned}a &= 0,1445 \cdot 5,31; \\a &= 0,7672; \\b &= 5,31 + 0,1445(5,31 - 4) - 25; \\b &= -19,501; \\c &= (5,31 - 4) - 25, \\c &= -23,69; \\n_0 &= \frac{19,501 + \sqrt{(-19,501)^2 + 4 \cdot 0,7672 \cdot 23,69}}{2 \cdot 0,7672}, \\n_0 &= 26,6\end{aligned}$$

2. Under given conditions, we will determine the ratio \underline{n} :

a) we will choose $n_1 = 20$ in the first approximation.

We will find $B_1 = 0.412$ according to the nomogram presented in Fig. 27 of the Appendix.

We will find $B_2 = 2.44$ according to the nomogram presented in Fig. 28 of the Appendix.

We will determine the coefficients of the equation:

$$a=1,180; b=-16,296; c=-21,25.$$

We will find $n_2 = 15.02$ according to Eq. (4.34);

b) we will determine n_3 according to Formula (4.36) in the second

approximation:

$$n_3 = \frac{2}{3} \cdot 15,02 + \frac{1}{3} \cdot 20;$$
$$n_3 = 16,67.$$

We will round off to $n_3 = 17,0$.

We will find $B_1 = 0,382$ according to the nomogram presented in Fig. 27 of the appendix.

We will find $B_2 = 1,83$ according to the nomogram presented in Fig. 28 of the Appendix.

We will determine the coefficients of the equation:

$$a=1,087; b=-17,024; c=-21,86.$$

We will find $n_4 = 16,85$;

c) we will find n_5 according to Formula (4.36) in the third approximation

$$n_5 = \frac{2}{3} \cdot 16,85 + \frac{1}{3} \cdot 17;$$
$$n_5 = 16,90.$$

We will find $B_1 = 0,381$ according to the nomogram presented in Fig. 27 of the Appendix.

We will find $B_2 = 1,82$ according to the nomogram presented in Fig. 28 of the Appendix.

We will determine the coefficients:

$$a=1,086; b=-17,037; c=-21,87.$$

According to Eq. (4.34), we will find

$$n_6 = 16,89.$$

Finally, we select $n = 16,89$.

At $n = 16,89$ we will find the partial pressures of the products of combustion.

Последовательность вычислений	Значения парциальных давлений газов <i>ата</i>	Значения парциальных давлений газов при расчете по точному методу в <i>ата</i>	Последовательность вычислений	Значения парциальных давлений газов <i>ата</i>	Значения парциальных давлений газов при расчете по точному методу в <i>ата</i>
1	2	3	1	2	3
$b' = 0,5(K_3n + K_4)$	0,931	—	P_{O_2}	0,685	0,6868
$a' = n+1$	17,89	—	P_{CO}	7,260	7,257
$c' = -0,5X_H$	-4	—	P_{CO_2}	17,740	17,743
$\sqrt{P_{H_2}}$	0,4472	—	$\sqrt{P_{N_2}}$	3,288	—
P_{H_2}	0,20	0,200	P_{NO}	0,3372	0,3388
P_{H_2O}	3,380	3,384	P_N	0,0049	0,0049
P_{OH}	0,7580	0,7577	P_{N_2}	10,829	10,883
P_H	0,0754	0,0754	Давление диссоциированных продуктов сгорания	41,367	41,425
P_O	0,0974	0,0976			
$\sqrt{P_{O_2}}$	0,828	—			

1) Calculation sequence; 2) values of the partial pressures of gases, atm abs; 3) values of the partial pressures of gases, in calculations according to the exact method, in atm abs; 4) pressure of dissociated products of combustion.

For purposes of comparison, the right-hand column shows the calculations of the chemical composition of the products of combustion from this propellant, said calculations carried out by the exact method. The results of the two methods in this example are virtually identical.

Example 4. Calculate the chemical composition of the products of combustion for the same propellant as in Example 3, at a pressure $p = 40$ atm abs for the dissociated products of combustion and at a temperature of $T = 3000^\circ$ abs.

1. We will determine the pressure increment Δp which corresponds to a pressure of $p = 40$ atm abs for the dissociated products of combustion.

a) We will find the parameter $\Delta i_{B,v,f}$ for $\alpha = 0.9018$.

Since $\alpha = 0.9018$ is extremely close to $\alpha = 0.90$, the value of $\Delta i_{B,v,f}$ for $\alpha = 0.9018$ will be found according to the following relationship:

$$\frac{0,95 - 0,90}{0,9018 - 0,90} = \frac{(\Delta i_{B,v,\phi})_{0,95} - (\Delta i_{B,v,\phi})_{0,90}}{(\Delta i_{B,v,\phi})_{0,9018} - (\Delta i_{B,v,\phi})_{0,90}}$$

Having determined the following quantities according to the nomograms presented in Figs. 9 and 10,

$$(\Delta i_{B,v,\phi})_{0,95} = 755;$$

$$(\Delta i_{B,v,\phi})_{0,90} = 596,$$

we will find

$$(\Delta i_{B,v,\phi})_{0,9018} = 602;$$

b) we will find m_B according to the nomogram presented in Fig. 7 of the Appendix

$$m_B = 2,90;$$

c) we will determine Δp according to Eq. (4.37):

$$\Delta p = 1,488.$$

2. We will find the pressure p_T^0 of the undissociated products of combustion:

$$p_T^0 = 40 - 1,488;$$

$$p_T^0 = 38,512 \text{ atm abs.}$$

3. We will find the parameters

$$X_{C_T} = 25 \frac{38,512}{40} = 24,08;$$

$$X_{H_T} = 8 \frac{38,512}{40} = 7,705;$$

$$X_{N_T} = 22 \frac{38,512}{40} = 21,19;$$

$$X_{O_T} = 48,69 \frac{38,512}{40} = 46,90;$$

$$A_s = 2 \cdot 24,08 + \frac{7,705}{2} - 46,90 = 5,112.$$

4. In Example 3 we obtained $n = 16.89$ for the same propellant, but for $p^0 = 40$ atm abs. We can expect that in our case the value of n

will be somewhat lower, since the total pressure of the products of combustion has diminished.

a) We will select $n_1 = 16.80$.

At $0.5 X_{H_T} = 3.852$ we will find $B_1 = 0.373$ according to the nomogram presented in Fig. 27 of the Appendix.

At $0.5 X_{N_T} = 10.60$ we will find $B_2 = 1.78$ according to the nomogram presented in Fig. 28 of the Appendix.

We will determine the coefficients of the equation:

$$a=1,051; b=-16,375; c=-21,04.$$

Having solved Eq. (4.34), we will obtain

$$n_2=16,77.$$

Finally, we assume $n = 16.78$.

5. At $n = 16.78$ we will determine the partial pressures of the products of combustion.

Последовательность вычислений 1	Значение парциальных давлений газов 2 атм	Значение парциальных давлений газов после пересчета в атм 3	Последовательность вычислений 1	Значение парциальных давлений газов 2 атм	Значение парциальных давлений газов после пересчета в атм 3
$b' = 0,5(K_3n + K_4)$	0,925	—	P_{O_2}	0,676	0,678
$a' = n + 1$	17,78	—	P_{CO}	7,030	7,052
$c' = -0,5X_H$	-3,852	—	P_{CO_2}	17,050	17,110
$\sqrt{P_{H_2}}$	0,4392	—	$\sqrt{P_{N_2}}$	3,226	—
P_{H_2}	0,1930	0,1936	P_{NO}	0,3287	0,3300
P_{H_2O}	3,239	3,2500	P_N	0,0048	0,0048
P_{OH}	0,7400	0,7420	P_{N_2}	10,428	10,475
P_H	0,0740	0,0742	4 Давление диссоциированных продуктов сгорания	39,8607	40,0000
P_O	0,0968	0,0973			
$\sqrt{P_{O_2}}$	0,822	—			

1) Calculation sequence; 2) value of the partial pressures of gases, atm abs; 3) value of the partial pressure of gases, after recalculation, in atm abs; 4) pressure of the dissociated products of combustion.

The pressure of the products of combustion was determined as equal 39.8607 atm abs. In view of the fact that the difference between 40 and 39.8607 atm abs is extremely small, no subsequent calculation procedure is required.

In this case, each of the partial pressures can be multiplied by the ratio 40/39.8607, assuming at the same time that the partial pressures of each of the components is a linear function of the total pressure of the products of combustion. The results of the recalculation are presented in the right-hand column.

20. Calculation of Heat Content of Fuel (Combustible), Oxidizer, and Propellant

On the whole, the heat content of the fuel (combustible), oxidizer, and propellant must be presented within the same system as was adopted for the heat content of the products of combustion.

We will denote by q_g and q_o the weight fractions of the fuel (combustible) and oxidizer in 1 kg of propellant, and by i_g and i_o , respectively, their heat content in kcal/kg.

In accordance with the additive law, the total heat content i_t of 1 kg of propellant is equal to:

$$i_t = i_g q_g + i_o q_o \quad (1)$$

The numerical values of the heat contents of the fuel (combustible) and oxidizer may be determined either through the heat of combustion Q_{sg} of these materials or through the heat of formation Q_{obr} of these materials, the values for which are determined from handbooks.

We will demonstrate the method employed in the calculation of the heat content of matter with the heat of combustion Q_{sg} known. By heat of combustion we refer to the quantity of heat generated on the complete combustion of organic compounds in oxygen. In this case it is assumed that the process takes place at identical pressure at the be-

ginning and end of the combustion, and the temperature of the products of combustion is equal to the temperature of the initial substance. The values for the heats of combustion of various substances are generally cited in handbooks for temperatures of $+20^{\circ}\text{C}$ or values close to this ($+18, +25^{\circ}\text{C}$).

The quantity of heat generated during the combustion of the substances under the conditions referred to above will be equal to the difference between the heat content of the fuel and the heat content of the products of combustion:

$$Q_{cr} = i + i_{o_2} - i_{n.c.} \quad (2)$$

whence

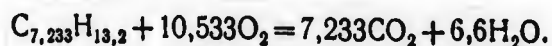
$$i = Q_{cr} + i_{n.c.} - i_{o_2} \quad (3)$$

Here i is the heat content of the substance at the same temperature at which was determined the heat of combustion Q_{sg} ; Q_{sg} is the heat of combustion of the substance (determined from handbooks); $i_{p.s}$ is the total heat content of the products of combustion; i_{O_2} is the heat content of oxygen.

The heat-content values for individual components of the products of combustion, necessary for the calculation of $i_{p.s}$, as well as the value of the heat content of oxygen (i_{O_2}) is found from a table showing the heat contents of the products of combustion at the same temperature at which the heat of combustion Q_{sg} was determined.

Example 1. Determine the heat content of kerosene (conventional formula $C_{7.233}H_{13.2}$) at a given heat of combustion $Q_{sg} = 11,000 \text{ kcal/kg} = 1100 \text{ kcal/g-mole}$ (per conventional gram-molecule of kerosene equal to 100 g) at 20°C ($T = 293^{\circ}$ abs).

We will write the equation of the complete combustion of kerosene in oxygen:



According to Table 1 of the Appendix, for 20°C abs, we will find

$$I_{O_2} = 2.03 \text{ kcal/g-mole};$$

$$I_{CO_2} = 2.18 \text{ kcal/g-mole};$$

$$I_{H_2O} = 2.31 \text{ kcal/g-mole.}$$

(water vapor)

The heat content of water in the liquid state is determined according to the following formula:

$$I_{H_2O} = I_{H_2O} + Q_{1sp},$$

(water vapor)

where Q_{1sp} is the heat of vaporization, equal to 10.57 kcal/g-mole, for water at the given temperature.

Consequently, the heat content of water

$$I_{H_2O} = 2.31 - 10.57 = -8.26 \text{ kcal/g-mole.}$$

The heat content of kerosene is determined according to Formula (3):

$$i_{ker} = 1100 + 7,233 \cdot 2.18 + 6,6(-8,26) - 10,533 \cdot 2,03;$$

$$i_{ker} = 1039.9 \text{ kcal/g-mole} = 10399 \text{ kcal/kg.}$$

We will demonstrate the method employed in the calculation of the heat content of various substances for a case in which the magnitude of the heat of formation Q_{obr} is known. The numerical value of Q_{obr} is a function of the state of the initial elements which go to make up the given substance. For the determinacy of the quantity Q_{obr} these elements are regarded as existing in a certain standard state, i.e., in the state in which they occur most frequently in nature. We have assumed a state in the form of molecular gases H_2 , N_2 , and O_2 as the standard states for hydrogen, nitrogen, and oxygen.

Carbon is assumed to be in the form of solid graphite.

Handbooks present the values for the heats of formation of sub-

stances composed of elements existing in their standard state. We will determine the sign of the heat of formation on the basis of the following considerations: if heat is required for the formation of a given substance, Q_{obr} is regarded as a negative quantity; if, however, the reaction resulting in the formation of substances whose elements are found in their standard state is accompanied by heat generation, Q_{obr} is assumed to be a positive quantity. Since certain tables may present contradictory interpretations of the sign of the heat of formation, it is necessary to establish whether heat is liberated or absorbed during the reaction resulting in the formation of the substance, and this must be done prior to the determination of the heat content of the material; after this has been established, the appropriate positive or negative sign should be applied to the parameter Q_{obr} .

The heat content of a substance can be determined as the difference between the heat content of the components of its chemical elements and the heat of formation (selected with the appropriate sign):

$$i = i_e - Q_{obr} \quad (4)$$

Here i is the heat content of the substance; i_e is the total heat content of the chemical elements, in their standard state; Q_{obr} is the heat of formation, in kcal/g-mole (determined from handbooks).

The heat content of the chemical elements oxygen, nitrogen, hydrogen, and carbon in their standard state ($T = 293^\circ$ abs) in the adopted system for heat content are presented in the following table.

1 Наименование элементов	2 Теплосодержание	
	3 ккал/г-мол	4 ккал/кг
5 Кислород (O ₂)	2,03	63,6
6 Азот (N ₂)	2,03	72,5
7 Водород (H ₂)	59,09	29 300
8 Углерод (β-графит)	94,23	7 884

1) Designation of element; 2) heat content; 3) kcal/g-mole; 4) kcal/kg; 5) oxygen (O₂); 6) nitrogen (N₂); 7) hydrogen (H₂); 8) carbon (β-graphite).

Example 2. Determine the heat content of nitric acid (HNO₃) if we know its heat of formation

$$Q_{obr} = 41.66 \text{ kcal/g-mole.}$$

In accordance with Formula (4)

$$\begin{aligned} i_{HNO_3} &= \frac{1}{2} i_H + \frac{1}{2} i_N + \frac{3}{2} i_O - Q_{obr} = \\ &= \frac{1}{2} \cdot 59,09 + \frac{1}{2} \cdot 2,03 + \frac{3}{2} \cdot 2,03 - 41,66 = \\ &= - 8.06 \text{ kcal/g-mole} = - 128 \text{ kcal/kg.} \end{aligned}$$

If the temperature of the fuel (combustible) or oxidizer is noticeably different from the temperature at which the parameters Q_{sg} or Q_{obr} were determined as they enter the combustion chamber of the engine, it is necessary to take this circumstance into consideration by reducing or increasing the heat content of the fuel (combustible) or oxidizer, respectively. Where they change in the aggregate state of the substance relative to the state for which the heat-content calculation according to Formulas (3) or (4) is being carried out, it is necessary to take into consideration the heat of the transition from one state to another.

The heat content of the propellant may also be determined according to the following formula

$$i_r = \frac{i_r + v i_o}{1 + v} \quad (5)$$

Tables 4 and 5 of the Appendix present some of the physicochemical properties of the better-known oxidizers and fuels (combustibles).*

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FOOTNOTES

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*G.E. Sinyarev and M.V. Dobrovol'skiy. Zhidkostnyye raketnyye dvigateli. Oborongiz [Liquid-Fuel Rocket Engines. Defense Industry Press], 1957.

[LIST OF TRANSLITERATED SYMBOLS]

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- 148 $Q_{cr} = Q_{sg} = Q_{sgoraniye} = Q_{combustion}$
- 148 $Q_{obr} = Q_{obrazovaniye} = Q_{formation}$
- 149 $i_{n.c} = i_{p.s} = i_{produkty\ sgoraniya} = i_{products\ of\ combustion}$
- 151 $i_a = i_e = i_{elementy} = i_{elements}$

A P P E N D I X

TABLE 1

Heat Content I of Gases, in kcal/g-mole

1) T° abs.	H ₂	O ₂	N ₂	CO	NO	OH	CO ₂	H ₂ O	H	O	N
0	57,113	0	0	66,760	21,400	34,444	0	0	79,797	58,68	84,61
300	59,149	2,082	2,085	68,845	23,612	36,563	2,256	2,387	81,286	60,170	86,100
400	59,843	2,792	2,781	69,543	24,326	37,272	3,197	3,203	81,783	60,666	86,596
600	61,241	4,280	4,198	70,969	25,788	38,684	5,327	4,879	82,776	61,659	87,589
800	62,649	5,854	5,667	72,459	27,321	40,102	7,696	6,662	83,769	62,652	88,582
1000	64,077	7,497	7,202	74,016	28,920	41,548	10,233	8,563	84,762	63,645	89,575
1200	65,540	9,185	8,793	75,628	30,569	43,027	12,886	10,583	85,755	64,638	90,568
1400	67,044	10,903	10,426	77,281	32,254	44,564	15,624	12,700	86,748	65,631	91,561
1600	68,584	12,654	12,090	78,963	33,967	46,147	18,424	14,918	87,741	66,624	92,554
1800	70,166	14,427	13,782	80,669	35,699	47,759	21,273	17,230	88,734	67,617	93,547
2000	71,785	16,220	15,500	82,397	37,447	49,401	24,159	19,622	89,727	68,610	94,540
2200	73,436	18,036	17,226	84,140	39,208	51,068	27,072	22,069	90,720	69,603	95,533
2400	75,113	19,876	18,966	85,891	40,980	52,775	30,014	24,566	91,713	70,596	96,526
2600	76,822	21,734	20,714	87,649	42,761	54,458	32,983	27,109	92,706	71,589	97,519
2800	78,555	23,610	22,475	89,417	44,549	56,240	35,967	29,686	93,699	72,582	98,512
3000	80,302	25,502	24,245	91,194	46,343	58,009	38,970	32,288	94,692	73,575	99,505
3200	82,061	27,416	26,023	92,975	48,142	59,799	41,987	34,910	95,685	74,568	100,498
3400	83,828	29,348	27,796	94,756	49,948	61,599	45,015	37,547	96,677	75,560	101,490
3600	85,606	31,299	29,576	96,536	51,800	63,409	48,056	40,197	97,671	76,554	102,480
3800	87,388	33,263	31,376	98,308	53,600	65,222	51,104	42,850	98,664	77,547	103,470

1) T° abs.

TABLE 2
 Constants of Chemical Equilibrium

T° a6c.	$K = \frac{P_{CO} P_{H_2}}{P_{CO} P_{H_2 O}}$	$K_1 = \frac{P_{CO} \sqrt{P_{O_2}}}{P_{CO_2}}$	$K_2 = \frac{P_{H_2} \sqrt{P_{O_2}}}{P_{H_2 O}}$	$K_3 = \frac{P_{OH} \sqrt{P_{H_2}}}{P_{H_2 O}}$	$K_4 = \frac{P_H}{\sqrt{P_{H_2}}}$	$K_5 = \frac{P_O}{\sqrt{P_{O_2}}}$	$K_6 = \frac{P_N}{\sqrt{P_{N_2}}}$	$K_7 = \frac{P_{NO}}{\sqrt{P_{N_2}} \sqrt{P_{O_2}}}$
300	88520,0	—	—	—	—	—	—	—
400	1469,0	—	—	—	—	—	—	—
600	27,10	—	—	—	—	—	—	—
800	4,074	—	—	—	—	—	—	0,000006
1000	1,403	—	—	—	—	—	—	0,000089
1200	0,7162	—	—	—	—	—	—	0,000541
1400	0,4560	0,000001	—	—	0,000006	0,000001	—	0,001968
1600	0,3273	0,000019	0,000006	0,000006	0,000061	0,000016	—	0,005188
1800	0,2630	0,000204	0,000054	0,000065	0,000397	0,000130	—	0,01099
2000	0,2198	0,001375	0,000392	0,000398	0,001787	0,000710	—	0,02018
2200	0,1919	0,006413	0,001231	0,001820	0,006138	0,002845	0,000007	0,03319
2400	0,1730	0,02250	0,003891	0,006457	0,01720	0,009142	0,000038	0,05012
2600	0,1607	0,06223	0,01	0,01820	0,04126	0,02433	0,000158	0,07080
2800	0,1517	0,1546	0,02345	0,04571	0,08760	0,05656	0,000525	0,09572
3000	0,1445	0,3389	0,04898	0,1003	0,1683	0,1178	0,001497	0,1239
3200	0,1400	0,6668	0,09333	0,2065	0,2982	0,2265	0,003759	0,1560
3400	0,1343	1,208	0,1622	0,3793	0,4943	0,3972	0,008433	0,1901
3600	0,1301	2,049	0,2666	0,6362	0,777	0,6583	0,0170	0,234
3800	0,1273	3,282	0,4179	1,034	1,164	1,032	0,0265	0,280

TABLE 3
 Entropy S of Gases, in kcal/g-mole·deg

T° a6c.	H ₂	O ₂	N ₂	CO	NO	OH	CO ₂	H ₂ O	H	O	N
300	10,648	0	0	20,786	2,947	9,106	0	0	17,117	13,718	13,46
400	12,646	2,060	2,005	22,795	4,996	11,146	2,694	2,321	18,564	15,216	15,31
600	15,480	5,056	4,873	25,683	7,931	13,990	7,024	5,713	20,561	17,355	17,33
800	17,494	7,320	6,987	27,825	10,131	16,030	10,410	8,274	21,988	18,639	18,773
1000	19,100	9,153	8,699	29,562	11,915	17,648	13,230	10,391	23,106	19,932	19,67
1200	20,432	10,680	10,146	31,029	13,421	19,002	15,655	12,229	24,005	20,852	20,42
1400	21,580	11,990	11,403	32,310	14,723	20,192	17,720	13,870	24,777	21,611	21,22
1600	22,610	13,190	12,500	33,410	15,870	21,255	19,590	15,340	25,440	22,308	21,91
1800	23,55	14,25	13,516	34,430	16,900	22,21	21,280	16,710	26,026	22,912	22,48
2000	24,405	15,173	14,421	35,344	17,830	23,075	22,810	17,980	26,553	23,436	22,99
2200	25,170	16,050	15,241	36,140	18,650	23,870	24,160	19,160	27,016	23,921	23,41
2400	25,910	16,87	15,999	36,925	19,433	24,611	25,457	20,234	27,451	24,385	23,89
2600	26,600	17,570	16,699	37,650	20,150	25,300	26,660	21,240	27,857	24,759	24,30
2800	27,250	18,27	17,373	38,310	20,813	25,946	27,764	22,199	28,225	25,136	24,68
3000	27,850	18,930	17,964	38,909	21,432	26,556	28,800	23,100	28,566	25,491	25,01
3200	28,42	19,55	18,538	39,484	22,012	27,134	29,773	23,946	28,886	25,844	25,33
3400	28,96	20,14	19,078	40,024	22,559	27,679	30,691	24,745	29,187	26,144	25,62
3600	29,48	20,70	19,558	40,536	23,076	28,194	31,561	25,496	29,470	26,432	25,91
3800	29,98	21,23	20,043	41,02	23,567	28,685	32,386	26,21	29,740	26,706	26,19

TABLE 4

Physicochemical Properties of Certain Oxidizers

1 Окислитель	2 Химическая формула	3 Молекулярный вес	4 Элементарный состав в кг/кг				5 Теплота образования $Q_{обр}$ ккал г-мол	6 Удельный вес при 15° С	7 $t_{плав}$ в °С при 1 атм	8 $t_{кип}$ в °С при 1 атм
			O _o	C _o	H _o	N _o				
9 Азотная кислота	HNO ₃	63,02	0,762	0	0,016	0,222	+41,66	1,52	-41,6	+86
10 Четырехокись азота	N ₂ O ₄	92,01	0,696	0	0	0,304	+6,80	1,47	-11,2	+21
11 Тетранитрометан	C(NO ₂) ₄	196,03	0,653	0,061	0	0,286	-8,80	1,65	+13	+126
12 Кислород жидкий	O ₂	32,00	1,000	0	0	0	0,00 (газ)	1,14 (при -183°С)	-227	-183
13 Перекись водорода	H ₂ O ₂	32,02	0,940	0	0,060	0	+44,84	1,46	-2	+151
14 Вода жидкая	H ₂ O	18,02	0,889	0	0,111	0	+68,35	1,00	0	+100

1) Oxidizer; 2) chemical formula; 3) molecular weight; 4) elementary composition, in kg/kg; 5) heat of formation Q_{obr} , kcal/g-mole; 6) specific weight at 15°C; 7) t_{plavl} in °C at 1 atm abs; 8) t_{kip} in °C at 1 atm abs; 9) nitric acid; 10) nitrogen tetroxide; 11) tetranitromethane; 12) liquid oxygen; 13) hydrogen peroxide; 14) liquid water.

TABLE 5

Physicochemical Properties of Certain Fuels (Combustibles)

1 Горючее	2 Химическая формула	3 Молекулярный вес	4 Элементарный состав в кг/кг				5 Теплота образования $Q_{обр}$ ккал г-мол	6 Удельный вес при 15°C	7 $t_{кип}$ в °C при 1 атм	8 $t_{плавл}$ в °C при 1 атм
			C _r	H _r	O _r	N _r				
9 Керосин тракторный	21 Смесь углеводородов	—	0,858	0,135	0,007	0	—	0,79±0,04	~170	-60
10 Спирт этиловый	C ₂ H ₅ OH	46,04	0,522	0,131	0,347	0	+66,36	0,789	78,3	-117,3
11 Спирт метиловый	CH ₃ OH	32,03	0,375	0,125	0,500	0	+57,02	0,791	64,6	-94,9
12 Анилин	C ₆ H ₅ NH ₂	93,08	0,774	0,076	0	0,150	-7,09	1,022	184,4	-6,2
13 Фурфуроловый спирт	C ₄ H ₃ OCH ₂ OH	98,06	0,614	0,061	0,325	0	+63,1	—	171	-32
14 Триэтиламин	(C ₂ H ₅) ₃ N	101,07	0,712	0,149	0	0,139	+42,33	0,728	89,5	-114,8
15 Ксилидин	(CH ₃) ₂ C ₆ H ₃ NH ₂	121,12	0,793	0,091	0	0,116	+46,2	0,98	210	-54
16 Диэтиламин	C ₄ H ₁₁ N	73,14	0,657	0,152	0	0,191	+29,3	0,70	56	-50
17 Диметилгидразин	(CH ₃) ₂ N ₂ H ₂	62,12	0,400	0,134	0	0,466	-11,28	0,83	63	-58
18 Гидразингидрат	(NH ₂) ₂ H ₂ O	50,06	0	0,122	0,318	0,560	+63,15	1,03	118,5	-40
19 Гидразин	N ₂ H ₄	32,03	0	0,125	0	0,875	-12,05	1,01	113,5	-2
Аммиак 20	NH ₃	17,03	0	0,177	0	0,823	+16,60	0,68	-33	-77

1) Fuel (combustible); 2) chemical formula; 3) molecular weight; 4) elementary composition, in kg/kg; 5) heat of formation $Q_{обр}$, kcal/g-mole; 6) specific weight at 15°C; 7) $t_{кип}$ in °C at 1 atm abs; 8) $t_{плавл}$ in °C at 1 atm abs; 9) tractor kerosene; 10) ethyl alcohol; 11) methyl alcohol; 12) aniline; 13) furfuryl alcohol; 14) diethylamine; 15) xylidene; 16) diethylamine; 17) dimethylhydrazine; 18) hydrazine hydrate; 19) hydrazine; 20) ammonia; 21) hydrocarbon mixture.

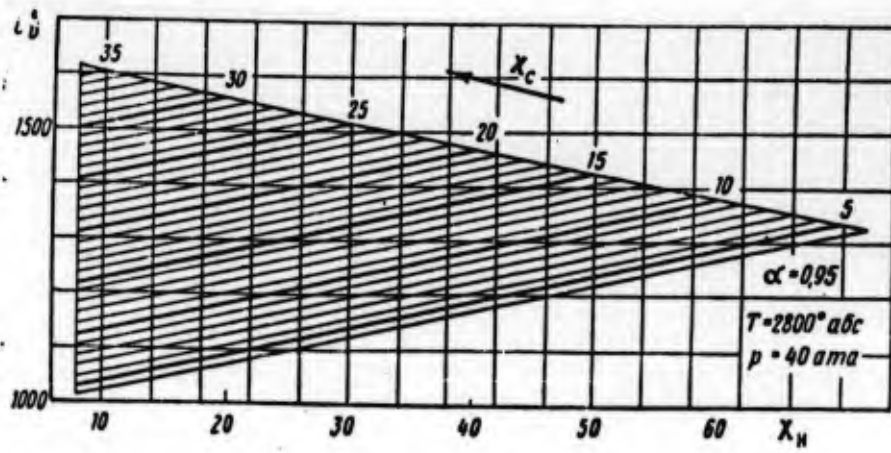


Fig. 1

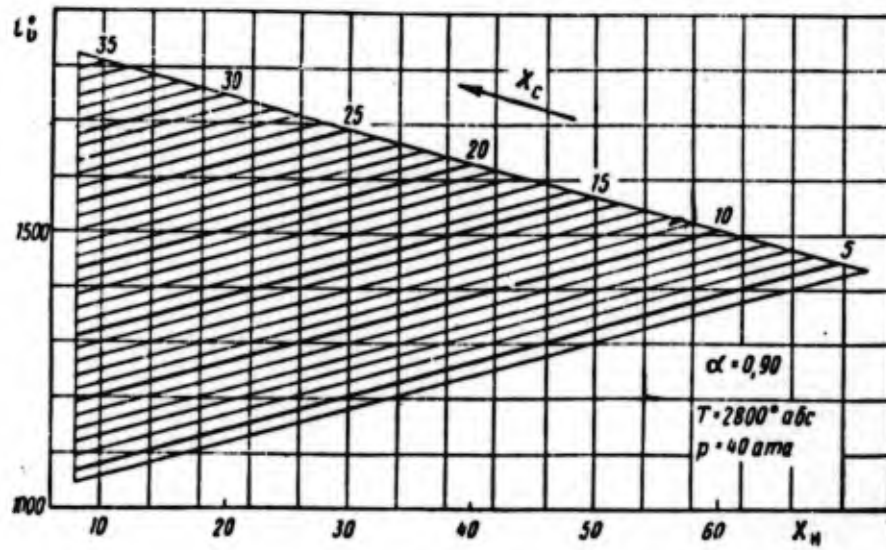


Fig. 2

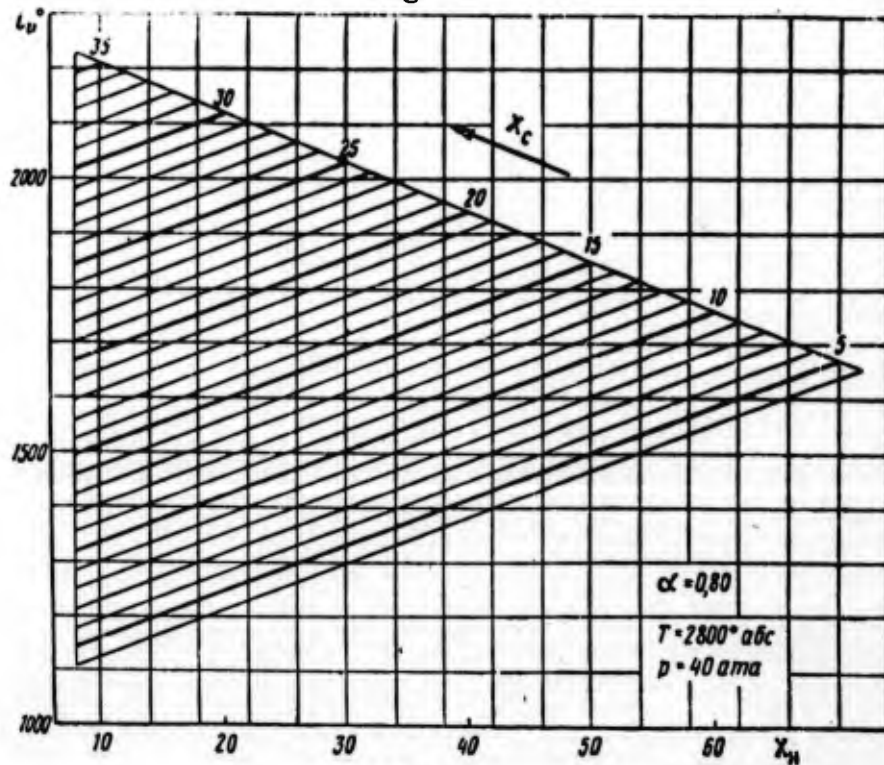


Fig. 3

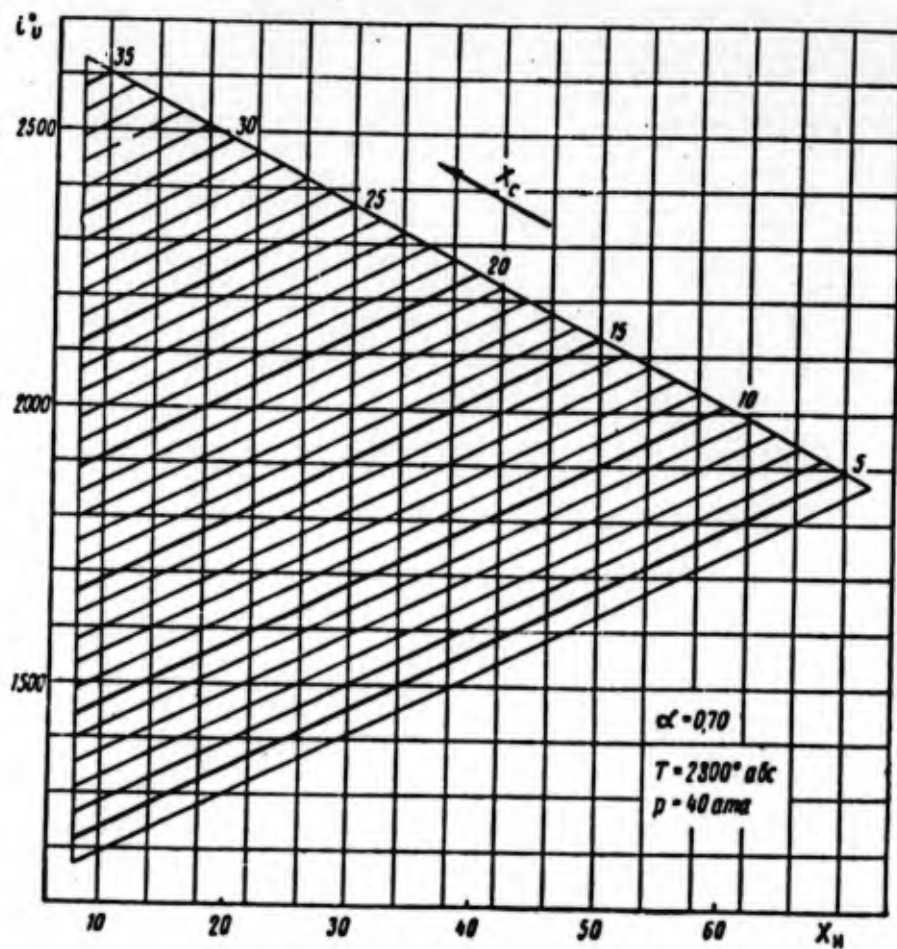


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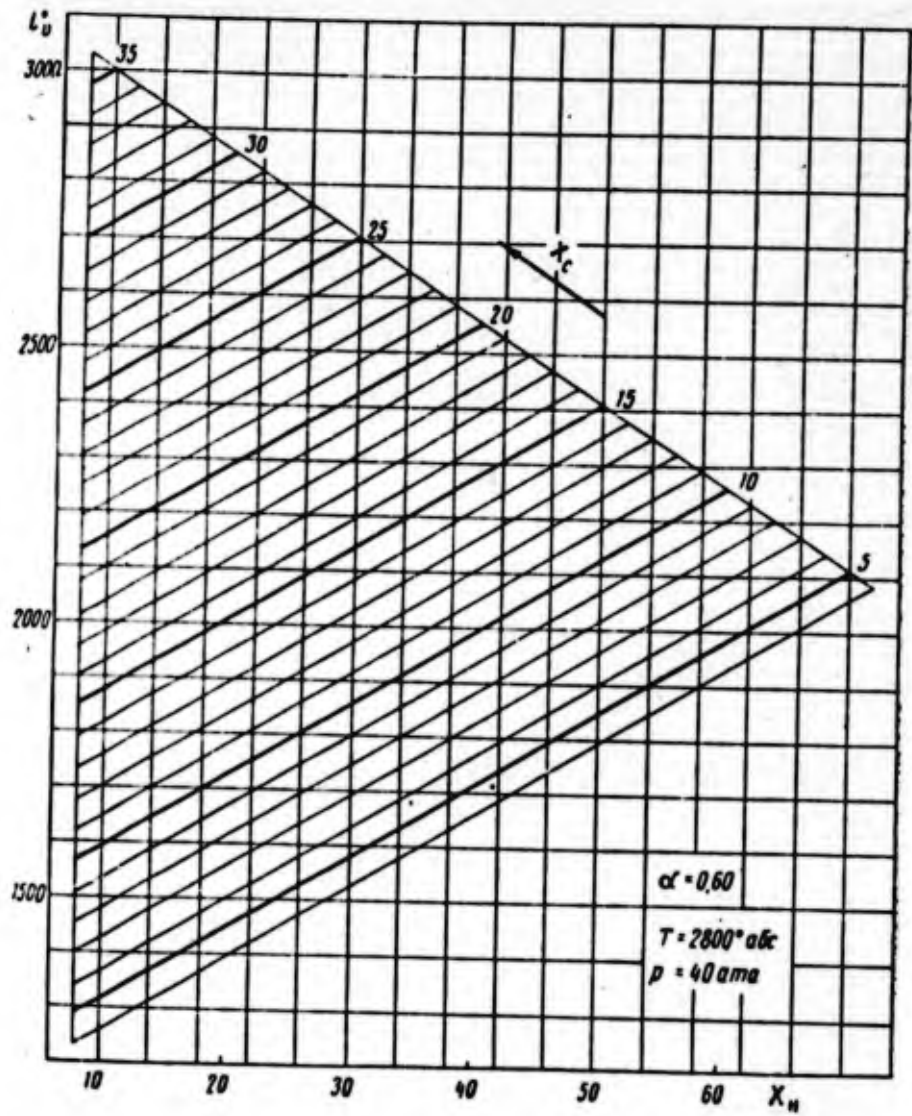


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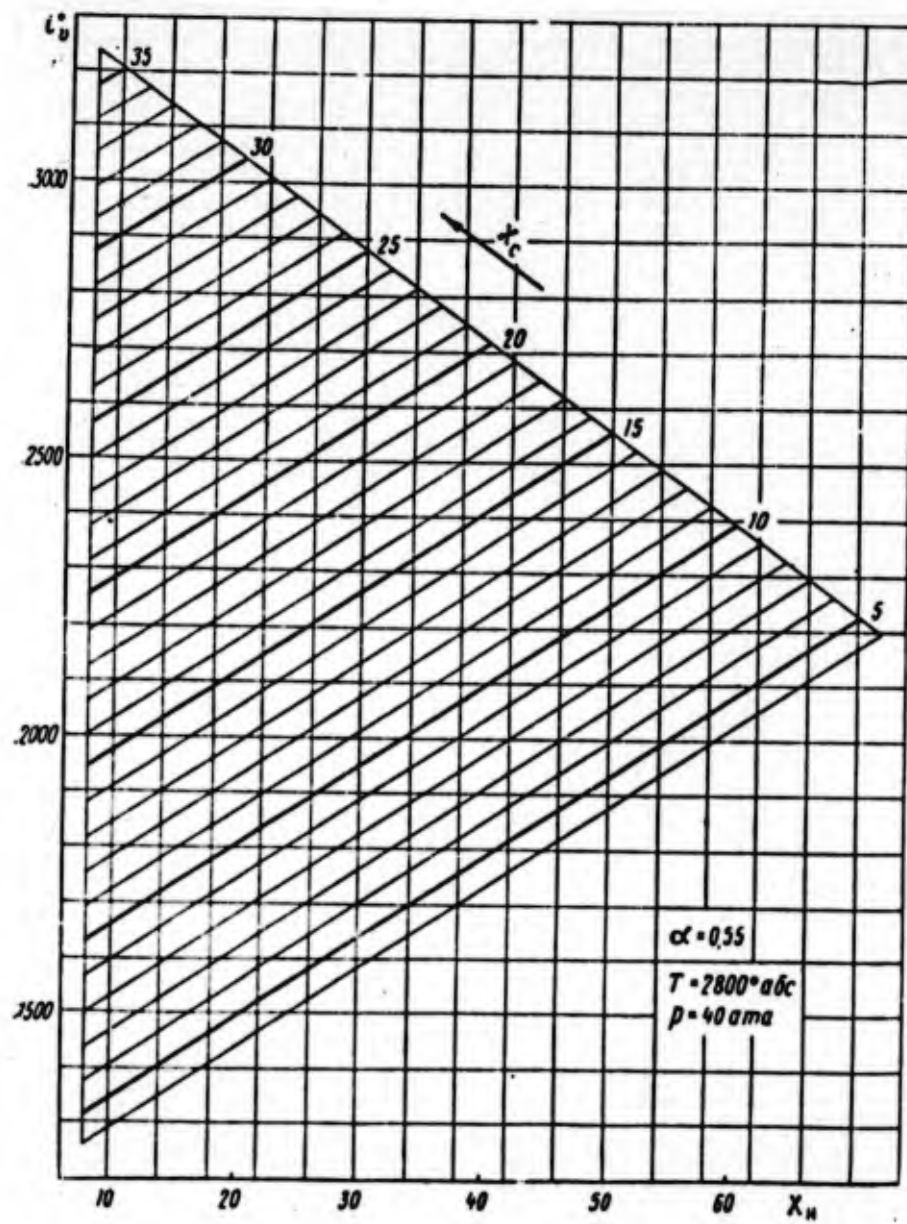


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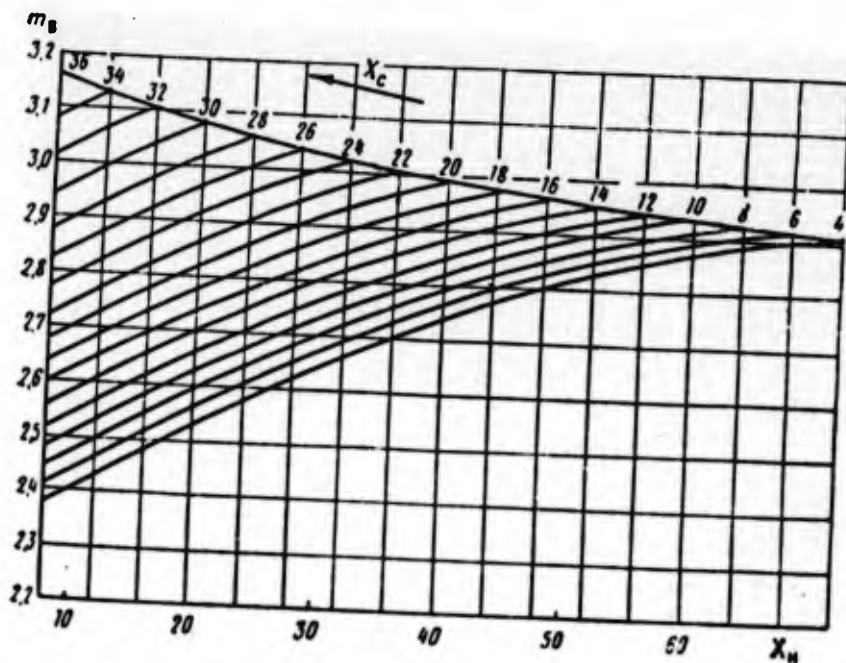


Fig. 7

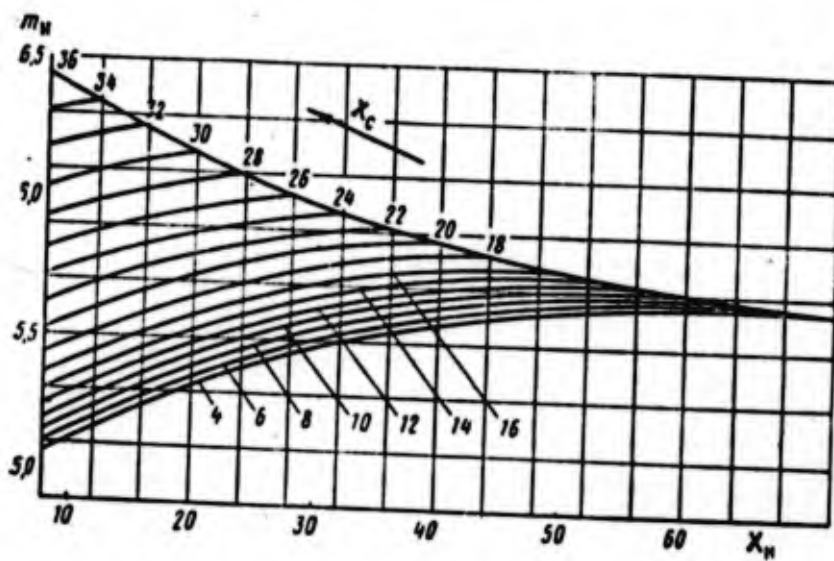


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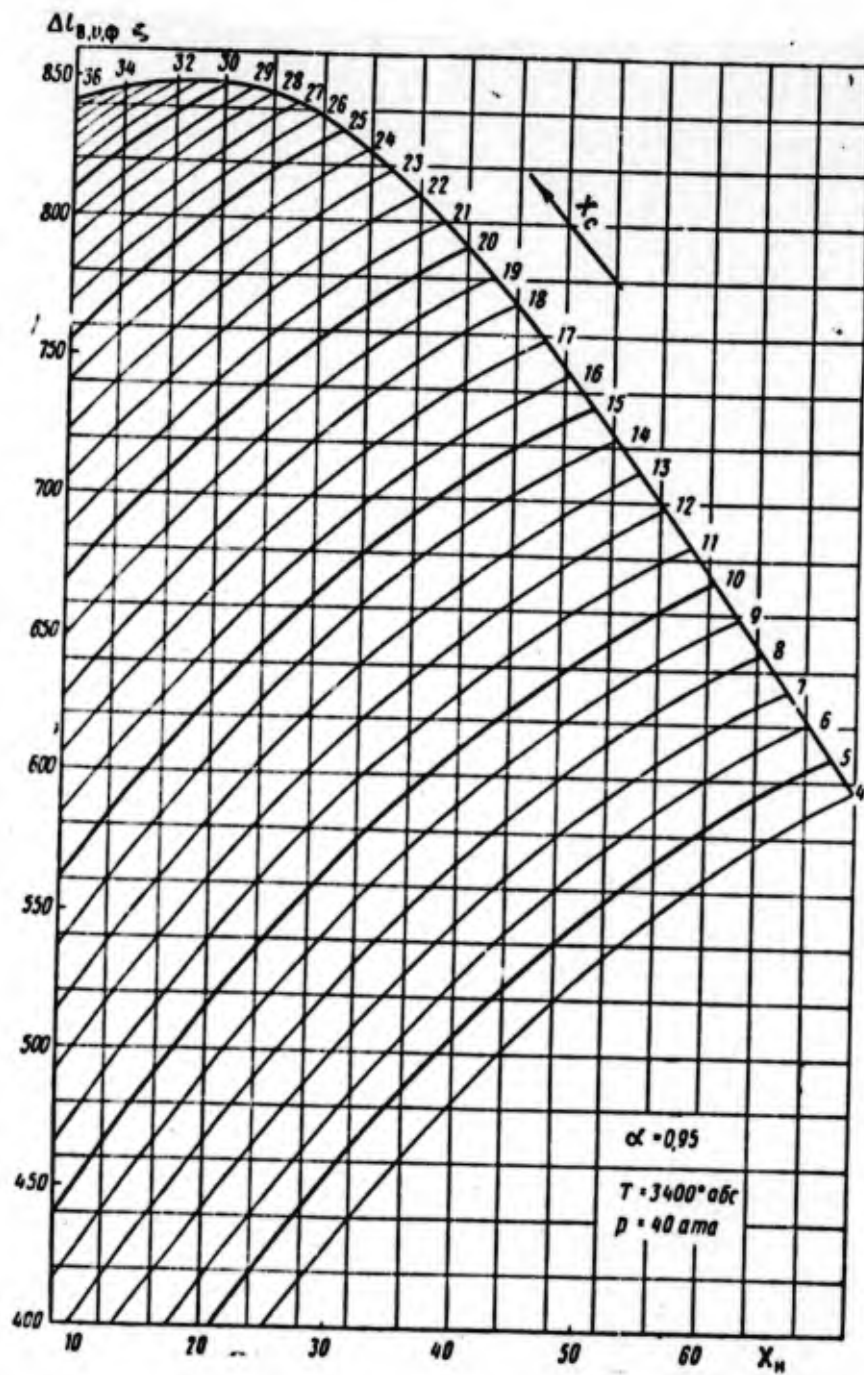


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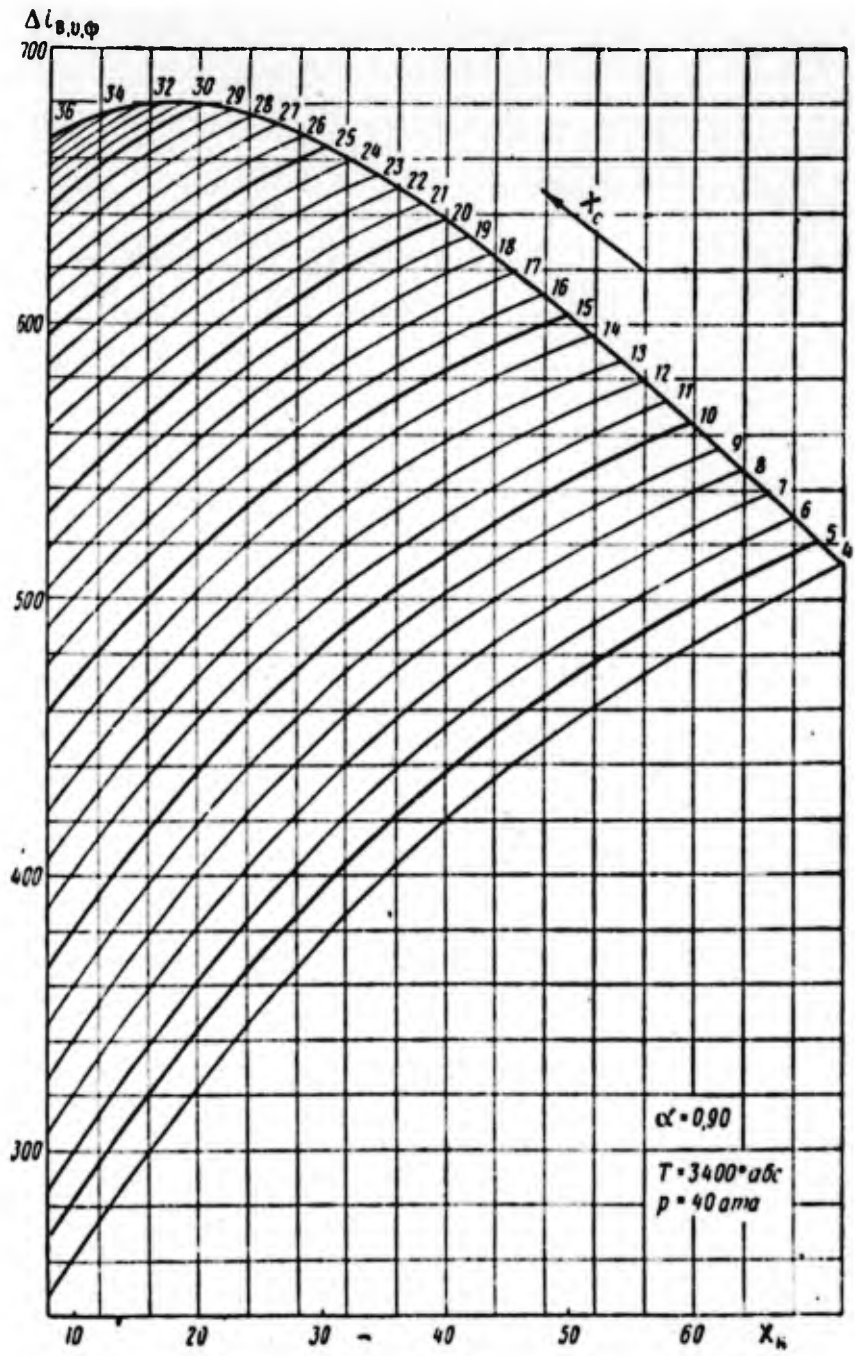


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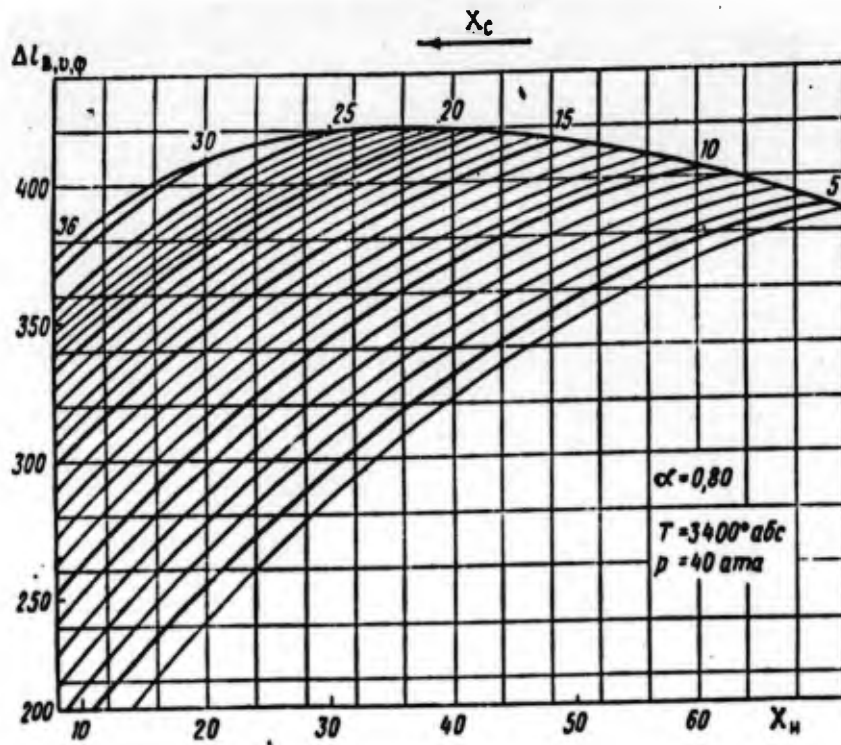


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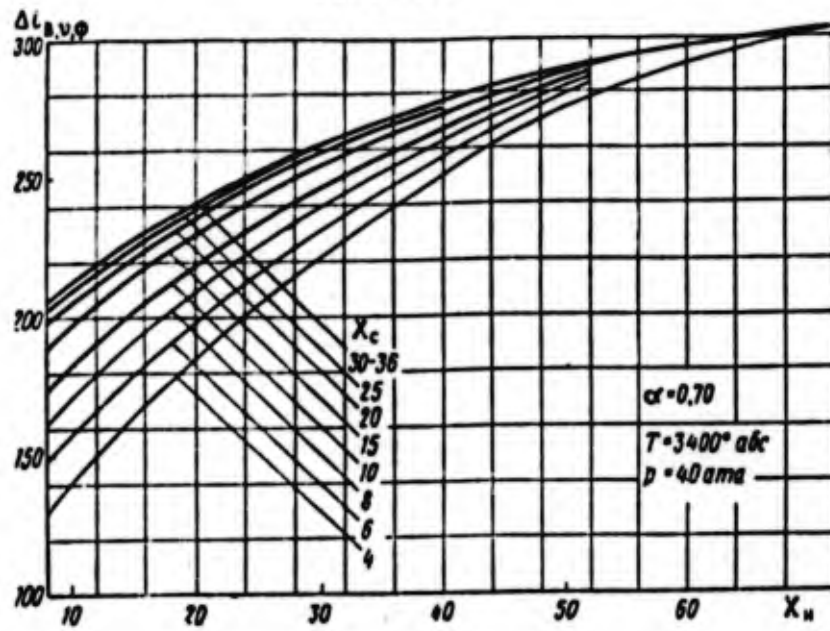


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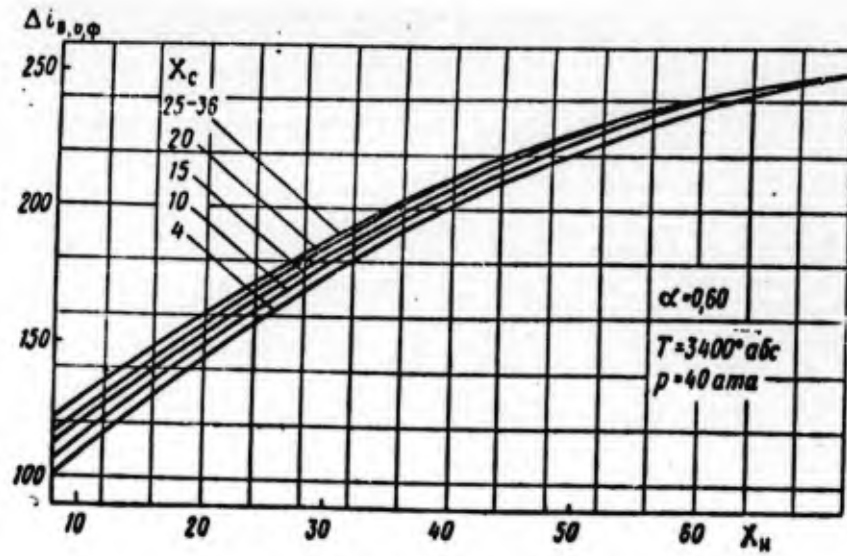


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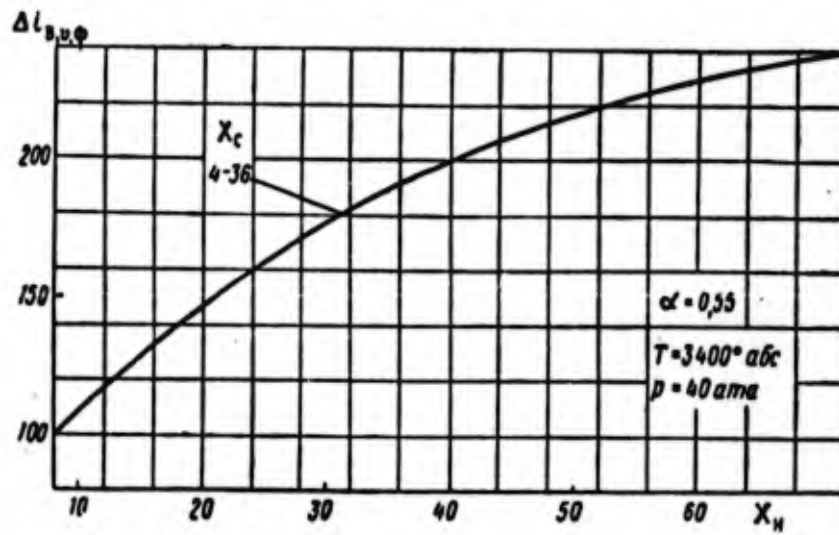


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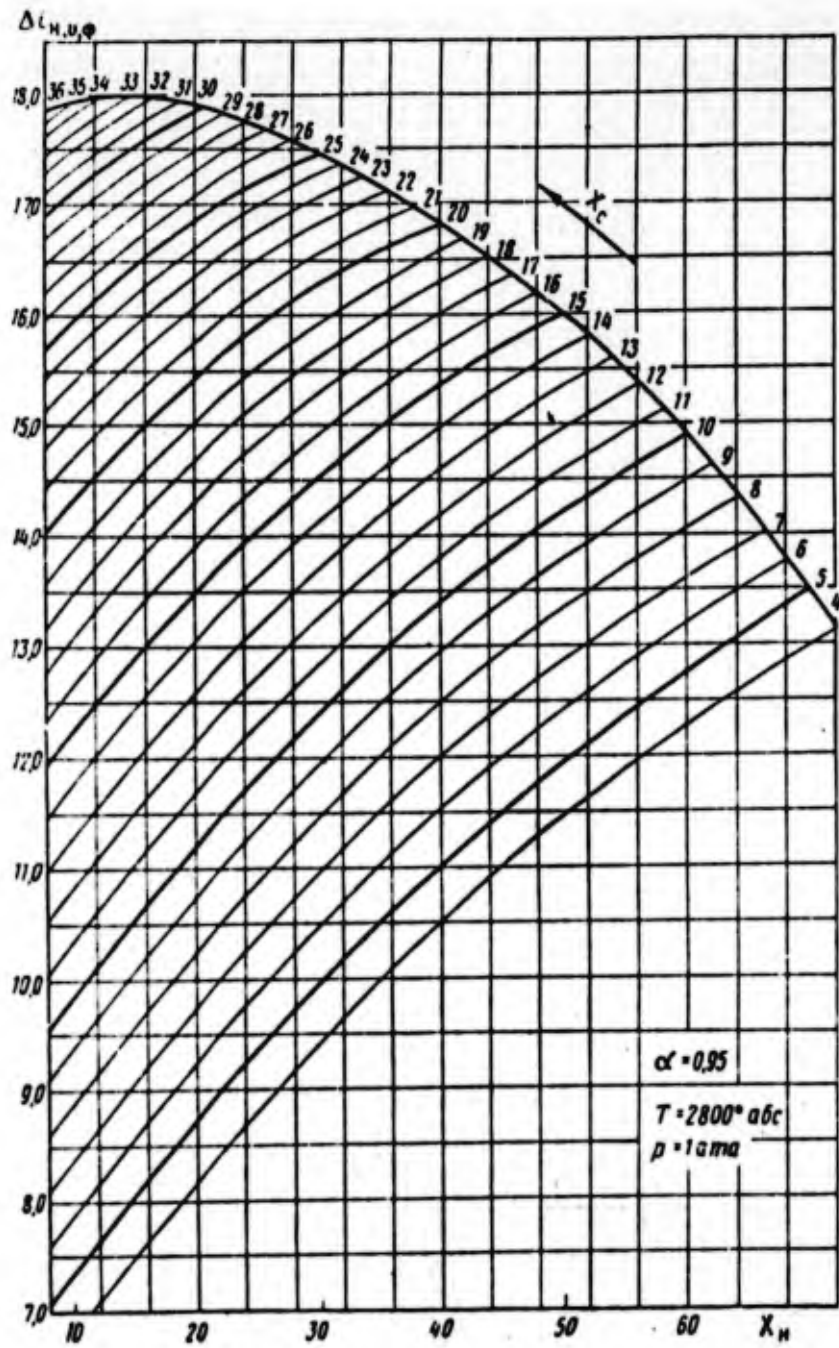


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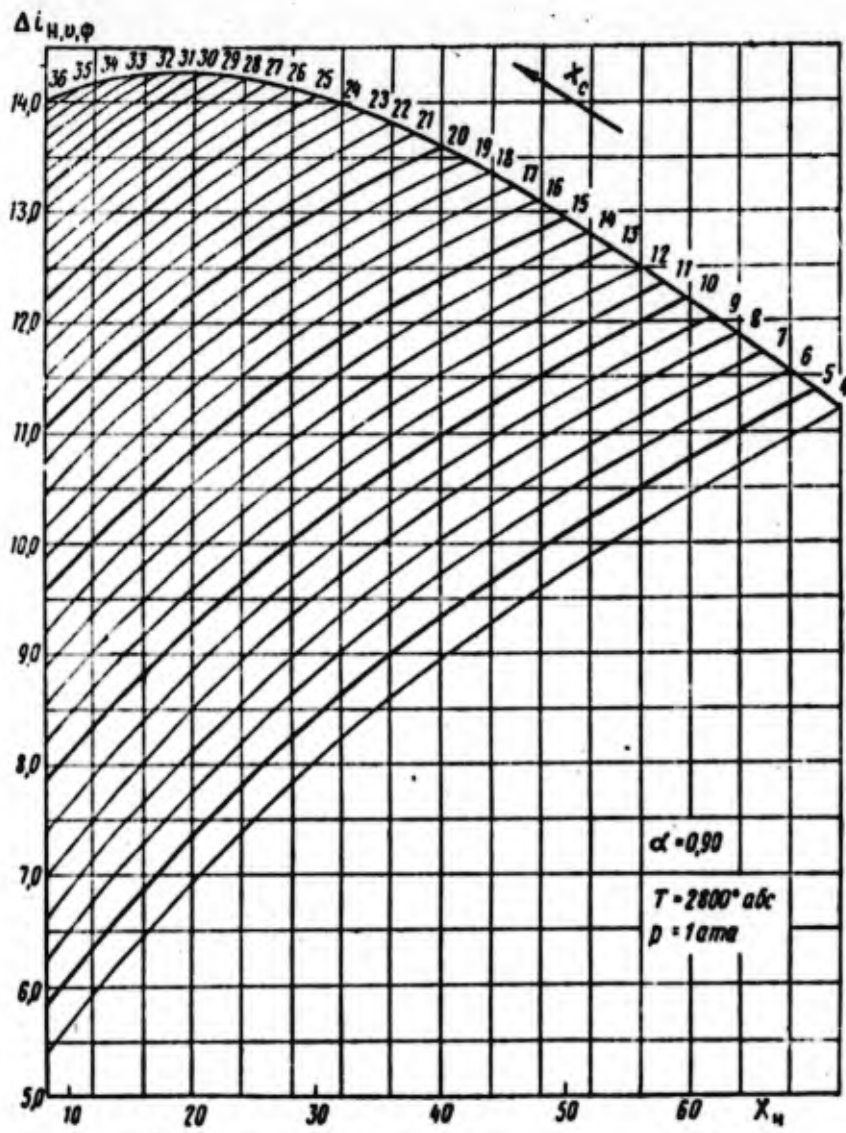


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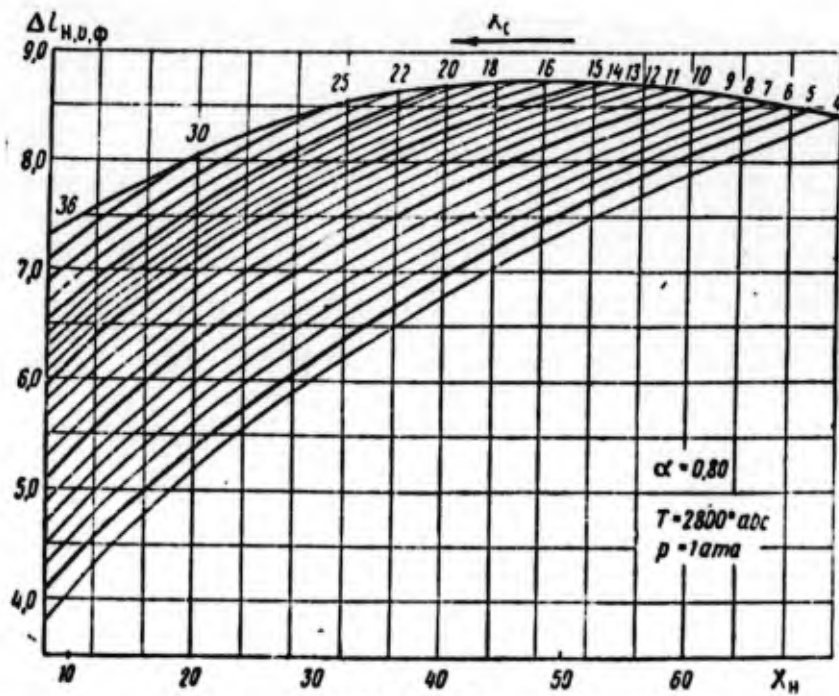


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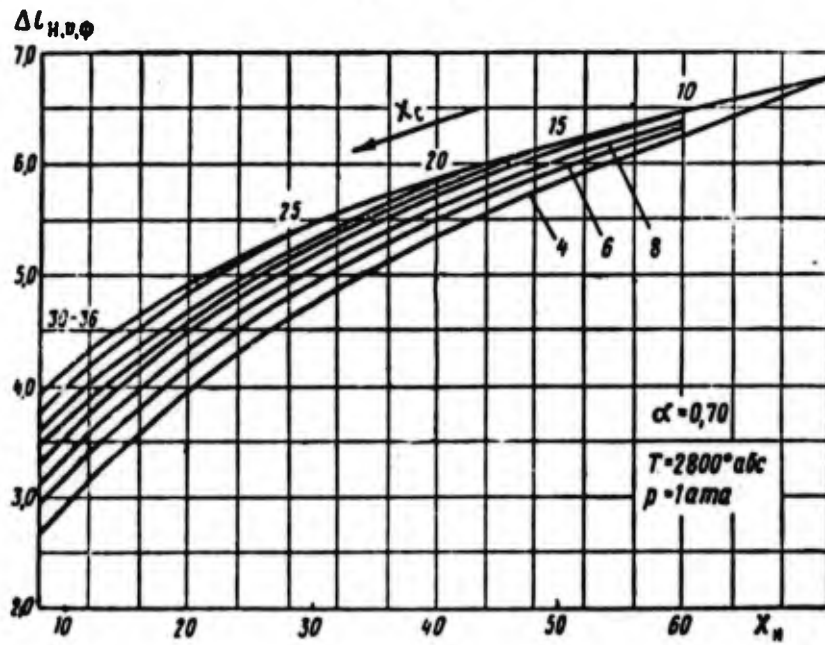


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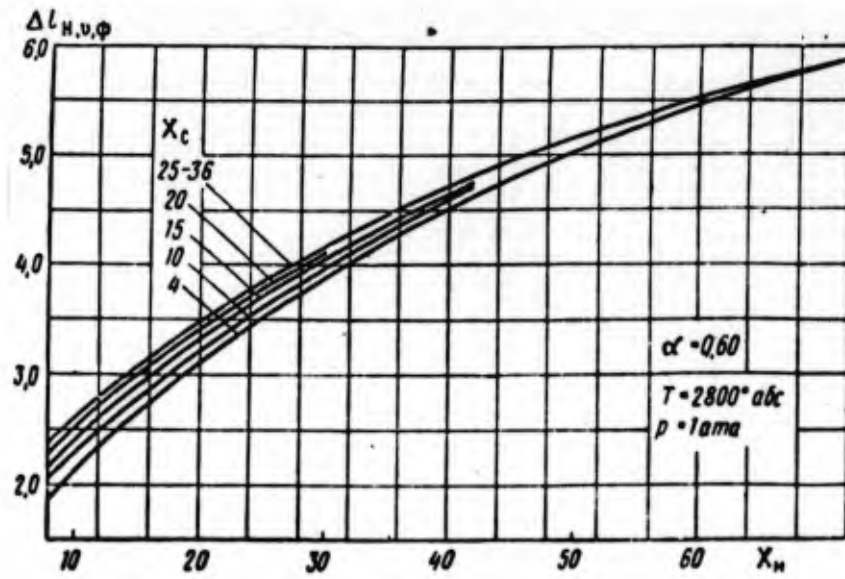


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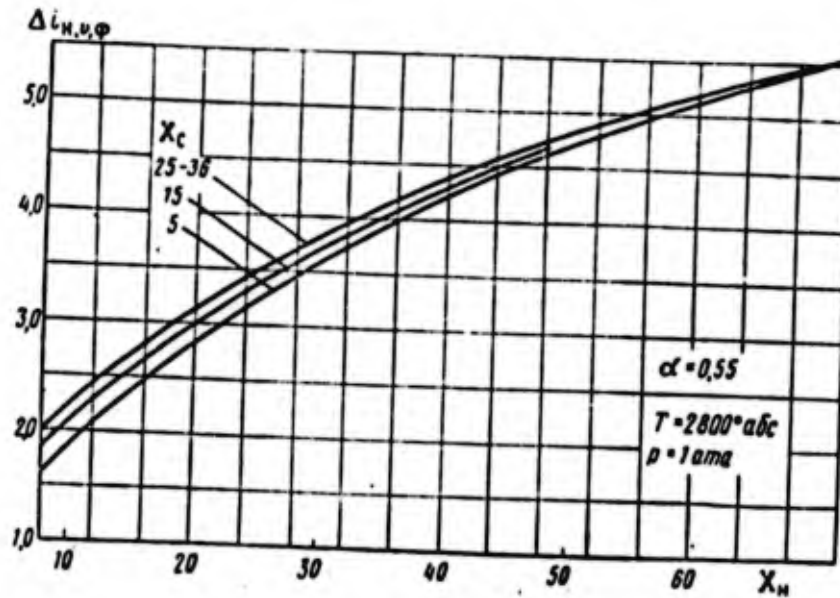


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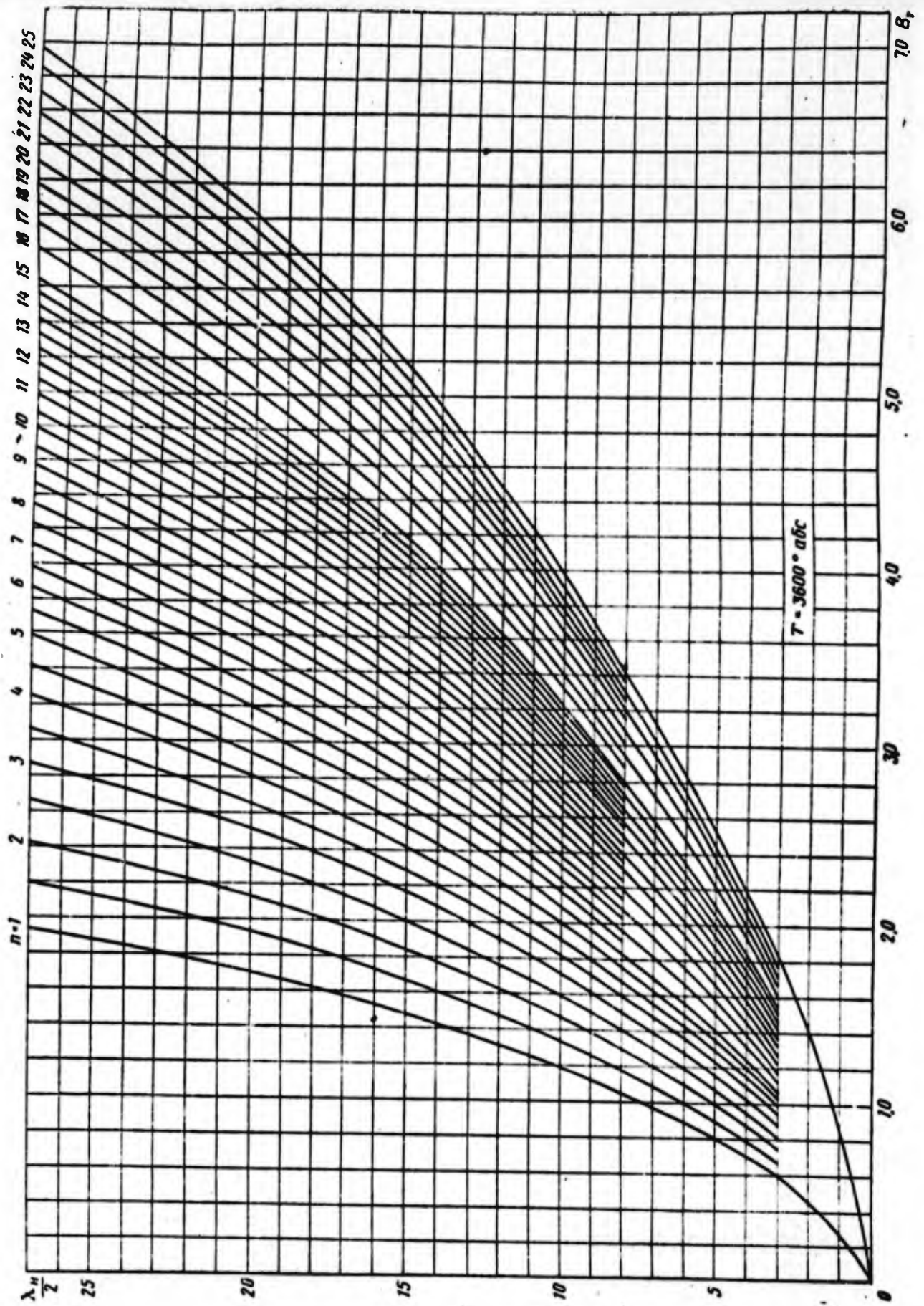


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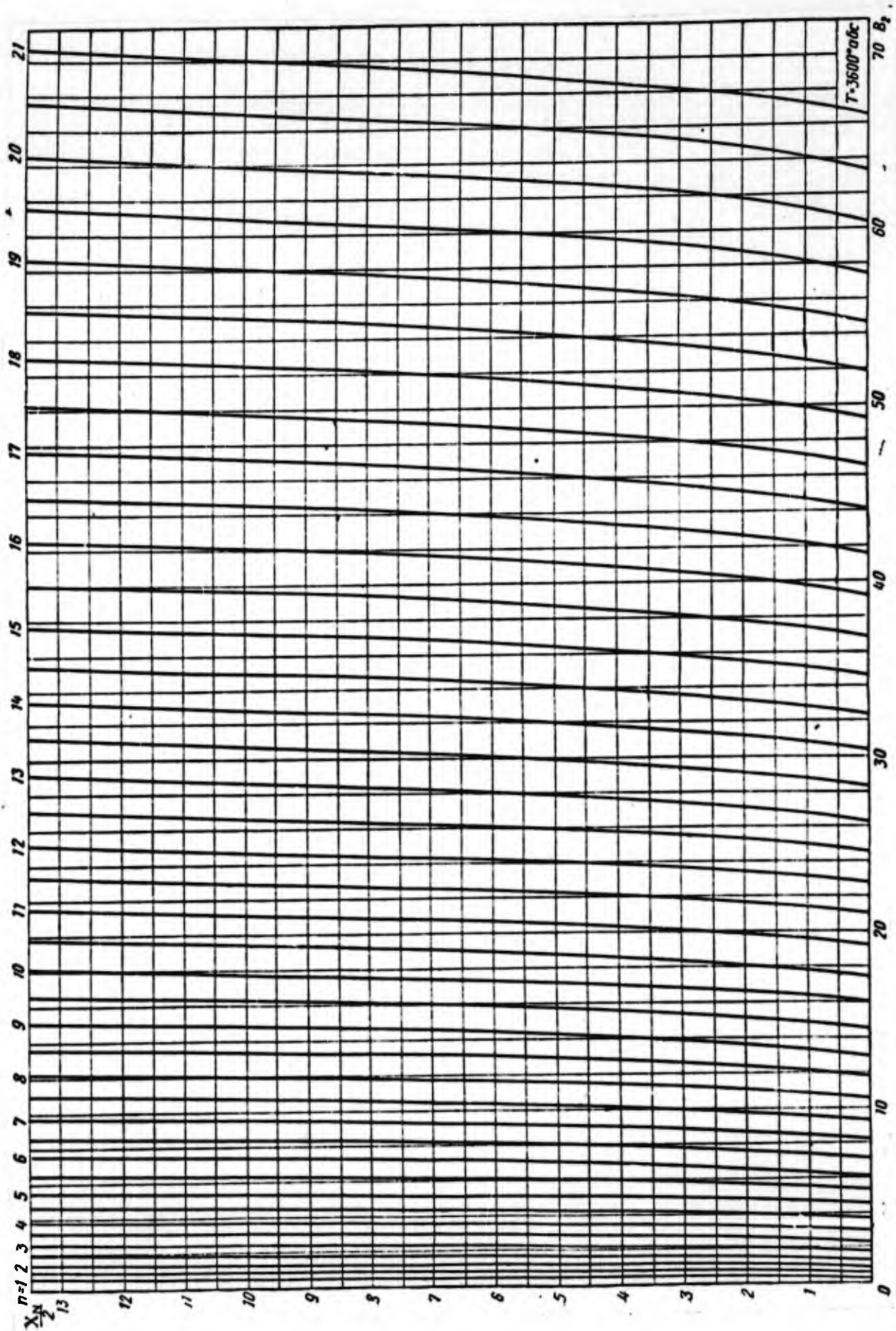


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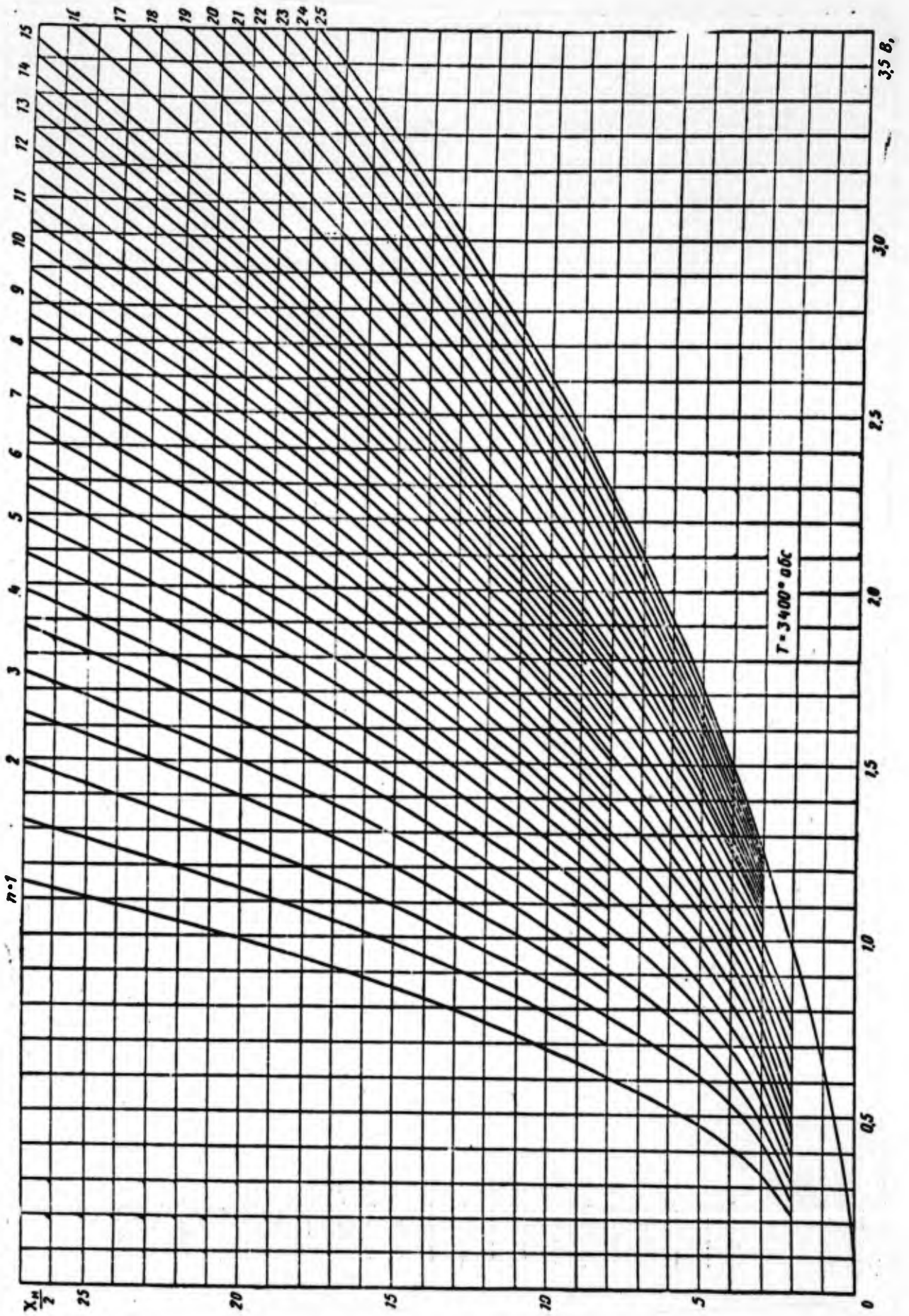


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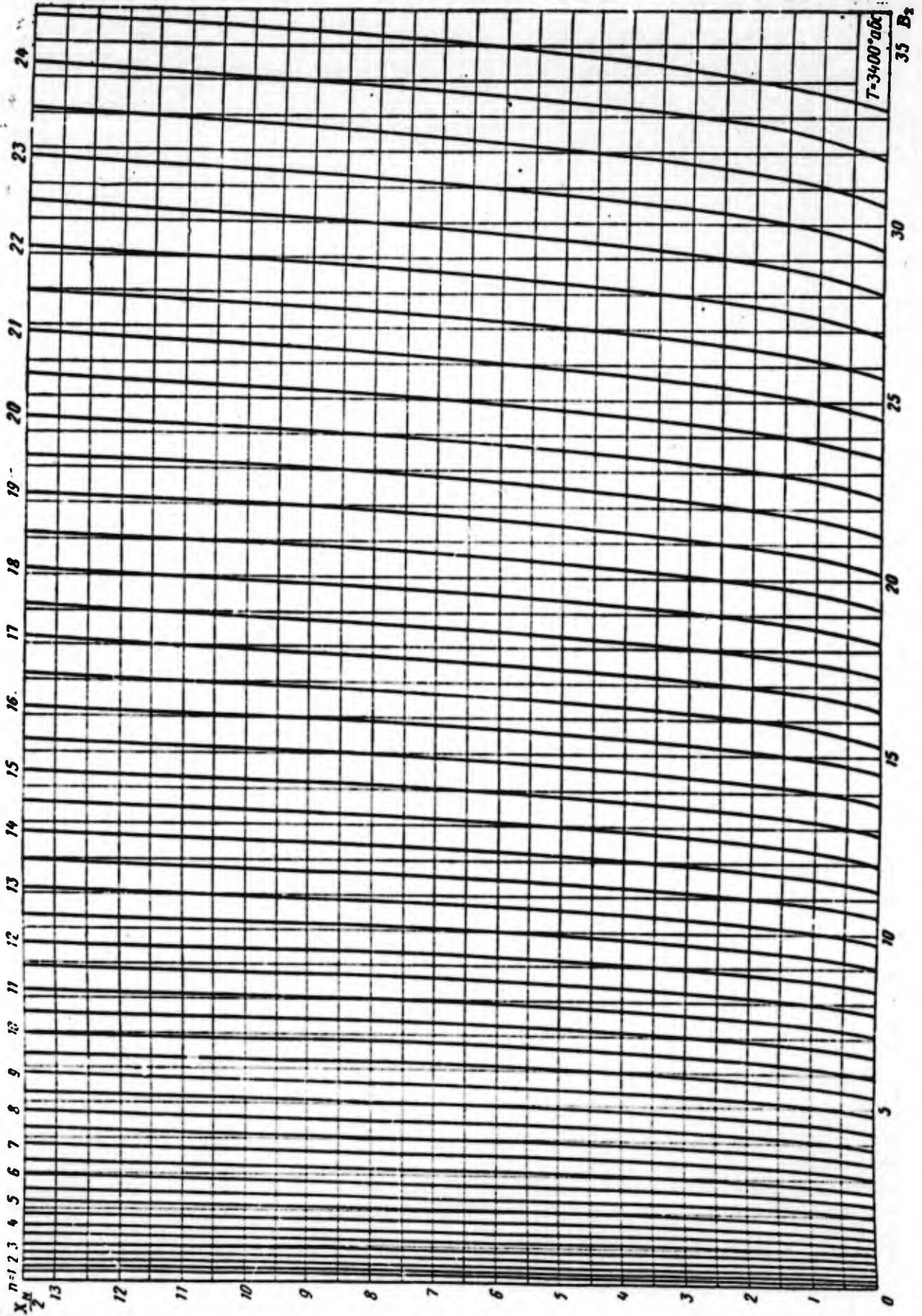


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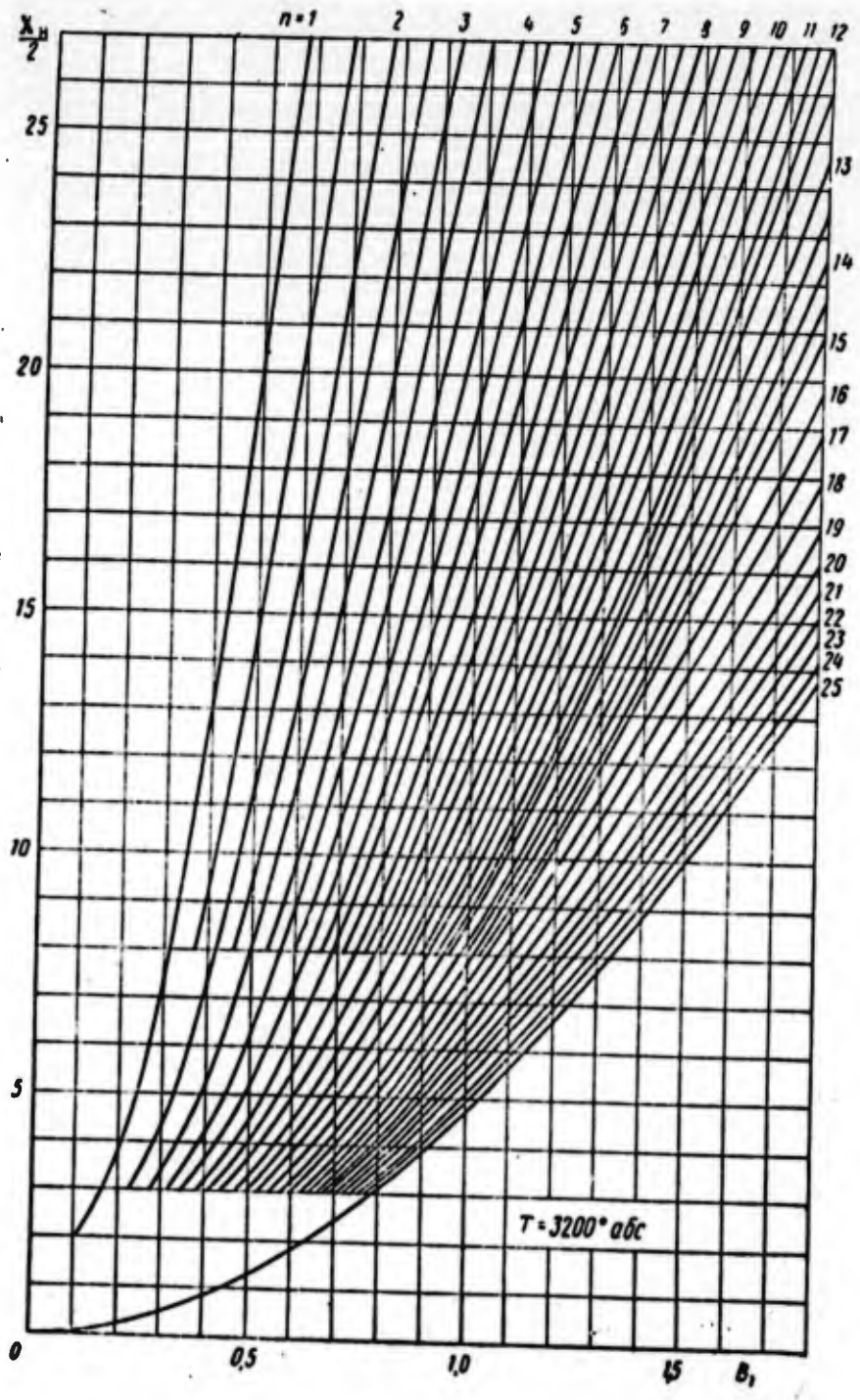


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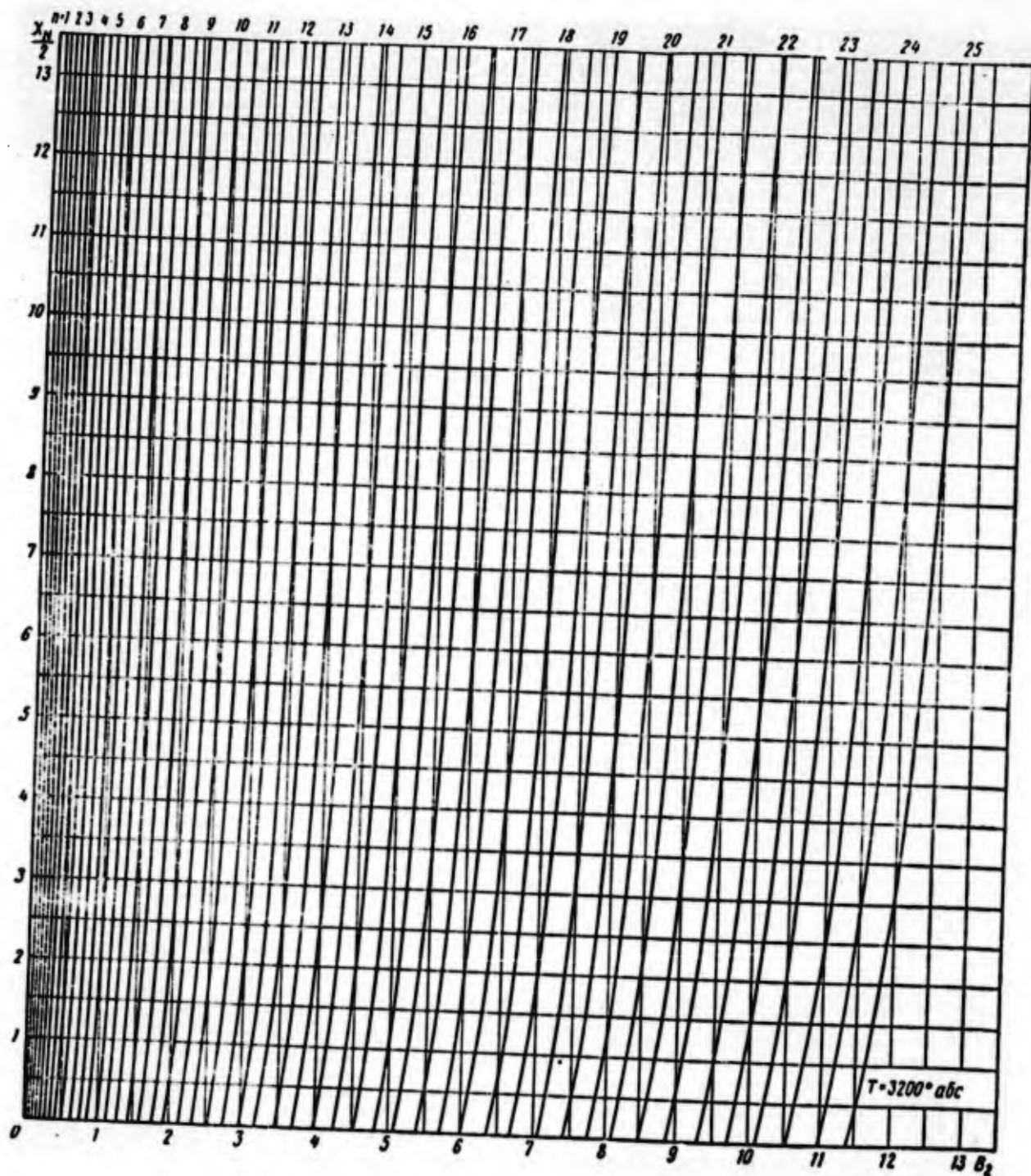


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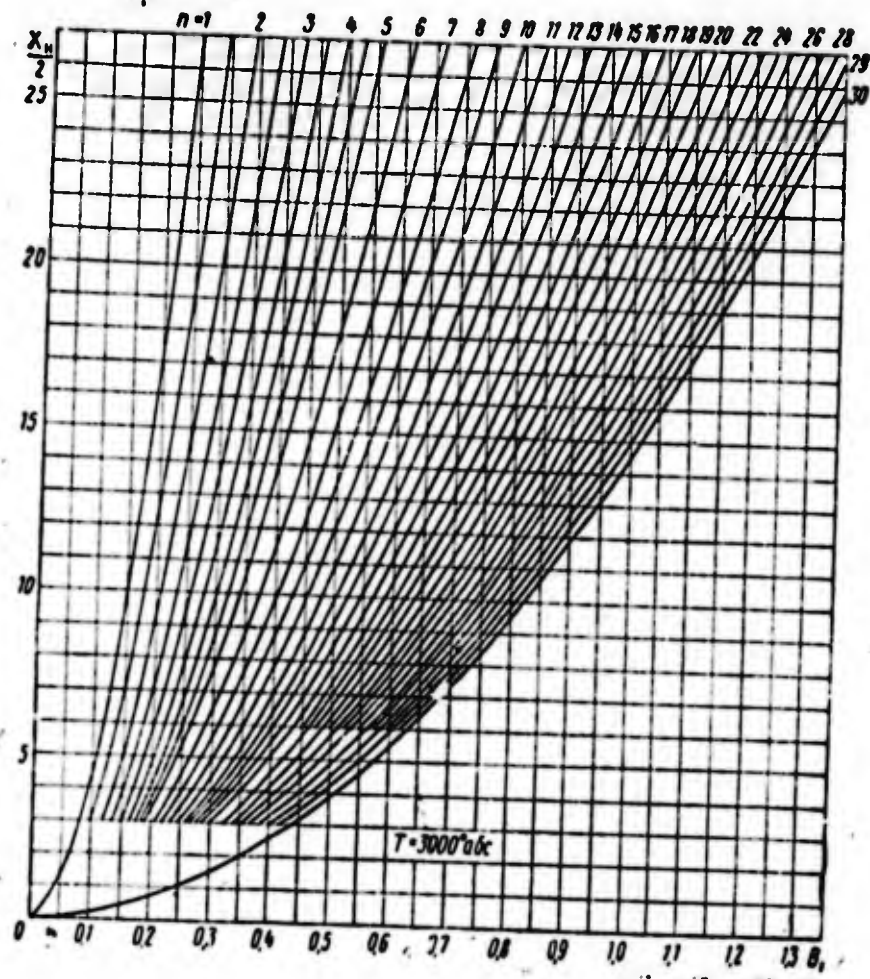


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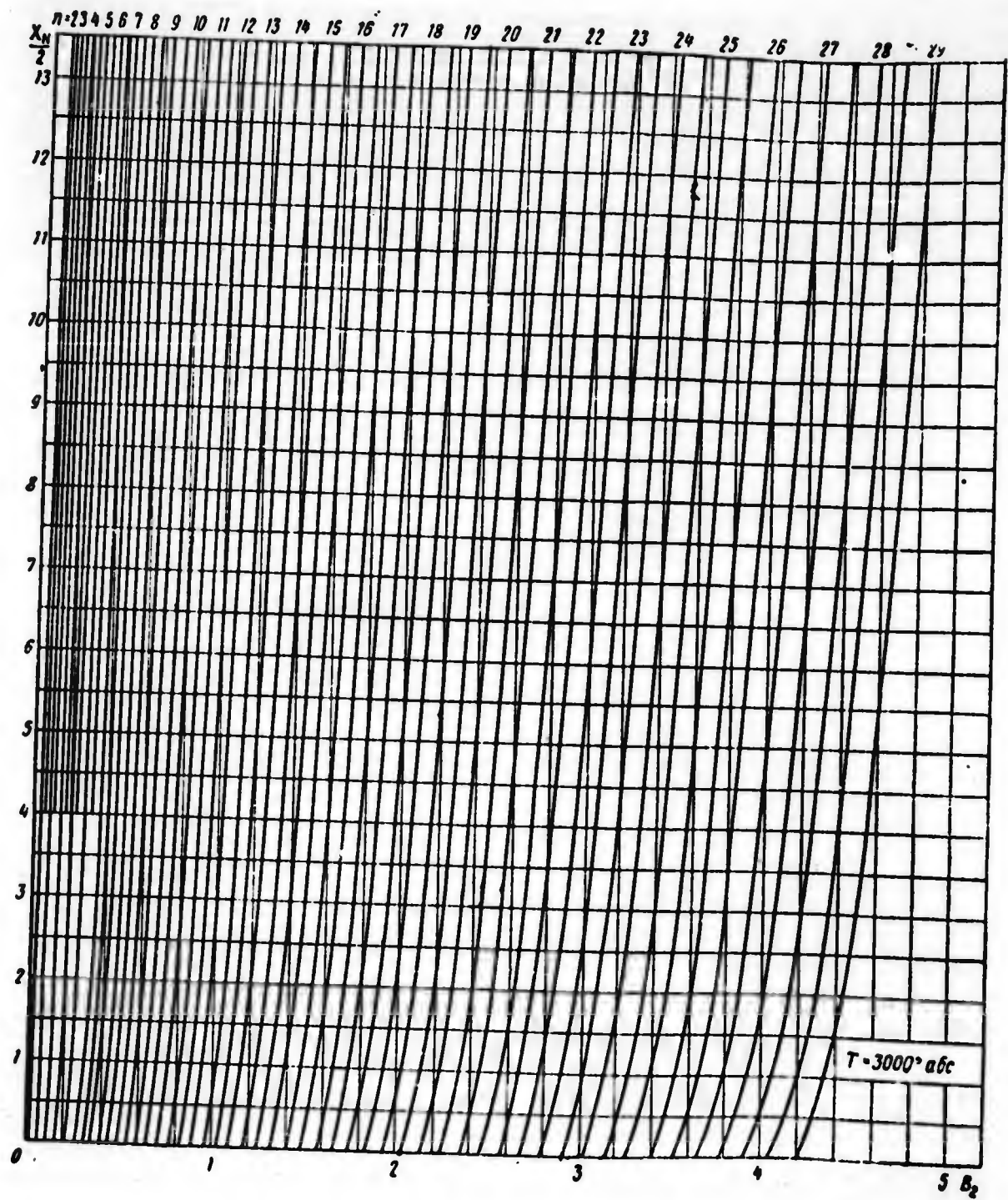


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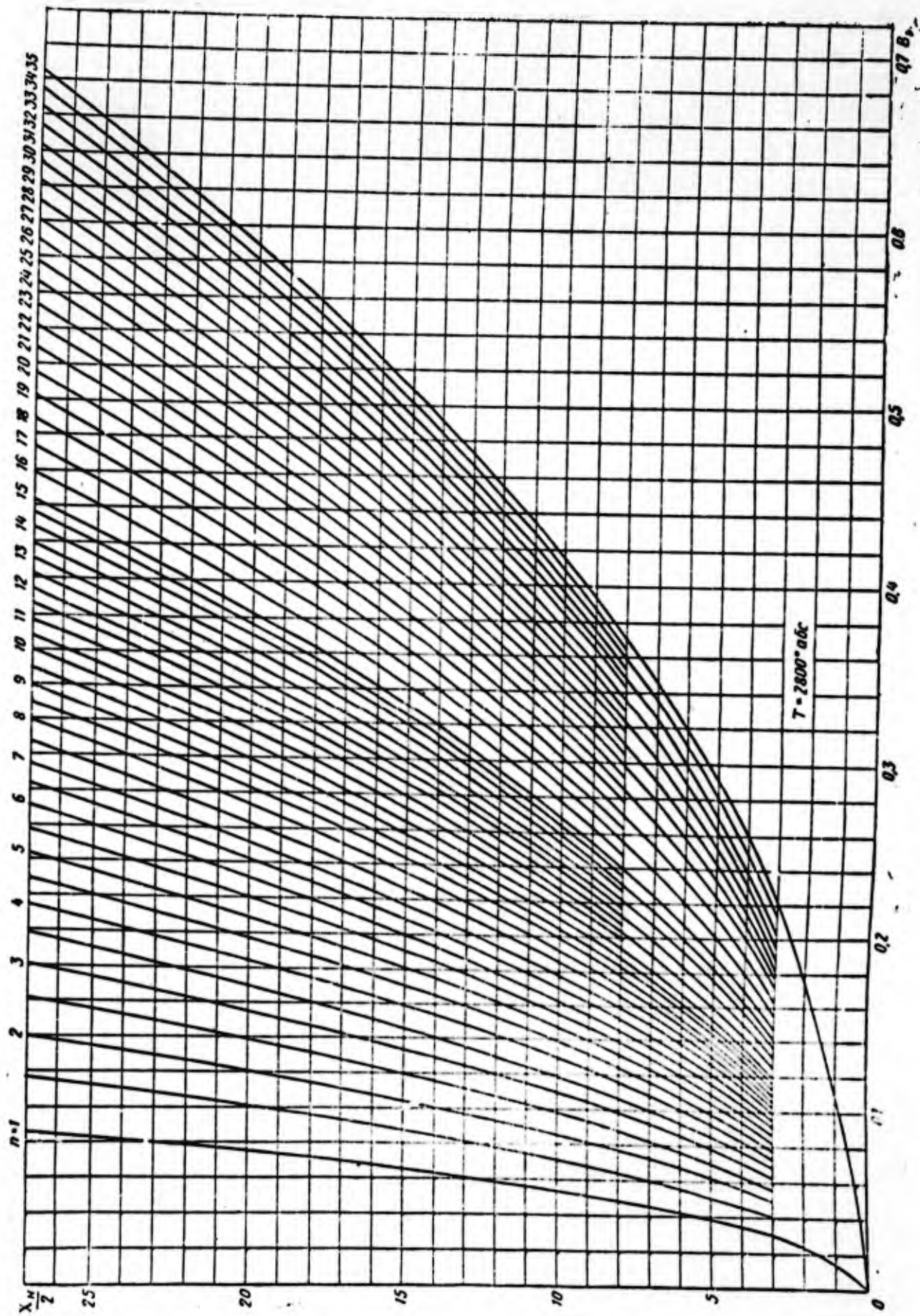


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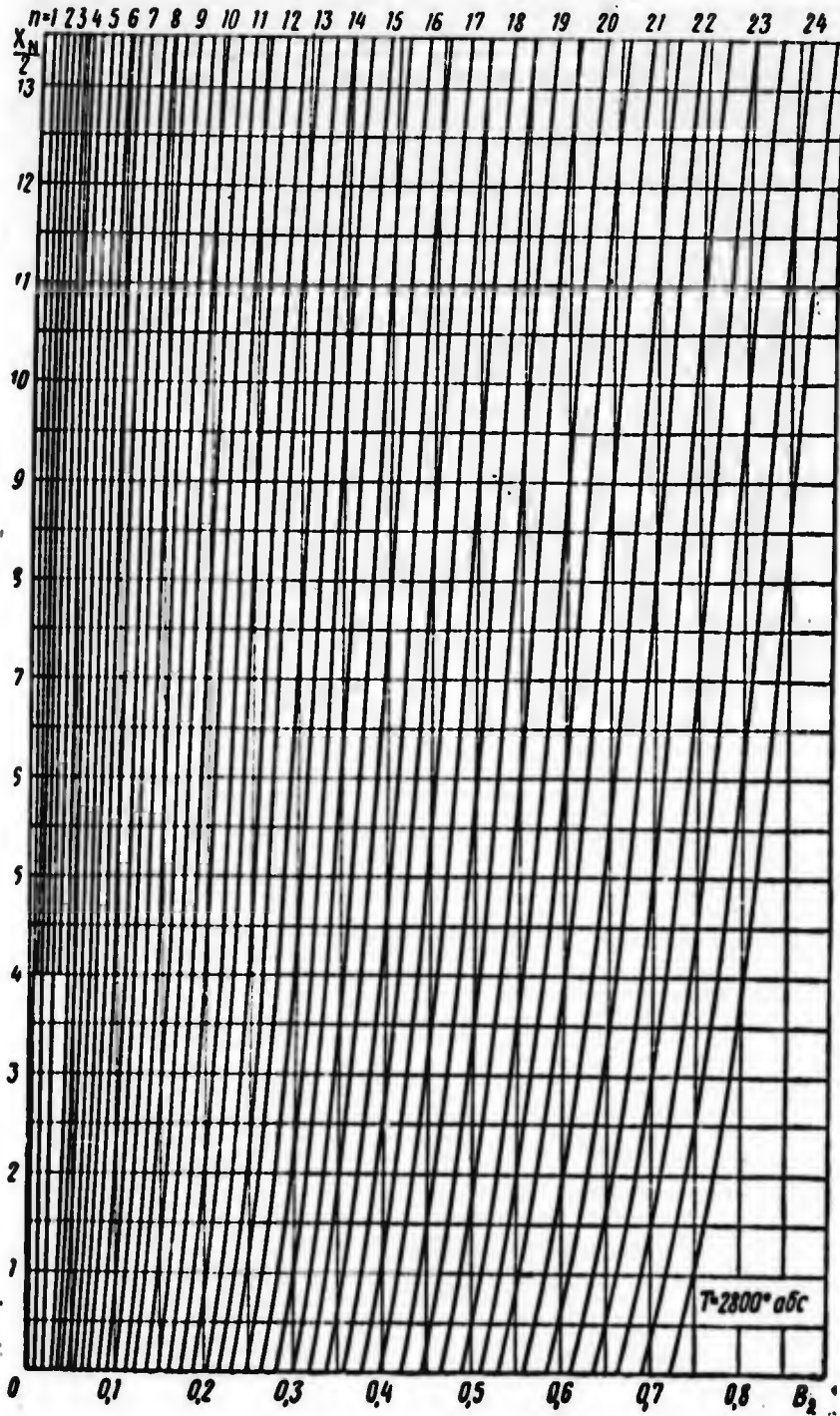


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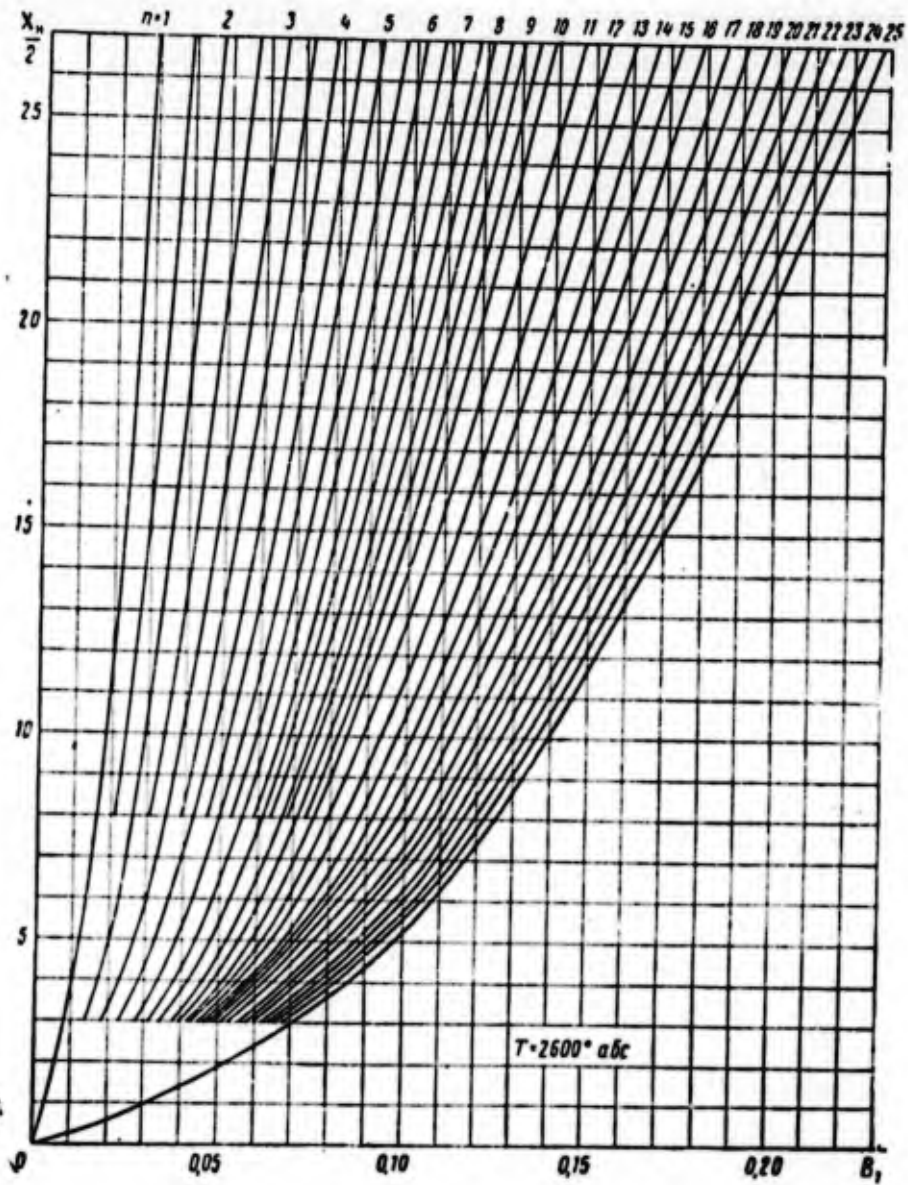


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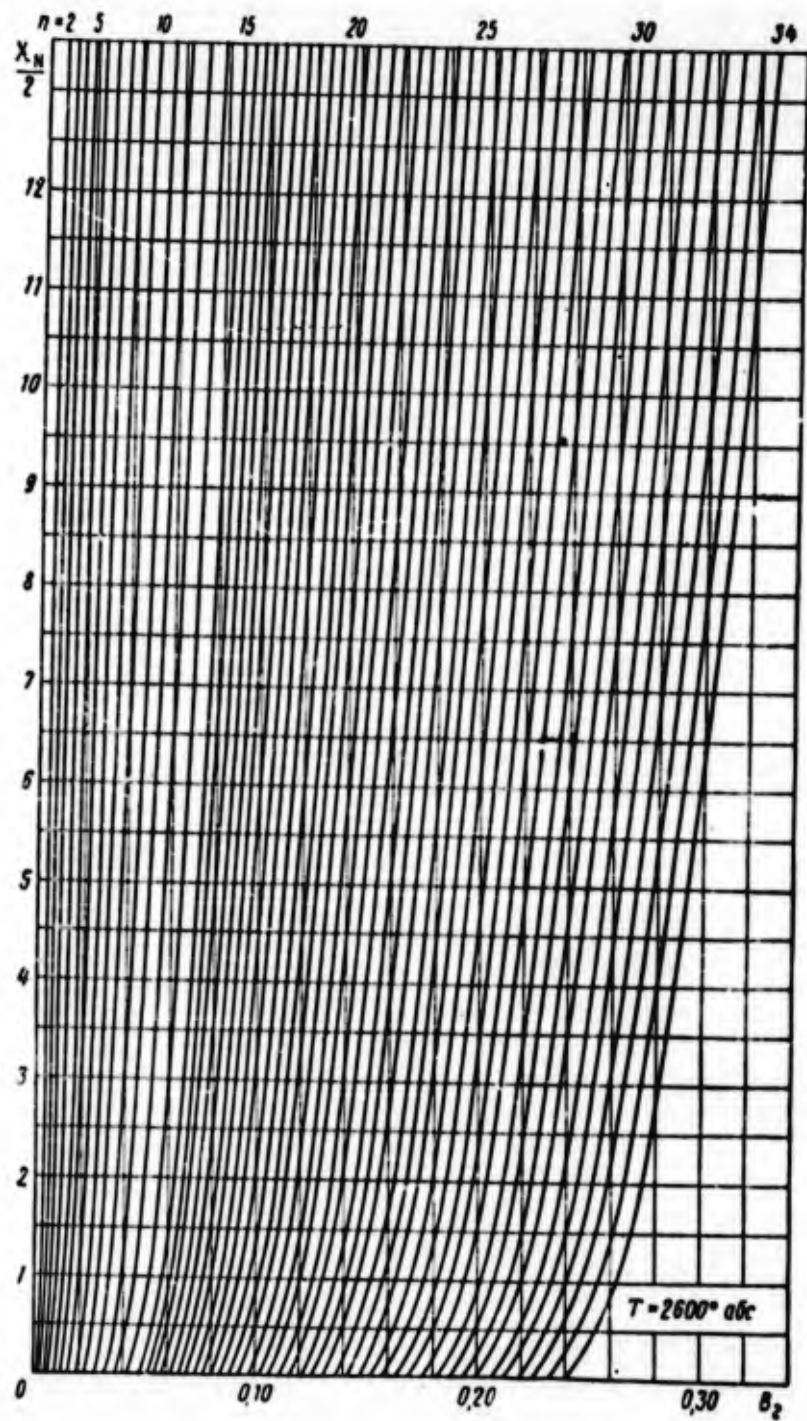


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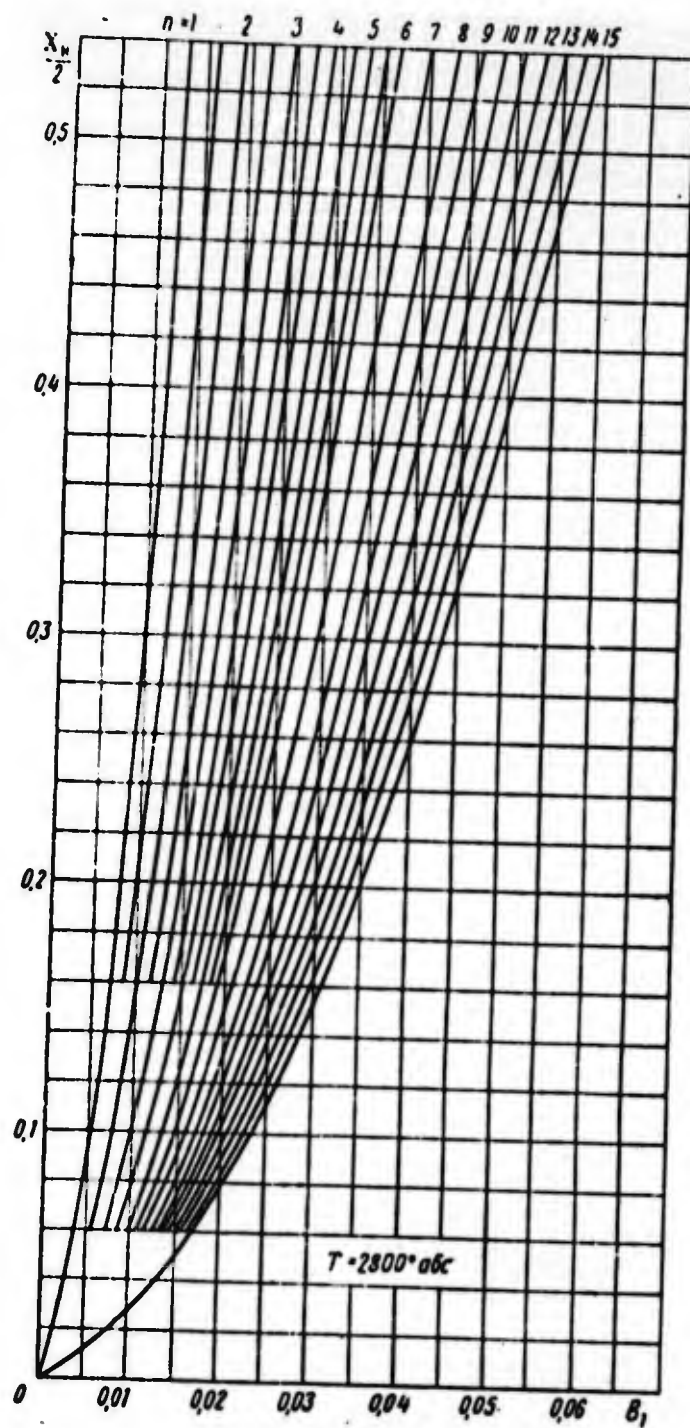


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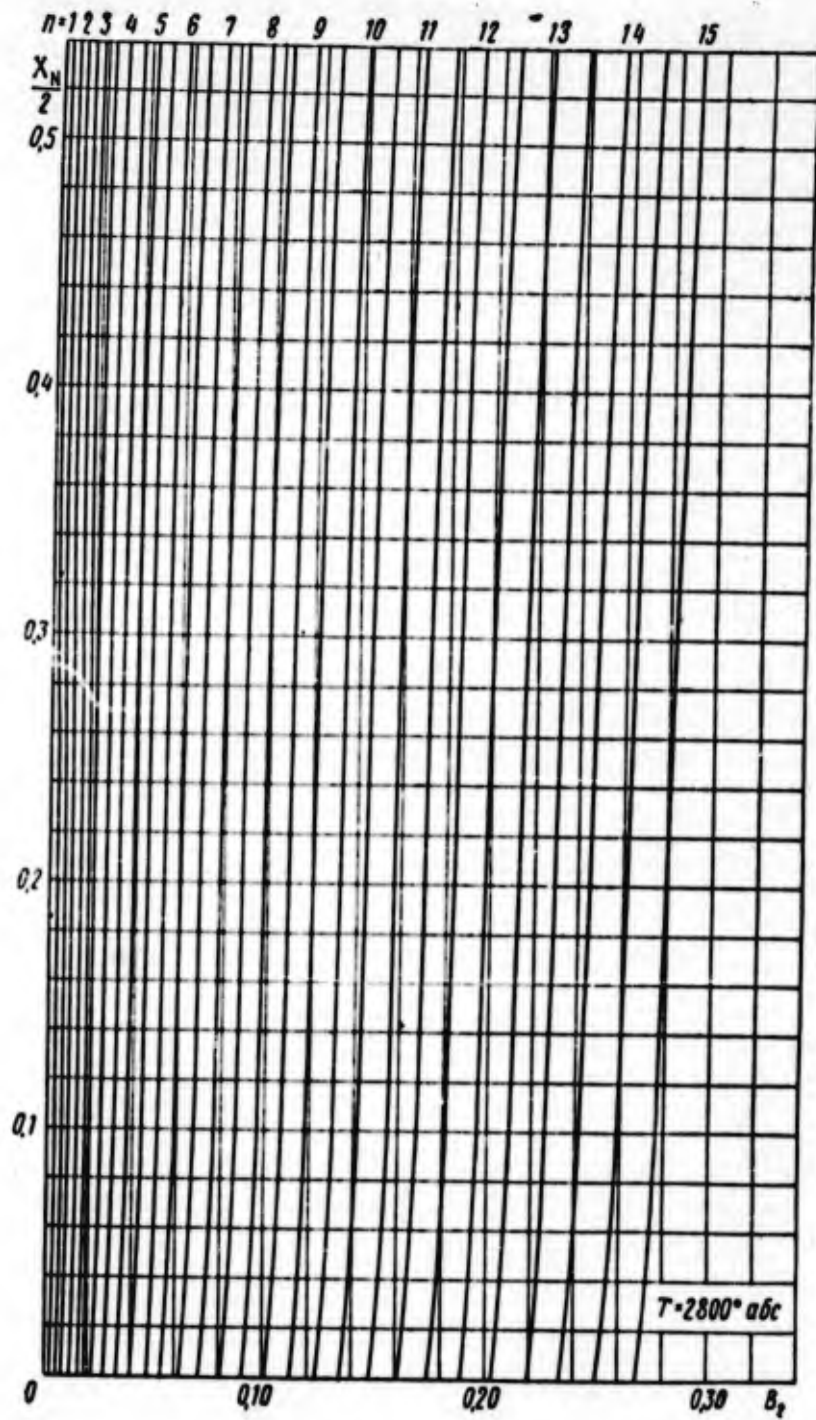


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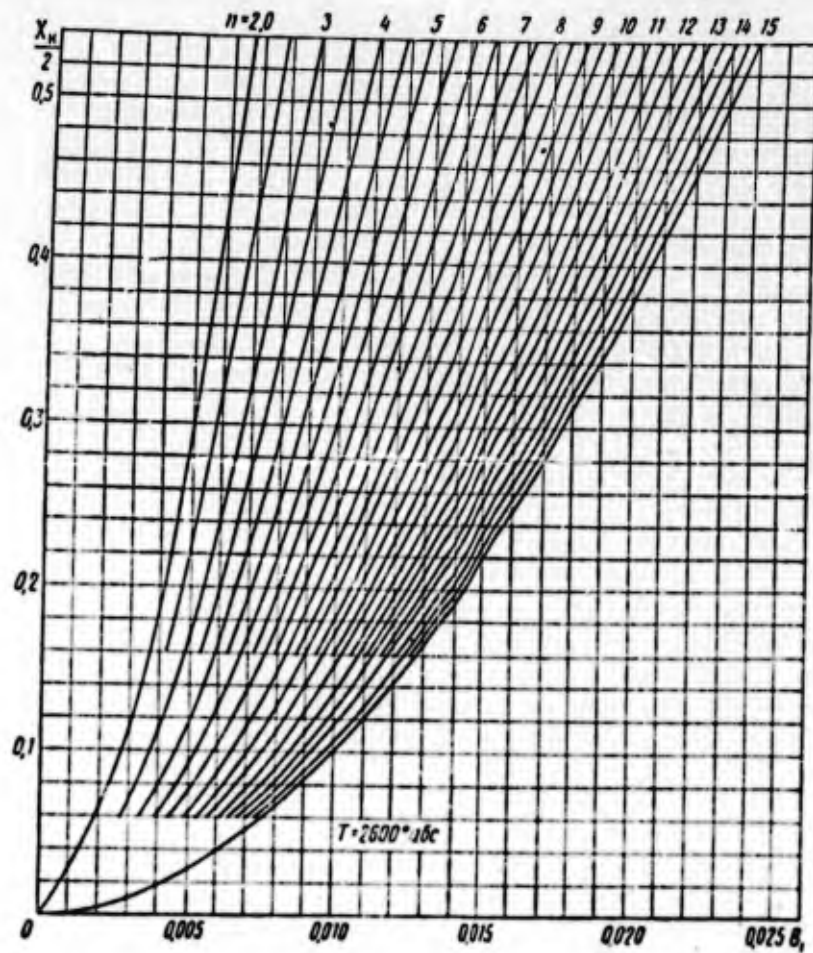


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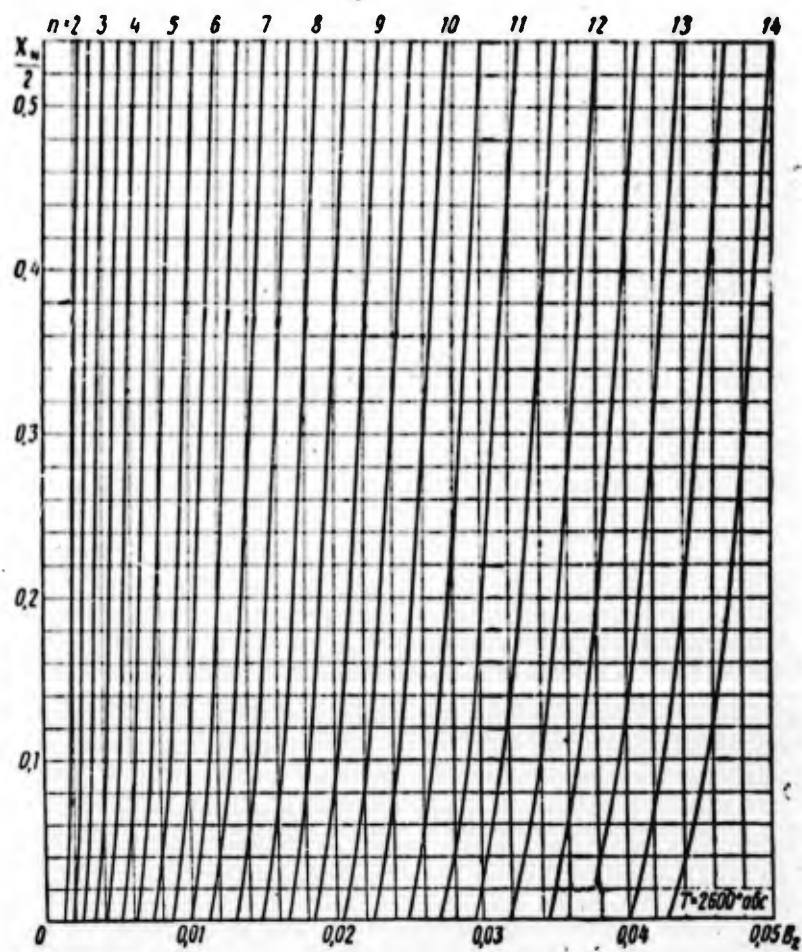


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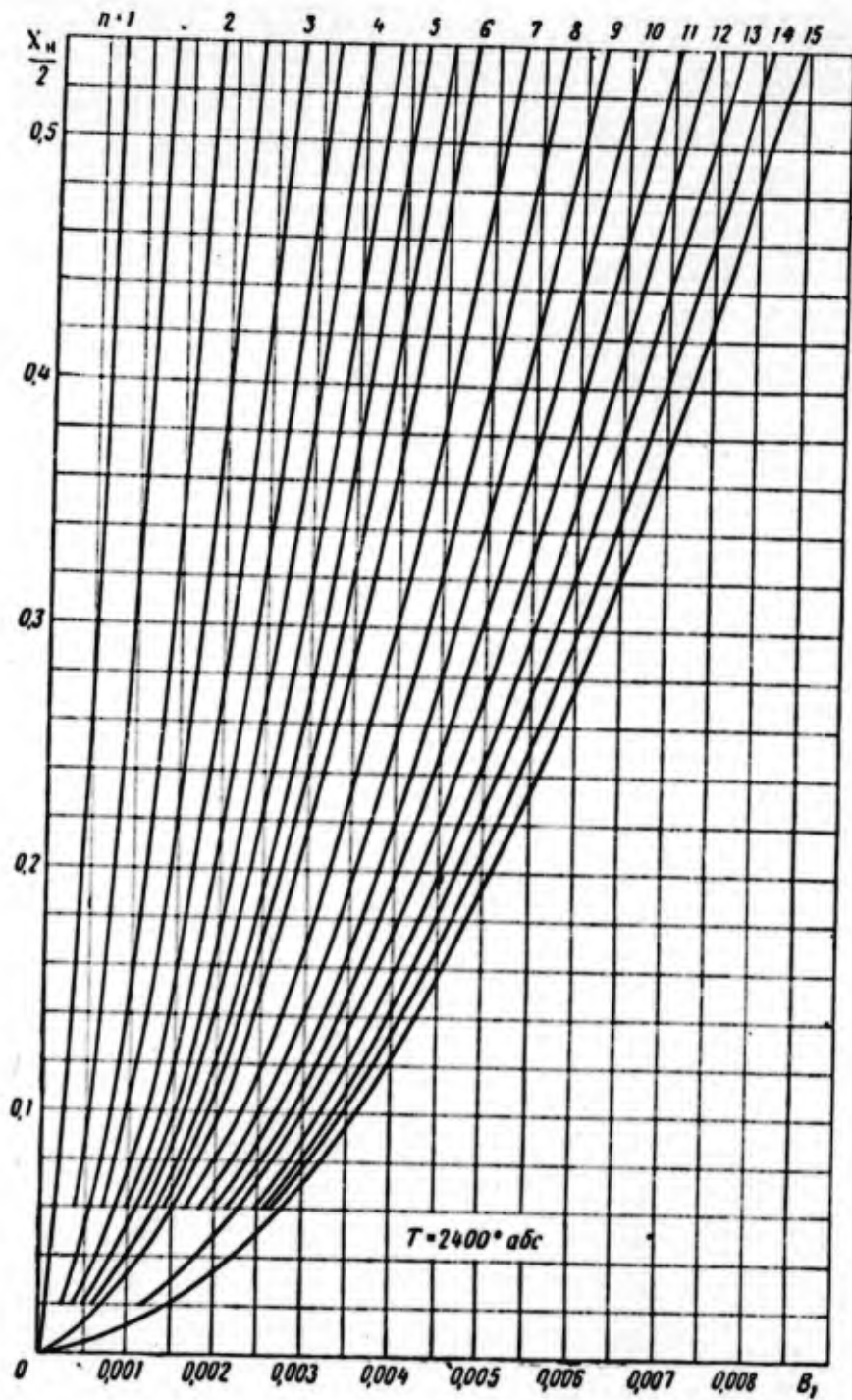


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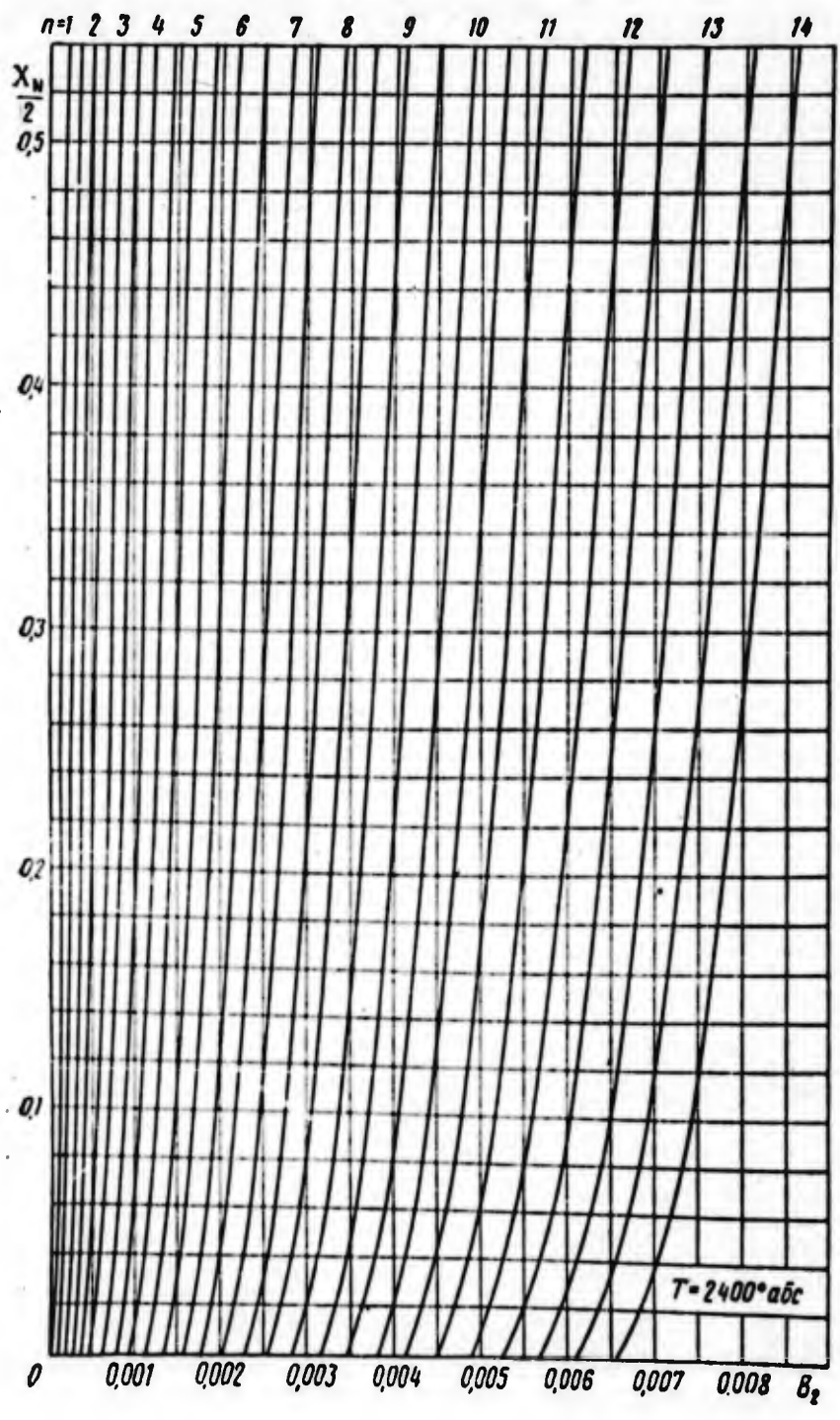


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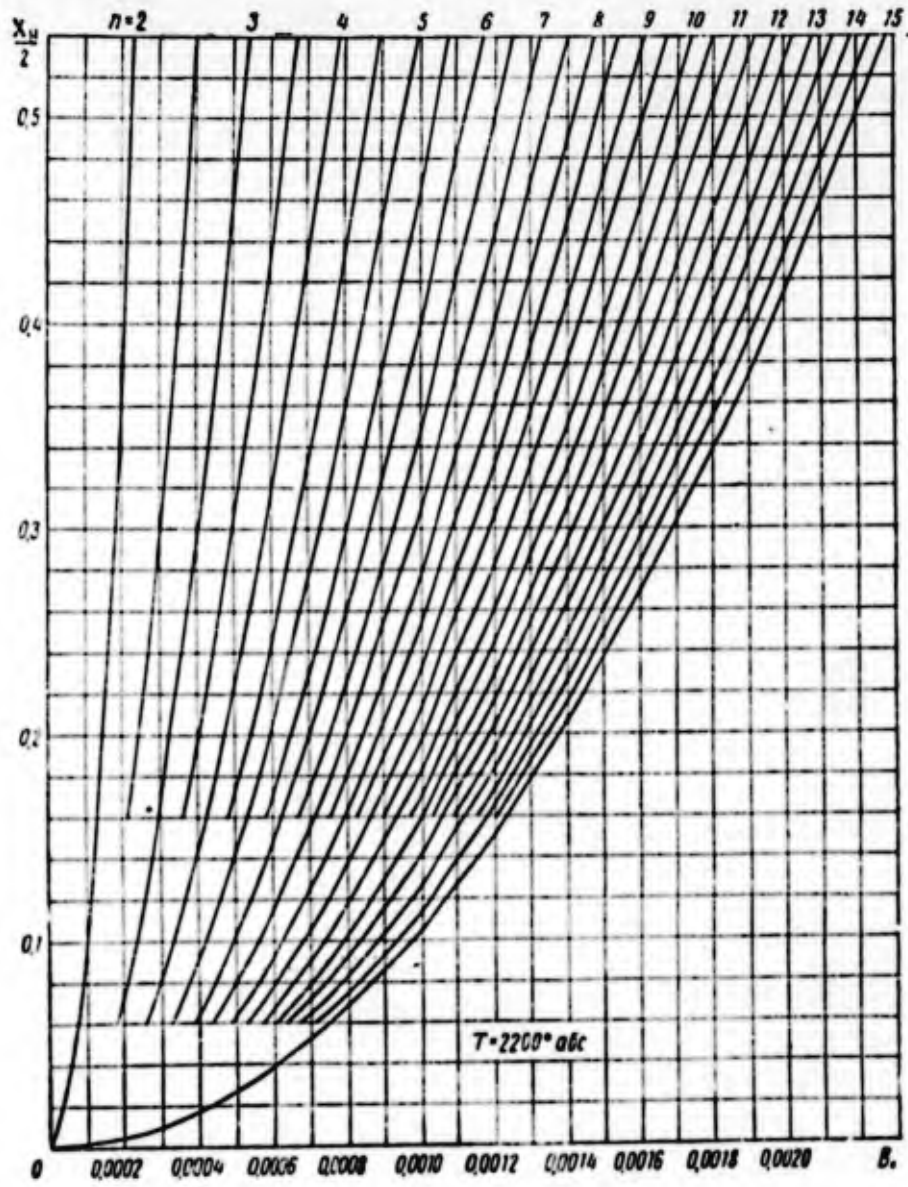


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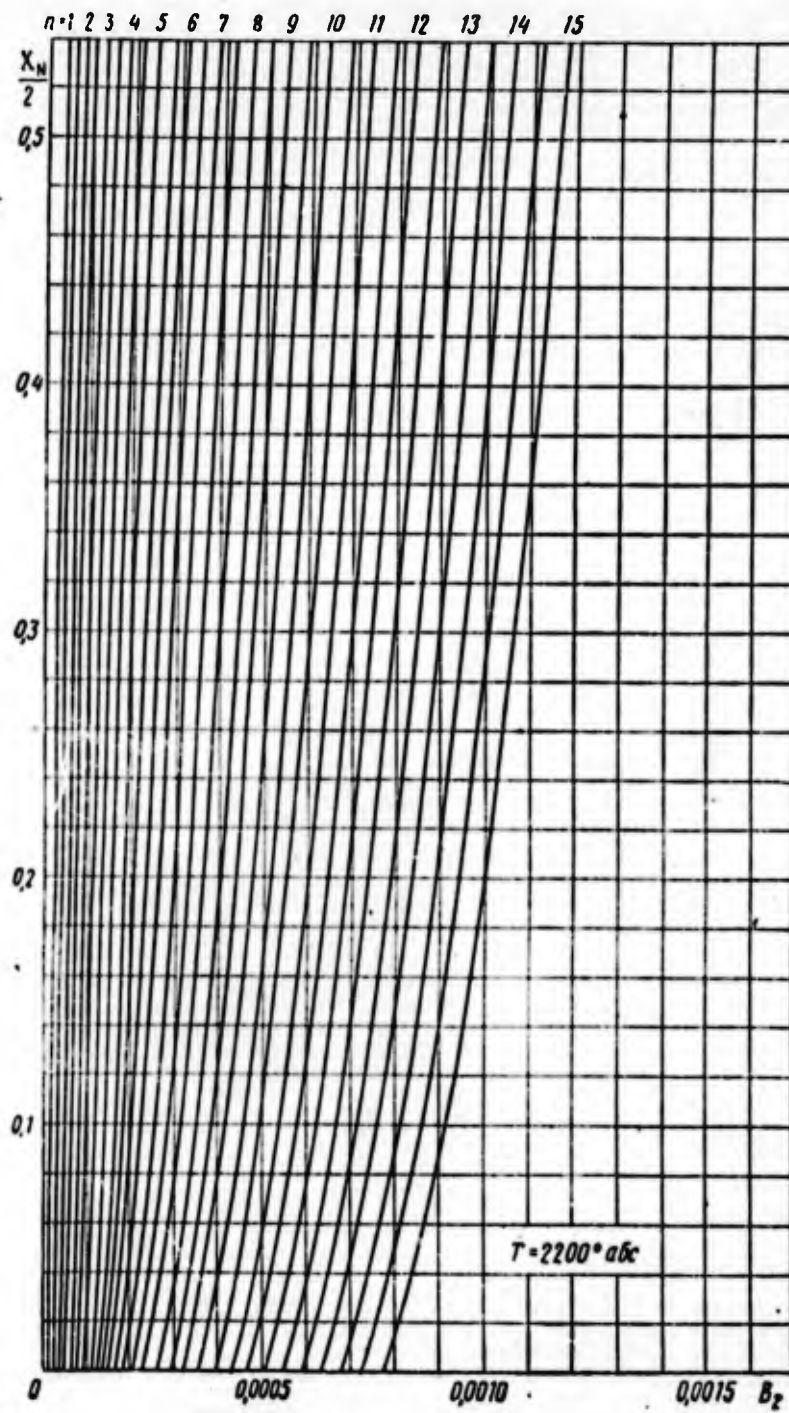


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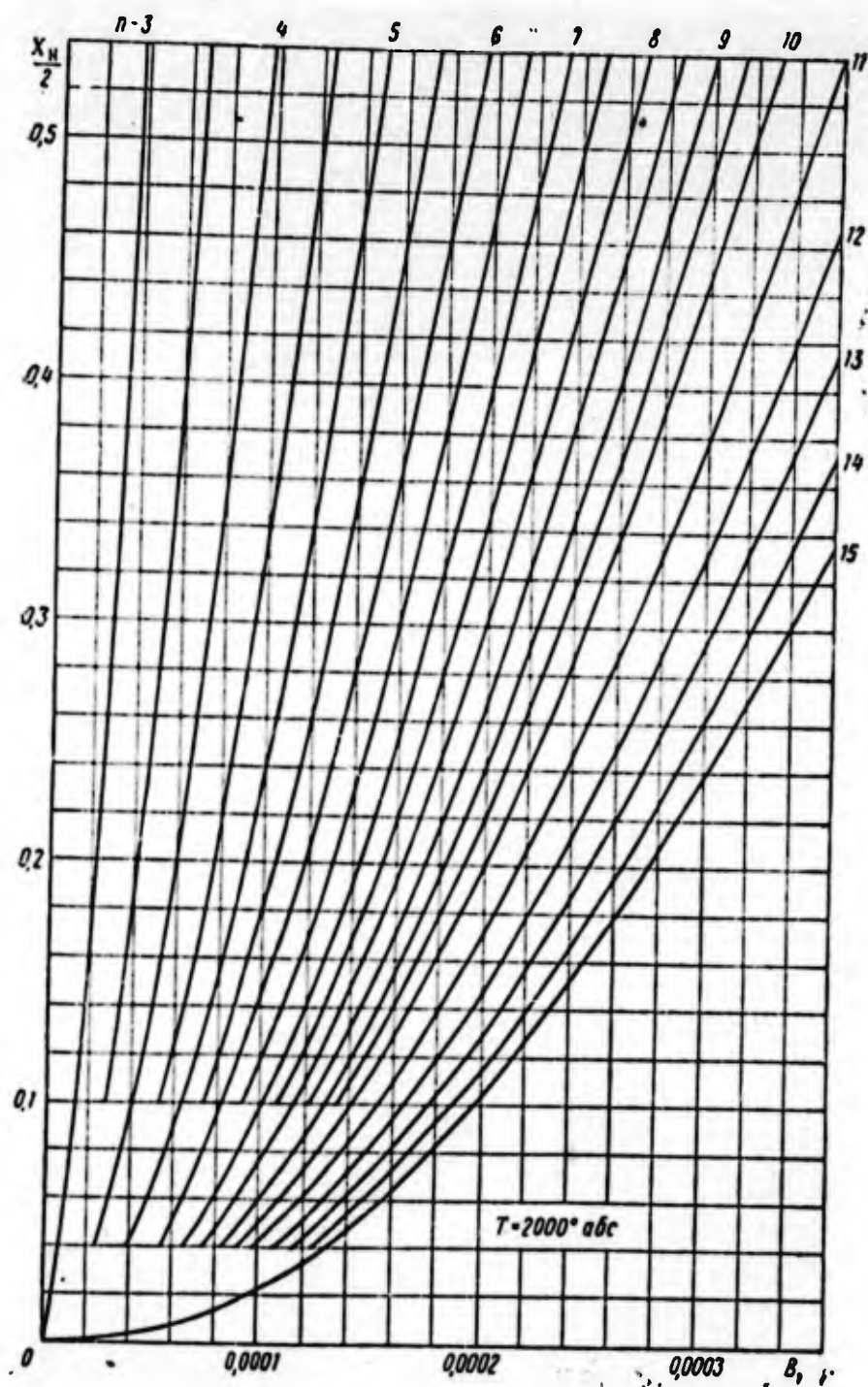


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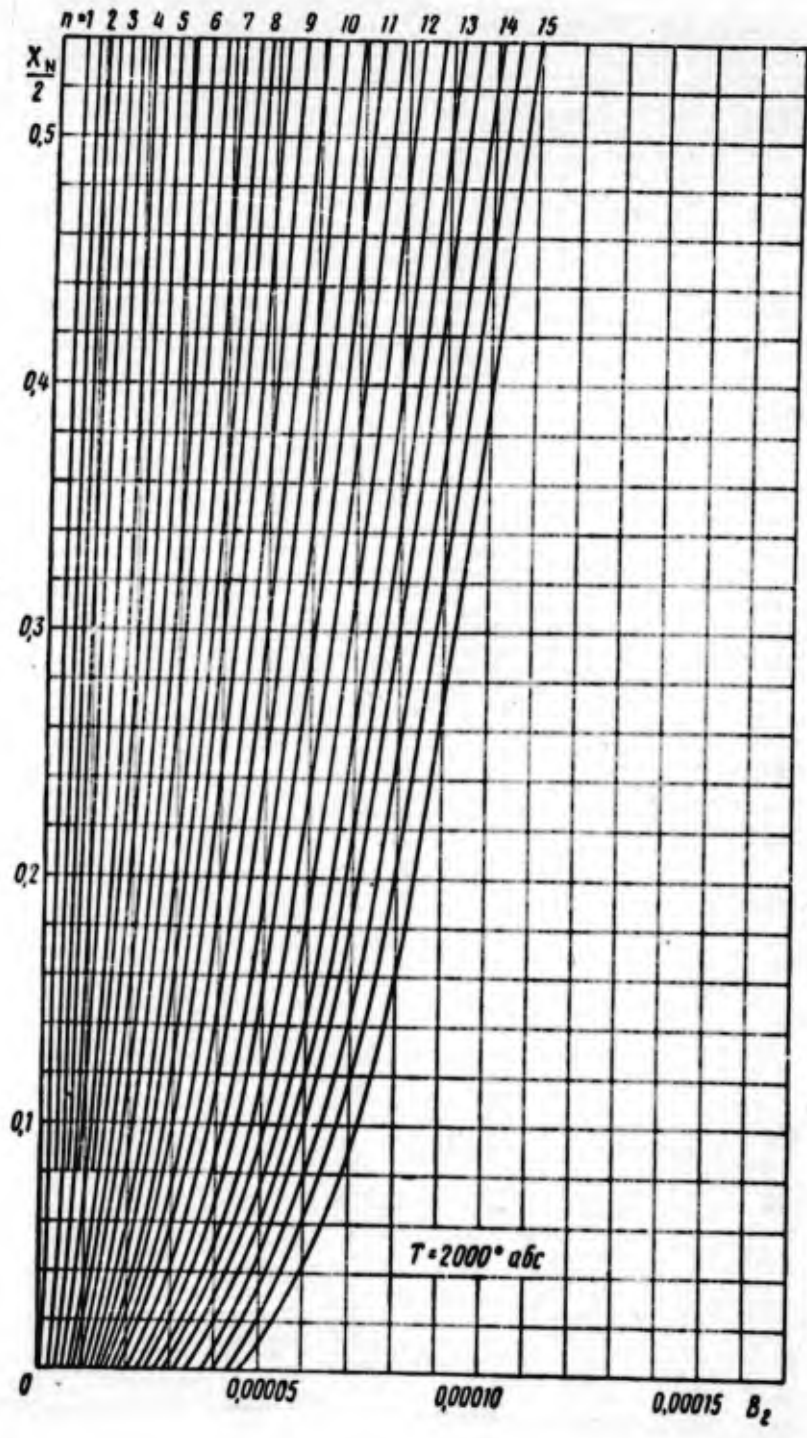


Fig. 42.

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