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ASYMMETRIC RELAXATION AND COMPLIANCE MATRICES

IN LINEAR VISCOELASTICITY

T. G. Rogers and A. C. Pipkin

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ASYMMETRIC RELAXATION AND COMPLIANCE MATRICES IN LINEAR
VISCOELASTICITY*

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Abstract

The restrictions imposed on anisotropic viscoelastic compliance matrices by material symmetry are derived, without using the matrix symmetry demanded by thermodynamic theories. Experiments to test the thermodynamic theories can be based on the differences between the forms obtained with and without the assumption of matrix symmetry. Transverse isotropy and cubic symmetry are treated in detail.

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1. Introduction.

The theory of linear isotropic viscoelasticity and its application to stress analysis of viscoelastic materials are now well developed (for reviews see Lee [1]* and Bland [2]). For such materials the constitutive equations relating the stress components σ_{1j} at time t to the strain components ϵ_{1j} up to and including time t may be written in the form**

$$s_{1j}(x,t) = \int_{-\infty}^t G_1(t-\tau) \dot{\epsilon}_{1j}(x,\tau) d\tau \quad (1.1)$$

and

$$\sigma_{11}(x,t) = \int_{-\infty}^t G_2(t-\tau) \dot{\epsilon}_{11}(x,\tau) d\tau ; \quad (1.2)$$

where $s_{1j}(x,t)$ and $\epsilon_{1j}(x,t)$ are the deviatoric components of stress and strain respectively, and $G_1(t)$ and $G_2(t)$ are the relaxation moduli in shear and dilatation. Equivalently, the strain components may be expressed in terms of the stress by the relations

* Numbers in square brackets refer to the bibliography at the end of the paper.

** Here, as in the sequel, the single symbol x represents the triad of cartesian coordinates x_1, x_2 and x_3 , and the dot represents partial differentiation with respect to the time variable. It is understood that all Latin subscripts, unless otherwise specified, can take the values 1, 2 and 3 independently and that a repeated index implies summation over these values.

$$e_{ij}(x, t) = \int_{-\infty}^t J_1(t-\tau) \dot{s}_{ij}(x, \tau) d\tau \quad (1.3)$$

and

$$\epsilon_{ii}(x, t) = \int_{-\infty}^t J_2(t-\tau) \dot{\delta}_{ii}(x, \tau) d\tau, \quad (1.4)$$

where $J_1(t)$ and $J_2(t)$ are the creep compliances in shear and dilatation respectively. Thus it may be seen that only two independent functions of time are required to describe linear isotropic viscoelastic materials.

In the general anisotropic case the constitutive relations may be written as

$$\sigma_{ij}(x, t) = \int_{-\infty}^t c_{ijkl}(t-\tau) \dot{\epsilon}_{kl}(x, \tau) d\tau, \quad (1.5)$$

or equivalently,

$$\epsilon_{ij}(x, t) = \int_{-\infty}^t s_{ijkl}(t-\tau) \dot{\sigma}_{kl}(x, \tau) d\tau, \quad (1.6)$$

where $c_{ijkl}(t)$ and $s_{ijkl}(t)$ are the relaxation moduli and creep compliances respectively. In the special case of isotropic materials, the symmetry of the material makes it necessary for the matrix of relaxation moduli to take the form*

$$c_{ijkl}(t) = \lambda(t) \delta_{ij} \delta_{kl} + \mu(t) (\delta_{ik} \delta_{jl} + \delta_{jk} \delta_{il}), \quad (1.7)$$

when the symmetry of the stress and strain tensors is taken into account. By setting $G_1 = 2\mu$ and $G_2 = 3\lambda + 2\mu$, (1.1) and (1.2) are

* δ_{ij} is the Kronecker delta, zero if $i \neq j$, unity if $i = j$.

obtained. A similar form is obtained for the creep compliances.

Thermodynamic theories of viscoelasticity have been developed by Biot [3],[4], Staverman and Schwarzl [5],[6], and Meixner [7],[8]. As a consequence of the use of Onsager's reciprocity relations [9],[10], these theories predict that the matrices of relaxation moduli and creep compliances are symmetric, i.e.

$$c_{ijkl}(t) = c_{klij}(t), \quad s_{ijkl}(t) = s_{klij}(t). \quad (1.8)$$

This result is important in stress analysis, since it means that the application of the Laplace transform reduces the constitutive equations to forms identical with those of classical anisotropic elasticity theory. This correspondence between viscoelasticity and elasticity has been fruitfully exploited in the case of isotropic materials (see Lee [11] and [1]).

In elasticity theory, the elastic constants are shown to satisfy the symmetry conditions (1.8) by an argument based on the fact that elastic deformation is non-dissipative. Derivation of the symmetries (1.8) in the case of dissipative materials is based on much more complicated arguments involving the application of Onsager's principle to latent variables. The symmetries (1.8) are thus not on such firm footing in viscoelasticity as the corresponding symmetries are in elasticity theory, and it is reasonable to ask for experimental verification of these symmetry relations.

The predictions (1.8) of the thermodynamic theories have no content in the case of isotropic materials, since

isotropy is itself sufficient to make the matrices symmetric, as (1.7) shows. In anisotropic materials, however, it is usually the case that certain of the relations (1.8) are consequences of material symmetry alone, while others are not. To know which are which, it is necessary to work out the restrictions implied by material symmetry, and this has apparently never been done without assuming matrix symmetry from the beginning.

The object of this paper is to determine the restrictions imposed by material symmetry alone, in the special cases of transverse isotropy and cubic symmetry. Experiments to test the thermodynamic theories can then be performed by measuring those relaxation or compliance functions which must vanish or be equal to others if the matrices are to be symmetric, but which can be non-zero or unequal insofar as material symmetry is concerned. The choice of transverse isotropy and cubic symmetry as cases for consideration is motivated by the belief that industrial forming processes such as drawing, rolling and extrusion will tend to introduce these types of asymmetry.

The general requirements imposed by material symmetry are reviewed in §2, starting from the integral operator representation (1.5) of the linear viscoelastic stress-deformation relations. Material symmetries associated with transverse isotropy are discussed in §3. There are four types of material symmetry which may be grouped under the general heading of transverse isotropy, differing from one another in their reflectional symmetries. These four types lead to two distinct

types of mechanical behavior. In one type it is found that there are eight independent relaxation moduli, of which two must be zero and two must be equal if the matrix is to be symmetric. In the other type, which has a higher degree of reflectional symmetry, there are six independent relaxation moduli, two of which must be equal for the matrix to be symmetric. In §4 cubic symmetry is examined, and similar results are obtained. The final section discusses the effects of matrix asymmetry on some simple experiments.

2. Material Symmetry Restrictions.

Equation (1.5) is a relation between six independent components of stress and six independent components of strain, since $\sigma_{ij} = \sigma_{ji}$ and $\epsilon_{ij} = \epsilon_{ji}$. The number of independent relaxation moduli is then thirty-six at most, and the eighty-one moduli $c_{ijkl}(t)$ are related to one another by

$$c_{ijkl}(t) = c_{jikl}(t) = c_{ijlk}(t). \quad (2.1)$$

Any symmetry in the structure of the material will give rise to further relations among the functions $c_{ijkl}(t)$, with consequent reduction in the number of independent moduli.

The relations (1.5) are assumed to be valid in a coordinate system x which has some particular orientation with respect to the material. Let \bar{x} be a second system, related to the system x by a time-independent orthogonal transformation

$$\bar{x}_i = a_{ij}x_j, \quad a_{ik}a_{jk} = a_{ki}a_{kj} = \delta_{ij}. \quad (2.2)$$

The relation between the components of stress $\bar{\sigma}_{ij}$ and of strain $\bar{\epsilon}_{ij}$ measured with respect to the system \bar{x} will involve a new set of relaxation moduli $\bar{c}_{ijkl}(t)$. Thus

$$\bar{\sigma}_{ij}(\bar{x}, t) = \int_{-\infty}^t \bar{c}_{ijkl}(t-\tau) \dot{\bar{\epsilon}}_{kl}(\bar{x}, \tau) d\tau, \quad (2.3)$$

where

$$\bar{\sigma}_{ij}(\bar{x}, t) = a_{ik}a_{jl}\sigma_{kl}(x, t) \quad (2.4)$$

and

$$\bar{\epsilon}_{ij}(\bar{x}, t) = a_{ik}a_{jl}\epsilon_{kl}(x, t). \quad (2.5)$$

By using (1.5) and (2.2) to (2.5), it can be shown that the two sets of relaxation moduli must be related by the equations

$$\bar{c}_{ijkl}(t) = a_{ip}a_{jq}a_{kr}a_{ls}c_{pqrs}(t). \quad (2.6)$$

If the structure of the material is the same when described with respect to the system \bar{x} as it is when described in the system x , then (2.2) is a symmetry transformation, and the moduli $\bar{c}_{ijkl}(t)$ must be identical with the moduli $c_{ijkl}(t)$. Thus, if $\|a_{ij}\|$ is the matrix of a symmetry transformation, then (2.6) implies that*

$$c_{ijkl} = a_{ip}a_{jq}a_{kr}a_{ls}c_{pqrs}. \quad (2.7)$$

The set of functions c_{ijkl} must be of such a form that (2.7) is satisfied for each symmetry of the material under consideration. In the following section we shall work out the consequences of (2.7) for the case of transversely isotropic materials, and in §4 we shall consider materials with cubic symmetry.

* We have omitted the argument t of $c_{ijkl}(t)$ for abbreviation, and we shall do so throughout the following two sections.

3. Transverse Isotropy.

Transversely isotropic materials are characterized by an axis of rotational symmetry. If the symmetry axis is taken to be the x_3 -axis of a system of rectangular cartesian coordinates, then the matrix of coefficients of a symmetry transformation (2.2) is of the form

$$\|a_{ij}\| = \begin{vmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{vmatrix}, \quad (3.1)$$

where the angle of rotation θ is arbitrary.

There are four distinct types of material symmetry which may be classified as transverse isotropy. All four types have the rotational symmetries (3.1), but they differ in their reflectional symmetries. The material may, or may not, be symmetric under reflection in the x_1x_2 -plane, and it may, or may not, be symmetric under reflections in planes containing the x_3 -axis. As we shall see below, however, presence or absence of reflectional symmetry in the x_1x_2 -plane (i.e. reversal of the direction of the x_3 -axis) is immaterial insofar as the relation between stress and strain is concerned. Thus there are only two essentially different cases to consider. In §3a, we will consider materials which possess only the rotational symmetries (3.1). This class of materials includes those which physically have an intrinsic right- or left-handedness of structure. In §3b we will consider materials which are also symmetric under

reflections in planes containing the x_3 -axis. These, of course, correspond to materials which have no handedness.

3a. Rotational Symmetry.

In a material which has the rotational symmetries (3.1), the relaxation moduli must satisfy equation (2.7) for all matrices $\|a_{ij}\|$ of the form (3.1). The simplest method of finding what these restrictions imply about the functions c_{ijkl} is an indirect method due to Smith and Rivlin [12]. For any group of symmetry transformations there are certain tensors whose components are invariant under all transformations of the group, and Smith and Rivlin call these the anisotropic tensors for the group considered. For example, the Kronecker delta δ_{ij} transforms into itself under any rotation or reflection of coordinates. If only rotations are considered, and not reflections, then the alternating symbol e_{ijk} transforms into itself. Equation (2.7) states that the tensor with components c_{ijkl} transforms into itself under any symmetry transformation of the group considered, and thus implies that it is an anisotropic tensor for that group.

Smith and Rivlin show that for any group of symmetries there are a finite number of basic anisotropic tensors, which are such that any anisotropic tensor can be represented as a linear combination of outer products of these basic tensors. For the group of rotations about one axis, characterized by $\|a_{ij}\|$ in (3.1), the basic anisotropic tensors are

$$\delta_{31}, \Delta_{1j} = \delta_{11}\delta_{1j} + \delta_{21}\delta_{2j}, \text{ and } e_{31j}. \quad (3.2)$$

Here δ_{31} is a unit vector directed along the symmetry axis, Δ_{1j} is a two-dimensional analog of the Kronecker delta, and e_{31j} is the two-dimensional counterpart of the alternating symbol.

Then c_{ijkl} must be a linear combination of the tensors

$$\Delta_{1j}\Delta_{kl}, \Delta_{1j}e_{3kl}, \Delta_{1j}\delta_{3k}\delta_{3l}, e_{31j}\delta_{3k}\delta_{3l}, \delta_{31}\delta_{3j}\delta_{3k}\delta_{3l}, \quad (3.3)$$

and those which can be obtained by permuting the subscripts.

Tensors of the type $e_{31j}e_{3kl}$ are omitted from the list (3.3) since they can be expressed in terms of those of the type $\Delta_{1j}\Delta_{kl}$:

$$e_{31j}e_{3kl} = \Delta_{1k}\Delta_{jl} - \Delta_{1l}\Delta_{jk}. \quad (3.4)$$

The set of relaxation moduli must be a linear combination of tensors of the types (3.3) which is symmetric under interchange of i and j , and also symmetric with respect to k and l , according to (2.1). The most general linear combination which satisfies these requirements is of the form

$$\begin{aligned} c_{ijkl} = & \alpha_1 \Delta_{1j}\Delta_{kl} + \alpha_2 \Delta_{1j}\delta_{3k}\delta_{3l} + \alpha_3 \Delta_{kl}\delta_{31}\delta_{3j} \\ & + \alpha_4 \delta_{31}\delta_{3j}\delta_{3k}\delta_{3l} + \alpha_5 (\Delta_{1k}\Delta_{jl} + \Delta_{jk}\Delta_{1l}) \\ & + \alpha_6 (\Delta_{1k}e_{3jl} + \Delta_{jk}e_{31l} + \Delta_{1l}e_{3jk} + \Delta_{jl}e_{31k}) \\ & + \alpha_7 (\Delta_{1l}\delta_{3j}\delta_{3l} + \Delta_{jk}\delta_{31}\delta_{3l} + \Delta_{1l}\delta_{3j}\delta_{3k} + \Delta_{jl}\delta_{31}\delta_{3k}) \\ & + \alpha_8 (e_{31k}\delta_{3j}\delta_{3l} + e_{3jk}\delta_{31}\delta_{3l} + e_{31l}\delta_{3j}\delta_{3k} + e_{3jl}\delta_{31}\delta_{3k}). \quad (3.5) \end{aligned}$$

Since c_{ijkl} is a function of t , the coefficients α_1 to α_8 are also functions of t . These eight functions give eight independent relaxation moduli for the material, and by making particular choices of i, j, k and l in (3.5) we may identify them as

$$\begin{aligned} \alpha_1 &= c_{1122}, & \alpha_2 &= c_{1133}, & \alpha_3 &= c_{3311}, & \alpha_4 &= c_{3333}, \\ \alpha_5 &= \frac{1}{2}(c_{1111} - c_{1122}), & \alpha_6 &= \frac{1}{2} c_{1112}, & \alpha_7 &= c_{2323}, & \alpha_8 &= c_{3123}. \end{aligned} \quad (3.6)$$

By using (3.6) in (3.5) we obtain expressions for all the functions c_{ijkl} in terms of those which appear in (3.6).

The moduli c_{ijkl} can be exhibited in a 6x6 matrix, as is the custom with elastic moduli for anisotropic materials. The rows are numbered with the double subscript ij , and the columns with kl , in the order 11,22,33,23,31,12. The matrix obtained by using (3.6) in (3.5) is

$$\begin{vmatrix} c_{1111} & c_{1122} & c_{1133} & 0 & 0 & c_{1112} \\ c_{1122} & c_{1111} & c_{1133} & 0 & 0 & -c_{1112} \\ c_{3311} & c_{3311} & c_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{2323} & -c_{3123} & 0 \\ 0 & 0 & 0 & c_{3123} & c_{2323} & 0 \\ -c_{1112} & c_{1112} & 0 & 0 & 0 & \frac{1}{2}(c_{1111} - c_{1122}) \end{vmatrix} \quad (3.7)$$

The thermodynamic theories state that the matrix (3.7) should be symmetric. Thus the predictions of the thermodynamic theories, beyond what are consequences of material symmetry alone, are that

$$c_{1112} = c_{3123} = 0 \quad \text{and} \quad c_{1133} = c_{3311}. \quad (3.8)$$

Suppose now that the material considered has reflectional symmetry in the x_1x_2 -plane in addition to the rotational symmetries (3.1). The tensor δ_{3i} transforms into $-\delta_{3i}$ when the direction of the x_3 -axis is reversed; hence δ_{3i} is no longer an anisotropic tensor. However, $\delta_{3i}\delta_{3j}$ is invariant under this reflection, and δ_{3i} appears in (3.5) only in the combination $\delta_{3i}\delta_{3j}$. Thus reflectional symmetry in the x_1x_2 -plane imposes no new restriction on the matrix of relaxation moduli.

3b. Reflectional Symmetry.

We now consider materials which are not only symmetric under the rotations (3.1) but also under reflections in planes containing the x_3 -axis. The tensor e_{3ij} is not invariant under such reflections, but transforms into $-e_{3ij}$. The combination $e_{3ij}e_{3kl}$ is invariant, but as (3.4) shows, it may be expressed in terms of Δ_{1j} . Thus the form of the matrix of relaxation moduli in the present case is obtained from (3.5) by omitting those terms which involve e_{3ij} . This amounts to setting

$$\alpha_6 = \alpha_8 = 0, \quad \text{or} \quad c_{1112} = c_{3123} = 0, \quad (3.9)$$

leaving only six independent relaxation moduli, of which two (c_{1133}, c_{3311}) must be equal if the thermodynamic theories are correct.

It is, perhaps, worth noting that the thermodynamic theories predict the same five independent functions in both types of material discussed in §3a and §3b, and thus make no distinction between materials which have a "handedness" and those which have not.

4. Cubic Symmetry.

For a material with three mutually perpendicular planes of symmetry, let the coordinate system x be chosen in such a way that the symmetry planes are coordinate planes. Then the symmetry transformations (2.2) have diagonal matrices $\|a_{ij}\|$ with diagonal components $(-1,1,1)$, $(1,-1,1)$ and $(1,1,-1)$ respectively. Equation (2.7) then implies that the function c_{ijkl} is zero if some index 1,2 or 3 appears an odd number of times among i,j,k and l . Thus the matrix of relaxation moduli is of the form

$$\begin{vmatrix} c_{1111} & c_{1122} & c_{1133} & 0 & 0 & 0 \\ c_{2211} & c_{2222} & c_{2233} & 0 & 0 & 0 \\ c_{3311} & c_{3322} & c_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{3131} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{1212} \end{vmatrix}, \quad (4.1)$$

i.e. the symmetry of the material allows twelve independent relaxation moduli. The matrix symmetry predicted by the thermodynamic theories reduces this number to nine by imposing the additional requirements:

$$c_{1122} = c_{2211}, \quad c_{1133} = c_{3311}, \quad \text{and} \quad c_{2233} = c_{3322}. \quad (4.2)$$

When a plate is formed by rolling in two perpendicular directions, the resulting material symmetry may be cubic symmetry with two equivalent directions. It was assumed above that the x_1 , x_2 and x_3 directions are non-equivalent. If the x_1 and x_2

directions are equivalent, the following further relations will hold amongst the relaxation moduli (4.1):

$$\begin{aligned} c_{1111} = c_{2222}, \quad c_{1122} = c_{2211}, \quad c_{1133} = c_{2233}, \quad c_{3311} = c_{3322}, \\ c_{2323} = c_{3131}. \end{aligned} \quad (4.3)$$

The matrix of relaxation moduli then has a form identical with that of a transversely isotropic material with reflectional symmetry, except that c_{1212} is not expressed in terms of c_{1111} and c_{1122} .

If the symmetry of the material is precisely that of a cube, so that the x_1 , x_2 and x_3 directions are equivalent, then symmetry requires that

$$\left. \begin{aligned} c_{1122} = c_{2211} = c_{1133} = c_{3311} = c_{2233} = c_{3322}, \\ c_{1111} = c_{2222} = c_{3333}, \\ c_{2323} = c_{3131} = c_{1212}. \end{aligned} \right\} \quad (4.4)$$

and

The thermodynamic theories cannot be tested in such materials, since the symmetry of the material is sufficient to make the matrix (4.1) symmetric.

5. Creep Tests.

Since the arguments in the preceding sections hold irrespective of whether the c_{ijkl} or s_{ijkl} are considered, the results obtained for the relaxation moduli may be immediately interpreted for the creep compliances by replacing the c by s . For example, the compliance matrix for a cubic material with equivalent x_1 and x_2 directions is

$$\begin{pmatrix} s_{1111} & s_{1122} & s_{1133} & 0 & 0 & 0 \\ s_{1122} & s_{1111} & s_{1133} & 0 & 0 & 0 \\ s_{3311} & s_{3311} & s_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{2323} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{1212} \end{pmatrix} \quad (5.1)$$

If $2s_{1212} = s_{1111} - s_{1122}$, (5.1) is also the form of the compliance matrix for a transversely isotropic material of the type discussed in §3b. There is some experimental evidence which suggests that some polymer films produced by drawing have this type of transverse isotropy with the axis of rotational symmetry in the plane of the film (Pinnock and Ward [13]). The symmetry of the matrix for this material may be confirmed or refuted experimentally by performing creep tests in simple extension parallel and perpendicular to the axis of rotational symmetry. The application of a constant stress $\sigma H(t)^*$ in the x_3 -direction, and zero stress in all others, gives from (1.6)

* $H(t)$ is the Heaviside step function, zero for negative argument, unity for positive argument.

and (5.1),

$$\epsilon_{11}(t) = \epsilon_{22}(t) = \sigma s_{1133}(t); \quad (5.2)$$

thus measurement of the resulting creep in either of the transverse directions immediately determines the creep compliance function s_{1133} . Similarly, measurement of the creep in the axial direction when a uniaxial stress is applied in either transverse direction will give s_{3311} directly. Hence a direct comparison will show whether or not s_{1133} is equal to s_{3311} .

If experimental errors are too large to distinguish a possible difference between the two creep compliances $s_{1133}(t)$ and $s_{3311}(t)$, the following visual experiment may be adequate. Consider the initial configuration of two long plates A and B of equal width, joined together along one edge as shown in Fig. 1. Let the y-axis be along this edge, the z-axis perpendicular to the plane of the plates and the x-axis perpendicular to both the y-axis and the z-axis. The two plates differ in that their axes of rotational symmetry are the x and y axes respectively. The plate is considered sufficiently thin for plane stress conditions to be assumed. If the system is now deformed by a uniaxial stress $\sigma_{xx} = \sigma H(t)$ applied to the ends of the strip, the deformation in the regions far from the join will be approximately homogeneous. In these two regions the lateral strain ϵ_{yy} will then be given by

$$\epsilon_{yy} = \begin{cases} \epsilon_{22} = \sigma s_{1133}(t) & \text{in A,} \\ \epsilon_{33} = \sigma s_{3311}(t) & \text{in B.} \end{cases} \quad (5.3)$$

Thus if the compliance matrix is symmetric, the y-dimensions of the two plates (or films) will be equal in the deformed state and the join at $x=0$ will remain a straight line. However, if the matrix is non-symmetric, the y-dimension of the deformed system will vary between the two regions of homogeneous deformation, as in Fig. 2. Since the shear stresses on the boundary are zero, equations (1.6) and (5.1) show that the shear strains must also be zero there. In particular, they must be zero at C and C', and thus the join will remain perpendicular to the stress-free boundaries at those points. Hence the join will be curved in the x-y plane. This curvature should be easily observable and reproducible if it exists.

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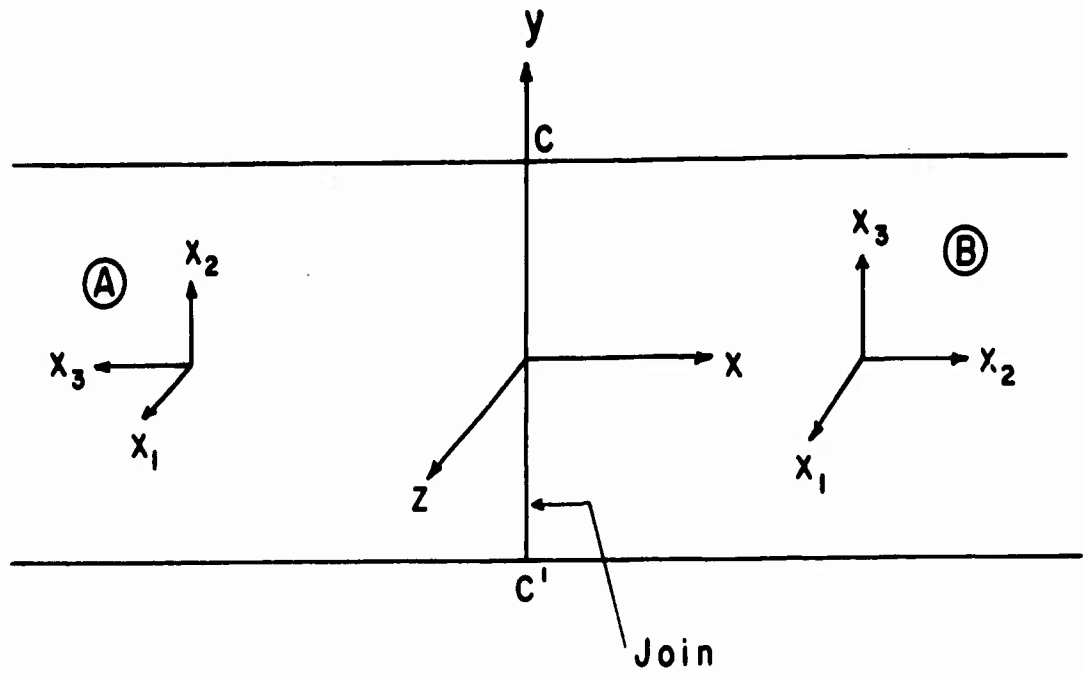


Fig.1 Two plates with mutually orthogonal symmetry axes (x_3 -axes) joined along the edge CC' .

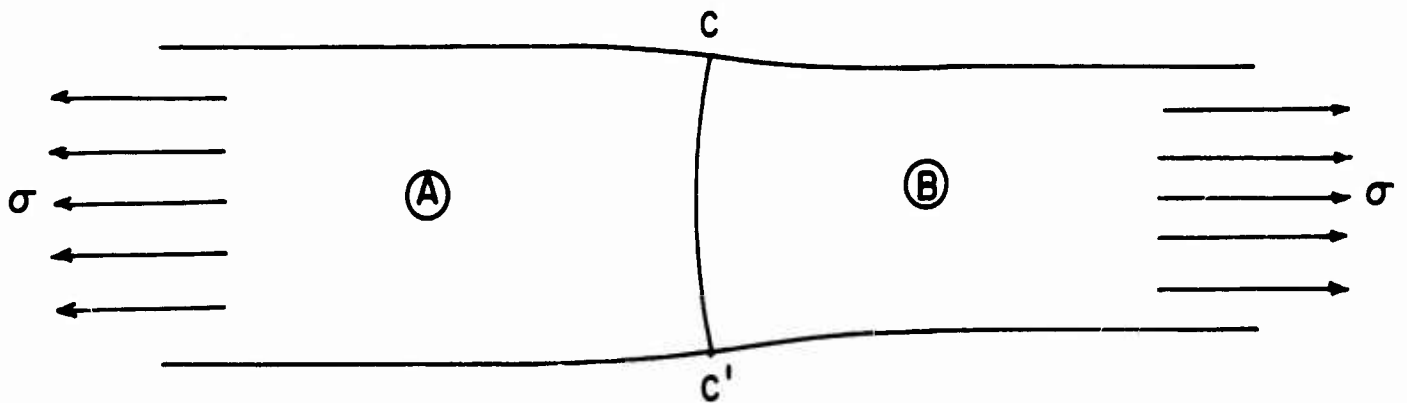


Fig.2 Deformation under uniaxial stress σ when the compliance matrix is not symmetric

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