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By

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EFFECT OF RADIAL FLOW BETWEEN THE ROTATING DISC AND HOUSING ON THEIR RESISTANCE AND HEAT TRANSFER

L. A. Dorfman

The problem under consideration was set up in connection with a study of air cooling of gas turbine rotors by a radial flow and also of their liquid cooling by means of shields placed on the ends [-4]. It is necessary to investigate the effect of a flow on the hydrodynamics of a stream close to a disc rotating in the housing when studying the operation of centrifugal pumps, compressors, turbines, etc. [5].

Certain results of the theoretical investigation of these problems are cited below and a comparison is made with known experimental data.

1. Solution of the basic equation. Let us examine the equation of momentum for a stream close to a disc rotating in the housing (Fig. 1) in the presence of a radial flow

$$\frac{d}{dr}\left\{r^2\int_0^s v_r v_\varphi\, dy\right\} = \frac{r^2}{\rho}\left(\tau_\varphi|_s - \tau_\varphi|_0\right) \qquad (1.1)$$

Here v_r, v_φ denote respectively the radial and peripheral components of velocity. The magnitudes of the friction on the disc $\tau_\varphi|_0$ and on the housing wall can be presented in the form

$$\tau_\varphi|_s = \rho v_{s*}^2, \qquad \tau_\varphi|_0 = \rho v_{0*}^2 \qquad (1.2)$$

Let us introduce the average discharge peripheral velocity in the gap v_{φ_0}, so that

$$2\pi\int_0^s v_r v_\varphi \, dy = v_{\varphi 0} 2\pi \int_0^s v_r \, dy = v_{\varphi 0}\frac{Q}{r} \qquad (1.3)$$

where Q is the per second volume discharge of the fluid in the gap. Then Eq. (1.1) will have the form

$$Q\frac{d}{dr}(rv_{\varphi 0}) = 2\pi r^2 (v_{s*}^2 - v_{0*}^2) \qquad (1.4)$$

The known experimental data [2, 4] show that for sufficiently small relative gaps $s:r_1$ and for discharges through the gap the values of v_{φ_0} are close to the magnitude of the peripheral velocity v_φ in the middle of the gap

$$v_{\varphi 0} = v_\varphi |_{y=s/2} \qquad (1.5)$$

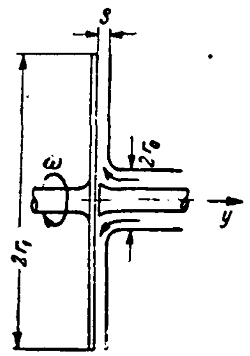

Fig. 1. Diagram of the problem.

It is necessary to note that the approximate solution of Eq. (1.1) for a turbulent regime was first derived by A. A. Lomakin [5]. However, in this case the values of the coefficients of the local resistance of the disc and housing were taken as constant along the radius regardless of the number $R = r^2\omega/\nu$; their numerical values were taken from the appropriate experimantal data for a disc in the absence of a discharge flow.

In order to obtain a more accurate solution we will use the known regularities of a turbulent flow which were obtained while studying streams in tubes and along a flat wall. As is known the use of these regularities for calculating flows around rotating bodies yields results which agree well with experimental data [6].

Experiments with rotating discs in the presence of a radial flow in the gap show [2] that there is no distinctly outlined flow core and we cannot speak about the existence of separated boundary layers with an intermediate potential core. Therefore, we will assume that starting with an initial radius r_o, the boundary layer at a fixed wall closes with the boundary layer on the disc.

Taking into consideration the comments made above we can represent the velocity profiles in the form of an exponential law: close to the rotating disc

$$\frac{r\omega - v_\varphi}{v_{0*}} = A\left(\frac{v_{0*}y}{\nu}\right)^m \qquad (1.6)$$

close to the housing wall

$$\frac{v_\varphi}{v_{s*}} = A\left[\frac{v_{s*}(s-y)}{\nu}\right]^m . \qquad (1.7)$$

Here, as is known [7],

$$A = 8.74, \quad m = \frac{1}{7} \text{ when } \quad R = \frac{r\omega s}{2\nu} \leqslant 10^5$$

Starting from the assumption (1.5) we will find the relation beteeen $v_{\varphi o}$, v_{o*} and v_{s*}

$$r\omega - v_{\varphi 0} = A\left(\frac{v_{0*}s/2}{\nu}\right)^m v_{0*}, \qquad v_{\varphi 0} = A\left(\frac{v_{s*}s/2}{\nu}\right)^m v_{s*} \qquad (1.8)$$

hence, we obtain

$$v_{0*}^2 = \left[\frac{r\omega - v_{\varphi 0}}{A}\nu^m\left(\frac{2}{s}\right)^m\right]^{\frac{2}{1+m}}, \qquad v_{s*}^2 = \left[\frac{v_{\varphi 0}}{A}\nu^m\left(\frac{2}{s}\right)^m\right]^{\frac{2}{1+m}} \qquad (1.9)$$

We introduce the designation: $z = v_{\varphi 0} / r\omega,\ x = r / r_0$; after substitution of (1.9) into (1.4) we obtain

$$\frac{d}{dx}(zx^2) = \beta x^{\frac{13}{4}}\left[(1-z)^{\frac{7}{4}} - z^{\frac{7}{4}}\right] \text{ when } m = \frac{1}{7}, \quad A = 8.74 \qquad (1.10)$$

The magnitude

$$\beta = 0.0268\, K_v\, R_1^{-1/4}\left(\frac{r_1}{s}\right)^{5/4}\left(\frac{r_0}{r_1}\right)^{11/4} \qquad \left(K_v = \frac{\omega r_1}{v_{r_1}} = 2\pi r_1^2 s\frac{\omega}{Q},\ R_1 = \frac{r_1^2\omega}{\nu}\right) \qquad (1.11)$$

is denoted in terms of β.

Here r_0 is the starting radius and r_1 is the final radius.

The value of β shows on what parameters depends the solution of the problem (it also depends on the condition at the starting radius $x = 1$): when the medium is sent to the disc, as shown in the diagram in Fig. 1, $z = 0$ when $x = 1$; if the medium is sent to the gas along the rotating shaft, $\underline{z}$ is close to unity.

From Eq. (1.10) we directly obtain the solutions for the limiting cases:

1) in the absence of a discharge $\beta = \infty$, so that

$$(1 - z)^{7/4} = z^{7/4}, \qquad z = 0.5$$

2) at very large discharges $K_V \to 0$, then

$$zx^2 = \text{const}, \qquad z \equiv 0 \text{ when } z|_{x=1} = 0$$

The result of numerical integration of Eq. (1.10) when $z_0 = z_{x=1} = 0$ is shown in Fig. 2.

The approximate solution of Eq. (1.10) can also be obtained if we note that

$$(1 - z)^{7/4} - z^{7/4} \approx 1 - 2z \qquad \text{for } 0 < z < 0.5$$

then the solution of the equation for an arbitrary starting condition $z = z_0$ (when $x = 1$) will have the form

$$z(x) = z^{\circ}(x) + z_0 \frac{1}{x^2} \exp\left[\frac{8}{11}\beta\left(1 - x^{11/4}\right)\right] \qquad (1.12)$$

where $z^{\circ}(x)$ is the solution for the starting condition $z_0 = 0$,

$$z^{\circ}(x) = \frac{\beta}{x^2} \exp\left[\frac{8}{11}\left(1 - x^{11/4}\right)\right] \int_1^x x^{11/4} \exp\left[\frac{8}{11}\beta\left(x^{11/4} - 1\right)\right] dx \qquad (1.13)$$

Calculation by this equation yields a result close to the calculation by Eq. (1.10) (Fig. 2).

Using Fig. 2 and Formula (1.12) we can find the values of $z(x)$ for any z_0.

Formula (1.12) shows that the effect of the initial twist abruptly decreases with distance from the starting radius; this effect also decreases with an increase of β.

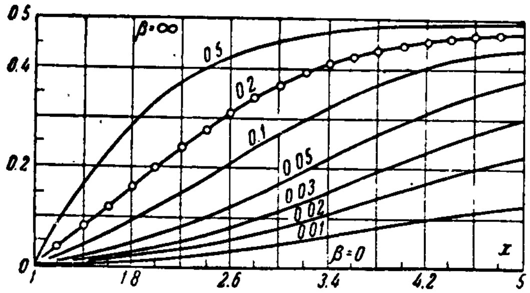

Fig. 2. The dependence of the relative twist of flow $z^o = v\varphi^o/r\omega$ in the middle of the gap on $x = r/r_o$ for different values of the parameter β at different relative distances from the axis of rotation (calculation by Formula (1.10)); the twist at the starting radius $x_1 = 1$ is 0 ($z_o = 0$, $_o$ is calculated by Formula (1.5).

2. Resistant moment. Comparison of the calculations with experimental data. Having determined the values of z (x), we can calculate the friction and resistant moment M of the disc surface being blown over

$$M = 2\pi \int_{r_0}^{r_1} r^2 \tau_\varphi |_0 \, dr$$

By means of Formulas (1.2) and (1.9) when m = 1/7 we will obtain for the coefficient of the resistant moment

$$c_M = \frac{M}{{}^1/_2 \rho r_1{}^5 \omega^2} = 0.337 x_1{}^{-11/4} R_1{}^{-1/4} \left(\frac{r_1}{s}\right)^{1/4} \int_1^{x_1} x^{11/4} (1 - z)^{10/4} dx \qquad (2.1)$$

The result of the calculations of c_m when $z_o = 0$ is shown in Fig. 3. Let us compare the results obtained above with the experimental data. The calculated velocity distributions on the gap derived from Formulas (1.6) and (1.7) are compared in Fig. 4 with the experimental [2]. As we see, as the discharge increases the agreement between the calculations and the experiment deteriorates. The agreement also deteriorates when the relative gap increases. In addition to the causes indicated in section 1, the nonuniformity of the air distribution throughout the section at the starting radius can also have a substantial effect.

Figure 5 shows the calculated and experimental [4] values of the relative magnitude of the peripheral velocities in the middle of the gap.

It is necessary to note that the initial twist was not measured; the estimate based on measurements of two relative radii x = 1.3 and 1.915 yields a magnitude $z_0 \approx 0.25$.

When $\beta = \infty$, i.e., in the absence of a discharge flow, the experimental values are close to 0.37 instead of 0.5 as calculated, which is due to the effect of the cylindrical rim of the housing and rim of the disc (see [6]).

We see from the graphs in Fig. 5 that the effect of the rims of the housing and disc decreases with an increase in discharge. Figure 5 also illustrates the fact that the effect of the initial twist decreases with an increase in β and with distance from the starting radio.

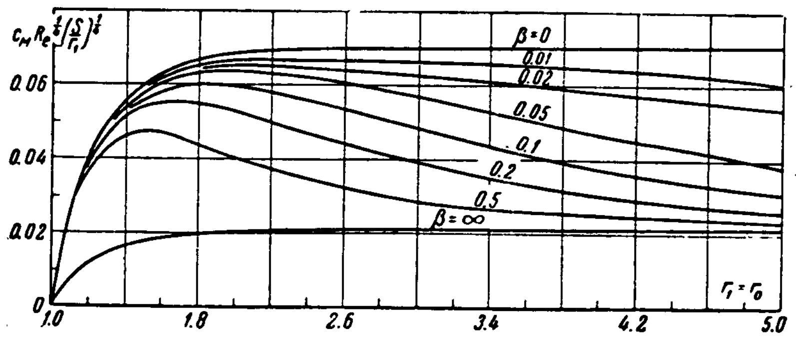

Fig. 3. Dependents of the coefficient of the resistant moment c* $= c_M R_1^{1/4} (s/r_1)^{1/4}$ of the rotating disc on $r_1 : r_0$; calculated by Formula (21)

The increase in the resistant moment of a rotating disc with an increase of the discharge of the medium through the gap was first investigated by V. S. Sedach [2]. Figure 6 shows the increase in the resistant moment of a disc rotating in air; the calculated values are plotted in the same place. In spite of the large magnitude of the relative gap there is a satisfactory agreement between the calculation and experiment. The increase in divergence with an increase of discharge can be explained by the fact that, due to the large gap during the feed of air through the central tube the picture of flow approaches the case of an air stream feed to the center of the rotating disc.

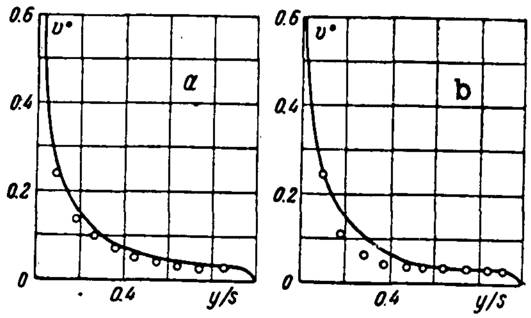

Fig. 4. Distrubution of peripheral velocities $v^{o} = v_{\varphi} / r\omega$ in the gap between the rotating disc and housing; the circles are the experimental data of V. S. Sedach [2], the curves are the calculated; in this case $R_1 = 10^6$, $s/r_1 = 0.055$; a) $K_v = 13(\beta_v = 0.0083)$; b) $K_v = 9(\beta_v = 0.00645)$.

The data of V. S. Sedach and A. N. Nespela [8] obtained for water differ substantially from the calculated and experimental values obtained for air. According to their opinion there is an effect of the mass forces which are characterized by the Galilei number $G = gr_1^3/\nu^2$.

3. Heat transfer of the disc. To calculate the radial cooling of a gas turbine disc rotating in a narrow chamber we must find the effect of the discharge on the heat-transfer coefficient of the disc.

At a Prandtl number value of P = 1 and a quadratic distribution of the mean temperature differences between the disc and the housing, as shown by the author [6], there occurs a similarity of the temperature distribution in the gap to the distribution of the peripheral velocities

$$(T^\circ - T_s) / (T_0 - T_s) = z \qquad (3.1)$$

Here T_s is the temperature at a fixed wall, T_o is the temperature of the disc, T^o is the temperature in the middle of the gap.

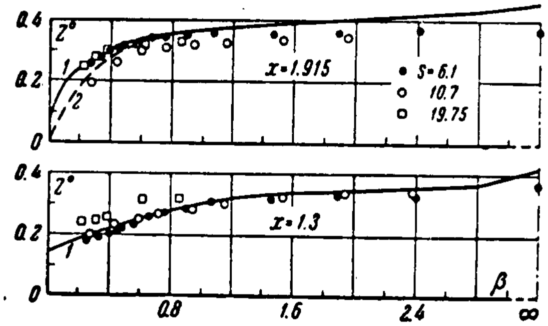

Fig. 5. Magnitude of the angle of the relative twist of flow $z^o = v_\varphi^o / r\omega$ in the middle of the gap on two radii; 1) calculation at $z_o = 0.25$; 2) calcuation at $z_o = 0$; experimental circles are based on the data of B. P. Mironov [4] (r_o = 117 mm)

The heat flux q from a unit of disc surface equals

$$q = c_p \tau_\varphi |_0 (T_0 - T_s) / r\omega \qquad (3.2)$$

By means of these formulas we can calculate the dimensionless coefficient of local heat transfer at radius r (Nusselt number)

$$N = qr / (T_0 - T^\circ) \lambda \qquad (3.3)$$

Having substituted $\tau_\varphi |_o$ from (1.2) into (3.2), we obtain

$$N = 0.0268 (1 - z)^{7/4} R^{3/4} (r / s)^{1/4} \qquad (3.4)$$

For the average (index m) dimensionless coefficient of heat transfer N_m we derive

$$N_m = \frac{q_m r_1}{(T_0 - T^\circ)_m \lambda}$$

Here

$$q_m = \frac{2\pi}{\pi (r_1^2 - r_0^2)} \int_{r_0}^{r_1} qr\,dr, \qquad (T_0 - T^\circ)_m = \frac{2\pi}{\pi (r_1^2 - r_0^2)} \int_{r_0}^{r_1} (T_0 - T^\circ)\, r dr$$

Taking into consideration that $(T_0 - T^o) \sim r^2$, we obtain finally

$$\frac{N_m}{R_1^{3/4}} \left(\frac{s}{r_1}\right)^{1/4} = 0.0268 \left(\int_1^{x_1} (1 - z)^{1/4} x^{11/4}\, dx\right) \Big/ x_1^{3/4} \int_1^{x_1} (1 - z)\, x^3\, dx \qquad (3.5)$$

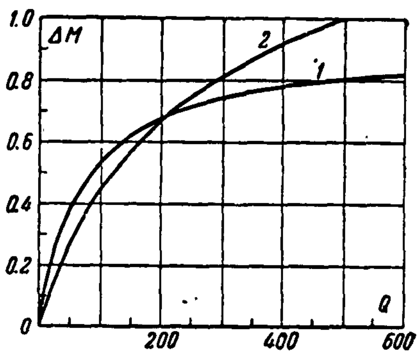

Fig. 6. Increase in the resistant moment ΔM kg cm of a rotating disc at different discharges Q kg/hr of air through the gap between the disc and the housing; 1) calculation by Formula (2.1); 2) from the experiment of V. S. Sedach [2] ($R_1 = 10^6$, $s/r_1 = 0.1$, $r_1 = 0.2$ m, $r_1/r_0 = 4$)

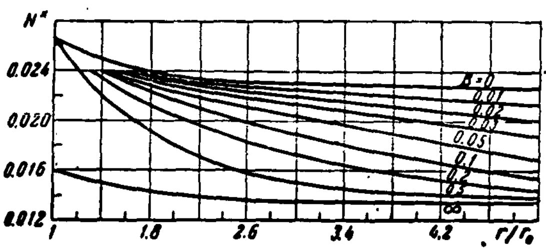

Fig. 7. Heat transfer of disc $N^* = N_m R^{-3/4} (s/r_1)^{1/4}$, rotating in the housing at P = 1 and a quadratic distribution of the mean temperature differences (calculation by Formula (3.5))

Figure 7 shows the result of rough calcuation by this formula.

Heat transfer of discs of gas turbines with radial cooling was investigated by V. M. Kapinos [3]. Although he measured the heat transfer of the disc not by calorimetry and the heat transfer coefficient found are in a certain sense conditional, it is nevertheless advantages to compare them with the calculation. In the experiments the distribution of the mean temperature differences had a quadratic character, therefore we can use directly Fig. 7 if we still take into account the effect of P number. This effect can be determined approximately from the data for a free disc [6] with the help of the multiplier $P^{0.6}$.

For the A series of experiments a good agreement with the calculated value is obtained (Fig. 8). On doubling the axial gap (A_d series with a thin disc) the experimental values substantially exceed those calculated which, in addition to making the air distribution worse, is associated mainly with the conditionality of the experimental coefficient N_m.

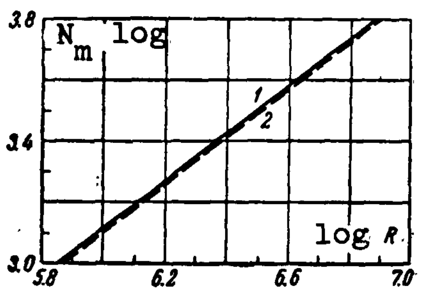

Fig. 8. Comparison of the heat coefficient of a rotating disc; 1]) based on the experiments of V.M. Kapinos (series A); 2) based on formula (3.5) (r_1 = 0.32 m, s = 10mm, K_v = 2, x_1 = 3.8)

The experiments confirmed the calculated character of dependence N_m on $x_1 = r_1/r_0$ and on K_v. For example a change of x_1 from 2 to 4 (for β = 0.2) yields, according to Fig. 7, a drop by 24.5%, and according to the experiments $N_m(\sim x_1^{-0.3})$ decreases by 23%.

Further when $R_1 = 10^6$, $r_1/s = 32$, $x_1 = 3.8$, a fourfold increase in K_v (from 2to 8) yields, according to the calculation, a decrease of N_m (by 8%) and a decrease (by 14%) is also obtained from the experiments.

Therefore, the calculated values are in complete agreement with the experimental. For a more accurate determination or the agreement between them, we must measure carefully the heat transfer coefficient of the disc by direct calorimetry.

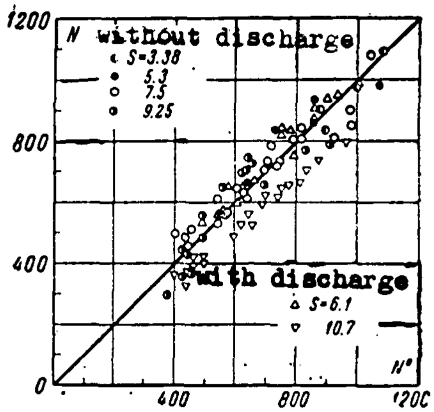

Fig. 9. A comparison of the calculated N and the experimental N^0 of the local heat transfer coefficient; calculation is based on formula (4.20) and the experiments are based on the data of B. P. Mironov for two radii r = 0.206 m and r = 0.243 m (r_0 =117 m)

4. Heat reception and Shields.

The heat flux q_s at the housing wall when P = 1 and a quadratic distribution of the mean temperature differences along the radius is determined in the same manner as that done to the disc:

$$q_s = c_p \tau_\varphi |_s (T_0 - T_s)/r\omega \qquad (4.1)$$

Then taking into account (3.1) and by means of (1.2) and (1.9) we obtain

$$N_s = 0.0268\, z^{4/5} R^{4/5} (r/s)^{1/5} \qquad (4.2)$$

The extensive data on the heat reception of shields in the presence of a discharge flux were obtained by B. P. Mirono [4]. A comparison of the experimental and a calculated values of local coefficients N is made in Fig. 9. For this purpose the calculation was performed by Formula (4.2) without consideration of the effect of the P number and the radiation exchange was not excluded from the experimental value. Because the effect of the P number can be taken into account approximately by the multiplier $P^{0.6} = 0.7^{0.6} = 0.806$ and the radiant heat based on the experiments was about 10%, we can consider that there is a satisfactory agreement between the experimental and calculated data. On an increase of the gap, as is apparent, the calculated values begin to lag behind the experimental. The same result is obtained with an increase of the discharge, particularily when $K_v < 20$.

In conclusion we will cite another formula for the average coefficient of the heat reception of the shield; it has a form similar to Formula (3.5)

$$\frac{N_m}{R^{4/5}}\left(\frac{s}{r_1}\right)^{1/4} = 0.0268\left(\int_1^{x_1} z^{7/4}x^{19/4}\,dx\right)\Big/\left(x_1^{7/4}\int_1^{x_1} zx^3\,dx\right) \quad (4.3)$$

We note that analogous calculations can be performed in the presence of a discharge flow in the gap between the rotating disc and housing from the peripheral radius toward the center of the disc.

REFERENCES

1. I. T. Shvets, Ye. P. Dyban. Air cooling of gas turbine rotors. Izd. Kiev Un., 1959.

2. V. S. Sedach. Kinematics of the flow of air cooling a gas turbine disc. Tr. KhPI, 1957, Vol. 24, No. 6.

3. V. M. Kapinos. Heat transfer of gas turbine disc with air cooling. Tr. KhPI, 1957, No. 6.

4. B. P. Mironov. Ingestigation of convection heat exchange on screens installed at the ends of a rotating disc. Izv. AN SSSR, OTN, Energetika i Avtomatika, 1960, No. 3.

5. A. A. Lomakin. Axial pressure in centrifugal pumps with consideration of the size of the gap in the packing rings. Sov. Kotloturbostroyeniye, 1940, No.12.

6. L. A. Dorman. Hydrodynamic resistance and heat exchange of rotating bodies. Fizmatgiz, 1960.

7. L. G. Loytsyanskiy. Fluid and gas mechanics. Fizmatgiz, 1959.

8. V. S. Sedach, A. N. Nespela. The effect of a discharge of fluid through a gap on the resistant moment of a rotating disc. Teploenergetika, 1958, No. 12.

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