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THE STATEMENT OF THE PROBLEM OF THE STATIONARY STRUCTURE OF THE BOUNDARY LAYER OF THE ATMOSPHERE

by

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The overall problem of the distribution of wind, temperature, humidity and of the turbulence coefficient in the boundary layer of the atmosphere under stationary conditions is mathematically formulated in this article.

The results of the calculations are compared with experimental data.

When an air mass moves above a homogeneous underlying surface as a result of turbulent mixing, the determined profiles of temperature, humidity, wind velocity, and of the turbulence coefficient are established. In a stationary condition all the characterisitcs mentioned are interrelated. This connection is caused first of all by processes on the active surface, as a result of which a fixed quantity of radiant energy is distributed along three channels: to the warming of the soil and air, and to evaporation and secondly by the mechanism of turbulence itself which is characterized by the fact that the intensity of turbulent mixing and the vertical temperature gradients and the wind velocities condition each other, in so far as the turbulent energy influx due to the energy of average motion and its expenditure on work against Archimedian forces are determined by the vertical gradients of temperature and wind velocity. At the same time, the intensification or weakening of turbulent mixing exerts a rapid and a completely determined effect on the vertical profile of any substance. In this connection, the correct statement of the problem of the stationary meteorological regime of the boundary layer of the atmosphere must be the common solution of a certain system of equations for determining the temperature, humidity, wind velocity, and turbulence coefficient.

Only four equations are known to find the five unknown functions: T, u, v, k, q, where T is the air temperature, q is the specific humidity of the air, u, v, are the components of the wind velocity, k is the turbulence coefficient; they are: two equations of motion, the equation of heat conductivity, and the equation for the diffusion of water vapor. In this connection, it was necessary to carry out a significant number of investigations with the assumption that the coefficient of turbulence is a known function of height.

Although important results of a purely practical aspect were obtained in a series of works, they yielded comparatively little for theory. An explanation of the variations in the vertical profiles of meteorological elements by any changes in the intensity of turbulent exchange, which result from such investigations, is hardly satisfactory. The phenomena mentioned are connected not as cause and effect, but rather are the manifestation of a single process, and, only because of this, are related to each other. On the same basis, it is possible to explain the variations of the temperature profile by the change of turbulence, and to explain changes in the intensity of turbulent exchange by changes of the temperature gradients. A considerably more correct statement is the application to the problem of methods of the theory of similarity and of an analysis of the measurements, which has been made recently in a series of investigations. Under the present condition of the problem, this method is completely logical and has already made it possible to obtain a series of results important for understanding the mechanism of the phenomenon. Unfortunately, with a more or less complete statement of the problem, the results are indefinite.

The application of the energy balance of turbulence as a non-satisfying equation seems fully expedient to us. One can easily understand that those same unknown functions enter this equation and it is not the result of the four mentioned equations. However the application of the indicated equation has many difficulties. First of all, there are difficulties of a principle order that arise from the fact that expressions for the diffusion of turbulence energy and for its dissipation into heat are presently unknown. Important difficulties of a mathematical nature also arise with a correct statement because the system of equations in the indicated statement becomes essentially non-linear.

Let us now indicate a certain method which makes it possible to obtain a number of important results in the solution of the general problem. Let us use the fact that it is possible to approximate the turbulence coefficient as a function of height as follows:

$$k(z) = \begin{cases} k_1\left(\frac{z}{z_1}\right)^{1-\varepsilon} & \text{for } z \leqslant h, \\ k_1\left(\frac{h}{z_1}\right)^{1-\varepsilon} & \text{for } z > h. \end{cases}$$

When there is equilibrium of stratification $\varepsilon = 0$, during the steady state $\varepsilon > 0$, and during the unsteady state $\varepsilon < 0$.

First let us examine the statement of the problem for the equilibrium state. If one is to consider that for sufficiently small values of z_1

$$k(z_1) = k_1 = \frac{0{,}14 c_1 z_1}{\ln \frac{z_1}{z_0}},$$

where c_1 is the modulus of the wind velocity at height z_1, z_0 is the roughness of the underlying surface, then instead of the unknown function $k(z)$, only the height of the cleavage h is unknown. In this connection, we can present the equation of the balance of the energy of turbulence in integral form, after having integrated it with respect to the entire boundary layer. In such a case, we can disregard, because of its relative smallness, the component which takes into account the diffusion of the energy of turbulence.

As to the dissipation of the turbulence energy into heat, we can obtain a convenient formula for it by taking advantage of certain physical considerations of the dissipation mechanism. The dissipation of turbulence energy into heat is equal to the work of the forces of friction, which occurs because the elements of turbulence move with a velocity that is different than the velocity of the surrounding medium.

We shall introduce the following designations: F is the force of friction, ρ is the air density, w' is the velocity pulsation, characteristic for a certain vortex, ν is the kinematic viscosity, ℓ is the characteristic dimension of the vortex.

Obviously

$$F = F(\rho, l, w', \nu) = \rho l^2 w'^2 f\left(\frac{\nu}{l w'}\right).$$

The work of the force of friction per unit time is Fw', and the work attributed to a unit of volume and equal to the dissipation of the turbulence energy into heat, can be expressed per unit of time per unit of volume as

$$D = \frac{\rho w'^3}{l} f\left(\frac{\nu}{l w'}\right). \qquad (1)$$

All values entering into (1) are averaged. If we take advantage of the fact that

$$w' = l \sqrt{\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2}$$

and

$$k = \ell w'$$

then

$$D = \rho k \left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2\right] f\left(\frac{\nu}{k}\right) \qquad (2)$$

In such a case, the balance of the turbulence energy can be written in the following form:

$$\frac{dk'}{dt} = \left[1 - f\left(\frac{\nu}{k}\right)\right] k\rho \left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2\right] - k\rho \frac{g}{T}\left(\frac{dT}{dz} + \gamma_a\right) - M. \qquad (3)$$

M is the diffusion of turbulence energy.

Here u, v are the two components of the vector of the averaged velocity.

Having integrated equation (3) with respect to the entire boundary layer, for the given condition we will obtain:

$$\int_{z_0}^{H} (1 - f)\, k\rho \left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2 - \frac{g}{T}\left(\frac{dT}{dz} + \gamma_a\right)\right] dz = 0 - M. \qquad (4)$$

Thus the problem of finding the average profiles of the meteorological elements and of the coefficient of turbulence in the boundary layer of the atmosphere reduces to the integration of the following system of equations:

$$\frac{d}{dz}\left[k_1 F\left(\frac{z}{z_1}\right)\frac{du}{dz}\right] + 2\omega_z v = 0, \qquad (5)$$

$$\frac{d}{dz}\left[k_1 F\left(\frac{z}{z_1}\right)\frac{dv}{dz}\right] - 2\omega_z (u - U) = 0, \qquad (6)$$

$$\frac{d}{dz}\left[k_1 F\left(\frac{z}{z_1}\right)\left(\frac{dT}{dz} + \gamma_a\right)\right] + \varphi(z) = 0, \qquad (7)$$

$$(1 - \delta)\int_{z_0}^{H} k\rho \left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2 - \frac{g}{T}\left(\frac{dT}{dz} + \gamma_a\right)\right] dz = 0, \qquad (8)$$

$$\frac{d}{dz}\left[k_1 F\left(\frac{z}{z_1}\right)\frac{dq}{dz}\right] = 0, \qquad (9)$$

[illegible] (10)

$$\frac{d}{dz}(u^2 + v^2)\Big|_{z=H} = 0, \tag{11}$$

$$k_1 = \frac{0{,}14\sqrt{u^2 + v^2}\, z_1}{\ln \frac{z_1}{z_0}} \tag{12}$$

Here θ is the soil temperature, ζ is the depth measured from the surface of the soil, (1 - δ) is the averaged value of the dissipation factor, H is the height of the boundary layer.

If we are to consider that the radiant flux is equal to zero or is given in the form of some interpolated formula, and

$$F\left(\frac{z}{z_1}\right) = \begin{cases} \left(\frac{z}{z_1}\right)^{1-\epsilon} & \text{for } z < h \\ \left(\frac{h}{z_1}\right)^{1-\epsilon} & \text{for } z > h \end{cases} \tag{13}$$

then to find the eight unknowns, u, v, T, q, k, H, h, θ we obtain eight equations, the differential equations being linear.

The boundary conditions are completely clear: on the surface layer, it is necessary to fulfill the equation of the heat balance, and to give the temperature continuity and the elasticity of the water vapor; at the upper edge of the boundary layer it is natural to give the temperature, the humidity and value of the geostrophic wind, and on the ground it is necessary to give the temperature at the depths where the annual variation practically disappears. These boundary conditions provide a well-defined solution for the problem.

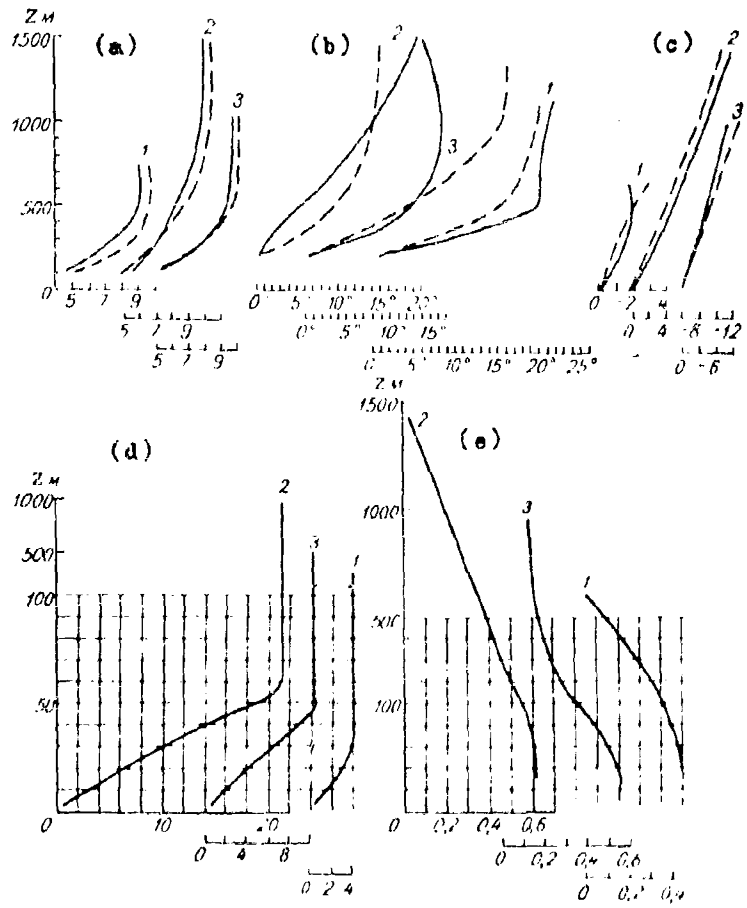
We can solve the formulated problem for a more particular, but sufficiently important case. The question is the distribution of temperature, wind velocity, and of the turbulence coefficient under different conditions of vertical stability. Then the system is closed by equations (5), (6), (7), (8), (12) and (13). In such a statement, the problem is examined in detail in the work of L. R. Orlenko and G. Kh. Tseitin (Tseitin, G. Kh. and L. R. Orlenko. "Statsionarnoe raspredelenie vetra, temperatury i turbulentnogo obmena v pogranichnom sloe vozdukha pri razlichnykh sostoianiiakh ustoichivosti" (Stationary distribution of the wind, temperature and turbulent exchange in the boundary layer of the air under various stability conditions), Glavnaia Geofizicheskaia Observatoriia Trudy, No. 94: 8-28, 1960 [AMS-T-R-387+]). The dissipation factor is determined on the basis of experimental data on the dependence of c_1/U on ε .

On the basis of the calculated profile k(z) and c(z) we can determine the wind strength by the formula

$$\sqrt{\overline{c'^2}} = l\frac{dc}{dz} = \sqrt{k\frac{dc}{dz}}\,.$$

Obviously, the average period of the gusts at different heights can be evaluated by the formula $\tau \sim 1/(dc/dz.)$

Figure 1 gives the results of the calculations of the average distribution of wind velocity and direction, and of the characteristics of strength and of the turbulence coefficient which G. Kh. Tseitin and L. R. Orlenko carried out according to the scheme cited above for roughness 1.8 cm. The mean monthly values of the wind velocity and direction and of the temperature obtained from observations in Pavlovsk are also given there. The good agreement between experimental and observed values points to the correctness of the assumed physical conceptions.



(a) - wind velocity (m/sec), (b) - wind direction (deviation from the direction of the wind at level 200 m), (c) air temperature (deviations from the surface values), (solid lines are the measured values - dashed lines are the calculated values); (d) is the coefficient of turbulence (m/sec), (e) is the average wind pulsation (m/sec); 1- is the steady state, 2- is the unstable state, 3 - is the equilibrium state.

Figure 1.

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