# THREE FLUID HEAT EXCHANGER DESIGN THEORY COUNTER- AND PARALLEL-FLOW

BY

TOR SORLIE

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STANFORD UNIVERSITY
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# THREE-FLUID HEAT EXCHANGER DESIGN THEORY
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For

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Stanford University
Stanford, California

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## ABSTRACT

A design theory for two flow arrangements of three-fluid heat exchangers has been developed. The dependent performance of the heat exchanger has been expressed in terms of two dimensionless quantities, $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$, termed temperature effectivenesses. The temperature effectivenesses are expressed as functions of five independent dimensionless exchanger variables, three representing operating conditions and two design conditions. This situation contrasts with one dependent and two independent dimensionless parameters for the two-fluid exchanger, a very much less complex problem. Graphs are presented showing $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ as functions of the five exchanger variables. The practical application of the design theory is shown in three examples.

Insight into the problems arising in designing three-fluid heat exchangers can be achieved by inspection of the temperature effectiveness curves.

TABLE OF CONTENTS

TABLE OF CONTENTS (CONT'D)

# LIST OF FIGURES

## LIST OF FIGURES (CONT'D)

## LIST OF TABLES

## NOMENCLATURE

### English Letter Symbols

| | |
|---|---|
| $A_1$ | area of one side of the heat transfer surface between the hot fluid and cold fluid No. 1, $ft^2$ |
| $A_2$ | area of one side of the heat transfer surface between the hot fluid and cold fluid No. 2, $ft^2$ |
| C | flow stream capacity rate, $(W \cdot c_p)$, Btu/(hr °F) |
| $c_p$ | specific heat at constant pressure, Btu/(lbs °F) |
| U | overall conductance for heat transfer, Btu/(hr °F $ft^2$ of A) |
| W | mass flow rate, lbs/hr |
| q | heat transfer rate, Btu/hr |

### Dimensionless Groupings

| | |
|---|---|
| $A^* \triangleq A_1/A_2$ | transfer area ratio (the area ratio between the two heat transfer surfaces) |
| $R^* \triangleq (A_2U_2)/(A_1U_1)$ | overall thermal resistance ratio (the ratio between the overall thermal resistances of the two heat transfer surfaces |
| $\Delta t^*_{in} \triangleq \frac{t_{h_{in}} - t_{c2_{in}}}{t_{h_{in}} - t_{c1_{in}}}$ | inlet cold fluid temperature ratio (the ratio between the inlet temperatures of the two cold fluids referred to $t_{h_{in}}$ as the datum) |
| $C^*_1 \triangleq C_{c1}/C_h$ | capacity rate ratio between cold fluid No. 1 and the hot fluid |
| $C^*_2 \triangleq C_{c2}/C_h$ | capacity rate ratio between cold fluid No. 2 and the hot fluid |

$Ntu_1 \triangleq (A_1U_1)/C_{c1}$ number of transfer units of the heat transfer surface between cold fluid No. 1 and the hot fluid

Greek Letter Symbols

$\epsilon_{q,o}$ overall heat exchanger effectiveness

$\epsilon_{t,c1} \triangleq \frac{t_{c1_{out}} - t_{c1_{in}}}{t_{h_{in}} - t_{c1_{in}}}$ temperature effectiveness for the heating of cold fluid No. 1

$\epsilon_{t,c2} \triangleq \frac{t_{c2_{out}} - t_{c2_{in}}}{t_{h_{in}} - t_{c2_{in}}}$ temperature effectiveness for the heating of cold fluid No. 2

Subscripts

| | |
|---|---|
| c1 | refers to cold fluid No. 1 |
| c2 | refers to cold fluid No. 2 |
| h | refers to the hot fluid |

# I. INTRODUCTION

Most processes of thermal energy recovery involve transfer of thermal energy between two fluids. However, in recent years some processes with heat transfer between three fluids have become important. One example is in air separation plants, which calls for an exchange of thermal energy between oxygen, nitrogen and air at low temperatures. Three-fluid heat exchangers also allow a more compact and economical design. Two two-fluid exchangers may, for example, be combined into a three-fluid unit with a saving in shell structure.

For two-fluid heat exchangers a large amount of material has been published on how to compute the relationship between heat transfer area and temperature difference between the fluids. Reference [4] presents one such treatment of this subject. However, there exists no general performance theory for three-fluid heat exchangers.

An approximate method of handling a three-fluid heat exchanger design problem is presented in Appendix V. This log-mean rate equation approach requires the following iteration procedure:

1. Estimate the two heat transfer rates between the hot and the two cold fluids. The outlet temperatures of the three fluids may then be calculated.
2. Calculate the two log-mean temperature differences.
3. Check initial estimate of the two heat transfer rates.
4. Repeat procedure as necessary.

The objection to this method lies in the degree of approximation involved in the use of the log-mean temperature differences.

In this report a general theory for three-fluid exchangers is developed for one flow arrangement each of parallel-flow and counter-flow. The performance of the exchanger is expressed as two temperature ratios, which are functions of five non-dimensional exchanger variables. Graphs of the performance expressions are provided for some values of the exchanger variables. Finally, some examples are given which illustrate the practical use of the theory in exchanger design.

## II. DESCRIPTION OF PROBLEM

### Three-Fluid Parallel-Flow

Figure 1 shows a schematic representation of a three-fluid parallel-flow exchanger. Heat is transferred from the hot fluid to both the cold fluids. There is no exchange of heat between the two cold fluids. The two cold fluids are numbered 1 and 2. The capacity rate of cold fluid No. 1 is $C_{c1}$ , the capacity rate of cold fluid No. 2 is $C_{c2}$ and the capacity rate of the hot fluid is $C_h$ . The capacity rate is defined as the product of the mass flow rate (lbs/hr) and the specific heat at constant pressure (Btu/lbs °F) of the fluid. $C = (W \cdot c_p)$, with units (Btu/hr °F).

The overall thermal conductance between the hot fluid and cold fluid No. 1 is termed $U_1$ , while the overall thermal conductance between the hot fluid and cold fluid No. 2 is termed $U_2$ . $U$ has units of (Btu/hr °F ft$^2$ of A). Then $U_1$ has units of (Btu/hr °F ft$^2$ of $A_1$) and $U_2$ has units of (Btu/hr °F ft$^2$ of $A_2$). The reciprocal of the overall thermal conductance $U$ is an overall thermal resistance which can be considered to have the following series components:

1. A hot side film convection component, including the temperature ineffectiveness of the extended area on this side.
2. A wall conduction component.
3. A cold side film convection component, including the temperature ineffectiveness of the extended area on this side.
4. Fouling factors to allow for scaling or fouling on both the hot and cold sides.

Reference [4], p. 8 , presents a detailed description of the method for calculating $U$ .

FIG. 1

SCHEMATIC REPRESENTATION OF A

THREE-FLUID PARALLEL-FLOW EXCHANGER
WITH TWO COLD AND ONE HOT FLUID

FIG. 2

SCHEMATIC REPRESENTATION OF A

THREE-FLUID COUNTER-FLOW EXCHANGER
WITH TWO COLD AND ONE HOT FLUID

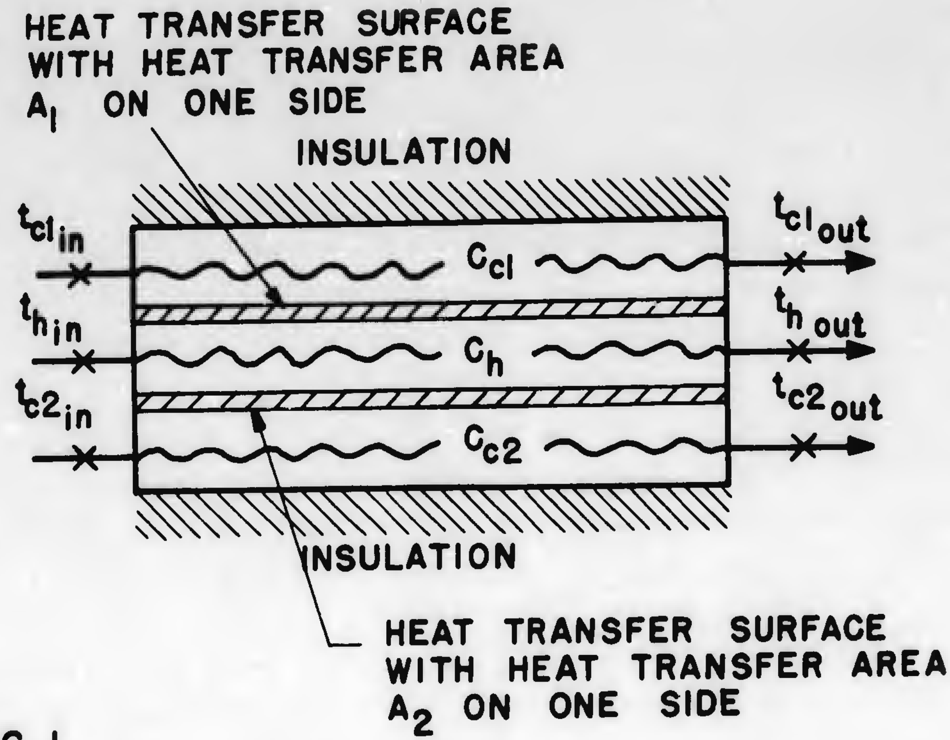

HEAT TRANSFER SURFACE
WITH HEAT TRANSFER AREA
A1 ON ONE SIDE
INSULATION
tc1 in
th in
tc2 in
Cc1
Ch
Cc2
tc1 out
th out
tc2 out
INSULATION
HEAT TRANSFER SURFACE
WITH HEAT TRANSFER AREA
A2 ON ONE SIDE

FIG. 1

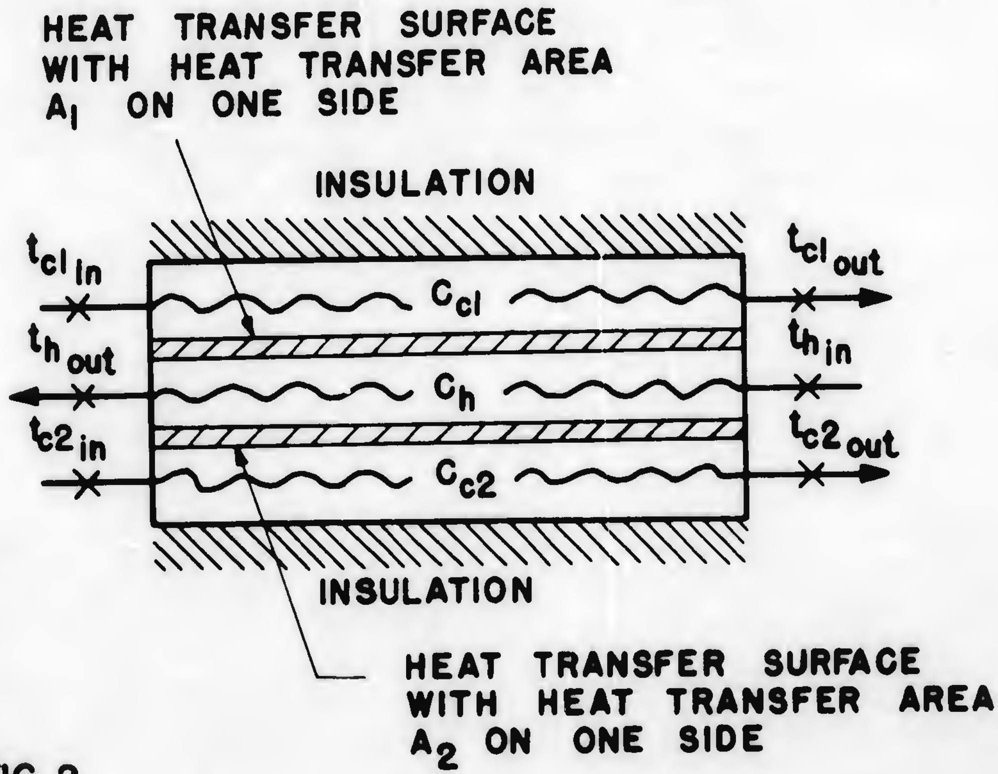

HEAT TRANSFER SURFACE
WITH HEAT TRANSFER AREA
A1 ON ONE SIDE
INSULATION
tc1 in
th out
tc2 in
Cc1
Ch
Cc2
tc1 out
th in
tc2 out
INSULATION
HEAT TRANSFER SURFACE
WITH HEAT TRANSFER AREA
A2 ON ONE SIDE

FIG. 2

## Three-Fluid Counter-Flow

Figure 2 shows a schematic representation of a three-fluid counter-flow exchanger. Similar to the parallel-flow exchanger, heat is transferred from the hot fluid to both the cold fluids, and there is no exchange of heat between the two cold fluids. The definitions of $U_1$, $U_2$, $C_{c1}$, $C_{c2}$, $C_h$, $A_1$, $A_2$ are the same as for the parallel-flow exchanger.

There are several other possibilities of flow arrangements and designs of three-fluid parallel- and counter-flow exchangers and these are illustrated in Fig. 3. These other possibilities will not be considered further in this report.

The problem is now to interrelate the heat exchanger parameters so as to produce an equation for the dependent heat exchanger performance in terms of the independent operating and design parameters. These parameters are:

$U_1, U_2$ - the overall conductances for heat transfer, (Btu/hr °F $ft^2$ of A)

$A_1, A_2$ - areas of one side of the heat transfer surface between the hot and the cold fluids, $ft^2$, the area on which $U_1$ and $U_2$ are based. (For details see ref. [4], p. 8.)

$C_{c1} \triangleq (Wc_p)_{c1}$ - cold fluid No. 1 capacity rate, (Btu/hr °F)

$C_{c2} \triangleq (Wc_p)_{c2}$ - cold fluid No. 2 capacity rate. (Btu/hr °F)

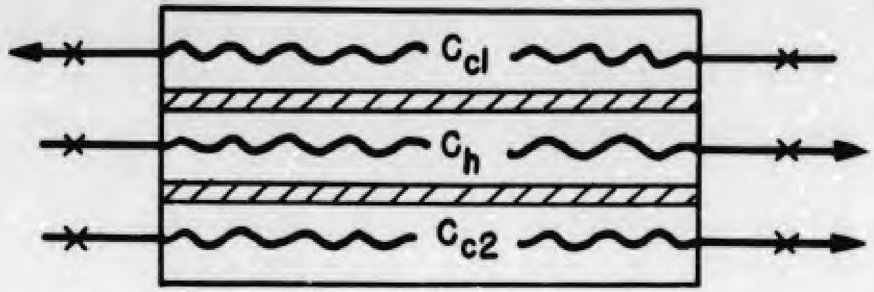
$C_h \triangleq (Wc_p)$ - hot fluid capacity rate, (Btu/hr °F)

$\left. \begin{array}{l} t_{c1_{in}} \\ t_{c1_{out}} \end{array} \right\}$ - cold fluid No. 1 terminal temperatures, °F

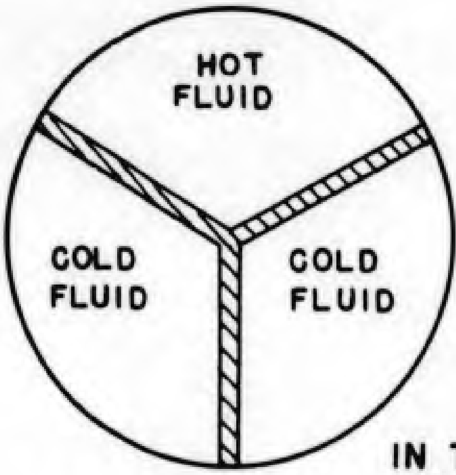
FIG. 3

OTHER POSSIBILITIES OF THREE-FLUID COUNTER-
AND PARALLEL-FLOW ARRANGEMENTS

PARALLEL-COUNTER-FLOW ARRANGEMENT




COUNTER- OR PARALLEL-FLOW ARRANGEMENT




IN THIS EXCHANGER THERE IS EXCHANGE OF HEAT BETWEEN THE TWO COLD FLUIDS

FIG. 3

$\left.\begin{array}{l} t_{c2_{in}} \\ t_{c2_{out}} \end{array}\right\}$ - cold fluid No. 2 terminal temperatures, °F

$\left.\begin{array}{l} t_{h_{in}} \\ t_{h_{out}} \end{array}\right\}$ - hot fluid terminal temperatures, °F

The outlet temperatures are dependent variables while the others are independent. The independent parameters, such as $C_{c1}$ and $t_{c1_{in}}$, are operating condition parameters, while ones, such as $A_1$ and $U_1$, are design parameters.

## III. IDEALIZATIONS

The following idealizations have been made in the analysis:

1. The heat exchangers, Figs. 1 and 2, are considered to be adiabatic, i.e., there is no heat loss to the surroundings. Also all heat exchange is from the hot fluid to the cold fluids.

2. The heat exchanger parameters $C_{c1}$ , $C_{c2}$ , $C_h$ , $U_1$ , $U_2$ are treated as constants with respect to temperature and position.

3. Perfect mixing in each passage, i.e., there is no temperature gradient normal to the flow direction.

4. Negligible longitudinal conduction in the walls or fluids.

## IV. DEVELOPMENT OF THE DESIGN THEORY

The thermodynamically limited maximum heat transfer rate is realized only in a counter-flow heat exchanger of infinite heat transfer area. Comparison of an actual heat exchanger to this infinite counter-flow exchanger will yield a useful measure of how well the performance compares with the thermodynamically limited performance of the exchanger. The overall heat transfer effectiveness of a heat exchanger can then be defined as follows.

$$\epsilon_{q,o} \triangleq \frac{\text{Actual heat transfer rate in exchanger}}{\text{Max possible heat transfer rate obtained in a counter-flow exchanger with infinite heat transfer area and same inlet temperatures and flow rates}} \triangleq \frac{q_{actual}}{q_{max}}$$

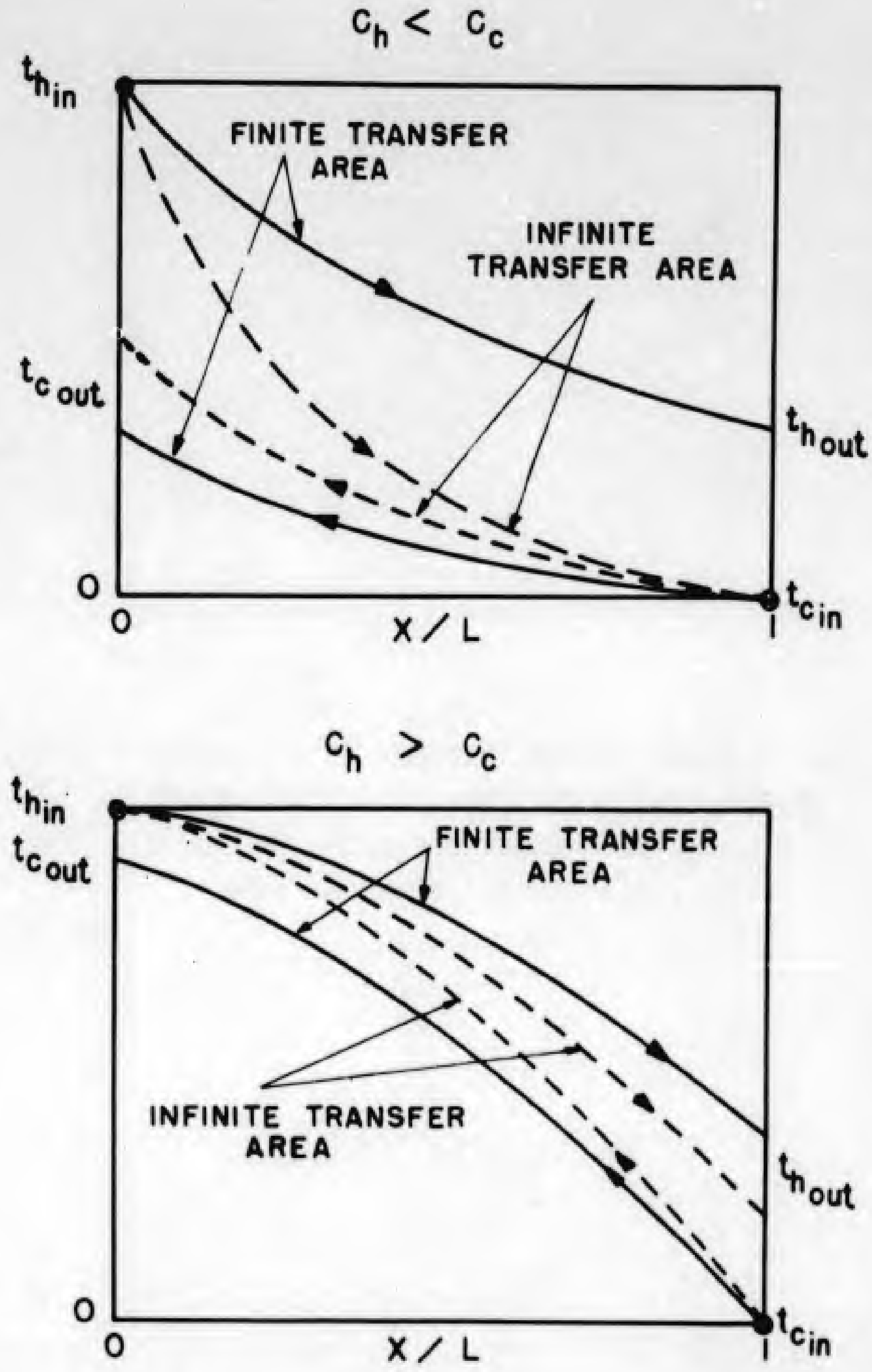
It is now necessary to derive an expression for the heat transfer rate in a three-fluid counter-flow exchanger with infinite heat transfer area.

Figure 4 describes schematically the temperature conditions in a two-fluid counter-flow exchanger with finite and infinite area. For the infinite area two-fluid exchanger with $C_h < C_c$ , $t_{h_{out}} = t_{c_{in}}$ ; and for $C_c < C_h$ , $t_{c_{out}} = t_{h_{in}}$ .

Analogous to Fig. 4, Fig. 5 describes schematically the temperature conditions in a three-fluid counter-flow exchanger. For the infinite area three-fluid exchanger with $(C_{c1} + C_{c2}) < C_h$ , $t_{h_{in}} = t_{c1_{out}} = t_{c2_{out}}$ . For the case when $(C_{c1} + C_{c2}) > C_h$ the temperature picture is more complex; the hot fluid outlet temperature lies somewhere between $t_{c1_{in}}$ and $t_{c2_{in}}$ . There exists a dynamic equilibrium condition for the hot fluid when the heat transfer rate from one of the cold fluids to the hot fluid is equal to the heat transfer rate

FIG. 4

SCHEMATIC DESCRIPTION OF THE FLUID TEMPERATURE CONDITIONS IN A TWO-FLUID COUNTER-FLOW EXCHANGER WITH FINITE AND INFINITE HEAT TRANSFER AREA



$C_h < C_c$
$t_{h_{in}}$
FINITE TRANSFER AREA
INFINITE TRANSFER AREA
$t_{c_{out}}$
$t_{h_{out}}$
0
0
X / L
1
$t_{c_{in}}$
$C_h > C_c$
$t_{h_{in}}$
$t_{c_{out}}$
FINITE TRANSFER AREA
INFINITE TRANSFER AREA
$t_{h_{out}}$
0
0
X / L
1
$t_{c_{in}}$


FIG. 4

FIG. 5

SCHEMATIC DESCRIPTION OF THE FLUID TEMPERATURE CONDITIONS IN A THREE-FLUID COUNTER-FLOW EXCHANGER WITH FINITE AND INFINITE HEAT TRANSFER AREA

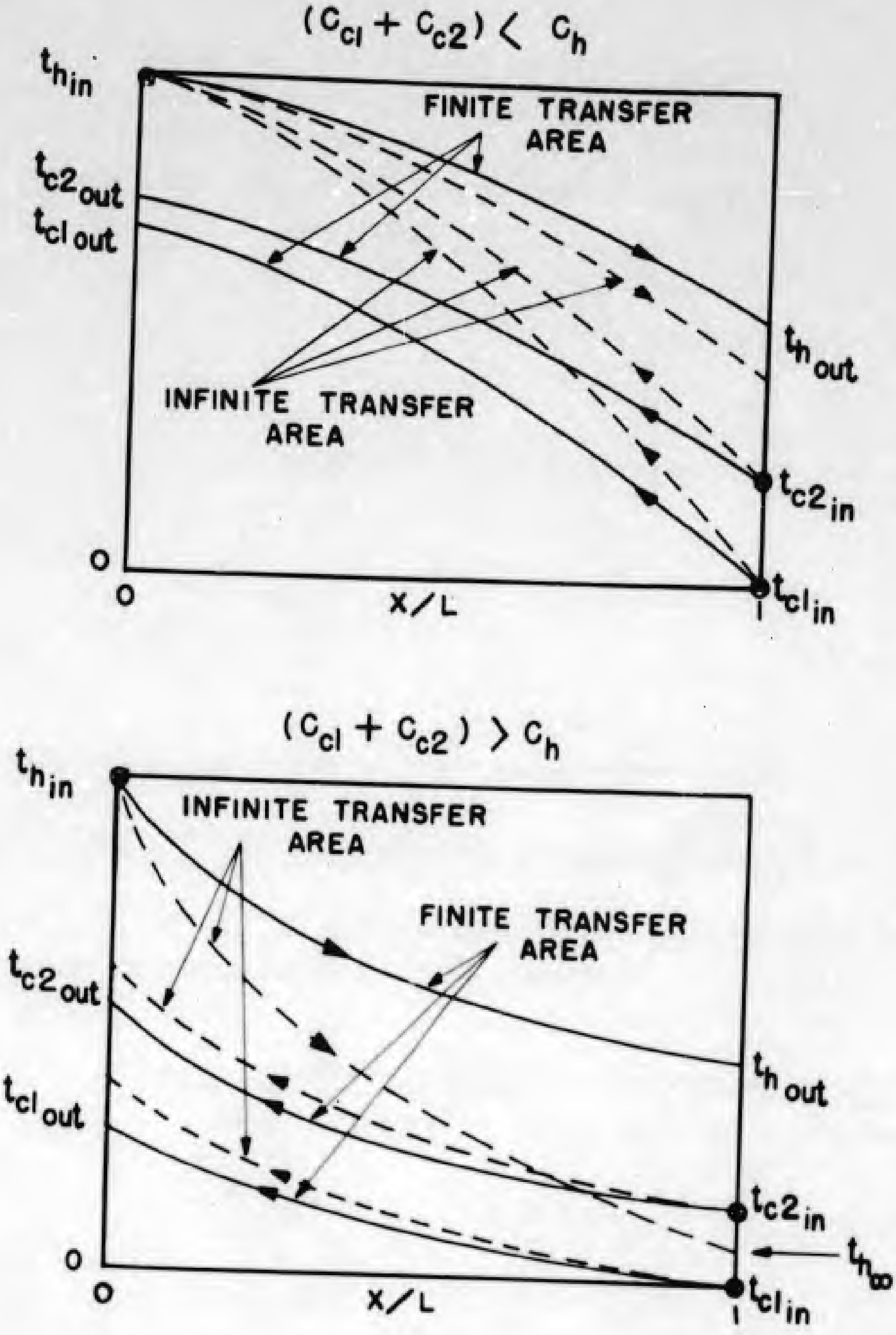

(Cc1 + Cc2) < Ch
th in
tc2 out
tc1 out
FINITE TRANSFER AREA
INFINITE TRANSFER AREA
th out
tc2 in
tc1 in
0
0
X/L
1
(Cc1 + Cc2) > Ch
th in
INFINITE TRANSFER AREA
FINITE TRANSFER AREA
tc2 out
tc1 out
th out
tc2 in
th∞
tc1 in
0
0
X/L
1

FIG. 5

from the hot fluid to the other cold fluid. This dynamic equilibrium temperature $t_{h\infty}$ is established as follows. Considering a differential element of the heat exchanger at $A = \infty$, the heat transfer rate equations can be written as

$$dq_{\text{from hot to cold fluid No. 1}} = U_1 dA_1 \, (t_{h\infty} - t_{c1_{in}})$$

$$dq_{\text{from cold No. 2 to hot fluid}} = U_2 dA_2 \, (t_{c2_{in}} - t_{h\infty})$$

Since:

$$dq_{\text{from hot to cold fluid No. 1}} = dq_{\text{from cold No. 2 to hot fluid}}$$

then:

$$U_1 dA_1 \, (t_{h\infty} - t_{c1_{in}}) = U_2 dA_2 \, (t_{c2_{in}} - t_{h\infty})$$

which yields:

$$t_{h\infty} = \frac{U_2 \, t_{c2_{in}} + U_1 \, A^* t_{c1_{in}}}{U_2 + A^* U_1} \quad \text{-----} \qquad (1)$$

where: $A^* \triangleq A_1/A_2$, the area ratio between the two heat transfer surfaces.

The overall heat transfer effectiveness expressions for three-fluid parallel- and counter-flow exchangers can now be derived.

<u>Overall Heat Transfer Effectiveness, Parallel-Flow</u>

An energy balance on the exchanger yields:

$$C_{c1}(t_{c1_{out}} - t_{c1_{in}}) + C_{c2}(t_{c2_{out}} - t_{c2_{in}}) =$$

$$C_h(t_{h_{in}} - t_{h_{out}}) = q_{actual}$$

For the case when $(C_{c1} + C_{c2}) < C_h$ (refer to Fig. 5),

$$q_{max} = C_{c1}(t_{h_{in}} - t_{c1_{in}}) + C_{c2}(t_{h_{in}} - t_{c2_{in}})$$

and:

$$\epsilon_{q,o} = \frac{q_{actual}}{q_{max}} = \frac{C_{c1}(t_{c1_{out}} - t_{c1_{in}}) + C_{c2}(t_{c2_{out}} - t_{c2_{in}})}{C_{c1}(t_{h_{in}} - t_{c1_{in}}) + C_{c2}(t_{h_{in}} - t_{c2_{in}})}$$

The following definitions are now introduced:

$$\epsilon_{t,c1} \triangleq \frac{t_{c1_{out}} - t_{c1_{in}}}{t_{h_{in}} - t_{c1_{in}}}$$

$$\epsilon_{t,c2} \triangleq \frac{t_{c2_{out}} - t_{c2_{in}}}{t_{h_{in}} - t_{c2_{in}}}$$

$$\Delta t^*_{in} \triangleq \frac{t_{h_{in}} - t_{c2_{in}}}{t_{h_{in}} - t_{c1_{in}}}$$

Then the overall heat exchanger effectiveness is obtained for the case of $(C_{c1} + C_{c2}) < C_h$

$$\epsilon_{q,o} = \frac{\left[\frac{C_{c1}}{C_{c2}} \cdot \epsilon_{t,c1} + \Delta t^*_{in} \cdot \epsilon_{t,c2}\right]}{\frac{C_{c1}}{C_{c2}} + \Delta t^*_{in}} \quad - - - - \quad (2)$$

For the other case, when $(C_{c1} + C_{c2}) > C_h$ (refer to Fig. 5),

$$q_{max} = C_h(t_{h_{in}} - t_{h\infty})$$

$$q_{actual} = C_{c1}(t_{c1_{out}} - t_{c1_{in}}) + C_{c2}(t_{c2_{out}} - t_{c2_{in}})$$

$$\epsilon_{q,o} = \frac{q_{actual}}{q_{max}} = \frac{C_{c1}(t_{c1_{out}} - t_{c1_{in}}) + C_{c2}(t_{c2_{out}} - t_{c2_{in}})}{C_h(t_{h_{in}} - t_{h\infty})}$$

Introducing Eq. (1) and rearranging, to obtain the overall heat exchanger effectiveness for the case when $(C_{c1} + C_{c2}) > C_h$.

$$\epsilon_{q,o} = \frac{\left[\frac{C_{c1}}{C_h} \cdot \epsilon_{t,c1} + \frac{C_{c2}}{C_h} \cdot \Delta t^*_{in} \cdot \epsilon_{t,c2}\right]\left[1 + \frac{1}{R^*}\right]}{\left[\Delta t^*_{in} + \frac{1}{R^*}\right]} \qquad (3)$$

where: $R^* \triangleq \frac{A_2U_2}{A_1U_1}$ , the ratio between the overall thermal resistances of the two heat transfer surfaces.

Overall Heat Transfer Effectiveness, Counter-Flow

The overall heat transfer effectiveness expressions for counter-flow exchangers are identical to the expressions obtained for the parallel-flow exchangers.

The heat transfer effectiveness expressions, $\epsilon_{q,o}$ , for three-fluid exchangers must reduce to the heat transfer effectiveness expressions for two fluid exchangers in the following limiting conditions.

1. $\epsilon_{t,c1} = \epsilon_{t,c2}$, i.e., when: $\Delta t^*_{in} = 1$, $C_{c1} = C_{c2} = \frac{C_{c_{total}}}{2}$ , $R^* = 1$
2. $U_1 = 0$ ; then $\epsilon_{t,c1} = 0$
3. $U_2 = 0$ ; then $\epsilon_{t,c2} = 0$
4. $C_{c1} = 0$
5. $C_{c2} = 0$

For a two fluid heat exchanger (refer to Fig. 4),

$$q_{actual} = C_c(t_{c_{out}} - t_{c_{in}}) = C_h(t_{h_{in}} - t_{h_{out}})$$

when $C_c < C_h$ ; $q_{max} = C_c(t_{h_{in}} - t_{c_{in}})$

When $C_h < C_c$ : $q_{max} = C_h(t_{h_{in}} - t_{c_{in}})$

Then:

$$\epsilon_{q,o} = \frac{q_{actual}}{q_{max}} = \frac{t_{c_{out}} - t_{c_{in}}}{t_{h_{in}} - t_{c_{in}}} \text{ ; when } C_c < C_h \text{ - - - -} \quad (4)$$

$$\epsilon_{q,o} = \frac{q_{actual}}{q_{max}} = \frac{C_c}{C_h} \frac{t_{c_{out}} - t_{c_{in}}}{t_{h_{in}} - t_{c_{in}}} \text{ ; when } C_h < C_c \quad (5)$$

It can easily be demonstrated that Eq. (2) reduces to Eq. (4) in the five limiting conditions, and that Eq. (3) reduces to Eq. (5) in all five limiting conditions.

The designer, given a specific heat exchanger, flow rates and inlet temperatures, is interested in being able to predict the outlet temperatures of the fluids; or, given inlet and outlet temperatures and flow rates specifications, he wants to be able to calculate the necessary heat transfer areas. As seen, the overall heat transfer effectiveness of a three-fluid exchanger is a function of the heat exchanger operating parameters and the two temperature effectiveness expressions $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$. If these temperature effectivenesses may be calculated, the performance of the exchanger is completely determined. Knowing $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$, the cold fluid outlet temperatures may be calculated, provided the inlet temperatures of all three fluids are known. Knowing the cold fluid outlet temperatures, the hot fluid outlet temperature is obtained from an energy balance on the exchanger.

The following sections give a description of the method for calculating $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ for the two three-fluid exchangers considered in this report.

Temperature Effectivenesses, Parallel-Flow

A detailed derivation of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ for three-fluid parallel-flow exchangers is presented in Appendix I. Due to the complexity of the algebra only a discussion of the analysis is given here.

Referring to Fig. A1, energy balance considerations on three differential elements of the exchanger yield:

$$dq_1 = C_{c1} \cdot dt_{c1} \; ; \; dq_2 = C_{c2} \cdot dt_{c2} \; ; \; dq_1 + dq_2 = - C_h \cdot dt_h$$

The rate equations for the heat transfer rates, $dq_1$ and $dq_2$, through the differential areas, $dA_1$ and $dA_2$, may be written as follows:

$$dq_1 = U_1 dA_1 (t_h - t_{c1}) \; ; \; dq_2 = U_2 dA_2 (t_h - t_{c2})$$

By combining the energy balance and rate equations, a set of three linear first order differential equations in the three temperatures $t_{c1}$, $t_{c2}$, and $t_h$ is obtained.

$$C_{c1} dt_{c1} + C_{c2} dt_{c2} = - C_h dt_h$$

$$C_{c1} dt_{c1} = U_1 dA_1 (t_h - t_{c1})$$

$$C_{c2} dt_{c2} = U_2 (dA_1/A^*)(t_h - t_{c2})$$

This set is solved for the three temperatures $t_{c1}$, $t_{c2}$, and $t_h$ by applying the standard procedure, outlined in most books on differential equations, for solution of a set of simultaneous linear equations, for instance Reference [1].

The constants of integration are determined by applying the boundary conditions, which are the inlet temperatures of the three fluids.

The solution yields $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ as a function of the exchanger parameters listed on page 5. In order to obtain a more compact description of the temperature effectivenesses as functions of these parameters, the parameters were combined into five appropriate non-dimensional groups. The following non-dimensional groupings were selected as being most convenient, and possessing the most readily grasped physical significance:

$$C_1^* \triangleq \frac{C_{c1}}{C_h}$$

- Capacity rate ratio between cold fluid No. 1 and the hot fluid; an operating conditions parameter.

$$C_2^* \triangleq \frac{C_{c2}}{C_h}$$

- Capacity rate ratio between cold fluid No. 2 and the hot fluid; an operating conditions parameter.

$$R^* \triangleq \frac{A_2U_2}{A_1U_1}$$

- Ratio between the overall thermal resistances of the two heat transfer surfaces; a design parameter.

$$\Delta t_{in}^* \triangleq \frac{t_{h_{in}} - t_{c2_{in}}}{t_{h_{in}} - t_{c1_{in}}}$$

- Ratio between the inlet temperatures of the two cold fluids, $t_{c2_{in}}$ is always greater than $t_{c1_{in}}$; an operating conditions parameter.

$$Ntu_1 \triangleq \frac{A_1U_1}{C_{c1}}$$

- Number of transfer units of the heat transfer surface between cold fluid No. 1 and the hot fluid. (For a discussion of the physical significance of this non-dimensional parameter see Reference [4], p. 10); a design parameter.

By introducing these five non-dimensional groupings the following expressions for the temperature effectivenesses, $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$, for parallel-flow exchangers are obtained, Eqs. (33, 34) in Appendix I.

$$\epsilon_{t,c1} = \frac{1 + C_2^*(1 - \Delta t_{in}^*)}{(1 + C_1^* + C_2^*)} + \frac{EXP(EX_2) - EXP(EX_3)}{(B_2 - B_3)}$$
$$+ \frac{[B_2 \cdot EXP(EX_3) - B_3 \cdot EXP(EX_2)][C_2^*(\Delta t_{in}^* - 1) - 1]}{[1 + C_1^* + C_2^*][B_2 - B_3]}$$

$$\epsilon_{t,c2} = \frac{1 - C_1^*[\frac{1}{\Delta t_{in}^*} - 1]}{[1 + C_1^* + C_2^*]}$$
$$- \left[\frac{1 + C_1^* + B_3}{C_2^*}\right]\left[\frac{B_2[C_2^* - \frac{1}{\Delta t_{in}^*}(1+C_2^*)] - [1 + C_1^* + C_2^*]\frac{1}{\Delta t_{in}^*}}{[1 + C_1^* + C_2^*][B_2 - B_3]}\right]EXP(EX_3)$$
$$- \left[\frac{1 + C_1^* + B_2}{C_2^*}\right]\left[\frac{[1 + C_1^* + C_2^*]\frac{1}{\Delta t_{in}^*} - B_3[C_2^* - \frac{1}{\Delta t_{in}^*}(1+C_2^*)]}{[1 + C_1^* + C_2^*][B_2 - B_3]}\right]EXP(EX_2)$$

Where:

$$B_2 = -\frac{1}{2}[R^* \frac{C_1^*}{C_2^*}(1 + C_2^*) + (1 + C_1^*)]$$
$$+ \frac{1}{2}\left[[R^* \frac{C_1^*}{C_2^*}(1 + C_2^*) + (1 + C_1^*)]^2 - 4R^* \frac{C_1^*}{C_2^*}(1 + C_1^* + C_2^*)\right]^{1/2}$$

$$B_3 = -\frac{1}{2}[R^* \frac{C_1^*}{C_2^*}(1 + C_2^*) + (1 + C_1^*)]$$
$$- \frac{1}{2}\left[[R^* \frac{C_1^*}{C_2^*}(1 + C_2^*) + (1 + C_1^*)]^2 - 4R^* \frac{C_1^*}{C_2^*}(1 + C_1^* + C_2^*)\right]^{1/2}$$

$$EX_2 = B_2 \cdot Ntu_1$$

$$EX_3 = B_3 \cdot Ntu_1$$

As seen the equations for $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ are complex algebraic expressions. A graphical representation of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ also presents a problem since both are functions of five independent variable parameters. A function of one independent variable is represented by a single curve; two independent variables require a "one page" family of curves; three independent variables require a "book" of curves; four independent variables require a "library of books" of curves; and finally five independent variables require a "set of libraries of books" of curves. A complete graphical representation of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ is, therefore, out of the question. However, a good representation of the behavior of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ can be obtained by drawing the curves for just a few selected values of the parameters. This is done in Figs. 6-13.

In Figs. 6-13 $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ are plotted on the ordinate and $Ntu_1$ is plotted on the abscissa. $Ntu_1$ is a measure of the "size" of the heat exchanger and has values in the range from zero to infinity. The parameters $C_1^*$, $C_2^*$, and $R^*$ have also magnitudes that range from zero to infinity. In drawing the graphs all three are assigned the two values 0.5 and 2.0. This is a four-fold range for each. $\Delta t_{in}^*$ can be defined so that it will have values in the range from zero to unity by naming the cold fluid with the smallest $t_{c_{in}}$, cold fluid No. 1. In plotting the graphs $\Delta t_{in}^*$ is given the values 0.25, 0.5, 0.75, 1.0. Each page of the graphs is plotted for constant values of $R^*$, $C_1^*$, and $C_2^*$ with $\Delta t^*_{in}$ as a parameter. For two assigned values, 0.5 and 2.0, of each of the parameters $R^*$, $C_1^*$ and $C_2^*$ $8 (= 2^3)$ graphs are presented. For three assigned values $27 (= 3^3)$

FIGS. 6-14

CURVES FOR PARALLEL-FLOW THREE-FLUID
HEAT EXCHANGER TEMPERATURE EFFECTIVENESSES
VERSUS NUMBER OF HEAT TRANSFER UNITS

Ranges Covered

| $R^*$ | $C_1^*$ | $C_2^*$ | |
|---|---|---|---|
| 0.5 | 0.5 | 0.5 | Fig. 6 |
| 0.5 | 2.0 | 0.5 | Fig. 7 |
| 0.5 | 0.5 | 2.0 | Fig. 8 |
| 0.5 | 2.0 | 2.0 | Fig. 9 |
| 2.0 | 0.5 | 0.5 | Fig. 10 |
| 2.0 | 0.5 | 2.0 | Fig. 11 |
| 2.0 | 2.0 | 0.5 | Fig. 12 |
| 2.0 | 2.0 | 2.0 | Fig. 13 |
| 1.0 | 0.5 | 0.5 | Fig. 14 |

FIG. 15

CURVES FOR COUNTER-FLOW THREE-FLUID
HEAT EXCHANGER TEMPERATURE EFFECTIVENESSES
VERSUS NUMBER OF HEAT TRANSFER UNITS

$R^* = 1.0$   $C_1^* = 0.5$   $C_2^* = 0.5$

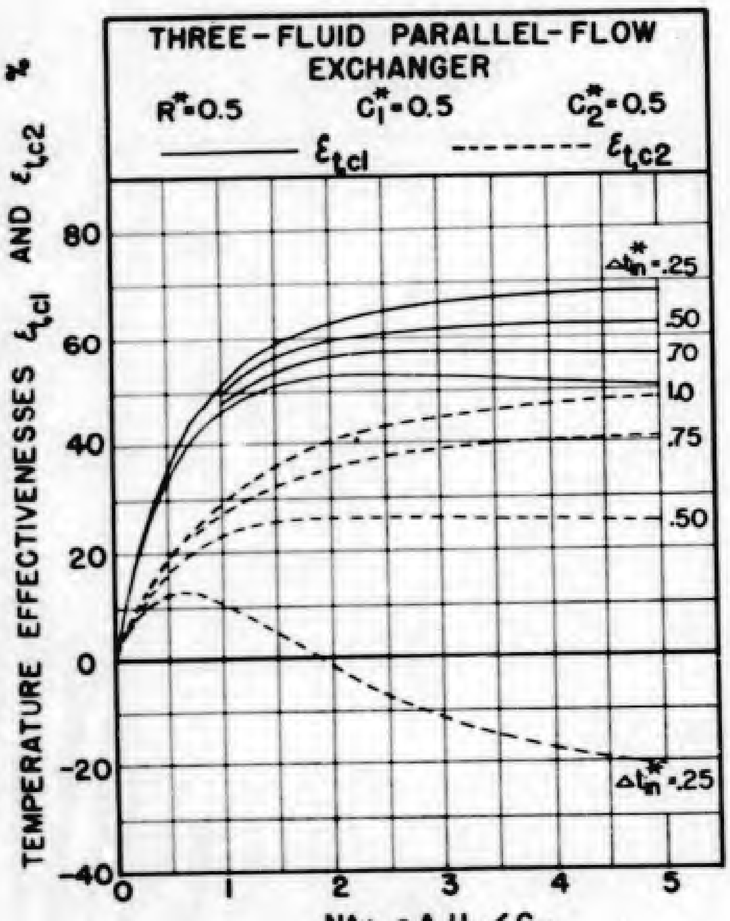

THREE-FLUID PARALLEL-FLOW EXCHANGER
R*=0.5
C₁*=0.5
C₂*=0.5
ε_tc1
ε_tc2
TEMPERATURE EFFECTIVENESSES ε_tc1 AND ε_tc2 %
Δt_in* =.25
.50
.70
1.0
.75
.50
Δt_in* =.25
Ntu₁ = A₁ U₁ / C_c1

FIG. 6

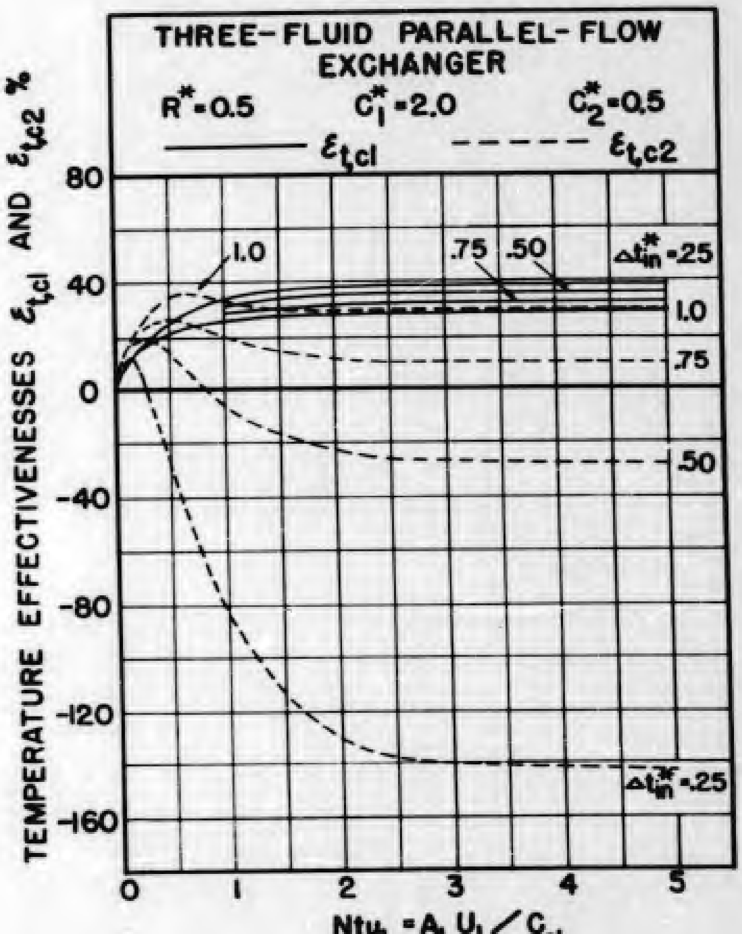
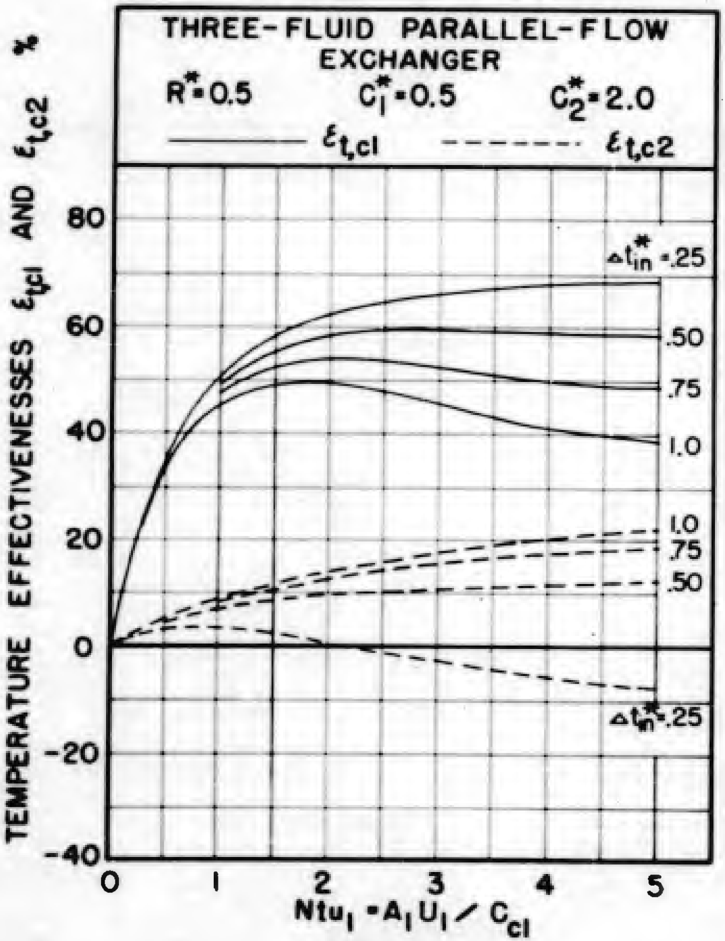
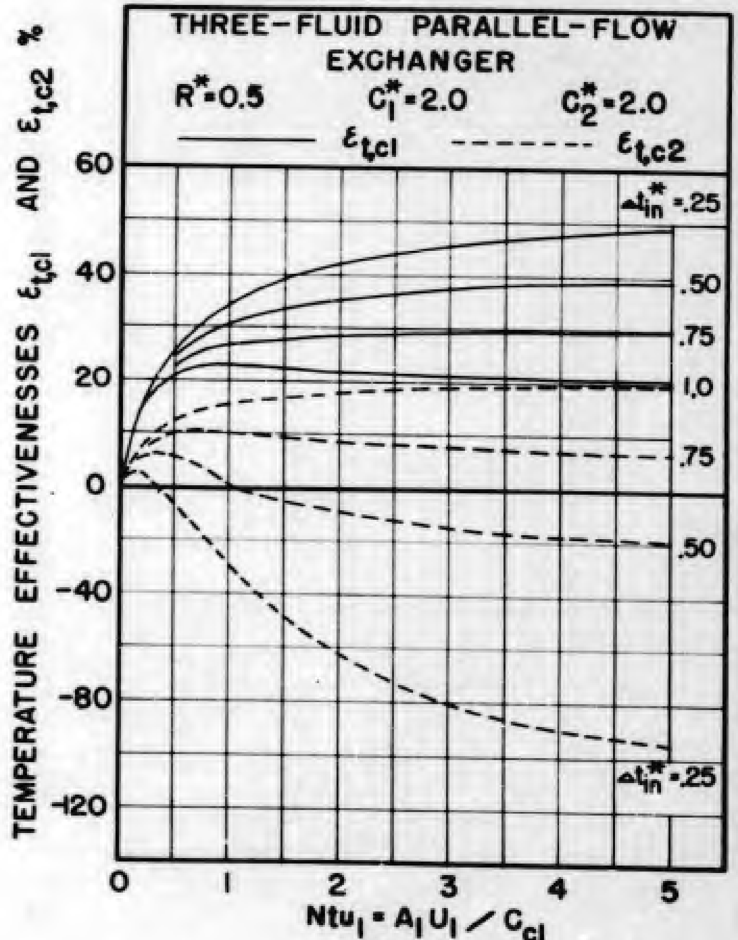

THREE-FLUID PARALLEL-FLOW EXCHANGER
R*=0.5
C₁*=2.0
C₂*=0.5
ε_tc1
ε_tc2
TEMPERATURE EFFECTIVENESSES ε_tc1 AND ε_tc2 %
1.0
.75 .50
Δt_in* =.25
1.0
.75
.50
Δt_in* =.25
Ntu₁ = A₁ U₁ / C_c1

FIG. 7


THREE-FLUID PARALLEL-FLOW EXCHANGER
R*=0.5 C1*=0.5 C2*=2.0
εt,c1 εt,c2
TEMPERATURE EFFECTIVENESSES εt,c1 AND εt,c2 %
80
60
40
20
0
-20
-40
Δtin* =.25
.50
.75
1.0
1.0
.75
.50
Δtin*=.25
0 1 2 3 4 5
Ntu1 = A1U1 / Cc1
FIG. 8


THREE-FLUID PARALLEL-FLOW EXCHANGER
R*=0.5 C1*=2.0 C2*=2.0
εt,c1 εt,c2
TEMPERATURE EFFECTIVENESSES εt,c1 AND εt,c2 %
60
40
20
0
-40
-80
-120
Δtin* =.25
.50
.75
1.0
.75
.50
Δtin*=.25
0 1 2 3 4 5
Ntu1 = A1U1 / Cc1
FIG. 9

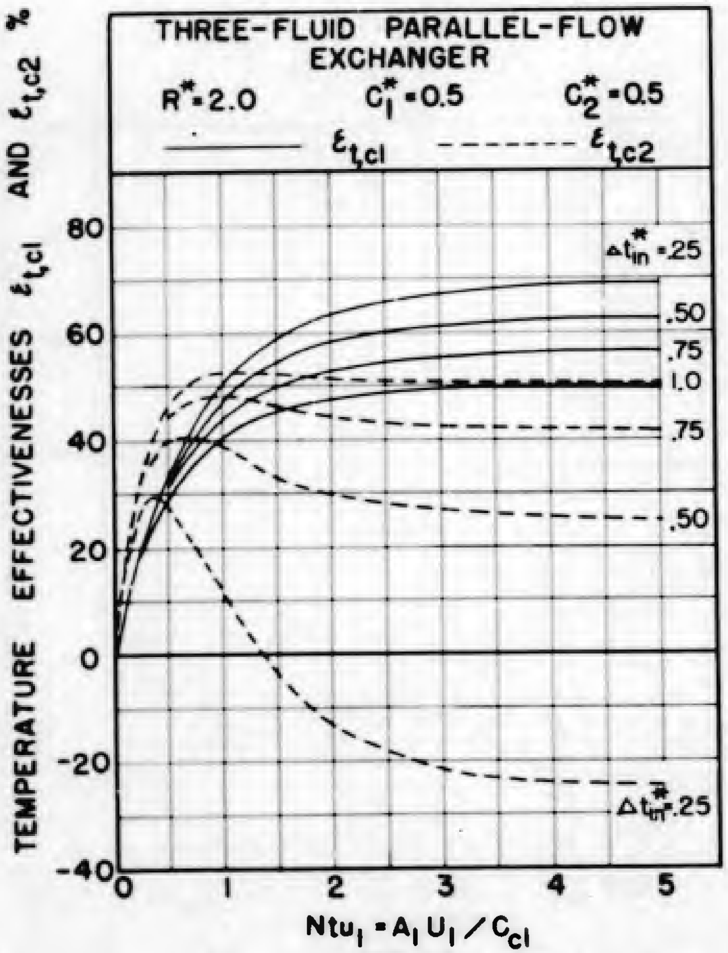

THREE-FLUID PARALLEL-FLOW EXCHANGER
R* = 2.0
C1* = 0.5
C2* = 0.5
ε t,c1
ε t,c2
Δt*in = .25
.50
.75
1.0
.75
.50
Δt*in = .25
TEMPERATURE EFFECTIVENESSES ε t,c1 AND ε t,c2 %
80
60
40
20
0
-20
-40
0
1
2
3
4
5
Ntu1 = A1 U1 / Cc1

FIG. 10

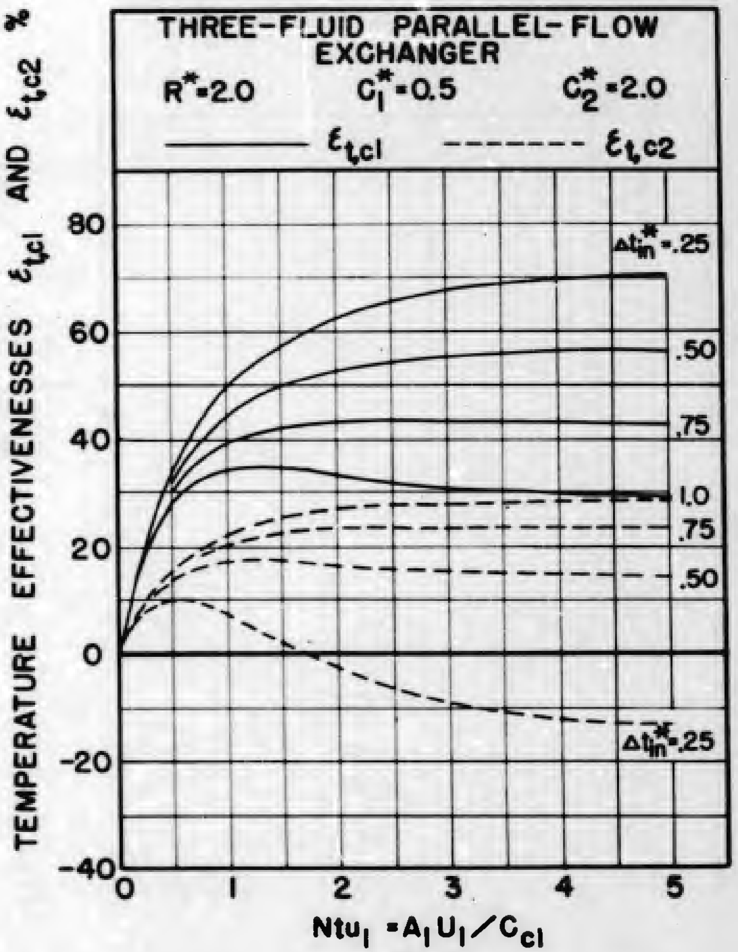
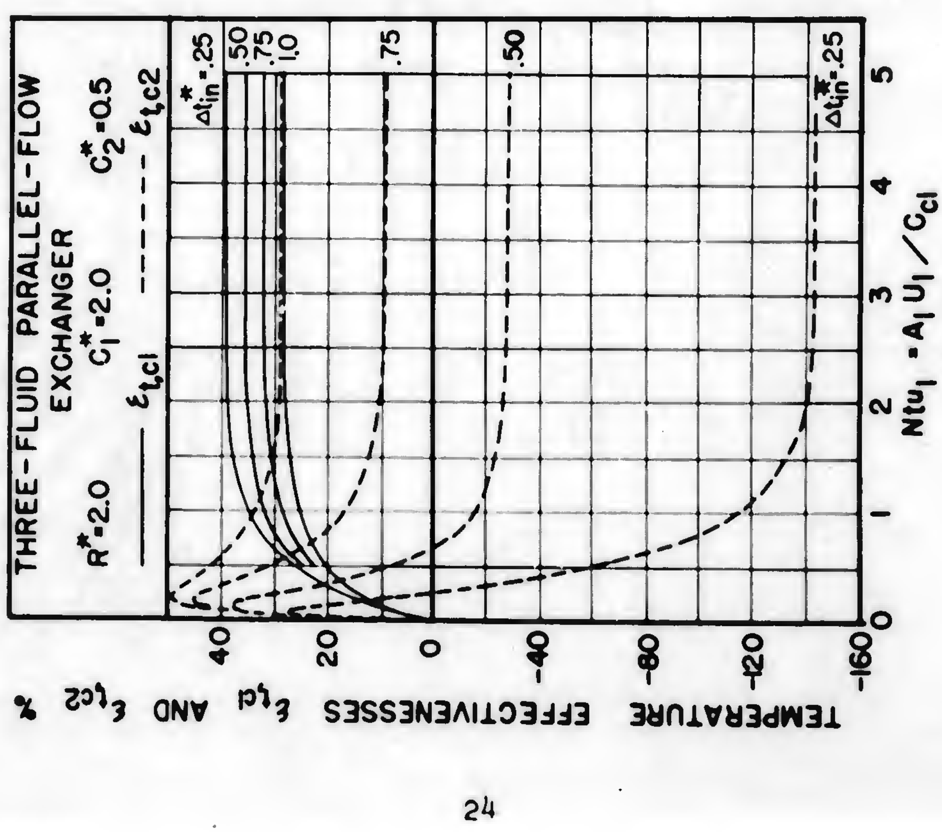
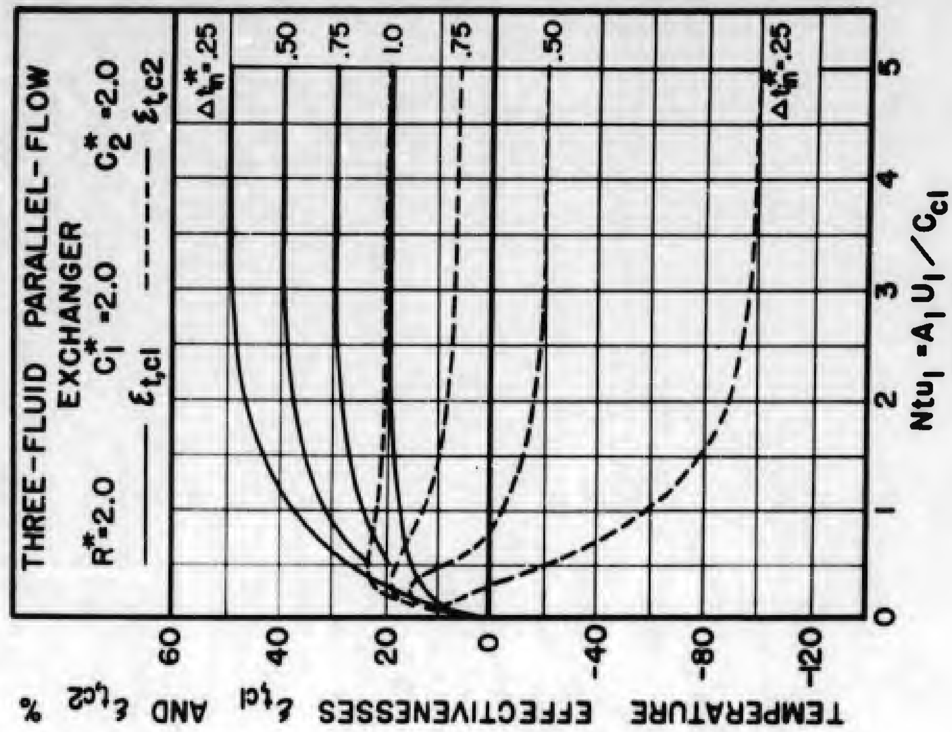
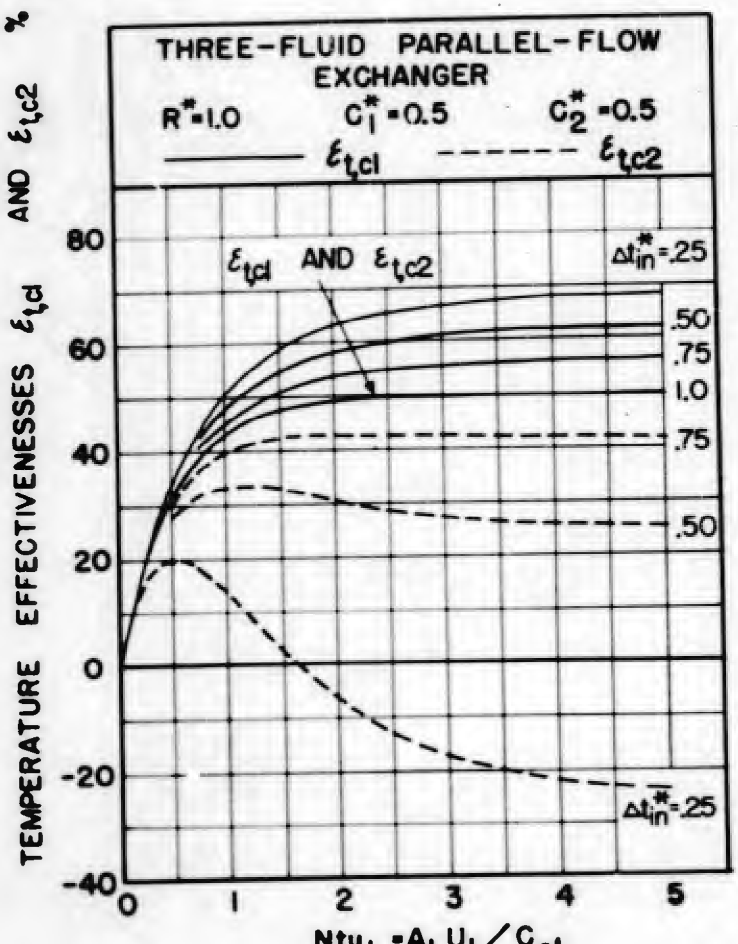
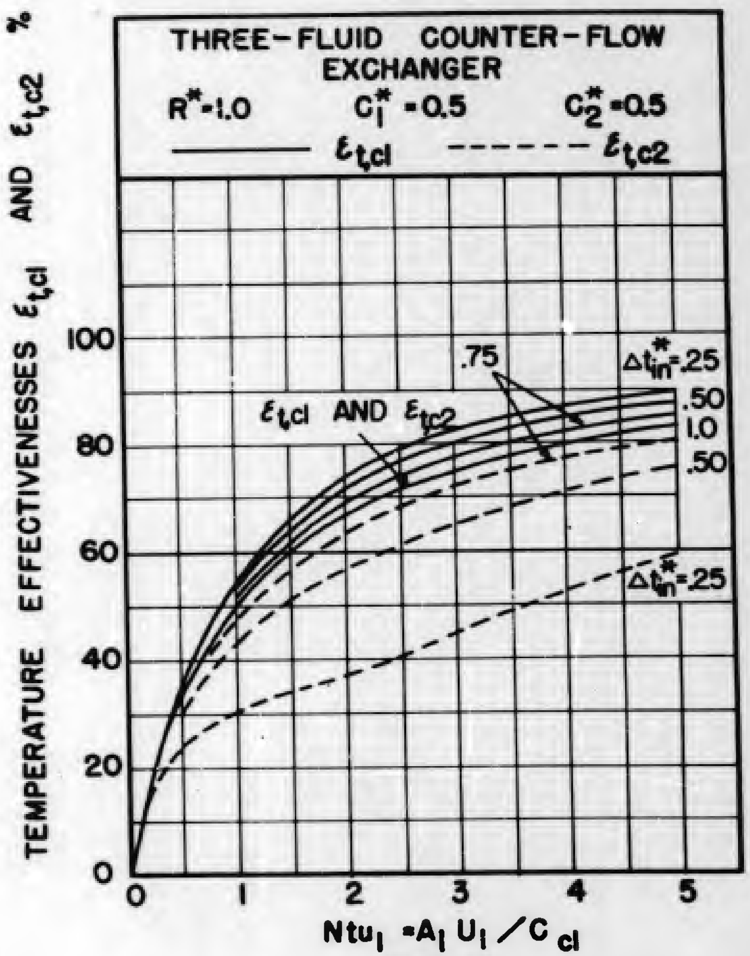

THREE-FLUID PARALLEL-FLOW EXCHANGER
R* = 2.0
C1* = 0.5
C2* = 2.0
ε t,c1
ε t,c2
Δt*in = .25
.50
.75
1.0
.75
.50
Δt*in = .25
TEMPERATURE EFFECTIVENESSES ε t,c1 AND ε t,c2 %
80
60
40
20
0
-20
-40
0
1
2
3
4
5
Ntu1 = A1 U1 / Cc1

FIG. 11


THREE-FLUID PARALLEL-FLOW EXCHANGER
R*=2.0
C*1=2.0
C*2=0.5
ε t,cl
ε t,c2
Δt*in=.25
.50
.75
1.0
.75
.50
Δt*in=.25
TEMPERATURE EFFECTIVENESSES ε t,cl AND ε t,c2 %
40
20
0
-40
-80
-120
-160
0
1
2
3
4
5
Ntu1 = A1 U1 / Ccl
FIG. 12


THREE-FLUID PARALLEL-FLOW EXCHANGER
R*=2.0
C*1=2.0
C*2=2.0
ε t,cl
ε t,c2
Δt*in=.25
.50
.75
1.0
.75
.50
Δt*in=.25
TEMPERATURE EFFECTIVENESSES ε t,cl AND ε t,c2 %
60
40
20
0
-40
-80
-120
0
1
2
3
4
5
Ntu1 = A1 U1 / Ccl
FIG. 13


THREE-FLUID PARALLEL-FLOW EXCHANGER
R* = 1.0
C*1 = 0.5
C*2 = 0.5
εt,c1
εt,c2
εt,c1 AND εt,c2
Δt*in = .25
.50
.75
1.0
.75
.50
Δt*in = .25
TEMPERATURE EFFECTIVENESSES εt,c1 AND εt,c2 %
80
60
40
20
0
-20
-40
0
1
2
3
4
5
Ntu1 = A1 U1 / Cc1
FIG. 14


THREE-FLUID COUNTER-FLOW EXCHANGER
R* = 1.0
C*1 = 0.5
C*2 = 0.5
εt,c1
εt,c2
εt,c1 AND εt,c2
.75
Δt*in = .25
.50
1.0
.50
Δt*in = .25
TEMPERATURE EFFECTIVENESSES εt,c1 AND εt,c2 %
100
80
60
40
20
0
0
1
2
3
4
5
Ntu1 = A1 U1 / Cc1
FIG. 15

graphs would be required.

There are certain limiting conditions that must be satisfied by the equations for $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$.

1. If the exchanger has infinitely large heat transfer area, i.e., $Ntu_1 \to \infty$, then $t_{c1_{out}} = t_{c2_{out}} = t_{h_{out}}$ = the calorimetric mixed mean temperature.

2. If $C_{c1} = 0$, cold fluid No. 1 will have no influence on the temperature picture in the exchanger. Then $\epsilon_{t,c2}$ should reduce to the effectiveness expression for two-fluid exchangers.

3. If $C_{c2} = 0$, cold fluid No. 2 will have no influence on the temperature conditions in the exchanger. Consequently $\epsilon_{t,c1}$ should reduce to the effectiveness expression for two-fluid exchangers.

4. When $C_{c1} = C_{c2} = C_{c,tot.}/2$; $R^* = 1$; $\Delta t^*_{in} = 1$, the three-fluid exchanger is equivalent to a two-fluid exchanger. In this case $\epsilon_{t,c1} = \epsilon_{t,c2} = \epsilon$ of a two-fluid exchanger.

It can be shown that $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ reduce to the proper form in all four limiting cases. However, the actual proof leads to fairly complicated algebra and will be omitted here. The reduction of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ in case (4) is illustrated in Fig. 14, where $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ are plotted for $C_1^* = 0.5$, $C_2^* = 0.5$, $R^* = 1.0$, and $\Delta t^*_{in}$ has values 0.25, 0.5, 0.75, 1.0.

It can also be argued that $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ should reduce to the two-fluid effectiveness expression when $U_1 = 0$, and when $U_2 = 0$. However, this is not the case since it has been assumed in the derivation of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ that both $U_1$ and $U_2$ are different from zero. In going back to the original set of differential equations with $U_1 = 0$ or $U_2 = 0$, it is easily shown that in both cases the set of three equations reduce to a set of two equations, identical to the set of equations obtained when analysing a two-fluid exchanger.

Temperature Effectivenesses, Counter-Flow

A detailed derivation of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ for the three-fluid counter-flow arrangement shown in Fig. 2 is presented in Appendix II. Due to the complexity of the algebra only a discussion of the analysis is given here. The analysis is in all aspects similar to the one presented for parallel-flow exchangers.

Referring to Figs. A4 and A5, an energy balance consideration on a differential element of the exchanger yields:

$$dq_1 = C_{c1} \cdot dt_{c1} ; dq_2 = C_{c2} \cdot dt_{c2} ; dq_1 + dq_2 = C_h \cdot dt_h$$

The rate equations for the heat transfer rates through the differential areas $dA_1$ and $dA_2$ are:

$$dq_1 = U_1 \cdot dA_1 \cdot (t_h - t_{c1}); dq_2 = U_2 \cdot dA_2 \cdot (t_h - t_{c2})$$

By combining the energy balance and rate equations the following set of linear differential equations is obtained.

$$C_{c1}dt_{c1} + C_{c2}dt_{c2} = C_h dt_h$$

$$C_{c1}dt_{c1} = U_1 dA_1(t_h - t_{c1})$$

$$C_{c2}dt_{c2} = U_2(dA_1/A^*)(t_h - t_{c2})$$

On solving the set of simultaneous differential equations, and determining the constants of integration by applying as boundary conditions the inlet temperatures of the three fluids, the temperature effectivenesses are obtained as functions of the exchanger parameters. The same non-dimensional groupings are selected as for the parallel-flow case, and the temperature effectiveness equations are as follows, Eqs. (67, 68) in Appendix II.

$$\epsilon_{t,c1} = \frac{[1 - EXP(EX_2)]}{[1 - (B_2 + 1) \cdot EXP(EX_2)]} + \left[\frac{B_3 \cdot EXP(EX_3)[1-EXP(EX_2)]-B_2 \cdot EXP(EX_2)[1-EXP(EX_3)]}{[(B_2 + 1)EXP(EX_2) - 1]}\right] \cdot K$$

$$\epsilon_{t,c2} = 1 - \frac{1}{\Delta t^*_{in}} + \left[\frac{\frac{1}{\Delta t^*_{in}}[(B_2 + 1) \cdot EXP(EX_2) - C^*_1 \cdot EXP(EX_2) - C^*_2]}{C^*_2[(B_2 + 1)EXP(EX_2) - 1]}\right]$$

$$+ \left[\frac{[(B_3 + 1)EXP(EX_3) - (B_2 +1)EXP(EX_2)]}{[(B_2 +1)EXP(EX_2) - 1]}\right] \cdot \frac{K}{\Delta t^*_{in}}$$

$$- \left[\frac{EXP(EX_2)[B_2 + 1 - C^*_1][(B_3 +1)EXP(EX_3) -1]}{C^*_2[(B_2 + 1)EXP(EX_2) - 1]}\right] \cdot \frac{K}{\Delta t^*_{in}}$$

$$+ \left[\frac{[(B_3+1)EXP(EX_3) - C^*_1 \cdot EXP(EX_3)][(B_2+1)EXP(EX_2) - 1]}{C^*_2[(B_2 + 1)EXP(EX_2) - 1]}\right] \cdot \frac{K}{\Delta t^*_{in}}$$

Where:

$$B_2 = -\frac{1}{2}[R^* \frac{C^*_1}{C^*_2}(1-C^*_2) + (1-C^*_1)] + \frac{1}{2}\left[[R^* \frac{C^*_1}{C^*_2}(1-C^*_2) + (1-C^*_1)]^2 - 4R^* \frac{C^*_1}{C^*_2}(1 - C^*_1 - C^*_2)\right]^{1/2}$$

$$B_3 = -\frac{1}{2}[R^* \frac{C_1^*}{C_2^*}(1-C_2^*) + 1-C_1^*)]$$

$$-\frac{1}{2}\left[[R^* \frac{C_1^*}{C_2^*}(1-C_2^*) + (1-C_1^*)]^2 - 4R^* \frac{C_1^*}{C_2^*}(1-C_1^*-C_2^*)\right]^{1/2}$$

$$EX_2 = B_2 \cdot Ntu_1$$

$$EX_3 = B_3 \cdot Ntu_1$$

$$K = \frac{C_2^* \cdot \Delta t_{in}^* [(B_2+1)EXP(EX_2)-1] + [(B_2+1)(1-C_2^* \cdot EXP(EX_2))-C_1^*]}{\left[\begin{array}{l} B_2 \cdot B_3[EXP(EX_3)-EXP(EX_2)] + B_3[1-EXP(EX_2)] - B_2[1-EXP(EX_3)] \\ \qquad + [1-C_1^*-C_2^*][(B_3+1)EXP(EX_3) - (B_2+1)EXP(EX_2)] \end{array}\right]}$$

As seen, the equations for $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ for the counter-flow exchanger are even more complex than for the parallel-flow exchanger. $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ are plotted in Figs. 16-23 with the same values of the five parameters as for the parallel-flow exchangers.

It should be noted that when $(C_1^* + C_2^*) = 1.0$, the two temperature effectiveness expressions for counter-flow exchangers are indeterminate. The derivation of the temperature effectiveness expressions for the case when $(C_1^* + C_2^*) = 1.0$ is presented in Appendix II (Eqs. (93,94), and are rewritten here.

$$\boxed{\epsilon_{t,c1} = \frac{Ntu_1}{1 + Ntu_1} + \left[EXP(EX_3) - 1 - \frac{[(B_3+1)EXP(EX_3)-1]Ntu_1}{[1 + Ntu_1]}\right] \cdot K}$$

$$\epsilon_{t,c2} = 1 - \frac{1}{\Delta t^*_{in}} + \left[1 - \frac{1}{R^*[1 + Ntu_1]}\right]\frac{1}{\Delta t^*_{in}}$$

$$+ \left[[\frac{B_3+1-C^*_1}{C^*_2}]EXP(EX_3)-1-[1-\frac{1}{R^*[1+Ntu_1]}][(B_3+1)\cdot EXP(EX_3)-1]\right]\frac{K}{\Delta t^*_{in}}$$

Where:

$$B_3 = -[R^* \frac{C^*_1}{C^*_2}(1 - C^*_2) + (1 - C^*_1)]$$

$$EX_3 = B_3 \cdot Ntu_1$$

$$K = \frac{[\frac{1}{R^*} + Ntu_1] - \Delta t^*_{in}[1 + Ntu_1]}{[(B_3+1)EXP(EX_3)-1][\frac{1}{R^*}+Ntu_1] - [(B_3+1)(EXP(EX_3) - \frac{1}{C^*_2}) + \frac{C^*_1}{C^*_2}][1+Ntu_1]}$$

The temperature effectivenesses used for plotting Figs. 15, 16, and 20 are obtained from these equations.

The temperature effectivenesses for counter-flow exchangers must satisfy the same four limiting conditions as listed on page 26 for parallel-flow exchangers. It can be shown that all these conditions are satisfied. However, the actual proof leads to very involved algebra and is omitted here. The reduction of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ in case (4) is illustrated in Fig. 15, where $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ are plotted for $C^*_1 = 0.5$, $C^*_2 = 0.5$, $R^* = 1.0$, and $\Delta t^*_{in}$ has values 0.25, 0.5, 0.75, 1.0.

Like the parallel-flow situation, for the $U_1 = 0$ or $U_2 = 0$ cases, the original three differential equations will reduce to a set of two differential equations, identical to the set of equations obtained when analysing a two-fluid counter-flow exchanger.

FIGS. 16-23

CURVES FOR COUNTER-FLOW THREE-FLUID
HEAT EXCHANGER TEMPERATURE EFFECTIVENESSES
VERSUS NUMBER OF HEAT TRANSFER UNITS

Ranges Covered

| $R^*$ | $C_1^*$ | $C_2^*$ | |
|---|---|---|---|
| 0.5 | 0.5 | 0.5 | Fig. 16 |
| 0.5 | 0.5 | 2.0 | Fig. 17 |
| 0.5 | 2.0 | 0.5 | Fig. 18 |
| 0.5 | 2.0 | 2.0 | Fig. 19 |
| 2.0 | 0.5 | 0.5 | Fig. 20 |
| 2.0 | 0.5 | 2.0 | Fig. 21 |
| 2.0 | 2.0 | 0.5 | Fig. 22 |
| 2.0 | 2.0 | 2.0 | Fig. 23 |

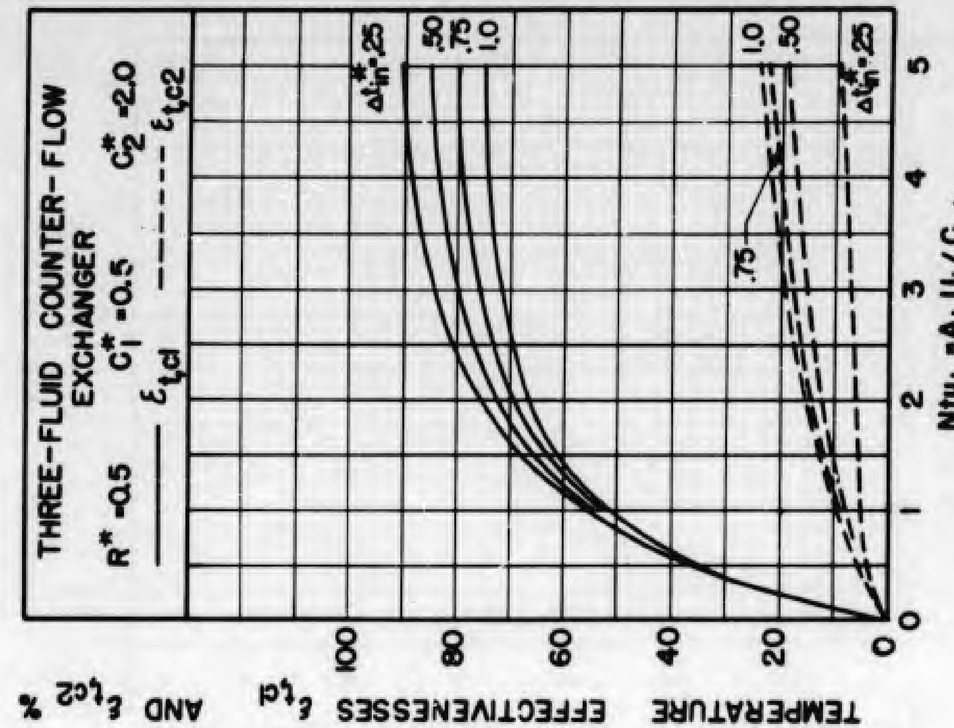

THREE-FLUID COUNTER-FLOW EXCHANGER
$R^* = 0.5$ $C_1^* = 0.5$ $C_2^* = 2.0$
—— $\varepsilon_{t,c1}$ ------ $\varepsilon_{t,c2}$
$\Delta t_{in}^* = .25$
.50
.75
1.0
1.0
.75
.50
$\Delta t_{in}^* = .25$
0
20
40
60
80
100
TEMPERATURE EFFECTIVENESSES $\varepsilon_{t,c1}$ AND $\varepsilon_{t,c2}$ %
0
1
2
3
4
5
$Ntu_1 = A_1 U_1 / C_{c1}$

FIG. 17

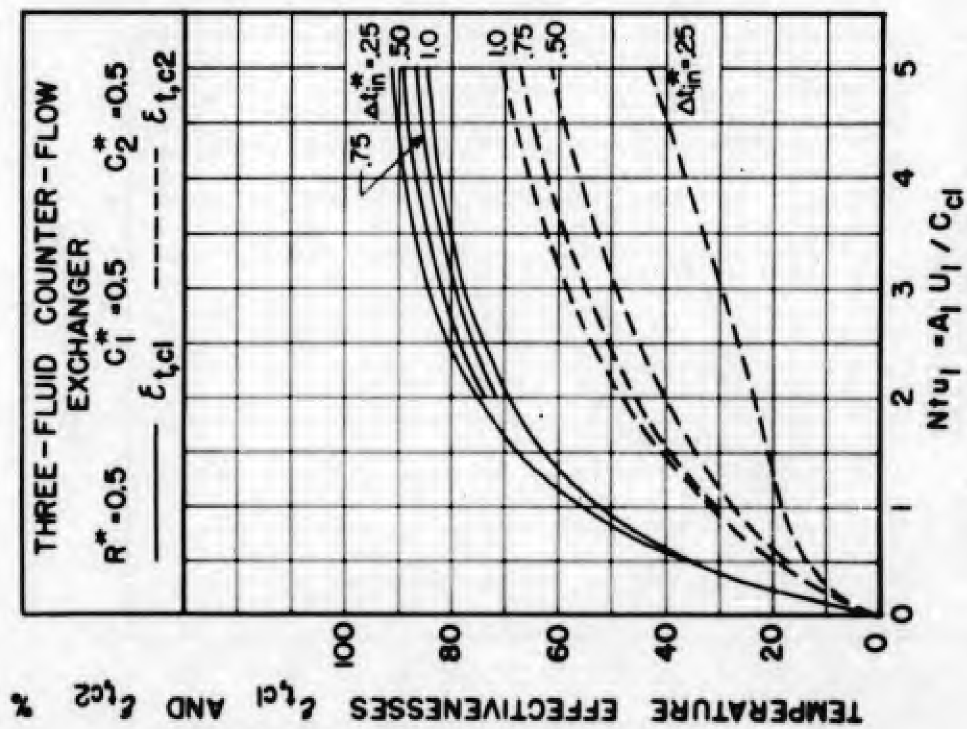

THREE-FLUID COUNTER-FLOW EXCHANGER
$R^* = 0.5$ $C_1^* = 0.5$ $C_2^* = 0.5$
—— $\varepsilon_{t,c1}$ ------ $\varepsilon_{t,c2}$
$\Delta t_{in}^* = .25$
.50
1.0
.75
1.0
.75
.50
$\Delta t_{in}^* = .25$
0
20
40
60
80
100
TEMPERATURE EFFECTIVENESSES $\varepsilon_{t,c1}$ AND $\varepsilon_{t,c2}$ %
0
1
2
3
4
5
$Ntu_1 = A_1 U_1 / C_{c1}$

FIG. 16

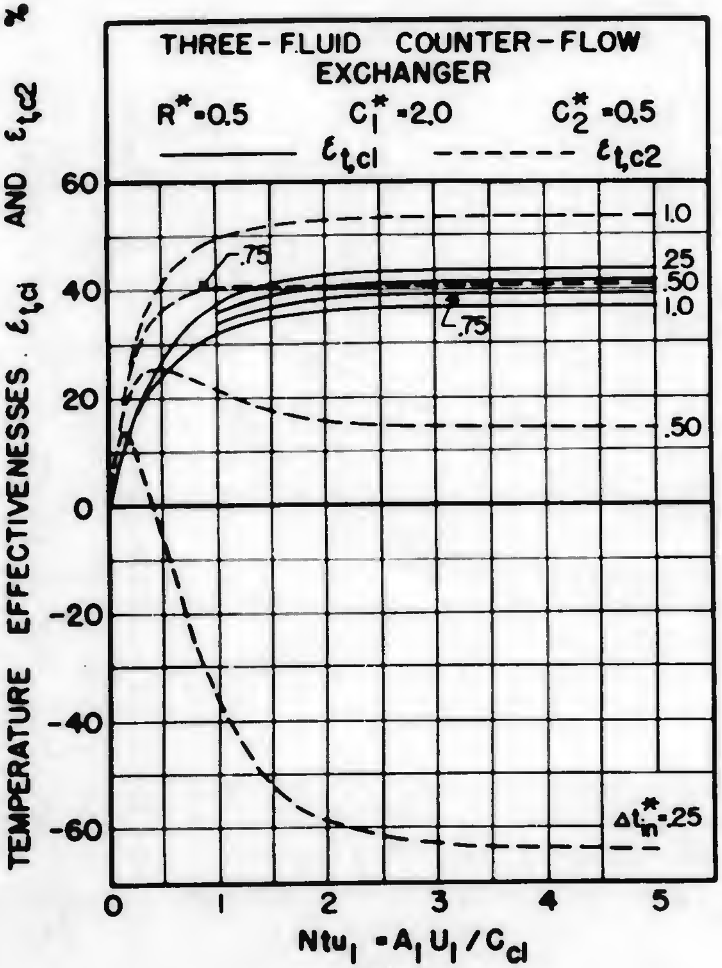

THREE-FLUID COUNTER-FLOW EXCHANGER
R* = 0.5 C1* = 2.0 C2* = 0.5
ε t,c1 ------ ε t,c2
TEMPERATURE EFFECTIVENESSES ε t,c1 AND ε t,c2 %
60
40
20
0
-20
-40
-60
1.0
.75
.25
.50
1.0
.75
.50
Δt*in = .25
0
1
2
3
4
5
Ntu1 = A1 U1 / Cc1

FIG. 18

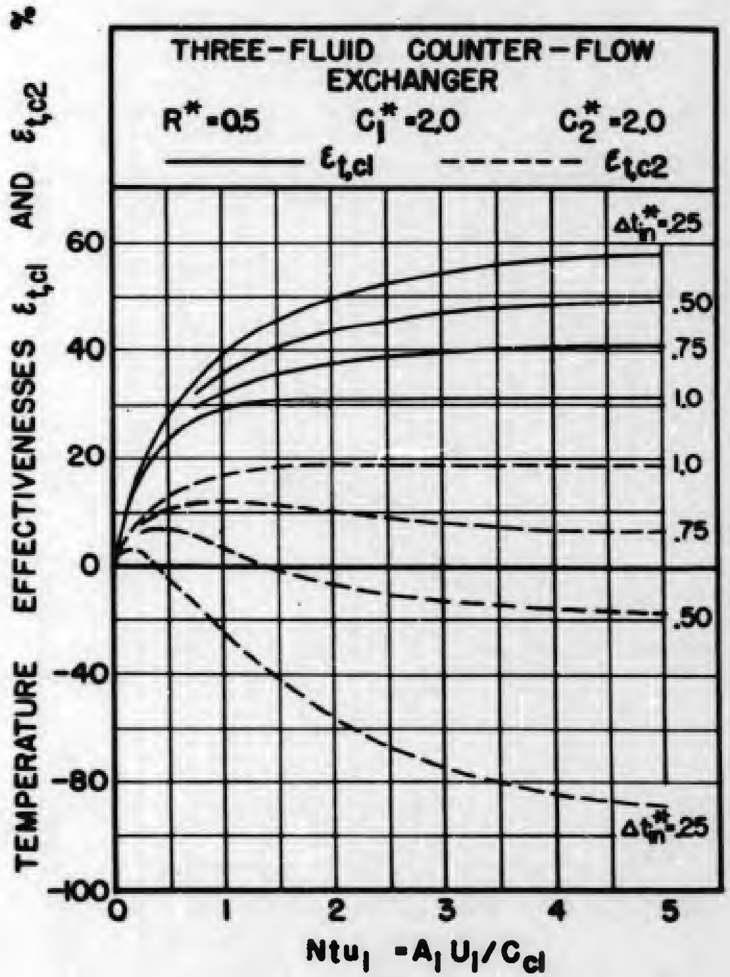

THREE-FLUID COUNTER-FLOW EXCHANGER
R* = 0.5 C1* = 2.0 C2* = 2.0
ε t,c1 ------ ε t,c2
TEMPERATURE EFFECTIVENESSES ε t,c1 AND ε t,c2 %
60
40
20
0
-40
-80
-100
Δt*in = .25
.50
.75
1.0
1.0
.75
.50
Δt*in = .25
0
1
2
3
4
5
Ntu1 = A1 U1 / Cc1

FIG. 19

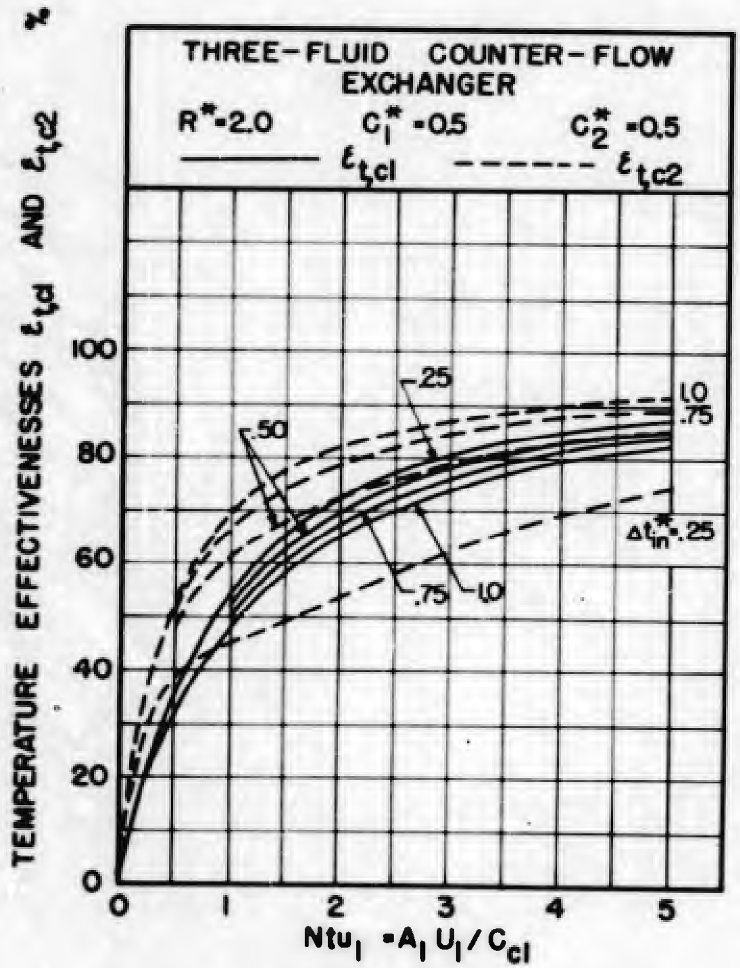

THREE-FLUID COUNTER-FLOW EXCHANGER
R*=2.0 C*1 =0.5 C*2 =0.5
ε t,c1 ------ ε t,c2
TEMPERATURE EFFECTIVENESSES ε t,c1 AND ε t,c2 %
Ntu1 = A1 U1 / Cc1
.25
.50
.75
1.0
1.0
.75
Δt*in=.25

FIG. 20

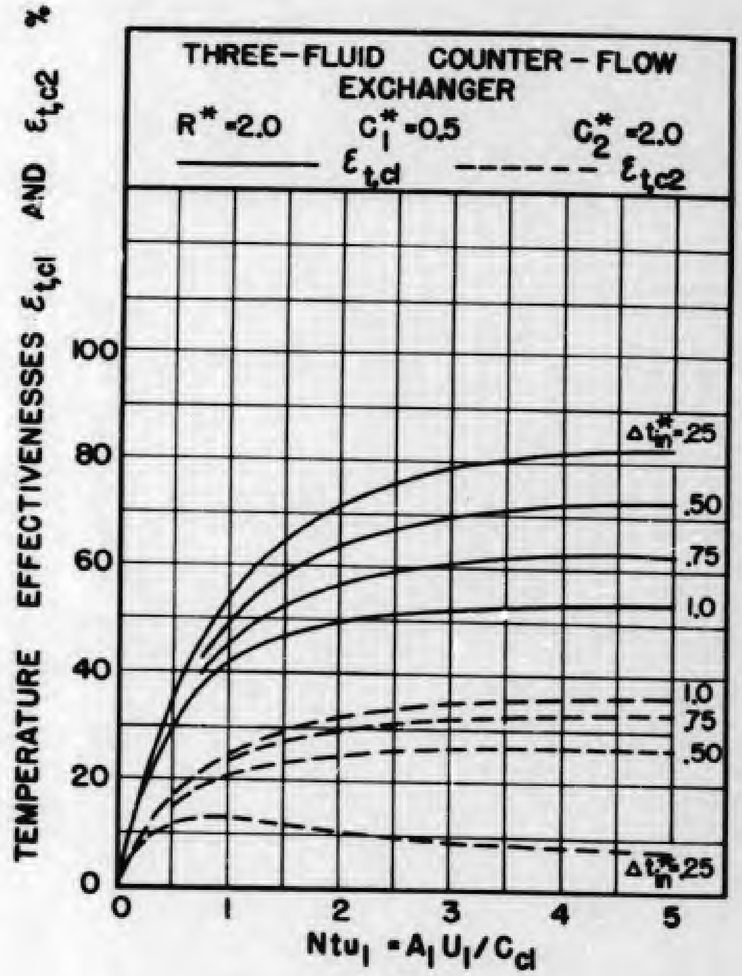

THREE-FLUID COUNTER-FLOW EXCHANGER
R* =2.0 C*1 =0.5 C*2 =2.0
ε t,c1 ------ ε t,c2
TEMPERATURE EFFECTIVENESSES ε t,c1 AND ε t,c2 %
Ntu1 = A1 U1 / Cc1
Δt*in=.25
.50
.75
1.0
1.0
.75
.50
Δt*in=.25

FIG. 21

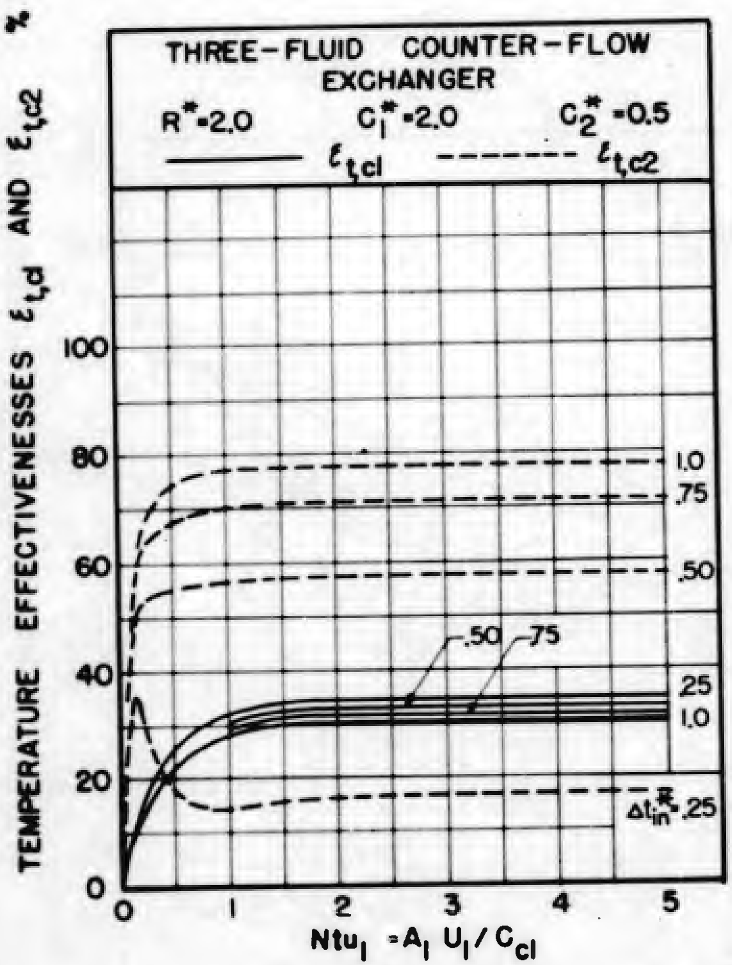

THREE-FLUID COUNTER-FLOW EXCHANGER
R* = 2.0
C1* = 2.0
C2* = 0.5
ε t,c1
ε t,c2
TEMPERATURE EFFECTIVENESSES ε t,c1 AND ε t,c2 %
1.0
.75
.50
.50
.75
.25
1.0
Δt*in = .25
Ntu1 = A1 U1 / Cc1

FIG. 22

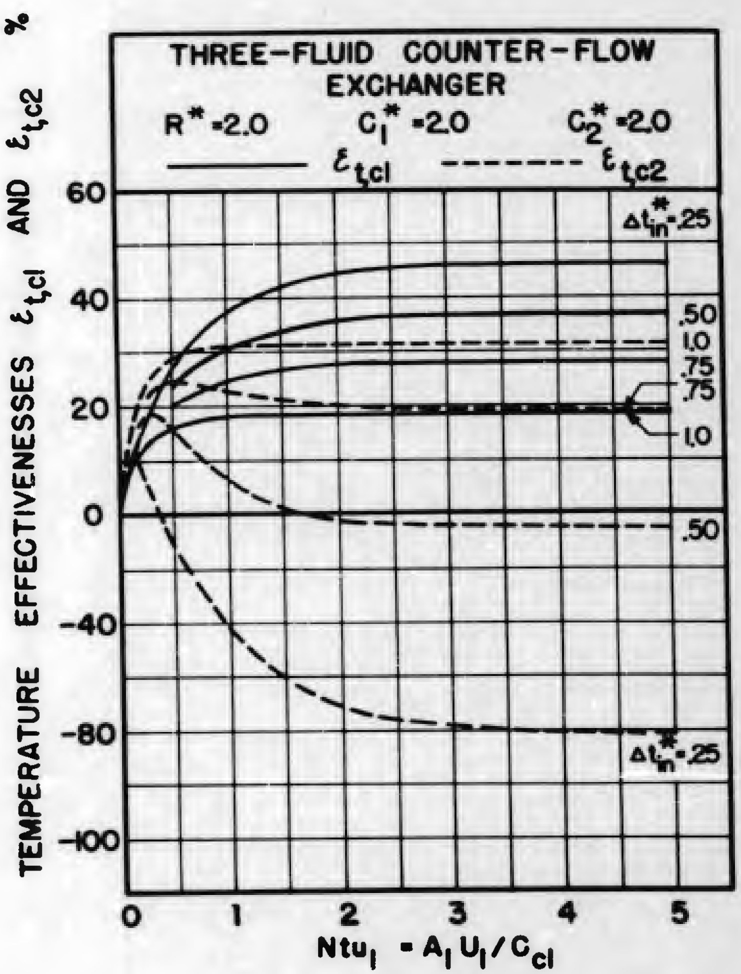

THREE-FLUID COUNTER-FLOW EXCHANGER
R* = 2.0
C1* = 2.0
C2* = 2.0
ε t,c1
ε t,c2
TEMPERATURE EFFECTIVENESSES ε t,c1 AND ε t,c2 %
Δt*in = .25
.50
1.0
.75
.75
1.0
.50
Δt*in = .25
Ntu1 = A1 U1 / Cc1

FIG. 23

## V. DISCUSSION OF THE TEMPERATURE EFFECTIVENESS EXPRESSIONS AND GRAPHS

Due to the algebraic complexity of the temperature effectiveness expressions for both counter- and parallel-flow exchangers, a Burroughs 220 digital computer was used for calculating the necessary values for drawing the curves. This insures an accuracy of calculation not obtainable by slide rule or desk calculator. The computer programs, together with the numerical values of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ used for plotting the graphs, are given in Appendices VI-IX.

Negative magnitudes of $\epsilon_{t,c2}$ as large as approximately - 1.5 are obtained for large $Ntu_1$ (large exchanger sizes) under certain circumstances for both parallel- and counter-flow. In effect, the colder fluid is then cooling both the hot fluid directly and the other less cold fluid indirectly For the case of parallel-flow, when $\Delta t^*_{in} = 0.25$, $C^*_1 = 2.0$, $C^*_2 = 2.0$, $R^* = 2.0$ and $Ntu_1 = 5.0$, $\epsilon_{t,c1}$ is calculated to be 0.4993 and $\epsilon_{t,c2}$ is calculated to be - 0.9977, Fig. 13. The mixed mean outlet temperature for the infinite transfer area exchanger is:

$$t_{mix.mean} = \frac{t_{h_{in}} + C^*_1 t_{c1_{in}} + C^*_2 t_{c2_{in}}}{1 + C^*_1 + C^*_2}$$

Then:

$$\epsilon_{t,c1} = \frac{t_{mix.mean} - t_{c1_{in}}}{t_{h_{in}} - t_{c1_{in}}} = \frac{1 + C^*_2 (1 - \Delta t^*_{in})}{1 + C^*_1 + C^*_2}$$

$$\epsilon_{t,c1} = \frac{1 + 2(1 - 0.25)}{5} = 0.5$$

$$\epsilon_{t,c2} = \frac{t_{mix.mean} - t_{c2_{in}}}{t_{h_{in}} - t_{c2_{in}}} = \frac{1 + C_1^* \left(1 - \frac{1}{\Delta t_{in}^*}\right)}{1 + C_1^* + C_2^*}$$

$$\epsilon_{t,c2} = \frac{1 + 2\left(1 - \frac{1}{0.25}\right)}{5} = - 1.0$$

As seen, this asymptotic behavior is in full agreement with the $Ntu_1 = 5$ calculation.

At first glance it may appear that some of the temperature effectiveness curves should be symmetrical in $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$. As an example, it may appear that Fig. 16 (with $C_1^* = 0.5$, $C_2^* = 0.5$ and $R^* = 0.5$) and Fig. 20 (with $C_1^* = 0.5$, $C_2^* = 0.5$ and $R^* = 2.0$) should be symmetrical. However, it must be stressed that there is no symmetry here. This is easily discovered by a closer investigation of the problem.

# VI. LIMITATIONS TO THE THEORY IMPOSED BY THE IDEALIZATIONS

The idealizations made in the development of the mathematical theory are listed on page 8.

The specification of an adiabatic heat exchanger is not considered to introduce any significant error in applying the results. In most heat exchangers the rate of heat lost to the surroundings is in the order of a few per cent of the heat rate transferred between the hot and the cold fluids. However, if it is found that the heat exchanger cannot be considered to be adiabatic, an analysis can be carried out assuming the exchanger to be adiabatic, and the result can then be corrected for heat loss to the surroundings.

The idealization of perfect mixing in each passage, i.e., that there is no temperature gradient across the passage normal to the flow direction, is made as a necessity for the analysis, but will not introduce a significant error in the results if the fluid temperature is treated as the mixed mean temperature at the section in question.

Both the idealizations of an adiabatic exchanger and of perfect mixing in each passage are also made in the analysis of two-fluid exchangers. Since these idealizations work out well in practice for two-fluid exchangers they should be equally applicable for the three-fluid exchanger.

The idealization that the exchanger variables $C_{c1}$, $C_{c2}$, $C_h$, $U_1$ and $U_2$ are constant with temperature, and calculated at a "suitably" averaged temperature impose a restriction on the theory developed in this report. In Reference [5] it is clearly demonstrated that this assumption is not valid for a three-fluid exchanger designed for streams of air, oxygen and nitrogen operating at high effectiveness and a wide temperature range as in an air separation plant. However, the idealization of constant thermal properties of the exchanger variables is a necessity in order to be able to solve the problem. If the termal properties vary with temperature, the differential equations become non-linear, and there is no hope of obtaining an analytical solution. One way to obtain a solution to such problems is then to make use of an analog computer. This is demonstrated in Reference [5] for a special case, but it must be emphasized that no general solution to the problem of variable properties three-fluid heat exchangers can be obtained using analog computer techniques.

It appears that the idealization of constant thermal properties of the exchanger variables present a serious limitation to the theory developed in this report. However, it should be realized that by calculating the exchanger variables at a "suitably" averaged temperature, the correct performance prediction can always be made. The only problem is now to find this "suitable" average. It is very easy to estimate this average temperature after the exchanger has been built, but to estimate a "suitable" average temperature at the design stage requires experience.

Another way of handling a design problem where the exchanger variables cannot be assumed to be constant with temperature, and where it is difficult to obtain a good estimate of the averaged temperature at which the exchanger variables are calculated, is described as follows: Analyze sections of the exchanger within which it is known that a suitable average of the thermal properties can be obtained. The outlet temperatures of one section are then the inlet temperatures of the next section. This method will usually involve a considerable amount of work, especially for a counter-flow exchanger where an iterative procedure must be used.

It should be pointed out that the idealization of constant thermal properties also are made in the development of the general theory for two-fluid heat exchangers.

# VII. APPLICATIONS OF THE DESIGN THEORY

As has been pointed out previously, a complete graphical representation of the design theory is out of the question due to the large number of independent exchanger variables. The graphs presented for the temperature effectivenesses for parallel- and counter-flow exchangers give only limited information about their dependence upon the five non-dimensional exchanger variables $C_1^*$, $C_2^*$, $R^*$, $\Delta t_{in}^*$ and $Ntu_1$. Nevertheless, the graphs are useful in preparing a preliminary design, using interpolation techniques, as will be demonstrated. Needless to say, a considerable amount of information about three-fluid exchangers in general can be obtained by inspection of the temperature effectiveness curves.

To demonstrate the practical application of the design theory, three specific examples are considered.

<u>Example No. 1</u>:

The purpose of this example is to demonstrate the use of the temperature effectiveness curves when no interpolation is necessary.

A three-fluid parallel-flow heat exchanger is available which has the following specifications and operating conditions:

$$C_1^* = 0.5,\ C_2^* = 2.0,\ R^* = 0.5,\ Ntu_1 = 0.75$$

$$t_{h_{in}} = 750°F,\ t_{c1_{in}} = 150°F,\ t_{c2_{in}} = 600°F$$

Thus

$$\Delta t_{in}^* = \frac{t_{h_{in}} - t_{c2_{in}}}{t_{h_{in}} - t_{c1_{in}}} = 0.25$$

It is desired to calculate the outlet temperatures of all three fluids.

Entering Fig. 8 or the tabulation in Appendix VII where $C_1^* = 0.5$, $C_2^* = 2.0$, $R^* = 0.5$, and with $\Delta t_{in}^* = 0.25$ and $Ntu_1 = 0.75$, the temperature effectivenesses are found to be:

$$\epsilon_{t,c1} = 0.444 \; ; \; \epsilon_{t,c2} = 0.037$$

and the outlet temperatures are calculated to be:

$$t_{c1_{out}} = t_{c1_{in}} + \epsilon_{t,c1} \cdot (t_{h_{in}} - t_{c1_{in}}) = 416°F$$

$$t_{c2_{out}} = t_{c2_{in}} + \epsilon_{t,c2} \cdot (t_{h_{in}} - t_{c2_{in}}) = 605.5°F$$

From an energy balance consideration on the exchanger:

$$t_{h_{out}} = t_{h_{in}} - C_1^*(t_{c1_{out}} - t_{c1_{in}}) - C_2^*(t_{c2_{out}} - t_{c2_{in}})$$

$$t_{h_{out}} = 606°F$$

As seen, the outlet temperatures are obtained readily, with a minimum of calculations, by using the temperature effectivenesses curves.

In most practical design problems the temperature effectiveness curves cannot be used without making interpolations. The following two examples, in which the calculations are carried out in greater detail than in the first example, demonstrate the interpolation procedure.

Example No. 2:

Problem Statement:

Consider the three-fluid counter-flow exchanger shown schematically in Fig. 24. It is desired to estimate the outlet temperatures of all three fluids when the exchanger is operating under the following conditions:

Cold fluid capacity rates: $\begin{cases} 525 \text{ Btu/hr °F with } t_{in} = 156\text{°F} \\ 280 \text{ Btu/hr °F with } t_{in} = 90\text{°F} \end{cases}$

Hot fluid capacity rate: 465 Btu/hr °F with $t_{in} = 560$°F

The first step is to calculate $\Delta t^*_{in} = (t_{h_{in}} - t_{c2_{in}})/(t_{h_{in}} - t_{c1_{in}})$. "Names" were assigned to the two cold fluid streams such that $\Delta t^*_{in}$ has a value in the range from zero to unity. The cold fluid with the lowest inlet temperature is then named cold fluid No. 1; the one with the highest inlet temperature is then cold fluid No. 2. The total areas of the two heat transfer surfaces are 11.0 ft$^2$ and 16.0 ft$^2$, as shown in Fig. 24. Both these areas are hot fluid side magnitudes. The overall coefficient of heat transfer, U, based on the hot fluid side areas, are estimated to be 32.5 Btu/hr ft$^2$ °F and 35.5 Btu/hr ft$^2$ °F, respectively.

Since the cold fluid with the lowest inlet temperature is named cold fluid No. 1:

$C_{c1} = 280$ Btu/hr °F, $t_{c1_{in}} = 90$°F, $A_1 = 11.0$ ft$^2$,

$U_1 = 32.5$ Btu/hr ft$^2$ °F

$C_{c2} = 525$ Btu/hr °F, $t_{c2_{in}} = 156$°F, $A_2 = 16.0$ ft$^2$,

$U_2 = 35.5$ Btu/hr ft$^2$ °F

$C_h = 465$ Btu/hr °F, $t_{h_{in}} = 560$ °F

The values of the five non-dimensional operating and design parameters can now be calculated:

$$C_1^* \triangleq C_{c1}/C_h = 280/465 = 0.602$$

$$C_2^* \triangleq C_{c2}/C_h = 525/465 = 1.13$$

$$R^* \triangleq A_2U_2/A_1U_1 = (16.0 \cdot 35.5)/(11.0 \cdot 32.5) = 1.59$$

FIG. 24

SKETCH OF THE THREE-FLUID COUNTER-FLOW EXCHANGER ANALYSED IN EXAMPLE NO. 2

FIG. 25

SKETCH OF A TWO TWO-FLUID HEAT EXCHANGER SYSTEM WITH COUNTER-FLOW ARRANGEMENT WHICH IS EQUIVALENT TO THE THREE-FLUID EXCHANGER ANALYSED IN EXAMPLE NO. 2

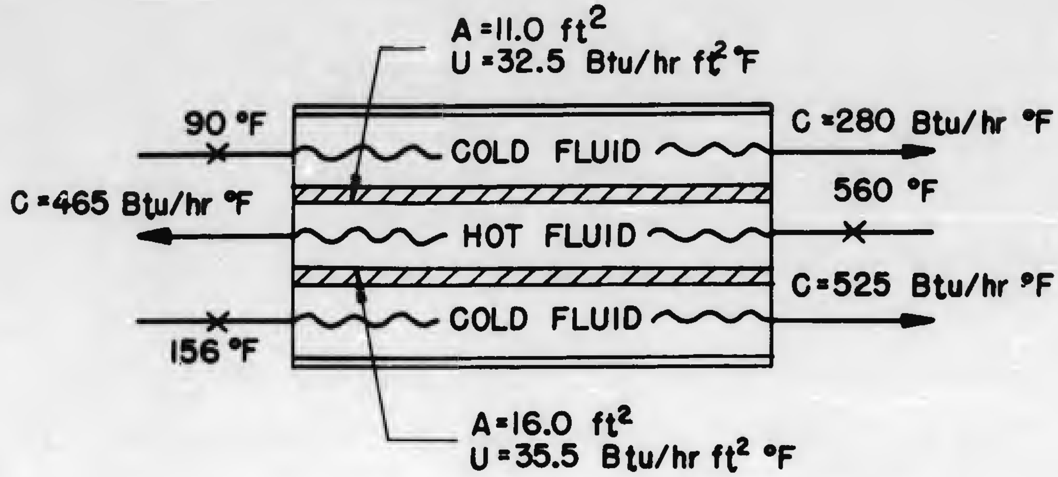

A = 11.0 ft²
U = 32.5 Btu/hr ft² °F
90 °F
COLD FLUID
C = 280 Btu/hr °F
C = 465 Btu/hr °F
560 °F
HOT FLUID
C = 525 Btu/hr °F
COLD FLUID
156 °F
A = 16.0 ft²
U = 35.5 Btu/hr ft² °F

FIG. 24

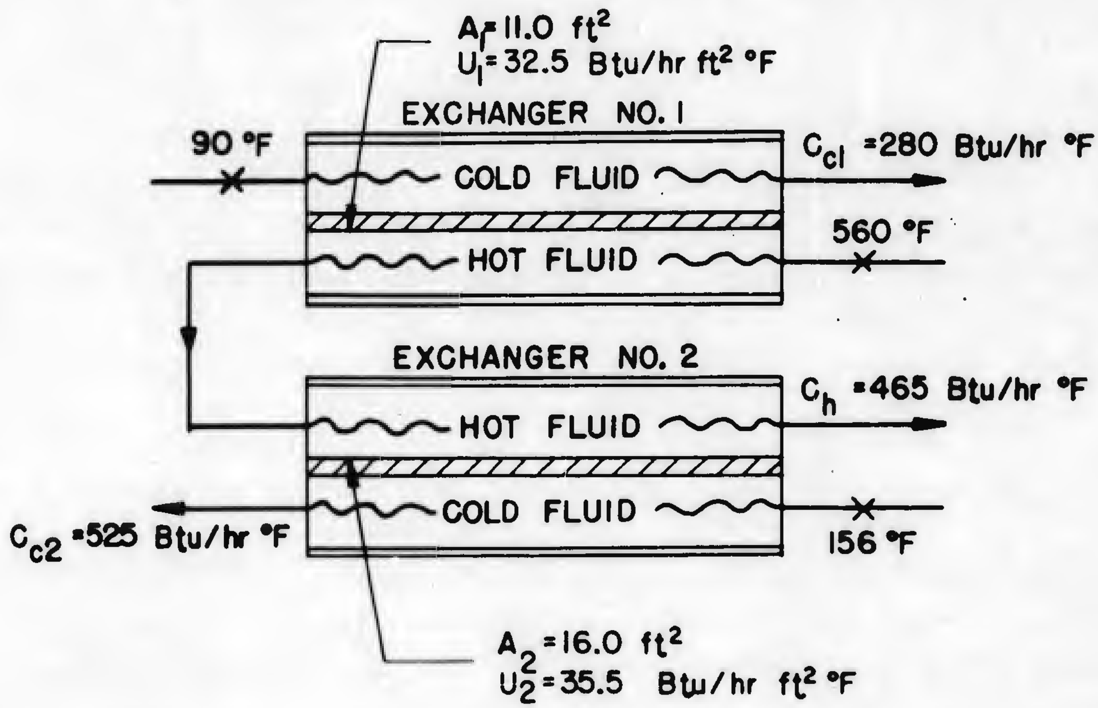

A1 = 11.0 ft²
U1 = 32.5 Btu/hr ft² °F
EXCHANGER NO. 1
90 °F
COLD FLUID
Ccl = 280 Btu/hr °F
560 °F
HOT FLUID
EXCHANGER NO. 2
Ch = 465 Btu/hr °F
HOT FLUID
COLD FLUID
Cc2 = 525 Btu/hr °F
156 °F
A2 = 16.0 ft²
U2 = 35.5 Btu/hr ft² °F

FIG. 25

$$\Delta t^*_{in} \triangleq (t_{h_{in}} - t_{c2_{in}})/(t_{h_{in}} - t_{c1_{in}}) = 0.86$$

$$Ntu_1 \triangleq A_1U_1/C_{c1} = (11.0)(32.5)/(280) = 1.278$$

Figure 26 illustrates the graphical three-way interpolation procedure between $R^*$, $C_1^*$, and $C_2^*$ used for obtaining $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$. This procedure yields:

$$\epsilon_{t,c1} = 0.49 \; ; \; \epsilon_{t,c2} = 0.41$$

Table 1 presents a linear three-way interpolation which yields:

$$\epsilon_{t,c1} = 0.521 \; ; \; \epsilon_{t,c2} = 0.461$$

As seen there is a substantial discrepancy introduced by a linear interpolation.

In order to check the values of the temperature effectivenesses obtained by the linear and the graphical interpolation procedures, $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ were calculated using the equations given on pages 28 and 29 . Due to the complexity of the algebra, it takes approximately one hour, using a sliderule, to calculate the two desired values. The calculation yields:

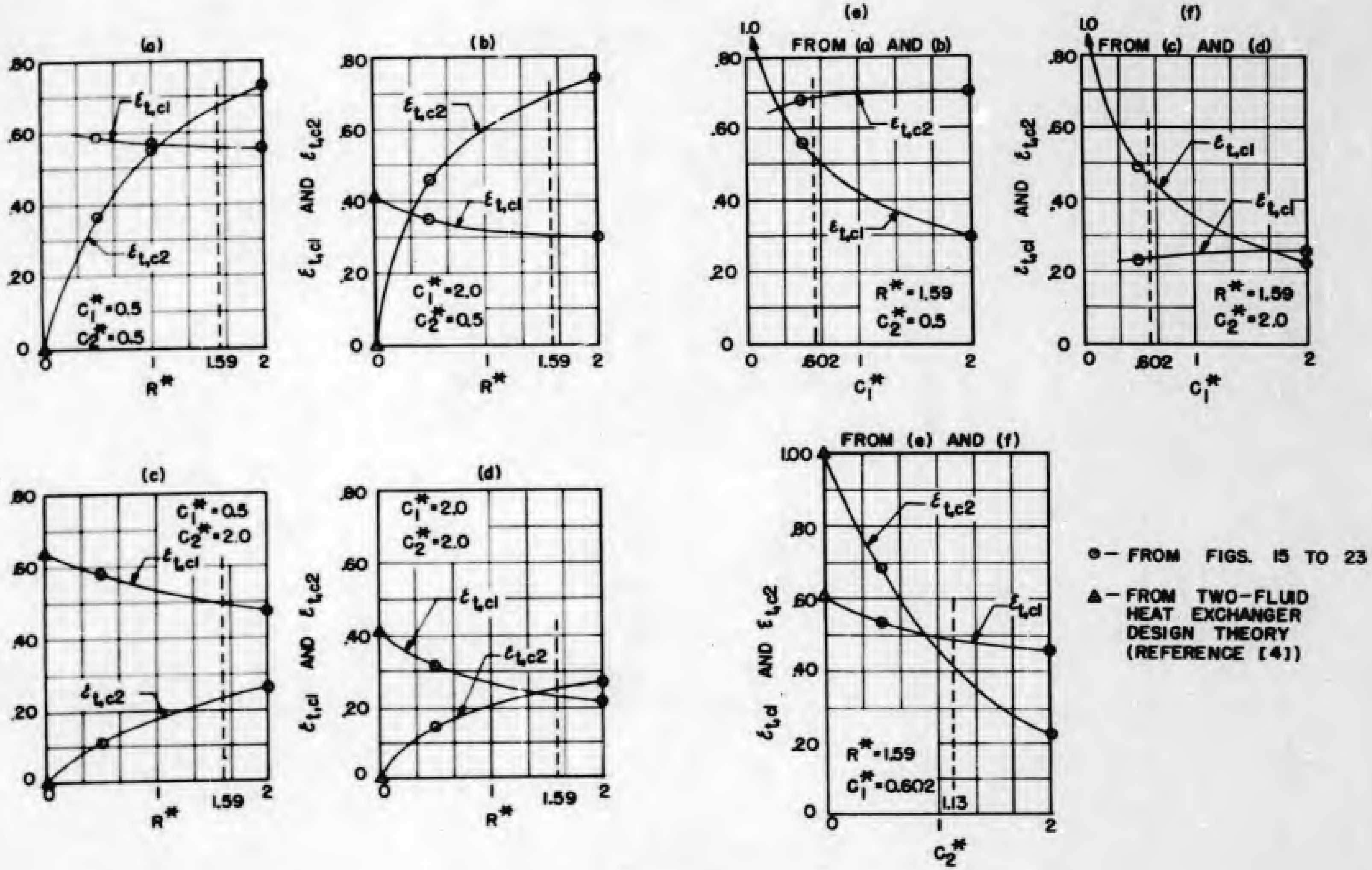
$$\epsilon_{t,c1} = 0.491 \; ; \; \epsilon_{t,c2} = 0.406$$

As seen, the values of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ obtained from the graphical interpolation procedure agree with the values obtained from the numerical calculation, while the values obtained from the linear interpolation procedure are only within 13 per cent of the true values. Since approximately equal amounts of work are involved in carrying out either interpolation procedure, the graphical procedure should be used.

It may seem that the time involved both in carrying out the interpolation and in carrying out the numerical calculations is approximately the same. However, it should

FIG. 26

GRAPHICAL THREE-WAY INTERPOLATION BETWEEN $R^*$, $C_1^*$, $C_2^*$, IN THAT ORDER, FOR THE ILLUSTRATIVE EXAMPLE NO. 2, FIG. 24. THESE GRAPHS DEMONSTRATE THAT A LINEAR INTERPOLATION BETWEEN FIGS. 15 TO 23 RESULTS IN A RATHER POOR APPROXIMATION.



(a)
(b)
(c)
(d)
(e)
(f)
FROM (a) AND (b)
FROM (c) AND (d)
FROM (e) AND (f)
C1* = 0.5
C2* = 0.5
C1* = 2.0
C2* = 0.5
C1* = 0.5
C2* = 2.0
C1* = 2.0
C2* = 2.0
R* = 1.59
C2* = 0.5
R* = 1.59
C2* = 2.0
R* = 1.59
C1* = 0.602
ε t,c1
ε t,c2
ε t,c1 AND ε t,c2
R*
C1*
C2*
1.59
.602
1.13
⊙ – FROM FIGS. 15 TO 23
▲ – FROM TWO-FLUID HEAT EXCHANGER DESIGN THEORY (REFERENCE [4])


FIG. 26

TABLE I

LINEAR INTERPOLATION FOR EXAMPLE NO. 2

| Row No. | | $R^*$ | $C_1^*$ | $C_2^*$ | $\Delta t_{in}^*$ | $Ntu_1$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | FIG. 16 | 0.5 | 0.5 | 0.5 | 0.86 | 1.278 | 0.588 | 0.363 |
| 2 | FIG. 20 | 2.0 | 0.5 | 0.5 | 0.86 | 1.278 | 0.555 | 0.73 |
| 3 | Interpol. on R* | 1.59 | 0.5 | 0.5 | 0.86 | 1.278 | 0.564 | 0.63 |
| 4 | FIG. 18 | 0.5 | 2.0 | 0.5 | 0.86 | 1.278 | 0.35 | 0.46 |
| 5 | FIG. 22 | 2.0 | 2.0 | 0.5 | 0.86 | 1.278 | 0.30 | 0.74 |
| 6 | Interpol. on R* | 1.59 | 2.0 | 0.5 | 0.86 | 1.278 | 0.313 | 0.66 |
| 7 | FIG. 17 | 0.5 | 0.5 | 2.0 | 0.86 | 1.278 | 0.58 | 0.11 |
| 8 | FIG. 21 | 2.0 | 0.5 | 2.0 | 0.86 | 1.278 | 0.475 | 0.267 |
| 9 | Interpol. on R* | 1.59 | 0.5 | 2.0 | 0.86 | 1.278 | 0.503 | 0.225 |
| 10 | FIG. 19 | 0.5 | 2.0 | 2.0 | 0.86 | 1.278 | 0.32 | 0.15 |
| 11 | FIG. 23 | 2.0 | 2.0 | 2.0 | 0.86 | 1.278 | 0.22 | 0.27 |
| 12 | Interpol. on R* | 1.59 | 2.0 | 2.0 | 0.86 | 1.278 | 0.25 | 0.24 |
| 13 | Interpol. between 3 and 6 | 1.59 | 0.602 | 0.5 | 0.86 | 1.278 | 0.547 | 0.632 |
| 14 | Interpol. between 9 and 12 | 1.59 | 0.602 | 2.0 | 0.86 | 1.278 | 0.486 | 0.226 |
| 15 | Interpol. between 13 and 14 | 1.59 | 0.602 | 1.13 | 0.86 | 1.278 | 0.521 | 0.461 |

be emphasized that the numerical calculations are algebraically very complex and that a hard to detect numerical mistake is easily made, while in the graphical procedure an error will usually show up and can then be corrected.

Having obtained the estimates for the temperature effectivenesses, the cold fluid outlet temperatures can be calculated.

$$t_{c1_{out}} = t_{c1_{in}} + \epsilon_{t,c1}(t_{h_{in}} - t_{c1_{in}}) = 90 + 0.49(560 - 90) = 320°F$$

$$t_{c2_{out}} = t_{c2_{in}} + \epsilon_{t,c2}(t_{h_{in}} - t_{c2_{in}}) = 156 + 0.41(560 - 156) = 321°F$$

The hot fluid outlet temperature is now obtained from an energy balance consideration on the exchanger.

$$C_h(t_{h_{in}} - t_{h_{out}}) = C_{c1}(t_{c1_{out}} - t_{c1_{in}}) + C_{c2}(t_{c2_{out}} - t_{c2_{in}})$$

$$t_{h_{out}} = t_{h_{in}} - C_1^*(t_{c1_{out}} - t_{c1_{in}}) - C_2^*(t_{c2_{out}} - t_{c2_{in}})$$

$$t_{h_{out}} = 234°F$$

In order to get an estimate of the thermodynamical performance of the exchanger, Eq. (3) is used for calculating the overall heat transfer effectiveness of the exchanger. Eq. (3) gives $\epsilon_{q,o}$ for the case when $(C_{c1} + C_{c2}) > C_h$ .

$$\epsilon_{q,o} = \frac{[C_1^* \cdot \epsilon_{t,c1} + C_2^* \cdot \Delta t_{in}^* \cdot \epsilon_{t,c2}][1 + 1/R^*]}{[\Delta t_{in}^* + 1/R^*]}$$

Introducing numerical values for the parameters and get:

$$\epsilon_{q,o} = 0.76$$

An overall heat transfer effectiveness of 76 per cent is then achieved in this specific exchanger operating under the specified conditions.

It would be interesting to compare the performance of this three-fluid exchanger with an exchanger system consisting of two two-fluid exchangers having the same heat transfer areas and the same overall conductances (i.e., the same $A_1U_1$ and $A_2U_2$) and the same operating conditions. One such system is illustrated schematically in Fig. 25 with series flow of the hot fluid through the two exchangers. The outlet temperatures of the two cold fluids, and the outlet temperature of the hot fluid after having passed through both heat exchangers, are calculated by means of the method described in Reference [4]. The following outlet temperatures are obtained:

$$t_{c1_{out}} = 384°F$$

$$t_{c2_{out}} = 262°F$$

$$t_{h_{out}} = 263°F$$

For the equivalent three-fluid exchanger the following outlet temperatures were obtained:

$$t_{c1_{out}} = 320°F$$

$$t_{c2_{out}} = 321°F$$

$$t_{h_{out}} = 234°F$$

The temperature effectivenesses for the two two-fluid exchanger system are:

$$\epsilon_{t,c1} \triangleq (t_{c1_{out}} - t_{c1_{in}})/(t_{h_{in}} - t_{c1_{in}}) = 0.625$$

$$\epsilon_{t,c2} \triangleq (t_{c2_{out}} - t_{c2_{in}})/(t_{h_{in}} - t_{c2_{in}}) = 0.262$$

In order to compare the thermodynamical performance of the two two-fluid and the three-fluid exchanger systems, the overall heat transfer effectiveness of the two-fluid exchanger system is calculated by using Eq. (3).

$$\epsilon_{q,o} = \frac{[C_1^* \cdot \epsilon_{t,c1} + C_2^* \cdot \Delta t_{in}^* \cdot \epsilon_{t,c2}][1 + 1/R^*]}{\Delta t_{in}^* + 1/R^*}$$

Substitution of numerical values for the parameters into this equation gives:

$$\epsilon_{q,o_{\text{two-fluid system}}} = 0.69$$

As seen, the two two-fluid exchanger system has in this case an overall effectiveness that is 9 per cent lower than for the three-fluid exchanger. It can then be concluded that in this particular case the savings obtained in shell structure by going to a three-fluid exchanger design is accompanied by a 9 per cent increase in overall heat transfer effectiveness. It must be emphasized that no general conclusion can be drawn from the fact that in this particular case the overall heat transfer effectiveness for a two two-fluid exchanger system is lower than for the equivalent three-fluid exchanger. It may prove that a parallel-flow arrangement for the hot fluid through the two two-fluid exchanger system can be optimised to yield a higher overall heat transfer effectiveness than the equivalent three-fluid exchanger. In making the decision of whether to build a two two-fluid exchanger system or a three-fluid exchanger, factors like savings in shell structure, gain or loss in overall heat exchanger effectiveness must all be considered.

This completes the analysis of this heat exchanger. As has been pointed out previously, the values for the temperature effectiveness expressions obtained by a graphical interpolation can be expected to be within a few per cent of the true values. As seen from the graphical interpolation in Fig. 26, a better interpolation would be possible if an additional set of four temperature effectiveness curves were available for an intermediate value of R*, e.g., R* = 1.0.

An interpolation by judgment will easily lead to erratic results and should only be used when a crude estimate of the temperature effectivenesses is required.

Example No. 3:

In most design problems, the designer, given the operating conditions, is required to estimate the "size" of the exchanger that will meet the specified operating conditions.

Problem Statement:

It is desired to design a three-fluid parallel-flow heat exchanger that will be operating under the following conditions:

Cold fluid capacity rates:

1050 Btu/hr °F, with $t_{in}$ = 35°F and $t_{out}$ = 210°F

625 Btu/hr °F, with $t_{in}$ = 120°F and $t_{out}$ = 175°F

The hot fluid capacity rate is 1850 Btu/hr °F, with $t_{in}$ = 475°F and $t_{out}$ = 357°F.

Fig. 27 illustrates the system that is to be analyzed. In this problem the quantity (A · U ) must be determined for both heat transfer surfaces.

The cold fluid with the lowest inlet temperature is named cold fluid No. 1. Then:

$C_{c1}$ = 1050 Btu/hr °F; $t_{c1_{in}}$ = 35°F; $t_{c1_{out}}$ = 210°F

$C_{c2}$ = 625 Btu/hr °F; $t_{c2_{in}}$ = 120°F; $t_{c2_{out}}$ = 175°F

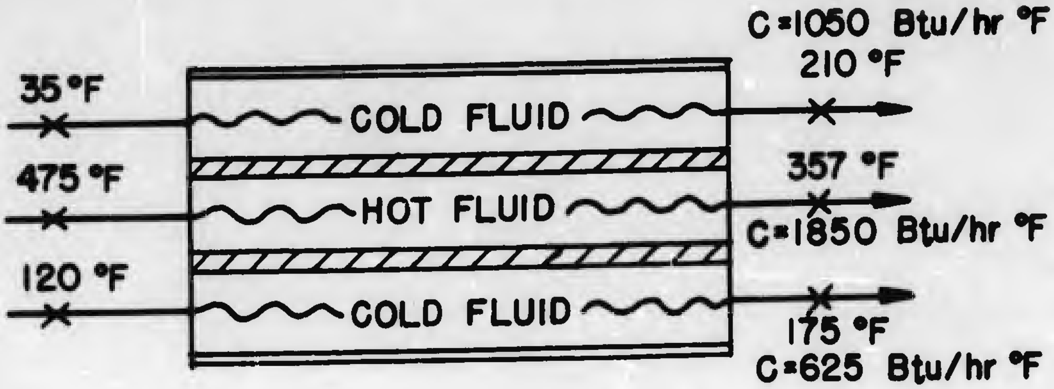
$C_h$ = 1850 Btu/hr °F; $t_{h_{in}}$ = 475°F; $t_{h_{out}}$ = 357°F

FIG. 27

SKETCH OF THE THREE-FLUID
PARALLEL-FLOW EXCHANGER ANALYZED
IN EXAMPLE NO. 3

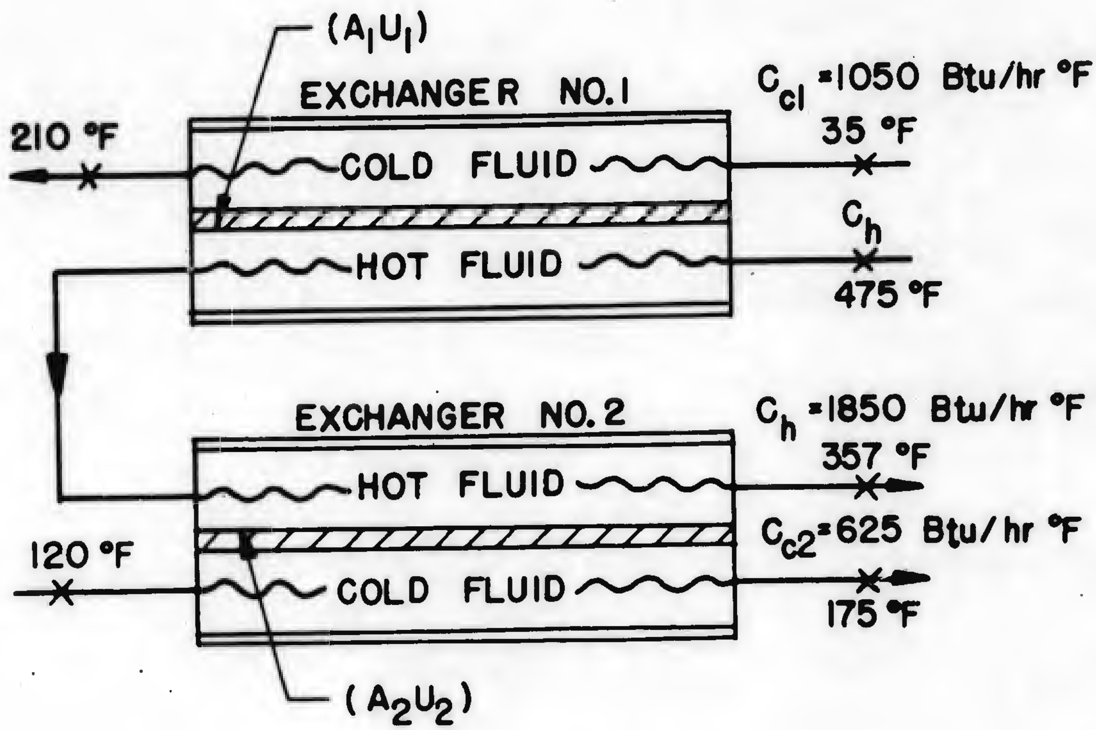
FIG. 28

SKETCH OF A TWO TWO-FLUID EXCHANGER
SYSTEM WITH PARALLEL-FLOW ARRANGEMENT
WHICH IS EQUIVALENT TO THE THREE-FLUID
EXCHANGER ANALYZED IN EXAMPLE NO. 3



C=1050 Btu/hr °F
210 °F
35 °F
COLD FLUID
475 °F
357 °F
HOT FLUID
C=1850 Btu/hr °F
120 °F
COLD FLUID
175 °F
C=625 Btu/hr °F


FIG. 27



($A_1U_1$)
EXCHANGER NO.1
$C_{c1}$=1050 Btu/hr °F
210 °F
35 °F
COLD FLUID
$C_h$
HOT FLUID
475 °F
EXCHANGER NO.2
$C_h$=1850 Btu/hr °F
357 °F
HOT FLUID
$C_{c2}$=625 Btu/hr °F
120 °F
COLD FLUID
175 °F
($A_2U_2$)


FIG. 28

The temperature effectivenesses can now be calculated.

$$\epsilon_{t,c1} \triangleq (t_{c1_{out}} - t_{c1_{in}})/(t_{h_{in}} - t_{c1_{in}}) = 0.398$$

$$\epsilon_{t,c2} \triangleq (t_{c2_{out}} - t_{c2_{in}})/(t_{h_{in}} - t_{c2_{in}}) = 0.155$$

The non-dimensional exchanger variables are:

$$C_1^* \triangleq C_{c1}/C_h = 0.568$$

$$C_2^* \triangleq C_{c2}/C_h = 0.338$$

$$\Delta t_{in}^* \triangleq (t_{h_{in}} - t_{c2_{in}})/(t_{h_{in}} - t_{c1_{in}}) = 0.806$$

In entering Fig. 6, where $R^* = 0.5$, $C_1^* = 0.5$, and $C^*_2 = 0.5$, with $Ntu_1 = 1.0$ and $\Delta t_{in}^* = 0.806$; find $\epsilon_{t,c1} = 0.475$ and $\epsilon_{t,c2} = 0.283$. Since the actual $C_2^* = 0.338$, the actual $\epsilon_{t,c2}$ will be larger than 0.283 for this set of values for the five parameters. Since the desired value of $\epsilon_{t,c2}$ is 0.155, it is clear that $R^*$ must be less than 0.5. At a smaller $R^*$, $\epsilon_{t,c1}$ will be larger and $\epsilon_{t,c2}$ will be smaller for the same values of the other four parameters. Therefore, the following values of $Ntu_1$ and $R^*$ are taken as a first estimate.

$$Ntu_1 = 0.75 \ ; \ R^* = 0.25$$

The temperature effectivenesses are then calculated, using the equations given on page 18, with the following numerical values of the five parameters: $C_1^* = 0.568$, $C_2^* = 0.338$, $\Delta t_{in}^* = 0.806$, $R^* = 0.25$, $Ntu_1 = 0.75$.

The calculations yield:

$$\epsilon_{t,c1} = 0.425 \ ; \ \epsilon_{t,c2} = 0.137$$

As seen, $\epsilon_{t,c1}$ is 6.8 per cent higher than the desired value and $\epsilon_{t,c2}$ is 11 per cent lower than the desired value. For this design example the estimates of $Ntu_1$ and $R^*$ are considered to be satisfactory for a first approximation. However, if a better accuracy is desired, an iterative procedure is used. $Ntu_1$ and $R^*$ are adjusted, the calculation of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ is repeated, and their values compared with the desired values. The number of iterations that are necessary will depend upon the skill of the designer.

Having the value of $Ntu_1$, $(A_1U_1)$ may be calculated.

$$Ntu_1 \triangleq (A_1U_1)/C_{c1}$$

$$\underline{(A_1U_1)} = C_{c1} \cdot Ntu_1 = \underline{788 \text{ Btu/hr °F}}$$

And knowing $(A_1U_1)$, $(A_2U_2)$ may be calculated.

$$R^* \triangleq (A_2U_2)/(A_1U_1)$$

$$\underline{(A_2U_2)} = R^* \, (A_1U_1) = \underline{195 \text{ Btu/hr °F}}$$

A comparison between the "size" of the heat transfer surface in a three-fluid exchanger and the size of the heat transfer surface in a two two-fluid exchanger system, operating under the same conditions, would be appropriate. Such a two two-fluid exchanger system is illustrated in Fig. 28. The two quantities $(A_1U_1)$ and $(A_2U_2)$ for the two two-fluid exchangers are calculated by means of the method described in Reference [4], and the following values were obtained.

$$(A_1U_1) = 630 \text{ Btu/hr °F} : (A_2U_2) = 156 \text{ Btu/hr °F}$$

As seen, the "size" of the heat transfer surface for the two two-fluid exchanger system is considerably less than for the three-fluid exchanger. $(AU)_{total} = (A_1U_1) + (A_2U_2)$ for the two-fluid system is approximately 20 per cent less than the total "size" of the heat transfer area in the three-fluid exchanger. It must again be emphasized that no general conclusion can be drawn from the fact that for this exchanger the total "size" of the heat transfer surface for the two-fluid system is less than for the three-fluid exchanger.

Savings in shell structure, and the size of the total heat transfer surface are the two main factors that must be considered when making the decision on whether to build a two two-fluid exchanger system or a three-fluid exchanger.

In this example it was demonstrated that the temperature effectiveness curves are helpful in obtaining the first estimate of the "size" of the two heat transfer surfaces in a three-fluid heat exchanger.

# VIII. SUMMARY AND CONCLUSIONS

In this thesis the general design theory for three-fluid parallel-flow exchangers, and for one arrangement of counter-flow exchangers has been developed. The performance of the three-fluid heat exchangers has been expressed as two temperature ratios, $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$, which are functions of five other non-dimensionalized exchanger variables; $C_1^*$, $C_2^*$, $R^*$, $\Delta t_{in}^*$ and $Ntu_1$. Due to the large number of exchanger variables a complete graphical description of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ is out of the question. However, a set of graphs is presented for two values (0.5 and 2.0) of $R^*$, $C_1^*$ and $C_2^*$; four values (0.25, 0.50, 0.75 and 1.0) of $\Delta t_{in}^*$ and for $Ntu_1$ in the range from 0 to 5.0. A good understanding and insight into the difficulties encountered in designing three-fluid heat exchangers can be achieved by a thorough inspection of the presented graphs.

In spite of the incompleteness of the graphical representation, the graphs can be extremely helpful to a designer, as some insight into the physical design problems of three-fluid exchangers is gained thereby. This is demonstrated in three design examples. These examples also illustrate the application of the theory to specific design problems.

An overall heat exchanger effectiveness expression is derived. This expression compares the actual heat transfer rate in the exchanger to the thermodynamically limited heat transfer rate; achieved in a counter-flow three-fluid exchanger with infinite heat transfer area, operating under the same conditions. This expression enables the designer to compare his design with the thermodynamically limited design.

Whenever applicable, the three-fluid exchanger has been compared with the two-fluid exchanger.

The idealizations made in deriving the theory may at first glance seem to put heavy restrictions on the practical use of the theory. However, it is shown that these restrictions can be relaxed by a skilled designer.

The design theory has been developed explicitly for two cold fluids and one hot fluid. Clearly, this same theory will apply to the case of two hot fluids and one cold fluid, since the hottest cold fluid is obviously the coldest hot fluid.

In order to test the adequacy of the theoretical analysis, an experimental test was performed on a three-fluid exchanger. This work is described in Appendices III and IV. Excellent agreement was found between the test results and the results predicted from the theoretical analysis, both for parallel- and counter-flow exchangers.

## IX. RECOMMENDATIONS FOR FURTHER WORK

Solutions have been obtained for one arrangement of parallel-flow, and for one arrangement of counter-flow. It may now be interesting to obtain solutions for the flow arrangements shown in Fig. 3.

In many design problems the exchanger parameters cannot be assumed to be constant with temperature. It would be very worthwhile to develop a technique for applying the presented design theory to such problems.

The designer may be interested in maximizing the overall heat exchanger effectiveness of his design for a given transfer area. The values of the independent and dependent exchanger variables -- $R^*$, $C_1^*$, $C_2^*$, $\Delta t_{in}^*$, $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ -- at which $\epsilon_{q,o}$ is a maximum should, therefore, be established.

The feasibility of using the log-mean rate equation approach, as illustrated in Appendix V, to three-fluid heat exchangers having "odd behavior" temperature conditions as in Figs. 18, 19, 22, 23 should be investigated.

## X. REFERENCES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# APPENDIX I

## MATHEMATICAL DEVELOPMENT OF THE TEMPERATURE EFFECTIVENESS EXPRESSIONS FOR THE PARALLEL-FLOW EXCHANGER

The objectives of this appendix are to present a detailed mathematical derivation of the temperature effectiveness expressions, $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$.

The three-fluid parallel-flow exchanger is described schematically in Fig. A1. Temperature conditions in the exchanger are described schematically in Fig. A2 (for $C_{c1} < C_{c2} < C_h$).

From an energy balance consideration on a differential element of the exchanger,

$$dq_1 = C_{c1}dt_{c1} \ ; \ dq_2 = C_{c2}dt_{c2} \ ; \ dq_1 + dq_2 = - C_h dt_h \quad (1)$$

In addition to the energy balance, two rate equations may be written for the heat transfer rates, $dq_1$ and $dq_2$, through the differential areas, $dA_1$ and $dA_2$.

$$dq_1 = U_1 dA_1(t_h - t_{c1}) \ ; \ dq_2 = U_2 dA_2(t_h - t_{c2}) \quad (2)$$

Combination of (1) with (2), and introducing the definition $A^* \triangleq A_1/A_2 = dA_1/dA_2$, yields

$$C_{c1}dt_{c1} + C_{c2}dt_{c2} = - C_h dt_h \quad (3)$$

$$C_{c1}dt_{c1} = U_1 dA_1(t_h - t_{c1}) \quad (4)$$

$$C_{c2}dt_{c2} = U_2 dA_1/A^*(t_h - t_{c2}) \quad (5)$$

FIG. A1

SCHEMATIC DESCRIPTION OF A THREE-FLUID PARALLEL-FLOW HEAT EXCHANGER WITH TWO COLD AND ONE HOT FLUID

FIG. A2

SCHEMATIC DESCRIPTION OF THE TEMPERATURE CONDITIONS IN A THREE-FLUID PARALLEL-FLOW HEAT EXCHANGER WITH TWO COLD AND ONE HOT FLUID

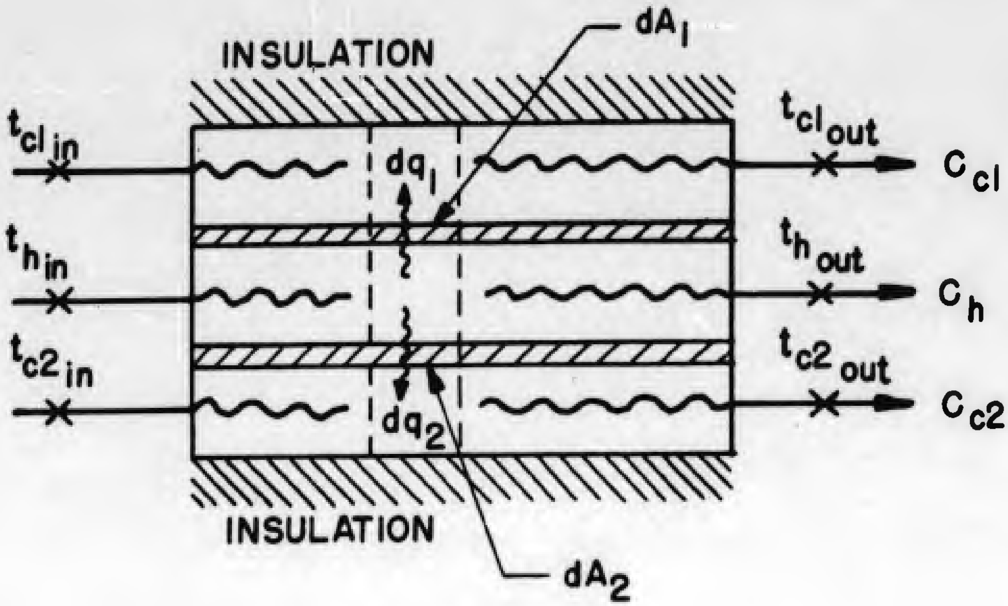

INSULATION
INSULATION
dA1
dA2
tcl in
th in
tc2 in
tcl out
th out
tc2 out
Ccl
Ch
Cc2
dq1
dq2

FIG. A1

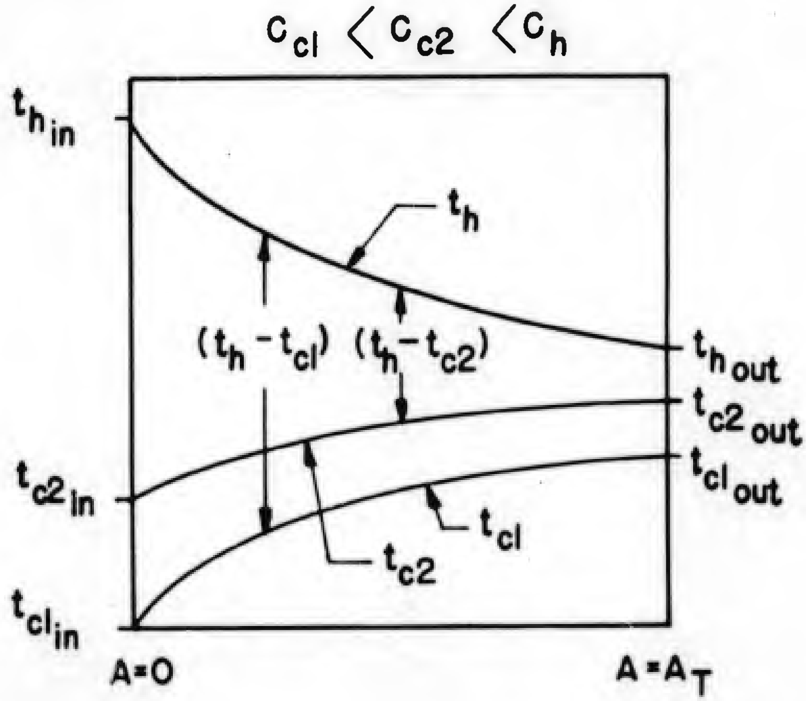

Ccl < Cc2 < Ch
th in
tc2 in
tcl in
th
(th - tcl)
(th - tc2)
th out
tc2 out
tcl out
tcl
tc2
A=0
A=AT

FIG. A2

Equations (3, 4, 5) are a simultaneous set of three linear first order differential equations for the temperatures $t_{c1}$, $t_{c2}$, and $t_h$. The method of solution of this set of equations follows the standard procedure for solving a set of linear differential equations as outlined in References [1, 2, 3].

Rearranging (3), (4), and (5)

$$C_{c1}\frac{dt_{c1}}{dA_1} + C_{c2}\frac{dt_{c2}}{dA_1} + C_h\frac{dt_h}{dA_1} = 0 \qquad (6)$$

$$\frac{C_{c1}}{U_1}\frac{dt_{c1}}{dA_1} + t_{c1} - t_h = 0 \qquad (7)$$

$$\frac{C_{c2}}{U_2}A^*\frac{dt_{c2}}{dA_1} + t_{c2} - t_h = 0 \qquad (8)$$

In operational form

$$C_{c1}Dt_{c1} + C_{c2}Dt_{c2} + C_hDt_h = 0$$

$$(D + \frac{U_1}{C_{c1}})t_{c1} - \frac{U_1}{C_{c1}}t_h = 0$$

$$(D + \frac{U_2}{C_{c2}A^*})t_{c2} - \frac{U_2}{C_{c2}A^*}t_h = 0$$

In order to have a nontrivial solution of the system of equations the determinant of the system must vanish.

$$\Delta = \begin{vmatrix} C_{c1}D & C_{c2}D & C_hD \\ D + \frac{U_1}{C_{c1}} & 0 & -\frac{U_1}{C_{c1}} \\ 0 & D + \frac{U_2}{C_{c2}A^*} & -\frac{U_2}{C_{c2}A^*} \end{vmatrix} = 0$$

Evaluation of the determinant leads to the following characteristic equation.

$$r^3 + [\frac{U_2}{C_{c2}A^*}(1 + \frac{C_{c2}}{C_h}) + \frac{U_1}{C_{c1}}(1 + \frac{C_{c1}}{C_h})]r^2$$

$$+ [\frac{U_2}{C_{c2}A^*}\frac{U_1}{C_{c1}}(1 + \frac{C_{c1}}{C_h} + \frac{C_{c2}}{C_h})]r = 0$$

Solution of this cubic equation in $r$ gives

$$r_1 = 0$$

$$\left.\begin{matrix} r_2 \\ r_3 \end{matrix}\right\} = -\frac{1}{2}[\frac{U_2}{C_{c2}A^*}(1 + \frac{C_{c2}}{C_h}) + \frac{U_1}{C_{c1}}(1 + \frac{C_{c1}}{C_h})]$$

$$\pm \frac{1}{2}\left[[\frac{U_2}{C_{c2}A^*}(1+\frac{C_{c2}}{C_h}) + \frac{U_1}{C_{c1}}(1+\frac{C_{c1}}{C_h})]^2 - 4(\frac{U_2}{C_{c2}A^*})(\frac{U_1}{C_{c1}})(1+\frac{C_{c1}}{C_h}+\frac{C_{c2}}{C_h})\right]^{1/2}$$

And the solutions to the set of three linear differential equations for $t_{c1}$, $t_{c2}$, and $t_h$ are:

$$t_{c1} = a_1 + a_2e^{r_2A_1} + a_3e^{r_3A_1} \qquad (9)$$

$$t_{c2} = b_1 + b_2e^{r_2A_1} + b_3e^{r_3A_1} \qquad (10)$$

$$t_h = d_1 + d_2 e^{r_2 A_1} + d_3 e^{r_3 A_1} \tag{11}$$

The nine constants of integration must be determined by the boundary conditions, which are the inlet temperatures of the three fluids.

i.e.: at $A_1 = 0$: $t_{c1} = t_{c1_{in}}$ ; $t_{c2} = t_{c2_{in}}$ ; $t_h = t_{h_{in}}$

Into Eqs. (9), (10), and (11)

$$t_{c1_{in}} = a_1 + a_2 + a_3 \tag{12}$$

$$t_{c2_{in}} = b_1 + b_2 + b_3 \tag{13}$$

$$t_{h_{in}} = d_1 + d_2 + d_3 \tag{14}$$

Furthermore, the solution for $t_{c1}$, $t_{c2}$ and $t_h$ must satisfy the original differential equations (6), (7) and (8). Substitution of (9), (10) and (11) back into (6), (7) and (8) yields the following eight relations between the nine constants of integration.

$$a_1 - d_1 = 0 \tag{15}$$

$$b_1 - d_1 = 0 \tag{16}$$

$$\frac{C_{c1}}{U_1} a_2 r_2 + a_2 - d_2 = 0 \tag{17}$$

$$\frac{C_{c2}A^*}{U_2} b_2 r_2 + b_2 - d_2 = 0 \tag{18}$$

$$C_{c1} a_2 + C_{c2} b_2 + C_h d_2 = 0 \tag{19}$$

$$\frac{C_{c1}}{U_1} a_3 r_3 + a_3 - d_3 = 0 \tag{20}$$

$$\frac{C_{c2}A^*}{U_2} b_3 r_3 + b_3 - d_3 = 0 \tag{21}$$

$$C_{c1}a_3 + C_{c2}b_3 + C_h d_3 = 0 \tag{22}$$

Six of the constants can now be expressed as functions of the remaining three constants:

From (15) and (16): $a_1 = b_1 = d_1$ (23)

From (17): $d_2 = a_2 \left(\frac{C_{c1}}{U_1} r_2 + 1\right)$ (24)

From (19): $b_2 = - a_2\left[\frac{C_{c1}}{C_{c2}} + \frac{C_h}{C_{c2}} \left(\frac{C_{c1}}{U_1} r_2 + 1\right)\right]$ (25)

From (20): $d_3 = a_3 \left(\frac{C_{c1}}{U_1} r_3 + 1\right)$ (26)

From (22): $b_3 = - a_3\left[\frac{C_{c1}}{C_{c2}} + \frac{C_h}{C_{c2}} \left(\frac{C_{c1}}{U_1} r_3 + 1\right)\right]$ (27)

Substitution of these expressions into (12), (13) and (14) gives:

$$t_{c1_{in}} = a_1 + a_2 + a_3 \tag{28}$$

$$t_{c2_{in}} = a_1 - \left[\frac{C_{c1}}{C_{c2}} + \frac{C_h}{C_{c2}} \left(\frac{C_{c1}}{U_1} r_2+1\right)\right]a_2 - \left[\frac{C_{c1}}{C_{c2}} + \frac{C_h}{C_{c2}} \left(\frac{C_{c1}}{U_1} r_3+1\right)\right]a_3 \tag{29}$$

$$t_{h_{in}} = a_1 + \left(\frac{C_{c1}}{U_1} r_2 + 1\right)a_2 + \left(\frac{C_{c1}}{U_1} r_3 + 1\right)a_3 \qquad (30)$$

Solution of (28), (29) and (30) for the constants $a_1$, $a_2$ and $a_3$ gives:

$$a_1 = t_{c1_{in}} + \frac{(t_{h_{in}} - t_{c1_{in}})\left(1 + \frac{C_{c2}}{C_h}\right) - \frac{C_{c2}}{C_h}(t_{h_{in}} - t_{c2_{in}})}{\left(1 + \frac{C_{c1}}{C_h} + \frac{C_{c2}}{C_h}\right)}$$

$$a_2 = \frac{(t_{h_{in}} - t_{c1_{in}})\left(1 + \frac{C_{c1}}{C_h} + \frac{C_{c2}}{C_h}\right)}{\left(1 + \frac{C_{c1}}{C_h} + \frac{C_{c2}}{C_h}\right)}$$

$$- \frac{\frac{C_{c1}}{U_1} r_3 \left[\frac{C_{c2}}{C_h}(t_{h_{in}} - t_{c2_{in}}) - (t_{h_{in}} - t_{c1_{in}})\left(1 + \frac{C_{c2}}{C_h}\right)\right]}{\left(1 + \frac{C_{c1}}{C_h} + \frac{C_{c2}}{C_h}\right)}$$

$$a_3 = \frac{\frac{C_{c1}}{U_1} r_2 \left[\frac{C_{c2}}{C_h}(t_{h_{in}} - t_{c2_{in}}) - (t_{h_{in}} - t_{c1_{in}})\left(1 + \frac{C_{c2}}{C_h}\right)\right]}{\left(1 + \frac{C_{c1}}{C_h} + \frac{C_{c2}}{C_h}\right)(r_2 - r_3) \cdot \frac{C_{c1}}{U_1}}$$

$$- \frac{(t_{h_{in}} - t_{c1_{in}})}{(r_2 - r_3) \cdot \frac{C_{c1}}{U_1}}$$

Eqs. (23), (25) and (27) give $b_1$, $b_2$, $b_3$ in terms of $a_1$, $a_2$, $a_3$.

$$b_1 = a_1$$

$$b_2 = - a_2 \left[\frac{C_{c1}}{C_{c2}} + \frac{C_h}{C_{c2}} \left(\frac{C_{c1}}{U_1} r_2 + 1\right)\right]$$

$$b_3 = - a_3 \left[\frac{C_{c1}}{C_{c2}} + \frac{C_h}{C_{c2}} \left(\frac{C_{c1}}{U_1} r_3 + 1\right)\right]$$

At the outlet section of the exchanger, i.e., at $A_1 = A_{1T}$, $t_{c1} = t_{c1_{out}}$ and $t_{c2} = t_{c2_{out}}$. Equations (9) and (10) then become:

$$t_{c1_{out}} = a_1 + a_2 e^{r_2 A_{1T}} + a_3 e^{r_3 A_{1T}} \qquad (31)$$

$$t_{c2_{out}} = b_1 + b_2 e^{r_2 A_{1T}} + b_3 e^{r_3 A_{1T}} \qquad (32)$$

Substitution of $a_1$, $a_2$, $a_3$ and $b_1$, $b_2$, $b_3$ into (31) and (32) yields, with some algebraic manipulation, the following equations for the temperature effectiveness expressions $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$:

$$\epsilon_{t,c1} = \frac{1 + C_2^*(1 - \Delta t_{in}^*)}{(1 + C_1^* + C_2^*)} + \frac{\mathrm{EXP}(EX_2) - \mathrm{EXP}(EX_3)}{(B_2 - B_3)} + \frac{[B_2 \cdot \mathrm{EXP}(EX_3) - B_3 \cdot \mathrm{EXP}(EX_2)][C_2^*(\Delta t_{in}^* - 1) - 1]}{[1 + C_1^* + C_2^*][B_2 - B_3]} \qquad (33)$$

$$\epsilon_{t,c2} = \frac{1 - C_1^* \left[\frac{1}{\Delta t_{in}^*} - 1\right]}{[1 + C_1^* + C_2^*]}$$

$$- \left[\frac{1 + C_1^* + B_3}{C_2^*}\right]\left[\frac{B_2[C_2^* - \frac{1}{\Delta t_{in^*}}(1 + C_2^*)] - [1 + C_1^* + C_2^*]\frac{1}{\Delta t_{in}^*}}{[1 + C_1^* + C_2^*][B_2 - B_3]}\right] EXP(EX_3) \qquad (34)$$

$$- \left[\frac{1 + C_1^* + B_3}{C_2^*}\right]\left[\frac{[1 + C_1^* + C_2^*]\frac{1}{\Delta t_{in^*}} - B_3[C_2^* - \frac{1}{\Delta t_{in^*}}(1 + C_2^*)]}{[1 + C_1^* + C_2^*][B_2 - B_3]}\right] EXP(EX_2)$$

Where the following definitions are made:

$$\Delta t_{in}^* \triangleq \frac{t_{h_{in}} - t_{c2_{in}}}{t_{h_{in}} - t_{c1_{in}}}$$

$$C_1^* \triangleq \frac{C_{c1}}{C_h}$$

$$C_2^* \triangleq \frac{C_{c2}}{C_h}$$

$$R^* \triangleq A_2U_2/A_1U_1$$

$$B_2 \triangleq \frac{C_{c1}}{U_1} r_2 = - \frac{1}{2}[R^* \frac{C_1^*}{C_2^*}(1 + C_2^*) + (1 + C_1^* )]$$

$$+ \frac{1}{2}\left[[R^* \frac{C_1^*}{C_2^*}(1 + C_2^*) + (1 + C_1^*)]^2 - 4R^* \frac{C_1^*}{C_2^*}(1 + C_1^* + C_2^*)\right]^{1/2}$$

$$B_3 \triangleq \frac{C_{c1}}{U_1} \cdot r_3 = -\frac{1}{2}[R^* \frac{C_1^*}{C_2^*}(1 + C_2^*) + (1 + C_1^*)]$$

$$- \frac{1}{2}\left[[R^* \frac{C_1^*}{C_2^*}(1 + C_2^*) + (1 + C_1^*)]^2 - 4R^* \frac{C_1^*}{C_2^*}(1 + C_1^* + C_2^*)\right]^{1/2}$$

$$EX_2 \triangleq B_2 \cdot Ntu_1$$

$$EX_3 \triangleq B_3 \cdot Ntu_1$$

$$Ntu_1 \triangleq A_1 U_1 / C_{c1}$$

In this appendix two equations for the temperature effectivenesses, $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$, for parallel-flow exchangers have been derived. $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ are found to be functions of five independent non-dimensional exchanger variables; $C_1^*$, $C_2^*$, $R^*$, $\Delta t_{in}^*$, and $Ntu_1$.

# APPENDIX II

## MATHEMATICAL DEVELOPMENT OF THE TEMPERATURE EFFECTIVENESS EXPRESSIONS FOR THE COUNTER-FLOW EXCHANGER

The objectives of this appendix are to present a detailed mathematical derivation of the temperature effectiveness expressions, $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ for the counter-flow exchanger.

The three-fluid counter-flow exchanger is described schematically in Fig. A3. Temperature conditions in the exchanger are described schematically in Fig. A4 (for $C_{c1} < C_{c2} < C_h$).

The method used for deriving the temperature effectiveness expressions for a counter-flow exchanger is identical to the method used in Appendix I for obtaining $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ for a parallel-flow exchanger. Some of the details in the mathematical development will, therefore, be omitted in deriving $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ for counter-flow exchangers, and the reader is referred to Appendix I for these details.

From an energy balance consideration on a differential element of the exchanger

$$dq_1 = C_{c1}dt_{c1} \; ; \; dq_2 = C_{c2}dt_{c2} \; ; \; dq_1 + dq_2 = C_h dt_h \qquad (35)$$

Rate equations for the heat transfer rates through the differential areas $dA_1$ and $dA_2$

$$dq_1 = U_1 dA_1(t_h - t_{c1}) \; ; \; dq_2 = U_2 dA_2(t_h - t_{c2}) \qquad (36)$$

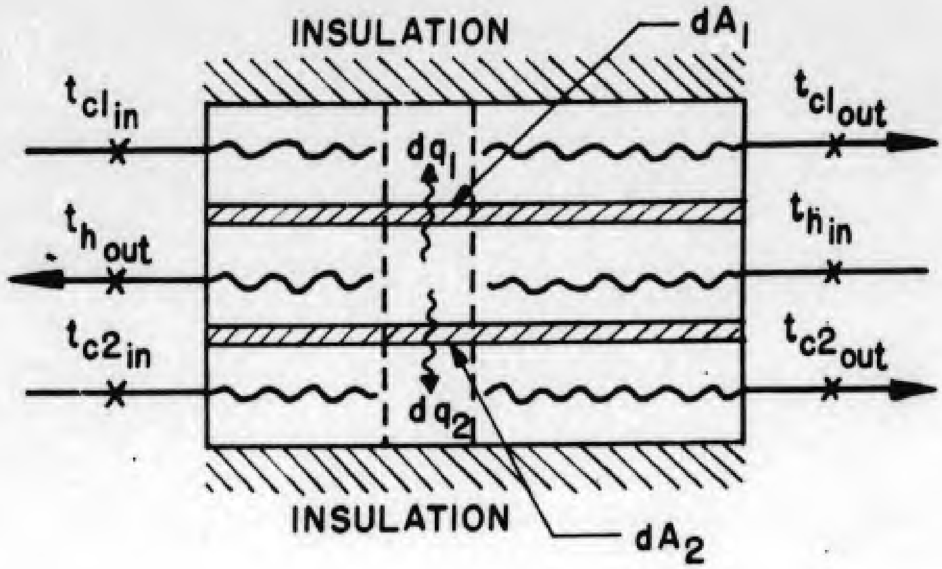
Combination of (35) and (36), and introducing the definition $A^* \triangleq A_1/A_2 = dA_1/dA_2$, yields.

FIG. A3

SCHEMATIC DESCRIPTION OF A THREE-FLUID COUNTER-FLOW HEAT EXCHANGER WITH TWO COLD AND ONE HOT FLUID

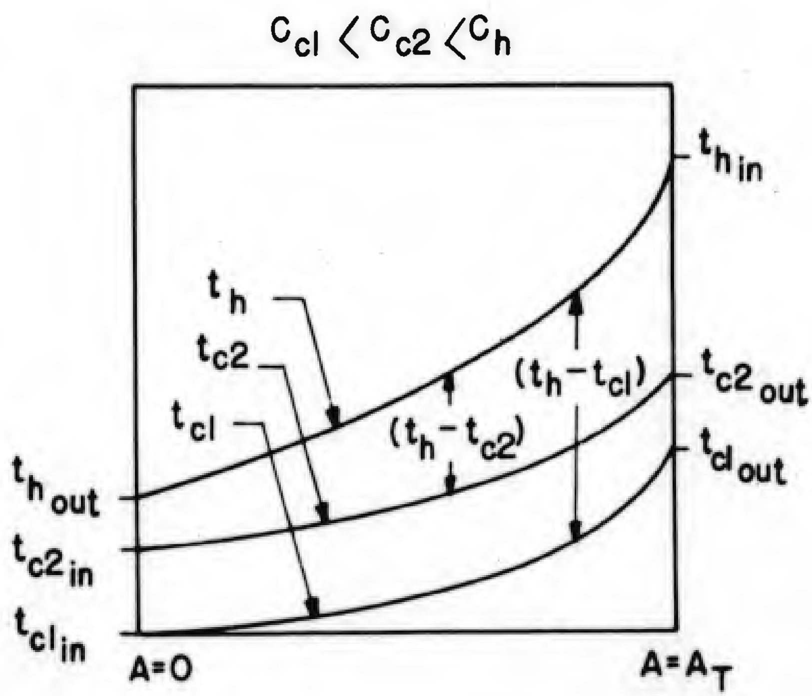
FIG. A4

SCHEMATIC DESCRIPTION OF THE TEMPERATURE CONDITIONS IN A THREE-FLUID COUNTER-FLOW HEAT EXCHANGER WITH TWO COLD AND ONE HOT FLUID



INSULATION
dA1
tcl in
tcl out
dq1
th out
th in
tc2 in
tc2 out
dq2
INSULATION
dA2


FIG. A3



Ccl < Cc2 < Ch
th in
th
tc2
tcl
(th - tcl)
tc2 out
(th - tc2)
tcl out
th out
tc2 in
tcl in
A=0
A=AT


FIG. A4

$$C_{c1}dt_{c1} + C_{c2}dt_{c2} = C_h dt_h \tag{37}$$

$$C_{c1}dt_{c1} = U_1 dA_1(t_h - t_{c1}) \tag{38}$$

$$C_{c2}dt_{c2} = U_2 \frac{dA_1}{A^*}(t_h - t_{c2}) \tag{39}$$

Rearranging (37), (38) and (39).

$$C_{c1}\frac{dt_{c1}}{dA_1} + C_{c2}\frac{dt_{c2}}{dA_1} - C_h\frac{dt_h}{dA_1} = 0 \tag{40}$$

$$\frac{C_{c1}}{U_1}\frac{dt_{c1}}{dA_1} + t_{c1} - t_h = 0 \tag{41}$$

$$\frac{C_{c2}A^*}{U_2}\cdot\frac{dt_{c2}}{dA_1} + t_{c2} - t_h = 0 \tag{42}$$

In operational form:

$$C_{c1}Dt_{c1} + C_{c2}Dt_{c2} - C_h Dt_h = 0$$

$$\left(D + \frac{U_1}{C_{c1}}\right)t_{c1} - \frac{U_1}{C_{c1}}t_h = 0$$

$$\left(D + \frac{U_2}{C_{c2}A^*}\right)t_{c2} - \frac{U_2}{C_{c2}A^*}t_h = 0$$

In order to have a non-trivial solution the determinant of the system must vanish.

$$\Delta = \begin{vmatrix} C_{c1}D & C_{c2}D & -C_h D \\ D + \frac{U_1}{C_{c1}} & 0 & -\frac{U_1}{C_{c1}} \\ 0 & D + \frac{U_2}{C_{c2}A^*} & -\frac{U_2}{C_{c2}A^*} \end{vmatrix} = 0$$

Evaluation of this determinant leads to the following characteristic equation.

$$r^3 + [\frac{U_2}{C_{c2}A^*}(1 - \frac{C_{c2}}{C_h}) + \frac{U_1}{C_{c1}}(1 - \frac{C_{c1}}{C_h})]r^2$$

$$+ [\frac{U_2}{C_{c2}A^*}\frac{U_1}{C_{c1}}(1 - \frac{C_{c1}}{C_h} - \frac{C_{c2}}{C_h})]r = 0$$

Solution of this cubic equation in $r$ gives.

$$r_1 = 0$$

$$\left.\begin{matrix} r_2 \\ r_3 \end{matrix}\right\} = -\frac{1}{2}[\frac{U_2}{C_{c2}A^*}(1 - \frac{C_{c2}}{C_h}) + \frac{U_1}{C_{c1}}(1 - \frac{C_{c1}}{C_h})]$$

$$\pm \frac{1}{2}\left[[\frac{U_2}{C_{c2}A^*}(1 - \frac{C_{c2}}{C_h}) + \frac{U_1}{C_{c1}}(1 - \frac{C_{c1}}{C_h})]^2 - 4\frac{U_2}{C_{c2}A^*}\cdot\frac{U_1}{C_{c1}}(1 - \frac{C_{c1}}{C_h} - \frac{C_{c2}}{C_h})\right]^{1/2}$$

And the solutions to the set of three linear differential equations for $t_{c1}$, $t_{c2}$ and $t_h$ are:

$$t_{c1} = k_1 + k_2 e^{r_2 A_1} + k_3 e^{r_3 A_1} \quad (43)$$

$$t_{c2} = \ell_1 + \ell_2 e^{r_2 A_1} + \ell_3 e^{r_3 A_1} \quad (44)$$

$$t_h = m_1 + m_2 e^{r_2 A_1} + m_3 e^{r_3 A_1} \quad (45)$$

The nine constants of integration must be determined by the boundary conditions, which are the inlet temperatures of the three fluids.

i.e.: at $A_1 = 0$: $t_{c1} = t_{c1_{in}}$ ; $t_{c2} = t_{c2_{in}}$

at $A_1 = A_{1T}$: $t_h = t_{h_{in}}$

Into Eqs. (43), (44) and (45).

$$t_{c1_{in}} = k_1 + k_2 + k_3 \tag{46}$$

$$t_{c2_{in}} = \ell_1 + \ell_2 + \ell_3 \tag{47}$$

$$t_{h_{in}} = m_1 + m_2 e^{r_2 A_{1T}} + m_3 e^{r_3 A_{1T}} \tag{48}$$

Substitution of (43), (44) and (45) back into the original differential equations (40), (41), and (42) yields the following eight relations between the nine constants of integration.

$$k_1 - m_1 = 0 \tag{49}$$

$$\ell_1 - m_1 = 0 \tag{50}$$

$$\frac{C_{c1}}{U_1} k_2 r_2 + k_2 - m_2 = 0 \tag{51}$$

$$\frac{C_{c2}A^*}{U_2} \ell_2 r_2 + \ell_2 - m_2 = 0 \tag{52}$$

$$C_{c1}k_2 + C_{c2}\ell_2 - C_h m_2 = 0 \tag{53}$$

$$\frac{C_{c1}}{U_1} k_3 r_3 + k_3 - m_3 = 0 \tag{54}$$

$$\frac{C_{c2}A^*}{U_2} \ell_3 r_3 + \ell_3 - m_3 = 0 \tag{55}$$

$$C_{c1}k_3 + C_{c2}\ell_3 - C_h m_3 = 0 \tag{56}$$

Six of the constants can now be expressed as functions of the remaining three constants.

From (49) and (50): $k_1 = \ell_1 = m_1$ (57)

From (51): $m_2 = k_2\left(\frac{C_{c1}}{U_1} r_2 + 1\right)$ (58)

From (53): $\ell_2 = k_2\left[\frac{C_h}{C_{c2}}\left(\frac{C_{c1}}{U_1} r_2 + 1\right) - \frac{C_{c1}}{C_{c2}}\right]$ (59)

From (54): $m_3 = k_3\left(\frac{C_{c1}}{U_1} r_3 + 1\right)$ (60)

From (56): $\ell_3 = k_3\left[\frac{C_h}{C_{c2}}\left(\frac{C_{c1}}{U_1} r_3 + 1\right) - \frac{C_{c1}}{C_{c2}}\right]$ (61)

Substitution of these expressions for the constants of integration into (46), (47) and (48) gives

$$t_{c1_{in}} = k_1 + k_2 + k_3 \qquad (62)$$

$$t_{c2_{in}} = k_1 + \left[\frac{C_h}{C_{c2}}\left(\frac{C_{c1}}{U_1}r_2+1\right) - \frac{C_{c1}}{C_{c2}}\right]k_2 + \left[\frac{C_h}{C_{c2}}\left(\frac{C_{c1}}{U_1}r_3+1\right) - \frac{C_{c1}}{C_{c2}}\right]k_3 \qquad (63)$$

$$t_{h_{in}} = k_1 + \left[\left(\frac{C_{c1}}{U_1}r_2+1\right) e^{r_2 \cdot A_{1T}}\right]k_2 + \left[\left(\frac{C_{c1}}{U_1}r_3+1\right) e^{r_3 \cdot A_{1T}}\right] k_3 \qquad (64)$$

Solution of (62), (63) and (64) for the constants $k_1$, $k_2$ and $k_3$ gives.

$$k_3 = \frac{KUP}{KLOW}$$

$$k_2 = \frac{(t_{h_{in}} - t_{c1_{in}}) - [(\frac{C_{c1}}{U_1} r_3 + 1)e^{r_3 A_{1T}} - 1] \cdot k_3}{[(\frac{C_{c1}}{U_1} r_2 + 1)e^{r_2 A_{1T}} - 1]}$$

$$k_1 = t_{c1_{in}} - \frac{(t_{h_{in}} - t_{c1_{in}}) - [(\frac{C_{c1}}{U_1} r_3 + 1)e^{r_3 A_{1T}} - (\frac{C_{c1}}{U_1} r_2 + 1)e^{r_2 A_{1T}}]k_3}{[(\frac{C_{c1}}{U_1} r_2 + 1)e^{r_2 A_{1T}} - 1]}$$

**Where:**

$$KUP = \frac{C_{c2}}{C_h} (t_{h_{in}} - t_{c2_{in}})[(\frac{C_{c1}}{U_1} r_2 + 1)e^{r_2 A_{1T}} - 1]$$

$$+ (t_{h_{in}} - t_{c1_{in}})[(\frac{C_{c1}}{U_1} r_2 + 1)(1 - \frac{C_{c2}}{C_h} e^{r_2 A_{1T}}) - \frac{C_{c1}}{C_h}$$

$$KLOW = \frac{C_{c1}^2}{U_1^2} r_2 r_3 [e^{r_3 A_{1T}} - e^{r_2 A_{1T}}]$$

$$+ \frac{C_{c1}}{U_1} r_3 [1 - e^{r_2 A_{1T}}] - \frac{C_{c1}}{U_1} r_2 [1 - e^{r_3 A_{1T}}]$$

$$+ [1 - \frac{C_{c1}}{C_h} - \frac{C_{c2}}{C_h}][(\frac{C_{c1}}{U_1} r_3 + 1)e^{r_3 A_{1T}} - (\frac{C_{c1}}{U_1} r_2 + 1)e^{r_2 A_{1T}}]$$

Equations (57), (59) and (61) give $\ell_1$, $\ell_2$, $\ell_3$, in terms of $k_1$, $k_2$, $k_3$.

$$\ell_1 = k_1$$

$$\ell_2 = [\frac{C_h}{C_{c2}} (\frac{C_{c1}}{U_1} r_2 + 1) - \frac{C_{c1}}{C_{c2}}]k_2$$

$$\ell_3 = [\frac{C_h}{C_{c2}} (\frac{C_{c1}}{U_1} r_3 + 1) - \frac{C_{c1}}{C_{c2}}] k_3$$

At the cold fluid outlet section of the exchanger $A_1 = A_{1T}$ and $t_{c1} = t_{c1_{out}}$, $t_{c2} = t_{c2_{out}}$.

Equations (43) and (44) then become

$$t_{c1_{out}} = k_1 + k_2 e^{r_2 A_{1T}} + k_3 e^{r_3 A_{1T}} \qquad (65)$$

$$t_{c2_{out}} = \ell_1 + \ell_2 e^{r_2 A_{1T}} + \ell_3 e^{r_3 A_{1T}} \qquad (66)$$

Substitution of $k_1$, $k_2$, $k_3$ and $\ell_1$, $\ell_2$, $\ell_3$ into (65) and (66) yields, with some algebraic manipulation, the following equations for the temperature effectiveness expressions $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$:

$$\epsilon_{t,c1} = \frac{[1 - \mathrm{EXP}(EX_2)]}{[1 - (B_2 + 1) \cdot \mathrm{EXP}(EX_2)]} + \left[\frac{B_3 \cdot \mathrm{EXP}(EX_3)[1 - \mathrm{EXP}(EX_2)] - B_2 \cdot \mathrm{EXP}(EX_2)[1 - \mathrm{EXP}(EX_3)]}{[(B_2 + 1) \cdot \mathrm{EXP}(EX_2) - 1]}\right] \cdot K \qquad (67)$$

$$\epsilon_{t,c2} = 1 - \frac{1}{\Delta t^*_{in}} + \frac{\frac{1}{\Delta t_{in^*}}[(B_2+1) \cdot EXP(EX_2) - C^*_1 \cdot EXP(EX_2) - C^*_2]}{C^*_2[(B_2 + 1) \cdot EXP(EX_2) - 1]}$$

$$+ \left[\frac{[(B_3 + 1)\ EXP(EX_3) - (B_2 + 1)\ EXP(EX_2)]}{[B_2 + 1)EXP(EX_2) - 1]}\right] \cdot \frac{K}{\Delta t_{in^*}}$$

$$- \left[\frac{EXP(EX_2)[B_2 + 1 - C^*_1][(B_3 + 1)EXP(EX_3) - 1]}{C^*_2[(B_2 + 1)EXP(EX_2) - 1]}\right] \cdot \frac{K}{\Delta t^*_{in}}$$

$$+ \left[\frac{[(B_3+1)EXP(EX_3) - C^*_1 \cdot EXP(EX_3)][(B_2+1)EXP(EX_2) - 1]}{C^*_2[(B_2 + 1)EXP(EX_2) - 1]}\right] \cdot \frac{K}{\Delta t^*_{in}}$$

(68)

Where:

$$B_2 = -\frac{1}{2}[R^* \frac{C^*_1}{C^*_2}(1 - C^*_2) + (1 - C^*_1)]$$

$$+ \frac{1}{2}\left[[R^* \frac{C^*_1}{C^*_2}(1 - C^*_2) + (1 - C^*_1)]^2 - 4R^* \frac{C^*_1}{C^*_2}(1 - C^*_1 - C^*_2)\right]^{1/2}$$

$$B_3 = -\frac{1}{2}[R^* \frac{C^*_1}{C^*_2}(1 - C^*_2) + (1 - C^*_1)]$$

$$- \frac{1}{2}\left[[R^* \frac{C^*_1}{C^*_2}(1 - C^*_2) + (1 - C^*_1)]^2 - 4R^* \frac{C^*_1}{C^*_2}(1 - C^*_1 - C^*_2)\right]^{1/2}$$

$$EX_2 = B_2 \cdot Ntu_1$$

$$EX_3 = B_3 \cdot Ntu_1$$

$$K = \frac{C_2^* \cdot \Delta t_{in}^* [(B_2+1)EXP(EX_2)-1] + [(B_2+1)(1-C_2^* \cdot EXP(EX_2))-C_1^*]}{\left[\begin{array}{l} B_2 \cdot B_3[EXP(EX_3)-EXP(EX_2)] + B_3[1-EXP(EX_2)] - B_2[1-EXP(EX_3)] \\ \qquad + [1-C_1^*-C_2^*][(B_3+1)EXP(EX_3) - (B_2+1)EXP(EX_2)] \end{array}\right]}$$

The two equations for $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ for the counter-flow exchanger are indeterminate for the case when $(C_1^* + C_2^*) = 1.0$. In order to obtain a solution for $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ when $(C_1^* + C_2^*) = 1.0$, it is necessary to go back to the solution of the characteristic equation on page 73.

When $(C_1^* + C_2^*) = 1.0$, $r_1 = 0$ and $r_2 = 0$, and the solutions to the set of linear differential equations are:

$$t_{c1} = k_1 + k_2 A_1 + k_3 e^{r_3 A_1} \tag{69}$$

$$t_{c2} = \ell_1 + \ell_2 A_1 + \ell_3 e^{r_3 A_1} \tag{70}$$

$$t_h = m_1 + m_2 A_1 + m_3 e^{r_3 A_1} \tag{71}$$

The remainder of the analysis is now identical to the analysis performed for the case when $(C_1^* + C_2^*)$ are different from 1.0.

Applying the boundary conditions

$$t_{c1_{in}} = k_1 + k_3 \tag{72}$$

$$t_{c2_{in}} = \ell_1 + \ell_3 \tag{73}$$

$$t_{h_{in}} = m_1 + m_2 A_{1T} + m_3 e^{r_3 A_{1T}} \tag{74}$$

Substitution of (69), (70) and (71) back into the original differential equations yields the following eight relations between the nine constants of integration:

$$k_2 - m_2 = 0 \tag{75}$$

$$\ell_2 - m_2 = 0 \tag{76}$$

$$C_{c1}k_2 + C_{c2}\ell_2 - C_h m_2 = 0 \tag{77}$$

$$C_{c1}k_3 + c_{c2}\ell_3 - C_h m_3 = 0 \tag{78}$$

$$\frac{C_{c1}}{U_1} k_2 + k_1 - m_1 = 0 \tag{79}$$

$$\frac{C_{c1}}{U_1} r_3 k_3 + k_3 - m_3 = 0 \tag{80}$$

$$\frac{C_{c2}}{U_2} A^* \ell_2 + \ell_1 - m_1 = 0 \tag{81}$$

$$\frac{C_{c2}}{U_2} A^* r_3 \ell_3 + \ell_3 - m_3 = 0 \tag{82}$$

Six of the constants are now expressed as functions of the remaining three constants.

$$k_2 = m_2 = \ell_2 \tag{83}$$

$$m_1 = k_1 + \frac{C_{c1}}{U_1} k_2 \tag{84}$$

$$m_3 = \left(\frac{C_{c1}}{U_1} r_3 + 1\right)k_3 \tag{85}$$

$$\ell_3 = \left[\frac{C_h}{C_{c2}} \left(\frac{C_{c1}}{U_1} r_3 + 1\right) - \frac{C_{c1}}{C_{c2}}\right]k_3 \tag{86}$$

$$\ell_1 = k_1 + \frac{C_{c1}}{U_1} k_2 - \frac{C_{c2}}{U_2} A^* k_2 \tag{87}$$

Substitution of these expressions for the constants of integration into (72), (73) and (74) gives:

$$t_{c1_{in}} = k_1 + k_3 \tag{88}$$

$$t_{c2_{in}} = k_1 + \frac{C_{c1}}{U_1} k_2 - \frac{C_{c2}}{U_2} A^* k_2 + \left[\frac{C_h}{C_{c2}}\left(\frac{C_{c1}}{U_1} r_3 + 1\right) - \frac{C_{c1}}{C_{c2}}\right] k_3 \tag{89}$$

$$t_{h_{in}} = k_1 + \frac{C_{c1}}{U_1} k_2 + k_2 \cdot A_{1T} + \left[\frac{C_{c1}}{U_1} r_3 + 1\right] k_3 \tag{90}$$

Solving (88), (89) and (90) for the three constants $k_1$, $k_2$, and $k_3$:

$$k_3 = \frac{(t_{h_{in}} - t_{c1_{in}})\left[\frac{1}{R^*} + Ntu_1\right] - (t_{h_{in}} - t_{c2_{in}})\left[1 + Ntu_1\right]}{\left[\begin{array}{l} \left[\left(\frac{C_{c1}}{U_1} r_3 + 1\right) e^{r_3 A_{1T}} - 1\right]\left[\frac{1}{R^*} + Ntu_1\right] \\ - \left[\left(\frac{C_{c1}}{U_1} r_3 + 1\right)\left(e^{r_3 A_{1T}} - \frac{C_h}{C_{c2}}\right) + \frac{C_{c1}}{C_{c2}}\right]\left[1 + Ntu_1\right] \end{array}\right]}$$

$$k_2 = \frac{(t_{h_{in}} - t_{c1_{in}}) - \left[\left(\frac{C_{c1}}{U_1} r_3 + 1\right) e^{r_3 A_{1T}} - 1\right] k_3}{\left[\frac{C_{c1}}{U_1} + A_{1T}\right]}$$

$$k_1 = t_{c1_{in}} - k_3$$

Equations (83), (86) and (87) give $\ell_1$, $\ell_2$, $\ell_3$ in terms of $k_1$, $k_2$, $k_3$.

$$\ell_1 = k_1 + \left(\frac{C_{c1}}{U_1} - \frac{C_{c2}}{U_2} A^*\right) k_2$$

$$\ell_2 = k_2$$

$$\ell_3 = \left[\frac{C_h}{C_{c2}}\left(\frac{C_{c1}}{U_1} r_3 + 1\right) - \frac{C_{c1}}{C_{c2}}\right] k_3$$

At the cold fluid outlet section $A_1 = A_{1T}$ and $t_{c1} = t_{c1_{out}}$, $t_{c2} = t_{c2_{out}}$. Equations (69) and (70) then become:

$$t_{c1_{out}} = k_1 + k_2 A_{1T} + k_3 e^{r_3 A_{1T}} \tag{91}$$

$$t_{c2_{out}} = \ell_1 + \ell_2 A_{1T} + \ell_3 e^{r_3 A_{1T}} \tag{92}$$

Substitution of $k_1$, $k_2$, $k_3$ and $\ell_1$, $\ell_2$, $\ell_3$ into (91) and (92) yields, with some algebraic manipulation, the following equations for $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ when $(C^*_1 + C^*_2) = 1.0$:

$$\epsilon_{t,c1} = \frac{Ntu_1}{1 + Ntu_1} + \left[ \mathbf{EXP}(\mathbf{EX}_3) - 1 - \frac{[(B_3+1)\mathbf{EXP}(\mathbf{EX}_3)-1]Ntu_1}{[1 + Ntu_1]} \right] \cdot K \tag{93}$$

$$\epsilon_{t,c2} = [1 - \frac{1}{\Delta t^*_{in}}] + [1 - \frac{1}{R^*[1 + Ntu_1]}] \frac{1}{\Delta t^*_{in}} + \left[ [\frac{B_3+1-C^*_1}{C^*_2}]\mathbf{EXP}(\mathbf{EX}_3)-1-[1- \frac{1}{R^*[1+Ntu_1]}][(B_3+1)\mathbf{EXP}(\mathbf{EX}_3)-1] \right] \frac{K}{\Delta t^*_{in}} \tag{94}$$

where:

$$B_3 = \frac{C_{c1}}{U_1} r_3 = - [R^* \frac{C^*_1}{C^*_2} (1 - C^*_2) + (1 - C^*_1)]$$

$$\mathbf{EX}_3 = r_3 A_{1T} = B_3 \cdot Ntu_1$$

$$K = \frac{[\frac{1}{R^*} + Ntu_1] - \Delta t^*_{in}[1 + Ntu_1]}{[(B_3+1)EXP(EX_3)-1][\frac{1}{R^*} + Ntu_1] - [(B_3+1)(EXP(EX_3) - \frac{1}{C^*_2}) + \frac{C^*_1}{C^*_2}][1+Ntu_1]}$$

It should be noted that when $C^*_1 = C^*_2 = 0.5$, $R^* = 1$, $\Delta t^*_{in} = 1$, then the three-fluid exchanger becomes a two-fluid exchanger with $C_c/C_h = 1.0$. The effectiveness expression for a two-fluid counter-flow exchanger with $C_c/C_h = 1.0$ is given in Reference [4].

$$\epsilon = \frac{Ntu}{1 + Ntu}$$

It is easily demonstrated that when $C^*_1 = C^*_2 = 0.5$, $R^* = 1$, $\Delta t^*_{in} = 1$; the temperature effectiveness expressions for the three-fluid counter-flow exchanger with $(C^*_1 + C^*_2) = 1.0$ becomes:

$$\epsilon_{t,c1} = \epsilon_{t,c2} = \frac{Ntu_1}{1 + Ntu_1}$$

This is identical to the effectiveness expression for two-fluid exchangers.

In this appendix the temperature effectiveness expressions for a three-fluid counter-flow exchanger have been derived. There are two cases which must be distinguished, $C^*_1 + C^*_2 \neq 1.0$ and $C^*_1 + C^*_2 = 1.0$. In both cases $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ are found to be functions of five independent non-dimensional exchanger variables: $C^*_1$, $C^*_2$, $R^*$, $\Delta t^*_{in}$ and $Ntu_1$.

## APPENDIX III

## VERIFICATION OF THE PARALLEL-FLOW HEAT EXCHANGER DESIGN THEORY BY EXPERIMENT

The objective of the experiment described here was to perform an experimental test for the adequacy of the theoretical analysis for the three-fluid parallel-flow exchanger.

The test was performed on a three-fluid exchanger available in the university laboratory. This water-to-water heat exchanger consists of three concentric tubes; the outer tube is insulated with 85 per cent magnesia insulation. The exchanger may be connected in parallel- or counter-flow arrangement by means of external hose connections. Fig. A5 shows a sketch of the three-fluid exchanger, and Fig. A6 shows a flow diagram of the parallel-flow test set-up.

More specifically, a description of the heat exchanger is:

| Tube | O.D. | I.D. | Wall Thickness | Free Flow Area | Hydraulic Dia. |
|---|---|---|---|---|---|
| #1 | 0.625 in. | 0.570 in. | 0.0275 in. | 0.00177 $ft^2$ | 0.0475 ft |
| #2 | 0.875 in. | 0.805 in. | 0.035 in. | 0.00141 $ft^2$ | 0.0150 ft |
| #3 | 1.125 in. | 1.055 in. | 0.035 in. | 0.00189 $ft^2$ | 0.0150 ft |

Effective length, i.e., length with fluid on both sides:

Tube #1: 127.4 in.

Tube #2: 120.75 in.

Effective heat transfer area, based on effective length and inside diameter of tubes.

Tube #1: 1.584 $ft^2$

Tube #2: 2.121 $ft^2$

The same three-fluid heat exchanger has in previous experiments been used in an undergraduate laboratory course

FIG. A5

THE THREE-FLUID CONCENTRIC
TUBE TEST HEAT EXCHANGER
SEE PAGE 84 FOR DIMENSIONS

FIG. A6

FLOW DIAGRAM FOR THE
PARALLEL-FLOW TEST SET-UP

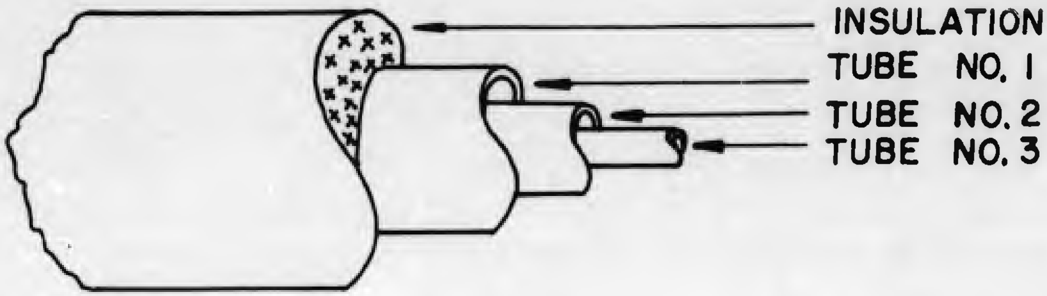

INSULATION
TUBE NO. 1
TUBE NO. 2
TUBE NO. 3

FIG. A5

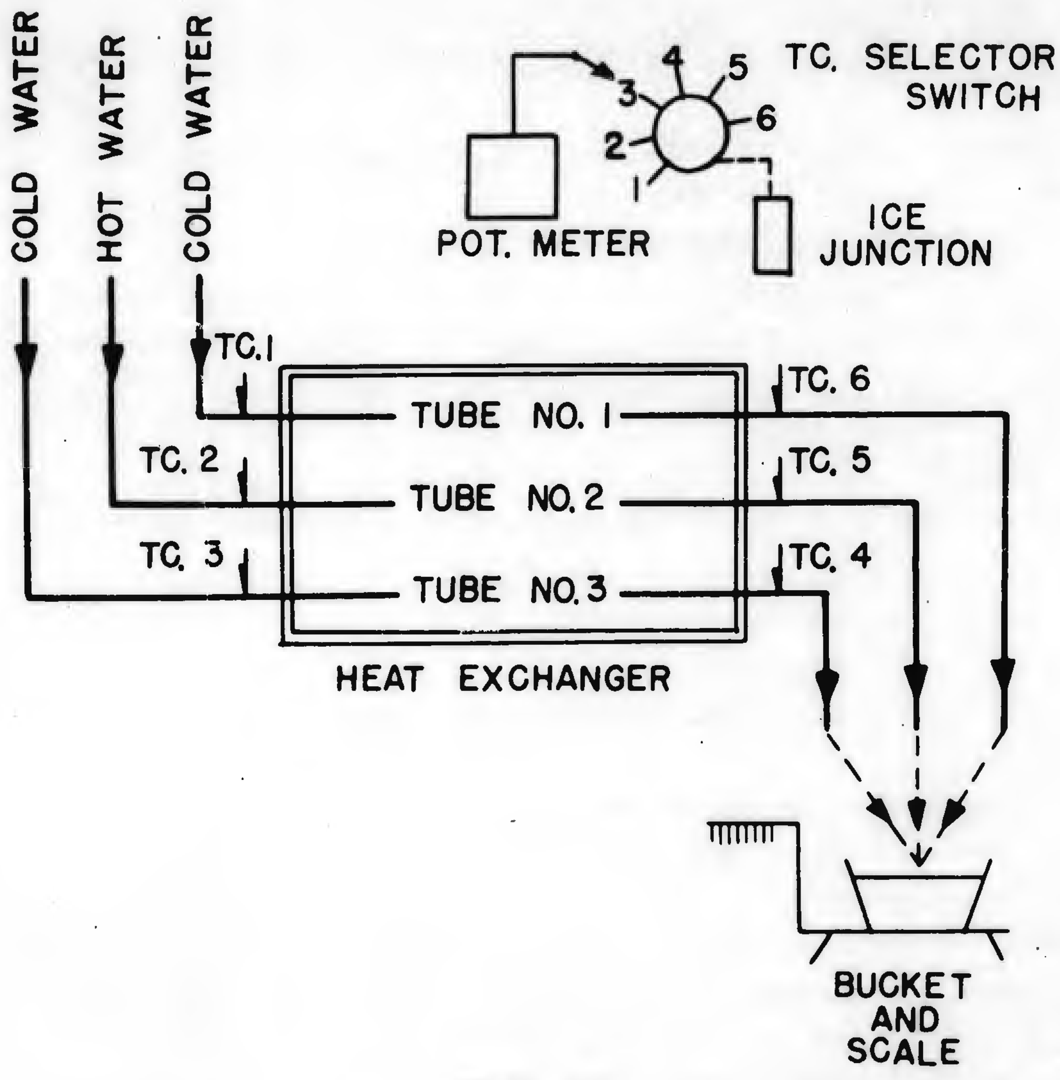

COLD WATER
HOT WATER
COLD WATER
TC. SELECTOR SWITCH
1
2
3
4
5
6
POT. METER
ICE JUNCTION
TC. 1
TC. 2
TC. 3
TC. 4
TC. 5
TC. 6
TUBE NO. 1
TUBE NO. 2
TUBE NO. 3
HEAT EXCHANGER
BUCKET AND SCALE

FIG. A6

to demonstrate the validity of the two-fluid exchanger design theory for counter- and parallel-flow (only two of the fluid passages were then used), and also for U-flow. Excellent agreement was then obtained between measured and predicted performance.

Test Procedure:

(a) Adjust the two cold streams and the hot stream flow rates to the desired values.

(b) Wait for steady-state to occur.

(c) Record inlet- and exit temperatures of all three fluids.

(d) Measure the flow rates of all three fluids using bucket, scale and a stopwatch.

As pointed out previously in this report, $\epsilon_{t,\,c1}$ and $\epsilon_{t,\,c2}$ are functions of the following five non-dimensional parameters; $Ntu_1$, $R^*$, $\Delta t^*_{in}$, $C_1^*$ and $C_2^*$; and in order to use the equations for $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ it is necessary to know the values of $Ntu_1 \triangleq (A_1U_1)/C_{c1}$ and $R^* \triangleq (A_2U_2)/(A_1U_1)$, or specifically, the values of $U_1$ and $U_2$ must be determined. $U_1$ is a function of the flow rates of cold fluid No. 1 and the hot fluid, and $U_2$ is a function of the flow rates of cold fluid No. 2 and the hot fluid.

$U_1$ and $U_2$ were obtained experimentally by using the following method:

(a) After the data for a run with three fluids was recorded, cold fluid No. 1 was disconnected from the exchanger by removing the hose connection. (No valve was touched in order to preserve the original flow rates.) Now the two-fluid exchanger with cold fluid No. 2 and the hot fluid flowing was tested using the following procedure:

(b) Wait for steady-state to occur.

(c) Record inlet and outlet temperatures of both fluids.

(d) Repeat measurement of the flow rates.

After this test was concluded, the cold fluid No. 1 was reconnected and cold fluid No. 2 was disconnected. Then the above three-step procedure was repeated for the two-fluid exchanger with cold fluid No. 1 and the hot fluid flowing.

The theory for two-fluid parallel- and counter-flow heat exchangers has been well established. Reference [4] gives the derivation of what is termed "Effectiveness-Ntu relations" for parallel- and counter-flow heat exchangers.

For parallel-flow:

$$\epsilon = \frac{1 - e^{-Ntu(1 + C_{min}/C_{max})}}{1 + C_{min}/C_{max}}$$

Where by definition:

$$\epsilon \triangleq q/q_{max\ possible}$$

and consequently

$$\epsilon = \frac{t_{h_{in}} - t_{h_{out}}}{t_{h_{in}} - t_{c_{in}}}, \quad \text{for } C_h = C_{min} < C_c$$

$$\epsilon = \frac{t_{c_{out}} - t_{c_{in}}}{t_{h_{in}} - t_{c_{in}}}, \quad \text{for } C_c = C_{min} < C_h$$

$$Ntu \triangleq (A \cdot U_{ave})/C_{min}$$

Knowing $\epsilon$ and $C_{min}/C_{max}$, $(AU_{ave})$ may be calculated.

$$(AU_{ave}) = \frac{\ell n\left[\frac{1}{1 - \epsilon(1 + C_{min}/C_{max})}\right] \cdot C_{min}}{(1 + C_{min}/C_{max})} \qquad (95)$$

Using equation (95), $(A_1U_1)$ may be calculated using the data obtained from the test of the two-fluid exchanger with cold fluid No. 1 and the hot fluid; and $(A_2U_2)$ may be calculated using the data obtained from the test of the two-fluid exchanger with cold fluid No. 2 and the hot fluid. Then $Ntu_1 \triangleq (A_1U_1)/C_{c1}$ and $R^* \triangleq (A_2U_2)/(A_1U_1)$, the two remaining parameters in the three-fluid temperature effectiveness expressions, may be calculated.

Five runs were taken with different values of the five parameters $Ntu_1$, $R^*$, $\Delta t^*_{in}$, $C^*_1$, and $C^*_2$. The measured values of the temperature effectivenesses, and the values calculated from equations (33) and (34) in Appendix I are tabulated in Table A1. The values of the overall heat exchanger effectiveness, calculated from equation (2) or (3), are also given in Table A1.

The predicted values of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ were obtained on a Burroughs 220 digital computer. As seen, an excellent agreement was obtained between the predicted and the measured temperature effectiveness expressions. The largest discrepancy was found in run no. 1, where $\epsilon_{t,c1}$ predicted was 2.2% higher than $\epsilon_{t,c1}$ measured, and $\epsilon_{t,c2}$ predicted was 2.5% lower than $\epsilon_{t,c2}$ measured. In run no. 1, the estimated uncertainty interval in the measured $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ was $\pm$ 4.0%. As seen, the predicted values are well within the uncertainty interval. For the other four runs the difference between the measured and the predicted values are within $\pm$ 1.6%.

It can then be concluded that the theoretical analysis is correct, since the predicted and the measured temperature effectivenesses are within the experimental uncertainties for all five runs. It also follows that the idealizations made in the derivation of the temperature effectiveness expressions are valid for the tested heat exchanger.

TABLE A1. PARALLEL-FLOW TEST RESULTS AND PREDICTED PERFORMANCE

| Run | $C_1^*$ | $C_2^*$ | $R^*$ | $Ntu_1$ | $\Delta t_{in}^*$ | Predicted from Theory | | Test Result | | $\epsilon_{q,o}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | |
| 1 | 1.50 | 1.60 | 1.025 | 0.706 | 0.979 | 0.2354 | 0.2240 | 0.230 | 0.220 | .695 |
| 2 | 1.485 | 1.44 | 1.025 | 0.665 | 0.826 | 0.2554 | 0.2140 | 0.251 | 0.214 | .745 |
| 3 | 0.499 | 0.961 | 1.462 | 1.15 | 0.662 | 0.4614 | 0.3034 | 0.455 | 0.304 | .648 |
| 4 | 0.506 | 1.005 | 1.462 | 1.13 | 0.976 | 0.4022 | 0.3361 | 0.406 | 0.330 | .545 |
| 5 | 0.512 | 0.536 | 1.070 | 1.11 | 0.950 | 0.4429 | 0.4355 | 0.434 | 0.431 | .460 |

## APPENDIX IV

### VERIFICATION OF THE COUNTER-FLOW HEAT EXCHANGER DESIGN THEORY BY EXPERIMENT

The objective of the experiment described here was to perform an experimental test for the adequacy of the theoretical analysis for the three-fluid counter-flow exchanger.

The test was performed on the same three-fluid heat exchanger as was used for verifying the parallel-flow theory. Fig. A7 shows a flow diagram of the counter-flow test set-up.

For a description of the exchanger, the reader is referred to Appendix III. The same test procedure was followed as for the three-fluid parallel-flow test.

$U_1$ and $U_2$ were obtained experimentally by means of the procedure used in Appendix III. Reference [4] gives the derivation of what is termed "Effectiveness-Ntu relations" for two-fluid counter-flow exchangers.

for two-fluid counter-flow:

$$\epsilon = \frac{1 - e^{-Ntu\,(1 - C_{min}/C_{max})}}{1 - C_{min}/C_{max} \cdot e^{-Ntu(1 - C_{min}/C_{max})}}$$

The definitions of $\epsilon$ and Ntu are given in Appendix III. Knowing $\epsilon$ and $C_{min}/C_{max}$, $(AU_{ave})$ may be calculated.

$$(AU_{ave}) = \frac{\ell n\,[\dfrac{1 - \epsilon \cdot C_{min}/C_{max}}{1 - \epsilon}] \cdot C_{min}}{(1 - C_{min}/C_{max})}$$

$A_1U_1$ and $A_2U_2$ are then calculated using the same method as in Appendix III.

FIG. A7

FLOW DIAGRAM FOR THE
COUNTER-FLOW TEST SET-UP

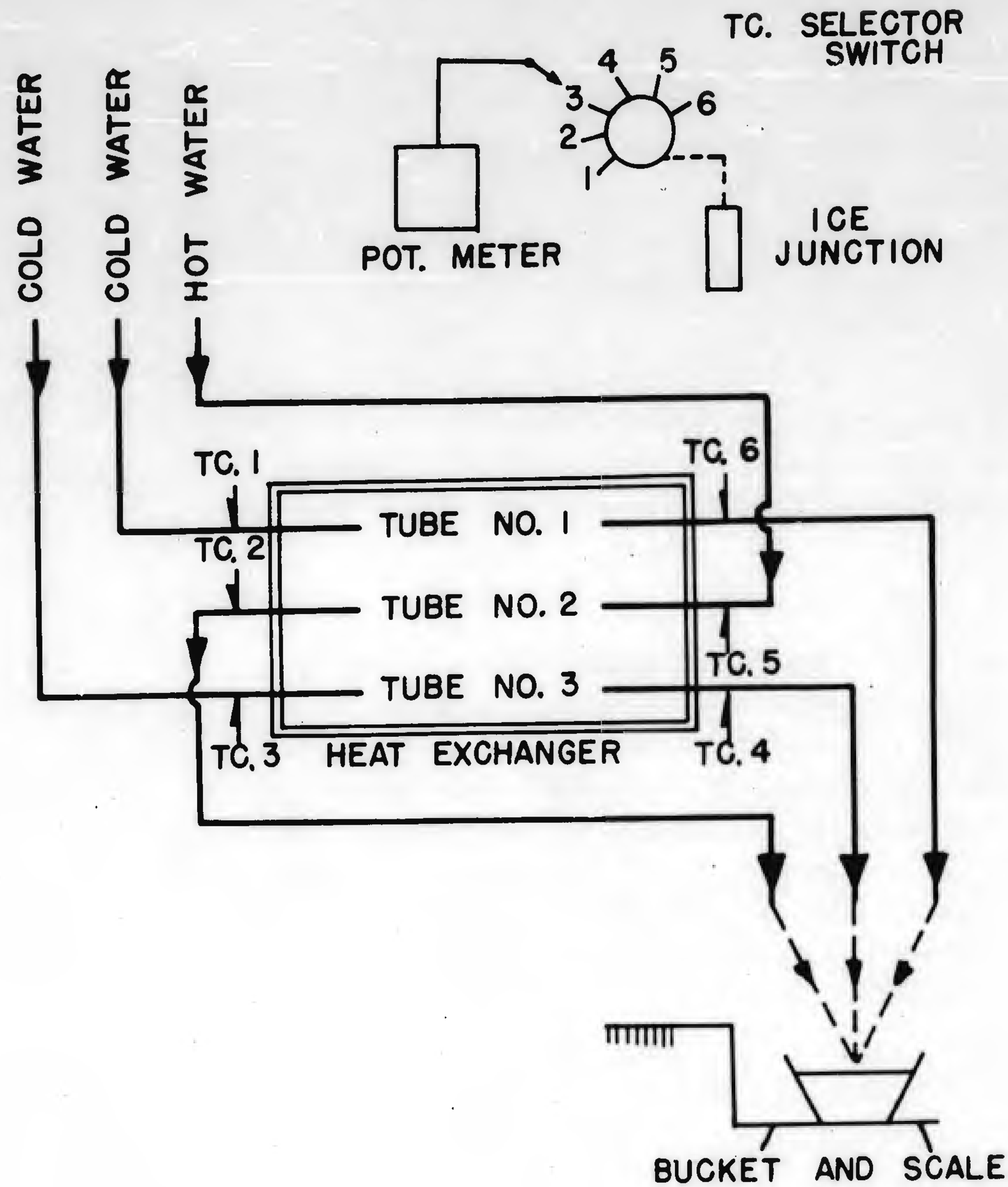

TC. SELECTOR SWITCH
1
2
3
4
5
6
POT. METER
ICE JUNCTION
COLD WATER
COLD WATER
HOT WATER
TC. 1
TC. 2
TC. 3
TC. 4
TC. 5
TC. 6
TUBE NO. 1
TUBE NO. 2
TUBE NO. 3
HEAT EXCHANGER
BUCKET AND SCALE

FIG. A7

Four runs were taken with different values of the five parameters $Ntu_1$, $\Delta t^*_{in}$, $C_1^*$, $C_2^*$ and $R^*$. The measured values of the temperature effectivenesses, and the values calculated from Eqs. (67,68) in Appendix II are tabulated in Table A2. The values of the overall heat exchanger effectiveness, calculated from Eq. (2) or (3), are also given in Table A2.

The predicted values of $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ were obtained on a Burroughs 220 digital computer. As seen, there is good agreement between the predicted and the measured temperature effectivenesses. The largest discrepancy is found in run no. 2, where $\epsilon_{t,c1}$ predicted was 2.7% higher than $\epsilon_{t,c1}$ measured, and $\epsilon_{t,c2}$ predicted was 3.9% higher than $\epsilon_{t,c2}$ measured. The estimated uncertainty interval for the temperature effectivenesses in run no. 2 was $\pm$ 4.0%. The predicted values are then within the uncertainty limits of the measured values. In the other three runs the difference between the predicted and the measured is less than $\pm$ 2.6%.

It can then be concluded that the theoretical analysis for the counter-flow exchanger is correct, since the predicted and the measured temperature effectivenesses are within $\pm$ 3.9% for all four runs.

TABLE A2. COUNTER-FLOW TEST RESULTS AND PREDICTED PERFORMANCE

| Run | $C_1^*$ | $C_2^*$ | $R^*$ | $Ntu_1$ | $\Delta t_{in}^*$ | Predicted from Theory | | Test Result | | $\epsilon_{q,o}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | |
| 1 | 1.125 | 0.628 | 0.732 | 0.862 | 0.978 | 0.3795 | 0.4501 | 0.369 | 0.442 | .700 |
| 2 | 1.33 | 1.65 | 0.970 | 0.704 | 0.972 | 0.2943 | 0.2372 | 0.286 | 0.228 | .768 |
| 3 | 0.684 | 0.446 | 0.796 | 1.04 | 0.970 | 0.4891 | 0.5462 | 0.478 | 0.543 | .576 |
| 4 | 0.709 | 0.519 | 1.04 | 0.828 | 0.717 | 0.4378 | 0.4901 | 0.427 | 0.481 | .646 |

## APPENDIX V

## AN APPROXIMATE METHOD OF HANDLING A THREE-FLUID HEAT EXCHANGER DESIGN PROBLEM

The objective of this appendix is to present a brief description of an approximate method of handling a three-fluid heat exchanger design problem. It is probable that this is the method currently used in industry.

### The Log-Mean Rate Equation Approach

For a description of a three-fluid parallel-flow exchanger the reader is referred to Figs. A1 and A2, and for a counter-flow exchanger to Figs. A3 and A4.

The heat transfer rate equations may be written as follows:

$$q_1 = A_1 U_1 \cdot \Delta_{mean,1} \tag{97}$$

$$q_2 = A_2 U_2 \cdot \Delta_{mean,2} \tag{98}$$

where:

$q_1$ = Heat transfer rate from the hot fluid to cold fluid No. 1.

$q_2$ = Heat transfer rate from the hot fluid to cold fluid No. 2.

$\Delta_{mean,1}$ = The true mean temperature difference for heat transfer between the hot fluid and cold fluid No. 1; in effect, defined by Eq. (97).

$\Delta_{mean,2}$ = The true mean temperature difference for heat transfer between the hot fluid and cold fluid No. 2; in effect, defined by Eq. (98).

The true mean temperature differences are functions of the terminal temperatures of the fluids for a given flow arrangement

$$\Delta_{mean,1} = \phi\,(t_h,\ t_{c1})_{terminal} \qquad (99)$$

$$\Delta_{mean,2} = \phi\,(t_h,\ t_{c2})_{terminal} \qquad (100)$$

The energy balance equations are:

$$q_1 + q_2 = C_h\,(t_{h_{in}} - t_{h_{out}}) \qquad (101)$$

$$q_1 = C_{c1}\,(t_{c1_{out}} - t_{c1_{in}}) \qquad (102)$$

$$q_2 = C_{c2}\,(t_{c2_{out}} - t_{c2_{in}}) \qquad (103)$$

The procedures for handling two typical design problems, assuming Eqs. (99,100) are available in explicit form, now follow:

Problem 1: Given $A_1$, $U_1$, $A_2$, $U_2$, $C_{c1}$, $C_{c2}$, $C_h$ and inlet temperatures; determine the outlet temperatures.

(1) Assume $q_1$ and $q_2$ and solve for the outlet temperatures from Eqs. (101), (102) and (103).

(2) Calculate $\Delta_{mean,1}$ and $\Delta_{mean,2}$ from Eqs. (99) and (100).

(3) Check initial assumption of $q_1$ and $q_2$ using Eqs. (97) and (98).

(4) Repeat as necessary.

Problem 2: Given $U_1$, $U_2$, $C_{c1}$, $C_{c2}$, $C_h$ and the terminal temperatures; determine the necessary $A_1$ and $A_2$.

(1) Calculate $\Delta_{mean,1}$ and $\Delta_{mean,2}$ from Eqs. (99) and (100).

(2) Calculate $q_1$ and $q_2$ from Eqs. (101), (102) and (103).

(3) Calculate $A_1$ and $A_2$ from Eqs. (97) and (98).

In design problems of the type of Problem 1, the mean temperature difference rate equation approach involves successive approximations, while the design theory presented in this report is straight forward.

In design problems of the type of Problem 2, both the mean temperature difference rate equation approach and the design theory presented in this report are straight forward.

The problem is now to determine the true mean temperature differences $\Delta_{mean,1}$ and $\Delta_{mean,2}$.

The log-mean temperature difference between the hot and the cold stream is defined as follows:

$$\Delta_{\ell m} \triangleq \frac{\Delta_{in} - \Delta_{out}}{\ell n(\Delta_{in}/\Delta_{out})} \qquad (104)$$

where:

$\Delta_{in}$ = Temperature difference between the two fluids at the hot fluid inlet section

$\Delta_{out}$ = Temperature difference between the two fluids at the hot fluid outlet section.

It should be emphasized that the log-mean temperature difference has been well established as the correct mean for heat transfer for two-fluid counter- and parallel-flow exchangers. Furthermore, correction factors have been developed analytically to be applied to $\Delta_{\ell m}$ for a variety of two-fluid flow arrangements to obtain the correct $\Delta_{mean}$. The present theory allows the development of correction factors to be applied to $\Delta_{\ell m}$ to obtain the correct $\Delta_{mean}$ for each of the two sides of a three-fluid exchanger. If these factors turn out to be close to unity $\Delta_{\ell m}$ can be used as the true mean temperature difference in practical problems.

Table A2 in Appendix IV presents the measured performance of a three-fluid counter-flow exchanger, together with the predicted performance obtained from the design theory. As a preliminary check on the feasibility of using the log-mean temperature difference as the correct mean difference for three-fluid counter-flow exchanger calculations, the performance of run 4 in Table A2 will be predicted using $\Delta_{\ell m}$ as $\Delta_{mean}$.

From the run 4 data sheet:

$$t_{h_{in}} = 174.4\ °F;\ t_{c1_{in}} = 63.5\ °F;\ t_{c2_{in}} = 94.9\ °F$$

$$C_{c1} = 694\ Btu/hr\ °F;\ C_{c2} = 508\ Btu/hr\ °F;\ C_h = 979\ Btu/hr\ °F$$

$$A_1U_1 = 594\ Btu/hr\ °F;\ A_2U_2 = 614\ Btu/hr\ °F$$

The procedure outlined in Problem 1 of this Appendix is now used for calculating the outlet temperatures of the fluids with:

$$\Delta_{mean,1} = \frac{\Delta_{in,1} - \Delta_{out,1}}{\ell n\ (\Delta_{in,1}/\Delta_{out,1})}$$

$$\Delta_{mean,2} = \frac{\Delta_{in,2} - \Delta_{out,2}}{\ell n\ (\Delta_{in,2}/\Delta_{out,2})}$$

where:

$$\Delta_{in} = (t_{h_{in}} - t_{c_{out}})$$

$$\Delta_{out} = (t_{h_{out}} - t_{c_{in}})$$

with matching subscripts 1 or 2.

The calculations yield after five iterations:

$$\Delta_{mean,1} = 58.5 \text{ °F} \quad ; \quad \Delta_{mean,2} = 32.0 \text{ °F}$$

$$q_1 = 34{,}700 \text{ Btu/hr}; \quad q_2 = 19{,}700 \text{ Btu/hr}$$

$$t_{c1_{out}} = 113.5 \text{ °F} \quad ; \quad t_{c2_{out}} = 134 \text{ °F} \quad ; \quad t_{h_{out}} = 119.9 \text{ °F}$$

Then:

$$\epsilon_{t,c1} = 0.451 \qquad \epsilon_{t,c2} = 0.492$$

As seen, there is very good agreement between these predicted temperature effectivenesses and the more rigorous theory results. (See Appendix IV, Table A2, run 4.) $\epsilon_{t,c1}$ is 3% higher than the theory result while $\epsilon_{t,c2}$ is 0.5% higher than the result obtained from the rigorous theory.

It can then be concluded that in this particular case it is feasible to use the log-mean temperature difference as the correct mean difference for three-fluid counter-flow exchanger calculations. In run 4 $C_1^* = 0.709$, $C_2^* = 0.519$, $R^* = 1.04$. Fig. 15, giving $\epsilon_{t,c1}$ and $\epsilon_{t,c2}$ for $R^* = 1.0$, $C_1^* = 0.5$, $C_2^* = 0.5$, illustrates then the temperature conditions in the exchanger in run 4. As seen, the temperature picture is "well behaved", and this is the reason why the log-mean rate equation approach works out well in this particular case. It is not expected that the log-mean approach will give good results if applied to an exchanger with "odd behavior" temperature conditions as in Figs. 18, 19, 22, 23.

APPENDIX VI

MACHINE PROGRAM FOR CALCULATING
THE TEMPERATURE EFFECTIVENESSES
FOR THE PARALLEL-FLOW EXCHANGER.

COMPUTER TYPE: BURROUGHS - 220
COMPUTER LANGUAGE: BALGOL
COMPUTER TIME: 7 MIN. (FOR 432 POINTS)

PROGRAM NOMENCLATURE:

C1 = $C_1^*$

C2 = $C_2^*$

R = $R^*$

TR = $\Delta t_{in}^*$

NTU1 = $Ntu_1$

ETC1 = $\epsilon_{t,c1}$

ETC2 = $\epsilon_{t,c2}$

```
2 S565          JOB  05/23/62,0007MIN   SORLIE          T
2               LOAD BALGOL
2 COMMENT THREE FLUID PARALLEL FLOW HEAT EXCHANGER ETC  CALCULATION $
2 WRITE ($$ HED)$
2 FORMAT HED (B6,*R*,B15,*C1*,B15,*C2*,B15,*TR*,B15,*NTU1*,B15,*ETC1*,
2  B15,*ETC2*,W2)$
2  FOR R=0.5,1.0,2.0 $
2  FOR C1=0.5,2.0 $
2  FOR C2=0.5,2.0 $
2  FOR TR=0.25,0.50,0.75,1.0 $
2  FOR NTU1=0.0,0.1,0.2,0.5,0.75,1.0,2.0,3.0,5.0 $
2  BEGIN
2 A=(R.(C1/C2).(1+C2))+(1+C1) $
2 B2=-(0.5).A+(0.5).SQRT(A*2-4.R.(C1/C2).(1+C1+C2)) $
2 B3=-(0.5).A-(0.5).SQRT(A*2-4.R.(C1/C2).(1+C1+C2)) $
2 EX2=(B2).(NTU1) $
2 EX3=(B3).(NTU1) $
2 ETC1=((-C2.TR+1+C2)/(1+C1+C2))+(EXP(EX2)-EXP(EX3))/(B2-B3)
2        +((B2.EXP(EX3)-B3.EXP(EX2)).(C2.TR-1-C2))/((1+C1+C2).(B2-B3)) $
2 D=(1+C1+B3)/C2 $
2 E=B2.(C2-(1+C2)/TR)-(1+C1+C2)/TR $
2 F=(1+C1+B2)/C2 $
2 G=(1+C1+C2)/TR-B3.(C2-(1+C2)/TR) $
2 H=(1+C1+C2).(B2-B3) $
2 ETC2=(1-C1/TR+C1)/(1+C1+C2)-(D.(E/H).EXP(EX3))-(F.(G/H).EXP(EX2)) $
2 WRITE($$OD,FORM) $
2 OUTPUT OD(R,C1,C2,TR,NTU1,ETC1,ETC2)$
2 FORMAT FORM(7F16.8,W2) $
2  END $
2 FINISH $
```

## APPENDIX VII

TABULATION OF THE NUMERICAL VALUES,
OBTAINED FROM THE COMPUTER PROGRAM IN APPENDIX VI,
USED FOR PLOTTING THE TEMPERATURE EFFECTIVENESS CURVES
FOR THE PARALLEL-FLOW EXCHANGER (FIGS. 6 - 14)

| | | (Fig. 6) $R^* = 0.50$ | | (Fig. 7) $R^* = 0.50$ | | (Fig. 8) $R^* = 0.50$ | |
|---|---|---|---|---|---|---|---|
| | | $C_1^* = 0.50$ | $C_2^* = 0.50$ | $C_1^* = 2.00$ | $C_2^* = 0.50$ | $C_1^* = 0.50$ | $C_2^* = 2.00$ |
| $\Delta t^*_{in}$ | $Ntu_1$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ |
| 0.25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0926 | .0435 | .0856 | .1076 | .0926 | .0111 |
| | .20 | .1718 | .0757 | .1487 | .0862 | .1718 | .0197 |
| | .50 | .3480 | .1222 | .2607 | - .2656 | .3477 | .0340 |
| | .75 | .4444 | .1240 | .3090 | - .5896 | .4437 | .0370 |
| | 1.00 | .5108 | .1072 | .3381 | - .8454 | .5096 | .0349 |
| | 2.00 | .6300 | - .0129 | .3820 | -1.3056 | .6268 | .0063 |
| | 3.00 | .6635 | - .1128 | .3906 | -1.4034 | .6615 | - .0265 |
| | 5.00 | .6815 | - .2083 | .3928 | -1.4275 | .6864 | - .0749 |
| 0.50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0923 | .0459 | .0846 | .1404 | .0923 | .0117 |
| | .20 | .1708 | .0843 | .1453 | .1952 | .1707 | .0219 |
| | .50 | .3425 | .1658 | .2480 | .1406 | .3419 | .0456 |
| | .75 | .4340 | .2067 | .2895 | .0336 | .4323 | .0597 |
| | 1.00 | .4952 | .2320 | .3135 | - .0601 | .4919 | .0703 |
| | 2.00 | .5952 | .2631 | .3486 | - .2375 | .5825 | .0950 |
| | 3.00 | .6167 | .2615 | .3554 | - .2758 | .5943 | .1079 |
| | 5.00 | .6237 | .2540 | .3571 | - .2853 | .5875 | .1229 |
| 0.75 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0920 | .0466 | .0836 | .1514 | .0920 | .0119 |
| | .20 | .1697 | .0872 | .1419 | .2315 | .1700 | .0226 |
| | .50 | .3370 | .1803 | .2353 | .2760 | .3361 | .0495 |
| | .75 | .4237 | .2342 | .2700 | .2414 | .4209 | .0673 |
| | 1.00 | .4796 | .2736 | .2890 | .2017 | .4742 | .0822 |
| | 2.00 | .5607 | .3551 | .3152 | .1186 | .5382 | .1245 |
| | 3.00 | .5700 | .3863 | .3202 | .1000 | .5271 | .1527 |
| | 5.00 | .5659 | .4082 | .3214 | .0954 | .4886 | .1888 |
| 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0917 | .0470 | .0826 | .1569 | .0917 | .0120 |
| | .20 | .1686 | .0886 | .1385 | .2497 | .1685 | .0230 |
| | .50 | .3316 | .1876 | .2226 | .3437 | .3303 | .0514 |
| | .75 | .4134 | .2480 | .2505 | .3453 | .4096 | .0711 |
| | 1.00 | .4640 | .2944 | .2645 | .3326 | .4565 | .0881 |
| | 2.00 | .5262 | .4011 | .2818 | .2966 | .4939 | .1393 |
| | 3.00 | .5231 | .4487 | .2849 | .2880 | .4599 | .1751 |
| | 5.00 | .5081 | .4852 | .2857 | .2858 | .3897 | .2218 |

| | | (Fig. 9) $R^* = 0.50$ | | (Fig. 10) $R^* = 2.00$ | | (Fig. 11) $R^* = 2.00$ | |
|---|---|---|---|---|---|---|---|
| | | $C_1^* = 2.00$ | $C_2^* = 2.00$ | $C_1^* = 0.50$ | $C_2^* = 0.50$ | $C_1^* = 0.50$ | $C_2^* = 2.00$ |
| $\Delta t_{in}^*$ | $Ntu_1$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ |
| 0.25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0856 | .0293 | .0919 | .1557 | .0918 | .0419 |
| | .20 | .1485 | .0273 | .1695 | .2419 | .1692 | .0702 |
| | .50 | .2603 | -.0701 | .3412 | .2768 | .3383 | .1011 |
| | .75 | .3106 | -.1849 | .4367 | .2028 | .4308 | .0924 |
| | 1.00 | .3445 | -.2976 | .5046 | .1122 | .4961 | .0703 |
| | 2.00 | .4210 | -.6292 | .6342 | -.1322 | .6271 | -.0300 |
| | 3.00 | .4588 | -.8062 | .6718 | -.2148 | .6761 | -.0900 |
| | 5.00 | .4887 | -.9471 | .6861 | -.2469 | .7065 | -.1319 |
| 0.50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0845 | .0379 | .0908 | .1643 | .0907 | .0442 |
| | .20 | .1447 | .0573 | .1658 | .2718 | .1651 | .0784 |
| | .50 | .2443 | .0581 | .3256 | .4016 | .3190 | .1400 |
| | .75 | .2834 | .0328 | .4109 | .4091 | .3956 | .1627 |
| | 1.00 | .3069 | .0031 | .4701 | .3882 | .4453 | .1719 |
| | 2.00 | .3538 | -.0918 | .5804 | .2988 | .5289 | .1663 |
| | 3.00 | .3759 | -.1434 | .6119 | .2647 | .5538 | .1546 |
| | 5.00 | .3934 | -.1846 | .6239 | .2513 | .5679 | .1453 |
| 0.75 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0834 | .0408 | .0900 | .1671 | .0895 | .0450 |
| | .20 | .1410 | .0674 | .1621 | .2818 | .1610 | .0812 |
| | .50 | .2283 | .1008 | .3100 | .4432 | .2995 | .1530 |
| | .75 | .2562 | .1054 | .3851 | .4779 | .3605 | .1861 |
| | 1.00 | .2693 | .1033 | .4356 | .4802 | .3945 | .2057 |
| | 2.00 | .2867 | .0873 | .5265 | .4424 | .4308 | .2317 |
| | 3.00 | .2931 | .0775 | .5519 | .4245 | .4315 | .2361 |
| | 5.00 | .2981 | .0696 | .5616 | .4174 | .4293 | .2377 |
| 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0824 | .0423 | .0886 | .1686 | .0884 | .0453 |
| | .20 | .1372 | .0724 | .1583 | .2868 | .1570 | .0826 |
| | .50 | .2123 | .1221 | .2944 | .4640 | .2800 | .1595 |
| | .75 | .2290 | .1417 | .3593 | .5123 | .3254 | .1978 |
| | 1.00 | .2317 | .1535 | .4011 | .5262 | .3437 | .2226 |
| | 2.00 | .2195 | .1769 | .4726 | .5143 | .3326 | .2645 |
| | 3.00 | .2102 | .1880 | .4920 | .5045 | .3092 | .2770 |
| | 5.00 | .2028 | .1967 | .4993 | .5004 | .2907 | .2840 |

| | | (Fig. 12) $R^* = 2.00$ | | (Fig. 13) $R^* = 2.00$ | | (Fig. 14) $R^* = 1.00$ | |
|---|---|---|---|---|---|---|---|
| | | $C_1^* = 2.00$ | $C_2^* = 0.50$ | $C_1^* = 2.00$ | $C_2^* = 2.00$ | $C_1^* = 0.50$ | $C_2^* = 0.50$ |
| $\Delta t_{in}^*$ | $Ntu_1$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ |
| 0.25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0841 | .2700 | .0836 | .0922 | .0923 | .0838 |
| | .20 | .1467 | .0895 | .1449 | .0582 | .1710 | .1402 |
| | .50 | .2655 | -.6003 | .2662 | -.2194 | .3451 | .1999 |
| | .75 | .3191 | -.9480 | .3317 | -.4294 | .4406 | .1796 |
| | 1.00 | .3502 | -1.1502 | .3784 | -.5867 | .5073 | .1327 |
| | 2.00 | .3881 | -1.3973 | .4668 | -.8871 | .6310 | -.0700 |
| | 3.00 | .3923 | -1.4250 | .4909 | -.9692 | .6681 | -.1784 |
| | 5.00 | .3929 | -1.4285 | .4993 | -.9977 | .6850 | -.2399 |
| 0.50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0810 | .3708 | .0799 | .1222 | .0918 | .0884 |
| | .20 | .1382 | .3604 | .1333 | .1508 | .1689 | .1566 |
| | .50 | .2443 | .0809 | .2286 | .0814 | .3354 | .2774 |
| | .75 | .2918 | -.0727 | .2771 | .0077 | .4232 | .3188 |
| | 1.00 | .3193 | -.1623 | .3113 | -.0492 | .4823 | .3324 |
| | 2.00 | .3529 | -.2718 | .3758 | -.1588 | .5843 | .3039 |
| | 3.00 | .3567 | -.2842 | .3934 | -.1888 | .6116 | .2730 |
| | 5.00 | .3571 | -.2857 | .3995 | -.1992 | .6233 | .2533 |
| 0.75 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0778 | .4043 | .0761 | .1322 | .0912 | .0899 |
| | .20 | .1297 | .4507 | .1217 | .1817 | .1669 | .1621 |
| | .50 | .2230 | .3079 | .1910 | .1816 | .3257 | .3032 |
| | .75 | .2644 | .2191 | .2224 | .1534 | .4058 | .3652 |
| | 1.00 | .2884 | .1670 | .2441 | .1299 | .4573 | .3990 |
| | 2.00 | .3177 | .1033 | .2848 | .0840 | .5376 | .4285 |
| | 3.00 | .3210 | .0961 | .2958 | .0714 | .5552 | .4235 |
| | 5.00 | .3214 | .0952 | .2997 | .0670 | .5616 | .4178 |
| 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0747 | .4211 | .0724 | .1372 | .0906 | .0906 |
| | .20 | .1213 | .4959 | .1101 | .1971 | .1648 | .1648 |
| | .50 | .2017 | .4215 | .1535 | .2317 | .3161 | .3161 |
| | .75 | .2371 | .3649 | .1678 | .2263 | .3884 | .3884 |
| | 1.00 | .2576 | .3316 | .1769 | .2195 | .4323 | .4323 |
| | 2.00 | .2825 | .2909 | .1937 | .2053 | .4908 | .4908 |
| | 3.00 | .2853 | .2863 | .1983 | .2015 | .4988 | .4988 |
| | 5.00 | .2857 | .2857 | .1999 | .2001 | .4999 | .4999 |

| | | (Fig. 9) $R^* = 0.50$ $C_1^* = 2.00$ | $C_2^* = 2.00$ | (Fig. 10) $R^* = 2.00$ $C_1^* = 0.50$ | $C_2^* = 0.50$ | (Fig. 11) $R^* = 2.00$ $C_1^* = 0.50$ | $C_2^* = 2.00$ |
|---|---|---|---|---|---|---|---|
| $\Delta t_{in}^*$ | $Ntu_1$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ |
| 0.25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0856 | .0293 | .0919 | .1557 | .0918 | .0419 |
| | .20 | .1485 | .0273 | .1695 | .2419 | .1692 | .0702 |
| | .50 | .2603 | - .0701 | .3412 | .2768 | .3383 | .1011 |
| | .75 | .3106 | - .1849 | .4357 | .2028 | .4308 | .0924 |
| | 1.00 | .3445 | - .2976 | .5046 | .1122 | .4961 | .0703 |
| | 2.00 | .4210 | - .6292 | .6342 | - .1322 | .6271 | - .0300 |
| | 3.00 | .4588 | - .8062 | .6718 | - .2148 | .6761 | - .0900 |
| | 5.00 | .4887 | - .9471 | .6861 | - .2469 | .7065 | - .1319 |
| 0.50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0845 | .0379 | .0908 | .1643 | .0907 | .0442 |
| | .20 | .1447 | .0573 | .1658 | .2718 | .1651 | .0784 |
| | .50 | .2443 | .0581 | .3256 | .4016 | .3190 | .1400 |
| | .75 | .2834 | .0328 | .4109 | .4091 | .3956 | .1627 |
| | 1.00 | .3069 | .0031 | .4701 | .3882 | .4453 | .1719 |
| | 2.00 | .3538 | - .0918 | .5804 | .2988 | .5289 | .1663 |
| | 3.00 | .3759 | - .1434 | .6119 | .2647 | .5538 | .1546 |
| | 5.00 | .3934 | - .1846 | .6239 | .2513 | .5679 | .1453 |
| 0.75 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0834 | .0408 | .0900 | .1671 | .0895 | .0450 |
| | .20 | .1410 | .0674 | .1621 | .2818 | .1610 | .0812 |
| | .50 | .2283 | .1008 | .3100 | .4432 | .2995 | .1530 |
| | .75 | .2562 | .1054 | .3851 | .4779 | .3605 | .1861 |
| | 1.00 | .2693 | .1033 | .4356 | .4802 | .3945 | .2057 |
| | 2.00 | .2867 | .0873 | .5265 | .4424 | .4308 | .2317 |
| | 3.00 | .2931 | .0775 | .5519 | .4245 | .4315 | .2361 |
| | 5.00 | .2981 | .0696 | .5616 | .4174 | .4293 | .2377 |
| 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0824 | .0423 | .0886 | .1686 | .0884 | .0453 |
| | .20 | .1372 | .0724 | .1583 | .2868 | .1570 | .0826 |
| | .50 | .2123 | .1221 | .2944 | .4640 | .2800 | .1595 |
| | .75 | .2290 | .1417 | .3593 | .5123 | .3254 | .1978 |
| | 1.00 | .2317 | .1535 | .4011 | .5262 | .3437 | .2226 |
| | 2.00 | .2195 | .1769 | .4726 | .5143 | .3326 | .2645 |
| | 3.00 | .2102 | .1880 | .4920 | .5045 | .3092 | .2770 |
| | 5.00 | .2028 | .1967 | .4993 | .5004 | .2907 | .2840 |

| | | (Fig. 12) $R^* = 2.00$ | | (Fig. 13) $R^* = 2.00$ | | (Fig. 14) $R^* = 1.00$ | |
|---|---|---|---|---|---|---|---|
| | | $C_1^* = 2.00$ | $C_2^* = 0.50$ | $C_1^* = 2.00$ | $C_2^* = 2.00$ | $C_1^* = 0.50$ | $C_2^* = 0.50$ |
| $\Delta t_{in}^*$ | $Ntu_1$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ |
| 0.25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0841 | .2700 | .0836 | .0922 | .0923 | .0838 |
| | .20 | .1467 | .0895 | .1449 | .0582 | .1710 | .1402 |
| | .50 | .2655 | - .6003 | .2662 | - .2194 | .3451 | .1999 |
| | .75 | .3191 | - .9480 | .3317 | - .4294 | .4406 | .1796 |
| | 1.00 | .3502 | -1.1502 | .3784 | - .5867 | .5073 | .1327 |
| | 2.00 | .3881 | -1.3973 | .4668 | - .8871 | .6310 | - .0700 |
| | 3.00 | .3923 | -1.4250 | .4909 | - .9692 | .6681 | - .1784 |
| | 5.00 | .3929 | -1.4285 | .4993 | - .9977 | .6850 | - .2399 |
| 0.50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0810 | .3708 | .0799 | .1222 | .0918 | .0884 |
| | .20 | .1382 | .3604 | .1333 | .1508 | .1689 | .1566 |
| | .50 | .2443 | .0809 | .2286 | .0814 | .3354 | .2774 |
| | .75 | .2918 | - .0727 | .2771 | .0077 | .4232 | .3188 |
| | 1.00 | .3193 | - .1623 | .3113 | - .0492 | .4823 | .3324 |
| | 2.00 | .3529 | - .2718 | .3758 | - .1588 | .5843 | .3039 |
| | 3.00 | .3567 | - .2842 | .3934 | - .1888 | .6116 | .2730 |
| | 5.00 | .3571 | - .2857 | .3995 | - .1992 | .6233 | .2533 |
| 0.75 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0778 | .4043 | .0761 | .1322 | .0912 | .0899 |
| | .20 | .1297 | .4507 | .1217 | .1817 | .1669 | .1621 |
| | .50 | .2230 | .3079 | .1910 | .1816 | .3257 | .3032 |
| | .75 | .2644 | .2191 | .2224 | .1534 | .4058 | .3652 |
| | 1.00 | .2884 | .1670 | .2441 | .1299 | .4573 | .3990 |
| | 2.00 | .3177 | .1033 | .2848 | .0840 | .5376 | .4285 |
| | 3.00 | .3210 | .0961 | .2958 | .0714 | .5552 | .4235 |
| | 5.00 | .3214 | .0952 | .2997 | .0670 | .5616 | .4178 |
| 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0747 | .4211 | .0724 | .1372 | .0906 | .0906 |
| | .20 | .1213 | .4959 | .1101 | .1971 | .1648 | .1648 |
| | .50 | .2017 | .4215 | .1535 | .2317 | .3161 | .3161 |
| | .75 | .2371 | .3649 | .1678 | .2263 | .3884 | .3884 |
| | 1.00 | .2576 | .3316 | .1769 | .2195 | .4323 | .4323 |
| | 2.00 | .2825 | .2909 | .1937 | .2053 | .4908 | .4908 |
| | 3.00 | .2853 | .2863 | .1983 | .2015 | .4988 | .4988 |
| | 5.00 | .2857 | .2857 | .1999 | .2001 | .4999 | .4999 |

## APPENDIX VIII

### MACHINE PROGRAMS FOR CALCULATING THE TEMPERATURE EFFECTIVENESSES FOR THE COUNTER-FLOW EXCHANGER

COMPUTER TYPE: BURROUGHS - 220

COMPUTER LANGUAGE: BALGOL

PROGRAM NOMENCLATURE:

C1 = $C_1^*$

C2 = $C_2^*$

R = R*

TR = $\Delta t_{in}^*$

NTU1 = $Ntu_1$

ETC1 = $\epsilon_{t,c1}$

ETC2 = $\epsilon_{t,c2}$

PAGE 106: PROGRAM FOR CASE WHEN $(C_1^* + C_2^*) \neq 1.0$

COMPUTER TIME: 6 MIN. (FOR 288 POINTS)

PAGE 107: PROGRAM FOR CASE WHEN $(C_1^* + C_2^*) = 1.0$

COMPUTER TIME: 3 MIN. (FOR 108 POINTS)

```
25565              JOB  05/23/62,0007MIN    SORLIE            T
2                  LOAD BALGOL
2COMMENT THREE FLUID COUNTER FLOW HEAT EXCHANGER CALCULATION $
2WRITE ($$ HED)$
2FORMAT,HED (B6,*R*,B15,*C1*,B15,*C2*,B15,*TR*,B15,*NTU1*,B15,*ETC1*,
2    B15,*ETC2*,W2)$
2 FOR R=0.5,2.0 $
2 FOR C1=0.5,2.0 $
2 FOR C2=0.5,2.0 $
2 FOR TR=0.25,0.50,0.75,1.0 $
2 FOR NTU1=0.0,0.1,0.2,0.5,0.75,1.0,2.0,3.0,5.0 $
2BEGIN
2A=(R.(C1/C2).(1-C2))+(1-C1) $
2B2=-(0.5).A+(0.5).SQRT(A*2-4.R.(C1/C2).(1-C1-C2))$
2B3=-(0.5).A-(0.5).SQRT(A*2-4.R.(C1/C2).(1-C1-C2))$
2EX2=B2.NTU1 $
2EX3=B3.NTU1 $
2D=(B2+1).EXP(EX2) $
2E=(B3+1).EXP(EX3) $
2F=(1-EXP(EX2)) $
2G=(1-EXP(EX3)) $
2H=((B3.EXP(EX3).F)-(B2.EXP(EX2).G))/(D-1) $
2KU=((C2.TR.(D-1))+((B2+1).(1-C2.EXP(EX2)))-C1) $
2KL=B2.B3.(EXP(EX3)-EXP(EX2))+ (1-C1-C2).(E-D)+(B3.F)-(B2.G) $
2K3=KU/KL   $
2ETC1=(F/(1-D))+H.K3 $
2I=C2.(E-D)-(D-C1.EXP(EX2)).(E-1) +(E-C1.EXP(EX3)).(D-1) $
2ETC2=(1-1/TR)+((1/TR).(D-C1.EXP(EX2)-C2))/(C2.(D-1))
2    +(1/(C2.(D-1))).(K3/TR) $
2WRITE($$OD,FORM) $
2OUTPUT OD(R,C1,C2,TR,NTU1,ETC1,ETC2) $
2FORMAT FORM(7F16.8,W2) $
2END $
2FINISH $
```

```
25565             JOB  05/23/62.0002MIN   SORLIE           T
2                 LOAD BALGOL
2COMMENT THREE FLUID COUNTER FLOW HEAT EXCHANGER CALCULATION $
2WRITE ($$ HED)$
2FORMAT HED (B6,*R*,B15,*C1*,B15,*C2*,B15,*TR*,B15,*NTU1*,B15,*ETC1*,
2   B15,*ETC2*,W2)$
2C1=0.5 $
2C2=0.5 $
2FOR R=0.5,1.0,2.0 $
2 FOR TR=0.25,0.50,0.75,1.0 $
2 FOR NTU1=0.0,0.1,0.2,0.5,0.75,1.0,2.0,3.0,5.0 $
2BEGIN
2A=(R.(C1/C2).(1-C2))+(1-C1) $
2B3=-(0.5).A-(0.5).SQRT(A*2-4.R.(C1/C2).(1-C1-C2))$
2EX3=B3.NTU1 $
2E=(B3+1).EXP(EX3) $
2K=(1/R+NTU1-TR.(1+NTU1))/((E-1).(1/R+NTU1)
2   -((B3+1).(EXP(EX3)-1/C2)+C1/C2).(1+NTU1)) $
2ETC1=NTU1/(1+NTU1)+(EXP(EX3)-1-(((E-1).NTU1)/(1+NTU1))).K $
2ETC2=(1-1/TR)+(1- 1/(R.(1+NTU1)))/TR
2+((((B3+1)/C2)-C1/C2).EXP(EX3)-1-((1-(1/(R.(1+NTU1)))).(E-1))).(K/TR)$
2WRITE($$OD,FORM) $
2OUTPUT OD(R,C1,C2,TR,NTU1,ETC1,ETC2) $
2FORMAT FORM(7F16.8,W2) $
2END $
2FINISH $
```

## APPENDIX IX

TABULATION OF THE NUMERICAL VALUES,
OBTAINED FROM THE COMPUTER PROGRAMS IN APPENDIX VIII,
USED FOR PLOTTING THE TEMPERATURE EFFECTIVENESS CURVES
FOR THE COUNTER FLOW EXCHANGER (FIGS. 15 - 23)

|  |  | (Fig. 15) $R^* = 1.00$ $C_1^* = 0.50$ $C_2^* = 0.50$ |  | (Fig. 16) $R^* = 0.50$ $C_1^* = 0.50$ $C_2^* = 0.50$ |  | (Fig. 17) $R^* = 0.50$ $C_1^* = 0.50$ $C_2^* = 2.00$ |  |
|---|---|---|---|---|---|---|---|
| $\Delta t_{in}^*$ | $Ntu_1$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ |
| 0.25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | .10 | .0925 | .0845 | .0927 | .0438 | .0927 | .0111 |
|  | .20 | .1721 | .1448 | .1729 | .0773 | .1729 | .0200 |
|  | .50 | .3559 | .2431 | .3586 | .1393 | .3583 | .0372 |
|  | .75 | .4657 | .2800 | .4702 | .1668 | .4696 | .0451 |
|  | 1.00 | .5495 | .3018 | .5558 | .1845 | .5549 | .0500 |
|  | 2.00 | .7409 | .3697 | .7547 | .2360 | .7529 | .0595 |
|  | 3.00 | .8251 | .4497 | .8443 | .2983 | .8414 | .0688 |
|  | 5.00 | .8933 | .5934 | .9148 | .4349 | .9063 | .0932 |
| 0.50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | .10 | .0920 | .0888 | .0925 | .0460 | .0925 | .0117 |
|  | .20 | .1703 | .1594 | .1719 | .0852 | .1718 | .0220 |
|  | .50 | .3484 | .3033 | .3542 | .1750 | .3533 | .0472 |
|  | .75 | .4533 | .3790 | .4623 | .2298 | .4602 | .0638 |
|  | 1.00 | .5330 | .4339 | .5447 | .2738 | .5410 | .0778 |
|  | 2.00 | .7162 | .5677 | .7343 | .3985 | .7226 | .1200 |
|  | 3.00 | .8001 | .6499 | .8207 | .4869 | .8003 | .1509 |
|  | 5.00 | .8733 | .7534 | .8926 | .6127 | .8561 | .1937 |
| 0.75 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | .10 | .0914 | .0902 | .0922 | .0467 | .0922 | .0119 |
|  | .20 | .1685 | .1642 | .1709 | .0878 | .1708 | .0227 |
|  | .50 | .3409 | .3233 | .3497 | .1869 | .3483 | .0506 |
|  | .75 | .4410 | .4121 | .4544 | .2508 | .4509 | .0700 |
|  | 1.00 | .5165 | .4780 | .5335 | .3036 | .5271 | .0871 |
|  | 2.00 | .6914 | .6337 | .7140 | .4526 | .6923 | .1402 |
|  | 3.00 | .7750 | .7166 | .7971 | .5497 | .7593 | .1783 |
|  | 5.00 | .8533 | .8067 | .8704 | .6720 | .8058 | .2272 |
| 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | .10 | .0909 | .0909 | .0919 | .0471 | .0919 | .0120 |
|  | .20 | .1667 | .1667 | .1700 | .0891 | .1698 | .0231 |
|  | .50 | .3333 | .3333 | .3452 | .1929 | .3433 | .0523 |
|  | .75 | .4286 | .4286 | .4466 | .2613 | .4416 | .0731 |
|  | 1.00 | .5000 | .5000 | .5223 | .3184 | .5132 | .0917 |
|  | 2.00 | .6667 | .6667 | .6937 | .4797 | .6621 | .1503 |
|  | 3.00 | .7500 | .7500 | .7736 | .5811 | .7182 | .1919 |
|  | 5.00 | .8333 | .8333 | .8481 | .7016 | .7556 | .2439 |

| | | (Fig. 18) $R^* = 0.50$ | | (Fig. 19) $R^* = 0.50$ | | (Fig. 20) $R^* = 2.00$ | |
|---|---|---|---|---|---|---|---|
| | | $C_1^* = 2.00$ | $C_2^* = 0.50$ | $C_1^* = 2.00$ | $C_2^* = 2.00$ | $C_1^* = 0.50$ | $C_2^* = 0.50$ |
| $\Delta t_{in}^*$ | $Ntu_1$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ |
| 0.25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0862 | .1137 | .0861 | .0300 | .0921 | .1581 |
| | .20 | .1515 | .1203 | .1514 | .0315 | .1709 | .2567 |
| | .50 | .2770 | -.0596 | .2795 | -.0448 | .3519 | .3884 |
| | .75 | .3363 | -.2290 | .3457 | -.1420 | .4594 | .4273 |
| | 1.00 | .3734 | -.3606 | .3932 | -.2428 | .5406 | .4501 |
| | 2.00 | .4281 | -.5881 | .4986 | -.5645 | .7217 | .5358 |
| | 3.00 | .4375 | -.6316 | .5461 | -.7440 | .8010 | .6238 |
| | 5.00 | .4395 | -.6407 | .5812 | -.8832 | .8712 | .7436 |
| 0.50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0852 | .1440 | .0851 | .0383 | .0911 | .1660 |
| | .20 | .1486 | .2146 | .1479 | .0596 | .1677 | .2821 |
| | .50 | .2671 | .2571 | .2650 | .0713 | .3408 | .4777 |
| | .75 | .3218 | .2364 | .3211 | .0547 | .4428 | .5601 |
| | 1.00 | .3556 | .2110 | .3591 | .0302 | .5203 | .6125 |
| | 2.00 | .4048 | .1573 | .4363 | -.0656 | .6981 | .7245 |
| | 3.00 | .4132 | .1461 | .4686 | -.1241 | .7806 | .7871 |
| | 5.00 | .4150 | .1437 | .4921 | -.1703 | .8570 | .8574 |
| 0.75 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0843 | .1540 | .0840 | .0411 | .0901 | .1686 |
| | .20 | .1456 | .2461 | .1444 | .0689 | .1646 | .2905 |
| | .50 | .2572 | .3627 | .2505 | .1100 | .3296 | .5074 |
| | .75 | .3073 | .3915 | .2966 | .1203 | .4262 | .6044 |
| | 1.00 | .3377 | .4015 | .3249 | .1212 | .5000 | .6667 |
| | 2.00 | .3815 | .4058 | .3739 | .1007 | .6745 | .7874 |
| | 3.00 | .3889 | .4053 | .3911 | .0825 | .7602 | .8416 |
| | 5.00 | .3905 | .4051 | .4030 | .0673 | .8428 | .8953 |
| 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0833 | .1591 | .0830 | .0425 | .0891 | .1700 |
| | .20 | .1427 | .2618 | .1409 | .0736 | .1614 | .2948 |
| | .50 | .2473 | .4155 | .2360 | .1293 | .3184 | .5223 |
| | .75 | .2927 | .4691 | .2720 | .1531 | .4096 | .6266 |
| | 1.00 | .3198 | .4968 | .2908 | .1668 | .4797 | .6937 |
| | 2.00 | .3582 | .5300 | .3115 | .1838 | .6509 | .8189 |
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| | | (Fig. 21) $R^* = 2.00$ | | (Fig. 22) $R^* = 2.00$ | | (Fig. 23) $R^* = 2.00$ | |
|---|---|---|---|---|---|---|---|
| | | $C_1^* = 0.50$ | $C_2^* = 2.00$ | $C_1^* = 2.00$ | $C_2^* = 0.50$ | $C_1^* = 2.00$ | $C_2^* = 2.00$ |
| $\Delta t_{in}^*$ | $Ntu_1$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ | $\epsilon_{t,c1}$ | $\epsilon_{t,c2}$ |
| 0.25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .10 | .0920 | .0422 | .0847 | .3349 | .0842 | .0987 |
| | .20 | .1704 | .0720 | .1476 | .3281 | .1468 | .0865 |
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