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Shells for Underwater Vehicles

September 28, 1962

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Shells for Underwater Vehicles

By J. D. Stachiw

ORDNANCE RESEARCH LABORATORY
The Pennsylvania State University
University Park, Pennsylvania

September 28, 1962

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Abstract

THE DESIGN criteria for underwater shells are discussed, and the basic shell designs are evaluated on the basis of these criteria. Internally pressurized shells and shell-pressurization methods are also discussed.

At present, the sandwich-shell design offers the highest pressure-to-weight ratio consistent with the shell-design criteria. The honeycomb or microballoon fiber glass sandwich shells could be best used in the lower pressure ranges and cellular sandwich shells, for a large band of intermediate pressure ranges; the cellular or solid sandwich shells would be better used at the high pressure ranges.

It is recommended that research be directed toward development of higher-strength materials and improved fabrication methods. It is also recommended that fluids of lower compressibility and density be developed for shell pressurization.

Acknowledgment

THE AUTHOR is indebted to P. C. Sweetland, Associate Professor of Engineering Research at ORL, for his help in the calculation and plotting of curves used in this report.

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Shells for Underwater Vehicles

ALL UNDERWATER vehicles require a shell structure primarily to prevent flooding of the vehicle cavity and to provide a skeleton on which the functional components are mounted. It also may provide the vehicle with a hydrodynamic shape. The basic shells for many underwater vehicles are the cylindrical ring-stiffened shell (Fig. 1) and the cylindrical sandwich shell (Fig. 2). The ring-stiffened shell consists of a smooth cylinder stiffened with rings; the sandwich shell consists of concentric cylinders separated by a spacer. There are several variations of the sandwich-shell design and each has its advantages, depending on the operating requirements of the shell. Despite the complexity of modern underwater vehicles, the basic criteria for shell design continue to be the same.

Design Criteria

There are four basic criteria that must be considered in the design of a successful shell. Listed in the order of their importance, they are:

1. resistance to collapse under external pressure;
2. rigidity of shell structure for mounting of propulsor and guidance components;
3. resistance to corrosion; and
4. fulfillment of all the above requirements with the least weight and the most internal space.

A shell should fulfill all these requirements. In addition, the shell may contribute to the reduction of drag forces and may provide some acoustic damping by using proper fabrication and design techniques. So far, no shell design satisfying all the basic requirements in one design has been found. As long as the fourth criterion is important, every shell design represents, at best, only a clever compromise.

Continuous demands for increased payload and increased speed will require that underwater vehicles shall continue to be weight-limited. There is no likelihood that this requirement will diminish; therefore, it is postulated that shell weight must be kept to an absolute minimum compatible with the other criteria.

RESISTANCE TO COLLAPSE

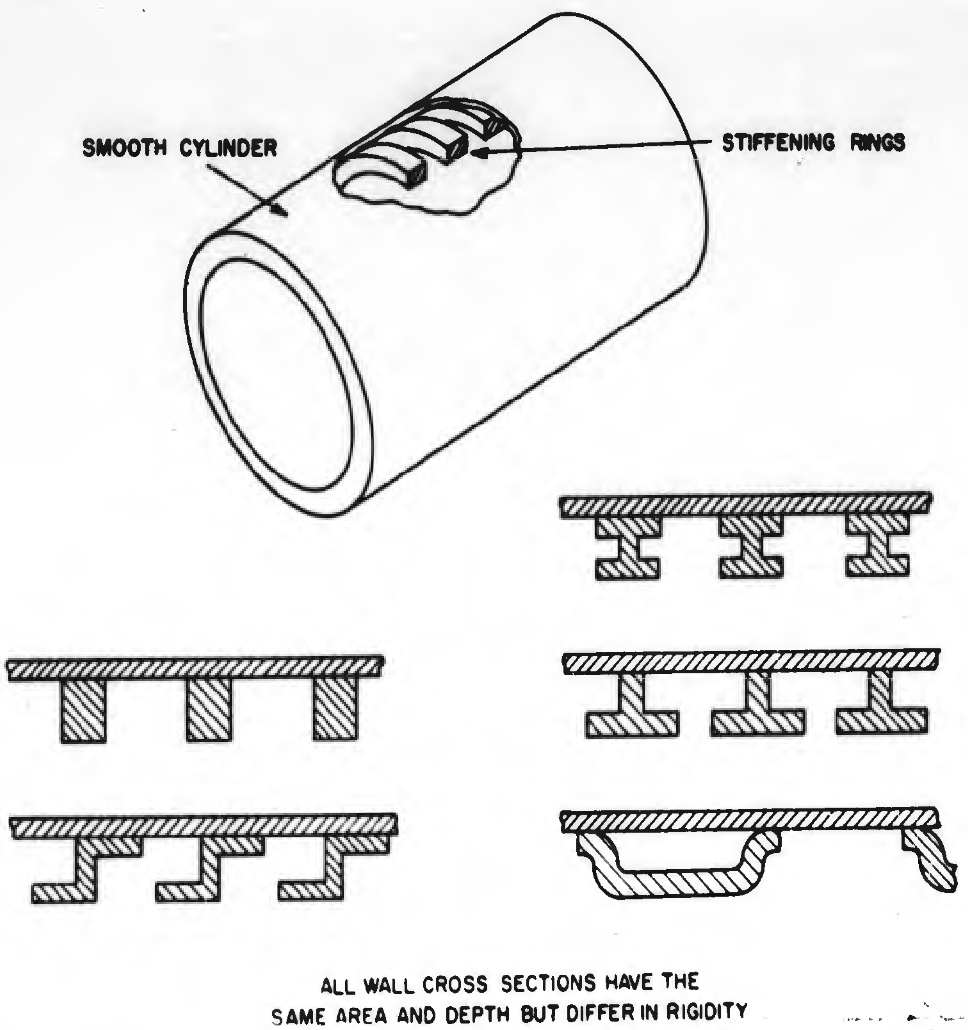
There are only three methods (discussed below) for providing the shell with sufficient strength and stability to operate under external hydrostatic pressure. Two of the methods have been used extensively; a third may find use in the future.

(1) The hydrostatic pressure acting on the outside of the shell must be counteracted by the compressive strength of shell material. To fully utilize the strength of the shell material, the shell must be built so that it will not fail because of elastic instability - a lower-energy-level failure.

(2) The external pressure can be neutralized by pressurizing the vessel cavity with gases or liquids so that the pressurized fluids, instead of the shell structure, carry the external load.

(3) The advantages of (1) and (2) can be judiciously combined, resulting in a superior shell design.

Resistance to collapse can be improved by designing a shell structure for external pressure having a spherical or double-curvature shape. However, many considerations preclude the use of a more implosion-resistant shape (such as the sphere) even though the strength of a spherical shell is considerably higher than that of a cylindrical shell of equal wall thickness. A spherical shell can be given a streamlined shape by means of fairings; but, when the weight of the fairing is included in the over-all vessel weight, the pressure-to-weight ratio of the vessel could equal or exceed that of a cylindrical vessel. It is possible to retain some of the implosion resistance of a sphere by means of a double-curvature pressure vessel. The strength advantages of double-curvature vessels decrease rapidly with an increase in length-to-



STIFFENERS FOR SHELLS

Fig. 1 - Cylindrical Ring-Stiffened Shell

diameter (L/D) ratio of the vessel, but some slight strength increase is present even in elliptical shells with an L/D ratio of 6. In any event these noncylindrical shapes are a special case to be used when the many considerations allow the necessary compromises.

SHELL RIGIDITY

The rigidity of an underwater vehicle is of great importance in the mounting of propulsor and guidance components inside the vehicle. When the shell is very rigid (less than 0.010-in. radial deflection under maximum operational pressure), most of the internal vehicle components can be rigidly mounted to the vehicle shell. Since the deflection is so minute, stresses induced in the equipment and structures secured directly to the inside surface are not likely to be excessively high. It may even be possible to mount such sensitive guidance system components as gyro platforms and yet not experience enough deflection to substantially affect the course stability of the vehicle.

As the rigidity of the shell decreases, the shell deflections become greater, and a completely different approach to the mounting of internal vehicle components must be devised. Elastic mountings must be provided to absorb the shell deflections rather than transmit them as a high compressive load to the internally mounted components.

Although the internal mounting problem may be solved by the use of proper mounting, further problems could arise since the perfect cylindrical shell assumes a different shape under high hydrostatic pressure. Because of unevenness in contraction the cylinder may become elliptical in cross section and have local un-

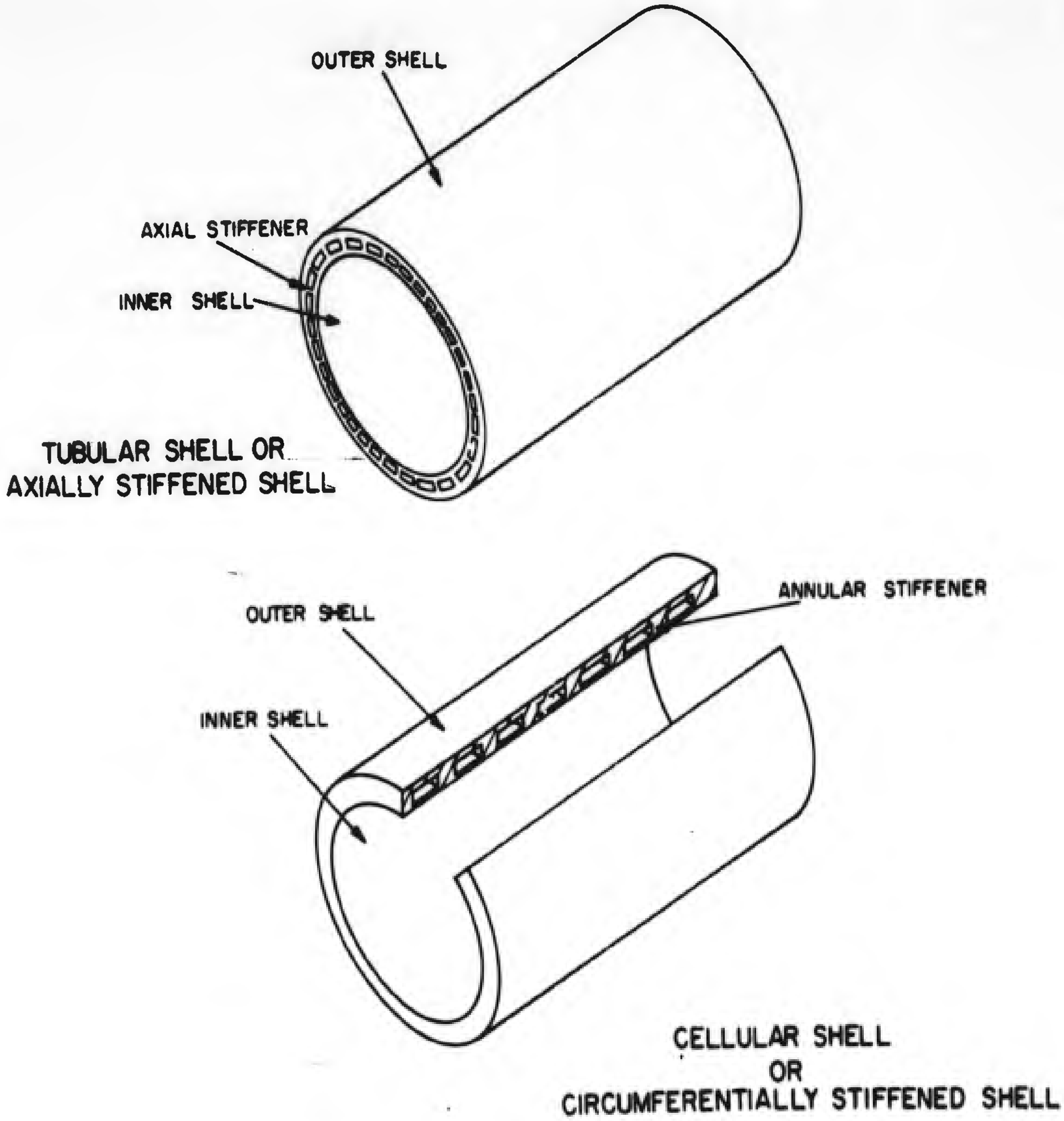


Fig. 2 - Cylindrical Sandwich Shells

evenness in the external shell surface, the inside and outside diameters varying from point to point along the length of the vessel.

The varying outside diameter is produced by the uneven rigidity of the shell. The shell-section joints, bulkheads, and hand-hole openings act like local stiffeners, increasing the rigidity of the shell. Since radial shell deflections are directly proportional to the rigidity of the shell, it follows that the deflections at stiffened shell-surface points will be less than those at points that do not have stiffeners. Because of the difference in radial deflections at various points on the shell, the originally smooth vehicle envelope develops a dimpled "toothpaste-tube" appearance. It is felt that this occurrence is undesirable for vehicles where hydrodynamic flow over the surface is critical.

The formation of local flat spots and local changes of outside diameter under external pressure can be minimized if the rigidity of the shell is kept constant along the entire length of the vessel. This can be accomplished by elimination of bulkheads, stiff shell joints, and hand-hole openings and covers. Considerable thought has been devoted to this subject, but not much has been done about it. However, as underwater vehicles descend to greater depths and attain greater speeds, the principles of uniform shell rigidity will become a prerequisite for successful shell design. Very rigid shells will need some internal pressurization to be successful under the weight limitation necessary. Unless some persuasive arguments are found for pressurization, the problems created by large shell deflections ~~must~~ be alleviated by uniformly rigid shells and by elastic mounting of components inside the shell.

RESISTANCE TO CORROSION

Sea-water corrosion tends to decrease the compressive strength of underwater vehicle shells in proportion to the length of time they are exposed to the salt water. The term corrosion, as used in this report, means the decrease of the compressive strength of the material - not just the discoloration of its surface. This distinction is important, for there are many materials that lose varying amounts of their strength without any detectable sign of surface corrosion. Conversely, some materials suffer surface damage, but their over-all strength diminishes very little, the corrosion being limited to only the surface layer of the material.

At this point, it may be helpful to classify all underwater vehicles arbitrarily according to their length of continuous submergence:

1. momentary immersion - 1 hr or less;
2. temporary immersion - 1 to 1000 hr; and
3. permanent immersion - 1000 hr or more.

It is difficult to find a structural material whose mechanical properties will not change at all under the corrosive action of salt water. It is often more practical to use a material that will lose only a specified amount of tensile and compressive strength when exposed to sea water for a given period of time.

An arbitrary material-corrosiveness scale is listed below. The corrosion resistance is based on the decrease of tensile strength in a 1/16-in. sheet-metal sample subjected to tidewater exposure for one year:

1. for corrosion-resistant materials, tensile-strength decrease 11 per cent or less;
2. for corrosion-retardant materials, tensile-strength decrease 11 to 45 per cent;
3. for corrosion-prone materials, tensile-strength decrease 45 to 90 per cent; and
4. for reactive materials, tensile-strength decrease 90 to 100 per cent.

Only corrosion-resistant materials are recommended for permanent immersion; those that are either corrosion-resistant or corrosion-retardant are recommended for temporary immersion. Materials that are either corrosion-resistant, corrosion-retardant, or corrosion-prone are recommended for momentary immersion. Reactive materials should not be used for momentary immersion unless a severe shortage of other materials exists.

The use of paint or other protective finishes is highly recommended, but it cannot be substituted for the corrosion-resistant properties of the material itself; protective finishes are often scratched in launching or handling.

WEIGHT AND SPACE LIMITATIONS

All the requirements for a successful shell could be fulfilled easily if it were not for weight and space limitations. The utopian dreams of the design engineer cannot be satisfied because underwater vehicles must meet maximum density requirements and provide sufficient internal space for fuel, propulsor machinery, and guidance equipment. Since weight and space are primary shell requirements that must be satisfied at the expense of other shell requirements, the achievement of a well designed shell becomes mostly a matter of judicious compromise between all the shell parameters.

Modes of Shell Failure

The collapse resistance of any shell depends basically on two parameters: the compressive strength of the material, and the elastic stability of the structure. The compressive strength of the vessel is maintained by sufficient facing and ring cross sections; the elastic stability depends on the moments of inertia of the facings and the stiffener rings. The collapse may be general or local, but both are disastrous to the structural integrity of the vessel.

The difference between a local and a general failure is primarily one of degree. General collapse is caused by the failure of the whole shell structure between bulkheads. It may be initiated by the failure of only one structural component, but the end result is a mass of twisted wreckage. Local failure is not as severe; the shell retains its cylindrical outline.

General collapse occurs when both the shell facings and the ring stiffeners collapse simultaneously because of insufficient compressive strength or elastic stability. Since the facings and the ring stiffeners form a unique structure, the stability of the structure is not the sum of the stabilities of its components but is the result of interaction between each shell component. For each diameter, external pressure, and bulkhead spacing, there exists an optimum combination of facing thicknesses, ring size, and ring spacing. This combination represents a shell that weighs less than any other shell made of the same material.

Selection of Parameters for High-Pressure Shells

Obtaining the optimum parameters involves a long process in which each parameter is varied while the others are held constant. (It is preferable to have these calculations performed by a computer.) Nevertheless some simplified generalizations can be made.

(1) The facings must be thick enough to carry compressive circumferential stresses and resultant longitudinal stresses between the rings.

(2) The spacing between the rings must stiffen the facings sufficiently to avoid buckling between stiffeners.

(3) The ring stiffeners must possess a sufficient moment of inertia to supplement the elastic stability of the facings between the stiffeners.

(4) The elastic stability of the facings and of the stiffeners must be achieved with a minimum of material.

To achieve high elastic stability with a minimum of material, designers have employed various stiffener shapes, some of which are shown in Fig. 1. Since the elastic stability of the ring is directly proportional to its moment of inertia, ring shapes that provide a large moment of inertia must be selected. This explains the wide use of T-rings, Z-rings, and Box-rings, all of which give a higher moment of inertia than a simple ring with a square or rectangular cross section.

In order to obtain lightweight collapse-resistant shells, optimum component proportions and premium materials with high strength-to-weight ratios must be selected. The selection of materials is limited to commercially available materials, but any acceptable design may be used. The use of premium materials and optimized parameters gives very attractive pressure-to-weight ratios.

To clarify some of the advantages gained from optimization of shell parameters, it is necessary to determine theoretical limits that no shell design, however well optimized, can exceed. To make the theoretical limits applicable to all shell designs, the limits must be made general. Such general limits are inherent in the compressive strength of the material and the elastic stability of a long smooth shell.

The compressive strength of a material is one of the limits that cannot be improved by shell design. The compressive strength of a long shell, when plotted against external pressure, can be used to illustrate the maximum buoyancy of the vessel for any given design pressure. Since the bulkheads of a long shell do not influence the collapse pressure of the shell between bulkheads, the designer can determine easily the lightest shell that will survive failure by compressive yielding. If close bulkhead spacing is a part of a given shell design, then the bulkhead weight must be included in the shell weight to put the shell comparison on a sound basis.

The compressive strength of the material provides the lower weight limit of the shell; that is, the lowest shell weight that will survive failure by compressive yielding. The elastic stability of a long smooth tube provides the upper weight limit of the shell. This upper limit is based on the premise that no shell design need be heavier than a long smooth shell, which possesses less elastic stability than any ring-stiffened or sandwich shell of the same diameter, weight, and pressure rating.

The strength-of-materials equation describing the lower weight limit of shells, when plotted on linear graph paper, is almost a straight line. The expression for the elastic stability of smooth tubes is a third-degree equation that, when plotted on the same coordinates as the strength-of-materials graph, takes the shape of a sharp curve intersecting the strength-of-materials line. The area between the two curves shows the possible weight savings and potential buoyancy gain for the shell with optimum parameters. The point at which these curves intersect gives the external pressure limit for the material - the limit beyond which no weight and, therefore, no buoyancy improvements over a smooth shell are possible (Fig. 3). The gain in buoyancy of a shell design over the buoyancy of a smooth tube made of the same material, having the same dimensions, and made for the same collapse pressure, is defined here as efficiency of shell design when it is compared to the maximum gain possible at that pressure.

Efficiency of Shell Designs

The efficiency of a shell design can now be described in terms of the two equations that serve as the upper and lower boundaries of shell-design efficiency. Since no design will surpass the strength-of-materials requirement represented by the almost straight line, it will

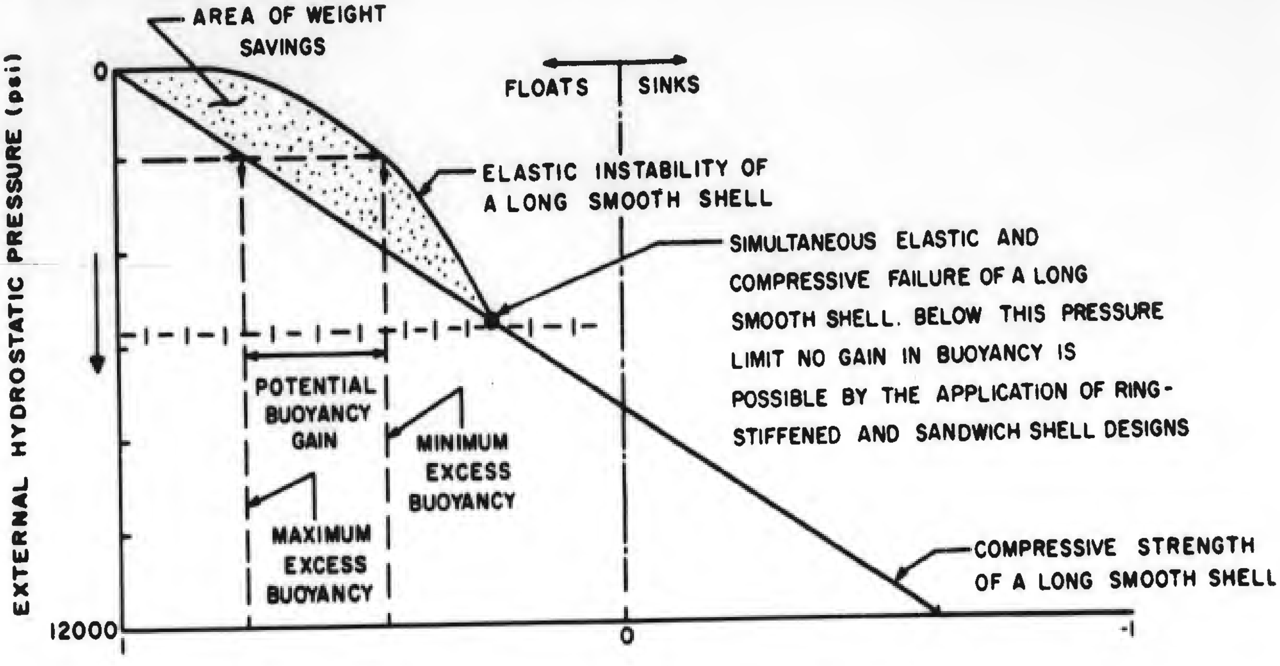


Fig. 3 - Area of Possible Improvements in Cylindrical Shell Design

be used as the upper buoyancy parameter (Fig. 4). The smooth-tube stability curve, describing the least efficient type of shell design, will be used as the lower buoyancy parameter. But a smooth infinitely long tube that fails by simultaneously buckling and yielding is given no rating since it represents the intersection point between the strength-of-materials and the elastic-stability curves where no gain in buoyancy is possible by better shell design. At pressures larger than the cross-over-point pressure, no advantages can be found for any other shell design but a smooth tube, because yielding of the material is the only mode of failure possible, and thus no design efficiency rating can be given to it.

Once upper and lower buoyancy parameter curves have been plotted, it is easy to determine the efficiency of any shell design provided the experimental data for the collapse of a long shell of this design are available. The efficiency is calculated in the following manner:

(1) Experimentally determine the collapse pressure of a long shell (short-shell collapse data may be used if the reinforcing action of the bulkheads or bulkhead weight is taken into consideration).

(2) Calculate and plot the strength-of-materials and elastic-stability curves for the construction material.

(3) Using the experimental collapse pressure as the starting point, draw a horizontal line to intersect both the strength-of-materials and elastic-stability curves (Fig. 4).

(4) Determine the coefficients of excess buoyancy (Fig. 3) for the intersection points and for the experimental shell. Excess buoyancy is defined as

$$E.B = 1 - \frac{W}{V\Delta},$$

where

W = weight of the shell structure

and

$V\Delta$ = weight of the sea water displaced by the enclosed shell.

The coefficients of excess buoyancy are then substituted in the design efficiency formula:

$$\text{Design efficiency} = \frac{E.B_e - E.B_l}{E.B_s - E.B_l} \times 100.$$

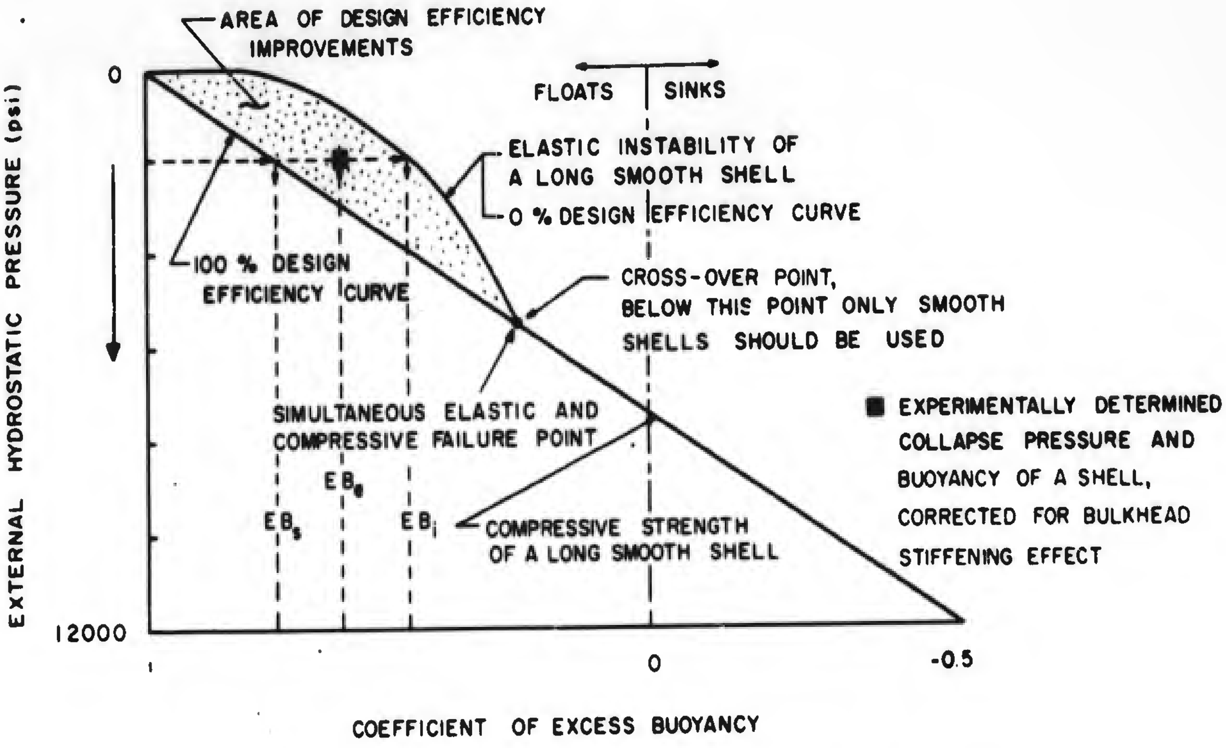


Fig. 4 - Method of Determining Shell-Design Efficiency

where

$E.B_e$ = coefficient of excess buoyancy of the shell at the experimentally determined collapse pressure, corrected for the bulkhead stiffening effect;

$E.B_s$ = coefficient of excess buoyancy of a 100 per cent efficient shell, shown on the graph by the intersection point between the experimental pressure line and the strength-of-materials curve;

and

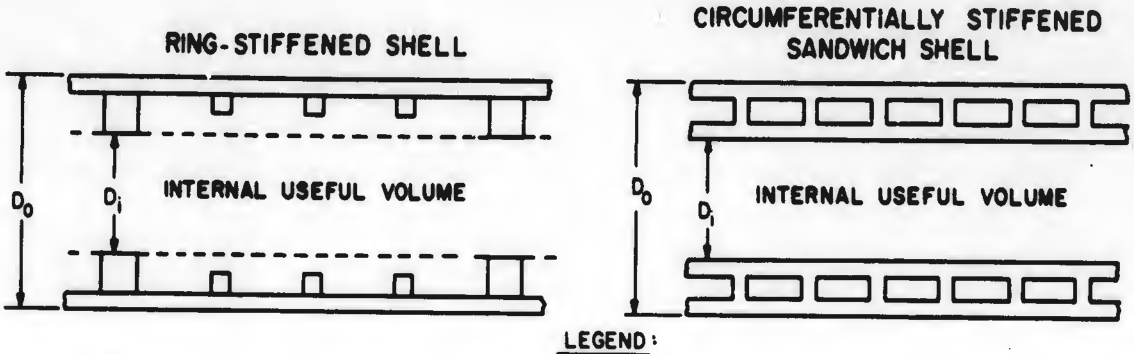
$E.B_i$ = coefficient of excess buoyancy of a 0 per cent efficient shell, shown on the graph by the intersection point between the experimental pressure line and the elastic-stability curve of a long smooth shell.

For example, the design efficiency of ORL cellular shell models A and A', calculated by this formula, was 100 per cent:

$$\text{Design efficiency} = \frac{0.92 - 0.54}{0.92 - 0.54} \times 100 = 100 \text{ per cent.}$$

The graphical representation of the strength-of-materials and elastic-stability equations permits comparison of the weight saving possible with other shell designs over the smooth shell in a given pressure range. Inspection of the curves shows that, for a given material, there is a limited pressure band in which it is advantageous to use shell designs other than a smooth tube. Not only that, it shows the relative and absolute magnitudes of the weight savings.

The design efficiency of ring-stiffened shells varies considerably as the position of the experimental collapse pressure of the shell varies with respect to the position of the material's cross-over pressure. Considerable data concerning the implosion testing of ring-stiffened shells have been collected, but unfortunately



LEGEND:

VOLUMES:

DISPLACEMENT VOLUME = $\pi D_o^2/4$

INTERNAL VOLUME = DISPLACEMENT - VOLUME OF SHELL MATERIAL

INTERNAL USEFUL VOLUME = $\pi D_i^2/4$

*ANNULAR VOLUME = $(\pi D_o^2/4 - \pi D_i^2/4)$ - VOLUME OF SHELL MATERIAL

COEFFICIENTS:

COEFFICIENT OF INTERNAL VOLUME = INTERNAL VOLUME/DISPLACEMENT VOLUME

COEFFICIENT OF INTERNAL USEFUL VOLUME = INTERNAL USEFUL VOLUME/DISPLACEMENT VOLUME

*APPLIES ONLY TO CIRCUMFERENTIALLY AND AXIALLY STIFFENED SANDWICH SHELLS

Fig. 5A - Utilization of Shell Volume in Underwater Shells

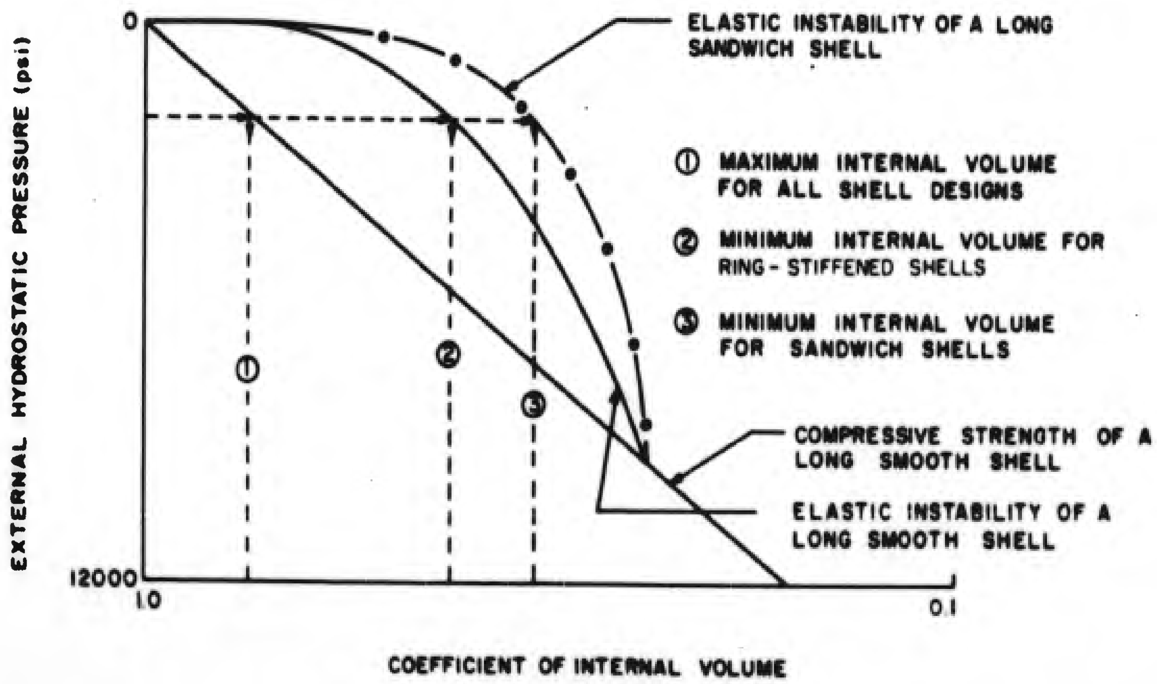


Fig. 5B - Internal Volumes of Cylindrical Shells

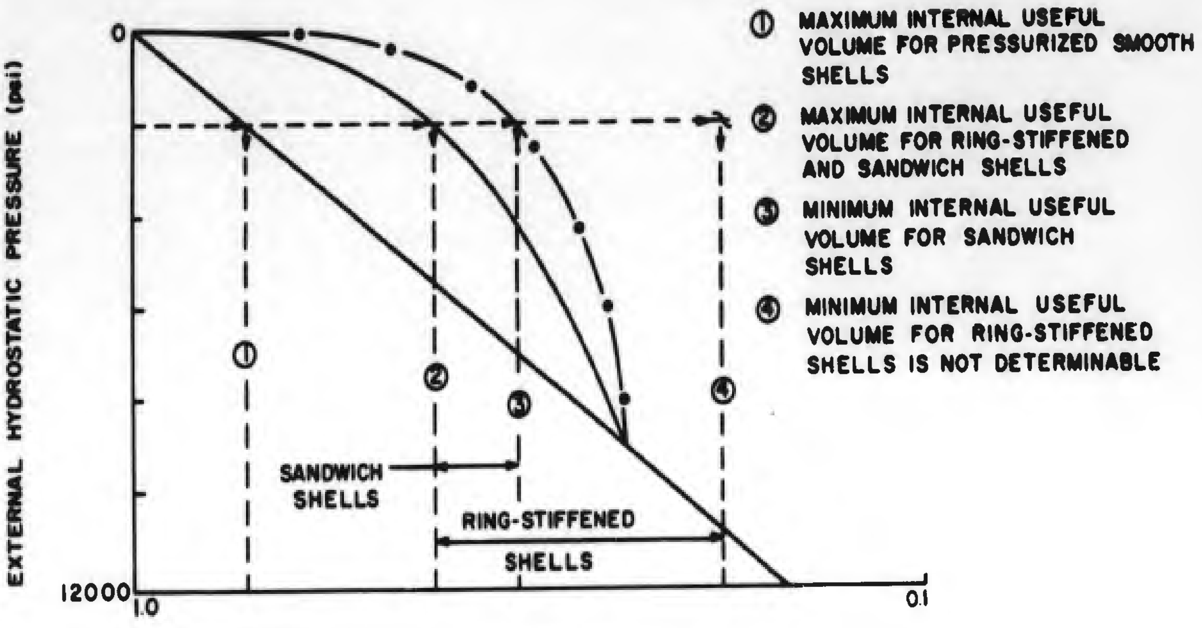


Fig. 5C - Internal Useful Volumes of Cylindrical Shells

most of the tests were conducted with shells having short bulkhead spacing. Failure to take into account the effect of the bulkheads, the elastic stability of which contributes considerably towards the over-all collapse resistance of the assembly, makes most of the plotted collapse data of dubious value for shell-efficiency evaluation purposes. Therefore, before comparisons of design efficiencies are made, the data must be carefully checked to ensure that they pertain either to long shells or to short shells whose bulkhead weights or reinforcing action have been taken into consideration.

A very useful yardstick for the comparison of shells is also the pressure-to-weight ratio:

$$\text{Pressure-to-weight ratio} = \text{collapse pressure} \times \frac{\text{displacement}}{\text{shell weight}} .$$

The pressure-to-weight ratio has been used for some time, and there are many references to it in the literature. This rating, in a brief expression, conveys the relationship between the weight of the shell, its displacement, and its experimentally determined ability to withstand pressure. However, it does not show the contributing factors, such as material strength, modulus of elasticity, or density, but gives the sum of all these properties as applied to a particular shell design. Briefly, this rating of the shell design concerns itself only with the over-all properties of the shell and not with the appropriateness of the shell design, the real measure of engineering achievement.

UTILIZATION OF INTERNAL SPACE

Ring-stiffened shells provide considerable internal volume, but the height and close spacing of the ribs reduce the useful volume (Figs. 5A to 5C). Since increasing the rib height increases the elastic stability of the shell without adding much weight, ring-stiffened shells are usually designed with high ribs. However, this severely restricts the size of any package that is to be inserted in the shell. To obtain more shell space, low ribs spaced at shorter intervals may be substituted, but the net result is an increase in the weight of the shell. Sandwich shells have the possibility of providing more internal space for the same weight and pressure because of the inherently higher moment of inertia of this sandwich wall structure. Although the weight of any rib is directly proportional to the height of the rib, the moment of inertia of the rib varies with the cube of the height. The use of T- and Z-shaped ribs (Fig. 1) has alleviated this problem somewhat, but the space limitation continues to be serious.

Ring-Stiffened Shells

Origin of Ring-Stiffened Shells

PRESSURE vessels subjected to external pressure received little attention in engineering circles until the late eighteen hundreds. It was felt that the smooth cylindrical tube provided a cheap and yet very satisfactory type of structure for vessels subjected to external pressure. The construction materials, mostly cast iron or steel, were of inferior quality; and relatively thick-walled shells were always used for underwater vessels. The thick walls prevented stressing of the tubes to their elastic-stability limits and presented no stimulus for research and experimentation in this area.

It was not until 1880 that Bresse derived an expression for the buckling of rings. In 1888 Bryan developed an accurate expression for the elastic buckling of infinitely long shells subjected to uniform external pressure; and Engesser extended this formula to include materials that do not follow Hooke's law, and to include the plastic region of materials that do follow Hooke's law. Until the outbreak of World War I, this simple formula satisfied the needs of the engineering profession.

World War I saw the development of the submarine as an instrument of war. The early submarines did not present structural problems because they were limited to shallow dives; but, near the end of the war, they had obtained depths of 250 ft and hull collapse caused by elastic buckling became prevalent. Although the reasons for these failures are obvious today, they were unknown to early investigators, who did not understand the complicated relationship between frame stiffness, plating thickness, and frame spacing. The stock remedy for hull collapse - that of increasing the design safety factor - could no longer be applied because the new submarines needed every pound of positive buoyancy. Spurred on by this crisis in submarine design, Von Mises developed a theory for the buckling of shells between stiffeners. Von Sanden and Gunther then developed a formula for the calculation of stresses in the shell at the stiffeners and midway between them. The size of the stiffeners and the collapse pressure of the shell caused by general elastic instability were not determined then, and it took another world war before these questions were answered.

In the lull between the two world wars, previously postulated theories were tested and information concerning ring-stiffened shells was gradually acquired. Trilling, Windenburg, Donnell, and Tokugawa performed many experiments in which shells were subjected to bending, and to compressive and implosive loading. The theories postulated by Von Mises and Von Sanden were found to predict the experimental results well, although they were not satisfactory in all areas. The theory of Von Mises, in particular, was found to accurately predict the number of lobes formed during buckling of tubes; but, according to Batdorf, it differed significantly from experimental collapse pressures in the low-curvature region (for $L^2/R_0 t \sqrt{1-\mu^2} < 100$, the theoretical values are up to 50 per cent higher than the experimental values).

Since World War II, the collapse of ring-stiffened cylinders has become more fully understood. The general elastic instability of ring-stiffened shells was explained by Kendrick; and the distribution of stresses in the shell and stiffeners, by Salerno and Pulos. With the contribution of Lunchick's theory on plastic failure, the failure of ring-stiffened cylinders, both in the elastic- and plastic-stress regions, became well understood. Ring-stiffened shells can now be designed to withstand a given pressure with a minimum of weight.

A selected bibliography of publications on ring-stiffened shells is given at the end of this report. This bibliography gives the primary sources listed here, as well as general information on ring-stiffened shells.

Structural Components of Ring-Stiffened Shells

The ring-stiffened shell is made up of three structural elements: the facing, the ring stiffeners, and the bulkheads.

The most important element is the facing, which serves as a barrier against the water outside the vessel. Although the facing contributes a large share of elastic stability and compressive strength to the vehicle, it must be stiffened by rings at close intervals to retain its cylindrical shape under pressure.

The ring stiffeners, or ribs, supply both elastic stability and compressive strength. The cross section and moment of inertia of the rings depend on the diameter of the vessel, maximum external pressure, spacing between the rings, and thickness of the facing - variables that thoroughly interact one with the other. Generally, the ring dimensions are such that, even when that part of the cylinder between the rings has been deformed and all load-carrying ability has been lost, the rings still retain their circular shape and prevent a general shell collapse.

The bulkheads, either solid discs or circular frames of at least twice the rigidity and strength of ordinary rings, divide the vessel structure into independent sections. All strength and stability calculations are based on the shell length between bulkheads, the bulkheads being considered perfectly rigid and uncollapsible.

The spacing of the bulkheads and rings gives the shell two independent parameters with which all other parameters must vary. To give calculations and figures a more nondimensional and general character, the ring and bulkhead spacings are given in terms of L/D and L_b/D, where L indicates ring spacing; L_b, bulkhead spacing; and D, the diameter of the smooth cylinder. The ring stiffeners come in a variety of forms, the form of the stiffener depending on the space and weight limitations.

Fabrication of Ring-Stiffened Shells

Ring-stiffened shells can be easily fabricated from almost any material. They can be cast, welded, or even machined from a single billet. Since most of the high-strength materials are either unweldable or require expensive postheat treatment, the ability to machine the shell from one billet is very important. The quality of the ring-stiffened shell can be easily controlled during manufacture, since both the external and internal shell surfaces are accessible. Because of these advantages, ring-stiffened shells are widely employed, except for a few applications where the pressure-to-weight ratio needed is extremely high.

Sandwich Shells

Origin of Sandwich Shells

THE pressure-to-weight ratio of the ring-stiffened smooth shell leaves much to be desired, even when all the shell parameters are optimized. The study of ring-stiffened smooth shells made it more and more apparent that the ability of the shell to withstand external pressure was dependent on two physical parameters: the strength of the construction material, and the stability of the stiffened wall. There is no shortage of high-strength materials; the problem is to find the proper shell structure with which to utilize these high-strength materials to maximum advantage. The talents of many engineers produced a number of ideas, but the high rigidity of the sandwich-wall design promised to significantly improve the pressure-to-weight ratio of shells subjected to external pressure. Although the idea for sandwich walls did not appear until the late nineteen forties, it has already become accepted as the optimum shell design for underwater vessels.

The idea for the sandwich-wall shell grew out of previous work with sandwich panels. Although sandwich panels were proposed a long time ago, they were not widely used until bonding and brazing techniques were perfected.

As engineers began to use the sandwich-type structure for different applications, it became imperative to derive, both on theoretical and experimental bases, stress-strain and stress-load relationships for sandwich structures of various configurations. In the short span of time from 1940 to 1960, the theory for flat sandwich plates became established and well supported by experimental data. However, the sandwich shell has not developed as rapidly. There are only a few researchers working on sandwich shells, and experimental data supporting the available theories are lacking. Most of the research in this field has been conducted at the Forest Products Laboratory, New York University, and at Soviet research institutions.

The earliest work in the sandwich-shell field was done by Leggett and Hopkins in England, Reissner in the United States, and Panov in the USSR. The major contribution was made by Reissner, who developed a nonbuckling theory for small deflections and strains in sandwich shells. His theory accounted for deflections resulting from compression of the sandwich core normal to the facings, as well as those caused by shear.

Following studies by Reissner, Stein and Mayers developed a linear small-deflection theory that does not consider core compression but includes average shear strains. Their theory, in terms of shears and deflections normal to the median surface, is expressed in three general equations with ten independent physical constants. If the simplification is introduced that the sandwich core is isotropic and does not carry the stresses directly, these equations can be reduced to Donnell's equation. Other workers in the field applied the equations of Stein and Mayers to various sandwich-shell configurations under different types of loadings.

The research group at New York University produced some outstanding results. They conducted many theoretical and experimental studies of sandwich shells. Their greatest contribution is a theory for the symmetrical buckling of sandwich shells under compressive end loads. Gerard and Wang derived a generalized nonlinear buckling theory, which is based on the principles of equilibrium and conservation of energy and includes shear effects. This theory neglects the rigidity of the facings, the basic assumptions being that the core behaves as a three-dimensional elastic medium in which stresses parallel to the facings are negligible when compared to the normal and transverse shear stresses.

The New York University group, using experimental data, compared their theoretical solutions with the Donnell equation as modified by Stein and Mayers. It can be stated that the modified Donnell equation predicts the axial bending load of weak-core cylinders well, but that the correlation between bending and torsional loads and the experimental data is not as good. However, until better theories are postulated, it must be stated that the linear buckling theory is acceptable for sandwich cylinders with soft cores under bending and torsional loading.

The Forest Products Laboratory, although primarily interested in the structural applications of wood-fiber board and plywood, has consistently contributed to analytical and experimental work on sandwich panels and cylinders. Raville, extending the work performed at the Forest Products Laboratory, concluded that when the sandwich facings are relatively thin an analysis in which the facings are treated as membranes is sufficiently accurate. However, he found that when the thickness of the facings is greater than one quarter of the core thickness the stiffness of the facings must be considered. The analytical work of Raville and others was mainly concerned with cylinders subjected to axial and lateral loads.

Soviet scientists, realizing the potential of sandwich structures, have spent considerable effort on sandwich-shell theory, as evidenced by the work of Prusakov, Vlasov, Grigolyuk, Korolev, Kurshin, Ambartsumian, and others. The work of Grigolyuk has produced the most generalized set of equations, and also provides for some plastic effects.

Bibliographies of publications on sandwich plates and sandwich shells are given at the end of this report. These bibliographies give the primary sources listed here, and are followed by a general shell bibliography. A review of the work done on sandwich shells will reveal how little is actually known about the detailed stress distribution in the sandwich cylinders and the various mechanisms of plastic and elastic collapse. Even when generalized sets of equations are available, like those of Grigolyuk and Donnell, it is quite difficult for the designer to use them for solutions of actual engineering problems. The one exception is Fulton's work, which graphically presents some of the equations for steel shells. Although there is an acute need for generalized stress-strain and deflection equations for orthotropic unsymmetric sandwich cylinders, there is also a very pressing need for special equations applicable to specific engineering applications.

Some experimentation has been conducted at the Ordnance Research Laboratory of The Pennsylvania State University toward development of a semiempirical expression for the general instability collapse of a sandwich shell. On the basis of theoretical considerations and the implosion data from four circumferentially stiffened cellular sandwich shells, the ring-buckling equation of Bresse has been modified to predict the general instability collapse of sandwich shells within ± 5 per cent. The modified Bresse equation has been used at ORL exclusively and has produced good results.* Most of the general elastic instability curves for sandwich shells have been plotted on the basis of this equation.

Sandwich-Shell Structure

The sandwich-shell structure satisfies two of the basic requirements for shell strength: it permits the use of high-strength materials, and it provides structural stability. The use of high-strength sandwich facings provides the desired compressive strength; the large moment of inertia of the widely separated facings supplies the required elastic stability. In the sandwich shell, the elastic stability is supplied by the wall itself, and not by rings and bulkheads, as in the ring-stiffened smooth shell. Since it is the wall itself that maintains the circular shape of the shell under load, it is quite easy to design the shell once the dimensions of the wall are determined. The shell becomes more homogeneous because of the uniformity of structure.

Although all sandwich shells are based on the same principle, the method of separating the sandwich facing differs considerably. The five basic methods for separating the sandwich facing are shown in Fig. 6. These are:

1. honeycomb matrix;
2. microballoon plastic matrix;
3. cellular matrix;
4. solid filler; and
5. tubular matrix.

Prototype shells employing these separation methods have been built. The David Taylor Model Basin and the Ordnance Research Laboratory have devoted most of their efforts to the cellular sandwich shell. Westinghouse has experimented with microballoon plastic sandwiches, and both ORL and the Hexcel Company have performed some exploratory work with honeycomb sandwich shells. So far, the microballoon, honey-

*Jaroslaw D. Stachiw, General Instability of Circumferentially Stiffened Sandwich Shells Subjected to Uniform External Pressure, Master's Thesis, The Pennsylvania State University, 1961, p. 96.

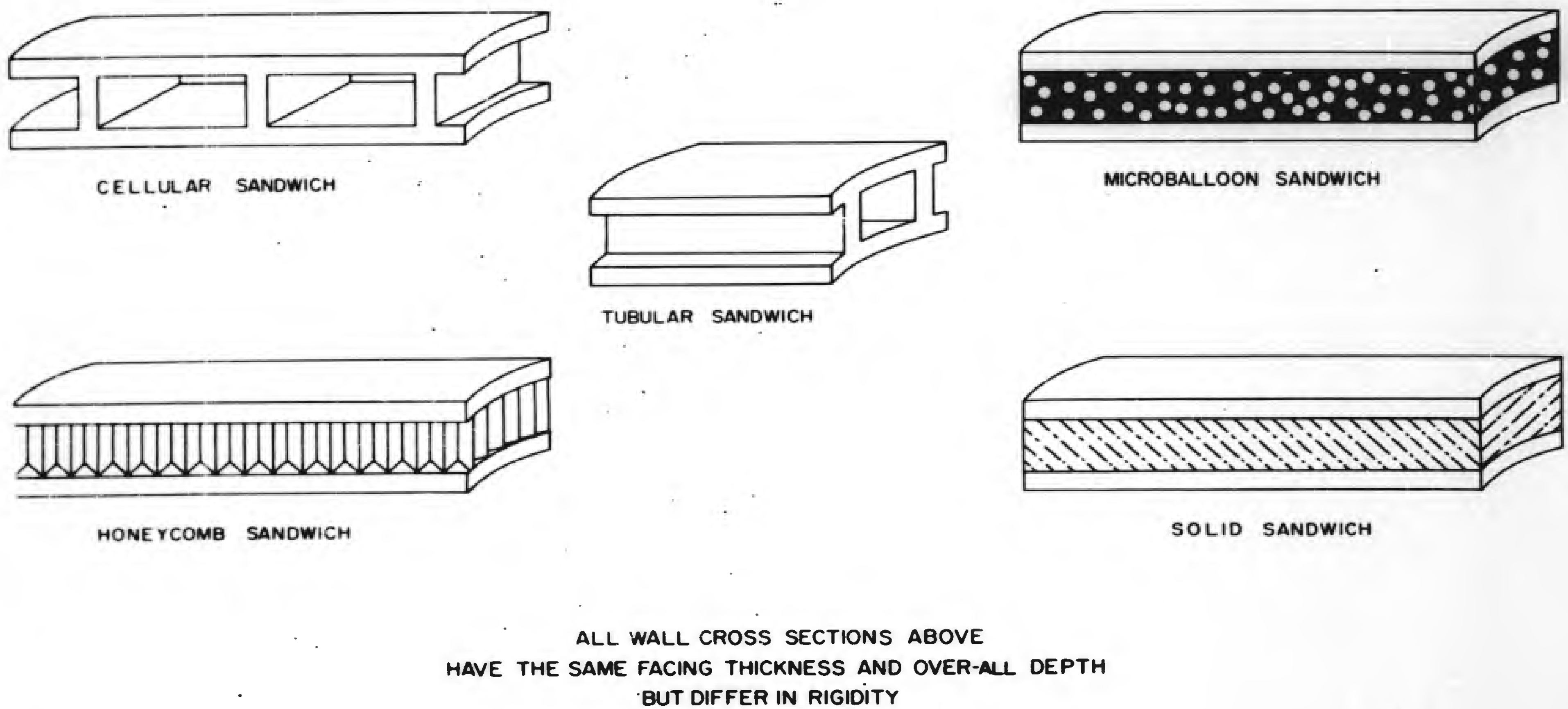


Fig. 6 - Methods of Separating Sandwich Facings

comb, tubular, and cellular sandwich shells have been subjected to external hydrostatic pressure. Of these, only the cellular sandwich type has been designed and tested for external pressures greater than 1500 psi.

Each of the sandwich-shell designs has its merits, and no one design is distinctly superior to the others. The honeycomb matrices have been made of fiber glass materials, and are available in a variety of thicknesses and compressive strengths. However, they are limited to a compressive strength of 1500 psi, and they present difficulties when formed into small cylinders or when used in cylinders having metal facings. The space between the facings is completely filled, and the shell wall cannot be used for heat-transfer purposes because of its heat-isolation properties. Nevertheless, honeycomb matrices faced with glass fiber laminations are inexpensive and can be used to construct lightweight, pressure-resistant vehicles (less than 1500 psi) with the possibility of an excellent structure.

Microballoon shell construction is a new development, but it has been tested experimentally and found to be satisfactory. The microballoon sandwich shell has a core of lightweight porous plastic, the porosity of which can be varied to meet the compressive strength requirements of the shell. The microballoon matrix is a viscoelastic material that may be useful for damping the shell wall. The space between the facings, however, cannot be utilized for storage of fluids or heat exchange. The latter limitation prohibits its use around propulsors that radiate appreciable heat.

The cellular sandwich shell relies on annular stiffeners for separating the shell facings. These stiffeners must be sufficiently thick and so spaced as to avoid local instability or yielding of the facings. This sandwich construction creates a convenient annular space that can be used for fluid storage (gases under pressure, or liquids), heat exchange functions, or other purposes. However, such construction creates problems in fabrication; but it is felt that they are amenable to solution by good product engineering practices. Adhesives, which are so admirably suited for honeycomb and microballoon sandwich shells, are unsuited for the cellular sandwich shell, which requires narrow joints and puts great strains on adhesives. It appears that only welded, cast, or laminate constructions are suitable for cellular sandwich shells. The annular stiffeners are designed to carry a considerable share of circumferential load, giving the cellular sandwich shell an inherently higher pressure-to-weight ratio than that of other types of sandwich shells, except for the solid sandwich. The only serious disadvantage of cellular construction is the difficulty involved in fabricating shell components of foil thickness. Although it is theoretically possible to construct very light cellular shells for low external pressures, there is a limit on the minimum thickness of the shell components.

The tubular shell has a lower pressure-to-weight ratio because the stiffeners are arranged axially instead of circumferentially. This type of construction will result in a shell having less stability than that of a cellular sandwich shell having the same weight. There may be some conditions regarding the use of the annular space in which tubular shells can have a distinct advantage and the extra weight can be absorbed. Fabrication problems for tubular shells appear to be similar to those for cellular type shells.

The solid sandwich shell utilizes either a metallic or nonmetallic spacer material of lower density than that of the facings. This design is particularly applicable to shells subjected to high external pressures. The high compressive strength of the spacer material eliminates the possibility of local failure. The use of a solid metallic spacer may make it easier to attach propulsor machinery and other heavy shell components to the wall. The fabrication of metal sandwich shells does not present any special difficulties. The solid spacer, however, eliminates the possibility of using the sandwich wall for storage of fluids or heat exchange.

Critical Comparison of Sandwich-Shell Designs

The five types of sandwich-shell designs - for small-diameter (10- to 30-in.) shells - can be grouped arbitrarily as follows:

1. designs applicable to low external pressure vehicles (0 to 1500 psi) - honeycomb, microballoon;
2. designs applicable to intermediate external pressure vehicles (1500 to 4000 psi) - cellular, solid, tubular; and
3. designs applicable to high external pressure vehicles (>4000 psi) - solid, cellular.

The honeycomb and microballoon sandwiches give very high pressure-to-weight ratios at low pressures. The extremely light honeycomb and microballoon spacers assure uniform support

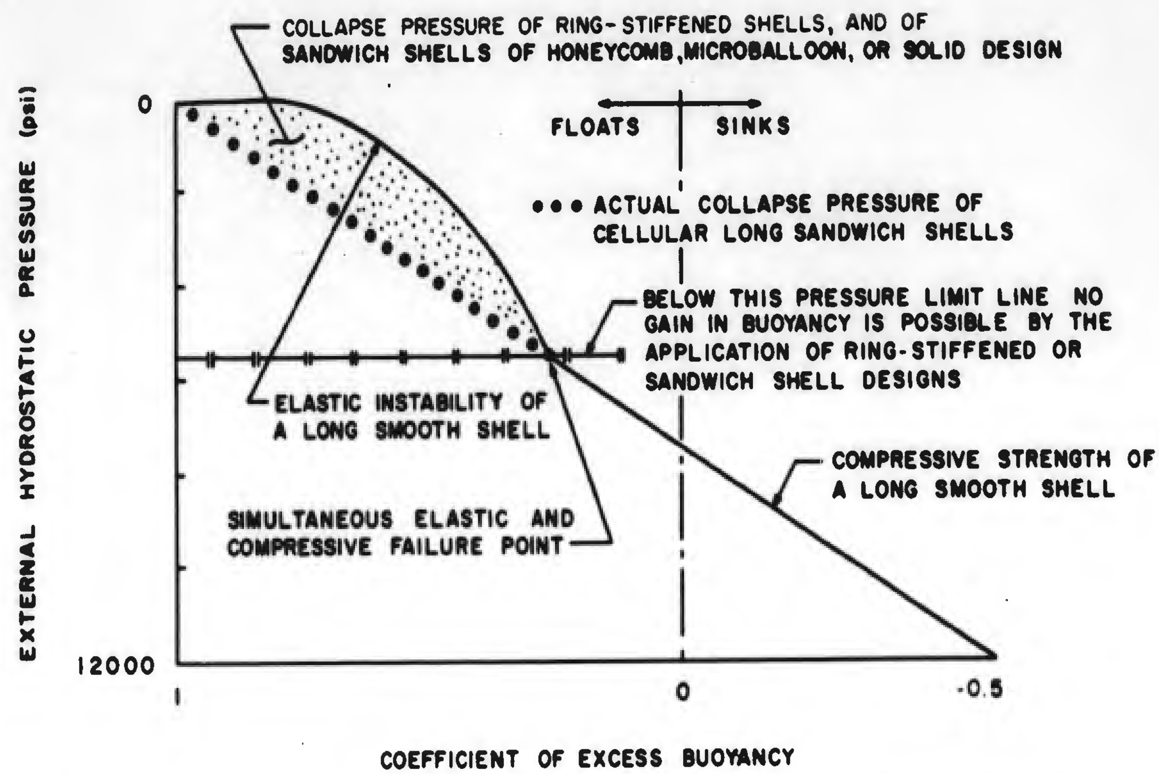


Fig. 7 - Buoyancy and Maximum Pressure Limits of Cylindrical Sandwich Shells

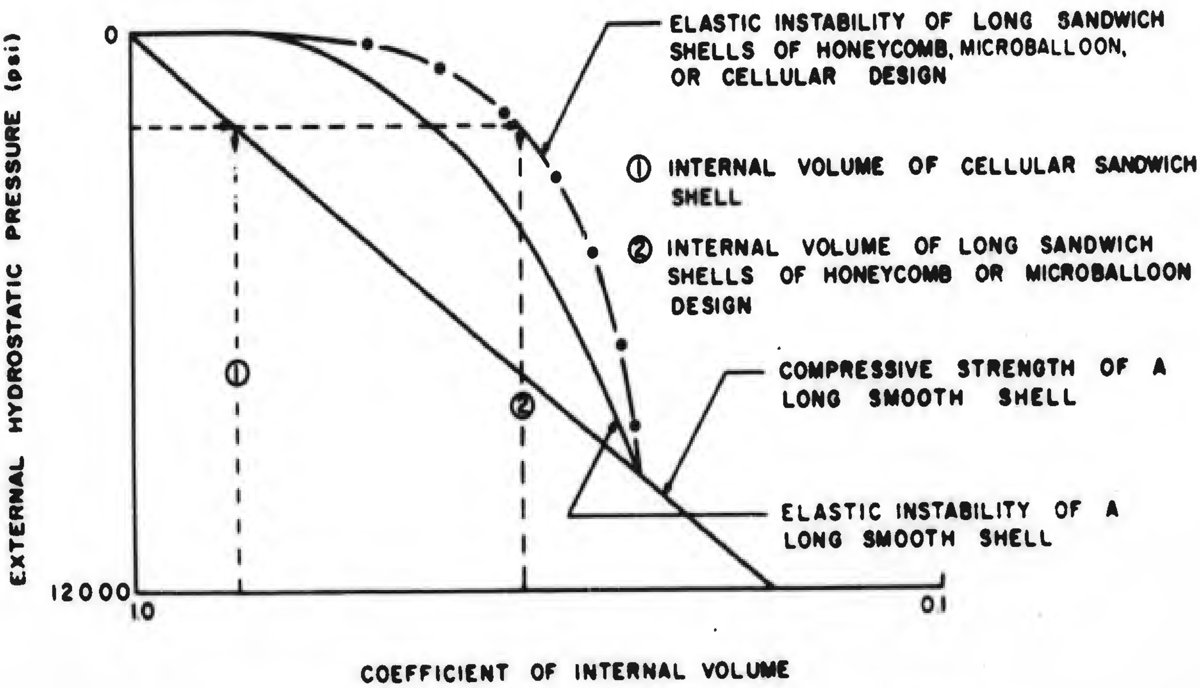


Fig. 8 - Internal Volumes of Cylindrical Sandwich Shells

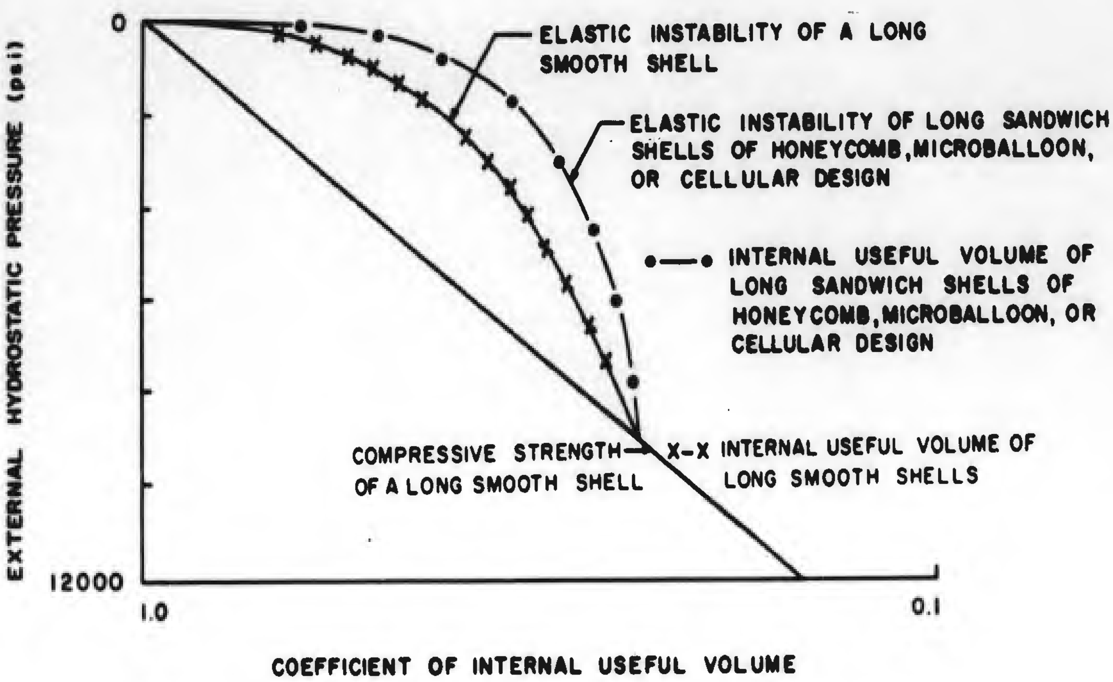


Fig. 9 - Internal Useful Volumes of Cylindrical Sandwich Shells

for the high-strength facings and make difficult the fabrication of small shells with an excess buoyancy factor of 0.9 below 1500 psi. It is impossible to fabricate a cellular sandwich shell with an excess buoyancy factor of 0.9 below 1500 psi. because the thin facings cannot withstand the pressure. The solid sandwich, even when constructed of extremely light metal and strengthened with solid nonmetallic spacers, cannot meet the 0.9 excess buoyancy criterion that is so easily met by the honeycomb or microballoon sandwiches.

At intermediate pressures, the cellular and solid sandwiches are recommended because of their ability to withstand high compressive loading. Also, the extra fluid-storage capacity in the walls of the cellular sandwich makes this design applicable to vessels that require an efficient heat exchange for the gases or liquids used or produced by the various subsystems making up the complete vehicle. At intermediate and high pressures, the facings and annular stiffeners are substantial, and cellular sandwich shells with excess buoyancy coefficients of 0.9 and 0.8 can be fabricated without difficulty.

The tubular sandwich shell has a relatively low pressure-to-weight ratio, but the axial passages between its stiffeners may provide a unique solution for certain problems involving heat exchange, fluid transfer, or even conduit passages. Coefficients of excess buoyancy in general will be less than those for the cellular sandwich shell.

If heat exchange and boundary-layer control are not required, the solid sandwich shell offers the best design for vessels subjected to high external pressures. This design assures elastic stability, good pressure-to-weight ratio, and resistance to local failure. However, design criteria and fabrication methods for this type of shell have not been explored. The cellular shell is also an attractive solution to the high external pressure requirement.

At present, underwater vehicles are limited to intermediate external pressures. The cellular sandwich design offers a very attractive answer to all the shell requirements in this pressure range, including a high coefficient of excess buoyancy, and fair utilization of internal vessel space. The cellular sandwich shell can be fabricated from a variety of structural materials and by existing fabrication methods. In addition to providing a good practical shell structure, the cellular sandwich shell can be used for fluid storage and heat transfer.

Figure 7 shows the buoyancies and maximum pressure limits for cylindrical sandwich shells; Figs. 8 and 9 show internal volumes and internal useful volumes, respectively, for cylindrical sandwich shells. Figures 10 and 11 give the same information as Figs. 7 to 9 for 6061-T6 aluminum sandwich shells, the collapse pressures of which have been adjusted for the stiffening action of bulkheads and shell joints. Volume-utilization and buoyancy curves for other shell materials are given in the Appendix.

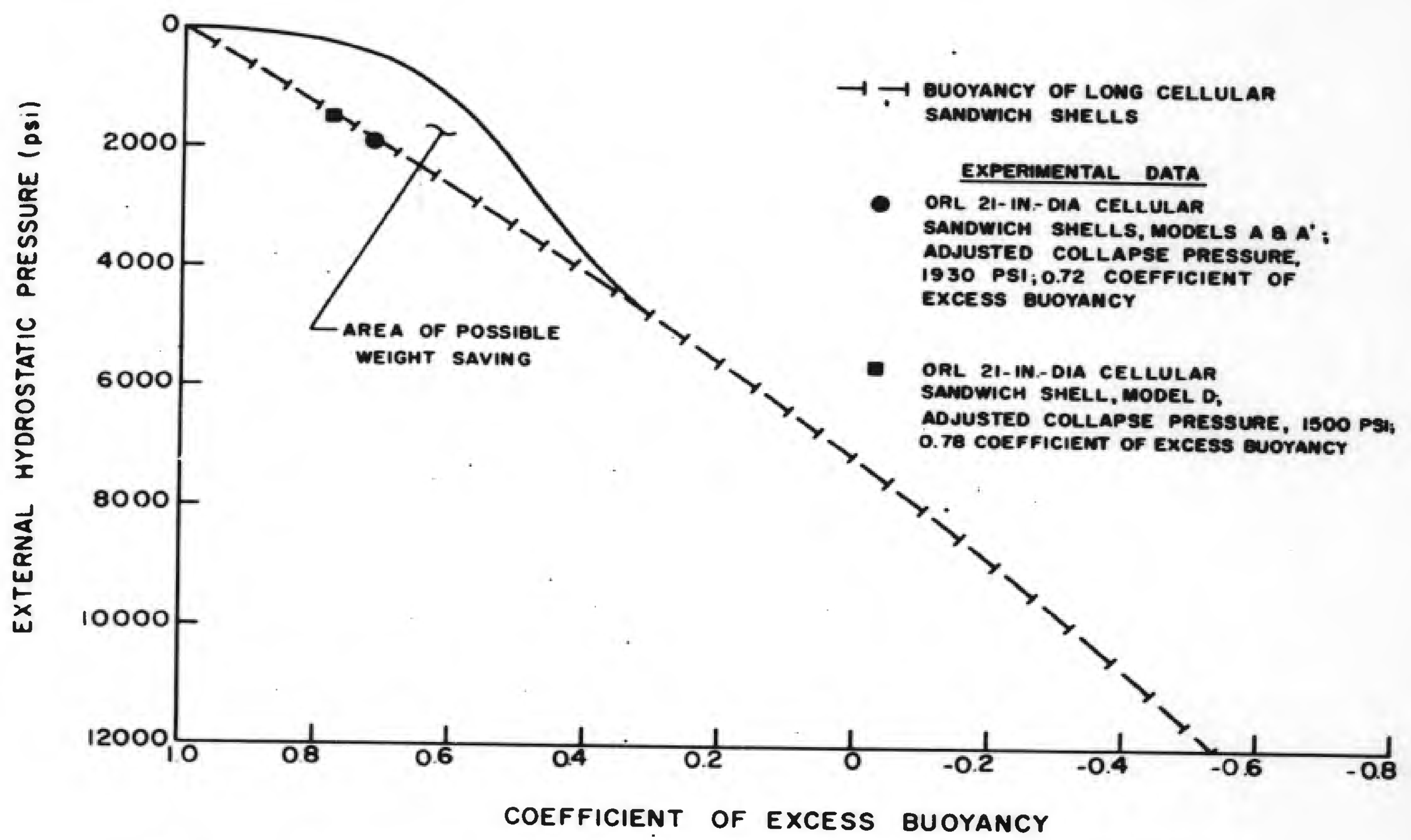


Fig. 10 - Buoyancies of Cylindrical Sandwich Shells Fabricated from 6061-T6 Aluminum

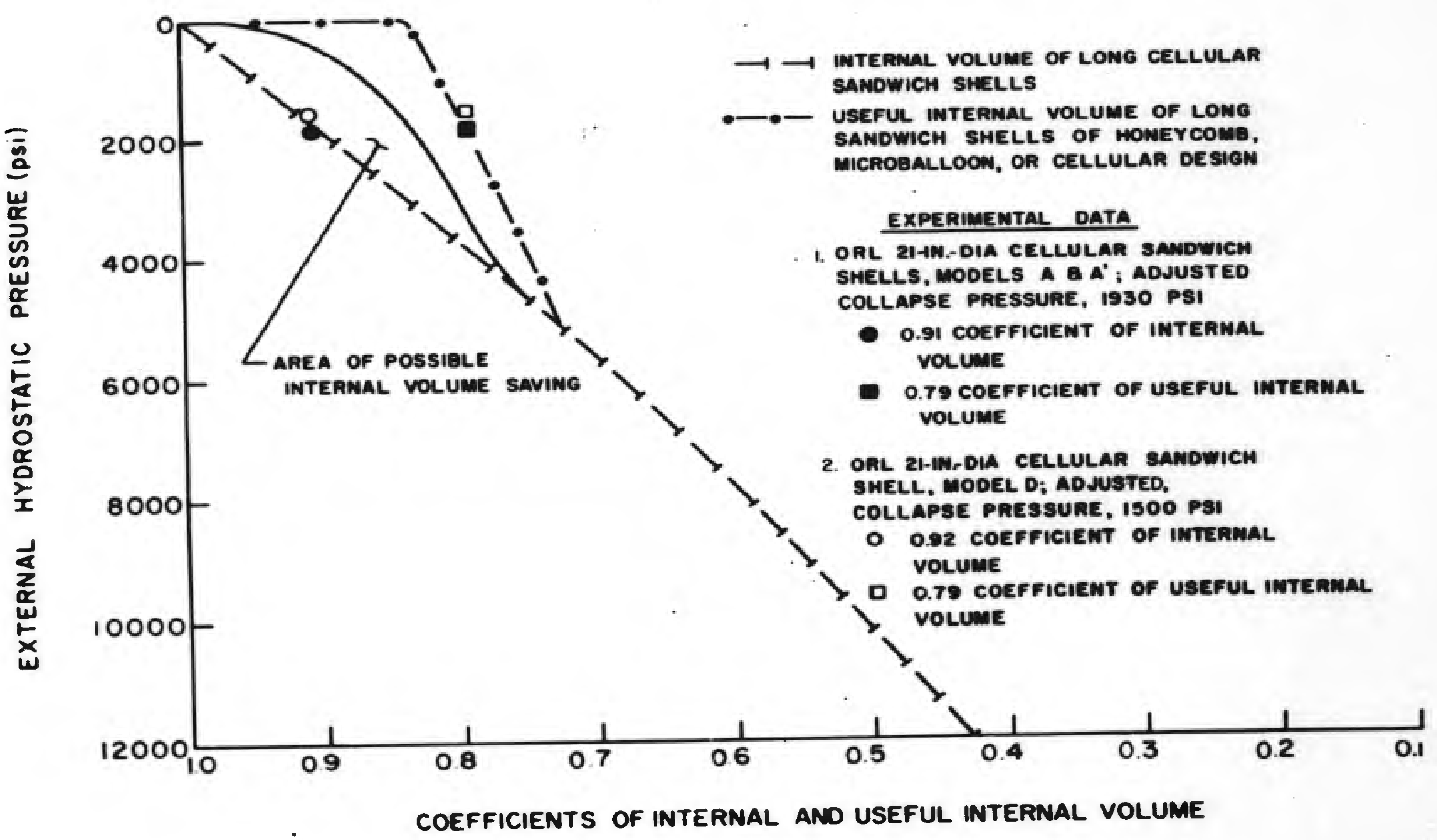


Fig. 11 - Shell Volume Utilization in 6061-T6 Aluminum Sandwich Shells

Internal Pressurization of Shells

INTERNAL pressurization is one method of increasing the implosion resistance of a shell without adding material to the shell structure. This method utilizes compressed gases or liquids to counteract the hydrostatic pressure acting on the outside of the shell (Fig. 12). Pressurizing the internal volume of the vessel makes it possible to design a vehicle shell that could operate at almost any external pressure. Internal pressurization for external pressure vessels has not been used to any great extent, but extension of pressure capability may require its use for future applications.

Pressurization Methods

Pressurization systems can be classified according to: (1) the pressurizing fluid, (2) the method by which the pressurized fluid carries the external load, and (3) the method by which the fluid is pressurized.

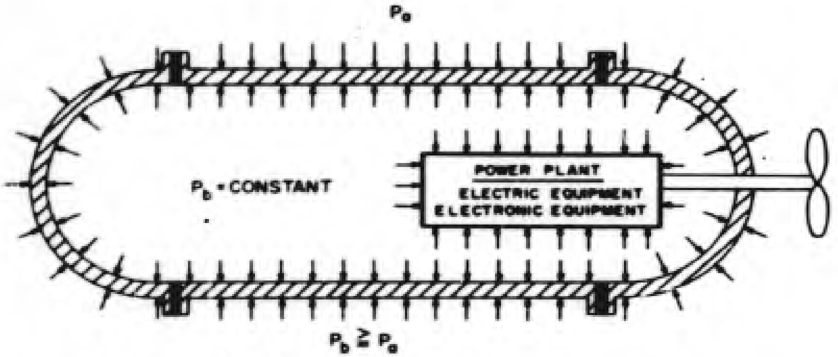
Pressurization systems may employ either liquid or gas as the pressurizing fluid. Gas-pressurization systems would be likely to use the low-density, inert gases as a pressurizing fluid. Liquid-pressurization systems would be apt to utilize a liquid that has a specific gravity lower than that of water, although water itself as a pressurizing fluid is a simple solution involving only free flooding of the cavity. The lower the specific gravity of the gas or liquid used, the higher will be the strength-to-weight ratio of the shell since pressurizing fluid weight must be charged to the shell weight.

Pressurization systems can also be classified according to the load-carrying methods employed. There are two basic systems: the first relies completely upon the internal fluid pressure to counteract the external pressure; the second is a hybrid system that uses both fluid pressure and shell structure to share the external load. In the first system, the shell serves as a membrane only, separating the sea water from the fluid inside the shell. The shell does not carry any external pressure, its compressive strength and elastic stability being completely disregarded in the internal-pressure calculations. The only strength requirement is that the shell withstand flexural loads imposed during use, handling, and storage. The hybrid system takes into consideration the compressive strength and elastic stability of the shell, as well as the forces exerted by the compressed fluid. This system is more economical, for it requires a lower internal fluid pressure and thus a smaller mass of pressurizing fluid.

The method by which the fluid is pressurized provides the third means of classifying pressurization systems. There are three ways of pressurizing the fluid: (1) pressurizing the shell cavity before launch; (2) pressurizing the vehicle by means of a pressure tank located within the vehicle; and (3) pressurizing the vehicle by means of the external pressure itself, which uses the external fluid for flooding or exerts pressure across a membrane. Each of these methods places special demands on the shell structure and the pressure-regulating system.

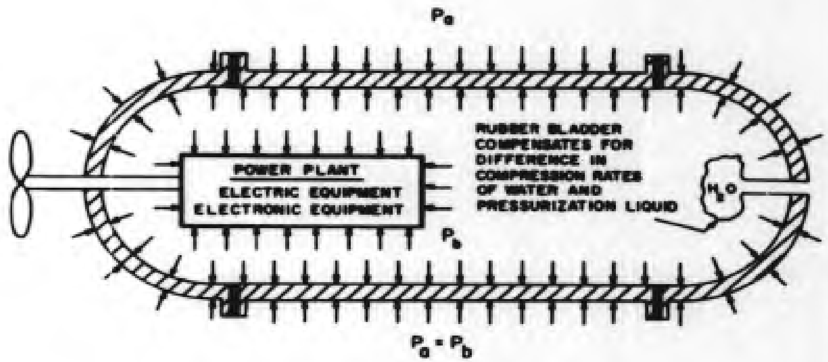
If the shell cavity is charged with a gas from some separate source, the shell must be capable of holding the high internal pressure. Since compressed gases are dangerous to operating personnel, a high safety factor must be used in the design of the shell. The absence of pressure regulators makes this type of pressurization system very reliable, but the shell becomes an internal pressure vessel that may be prohibitively heavy ~~because of the~~ thick shell walls.

When the vehicle uses gas from a pressure tank inside the shell, the only requirement is that the vehicle shell withstand those stresses caused by use, handling, or storage. As the external pressure varies with the operational depth of the vehicle, the pressure-regulator mechanism meters out the gas accordingly so that a set pressure differential is always maintained between the outside and the inside of the vehicle. The disadvantage of this system is the



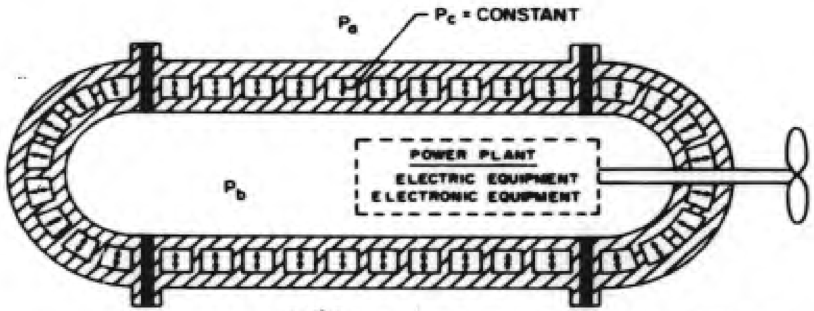
VESSEL PRECHARGED WITH COMPRESSED GAS

EQUIPMENT MUST BE ENCAPSULATED TO PROTECT IT FROM THE HIGH PRESSURE GAS ATMOSPHERE



VESSEL FILLED WITH LIQUID

EQUIPMENT MUST BE ENCAPSULATED TO PROTECT IT FROM COMING IN CONTACT WITH THE HIGH PRESSURE LIQUID



DOUBLE WALLED VESSEL WITH PRESSURIZED ANNULAR SPACE

EQUIPMENT DOES NOT NEED ANY SPECIAL PROTECTION FROM THE AIR AT ATMOSPHERIC PRESSURE

Fig. 12 - Methods of Pressurizing Underwater Vessels

necessity for venting the excess gas pressure so that it will not rupture the vehicle. The alternating pressurization and venting operations may deplete the compressed gas in the tank rapidly, limiting the depth variations of the vehicle.

In vehicles pressurized with liquid, the pressure of the liquid can be adjusted to the external pressure by means of flexible diaphragms that transmit the external pressure to the pressurizing liquid. Such an arrangement is very simple and yet very effective. There are no moving parts, and the vehicle shell serves as a membrane only. However, the shell must withstand the flexural stresses during prelaunch handling.

Comparison of Pressurization-System Weights

The weight of a pressurized underwater vehicle is the sum of the weights of the shell, shell components, component encapsulations, and pressurizing fluid. Once the pressurization system has been selected, the only way to lighten the vehicle is to obtain a gas or liquid having a lower specific gravity. Figures 13 and 14 show the excess buoyancy coefficients of shells pressurized with various gases and liquids under the conditions stated thereon.

It is easy to determine the best gas- or liquid-pressurization system for a given pressure. The weight of the gas-type system varies with the maximum operational depth for which it is designed; the weight of the liquid-type system is almost independent of the maximum operational depth. When the buoyancies of both systems are plotted as a function of external hydrostatic pressure, they intersect at some external-pressure coordinate. At any pressure less than that of the intersection point, a certain gas system is preferred because it possesses better pressure-to-weight characteristics than the liquid system. At any pressure greater than that of the intersection point the liquid system has a much better pressure-to-weight ratio. Generally, the gas-pressurization system will be better than the liquid-pressurization system, except where extremely great pressures are involved.

In addition to the weights of the shell structure and the pressurizing medium, the weight of the necessary protective encapsulation is a factor. Encapsulation is required to protect components from the effects of high pressure, or from the damaging liquid environment. These enclosures could also complicate placement and maintenance of the various internal components of the vehicle.

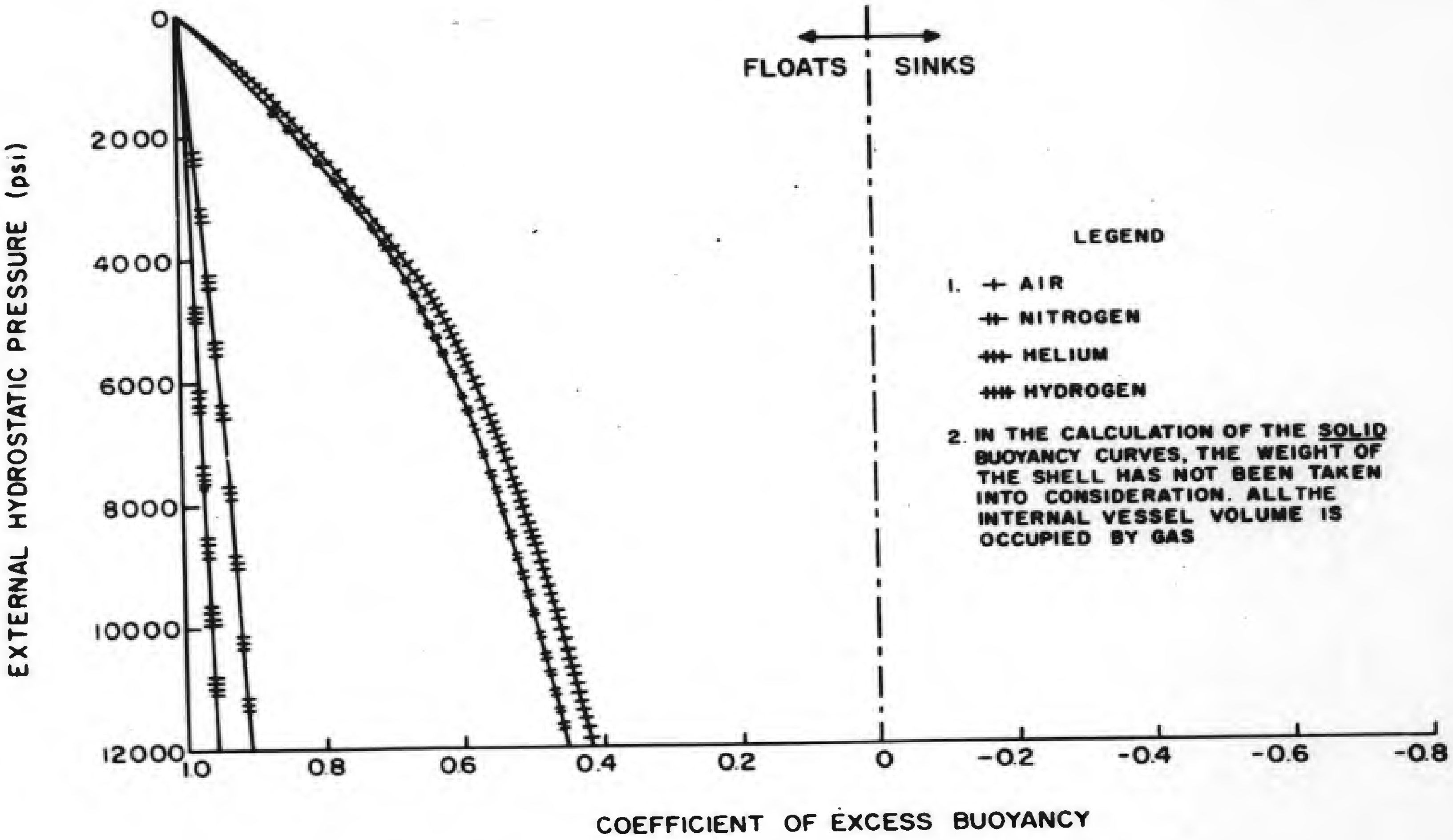


Fig. 13 - Buoyancies of Vessels Pressurized with Gas

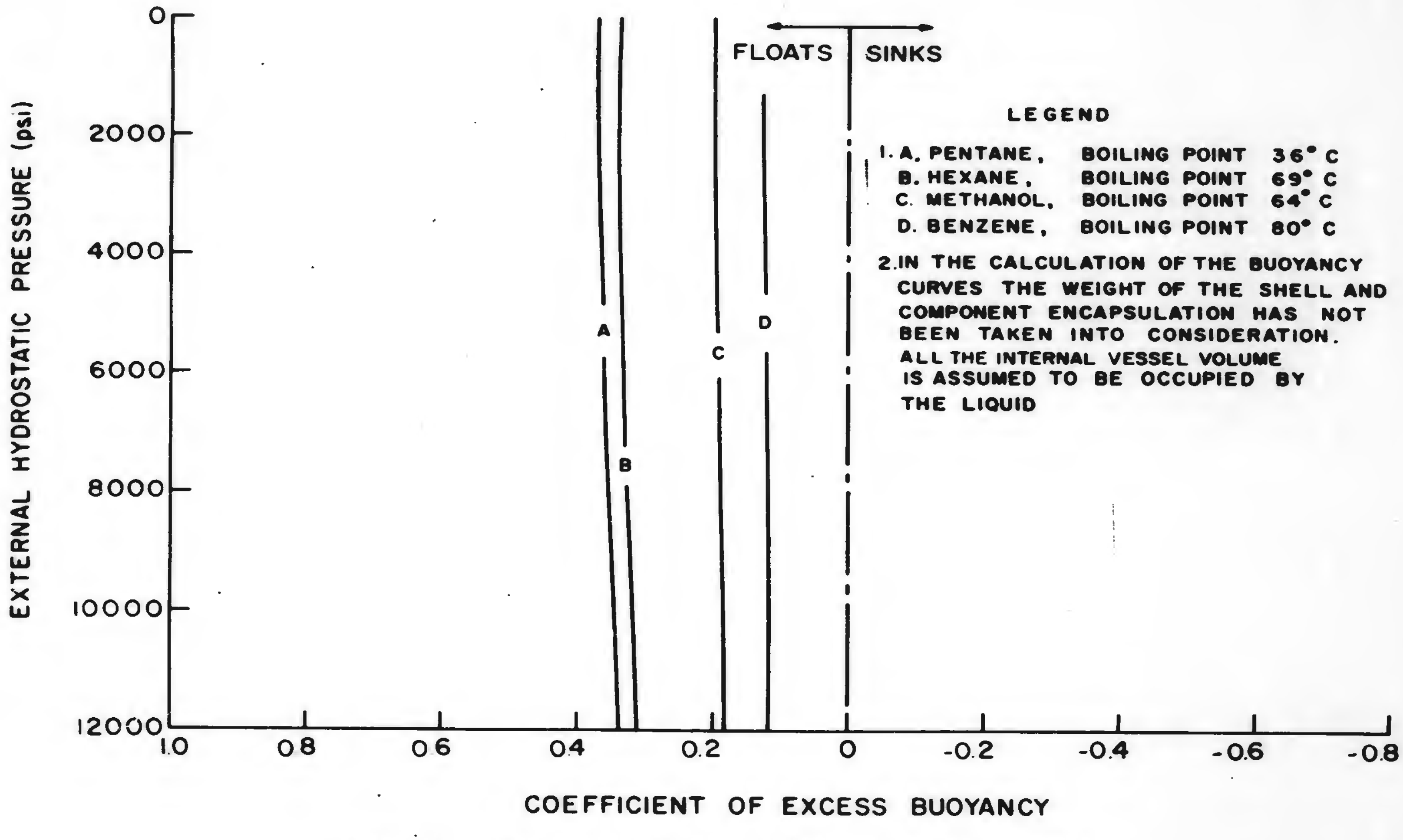


Fig. 14 - Buoyancies of Vessels Pressurized with Liquids

Summary and Evaluation of Shell Designs

Classification of Shell Designs

SHELLS for underwater vehicles can be grouped into two categories: (1) shells that utilize their structural characteristics to withstand external pressure; and (2) shells that utilize an internal pressurized fluid to withstand external pressure. Examples of the first category are ring-stiffened shells and sandwich shells. Examples of the second category are those structures, such as buoyancy or trim chambers or others, which are flooded as a necessary part of their operational functions.

Ring-stiffened shells and sandwich shells utilize the compressive strength of the material and the elastic stability of the shell structure to maintain the hydrodynamic shape of the vehicle. The ring-stiffened shell was developed in the late decades of the nineteenth century, and has been improved over the years. The sandwich-shell design was not feasible until the late nineteen forties, but it has already become accepted as the optimum shell design.

Present State of Shell Design

At the present time, the most reliable shell design is the ring-stiffened smooth cylinder. Mathematical expressions have been developed for the accurate calculation of stresses in the structure of the ring-stiffened shell. Using general equations, the designer can calculate the general instability, local instability, and local yielding of the ring-stiffened shell within ±5 per cent of experimentally determined values.

The ring-stiffened shells derive most of their stability from ring stiffeners and bulkheads. When the weight of all the structural components, including the bulkheads, is taken into consideration, the average weight of the shell is much heavier than the weight of the material necessary to withstand simple circumferential compressive stresses in the shell. The difference in weight between the actual shell weight and the weight specified by the average circumferential stress formula $t = pD/2\sigma$ is caused by inefficient stiffening of the facing against elastic-instability collapse. This means that the ring-stiffened shells do not utilize all the compressive strength available in the material. The utilization of internal useful volume is not very good either, because of the large stiffening rings and bulkheads. For these reasons, the ring-stiffened shell is no longer considered an efficient shell design.

Sandwich shells consist of concentric cylindrical facings separated by a lightweight spacer. Depending on the type of spacer, the sandwich structure is called cellular (circumferentially stiffened), tubular (longitudinally stiffened), microballoon (material with high porosity), or solid (bimetallic). The facings carry the circumferential and axial stresses; the wide spacing between the facings provides the shell with sufficient elastic stability. Unlike the ring-stiffened shell, the sandwich shell does not require additional material, and the simple compression-strength formula predicts its weight with reasonable accuracy.

All the sandwich-shell types have been built and tested on a limited basis. Sufficient data have been accumulated to design and fabricate a sandwich shell for a given external pressure. The uniformity of shell wall thickness and the smooth uncluttered interior permit efficient utilization of internal volume. Some of the sandwich-shell walls possess annular cavities that can be used for fluid-storage or heat-exchange purposes.

Since internally pressurized shells of the ring-stiffened or sandwich type have not been utilized to any great extent, there is a lack of experimental data for design purposes. However, the derivation of mathematical expressions for the calculation of the structural dimensions of the shell does not present any difficulty.

Future Research

Each of the methods for adapting underwater vehicle shells to high external pressures has its limitations and represents a compromise among all the parameters present. The ring-stiffened shell does not provide sufficient elastic stability and does not utilize all the available compressive strength of the shell material.

The sandwich-type shells are limited by the compressive-yield strength and the density of the material. The internally pressurized shells, on the other hand, are limited by the compressibility and specific gravity of the pressurizing medium.

The limitations of these shell designs point the way to future research. The possible areas of research are:

1. development of better stiffening methods for ring-stiffened smooth shells;
2. development of light shell materials with higher compressive strengths and moduli of elasticity, including the fabrication procedures; and
3. development of pressurizing media with lower compressibility and density.

Since sandwich shells have already overcome the elastic-stability limitations of the ring-stiffened shell, improvement of ring-stiffened smooth shells does not appear to be remunerative. At best, the elastic stability of improved ring-stiffened smooth shells would only equal the elastic stability of the sandwich shell.

The most promising area of research is the development of higher-strength materials. At present, the compressive strength of materials is below 300,000 psi, but fabrication of sandwich shells from these materials presents great difficulties. The best shells that have been built utilize material with only 150,000-psi compressive strength, but these are not reliable production-type shells. Thus, research in this area must also include methods of fabricating sandwich shells from existing high-strength materials.

The development of improved shell-pressurizing techniques could make pressurized shells competitive with sandwich shells. At present, the pressurizing techniques are in their primitive stage of development and not widely used.

Recommended Shell Designs

The sandwich shells possess the best possible strength-to-weight ratio because of their ability to fully utilize all the material used in the structure. They satisfy more of the requirements for a successful shell design than do other designs for a given pressure range. There is no particular sandwich-shell design, or construction material, however, that is best for the whole pressure range. Certain materials and construction designs among the cellular shells will result in a better shell for a given pressure range. At the present time, there is some question about the fabrication of certain of these designs; but, disregarding this factor since it is an engineering fabrication problem, materials and designs of sandwich shells can be recommended. The honeycomb or microballoon sandwich using fiber glass laminates, or their equivalent, is best for the lower pressure ranges; the cellular sandwich is best for a large range of intermediate pressures; and either cellular or solid sandwich shells are best for the higher pressures when the commonly used construction materials are considered. There is a considerable overlap of pressure ranges; in these "gray" areas only the use of good judgment will be of any help in selecting a design and material.

Some newer and untried materials, like ceramics and glass, show promise of being useful for construction of external-pressure vessels. These materials, if successfully exploited, may revolutionize the whole external-pressure-vessel art.

Appendix

FIGURES 15 through 35 give information on volume utilization and buoyancy for cylindrical shells. To describe the limitations of these data, the equations on which they are based are also presented (Fig. 15). Some of these equations have been obtained from handbooks; the others have been developed by the author on a semiempirical basis.

Figures 16 through 35 are substantiated by tests, performed at ORL, of model and full-scale acrylic resin and aluminum shells. These data have been adjusted for the stiffening action of the bulkheads and shell joints, and the plotted pressure actually represents the collapse pressure of a long cellular sandwich shell; that is, a shell whose bulkheads are widely spaced so that they do not substantially contribute to the collapse resistance of the shell. If shorter bulkhead spacing is considered, the curves cannot be read directly, but must be adjusted for the bulkhead strengthening effect.

The volume-utilization and buoyancy curves should be used for a general comparison of different materials and shell designs. They should never be used for actual design because greater accuracy can be obtained by detailed calculations based on shell application. In such detailed calculations, it is possible to take into account the effect of facing thickness on the stress distribution in the shell wall, the stress concentrations caused by stress raisers, and other factors that cannot be taken into consideration when plotting curves on nondimensional scales.

Figures 18 through 35 show only the maxima and minima of buoyancy and internal volume coefficient for cellular sandwich shells. Figures 18 through 23 are graphs of the cellular shell's coefficient of useful internal volume for selected premium materials. Figures 24 through 29 are curves of available buoyancy and internal volume utilization for all commonly used construction materials.

Figures 18 through 35 can also be used for the approximate determination of excess buoyancy, internal volume utilization, and useful internal volume for tubular, honeycomb, and microballoon sandwich shells. The shell properties for these sandwich shells can be only approximately determined from the plotted curves because the structural components add weight to the shell but do not carry any circumferential stresses. For this reason, the excess buoyancy of honeycomb, tubular, and microballoon shells will be less than that shown for cellular shells, but the useful internal volume will be approximately the same. The excess buoyancy will be approximately 30 per cent below that of the cellular shell.

No curves have been plotted for the solid sandwich shell because of the large selection of sandwich spacer materials, but the formulas for calculation of solid sandwich shell data are available. Depending on the materials used for the sandwich facings and sandwich filler, shells with widely varying collapse resistances and buoyancies can be fabricated. Particularly, sandwich designs with a light metal spacer material and high strength facing would result in shells with high pressure-to-weight ratios.

Although Figures 16 through 35 include data for various sandwich-shell materials, there is no assurance that the sandwich shell can be built from the material for a given external pressure because of fabrication limitations. For each material, sandwich-shell design, and shell diameter, there are minimum thicknesses for shell facing and cellular stiffeners below which a shell cannot be built by existing fabrication methods. This, of course, imposes a limit on the coefficient of buoyancy for which the shells can be economically designed. Figure 17 shows the maximum coefficient of buoyancy for which cellular shells of 36-in. diameter can be designed; similar curves can be plotted for each shell design, material, and diameter.

I COMPRESSIVE STRENGTH OF A LONG SMOOTH SHELL

$$\boxed{pD = 2A\sigma_{yp}}$$

WHERE p = EXTERNAL HYDROSTATIC PRESSURE (psi)
D = EXTERNAL VESSEL DIAMETER (in.)
A = WALL CROSS SECTION (in^2 per in.)
σ_{yp} = YIELD STRENGTH OF THE MATERIAL (psi)

II ELASTIC INSTABILITY OF A LONG SMOOTH SHELL

$$\boxed{p = \frac{2E_t}{(1-\mu^2)}\left(\frac{t}{D}\right)^3}$$

WHERE E_t = TANGENT MODULUS OF ELASTICITY, VARIES WITH σ (psi)
μ = POISSON'S RATIO
t = WALL THICKNESS (in.)

III COEFFICIENT OF EXCESS BUOYANCY = 1 − SHELL WEIGHT/DISPLACEMENT

IV COEFFICIENT OF INTERNAL VOLUME = [DISPLACEMENT − SHELL MATERIAL VOLUME]/DISPLACEMENT

V COEFFICIENT OF USEFUL INTERNAL VOLUME

$$= D_i^2/D_o^2$$

VI ELASTIC INSTABILITY OF A LONG SANDWICH SHELL

$$\boxed{p = \frac{2E_t}{(1-\mu^2)}\left[\frac{h^3-(h-t)^3}{D_c^2 D_o}\right]}$$

WHERE h = OVER-ALL DEPTH OF THE WALL (in.)
$t = t_o + t_i$
t_o = EXTERNAL SANDWICH FACING THICKNESS (in.)
t_i = INTERNAL SANDWICH FACING THICKNESS (in.)
D_c = DIAMETER OF THE NEUTRAL WALL PLANE; IN CASE OF ORL SHELLS $D_c = (D_o + D_i)/2$
E_t = WAS ASSUMED TO BE CONSTANT, AND ITS MAGNITUDE EQUAL TO E_t AT σ_{yp}

Fig. 15 - Equations for Volume-Utilization and Buoyancy Curves

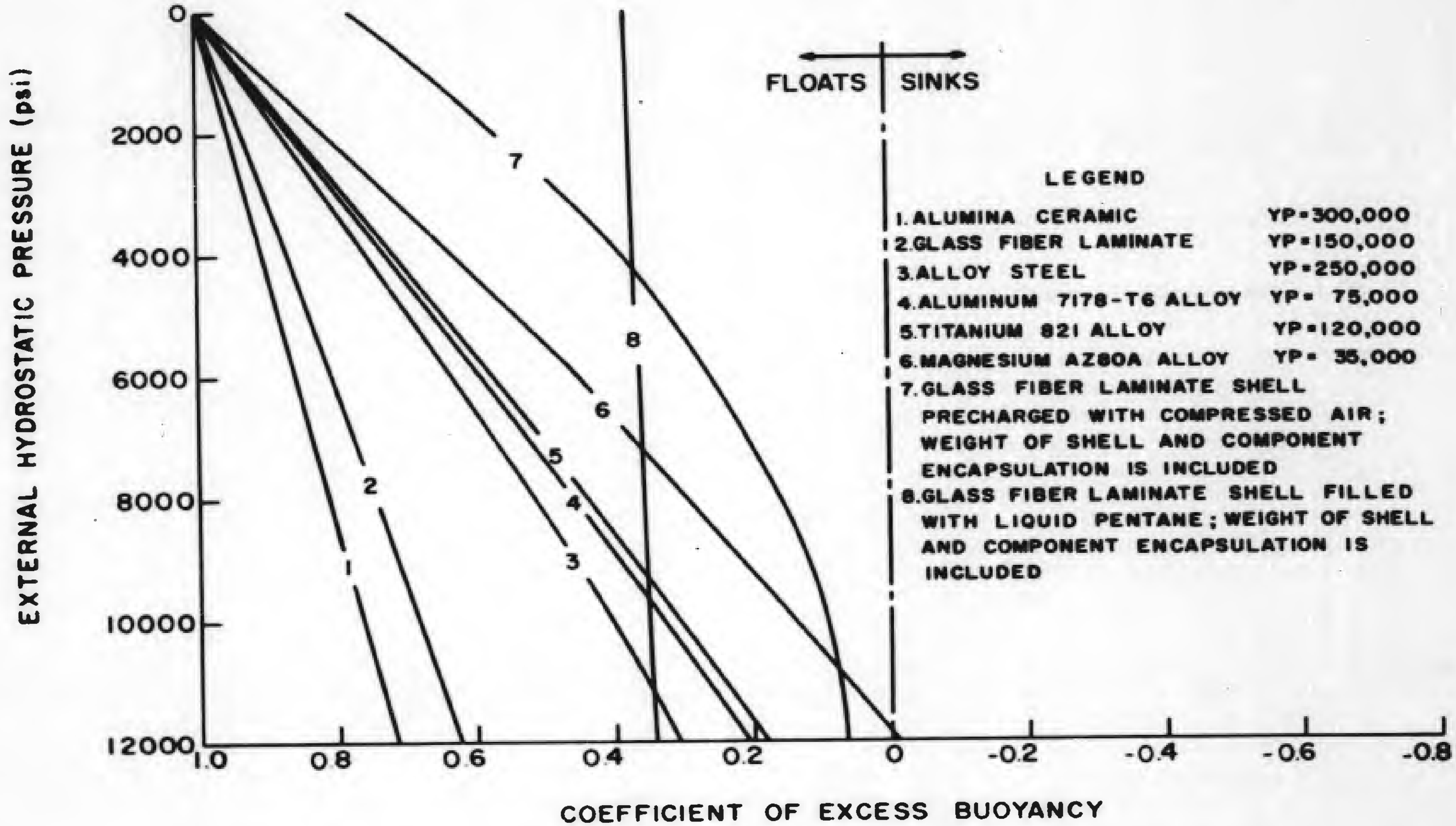


Fig. 16 - Limits of Theoretically Attainable Buoyancies for Cylindrical Shells Fabricated from Premium Materials

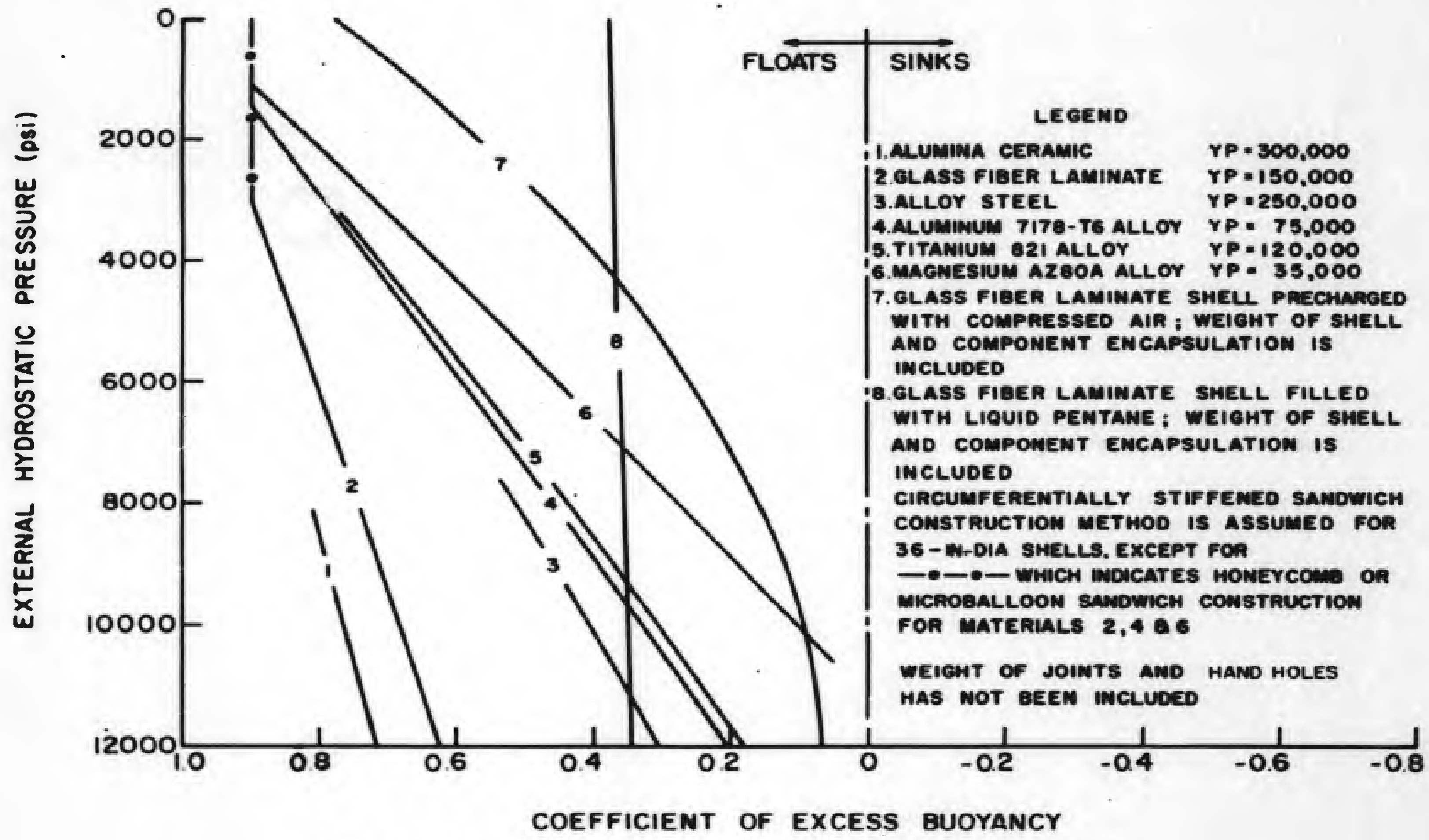


Fig. 17 - Actually Attainable Buoyancies for Cylindrical Shells Fabricated from Premium Materials

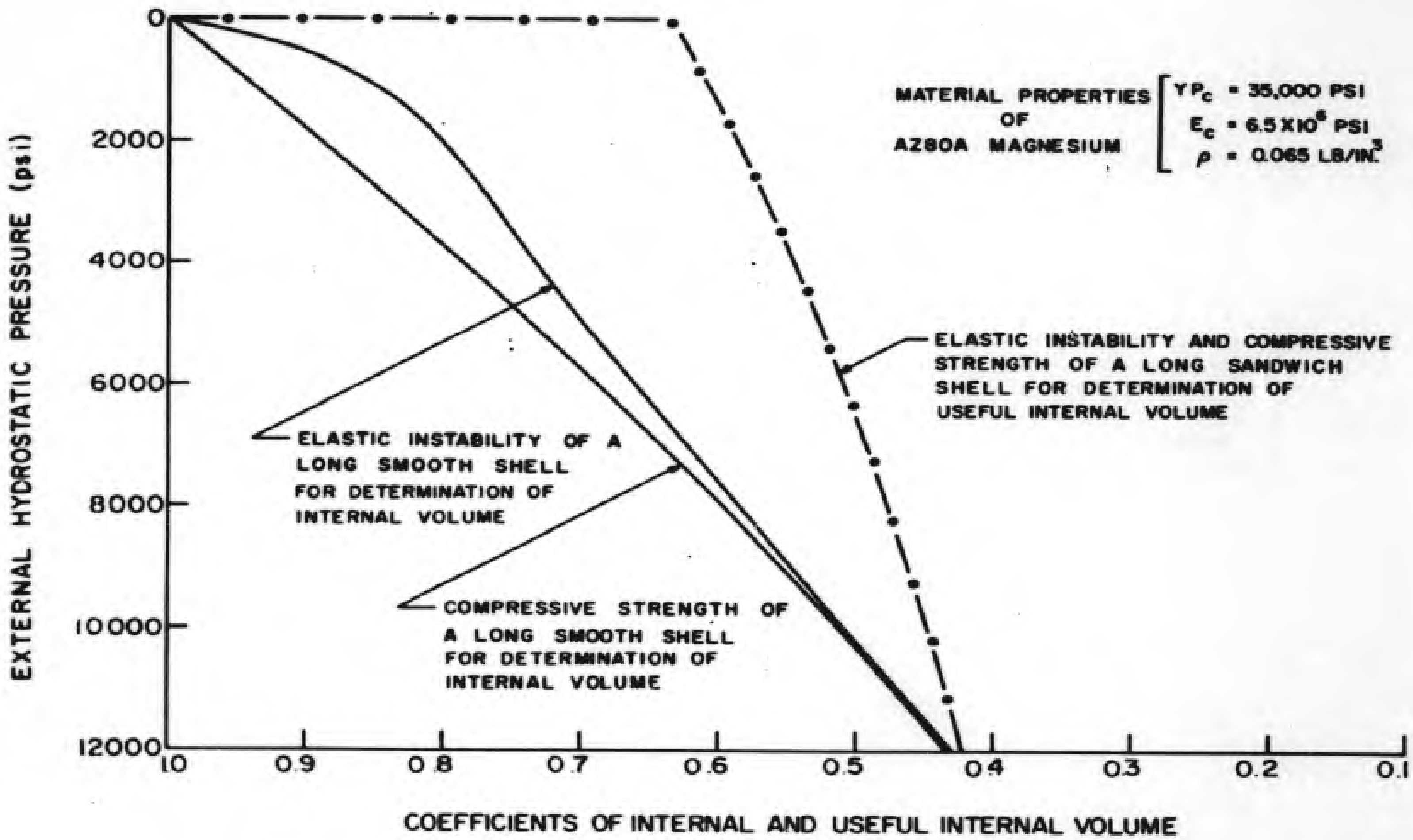


Fig. 18 - Internal and Useful Internal Volumes for Cylindrical Shells Fabricated from Premium Magnesium

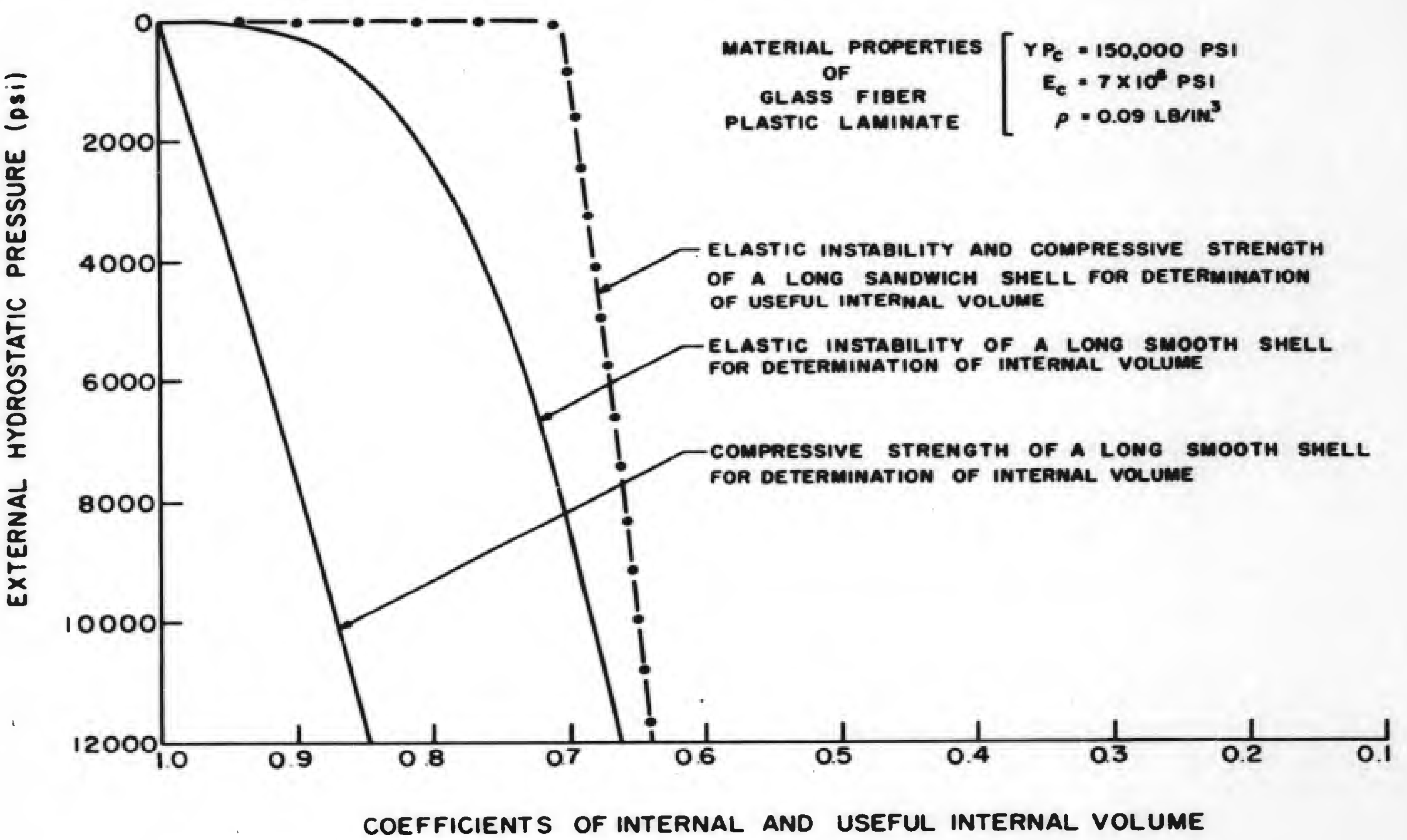


Fig. 19 - Internal and Useful Internal Volumes for Cylindrical Shells Fabricated from Premium Glass Fiber Plastic Laminate

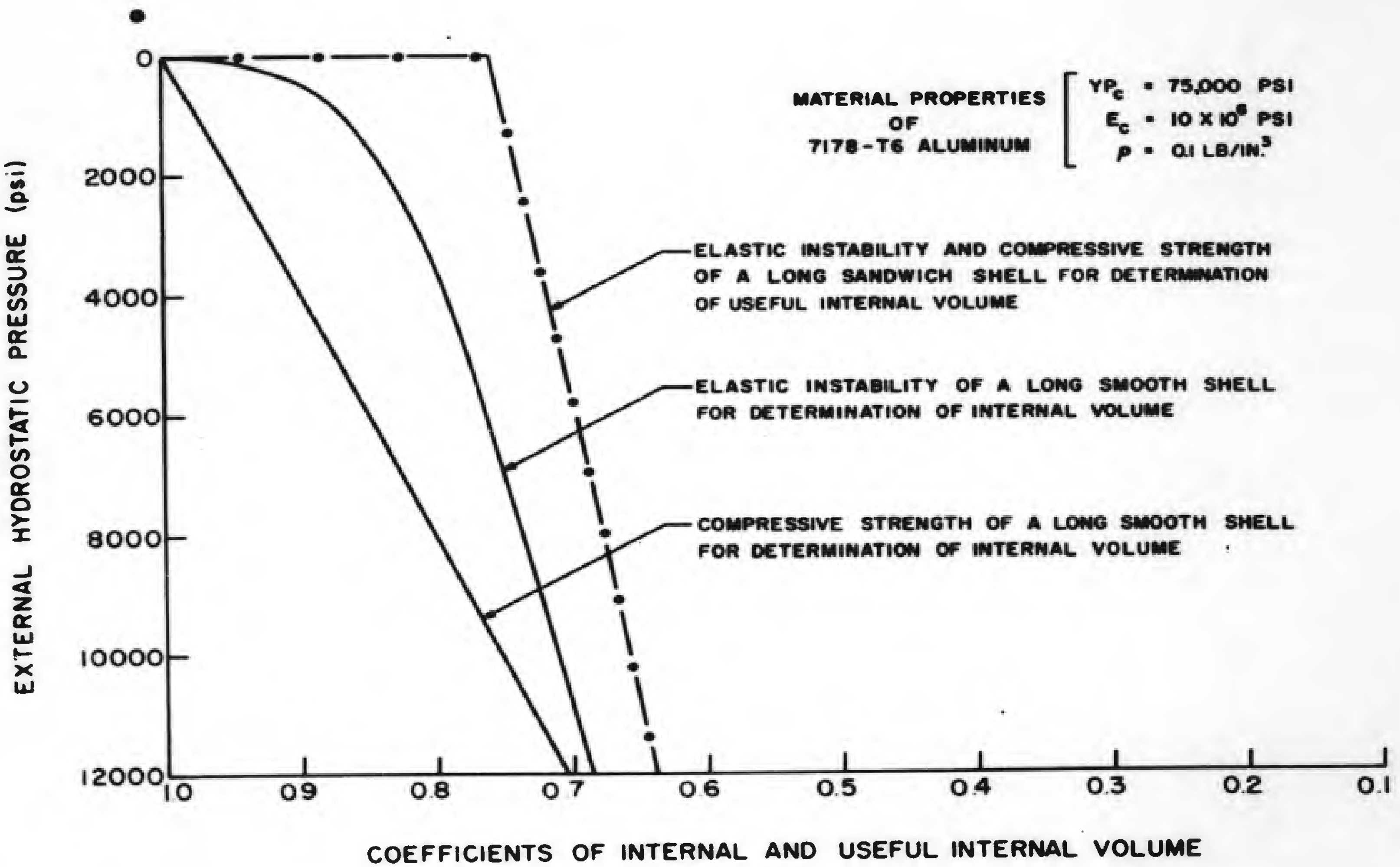


Fig. 20 - Internal and Useful Internal Volumes for Cylindrical Shells Fabricated from Premium Aluminum

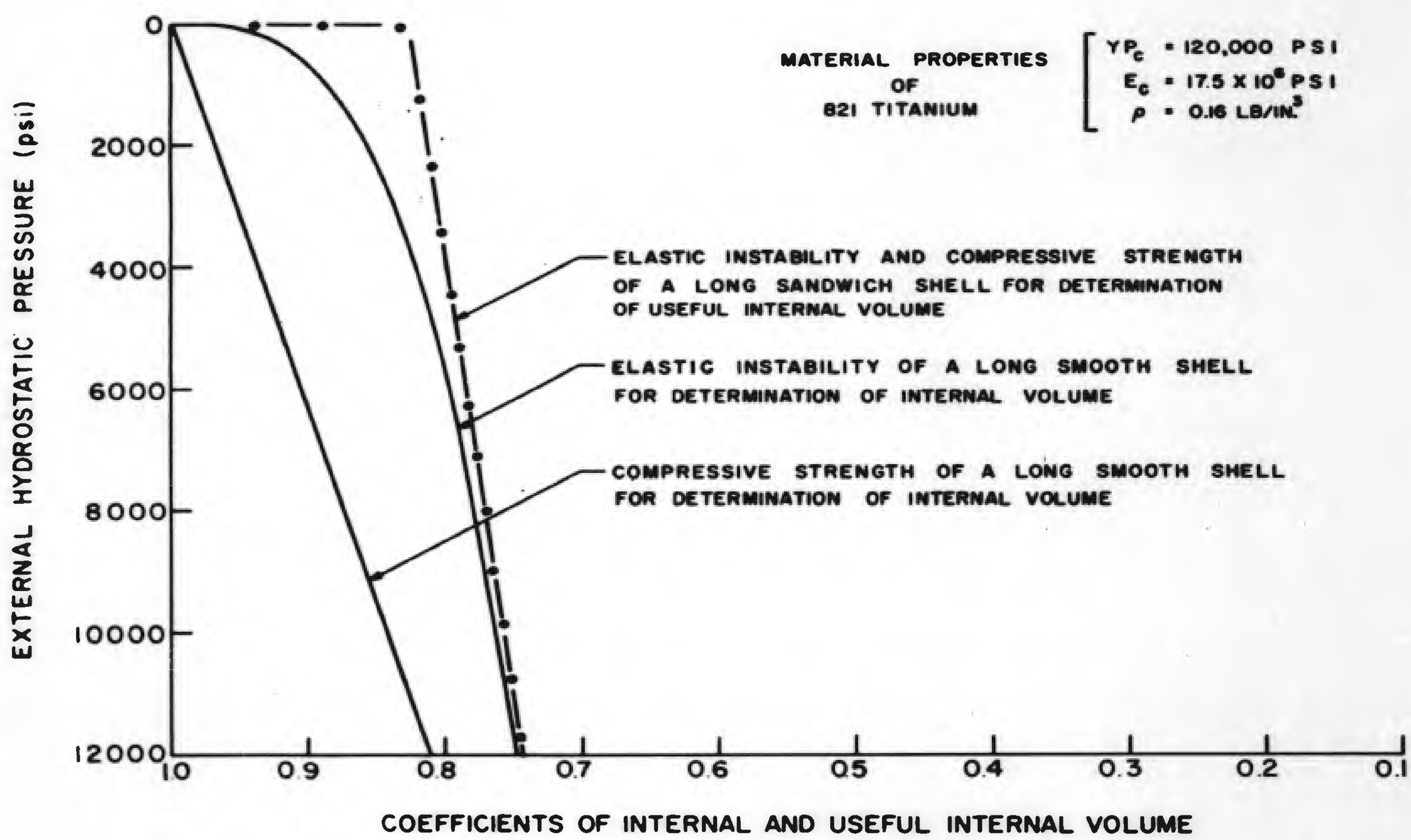


Fig. 21 - Internal and Useful Internal Volumes for Cylindrical Shells Fabricated from Premium Titanium

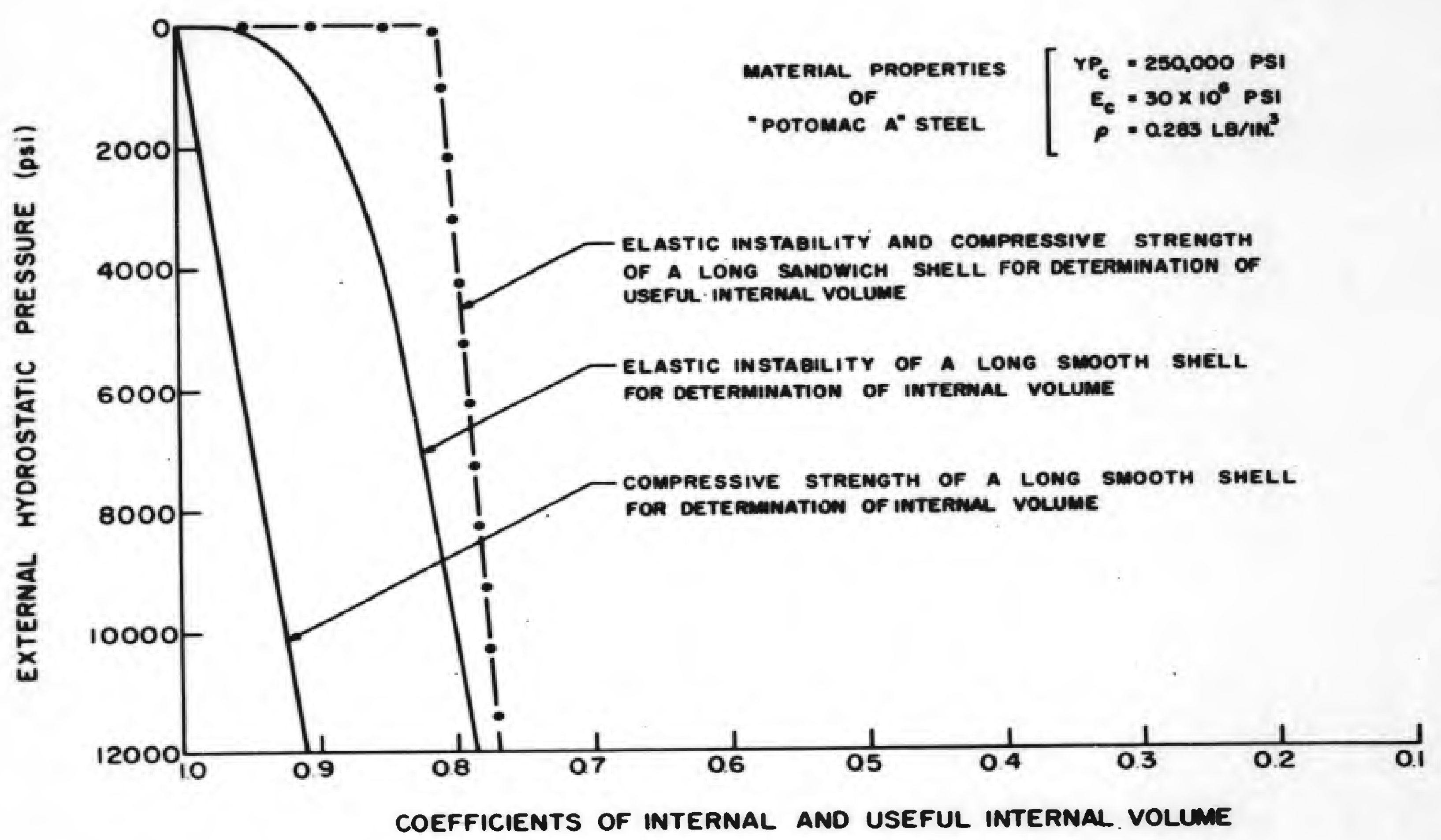


Fig. 22 - Internal and Useful Internal Volumes for Cylindrical Shells Fabricated from Premium Steel

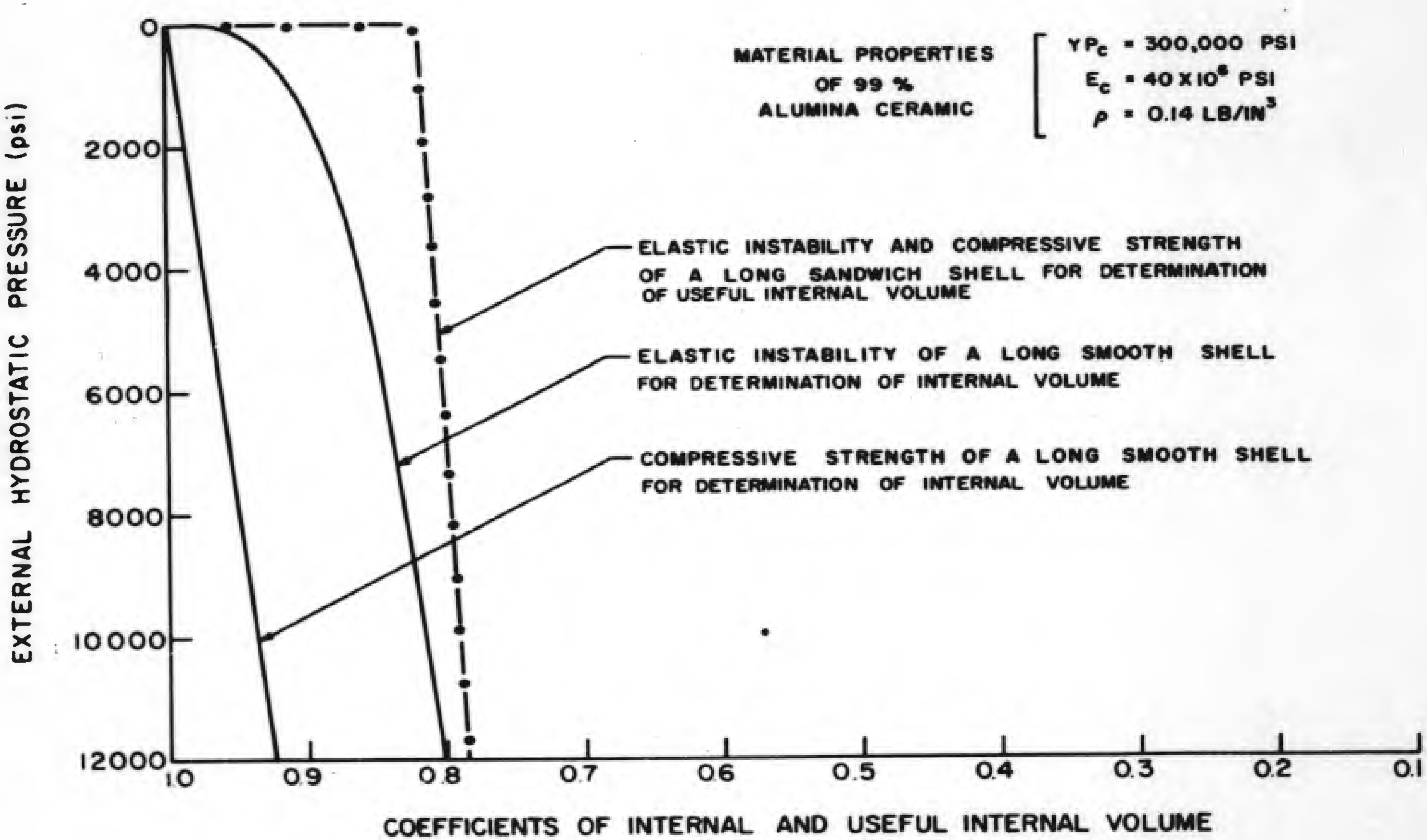


Fig. 23 - Internal and Useful Internal Volumes for Cylindrical Shells Fabricated from Premium Alumina Ceramic

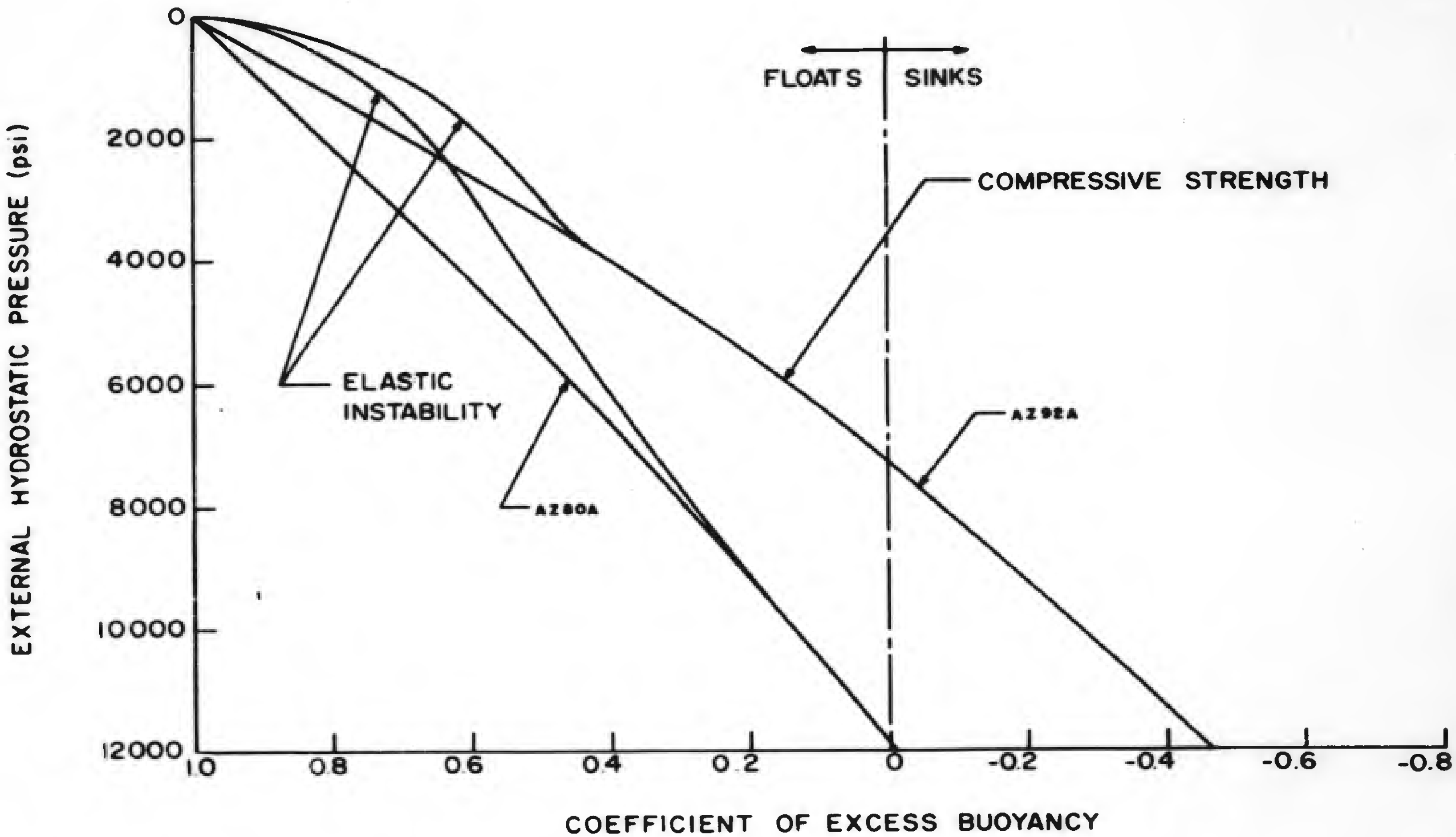


Fig. 24 - Buoyancies of Magnesium Cylindrical Shells

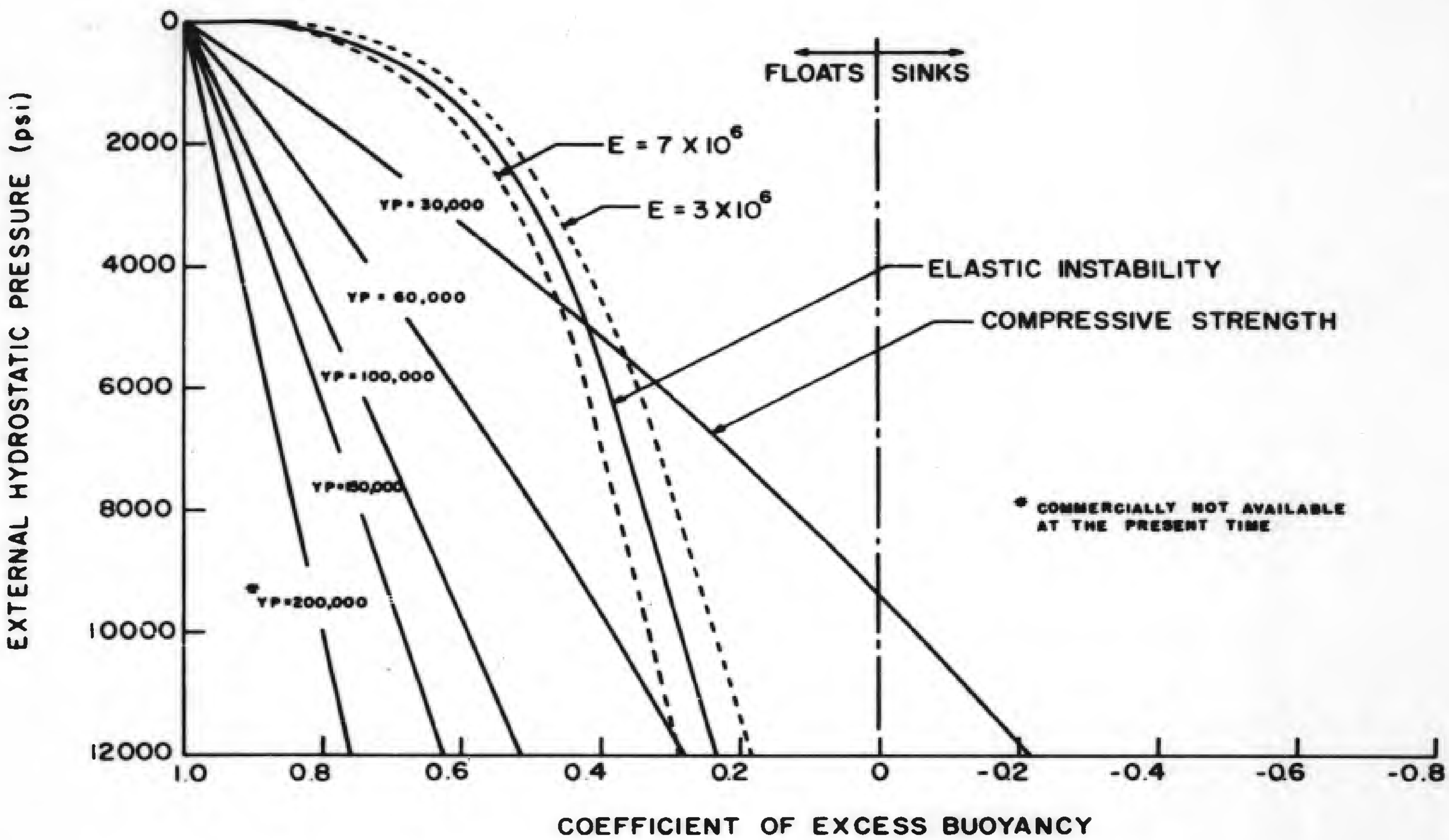


Fig. 25 - Buoyancies of Glass Fiber Plastic Laminate Cylindrical Shells

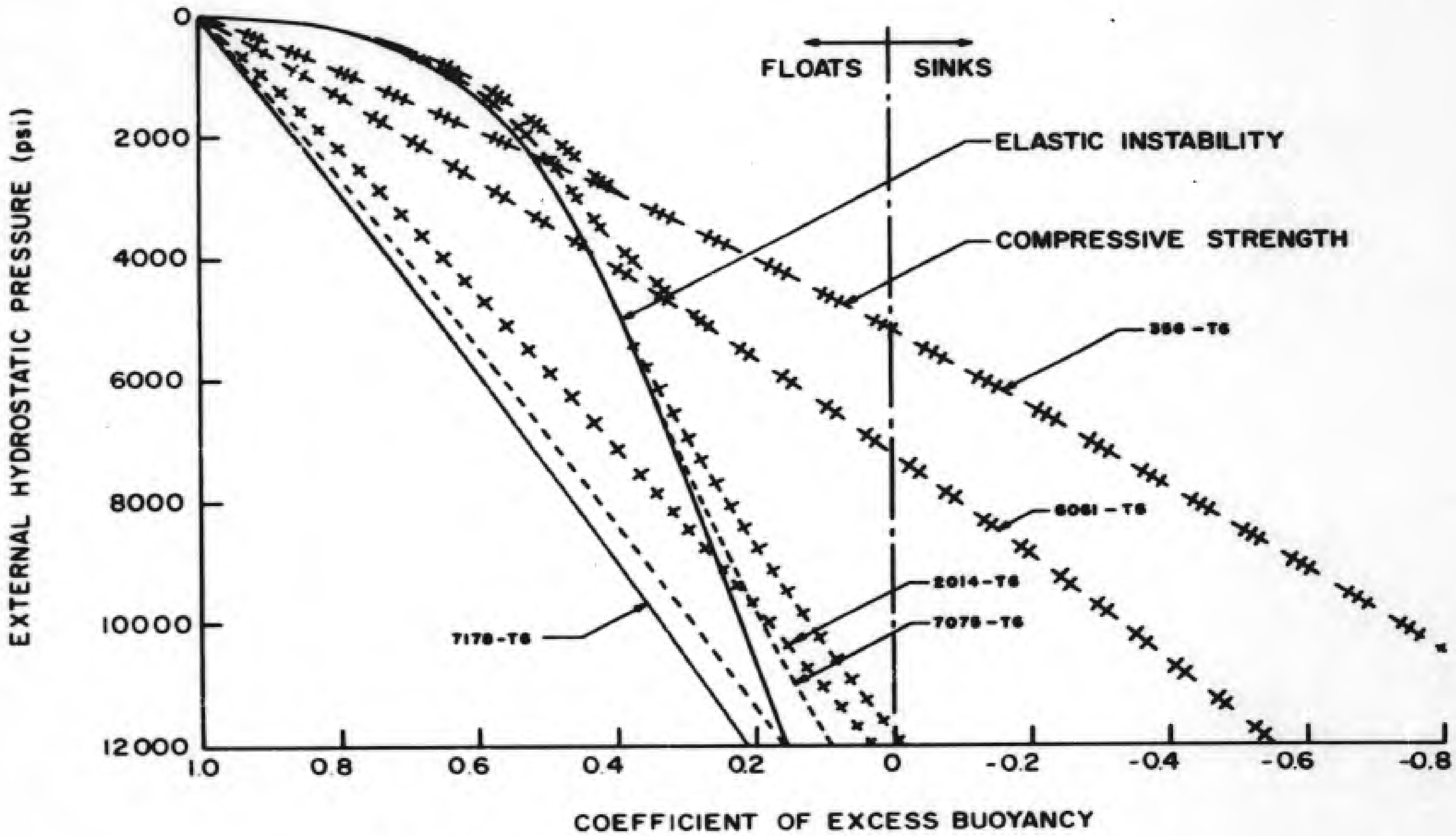


Fig. 26 - Buoyancies of Aluminum Cylindrical Shells

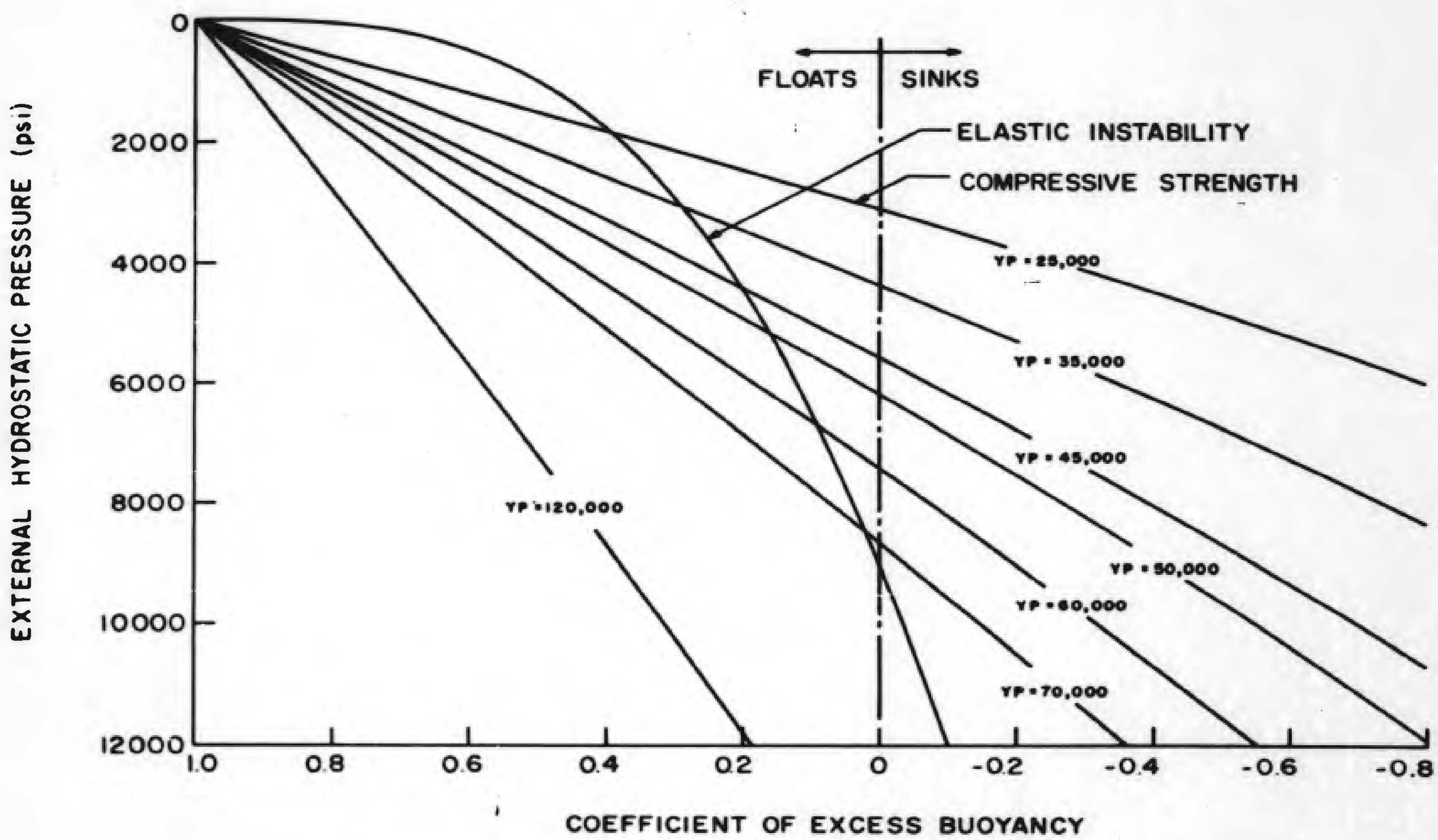


Fig. 27 - Buoyancies of Titanium Cylindrical Shells

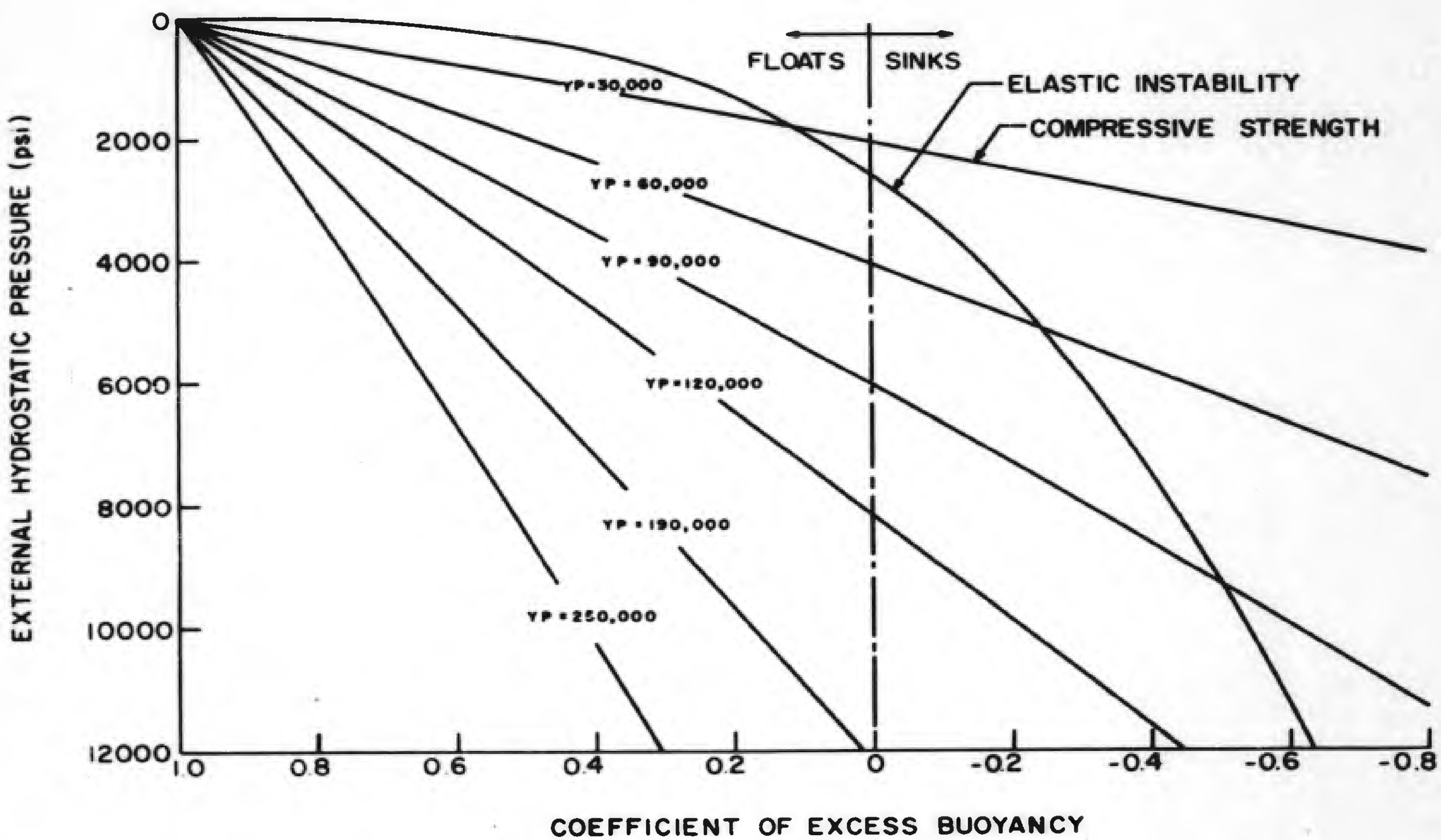


Fig. 28 - Buoyancies of Steel Cylindrical Shells

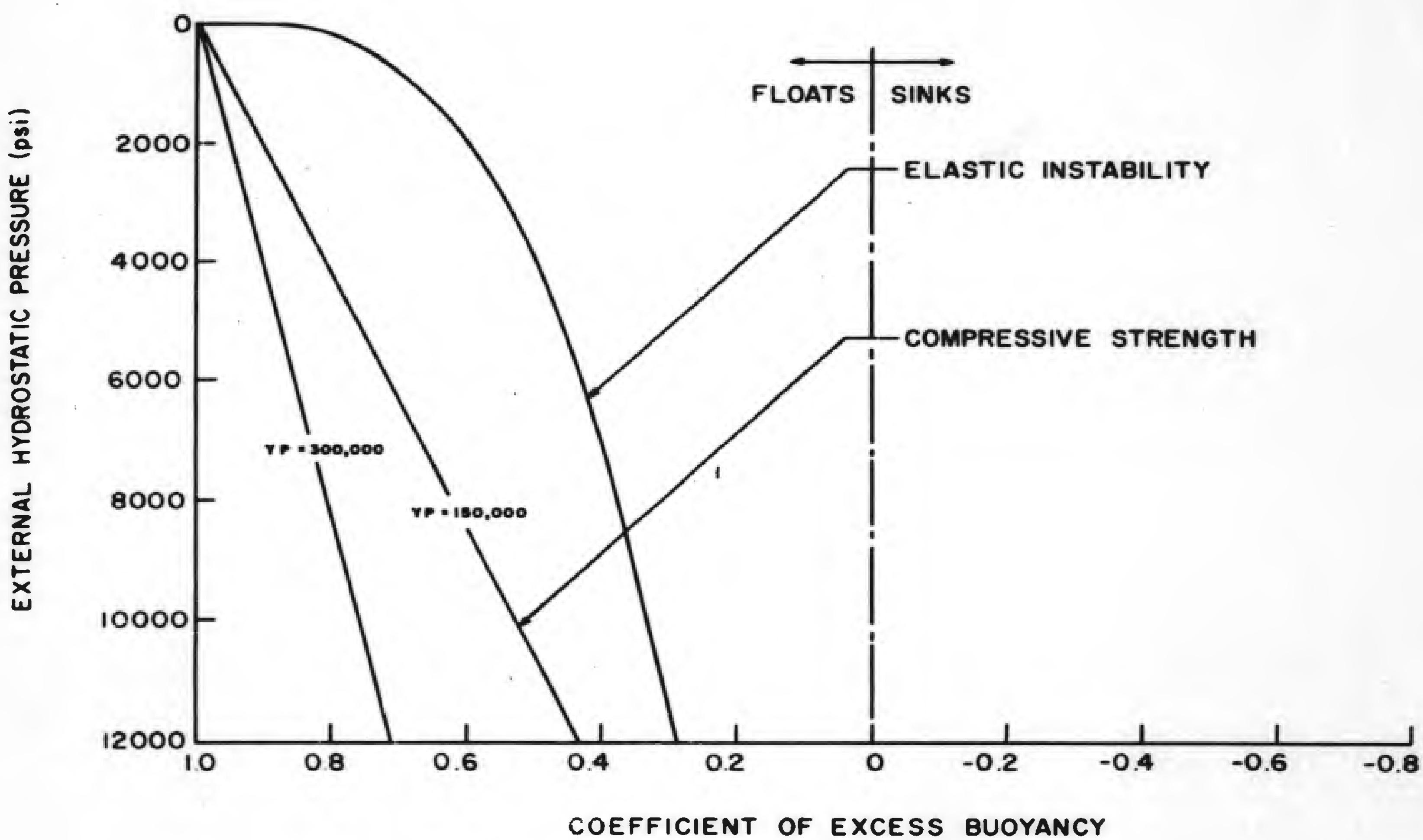


Fig. 29 - Buoyancies of Alumina Ceramic Cylindrical Shells

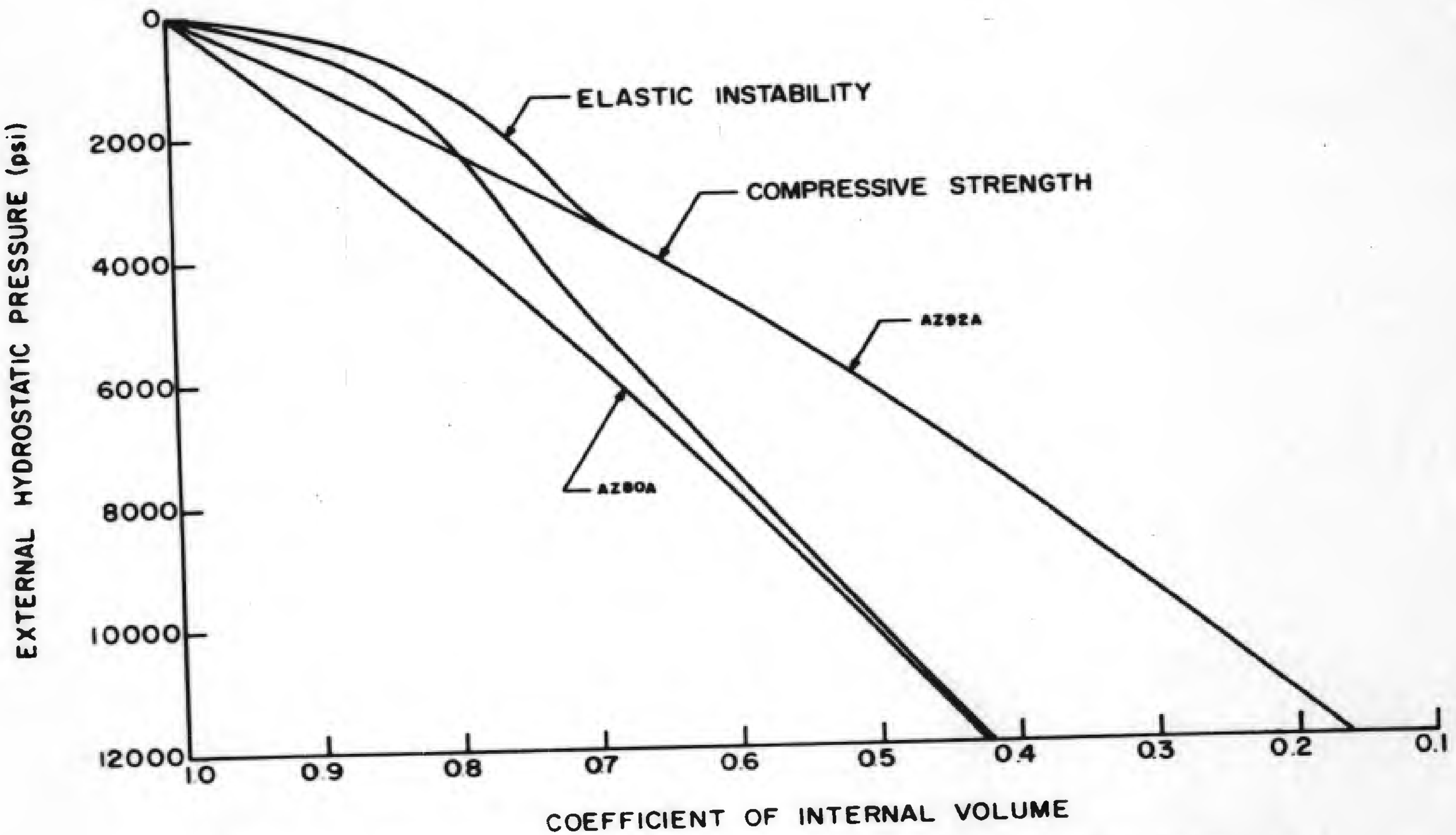


Fig. 30 - Internal Volumes of Cylindrical Shells Fabricated from Magnesium

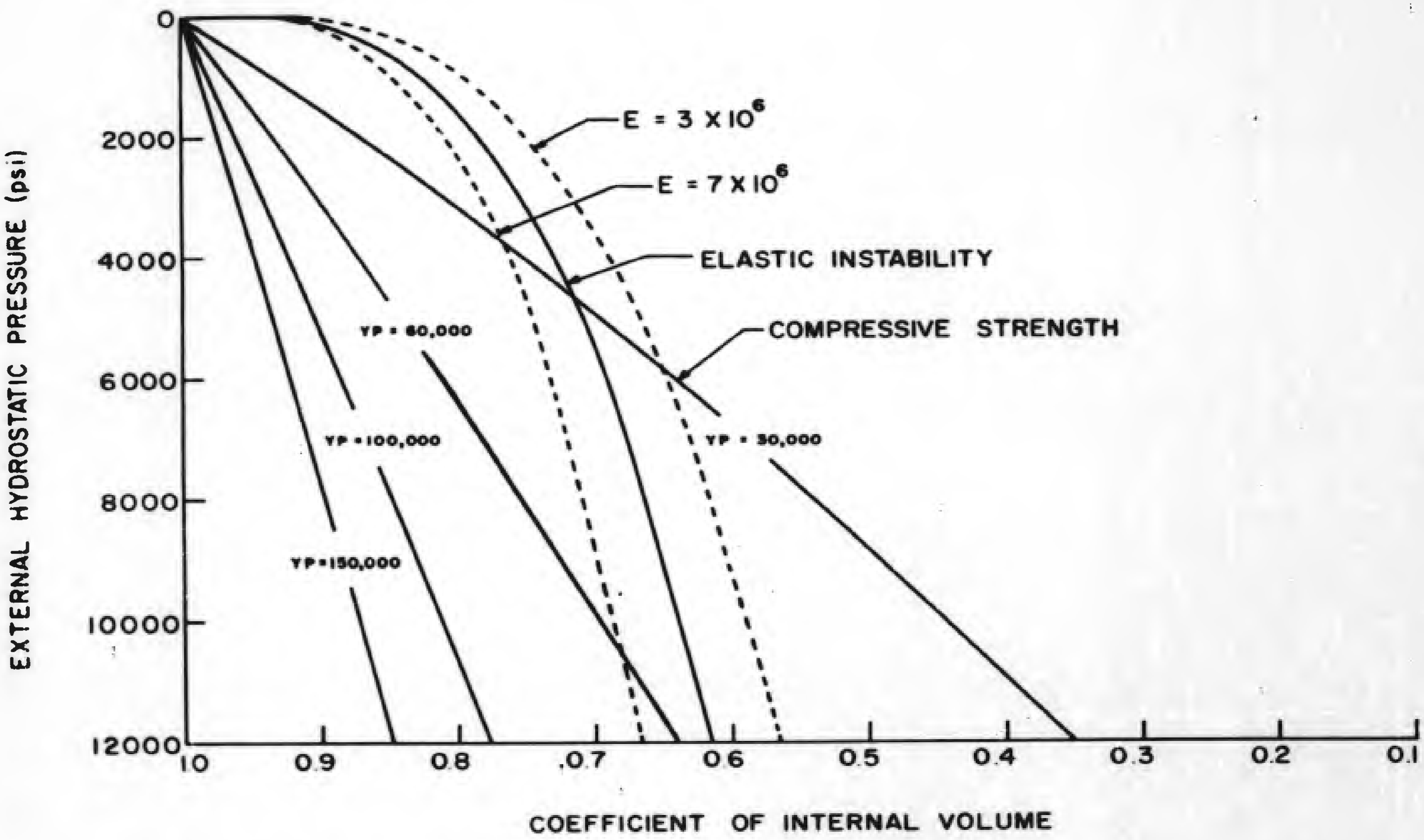


Fig. 31 - Internal Volumes of Cylindrical Shells Fabricated from Glass Fiber Plastic Laminate

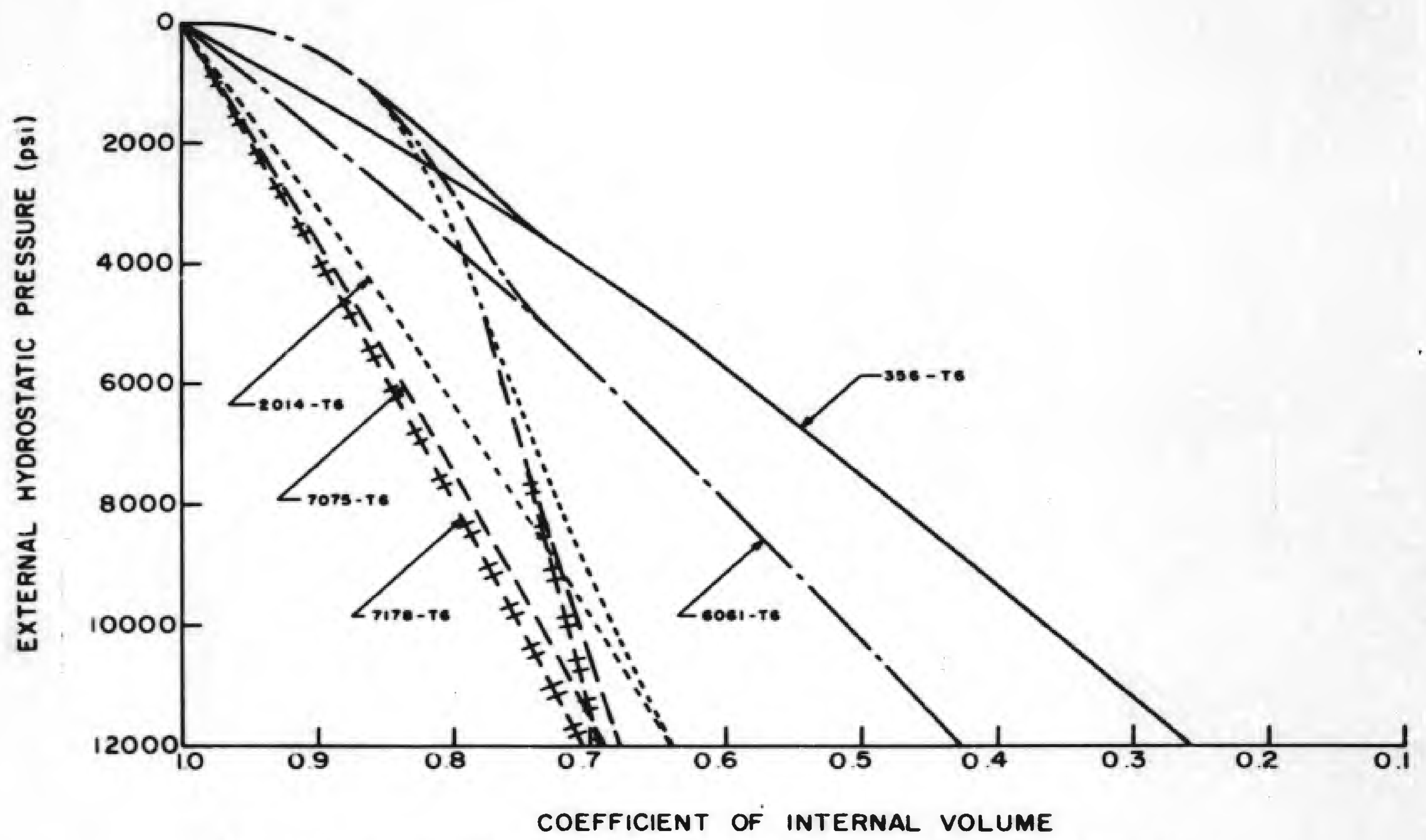


Fig. 32 - Internal Volumes of Cylindrical Shells Fabricated from Aluminum

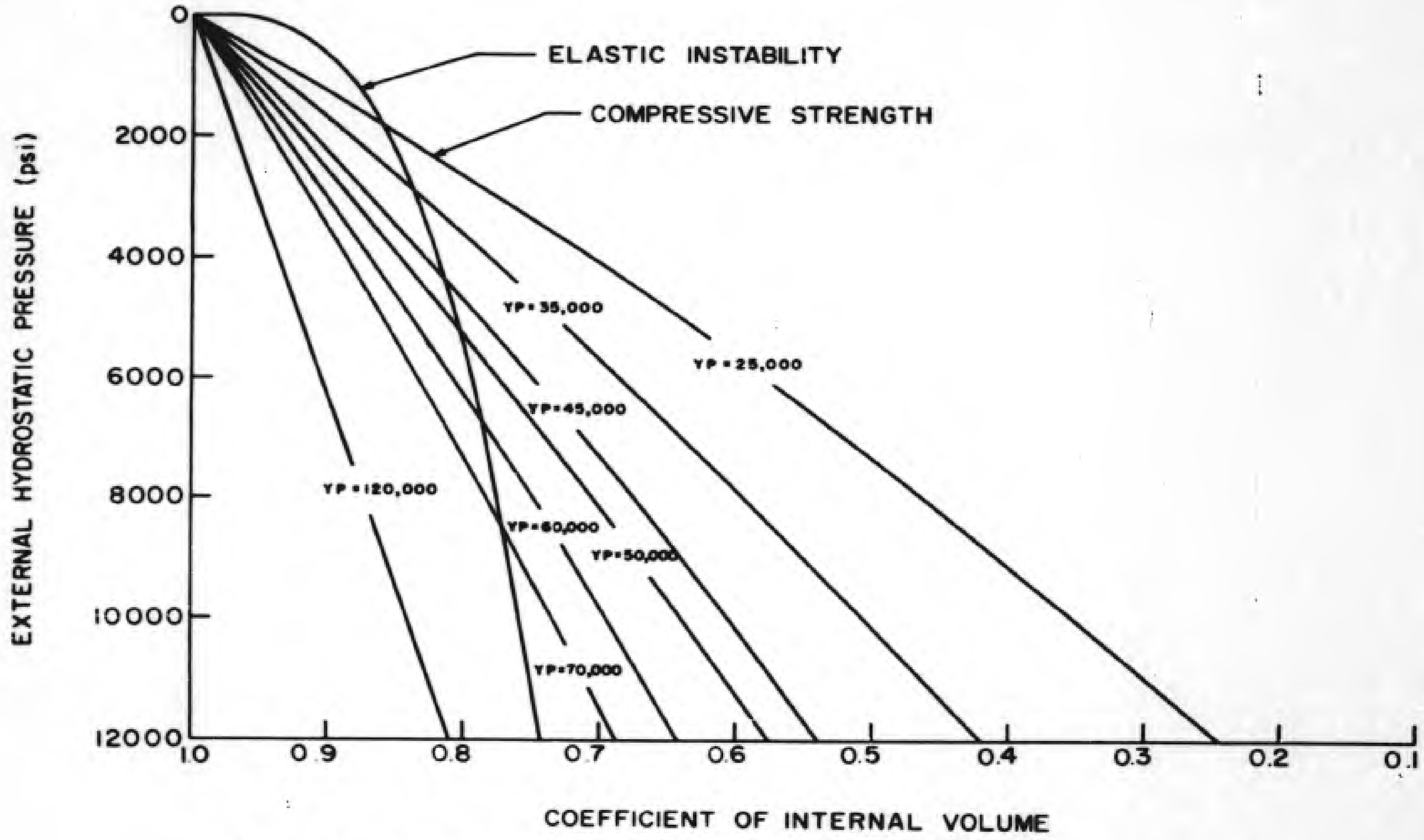


Fig. 33 - Internal Volumes of Cylindrical Shells Fabricated from Titanium

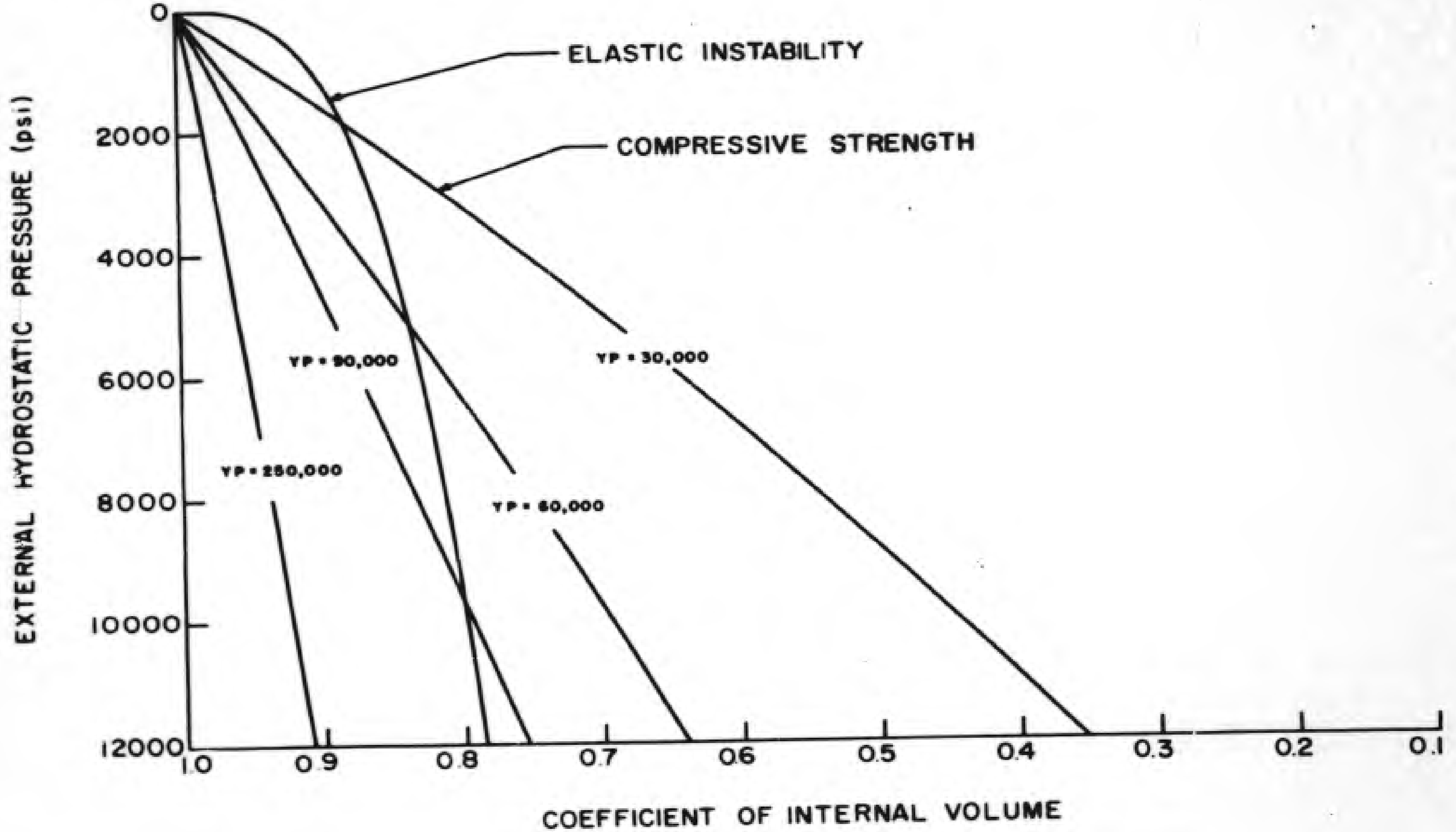


Fig. 34 - Internal Volumes of Cylindrical Shells Fabricated from Steel

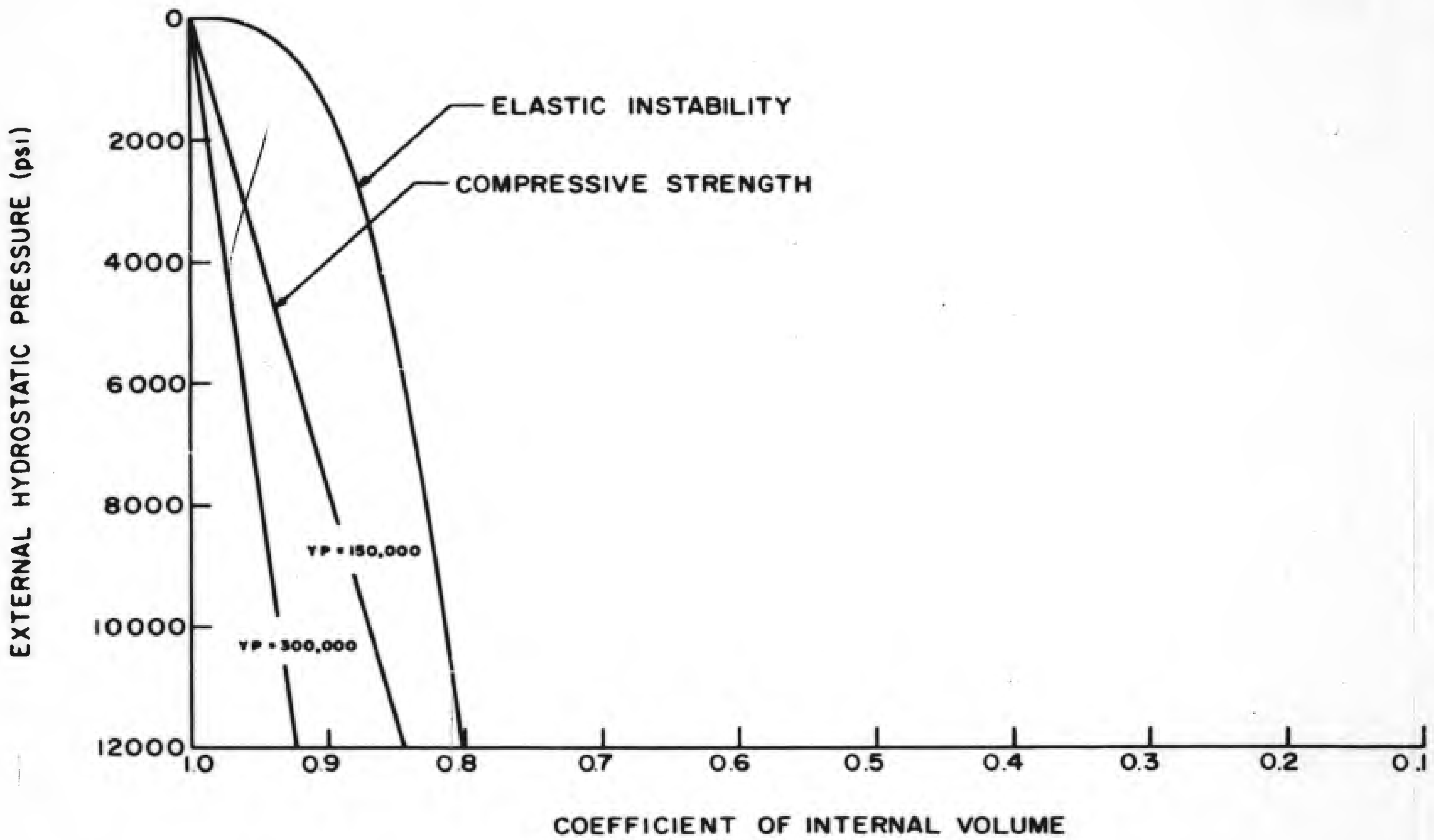


Fig. 35 - Internal Volumes of Cylindrical Shells Fabricated from Alumina Ceramic

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SHELLS FOR UNDERWATER VEHICLES

J. D. Stachiw

September 28, 1962; 56 pp. & figs.

The design criteria for underwater shells are discussed, and the basic shell designs are evaluated on the basis of these criteria. Internally pressurized shells and shell-pressurization methods are also discussed.

At present, the sandwich-shell design offers the highest pressure-to-weight ratio consistent with the shell-design criteria. The honeycomb or microballoon fiber glass sandwich shells could be best used in the lower pressure ranges and cellular sandwich shells, for a large band of intermediate pressure ranges; the cellular or solid sandwich shells would be better used at the high pressure ranges.

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