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62-590

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By

G. M. Bazhenov

FTD-TT- 62-590/1+2+4

UNEDITED ROUGH DRAFT TRANSLATION

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English Pages: 13

Source: Byulleten' Instituta Teoreticheskoy Astronomii, No. 10 (93), V. 7, 1960 pp. 757 to 765

SC-1367
SOV/35-61-0-7-7/21

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FTD-TT-62-590/1+2+4 Date 22 August 19 62

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THE DETERMINATION OF THE ORBIT OF AN ARTIFICIAL EARTH SATELLITE FROM THREE OBSERVATIONS

G. M. Bazhenov

This paper explains a method for determining the initial orbit of an artificial earth satellite from three observations separated by sufficiently long intervals of time.

To determine the orbit of an AES (artificial earth satellite) we can use Gauss's well-known method which is used to determine the orbits of small planets and comets. The successful application of this method is possible only under conditions in which the three observations of a heavenly body are close with respect to time, or more accurately, under conditions in which the change in the mean anomaly in the period of time between observations will be of the order of 5 to 15°. In this case, however, the determination of the orbit is unreliable due to the lack of precision in the initial data obtained from the observations. For those cases in which the change in the mean anomaly between observations is great, the Gauss method becomes unwieldy and inconvenient.

The author of the present article published a work in 1930 [1]

(Bazhenov, 1930) in which he described a rather simple method for determining an elliptical orbit from three arbitrary observations which may be separated by any time intervals desired, as long as they are not too small.

This method is satisfactory in determining the undisturbed Keplerian orbit of an AES in cases where the change in the mean anomaly will be anywhere from 5 to 1000° for the period of time between observations. This method is also used in cases where the variations in the mean anomaly are higher than 1000°, however, in these cases we cannot disregard the disturbances and assume that the orbit is Keplerian.

We present below the above-indicated method for determining the orbit of an AES. The observations of the AES give the following values:

$$t_i,\ \alpha_i,\ \delta_i\ (i=1,\ 2,\ 3),$$

i.e. the moments of the observations, the right ascensions and declinations of the AES corresponding to these moments and referring to a certain period (e.g. 1950.0). In addition, the values φ, λ and h should be known, i.e. the geographical latitude, longitude and altitude (in meters) above the sea level of the observation points.

As the unit of measurement for distances in this work, we take the equatorial radius of the earth according to Krasovskiy (a_0 = 6,378,245 m).

The equatorial rectangular geocentric coordinates of the observation point are determined from the formulas:

$$\left.\begin{aligned} X &= \left(1+\frac{h}{a_0}\right)\rho'\cos\varphi'\cos S, \\ Y &= \left(1+\frac{h}{a_0}\right)\rho'\cos\varphi'\sin S, \\ Z &= \left(1+\frac{h}{a_0}\right)\rho'\sin\varphi', \end{aligned}\right\} \quad (1)$$

ρ' and φ are the geocentric radius-vector and latitude of the observation point, while S is the local sidereal time (more accurately, the hour angle of the point of the vernal equinox of the period to which the coordinate system refers). The following relations are valid:

$$\left.\begin{aligned}\rho'\cos\varphi' &= 1.000839\ \cos\varphi - 0.000840\ \cos 3\varphi + 0.000001\ \cos 5\varphi,\\ \rho'\sin\varphi' &= 0.995811\ \sin\varphi - 0.000836\ \sin 3\varphi + 0.000001\ \sin 5\varphi.\end{aligned}\right\} \qquad (2)$$

The sidereal time may be determined with a sufficient degree of accuracy from the formula

$$S = S_0 + 0^\circ.98565\,N + 360^\circ.98565\,n + \lambda, \qquad (3)$$

where S_o is the sidereal time (in degrees) at midnight GMT for the zero number of the month of the observation taken from the table, N is the datum of the moment of observation and n is that fraction of the twenty-four hour periods from Greenwich midnight to the moment of observation.

Sidereal time at average midnight GMT for the zero number of each month in 1957, 1958, 1959 and 1960

	1957	1958	1959	1960
January	99°.3940	99°.1542	98°.9142	98°.6740
February	129.9492	129.7094	129.4694	129.2292
March	157.5472	157.3074	157.0673	157.8128
April	188.1019	187.8621	187.6220	188.3673
May	217.6710	217.4312	217.1911	217.9364
June	248.2269	247.9862	247.7461	248.4915
July	277.7957	277.5558	277.3156	278.0611
August	308.3509	308.1110	307.8708	308.6162
September	338.9016	338.6658	338.4257	339.1710
October	8.2248	8.2348	7.9946	8.7401
November	39.0295	38.7895	38.5493	39.2948
December	68.5990	68.3589	68.1187	68.6641

If we take into consideration the shift in the point of the vernal equinox along the equator in the time $\tau = t - 1950$ (in tropical years) which extends from the moment of 1950.0 to the moment of observation t, then from S, obtained from formula (3), we must subtract the value

$$\psi = 0^{s}.0128053\,\tau + 0^{s}.000000038833\,\tau^2 \tag{4}$$

and use in the formulas (1) the difference obtained instead of S.

A more precise calculation of the precession can be made in the following way. Form formulas (1) we determine the coordinates X, Y and Z in the coordinate system corresponding to the moment of observation, and then from the formula

$$\begin{pmatrix} X_{1950.0} \\ Y_{1950.0} \\ Z_{1950.0} \end{pmatrix} = \begin{pmatrix} X_x & -Y_x & -Z_x \\ -X_y & Y_y & Z_y \\ -X_z & Y_z & Z_z \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \tag{5}$$

we obtain the coordinates of the observation point in the system corresponding to moment 1950.0. The values X_x, Y_x,... can be obtained from Table 5 in the work of Zagrebin and Shumikhina [3] (1954).

The idea for the method of calculation of sidereal time from formula (3) was borrowed from the work of Zhongolovich, Amelin and Sabanina [2] (1959).

The geocentric rectangular equatorial coordinates of the AES are determined from the formulas:

$$x = \lambda\rho + X,\ y = \mu\rho + Y,\ z = \nu\rho + Z, \tag{6}$$

in which ρ is the distance from the observation point to AES (the unknown value), and

$$\lambda = \cos\delta\cos\alpha,\ \mu = \cos\delta\sin\alpha,\ \nu = \sin\delta. \tag{7}$$

Evidently $\lambda^2 + \mu^2 + \nu^2 = 1.$

In the future we will use of these relations:

$$\left.\begin{array}{c} r^2 = x^2 + y^2 + z^2,\ R^2 = X^2 + Y^2 + Z^2, \\ C = \lambda X + \mu Y + \nu Z,\ S^2 = R^2 - C^2, \\ \rho = -C + \sqrt{r^2 - S^2}, \end{array}\right\} \tag{8}$$

r and R are the geocentric distances of the AES and of the observation point.

In the method proposed in this paper for determining the AES orbit, we adopt the following as the main unknowns: a - the major simiaxis of the orbit and the two values:

$$g = \tan i \sin \Omega, \quad h = -\tan i \cos \Omega, \tag{9}$$

where i is the angle of inclination of the plane of the orbit of the AES to the plane of the earth's equator, Ω is the right ascension of the ascending node of the AES at the equator.

Since the undisturbed AES orbit is planar, the following relation is valid

$$xg + yh + z = 0. \tag{10}$$

The following equality follows from expression (10)

$$\rho = -\frac{Xg + Yh + Z}{m}, \tag{11}$$

where $m = \lambda g + \mu h + \nu$.

Formula (11) cannot be used when the plane of the AES orbit coincides or almost coincides with the plane of the earth's equator. The method for determining the AES orbit described in this work is inapplicable in these cases. As is known, for Soviet AES's $i \cong 65°$.

To determine the three unknowns a, g and h, we can use the three Lambert equations:

$$n\tau_i = (\varepsilon_i - \sin \varepsilon_i) - (\delta_i - \sin \delta_i), \tag{12}$$

where

$$\left. \begin{aligned} & n = \frac{k}{a\sqrt{a}}, \quad \tau_i = t_k - t_j, \\ & \Delta x_i = x_k - x_j, \quad r_j^2 = x_j^2 + y_j^2 + z_j^2, \\ & \Delta y_i = y_k - y_j, \quad r_k^2 = x_k^2 + y_k^2 + z_k^2, \\ & \Delta z_i = z_k - z_j, \quad c_i^2 = \Delta x_i^2 + \Delta y_i^2 + \Delta z_i^2. \end{aligned} \right\}$$

k = 107.0892 (a value corresponding to the Gauss constant in the problem of the determination of planetary orbits).

$$\sin\frac{\varepsilon_i}{2}=\pm\sqrt{\frac{r_j+r_k+\sigma_i}{4a}},\quad \sin\frac{\delta_i}{2}=\pm\sqrt{\frac{r_j+r_k-\sigma_i}{4a}}.$$

The subscripts i, j, and k = 1,2 and 3 must also satisfy the conditions $k > j$ and $k \neq j \neq i$.

The + or - sign for $\sin\frac{\varepsilon_1}{2}$ are assumed to be dependent on the fact that the AES makes an even or odd number of complete rotations about the center of the earth in the interval of time between the j and k observations. The choice of the sign $\sin\frac{\delta_1}{2}$ is made in accordance with the rules which were indicated in the work of Subbotin [4] (1941). The same work also gives the selection rules for that quadrant in which it is necessary to take the value ε_1.

The system of three equations (12) can be solved by Newton's method, if the approximate values of the unknowns a, g and h are known.

The corrections to the unknowns Δa, Δg and Δh satisfy the system of linear equations:

$$A_i\Delta a+G_i\Delta g+H_i\Delta h=F_i, \tag{14}$$

in which

$$\left.\begin{aligned}
F_i&=n\tau_i-\varepsilon_i+\delta_i+\sin\varepsilon_i-\sin\delta_i,\\
A_i&=-\frac{\partial F_i}{\partial a}=\frac{1}{a}\,[1.5\,n\tau_i-M_i(r_j+r_k+\sigma_i)+N_i(r_j+r_k-\sigma_i)],\\
G_i&=-\frac{\partial F_i}{\partial g}=R_{ij}x_j+R_{ik}x_k,\\
H_i&=-\frac{\partial F_i}{\partial h}=R_{ij}y_j+R_{ik}y_k.
\end{aligned}\right\} \tag{15}$$

Here

$$\left.\begin{aligned}
M_i&=\frac{1}{2a}\operatorname{tg}\frac{\varepsilon_i}{2},\quad P_i=N_i-M_i,\\
N_i&=\frac{1}{2a}\operatorname{tg}\frac{\delta_i}{2},\quad Q=N_i+M_i,\\
R_{ij}&=\frac{1}{m_j}(P_iq_j+Q_iq_{ij}),\\
R_{ik}&=\frac{1}{m_k}(P_iq_k-Q_iq_{ik}).
\end{aligned}\right\} \tag{15a}$$

where

$$q_j = \frac{1}{r_j}(x_j\lambda_j + y_j\mu_j + z_j\nu_j),$$
$$q_{ij} = \frac{1}{\sigma_i}(\Delta x_i\lambda_j + \Delta y_i\mu_j + \Delta z_i\nu_j).$$

In formulas (12), (13), (14) and (15) all the angles are expressed in radians.

To determine the approximate values of the unknowns a, g and h, we recommend this method. If we assume that the orbit of the AES is circular, then $r_1 = r_2 = r_3 = a$ and Lambert's equations are exchanged for:

$$\frac{x_jx_k + y_jy_k + z_jz_k}{a^2} = \cos\frac{k\tau_i}{a\sqrt{a}}. \qquad (16)$$

The coordinates x, y, and z are calculated from formulas (6); ρ appearing in these formulas is determined from the last of formulas (8) in which r = a.

Equation (16) may be solved graphically. It is necessary to construct a curve from the parametric equations:

$$\eta_i = \frac{x_jx_k + y_iy_k + z_jz_k}{a^2} = \eta_i(a),$$
$$\xi_i = \frac{k\tau_i}{a\sqrt{a}} = \xi_i(a).$$

This curve at high values of $k\tau_1$ is almost linear and the scale of values of parameter a on it is almost uniform. The points of intersection of this curve with the curve corresponding to equation

$$\eta = \cos\xi,$$

are the desired values of the unknown a. After solving equations (16) we will have three systems of values of the unknown a. For the approximate value of the major semiaxis of the AES orbit a we take the number close to the almost coinciding solutions in the three systems of solutions of equations (16).

The approximate values of the unknowns g and h are obtained by the method of least squares from the system:

$$x_i g + y_i h + z_i = 0 \quad (i = 1, 2, 3).$$

The coordinates x_1, y_1 and z_1 are determined from formulas (6); ρ_1 is found from the last of formulas (8) in which for r_1 we take the above adopted approximate value of the major semiaxis of the AES orbit.

After we have found the approximate values of all three unknown values a_0, g_0 and h_0 from equations (14), we determine the corrections for these approximate values Δa_0, Δg_0 and Δh_0.

The new approximate values of the unknown values

$$a_1 = a_0 + \Delta a_0, \quad g_1 = g_0 + \Delta g_0, \quad h_1 = h_0 + \Delta h_0$$

(first approximation) are substituted in equations (12) and if they do not satisfy these equations with the required precision, then using equations (14) we find the corrections Δa_1, Δg_1 and Δh_1 in the second approximation satisfies equations (12) with the necessary precision.

The remaining elements of the AES orbit are found in the following way. The set of formulas:

$$\left.\begin{array}{c} q_i = 1 - \frac{r_i}{a}, \quad e\cos E_2 = q_2, \\ 2g_i = \varepsilon_i - \delta_i, \quad e\sin E_2 = \frac{q_2\cos 2g_1 - q_3}{\sin 2g_1}, \\ E_1 = E_2 - 2g_3, \quad E_3 = E_2 + 2g_1, \\ M_i = E_i - e\sin E_i = n(t_i - t_\pi) \end{array}\right\} \qquad (17)$$

enables us to find the eccentricity e of the AES orbit and the moment the AES passes the perigee of its orbit t_π.

The matrix of the projected coefficients are obtained from formula

$$\begin{pmatrix} P_x & Q_x \\ P_y & Q_y \\ P_z & Q_z \end{pmatrix} = \begin{pmatrix} x_2 & x_3 \\ y_2 & y_3 \\ z_2 & z_3 \end{pmatrix} \begin{pmatrix} \xi_2 & \xi_3 \\ \eta_2 & \eta_3 \end{pmatrix}^{-1} = \begin{pmatrix} x_2 & x_3 \\ y_2 & y_3 \\ z_2 & z_3 \end{pmatrix} \frac{\begin{pmatrix} \eta_3 & -\xi_3 \\ -\eta_2 & \xi_2 \end{pmatrix}}{[r_2 r_3]}. \qquad (18)$$

where

$$[r_2r_3] = \xi_2\eta_3 - \xi_3\eta_2.$$

The element ω is the angular distance of the perigee of the AES orbit from its ascending node at the equator and is determined from equations:

$$P_z = \sin i \sin \omega, \quad Q_z = \sin i \cos \omega. \quad (19)$$

During the calculations it is desirable to check the results in passing.

Appendix

Determination of the Orbit of the Rocket of the Third AES (1953 δ,) from Three Observations on 29 July 1958.

All the calculations given below (1-7) were made by M. V. Kuznetsova.

1. Observation Data Taken From Bulletins of the AES Visual Observation Station

	1	2	3
	Tashkent, Astronomical Observatory of the AN Uzbek. SSR (Bulletin No. 4, p. 19)	Pulkovo, Main Astronomical Observatory, (Bulletin No. 10, p. 21)	Kiev, Main Astronomical Observatory, AN SSSR (Bulletin No. 9, p. 20)
UT	0.826447	0.893813	0.969969
$\alpha_{1950.0}$	46°.6079	250°.0400	334°.4738
$\delta_{1950.0}$	+50.6283	+77.0811	+15.1700

2. Auxilliary Values Depending on Basic Values Obtained from Observations

	1	2	3
$\rho' \cos \varphi'$	0.752065	0.504727	0.638839
$\rho' \sin \varphi'$	0.656881	0.860386	0.766170
$1 + \frac{h}{a_0}$	1.000078	1.000012	1.000029
S	313°.7688	299°.1208	326°.7825
Ψ	0.1098	0.1098	0.1098
$S - \Psi$	313.6590	299.0110	326.6727
λ	0.43579	−0.07632	0.87094
μ	0.46096	−0.21014	−0.41591
ν	0.77305	0.97469	0.26168
X	0.51924	0.24478	0.53379
Y	−0.54413	−0.44140	−0.35100

(cont.)

	1	2	3
Z	0.65693	0.86040	0.76619
R	0.99862	0.99752	0.99759
R^2	0.99725	0.99504	0.99518
C	0.48330	0.91269	0.81138
S^2	0.76368	0.16203	0.33684
(in twenty four hour periods)	0.076156	0.143522	0.067366
(in radians)	8.15549	15.36966	7.21417

3. Graphic Determination of the Major Semiaxis of the Orbit

	1	2	3
	$a = 1.1$		
ρ	0.18477	0.11101	0.12305
x	0.59976	0.23631	0.64096
y	−0.45896	−0.46473	−0.40218
z	0.79977	0.96890	0.79839
η	0.91875	0.99796	0.93362
ξ	7.06905	13.32218	6.25313
ρ	0.33209	0.21778	0.23893
x	0.66701	0.22816	0.74188
y	−0.38782	−0.48716	−0.45037
z	0.91906	1.07267	0.82871
η	0.88722	0.99385	0.92150
ξ	6.20411	11.69213	5.48802

The drawing gives the points of intersection of the cosine curve $\eta = \cos \xi$ and the straight line connecting the points with the coordinates ξ and η calculated above for a = 1.1 and a = 1.2. It follows from the fact that the scale of values of a on the straight line is almost uniform that we may find the values of a at the points of intersection. In the given case for all three combinations of the three observations, two at a time, we obtain the approximate general solution a = 1.14. The general solution is also taken to be the zero approximation of the major semiaxis of the orbit.

4. The Determination of g and h in zero approximation a = 1.14

	1	2	3
ρ	0.24877	0.15388	0.16982
x	0.62765	0.23304	0.68169
y	−0.42946	−0.47374	−0.42163
z	0.84924	1.01038	0.81063

* $k = 107.0892$

$$0.91295\,g_0 \quad -0.66737\,h_0 = -1.32108$$
$$-0.66737\,g_0 \quad +0.58664\,h_0 = \quad 1.18516$$
$$g_0 = 0.17679$$
$$h_0 = 2.22138$$

5. The Determination of the Corrections Δa_0, Δg_0 and Δh_0 to the Zero Approximation and Corrections Δa_1, Δg_1 and Δh_1 to the first Approximation

The calculation diagrams are the same in both cases. Below we present the complete calculation for the second case

	1	2	3
m	1.84375	0.50559	−0.50801
p	0.24211	0.12266	0.16804
x	0.62475	0.23542	0.68014
y	−0.43253	−0.46718	−0.42089
z	0.84409	0.97996	0.81016
r^2	1.28988	1.23400	1.29610
r	1.13573	1.11086	1.13846
Δx	0.44472	0.05539	−0.38933
Δy	0.04629	0.01164	−0.03465
Δz	−0.16980	−0.03393	0.13587
s	0.47828	0.06599	0.41381
$r+r'+s$	2.72760	2.34018	2.66040
$r+r'-s$	1.77104	2.20820	1.83278
$\sin\frac{\varepsilon}{2}$	−0.77333	0.71631	0.76374
$\sin\frac{\delta}{2}$	0.62314	0.69581	−0.63391
$\frac{\varepsilon}{2}$	4.02567	7.08168	2.27251
$\frac{\delta}{2}$	0.67276	0.76954	−0.68660
$\tan\frac{\varepsilon}{2}$	1.21977	1.02656	−1.18313
$\tan\frac{\delta}{2}$	0.79676	0.96878	−0.81964
$n\tau$	6.69827	12.62342	5.92515
$-\varepsilon$	−8.05134	−14.16336	−4.54502
δ	1.34552	1.53908	−1.37320
$\sin\varepsilon$	0.98057	0.99966	−0.98602
$-\sin\delta$	−0.97473	−0.99949	0.98054
F	−0.00169	−0.00069	+0.00147
M	0.53488	0.45016	−0.51881
N	0.34939	0.42482	−0.35942
P	−0.18549	−0.02534	0.15939
Q	0.88427	0.87498	−0.87823
q	0.63871	0.93204	0.86030
q_{ij}	−0.43734	0.04962	−0.19478
q_{ik}	0.67667	0.52313	0.40943
R_{ij}	−1.10085	0.01477	0.14799
R_{ik}	1.49197	0.94393	1.00503
A	8.07493	16.50528	8.42747
G	0.75417	0.65123	0.32906
H	−0.11086	−0.40368	−0.53354

The corrections to the first approximation:

$$\Delta a_1 = -0.000099,$$
$$\Delta g_1 = -0.00200,$$
$$\Delta h_1 = -0.00554.$$

The second approximation is taken as final:

$$a_2 = 1.140128,$$
$$g_2 = 0.15334,$$
$$h_2 = 2.17036.$$

6. Determination of the Values ρ, x, y, z, r, σ, ε, δ and F

With the obtained values of a_2, g_2 and h_2 we calculate the values ρ, x, y, z, r, σ, ε, δ and F from a diagram identical with the one given above. In the case being considered we obtained these values of the basic magnitudes:

	1	2	3
x	0.62447	0.23574	0.68181
y	−0.43282	−0.46630	−0.42169
z	0.84361	0.97589	0.81067
r	1.13533	1.10696	1.14012
σ	0.47777	0.06706	0.41198
$n\tau$	6.69915	12.62507	5.92592
$-\varepsilon$	−8.05021	−14.16445	−4.54760
δ	1.34482	1.53928	−1.37216
$\sin \varepsilon$	0.98081	0.99963	−0.98645
$-\sin \delta$	−0.97458	−0.99950	0.98034
F	−0.00001	0.00003	0.00005

To check the calculations we may use the relation:

$$\varepsilon_2 - \delta_2 = (\varepsilon_1 - \delta_1) + (\varepsilon_3 - \delta_3); \ (2g_2 = 2g_1 + 2g_3).$$

The smallness of the values F indicates that the desired magnitudes obtained in the second approximation need not be improved in the future.

7. Elements of the Orbit

In conclusion from formulas (17), (18) and (19) we obtain the remaining elements of the orbit and the matrix of the projected coefficients.

For the 1958 δ_1 satellite for the moment of the second observation i.e. for 1958, July 29.893813 U. T., we obtained the following elements of orbit:

$$a = 1.140128, \quad e = 0.07098, \quad M_2 = 62\ 0943,$$
$$i = 65^\circ 316, \quad \Omega = 175.958, \quad \omega = 34^\circ 440.$$

and this matrix of projected coefficients:

$$\begin{pmatrix} P_x & Q_x & R_x \\ P_y & Q_y & R_y \\ P_z & Q_z & R_z \end{pmatrix} = \begin{pmatrix} -0.83930 & 0.53989 & 0.06404 \\ -0.17747 & -0.38341 & 0.90636 \\ 0.51388 & 0.74934 & 0.41761 \end{pmatrix}$$

The example given above of the determination of the orbit of an AES indicates that the method proposed by the author makes it relatively easy to find the AES orbit from three essentially arbitrary observations.

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Submitted 4 July 1960.

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