

UNCLASSIFIED

---

AD **286 920**

---

*Reproduced  
by the*

ARMED SERVICES TECHNICAL INFORMATION AGENCY  
ARLINGTON HALL STATION  
ARLINGTON 12, VIRGINIA



---

UNCLASSIFIED

63-1-3

CATALOGED BY ASTIA  
AS AD NO.

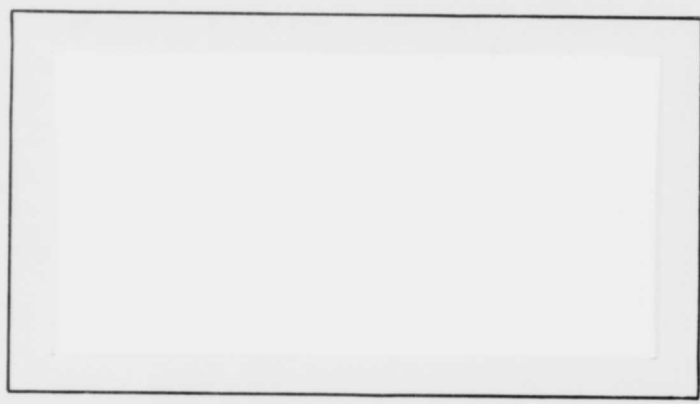
28 6920

286 920

# AIR FORCE INSTITUTE OF TECHNOLOGY



AIR UNIVERSITY  
UNITED STATES AIR FORCE



## SCHOOL OF ENGINEERING

WRIGHT-PATTERSON AIR FORCE BASE, OHIO

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

THE EFFECT OF FREQUENCY TRACKING, THE USE  
OF A PHASE-LOCK LOOP, AND PREDICTED  
TRACKING ON RECEIVER SENSITIVITY

GA/EE/62-2

EMIL G. RIEDEL JR  
Captain, USAF

THESIS

Presented to the Faculty of the School of Engineering of  
the Institute of Technology  
Air University  
in Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science

THE EFFECT OF FREQUENCY TRACKING, THE USE  
OF A PHASE-LOCK LOOP, AND PREDICTED  
TRACKING ON RECEIVER SENSITIVITY

GA/EE/62-2

EMIL G. RIEDEL JR  
Captain, USAF

Graduate Astronautics

August 1962

Preface

This report is an evaluation of three special techniques used by radio receivers in the tracking of satellites. My formal training, both at the undergraduate level at the U.S. Naval Academy, and at the graduate level in the Astronautics program at AFIT, has been broad and lacking in specialization. I chose this thesis topic so that I might have the opportunity to study in detail at least one small portion of the field of Astronautics.

I wish to acknowledge the patient encouragement and helpful suggestions given to me by Professor T.L. Regulinski. I also wish to express my gratitude to my wife for her forbearance and help in typing this report. I should also like to acknowledge the begrudged quiet given by the four little men who make all work worthwhile.

Contents

|                                                                  | Page |
|------------------------------------------------------------------|------|
| Preface                                                          | ii   |
| List of Figures                                                  | iv   |
| List of Symbols                                                  | vi   |
| Abstract                                                         | viii |
| I. Introduction                                                  | 1    |
| Background                                                       | 1    |
| Assumptions                                                      | 4    |
| Statement of the Problem                                         | 5    |
| II. Determining Receiver Sensitivity                             | 7    |
| III. Frequency Tracking                                          | 10   |
| IV. The Phase-Lock Loop                                          | 20   |
| V. Predicted Tracking                                            | 31   |
| VI. Comparison and Summary                                       | 38   |
| Bibliography                                                     | 43   |
| Appendix A: Automatic Frequency Tracking Circuit                 | 44   |
| Appendix B: Calculation of Bandwidth for Phase-Lock<br>Technique | 46   |
| Vita                                                             | 48   |

List of Figures

| Figure |                                                                                                                       | Page |
|--------|-----------------------------------------------------------------------------------------------------------------------|------|
| 1      | An Elementary Automatic Frequency Tracking Circuit                                                                    | 12   |
| 2      | Bandwidth for Entire Doppler Shift versus Distance for Various Transmitter Frequencies                                | 15   |
| 3      | Second Order Frequency Tracking System Bandwidth versus Distance for Various Transmitter Frequencies                  | 16   |
| 4      | Third Order Frequency Tracking System Bandwidth versus Distance for Various Transmitter Frequencies                   | 17   |
| 5      | Frequency Tracking System S/N Improvement versus Distance for Various Transmitter Frequencies                         | 18   |
| 6      | Frequency Tracking System: Signal Power versus Distance for Various Transmitter Frequencies                           | 19   |
| 7      | A Typical Phase-Lock Loop                                                                                             | 20   |
| 8      | Phase-Lock System: Second Order Bandwidth versus Distance for Various Transmitter Frequencies                         | 25   |
| 9      | Phase-Lock System: Third Order Bandwidth versus Distance for Various Transmitter Frequencies                          | 26   |
| 10     | Phase-Lock System: 30° Maximum Phase Error S/N Improvement versus Distance for Various Transmitter Frequencies        | 27   |
| 11     | Phase-Lock System: 50° Maximum Phase Error S/N Improvement versus Distance for Various Transmitter Frequencies        | 28   |
| 12     | Phase-Lock System: Second Order Signal Power versus Distance for Various Transmitter Frequencies                      | 29   |
| 13     | Phase-Lock System: Third Order Signal Power versus Distance for Various Transmitter Frequencies                       | 30   |
| 14     | Frequency Tracking System with Predicted Tracking S/N Improvement versus Distance for Various Transmitter Frequencies | 34   |

List of Figures

| Figure |                                                                                                                           | Page |
|--------|---------------------------------------------------------------------------------------------------------------------------|------|
| 15     | Phase-Lock System with Predicted Tracking:<br>S/N Improvement versus Distance for Various<br>Transmitter Frequencies      | 35   |
| 16     | Frequency Tracking System with Predicted<br>Tracking: Signal Power versus Distance for<br>Various Transmitter Frequencies | 36   |
| 17     | Phase-Lock System with Predicted Tracking:<br>Signal Power versus Distance for Various<br>Transmitter Frequencies         | 37   |
| 18     | Second Order Systems: Signal Power versus<br>Distance for Various Transmitter Frequencies                                 | 41   |
| 19     | Third Order Systems: Signal Power versus<br>Distance for Various Transmitter Frequencies                                  | 42   |
| 20     | Automatic Frequency Tracking Circuit                                                                                      | 44   |

List of Symbols

|                |                                                                                            |
|----------------|--------------------------------------------------------------------------------------------|
| a              | satellite acceleration (cm/sec <sup>2</sup> )                                              |
| AGC            | Automatic Gain Control Circuit                                                             |
| B              | bandwidth (cycles)                                                                         |
| d              | distance (Km)                                                                              |
| f <sub>d</sub> | Doppler frequency (cycles/sec)                                                             |
| f <sub>o</sub> | Satellite transmitter frequency (cycles/sec)                                               |
| F              | receiver noise figure                                                                      |
| G <sub>t</sub> | gain of transmitter                                                                        |
| G <sub>r</sub> | gain of receiver                                                                           |
| i              | input condition to receiver                                                                |
| J              | rate of change of acceleration (cm/sec <sup>3</sup> )                                      |
| K              | Boltzmann's constant (1.38 x 10 <sup>-23</sup> joules/degree Kelvin)                       |
| γ              | earth gravitational constant (3.987 x 10 <sup>20</sup> cm <sup>2</sup> /sec <sup>3</sup> ) |
| P <sub>r</sub> | required input signal power (watts)                                                        |
| P <sub>R</sub> | power received over a line of sight transmission path (watts)                              |
| P <sub>t</sub> | power transmitted by the satellite (watts)                                                 |
| o              | output condition of receiver                                                               |
| $\frac{S}{N}$  | signal to noise ratio (db)                                                                 |
| T              | noise temperature (degrees Kelvin)                                                         |
| V              | satellite velocity (cm/sec)                                                                |
| V <sub>r</sub> | satellite relative velocity (cm/sec)                                                       |

List of Symbols

|            |                                          |
|------------|------------------------------------------|
| $\lambda$  | wavelength of transmitted frequency (cm) |
| $\Theta_1$ | input signal (Phase-Lock Loop)           |
| $\Theta_2$ | output signal (Phase-Lock Loop)          |

Abstract

The purpose of this study is to quantitatively compare frequency tracking, the phase-lock loop, and predicted tracking when used to improve the effectiveness of radio receivers in the tracking of earth satellites. The techniques are evaluated in terms of their effect on receiver sensitivity. Sensitivity is defined as the input signal power required to produce the output signal to noise ratio deemed necessary, by the system designer, for detection. The detection bandwidth for each technique is smaller than that of the conventional receiver. By using the smaller bandwidth much extraneous noise is removed and the output signal to noise ratio is improved. A lower value of input signal power is required to produce the output signal to noise ratio and the sensitivity is increased. The detection bandwidths for frequency tracking and the phase-lock loop are nearly the same. Because of the lower threshold of the phase-lock loop, the receiver sensitivity is greater by two orders of magnitude than that of frequency tracking. Prediction tracking is a form of correlation detection which makes more effective use of the reduced detection bandwidth. When used in conjunction with the other two techniques, the sensitivity is increased by another order of magnitude. The techniques, developed for tracking earth satellites, should prove of even greater value for tracking inter-planetary probes.

THE EFFECT OF FREQUENCY TRACKING, THE USE OF  
A PHASE-LOCK LOOP, AND PREDICTED TRACKING ON  
RECEIVER SENSITIVITY

I. Introduction

The purpose of this study is to quantitatively compare three techniques used to improve the effectiveness of radio receivers in the tracking of earth satellites. Since the power transmitted by a satellite is limited by weight considerations and the distances involved are comparatively large, the received power will be very small in magnitude. The most effective improvement any special technique could make would be to increase the sensitivity of the receiver. Therefore the techniques of frequency tracking, the use of a phase-lock loop and predicted tracking are evaluated in this study in terms of their effect on receiver sensitivity.

Background

The configuration considered in this study is that of a satellite, continuously transmitting an un-modulated, radio frequency signal to a fixed receiver on the earth. The relative motion of the satellite with respect to the receiver causes the received frequency to differ from the transmitted frequency by a varying value known as the Doppler frequency. The Doppler frequency for a particular orbit is unique and the accurate determination of this frequency is a primary method of tracking satellites.

The Doppler frequency for a transit from horizon to horizon will vary over a bandwidth of several kilocycles, while for a given increment of time the Doppler frequency may change only a few cycles. To accurately determine the orbit it is necessary to observe the entire pattern of the Doppler shift. The conventional receiver accomplishes this by being tuned to receive the entire possible shift. A detection bandwidth of several kilocycles is processed to extract an intelligence of a few cycles per second. The result is a signal buried in a sea of noise, requiring the signal strength to be much greater than the noise power of the entire Doppler shift bandwidth.

The problem is further complicated by the relatively small signal power transmitted by the satellite. Weight limitations placed upon the transmitter power supply limit the transmitted signal power to typical values of one watt or several watts at best. The power received over a line of sight transmission path is governed by the equation:

$$P_R = \frac{P_t G_t G_r \lambda^2}{16 \lambda^2 d^2} \quad (\text{Ref 7:445}) \quad (1.)$$

Assuming

$$P_t = 1 \text{ watt}$$

$$G_t = 1$$

$$G_r = 1$$

$$f_0 = 1000 \text{ Mcs and } \lambda = 0.3 \text{ m}$$

$$d = 1000 \text{ Km}$$

For these conditions the power received would be  $5.7 \times 10^{-16}$  watts.

To clearly illustrate the problem this value is contrasted with the power required by a conventional receiver, using the entire Doppler shift bandwidth, to produce the output signal to noise ratio necessary for detection. It can be shown that the power required to produce a given output signal to noise ratio in a conventional receiver is governed by the equation:

$$P_r = F \frac{S_o}{N_o} K T B \quad (\text{Ref 7:445}) \quad (2.)$$

It can also be shown that the bandwidth for the entire Doppler shift is given by the equation:

$$B = \frac{2 V_r}{\lambda} \quad (\text{Ref 6:90}) \quad (3.)$$

For a distance of 1000 Km and a transmitting frequency of 1000 Mcs:

$$V_r = 7.17 \times 10^5 \text{ cm/sec}$$

$$B = 47.2 \text{ Kcs}$$

$$\lambda = 0.3 \text{ m}$$

$$F = 1$$

$$KT = 4 \times 10^{-21}$$

$$\frac{S_o}{N_o} = 26 \text{ db} = 400$$

Under these assumptions the power required is  $7.53 \times 10^{-11}$  watts.

Using a conventional receiver and the entire Doppler shift bandwidth

the power required exceeds the power available by two orders of magnitude.

#### Assumptions

This study has been carried out utilizing the following assumptions. It is assumed that the motion of the satellite takes place in a plane defined at any instant by the satellite, the receiver, and the center of the earth. The term distance is assumed to mean the height of the satellite above the earth when over the receiver. The distances considered are 600, 800, 1000, and 1200 Km. These distances are arbitrary but have been chosen to be representative of the actual area of interest.

The observed velocity differs from the inertial velocity because of the earth's rotation. The observed velocity, denoted as  $V_r$ , is referred to as the relative velocity. The values of relative velocity for the distances considered were extracted from the Pickard and Burns Inc. report. (Ref 6:11) Golay states that the acceleration of a satellite is given by the equation:

$$a = \frac{K}{d^2} \quad (\text{Ref 3:191}) \quad (4.)$$

and the rate of change of acceleration is given by the equation:

$$J = \frac{vK}{d^3} \quad (\text{Ref 3:191}) \quad (5.)$$

The extracted values of velocity and the computed values of acceleration and rate of change of acceleration are shown in Table I.

Table I  
Velocity, Acceleration, and Rate of Change of Acceleration

| Distance (Km) | Vel. (cm/sec)      | Acc. (cm/sec <sup>3</sup> ) | J (cm/sec <sup>3</sup> ) |
|---------------|--------------------|-----------------------------|--------------------------|
| 600           | $7.58 \times 10^5$ | $1.11 \times 10^5$          | $1.38 \times 10^3$       |
| 800           | $7.37 \times 10^5$ | $6.24 \times 10^4$          | $5.76 \times 10^2$       |
| 1000          | $7.17 \times 10^5$ | $3.99 \times 10^4$          | $2.86 \times 10^2$       |
| 1200          | $6.96 \times 10^5$ | $2.77 \times 10^4$          | $1.60 \times 10^2$       |

All receiver noise factors are assumed to be unity. This is an optimistic assumption but this is a comparative study and the particular value of this constant will not affect the results. The temperature used for the calculation of noise power is 290° K. The transmitter frequencies considered are 100, 500, 1000, 1500, and 2000 Mcs. Again these are arbitrary, but representative of the actual span of frequencies used in satellite transmission.

#### Statement of the Problem

Two problems are proposed for solution in this study. The first is to develop a common basis for comparing the effect of the three techniques on receiver sensitivity. The second problem is to analyze each technique and to express the characteristics of the technique in terms relating to the common basis of comparison. The systems analyzed are; the second and third order systems of the frequency tracking technique,

GA/EE/62-2

the second and third order systems of the phase-lock loop, and the effect of predicted tracking on all four systems.

## II. Determining Receiver Sensitivity

Sensitivity is nominally defined as the input signal power required by a receiver to produce a standard output signal to noise ratio. In this study the standard output signal to noise ratio is that value deemed necessary, by the system designer, for detection.

The main purpose of frequency tracking and the phase-lock loop is to reduce the detection bandwidth. The techniques remove the small segment of bandwidth containing the intelligence while neglecting the broad bandwidth noise power. The output signal to noise ratio for the special techniques will be greater than for the conventional receiver which must pass the broad bandwidth noise power. The improvement in the signal to noise ratio caused by using the special technique is a function of the bandwidth reduction.

If one knows the required output signal to noise ratio, the detection bandwidth, and its effect on the signal to noise ratio, the required input signal to noise ratio can be determined. The noise power associated with the input signal to noise ratio is that contained in the input bandwidth, which is considered to be the entire Doppler shift. The noise power is given by the equation:

$$N = F K T B \quad (\text{Ref 7:435}) \quad (6.)$$

By multiplying the noise power by the required input signal to noise ratio one obtains the required input signal power. The required input signal power is a measure of the sensitivity.

The following example illustrates the procedure. At a distance of 600 km and using a transmitter frequency of 100 mcs the bandwidth for the entire Doppler shift is given by:

$$B = \frac{2V_r}{\lambda} = \frac{(2)(7.58 \times 10^5)}{(3 \times 10^2)} = 5.04 \times 10^3 \text{ cycles}$$

Under the assumption that the detection bandwidth is 25 cycles and the relationship between the bandwidths and the signal to noise ratios is:

$$\frac{\frac{S_0}{N_0}}{\frac{S_1}{N_1}} = \left( \frac{B_1}{B_0} \right)^{\frac{1}{2}} = \left( \frac{5.04 \times 10^3}{25} \right)^{\frac{1}{2}} = 14.2$$

Under the assumption that the required output signal to noise ratio is 26 db, which is the number 400, the required input signal to noise ratio is:

$$\frac{S_1}{N_1} = \frac{400}{14.2} = 28.1$$

The input noise power is given by:

$$N = F K T B$$

Under the assumptions that

$$F = 1$$

$$K = 1.39 \times 10^{-23} \text{ joules/degree K}$$

GA/EE/62-2

$$T = 290^{\circ} \text{ K}$$

$$B = 5.04 \times 10^3 \text{ cycles}$$

gives  $N = 2.03 \times 10^{-17}$  watts.

Then

$$S_i = (28.1) (2.03 \times 10^{-17}) = 5.7 \times 10^{-16} \text{ watts}$$

In summary, the analysis of the techniques requires determination of the detection bandwidth, the required output signal to noise ratio and the relationship between the bandwidths and the signal to noise ratios.

### III. Frequency Tracking

The object of frequency tracking is to follow within narrow limits the incoming Doppler frequency. The basic procedure is to select a small element of bandwidth, for example 50 or 100 cycles, and maintain the center of this element aligned with the center of the intelligence bandwidth. The bandwidth element must be wide enough to include the entire change of the frequency per unit time and in normal practice will exceed this width several times to allow for noise or extraneous frequency modulation. The small bandwidth element becomes the detection bandwidth rather than the entire possible frequency change. Tracking can be done either manually or automatically.

There are many possible schemes that can be used for manual frequency tracking. Only one scheme will be discussed in this study and it is included only to convey the basic idea of the procedure. Manual tracking has two severe limitations, one of which is the inability of the human operator to precisely deal with such small quantities as parts of seconds or several cycles per second. The second limitation is that the signal power must exceed the noise power of the entire Doppler shift bandwidth in order for the signal to be detected.

In one manual scheme the Doppler audio frequency signal, obtained from the receiver detector output, is fed into one pair of deflection plates (x-axis) of an oscilloscope. The other plates

(y-axis) are connected to an audio oscillator output. The oscillator is adjusted to give a 1:1 Lissajous pattern, nominally a circle. The audio oscillator also intensity modulates the oscilloscope beam (z-axis), so the circular pattern appears as a "half-moon". If the frequency of the audio oscillator is low the "half-moon" rotates in one direction. If the frequency is high it rotates in the opposite direction. The operator tries to keep the "half-moon" stationary by varying the output frequency of the audio frequency oscillator. The result is that the weak, varying amplitude Doppler signal is replaced by a strong, constant amplitude signal from the audio oscillator. As previously pointed out the accuracy of this system is limited to the human operator's ability to track the signal. To be competitive with other systems it must be accurate to one-fourth of a cycle and this is beyond human capability. Also the Lissajous figure is subject to fading and collapsing because of the weak Doppler signal.

Figure 1 shows a schematic representation of an elementary automatic frequency tracking circuit. This circuit represents the basic principles involved in any automatic frequency tracking system. A local oscillator generates a signal of frequency  $f_0 + f_s$  which closely approximates  $f_0 + f_d$ . The servo circuit causes the frequency  $f_0 + f_s$  to follow  $f_0 + f_d$  as closely as the limitations of the circuit will allow. The useful output of the circuit is the constant amplitude signal  $f_0 + f_s$ .

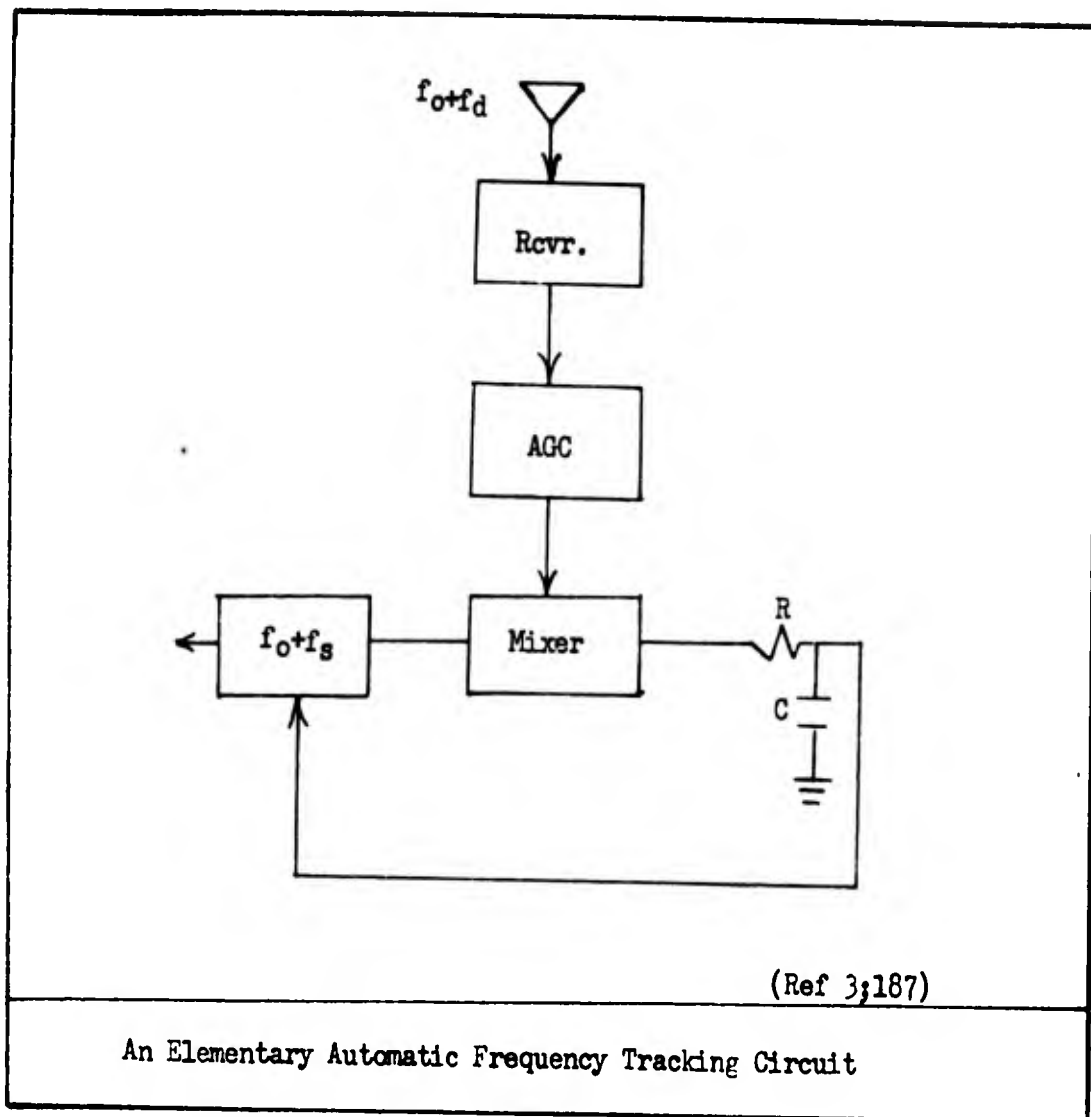


Figure 1

There are two disadvantages of this circuit. First the signal power must be greater than the noise power within the receiver bandwidth so that the signal controls the action of the AGC circuit. Secondly, at the optimum damping ratio the allowable frequency excursion is too

small to accurately track the changing Doppler frequency. (Ref 3:188)

The elementary system can be modified and refined to overcome these disadvantages. A detailed description of one configuration presented by Golay is included in Appendix A. This system will accurately track the frequency to one-fourth of a cycle. The noise can exceed the signal by several orders of magnitude and the system can track a rapidly varying signal. The significant result is that the detection bandwidth for a second order system, as shown by Golay is given by the equation:

$$B = 1.275 \left( \frac{a}{\lambda} \right)^{\frac{1}{2}} \quad (7.)$$

and the detection bandwidth required by a third order system is given by the equation:

$$B = 1.747 \left( \frac{J}{\lambda} \right)^{\frac{1}{3}} \quad \begin{matrix} \text{(Ref 3:187-191)} \\ \text{(8.)} \end{matrix}$$

The bandwidths for the entire Doppler shift, for the distances and frequencies considered are shown in figure 2. The bandwidths for these conditions range from 4.64 kcs to 101.2 kcs. The bandwidths for the second and third order frequency tracking systems are shown in figures 3 and 4 respectively. The bandwidths for the second order system range from 12.5 cycles to 109 cycles, and for the third order system from 1.4 cycles to 7.8 cycles. The second order system provides

a decrease of bandwidth by two orders of magnitude and the third order system reduces the bandwidth by three orders of magnitude.

It can be shown that the improvement in the signal to noise ratio is given by the equation:

$$\frac{\frac{S_0}{N_0}}{\frac{S_1}{N_1}} = \left( \frac{B_1}{B_0} \right)^{\frac{1}{2}} \quad (\text{Ref 1:27}) \quad (9.)$$

The improvement in the signal to noise ratios, for the second and third order systems are shown in figure 5. The second order system gives an improvement varying from 11.5 db to 16.2 db. The third order system gives an improvement varying from 16.2 db to 21.9 db.

Golay states that the output signal to noise ratio required by both the second and third order systems is 26 db. (Ref 3:192) All of the required information for computation of the required input signal power has now been stipulated. The results of the computations are shown graphically in figure 6. The input signal power for the second order system varies from  $3.82 \times 10^{-16}$  watts to  $51.1 \times 10^{-16}$  watts. The power required by the third order system varies from  $1.25 \times 10^{-16}$  watts to  $14.2 \times 10^{-16}$  watts. It should be noted that for the illustrative conditions in Section I, i.e. a distance of 1000 km and transmitting frequency of 1000 mcs that the power available exceeds the power required by the third order system but is less than that required by the second order system.

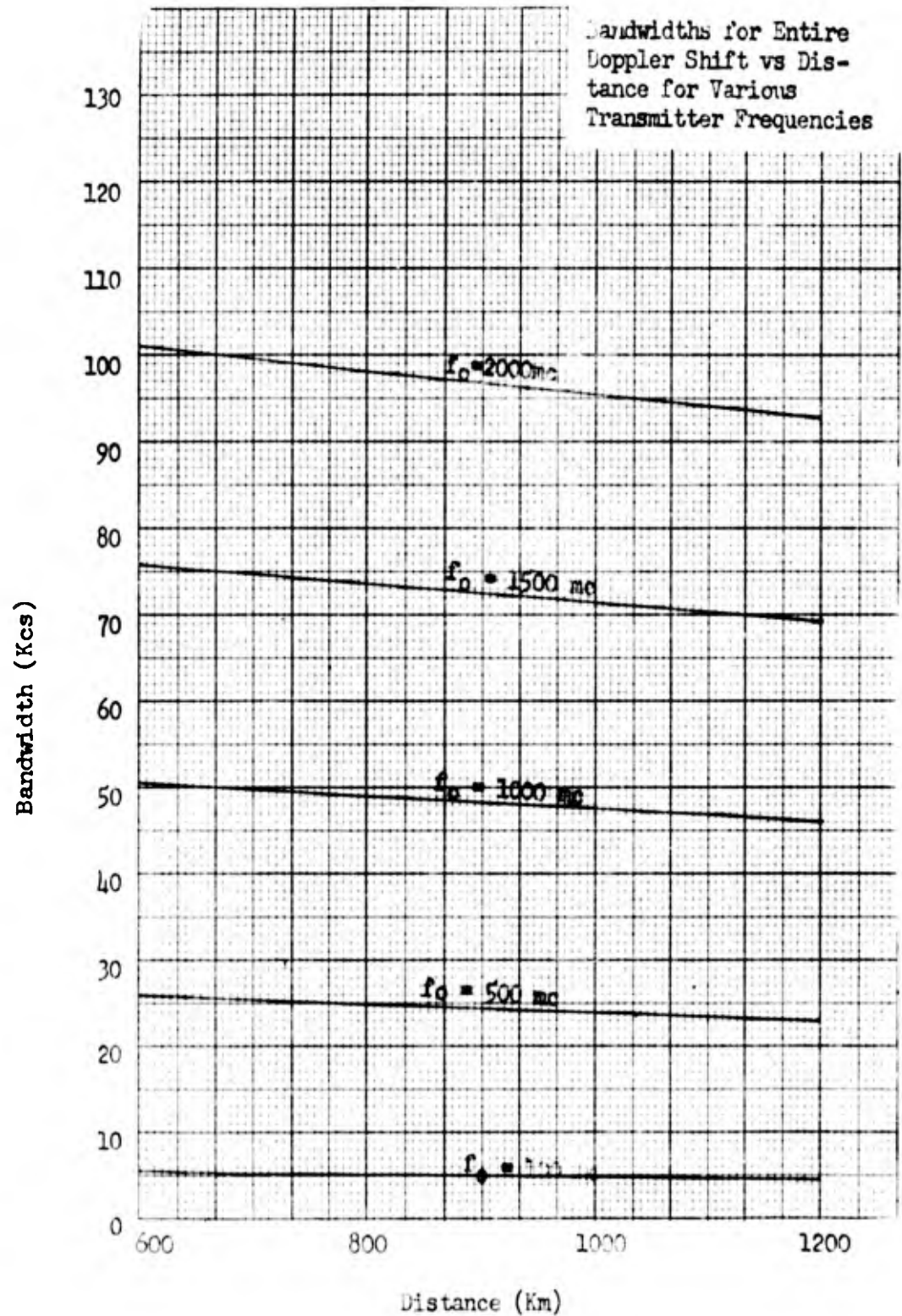


Figure 2

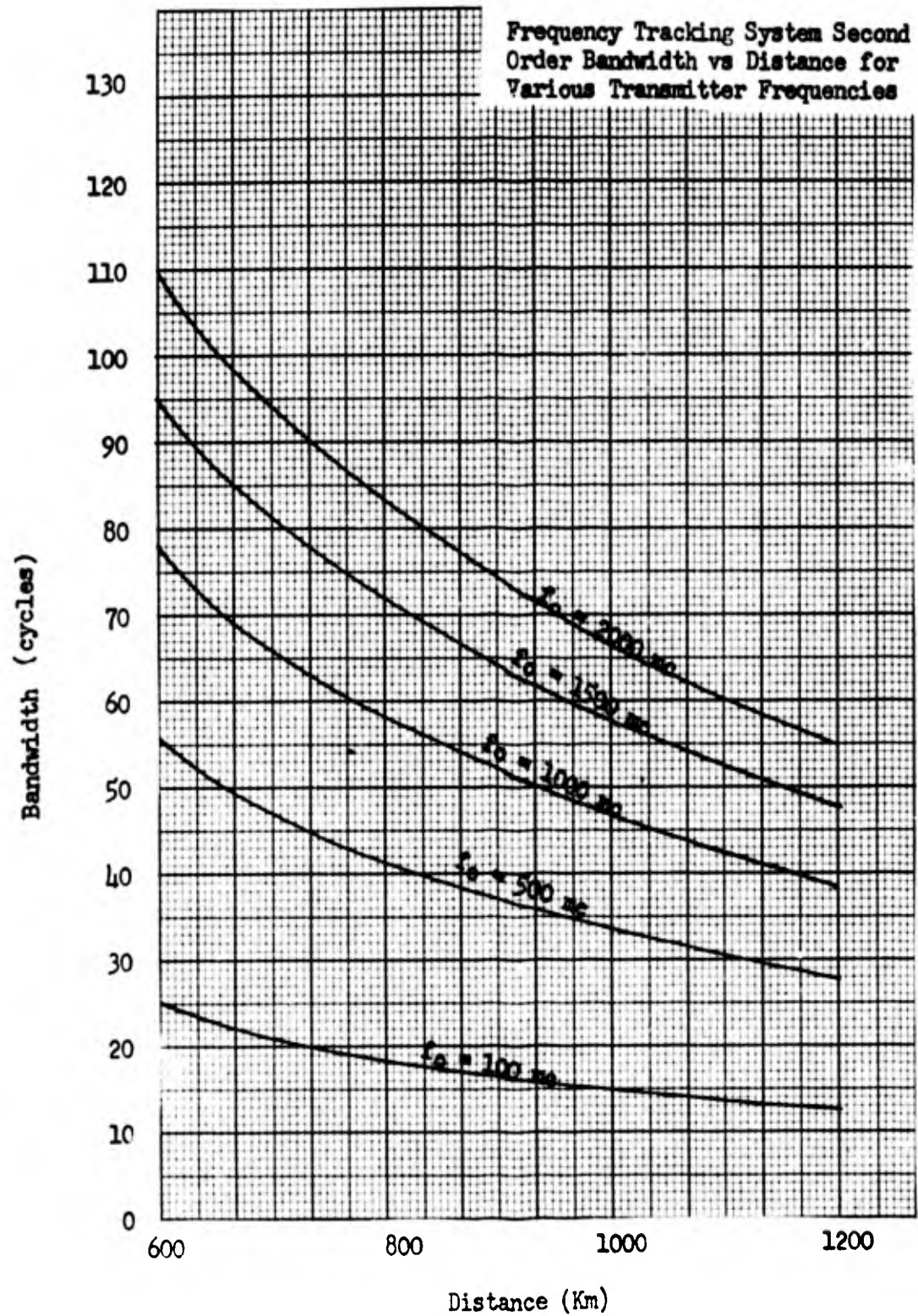


Figure 3

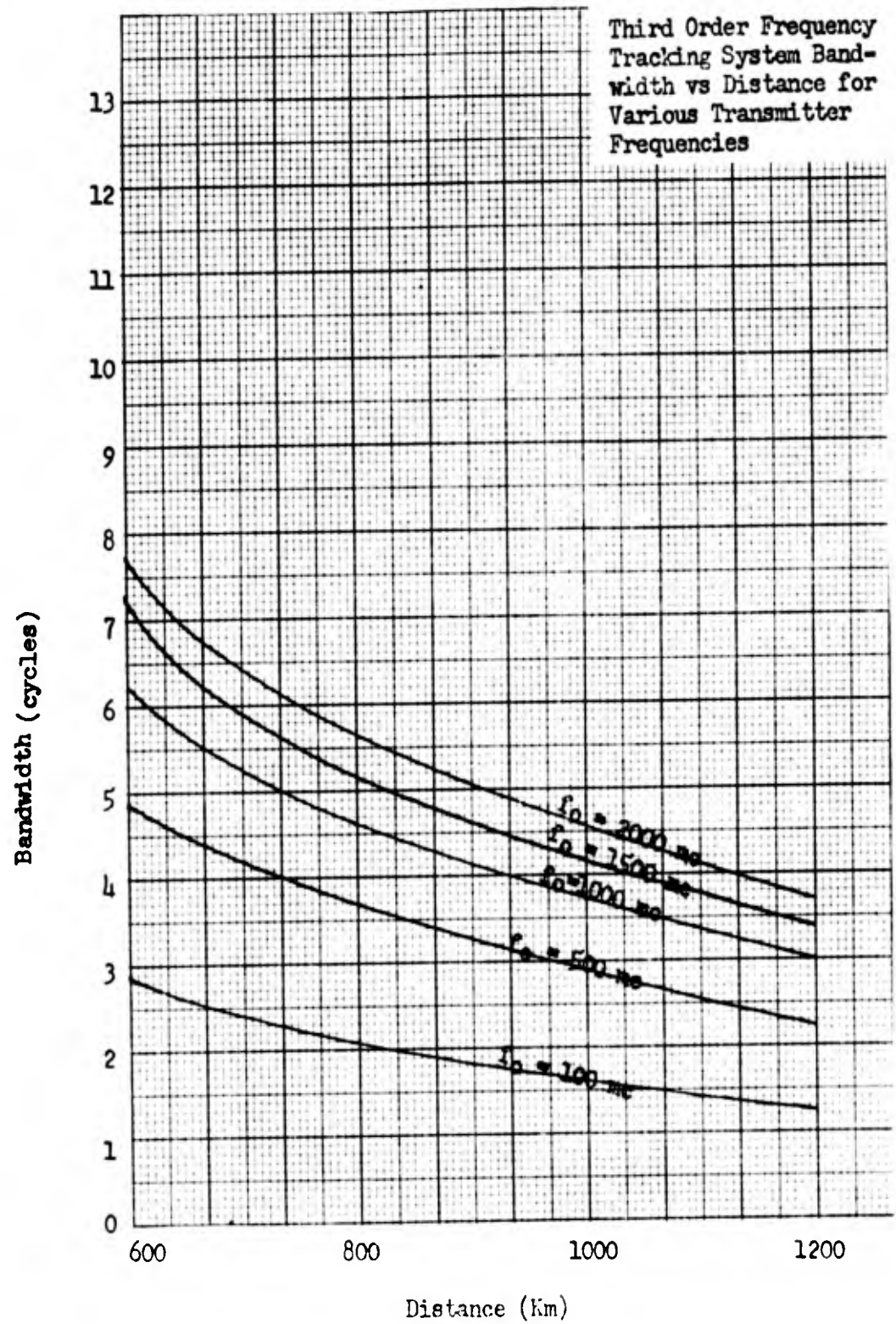


Figure 4

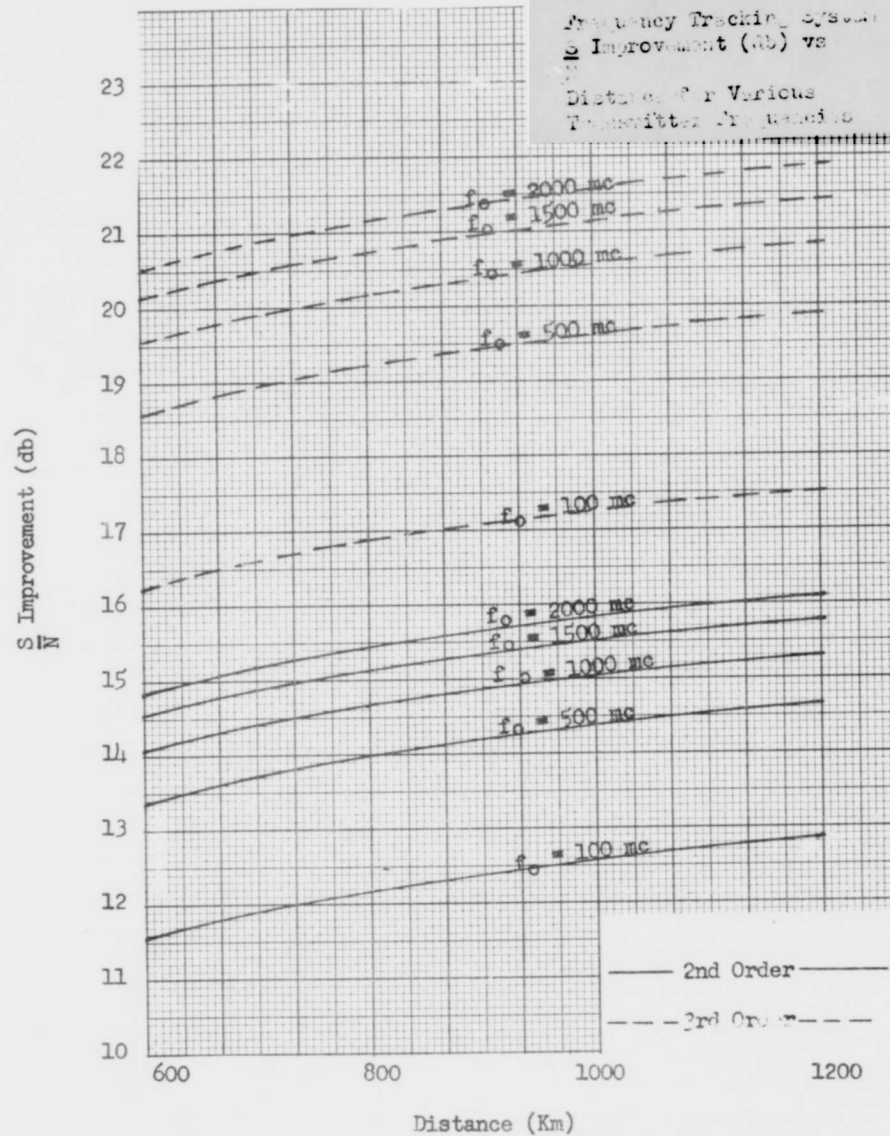


Figure 5

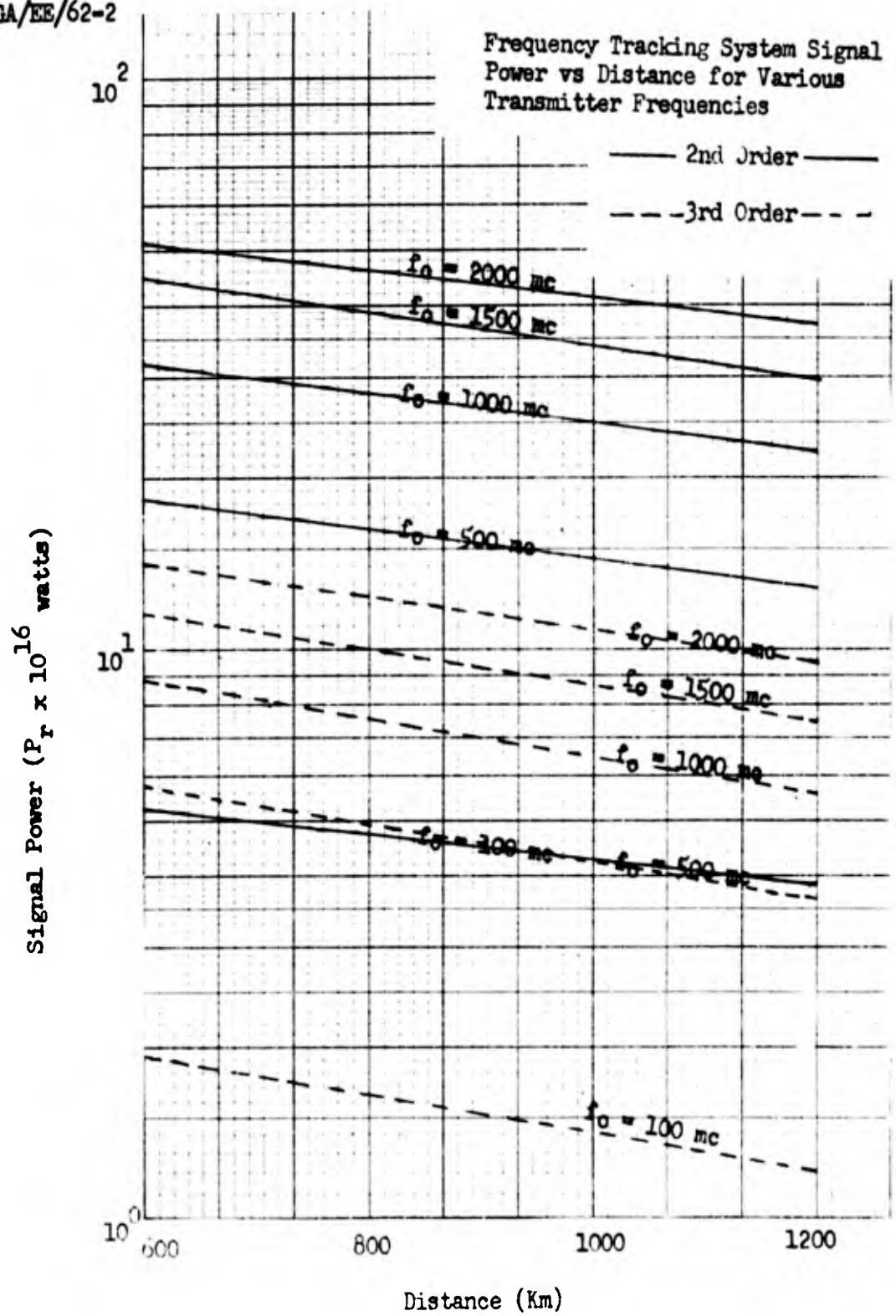


Figure 6

#### IV. The Phase-Lock Loop

A second technique for reducing the detection bandwidth and the required input signal power is by the use of a phase-lock loop. A schematic representation of the loop is shown in figure 7. The multiplier beats the signal input and the output of the voltage con-

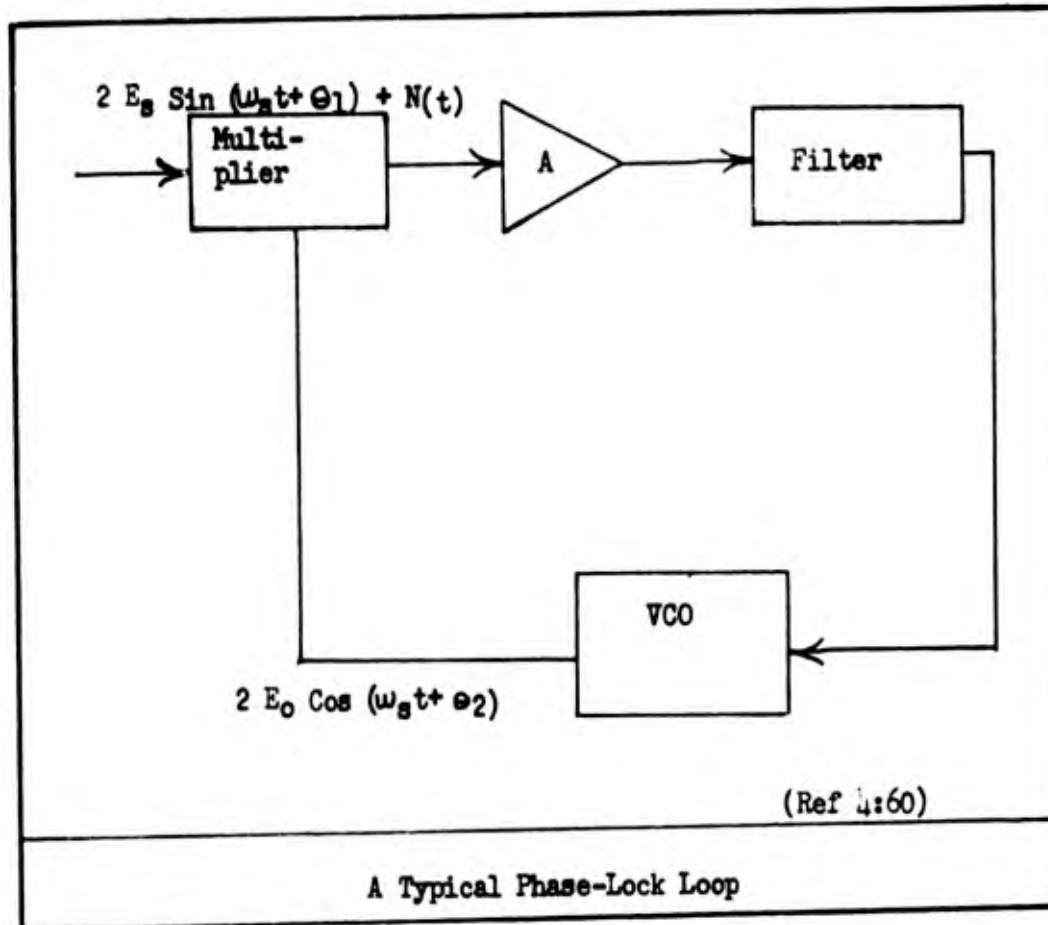


Figure 7

trolled oscillator together giving several resultant terms, including a low frequency output proportional to  $(\theta_1 - \theta_2)$ . The loop filter passes only the low frequency term, which is applied to the voltage controlled oscillator and which forces the output signal,  $\theta_2$ , to be equal to the input signal  $\theta_1$ . The bandwidth required by the phase-

lock loop must be large enough to pass the difference between the input and output signals. Since this difference varies less than the signal frequency the bandwidth is reduced and the amount of noise passed into the loop is reduced. (Ref 8:66)

The signal proportional to  $(\Theta_1 - \Theta_2)$  is known as the transient phase error and is necessary to develop the voltage required to keep the VCO at the same frequency as the signal. However the phase transient cannot exceed  $90^\circ$  for at this point the sign of the open loop gain will change and the loop will become unstable. The value of  $\Theta_1$  can fluctuate very rapidly because of random noise added to the signal, therefore it is necessary to limit the peak value of  $(\Theta_1 - \Theta_2)$  to  $90^\circ$ , and the mean value to less than  $90^\circ$ . (Ref 1:27)

It is shown in Appendix C that when the maximum mean phase transient error, hereafter called the phase error, is limited to  $30^\circ$  the bandwidth required by a second order system is given by the equation:

$$B = 1.760 \left( \frac{a}{\lambda} \right)^{\frac{1}{2}} \quad (10.)$$

and the bandwidth for a third order system is given by the equation:

$$B = 1.170 \left( \frac{J}{\lambda} \right)^{\frac{1}{3}} \quad (11.)$$

If the phase error is limited to  $30^\circ$  the output signal to noise ratio necessary for maintaining lock is 3 db. (Ref 1:27)

As the phase error is increased the ability of the loop to follow a varying signal is increased and the required detection bandwidth is decreased. If the phase error is increased to  $50^\circ$  the detection bandwidth, as shown in Appendix B, for the second order system is given by the equation:

$$B = 1.367 \left( \frac{a}{\lambda} \right)^{\frac{1}{2}} \quad (12.)$$

and for the third order system by the equation:

$$B = 0.968 \left( \frac{J}{\lambda} \right)^{\frac{1}{3}} \quad (13.)$$

The detection bandwidths for the second order system for both the  $30^\circ$  and  $50^\circ$  phase errors are shown in figure 8. For the  $30^\circ$  phase error the bandwidths vary from 17 cycles to 150 cycles and for the  $50^\circ$  error from 13.2 cycles to 117.1 cycles. The detection bandwidths for the third order system for both the  $30^\circ$  and  $50^\circ$  phase error are shown in figure 9. For the  $30^\circ$  phase error the bandwidths vary from 0.946 cycles to 5.26 cycles and for the  $50^\circ$  phase error the bandwidths vary from 0.785 cycles to 4.36 cycles. The detection bandwidths are reduced, from the entire Doppler shift, by two orders of magnitude for the second order system and by three orders of magnitude for the third order system.

The improvement in the signal to noise ratio is given by the equation:

$$\frac{\frac{S_0}{N_0}}{\frac{S_1}{N_1}} = \left( \frac{B_1}{B_0} \right)^{\frac{1}{2}} \quad (\text{Ref 1:27}) \quad (9.)$$

The improvement in the signal to noise ratio for the  $30^\circ$  phase error is shown in figure 10 and for the  $50^\circ$  phase error in figure 11. The second order system shows an improvement of 10.8 db to 15.4 db for the  $30^\circ$  phase error and 11.4 db to 16.0 db for the  $50^\circ$  phase error. The third order system shows an improvement of 17.1 db to 22.8 db for the  $30^\circ$  phase error and 11.4 db to 23.2 db for the  $50^\circ$  phase error.

As the maximum phase error is increased, the filter bandwidth required to pass the difference signal is increased and the internal noise power is increased. As a result of this increase the threshold required to remain inlock is increased. The relationship between the maximum phase error and the system threshold is non-linear and is best determined by experiment. Weaver shows that if the phase error is increased from  $30^\circ$  to  $50^\circ$  the threshold, expressed in decibels, doubles. (Ref 7:64) Thus the output signal to noise ratio necessary for maintaining lock in the  $50^\circ$  phase error system is 6 db.

All of the information for computation of the required input signal power is stipulated. The results of the computations are shown graphically in figures 12 and 13. The second order system requires power

ranging from  $2.24 \times 10^{-18}$  watts to  $3.12 \times 10^{-18}$  watts for the  $30^\circ$  phase error and from  $39.1 \times 10^{-18}$  watts to  $54.2 \times 10^{-18}$  watts for a phase error of  $50^\circ$ . The signal power required by the third order system varies from  $0.529 \times 10^{-18}$  watts to  $5.82 \times 10^{-18}$  watts for a  $30^\circ$  phase error and from  $9.47 \times 10^{-18}$  watts to  $100 \times 10^{-18}$  watts for a  $50^\circ$  phase error.

Both the second and third order systems, using both a  $30^\circ$  and  $50^\circ$  phase error, are capable of detecting the signals discussed in the illustrative example of section I. Also it is to be noted that even though the  $50^\circ$  phase error system requires a smaller detection bandwidth than the  $30^\circ$  phase error system, it is less sensitive because of the increased threshold.

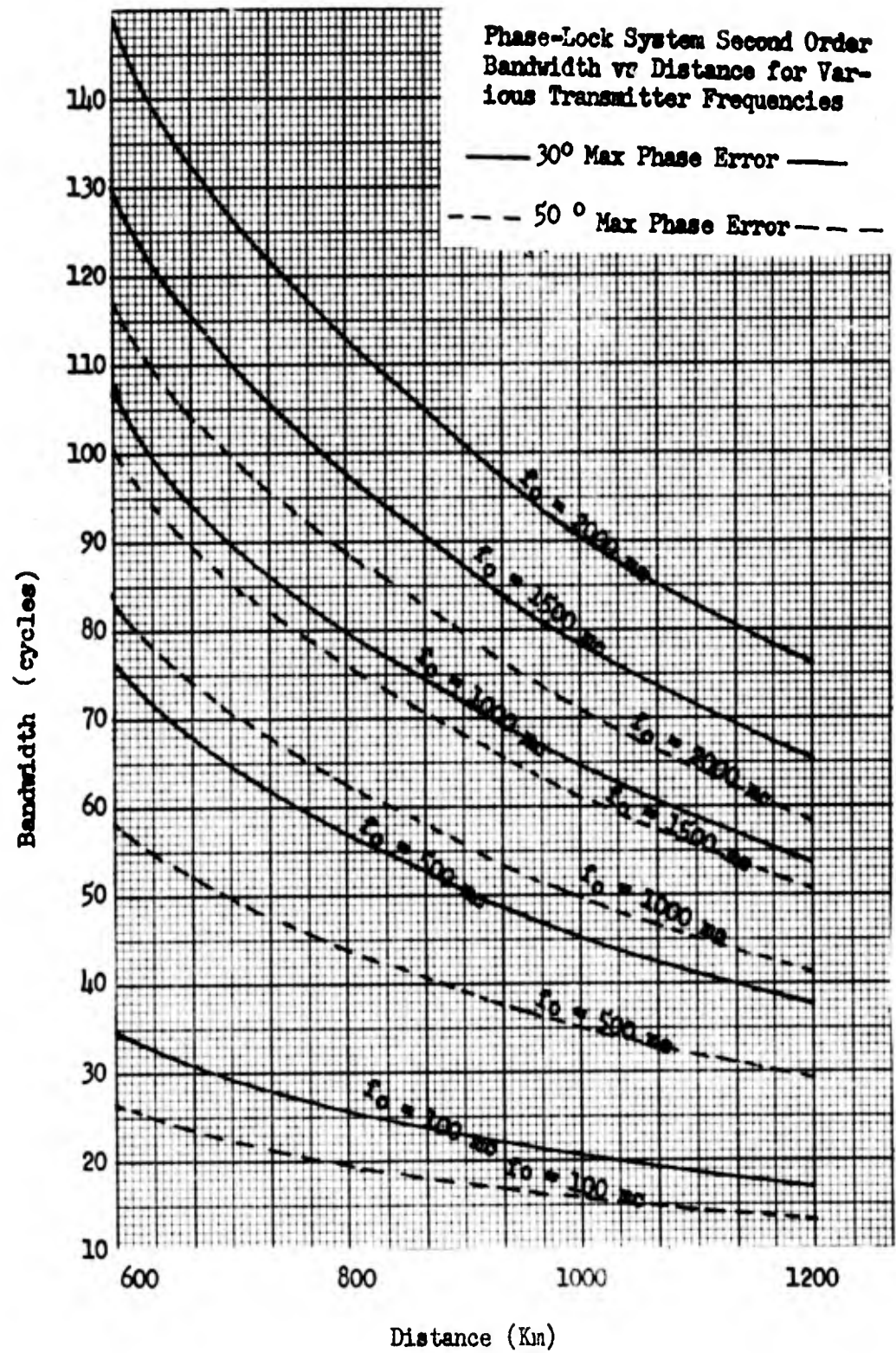


Figure 8

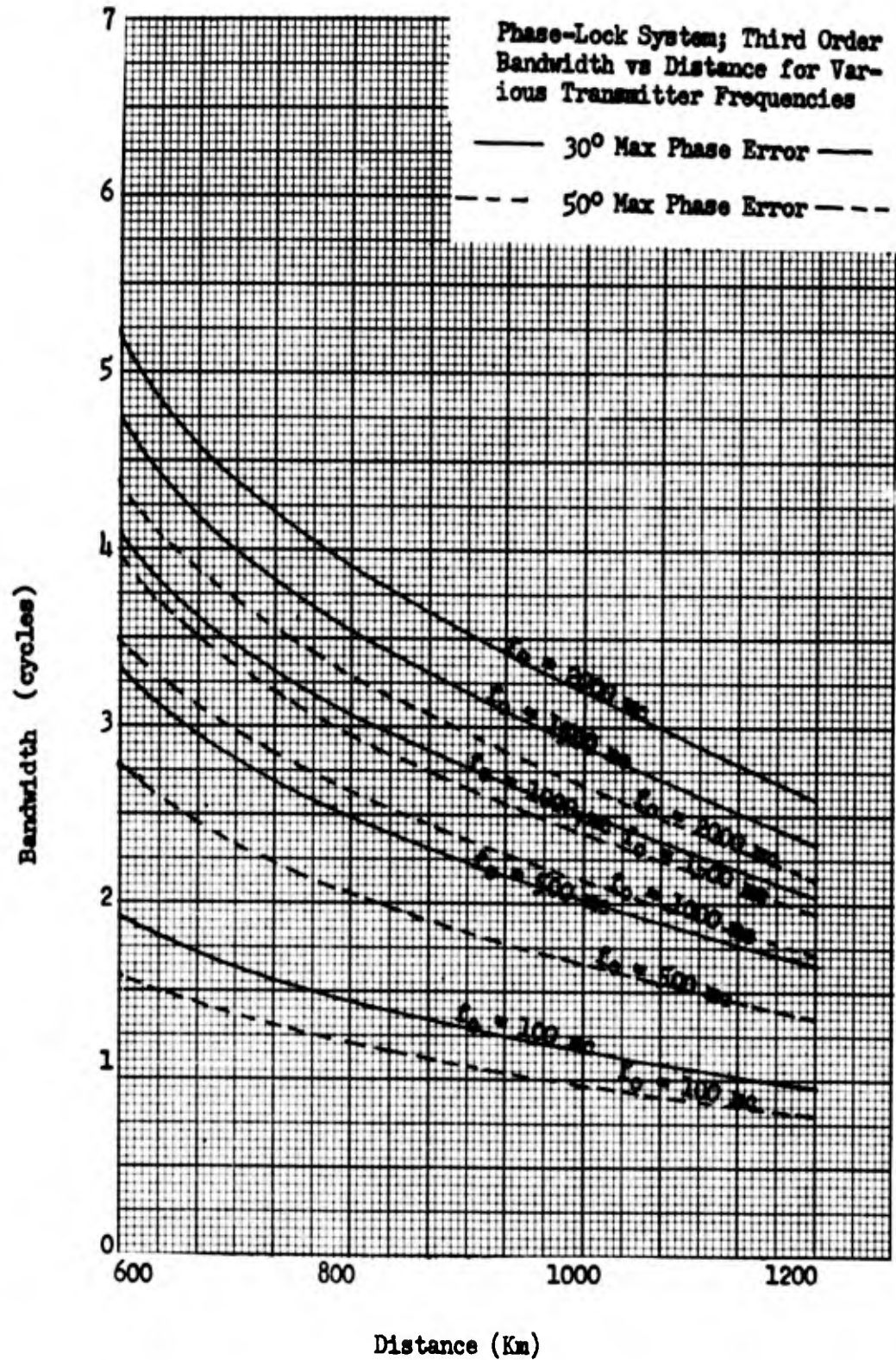


Figure 9

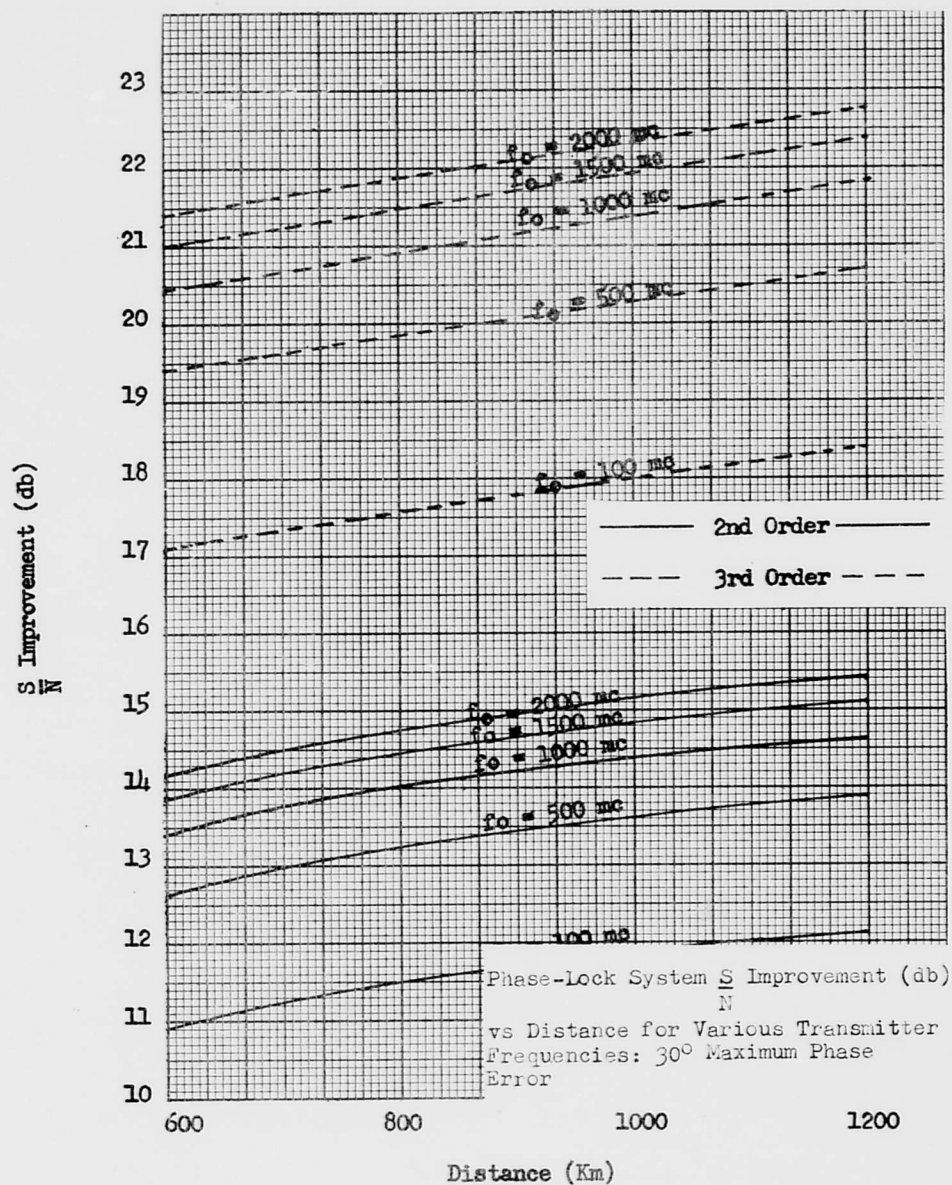


Figure 10

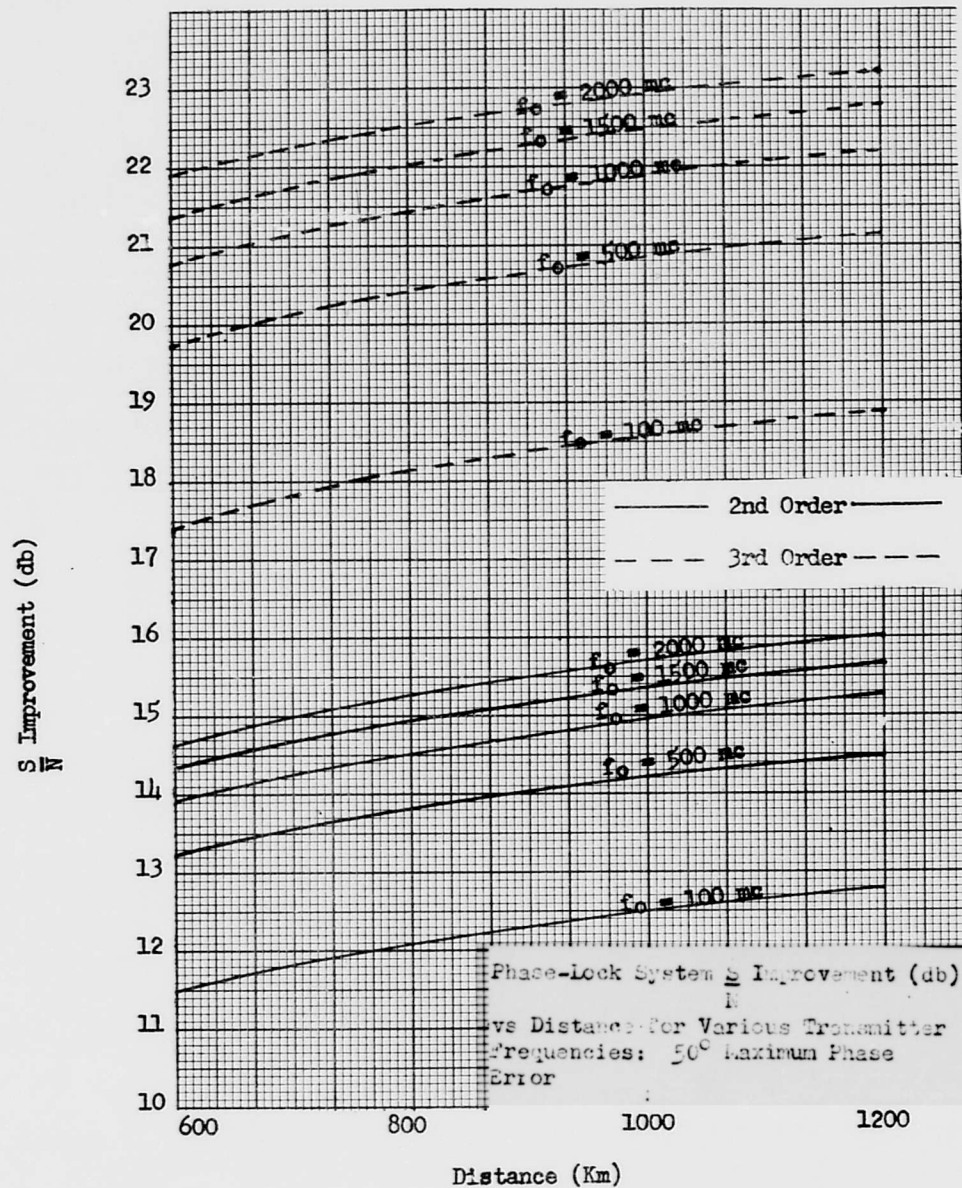


Figure 11

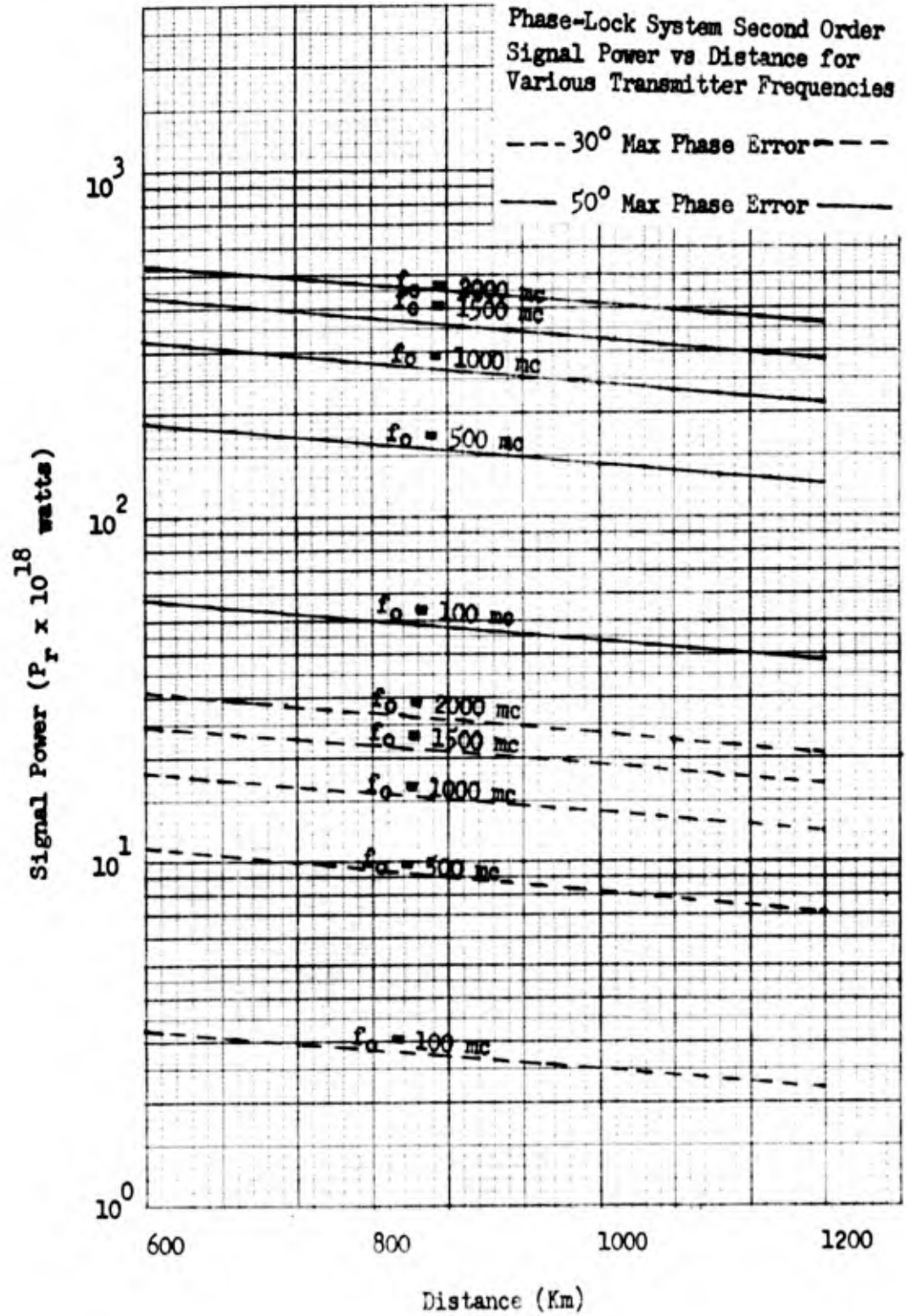


Figure 12

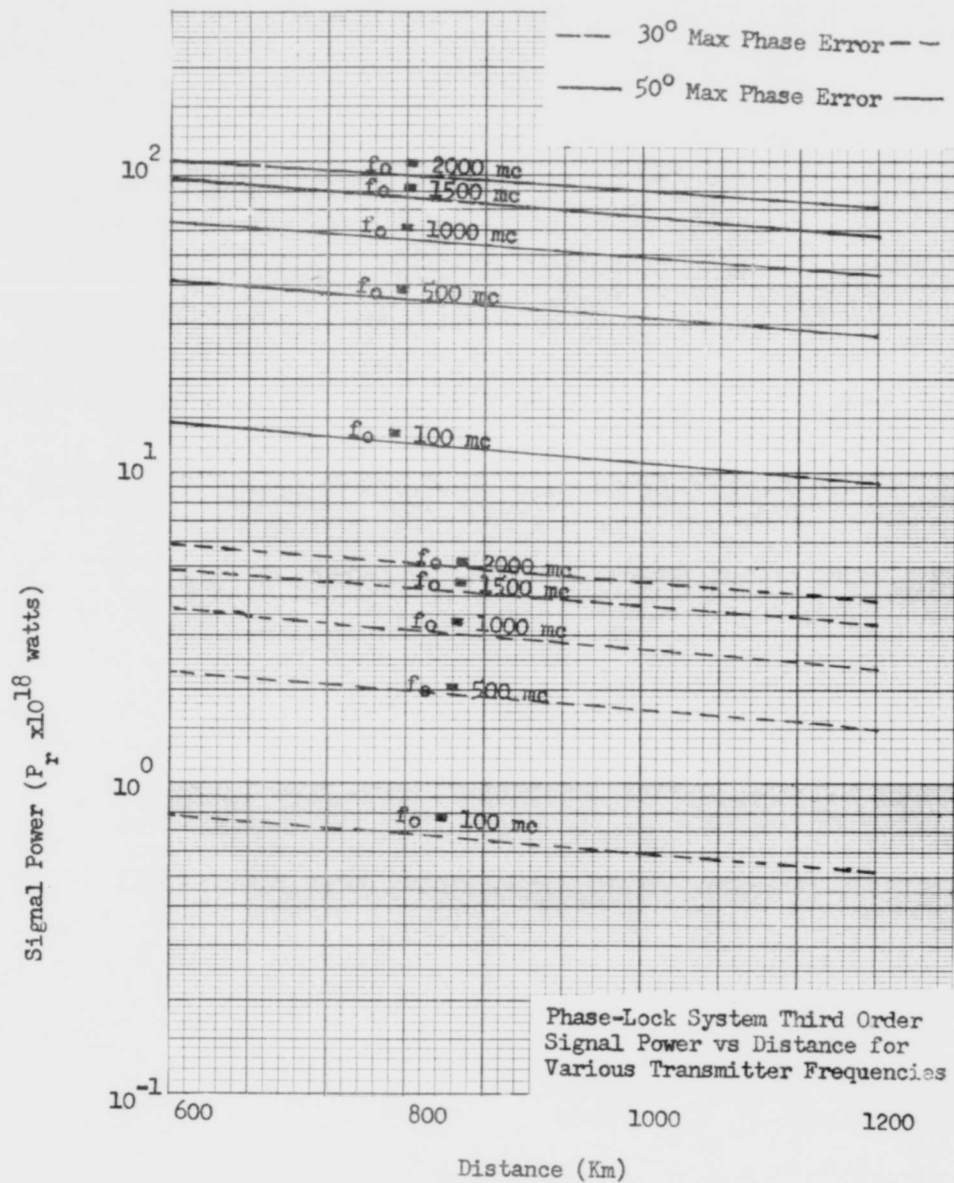


Figure 13

### V. Predicted Tracking

The third technique for reducing the required input signal power is predicted tracking. The orbit of a satellite is a relatively stable ellipse. The orbit decays over long periods of time but the decay is very slow and can be accurately predicted. The orbit will change position with respect to a fixed receiver because of the earth's rotation. This motion is also well known and can be accurately predicted. Thus by accounting for the orbit decay and the rotation of the earth, the position and motion of the satellite with respect to a fixed receiver can be accurately predicted. If the predicted motion and the known transmitter frequency are combined, the Doppler frequency can be predicted. The use of predicted Doppler frequency in the detection process is an example of correlation detection. Golay describes predicted tracking as, "Cranking into the circuit everything which can be predicted about the flight of the vehicle, as well as the presumably well-known motion of the ground station with respect to the center of gravity of the solar system." (Ref 3:191) In this study the uses of correlation detection will be limited to its effect upon frequency tracking and the phase-lock loop.

In actual practice the inserting of available data is done by computer and can be performed in various ways. The significant result is that the receiver sensitivity is increased in direct proportion to the reduction in bandwidth. (Ref 2:42)

The improvement in the signal to noise ratio is given by the equation:

$$\frac{\frac{S_0}{N_0}}{\frac{S_1}{N_1}} = \frac{B_1}{B_0} \quad (\text{Ref 2:42}) \quad (14.)$$

Applying this relationship to the detection bandwidths for the frequency tracking and phase-lock loop techniques gives even greater improvement in the signal to noise ratio. The improvement in the signal to noise ratio for the frequency tracking system is shown in figure 14. The second order system improvement ranges from 23.1 db to 32.3 db. The third order system improvement ranges from 32.4 db to 43.7 db. The improvement for the phase-lock technique is shown in figure 15. The second order system shows an improvement ranging from 21.7 db to 30.8 db. The third order system shows an improvement ranging from 38.7 db to 45.5 db. For both techniques, predicted tracking doubles, when expressed in decibels, the improvement in the signal to noise ratio.

The input power required is computed in the same manner as in the previous two sections. The power required by the frequency tracking system is shown in figure 16. The second order system required power ranges from  $15.5 \times 10^{-18}$  watts to  $172 \times 10^{-18}$  watts. The third order system required power ranges from  $2.3 \times 10^{-18}$  watts to  $12.7 \times 10^{-18}$  watts. The power required by the phase-lock technique

GA/EE/62-2

is shown in figure 17. The second order system required power ranges from  $13.8 \times 10^{-20}$  watts to  $121 \times 10^{-20}$  watts. The third order system required power ranges from  $0.76 \times 10^{-20}$  watts to  $4.2 \times 10^{-20}$  watts.

The lower values for the third order system is not shown in figure 17 but has been separately calculated. The required input signal power, for both techniques, is reduced by at least one order of magnitude by the addition of predicted tracking.

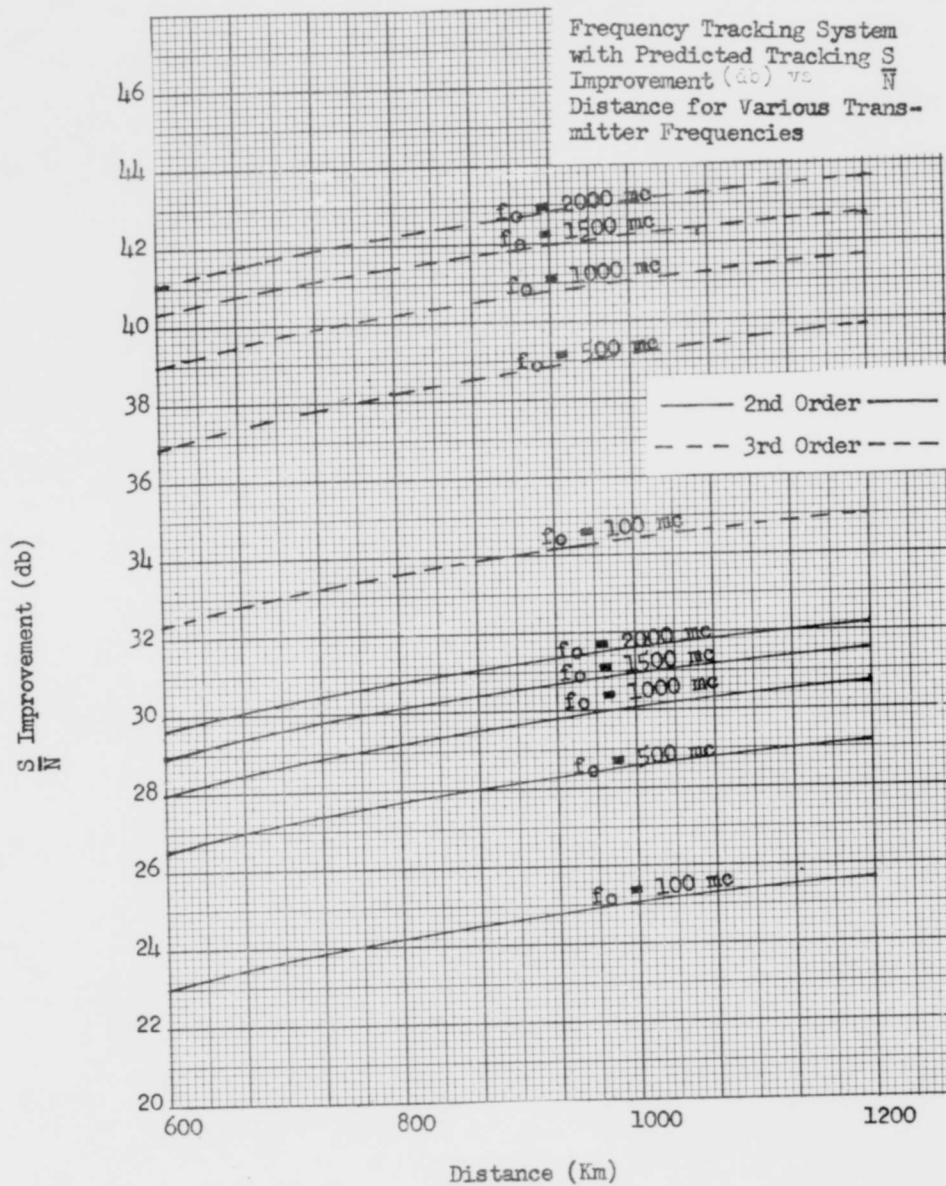


Figure 11

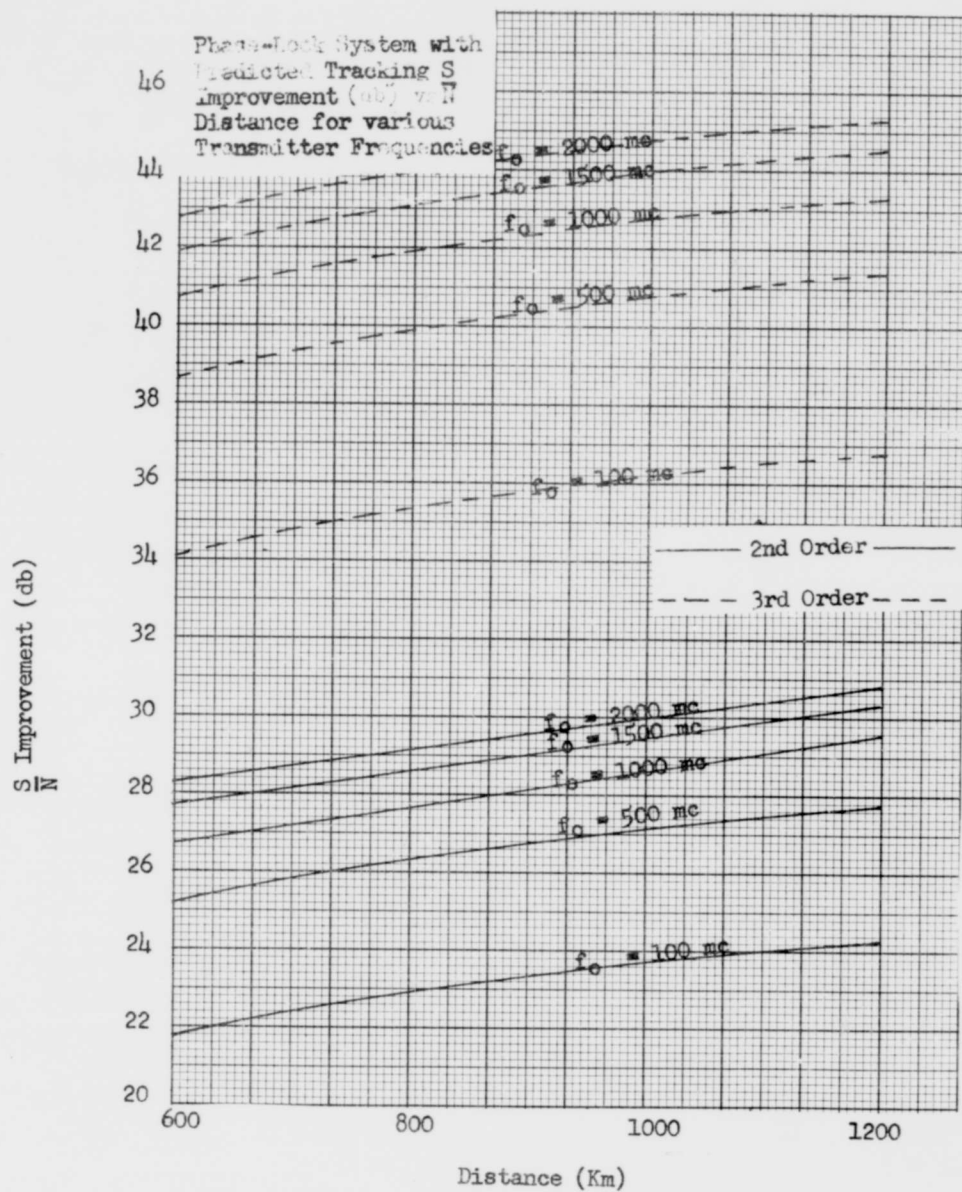


Figure 15

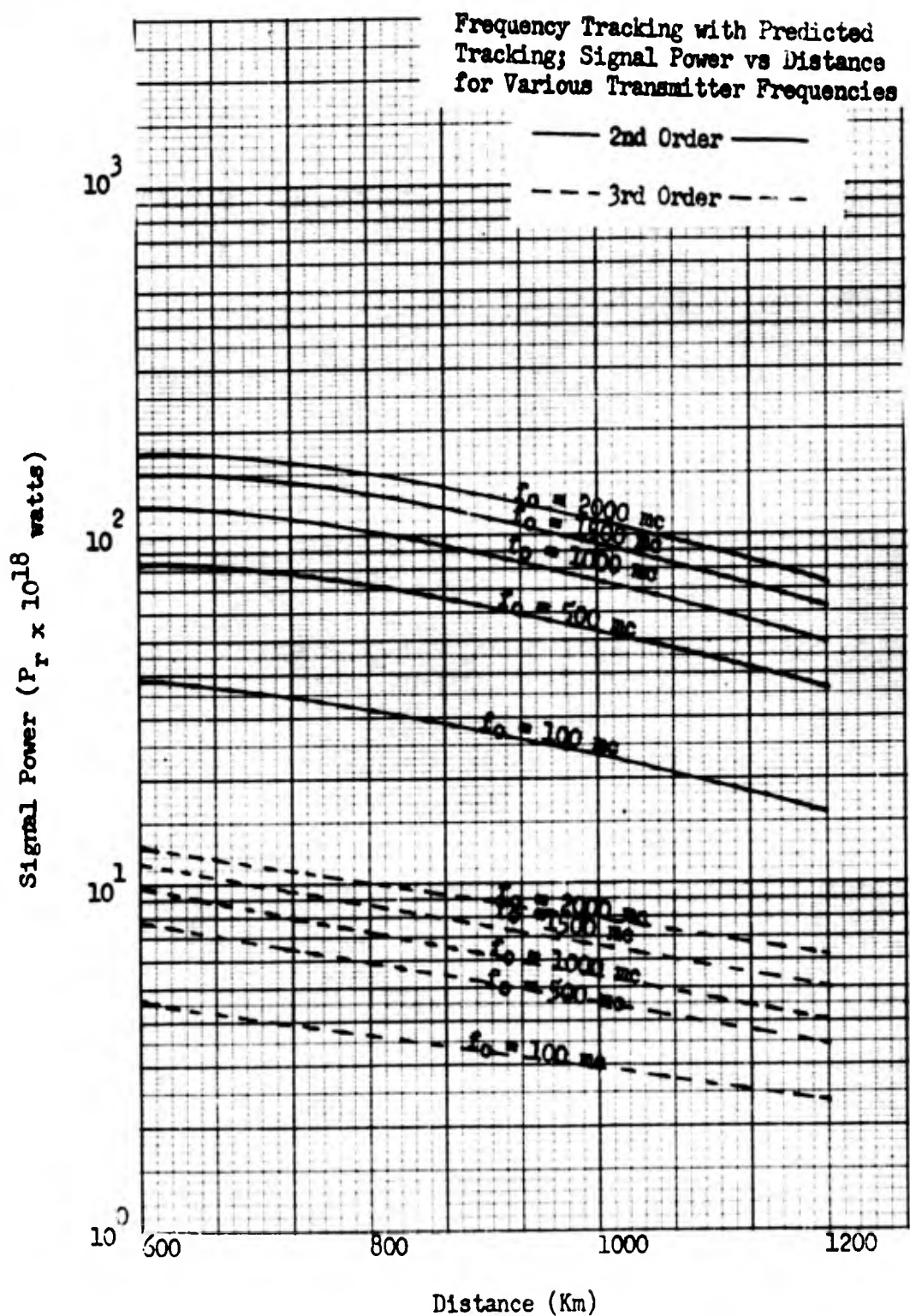


Figure 16

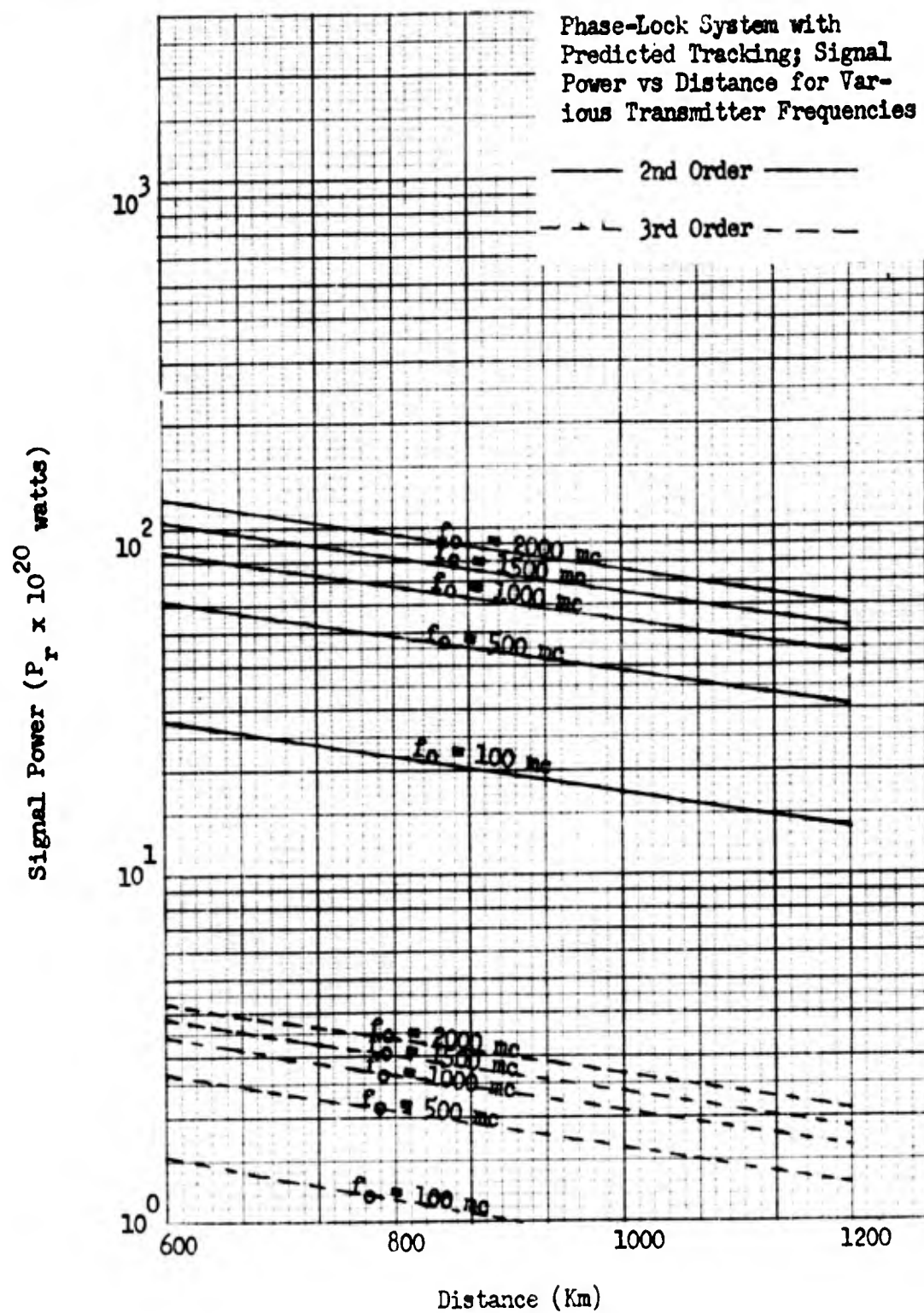


Figure 17

## VI. Comparison and Summary

It has been shown that by using the techniques of frequency tracking and the phase-lock loop that the detection bandwidth for a receiver processing a varying frequency can be considerably reduced. As a result of this reduction the output signal to noise ratio is greater than that of a conventional receiver. The conventional receiver must process the entire range of possible frequencies whereas the special techniques can select only the bandwidth containing the intelligence and a major portion of the noise power is eliminated from the system. The end result of the bandwidth reduction is that a weaker signal can be detected because the sensitivity of the receiver is improved.

In the previous three sections the three techniques have been compared to a conventional receiver, processing the entire Doppler shift bandwidth. In this section the techniques are compared to each other. Only the phase-lock system using the  $30^\circ$  phase error will be considered in this comparison as it has been shown to be more sensitive than the  $50^\circ$  phase error system.

Figure 18 shows the power required for the second order system of the four possible combinations of the techniques considered. In figures 18 and 19 the values of the signal power required for three transmitter frequencies 100, 1000, and 2000 mcs are plotted. The values for 500 and 1500 mcs are omitted to avoid excess cluttering of the plot but they would fall between the immediate higher and lower values of the transmitted frequency. The phase-lock technique requires less input signal

power than the frequency tracking system by an order of two magnitudes. The use of predicted tracking reduces the required power of each system by an order of magnitude. Figure 19 shows the input signal power required for the third order system of the four possible combinations. The phase-lock system requires less power than the frequency tracking system by an order of three magnitudes. The use of predicted tracking reduces the power required by each system by two orders of magnitude. It is also to be noted that the third order system is generally one order of magnitude better than the second order system in all comparable systems.

The detection bandwidths for the frequency tracking system and the phase-lock loop system are very close in magnitude. By appropriate choice of system parameters the bandwidths for a given order could be made to be the same. It must be concluded that detection bandwidth is not the critical criterion in determining system sensitivity. The determining factor is system threshold and the low threshold of the phase-lock loop system makes it more sensitive than the frequency tracking system. The predicted tracking technique improves both system in equal magnitude and thus does not alter the basic conclusion.

One conclusion that cannot be drawn from this study is the optimum frequency for the satellite transmitter. Of the frequencies considered in the study it would appear that 100 mcs would be the most desirable. However there are many factors involved in this choice. Among the factors are the effect on antenna gain, the effect on noise temperature,

atmospheric and ionospheric attenuation and system fabrication considerations. The determination of the optimum frequency is certainly a study by itself.

The techniques considered in this study were developed for use in satellite tracking. However they do not exploit their full potentialities in this limited area. The techniques are of even greater value when used in interplanetary tracking. As the velocity of the vehicle increases to reach the necessary escape and trajectory velocities the Doppler shift will become very large. It is under these circumstances that the sensitive phase-lock loop and the use of predicted tracking will make the best utilization of the available power. This study has been carried out from the viewpoint of the effects of the technique on receiver sensitivity. Very little adjustment would be required to determine the effect on the useful range for a given transmitted power. It is obvious that the techniques considered extend the useful range considerably.

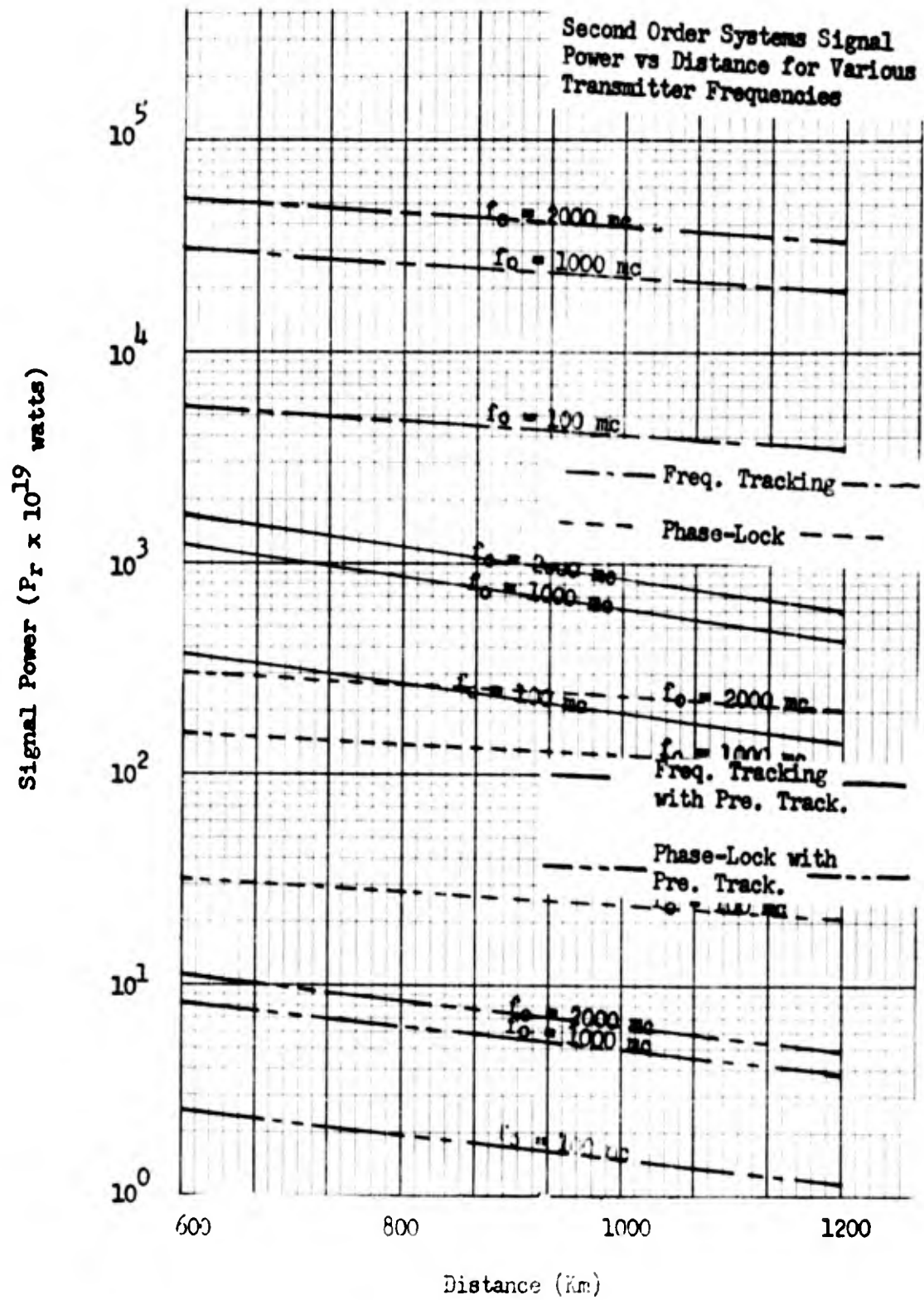


Figure 18

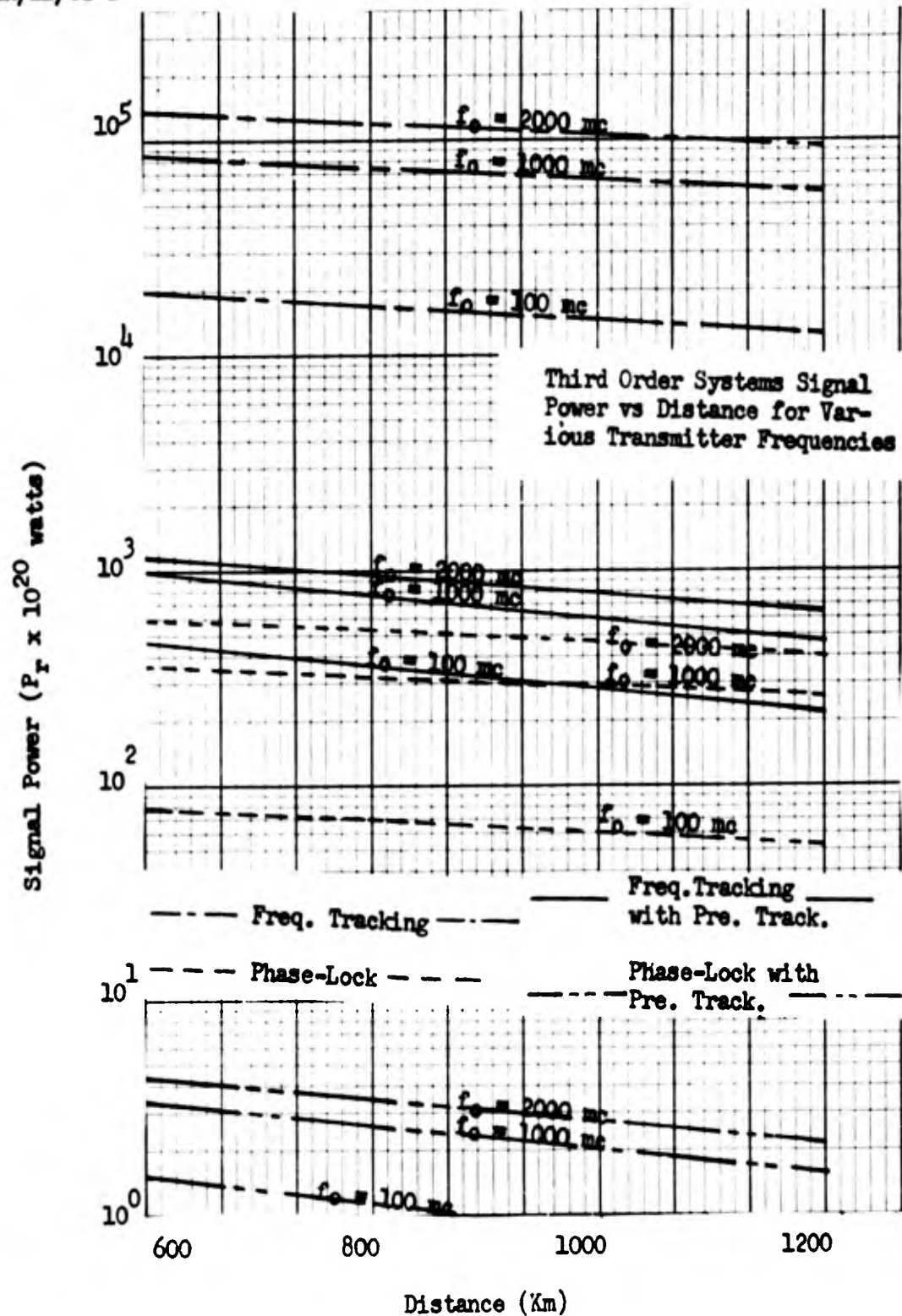


Figure 19

Bibliography

1. Anderson, Roy E. et al "Satellite Tracking by Radio" ASTIA AD 263168 (August 1961)
2. Ashbrook, F.M. and Stevenson, D.D. "The Navy's Portable Satellite Tracking Stations" IRE-Transactions on Space Electronics and Telemetry v SET-6, n. 1:41-45 (March 1960)
3. Golay, Marcel J.E. "The Application of Radio Interferometry to Extraterrestrial Metrology" IRE-Transactions on Space Electronics and Telemetry v SET-5 n. 4:186-193 (December 1959)
4. Jaffee, R. and Rechtin E. "Design and Performance of Phase-Lock Circuits Capable of Near-Optimum Performance Over a Wide Range of Input Signals and Noise Levels" IRE-Transactions on Information Theory v IT-1 n. 1:66-76 (March 1955)
5. Kamm, J.M. Jr The Effect of Phase-Lock Circuitry on Range of Communication Systems (unpublished) Dayton, Ohio :Air Force Institute of Technology (August 1961)
6. Pickard and Burns Inc. "Scientific Report #1 on Radio Tracking of Earth Satellites" ASTIA AD 160846 (June 1958)
7. Rideout, Vincent C. Active Networks New York: Prentice Hall Inc., 1954
8. Weaver, C.S. "Thresholds and Tracking Ranges in Phase-Locked Loops" IRE-Transactions on Space Electronics and Telemetry v SET-7 n. 3:60-70 (September 1961)

## Appendix A

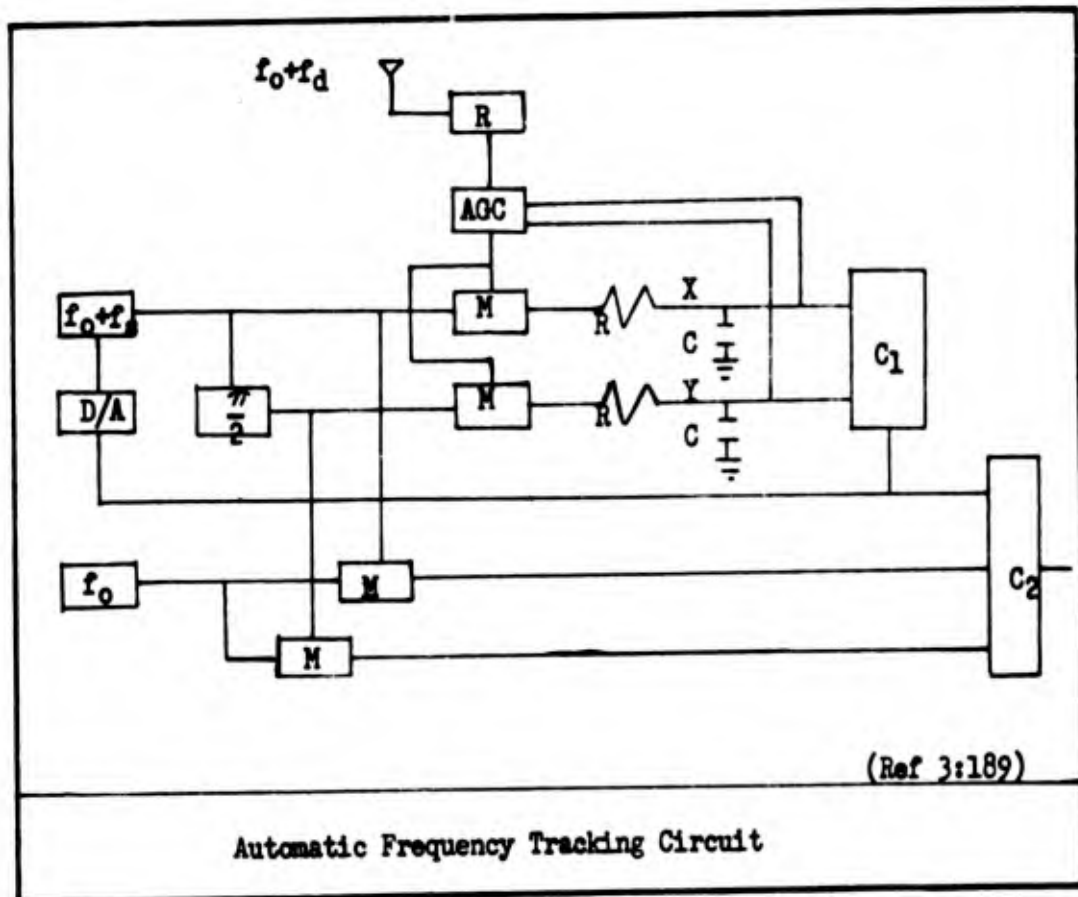
Automatic Frequency Tracking Circuit

Figure 20

The incoming signal,  $f_o + f_d$ , is mixed with two quadrature CW signals of the servoed frequency  $f_o + f_s$ . The mixer outputs,  $x$  and  $y$ , are filtered in two separate RC filters characterized by the same constant,  $2RC$ . The filter outputs,  $x$  and  $y$ , are the two components of a voltage vector and the angular coordinate of the voltage vector is the phase difference between the incoming and servoed signals.

The x and y signals constitute the input of an n-stage bidirectional counter,  $C_1$ , without short time internal memory, which registers every quarter turn of the xy vector. The output of the  $C_1$  counter is connected to a digital-to-analog (D/A) converter which, starting with the count zero produces uniform step-wise voltage increases for every  $\pi/2$  change of relative phase of  $f_d$  and  $f_s$ . The counter  $C_2$  is designed to count the total number of  $1/4$  cycles of the Doppler shift  $f_d$ . It does this by adding to the  $1/4$  cycles of  $f_s$  the  $1/4$  cycles of  $f_d - f_s$  obtained from the xy vector and counted by  $C_1$ . Thus the count registered by  $C_2$  is the number of  $\lambda/8$ 's traveled by the vehicle. (Ref. 3:188-189)

Appendix BCalculation of Bandwidth for Phase-Lock Technique

Kamm has shown that the maximum phase error for a second order system is given by the equation:

$$E(t)_{\max} = \frac{\Delta\alpha}{(B_2)^2} (1 + \sqrt{2} e^{-\frac{\pi}{B_2}} \sin \frac{\pi}{B_2}) \quad (\text{Ref 5:15})$$

where

$$\Delta\alpha = \dot{f}_d = \frac{a}{\lambda}$$

$$B_2 = \text{Bandwidth Parameter} = 2B \text{ (Detection Bandwidth)}$$

When  $E(t)_{\max} = 30^\circ = 0.0833$  cycles the equation becomes:

$$E(t)_{\max} = \frac{a}{\lambda (B_2)^2} (1.04) = 0.0833$$

solving for  $B_2$  gives:

$$B_2^2 = \frac{1.04}{0.0833} \frac{a}{\lambda} = 12.5 \frac{a}{\lambda}$$

$$B_2 = 3.53 \left( \frac{a}{\lambda} \right)^{\frac{1}{2}}$$

$$B = 1.765 \left( \frac{a}{\lambda} \right)^{\frac{1}{2}}$$

If  $E(t)_{\max} = 50^\circ = 0.139$  cycles then:

$$B_2 = 2.735 \left( \frac{a}{\lambda} \right)^{\frac{1}{2}}$$

$$B = 1.367 \left( \frac{a}{\lambda} \right)^{\frac{1}{2}}$$

The maximum phase error for a third order system is given by the equation:

$$E(t)_{\max} = 1.081 \frac{\Delta\beta}{(B_3)^3} \quad (\text{Ref 5:23})$$

where

$$B_3 = \text{Bandwidth Parameter} = 2B$$

$$\Delta\beta = \dot{f}_d = \frac{J}{\lambda}$$

for

$$E(t)_{\max} = 30^\circ = 0.0833 \text{ cycles}$$

$$B_3^3 = 13.0 \left( \frac{J}{\lambda} \right)$$

$$B_3 = 2.35 \left( \frac{J}{\lambda} \right)^{\frac{1}{3}}$$

$$B = 1.175 \left( \frac{J}{\lambda} \right)^{\frac{1}{3}}$$

For

$$E(t)_{\max} = 50^\circ = 0.139 \text{ cycles}$$

$$B_3 = 1.937 \left( \frac{J}{\lambda} \right)^{\frac{1}{3}}$$

$$B = 0.968 \left( \frac{J}{\lambda} \right)^{\frac{1}{3}}$$

Vita

Emil George Riedel Jr. was born on [REDACTED]  
[REDACTED], the son of Mr. and Mrs. Emil George Riedel. After  
completing his work in 1950 at [REDACTED],  
he enrolled at Ohio University. He entered the United States Naval  
Academy in 1951 from which he was graduated in 1955 with the degree  
of Bachelor of Science. His military assignment prior to coming to  
the Institute of Technology was as navigator in Air Refueling Operations  
of the Strategic Air Command.

Permanent address: [REDACTED]

This thesis was typed by Mrs. E.G. Riedel Jr.

UNCLASSIFIED

UNCLASSIFIED