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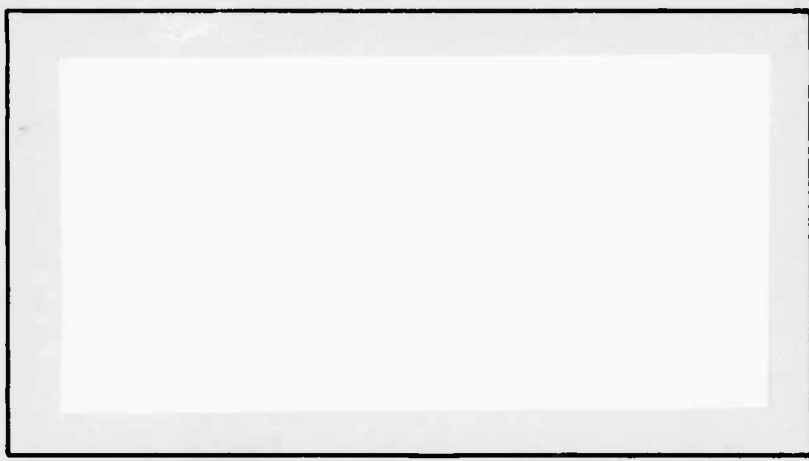
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**An Investigation of the Application
of the Mossbauer Effect to
Accelerometers Used for Inertial Navigation**

Lt. Jack E. Hesse, Jr.

GA/Phys/62-7

AN INVESTIGATION OF THE APPLICATION
OF THE MOSSBAUER EFFECT
TO ACCELEROMETERS USED FOR INERTIAL NAVIGATION

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

By

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Preface

The study involved in this thesis has been an attempt to find out whether there was some possible application of the Mossbauer Effect to accelerometers used in inertial navigation systems. After one year of research in this area I can be neither comprehensive nor conclusive in the presentation of this problem. In writing the thesis it has been my hope that the material presented will serve the following purposes:

First, to provide the applications engineer with a succinct, but not oversimplified, introduction to the Mossbauer Effect.

Second, to outline clearly some of the major limitations in the application of the Mossbauer Effect and the fundamental reasons for these limitations.

Third, to bring into focus by means of an order of magnitude analysis some of the more promising areas of investigation which have come to my attention.

At one point in the study I considered introducing classified performance figures from the more recent advanced techniques in inertial navigation. This would have provided a more precise basis for comparing the suggested techniques to the present state of the art. However, in view of the generality of my presentation, I concluded that the inclusion of the classified information would not add

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so much to the Thesis as to justify the concomitant administrative burden.

The area of investigation was suggested by Mr. Paul Polishuk of the Flight Control Laboratory, Aeronautical Systems Division. Mr. Polishuk was most helpful in providing reference material and in periodically reviewing the study. Dr. Bernard Kaplan acted as my thesis adviser, providing periodic counseling and review of my efforts. Finally, I am grateful to my wife, ■■■, for typing my notes as well as assembling and typing the final manuscript.

Jack E. Hesse, Jr.

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List of Symbols

Symbol

A	acceleration
A	gram atomic weight (Page 19 only)
C	velocity of an electromagnetic wave (Chapter II only)
C	coefficient of viscous friction (Chapter V only)
C_1	specific heat
E	energy, in general
E_γ	energy of a gamma photon
E_t	energy involved in a nuclear transition
F	$I(x)/I_0$
\vec{H}	Angular momentum (vector)
I	moment of inertia
I_0	photon flux incident upon an absorber
$I(x)$	photon flux as a function of absorber thickness
K	elastic constant, force/displacement
L	length, as defined
M	mass, as defined
N	number, as defined
N_0	number of absorber atoms per cubic centimeter
P	momentum of a nucleus
Q	E_γ/Γ , a resonance parameter
R	radius, as defined

Symbol

R	resolution (Page 46)
R	recoil energy (Chapter II, Appendix B)
S	shift in energy
\bar{S}	displacement (vector)
T	ambient temperature
T _{applied}	
	torque applied
T _d	driving torque
T _m	mean lifetime of the nuclear excited state
T _{1/2}	half-life of a nuclide
V _a	amplitude of the velocity wave of an absorber
V _e	amplitude of the velocity wave of an emitter
X	angular displacement, as defined
a	isotopic abundance
f	fraction of recoilless events
g	gravity (units of acceleration or specific force)
g _s	gravity at the earth's surface
h	Planck's constant
\hbar	$h/2\pi$
k	Boltzmann's constant
l	length, as defined
n	number, as defined
p	momentum of a photon

Symbol

q	S/Γ
r	radius, distance, as defined
t	time
v	velocity, as defined
v_a	velocity of an absorber
v_e	velocity of an emitter
v_r	relative velocity between an absorber and an emitter
w	angular velocity of an absorber
x	some distance or length, as defined
α	an angle, as defined
α	internal conversion coefficient (Chapter II, Appendix B)
β	an angle, as defined
Δ	increment of - - - -
Γ	uncertainty in energy, linewidth
λ	photon wavelength
ν	frequency of an electromagnetic wave
σ	cross section for resonant absorption
σ_n	standard deviation
σ_o	electromagnetic cross section
θ	angular displacement, as defined
θ_d	Debye temperature
θ_o	angular interval, as defined

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Symbol

- ϕ phase angle, as defined
- ω angular velocity, as defined
- ω_a a constant angular velocity

Abstract

An analytical investigation of the application of the Mossbauer Effect to accelerometers is presented. The Mossbauer Effect is the emission and absorption of gamma photons without energy losses due to the recoil of the emitting and absorbing nuclei. Resonant absorption so obtained is sensitive to changes in photon energy of one part in 10^{12} - 10^{15} . Fundamental limitations in the application of this phenomenon are the need for statistical counting of photons and the uncertainty in the energy of the nuclear gamma decay. An order of magnitude analysis indicates that the isotope Fe^{57} , which has favorable characteristics such as a relatively high cross section for resonant absorption and a relatively high ratio of photon energy to uncertainty in photon energy (3×10^{12}), is the best available isotope. The Mossbauer Effect is most sensitive to a shift in energy due to relative velocity between the source and absorber, allowing sensing of velocities of 10^{-2} - 10^{-5} cm/sec. Techniques are presented and analyzed that use the latter phenomenon to measure angular and linear velocity and angular position of instrument components. A typical estimate for one second of counting shows that angular velocity may be measured to within 10^{-6} radians/sec and angular position to within 2 seconds of arc. An application of these techniques to selected accelerometers does not indicate a promising increase in

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accuracy. Uncertainties in parameters not related to the Mossbauer Effect such as friction, hysteresis, mass unbalance, and angular measurement limit the possible accuracy to about one part in 10^5 .

A promising application is found in the Rotating Pendulum Accelerometer which offers the possibility of a low threshold sensitivity, 10^{-8} g or less.

AN INVESTIGATION OF THE APPLICATION
OF THE MOSSBAUER EFFECT
TO ACCELEROMETERS USED FOR INERTIAL NAVIGATION

I. Introduction

This thesis presents the results of an investigation to determine whether there are possible applications of the Mossbauer Effect to accelerometers used in inertial navigation systems. The study was motivated by the following factors:

First, the approaching era of space travel will place requirements of extreme accuracy on navigation systems. This is particularly true for inertial components such as accelerometers.

The Mossbauer Effect, although discovered in 1958, has already found wide application in scientific experiments requiring extreme sensitivity and precision. It is then natural to question whether the Mossbauer Effect can be used to advantage in other applications requiring unusual accuracy, such as the measurement of acceleration during space missions.

In this chapter the above two statements will be expanded in some detail, and the plan of the thesis will be outlined.

Requirements of Accelerometers Used for Inertial Navigation

The requirements for extreme accuracy in space navigation stem from the great distances and length of time involved in the

flights. A relatively small error expressed as a fraction of the total distance from, say Earth to Mars, will represent an error of many thousands of miles. There are several approaches to the problem of space navigation, including tracking and controlling the vehicle from an earth based command station, celestial navigation, and inertial navigation. Future space flights may employ any one of these methods, or more likely, a combination of all three. Tracking and controlling from an earth station saves weight in the assembly of navigation components carried by the vehicle; however, when one considers that it will take several minutes for an electronic signal to reach earth from a vehicle in the vicinity of one of the nearer planets, the disadvantages of this method become apparent. Using celestial navigation, a system will require several "fixes" taken over a considerable interval of distance and time in order to determine important flight parameters such as velocity. Thus it will be difficult to monitor and control the characteristically short bursts of powered flight by the use of celestial navigation. Therefore, it may be necessary, or at least desirable, to revert to an inertial system for some part of the total space navigation problem. The chief limitation in the use of inertial systems for space flight is the requirement for extreme accuracy.

An inertial system determines the position of a vehicle in space by twice integrating the accelerations incident upon the vehicle.

The measurement of these accelerations is accomplished by the use of accelerometers. The requirement for extreme accuracy is illustrated by the following:

Let

$$\bar{S} = \int \int \bar{A} dt \quad (1)$$

where \bar{S} is the position of the vehicle, \bar{A} is the acceleration of the vehicle, and dt is a differential element of time. If $\Delta\bar{A}$ is a constant error present in the measurement of acceleration, then $\Delta\bar{S}$ will be the resultant error in position, given by

$$\Delta\bar{S} = \Delta\bar{A} \frac{t^2}{2} \quad (2)$$

The significance of the t^2 term for space flights of long duration can be readily appreciated.

In addition to a requirement for extreme accuracy, it is desirable for purposes of space navigation to have accelerometers that measure very small values of acceleration. One reason for this requirement is the advent of small thrust propulsion devices such as ion engines and solar sails. For example, consider a device which produces 10^{-3} lbs of thrust on a 1000 lb vehicle. The resultant acceleration will be 10^{-6} times the value of acceleration due to gravity at the earth's surface. Another source of motivation for developing sensitive accelerometers is the possible use of gravity gradient techniques for space navigation. These methods ensue from the fact that

even in "free fall" only the center of mass of a vehicle is subject to an exact weightless condition. Positions of the vehicle at a greater or smaller distance from the attracting body than the distance from that body to the center of mass of the vehicle are subject to small values of specific force. These small specific forces are due to the gradient of gravity, which acts in accordance with the inverse square law. A description of some possible gravity gradient techniques is contained in Appendix A.

Applications of the Mossbauer Effect

A complete description of the Mossbauer Effect will be given in Chapter II. In this section it will suffice to say that the Mossbauer Effect is a form of nuclear resonance fluorescence of gamma rays. It is distinguished from other forms of nuclear resonance fluorescence in that gamma rays are emitted and absorbed without recoil of the individual emitting and absorbing nuclei.

Rudolf L. Mossbauer, a young German Physicist, discovered the effect named after him in 1958 (Ref 16). His discovery rapidly proved to be an invaluable tool for scientific research. Using the Mossbauer Effect, R. V. Pound and G. A. Rebka, Jr. were able to measure the effect of gravity on the energy of gamma photons "falling" through laboratory distances. This is a direct confirmation of Einstein's predicted red shift of spectral lines in a gravitational field (Ref 19, 20). The Mossbauer Effect has been used to study lattice

vibrations (Ref 30), nuclear magnetic effects (Ref 4, 6), and lifetimes of excited nuclei (Ref 15:20). As will be demonstrated in Chapter II, by using the Mossbauer Effect one can detect relative velocities of the orders 10^{-2} to 10^{-5} cm/sec. For comparison, the end of the minute hand on a watch moves at about 2×10^{-3} cm/sec.

In view of the above, one would not be surprised to see the Mossbauer Effect go into space along with the MASER* and the LASER, which, like the Mossbauer Effect, exhibit sharply resonant phenomena. However, in contrast to the LASER and the MASER, little or no work has been done to realize an engineering application from the Mossbauer Effect. Therefore, this study must start with the theory and scientific applications of the Mossbauer Effect and proceed in a very general and tortuous way to seek an engineering application in the field of accelerometers used for inertial navigation.

Thesis Plan

Chapter II provides a review of the theory and major experiments involved in the Mossbauer Effect. The most important isotopes are discussed, and relationships showing the effect of different physical phenomena on the Mossbauer Effect are developed. In Chapter III the thesis problem is initiated by outlining and explaining general limitations

* MASER or LASER, respectively Microwave or Light Amplification by Stimulated Emission of Radiation.

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applicable to any experiment performed with the Mossbauer Effect.

Chapter IV contains a description and an order of magnitude analysis of several hypothetical schemes for using the Mossbauer Effect to measure angular position and angular or linear velocity of an instrument component. In Chapter V the various types of accelerometers are classified, and selected accelerometers are analyzed with respect to an application of the Mossbauer Effect. Chapter VI contains the conclusions and recommendations derived from the study.

II. A Brief Review of the Mossbauer Effect

As stated previously, the Mossbauer Effect is a form of nuclear resonance fluorescence of gamma rays. Nuclear resonance fluorescence implies the excitation of a particular energy transition in one nucleus by a source of energy resulting from an identical energy transition in another nucleus. In the Mossbauer Effect, nuclear resonance fluorescence is accomplished by emission, absorption, and reemission of gamma rays. Further, these gamma rays are emitted and absorbed without recoil of the emitting and absorbing nuclei.

This chapter contains the explanation and significance of the Mossbauer Effect. The basic theory is presented; then a brief review of the major experiments involved in the development of the Mossbauer Effect. Two of the most significant Mossbauer isotopes are discussed. Finally, relationships showing the effect of different physical phenomena on the Mossbauer Effect are developed.

Theory

The theory of the Mossbauer Effect consists of an understanding of first, why energy losses due to recoil of a nucleus occur during the emission or absorption of a gamma ray, and second, how these recoil losses can be eliminated.

Consider a free nucleus, at rest. If the nucleus undergoes an energy transition, E_t , and thereby emits a gamma photon, the

the law of conservation of momentum demands that part of the energy of transition be given up to the recoil of the nucleus. This energy lost to recoil is denoted R , given by

$$R = \frac{P^2}{2M} \quad (3)$$

where P is the momentum, and M is the mass of the nucleus. By momentum conservation,

$$P = p = \frac{E_\gamma}{C} \quad (4)$$

where p is the photon momentum, E_γ is the photon energy, and C is photon velocity. By substituting (4) into (3), the following results:

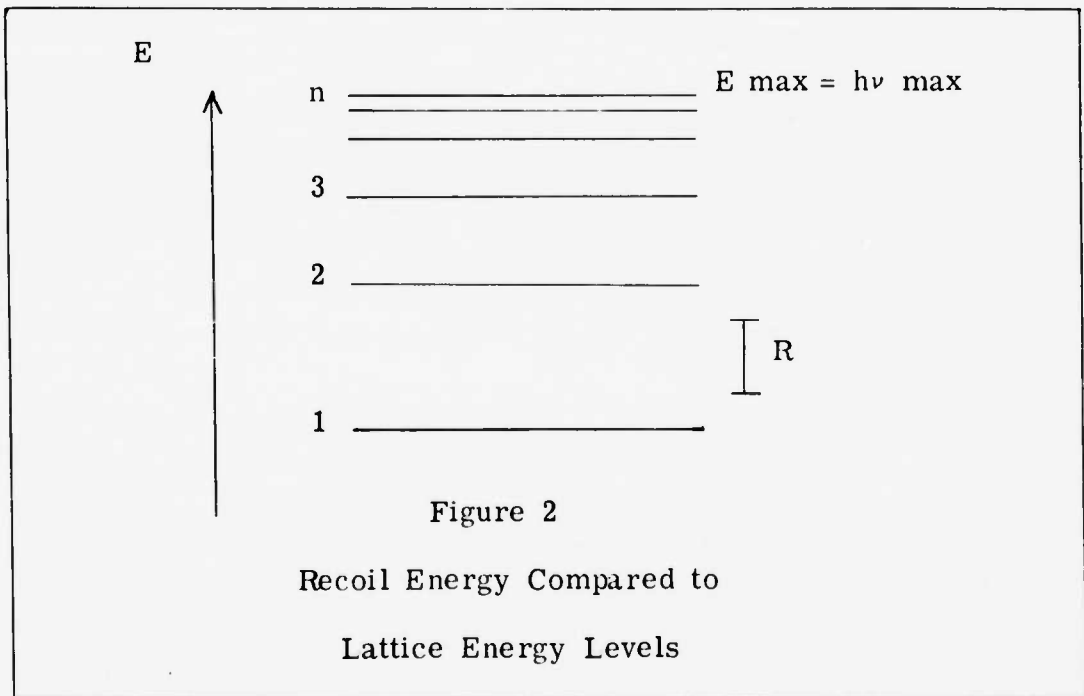
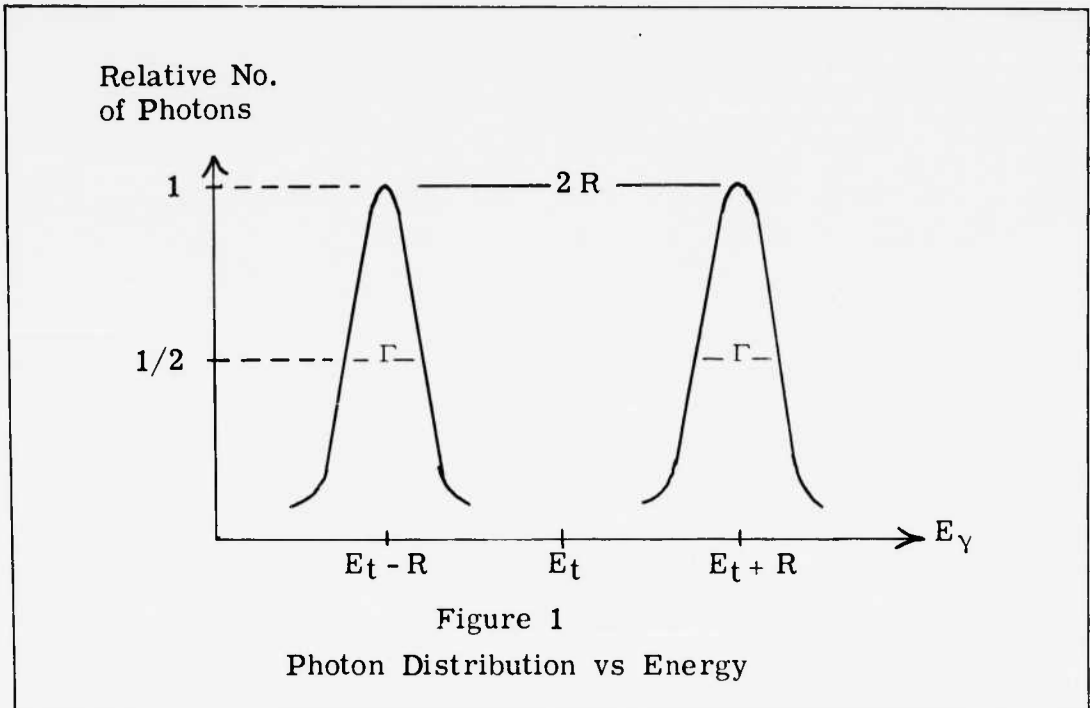
$$R = \frac{E_\gamma^2}{2MC^2} \quad (5)$$

Since energy is conserved, $E_\gamma = E_t - R$. A similar argument can be made to show that for absorption to take place, the energy required of the photon is given by $E_\gamma = E_t + R$.

The value of E_t is not an exact quantity, but is indeterminate in the amount Γ , given by the Heisenberg Uncertainty Principle

$$\Gamma = \frac{\hbar}{T_m} \quad (6)$$

where \hbar is Planck's constant divided by 2π , and T_m is the mean lifetime of the excited state of the nucleus (Ref 29:113). For nuclear transitions, $R \gg \Gamma$, therefore the energy separation between the emitted photon and the energy required for resonant absorption is approximately $2R$, as illustrated in Figure 1. If this were the only



consideration, then the energy difference $2R$ could be supplied by some artificial means such as by moving the source, thereby imparting a Doppler shift in the energy of the emitted photons. However, in nature the nuclei involved are not at rest as implied by the previous model, but are moving with respect to one another. In a gas the atoms containing the nuclei are in random thermal motion and in a solid the atoms vibrate about points in a crystal lattice. These motions induce a broadening in the energy distribution of the emitted photons, rendering artificial compensation for recoil losses a difficult task.

The essential feature of Mossbauer's discovery was that, under certain conditions, atoms in a solid emit or absorb gamma photons without recoil of the individual nuclei. It is possible to appreciate how this happens from fundamental results of quantum mechanics. The atoms in a solid oscillate with discrete energies, given by $E = h \nu$ (Ref 10:122). E is the energy of the oscillators and ν is the frequency of oscillation. The allowed energies are discrete, therefore only certain specific transitions are permitted. Suppose, then that all the atoms in a crystal are in state E_1 and the next allowed state is E_2 . Suppose further that the transition energy, $E_2 - E_1$, is twice the value of R , the recoil energy from a gamma photon emission. If a number of atoms, N , emit photons, and $N/2$ atoms undergo the transition from state 1 to state 2, then

the total energy allotted from recoil of all N atoms will be accounted for, since $E_2 - E_1 = 2R$. This will leave $N/2$ atoms to emit photons without recoil. Momentum is conserved because eventually, the recoil from a single event is absorbed by the entire crystal.

Application of the preceding to a more realistic model of a crystal makes the mathematics considerably more complicated, but the physical principles are the same. The emission of a photon without recoil is referred to as a zero phonon process. A phonon is the quantum of energy (or frequency) involved in the transition of an atom in a lattice from one state to another state. The fraction, f , of zero phonon processes that occur is approximated by

$$f = \exp \left[- \frac{3}{2} \frac{R}{k \theta_d} \left(1 + \frac{2\pi^2 T^2}{3\theta_d^2} \right) \right] \quad (7)$$

where k is Boltzman's constant, T is the ambient temperature of the crystal, and θ_d is the Debye Temperature. This is a good approximation where $T \ll \theta_d$ (Ref 20:440).

The parameter θ_d is defined by

$$\theta_d = \frac{h\nu_{\max}}{k} \quad (8)$$

where ν_{\max} is the maximum frequency at which any atom oscillates in the crystal lattice (Ref 5:39-45). With this definition in mind, one can appreciate the following requirements for obtaining a detectable Mossbauer Effect. A high θ_d is necessary in order that there

be lattice transitions whose energy is larger than the free recoil energy of the nucleus. The requirement, as seen from (7), is that $R \ll k \theta_d$. Another requirement is that $T \ll \theta_d$. Intuitively, a low T will mean that the atoms in the crystal are in the lower energy states where the transition energies are larger. These requirements are diagrammed in Figure 2.

If, in fact, the above requirements are met, then a significant fraction of the gamma photons can be emitted or absorbed by zero phonon processes. The cross section, σ , for resonant absorption is given by

$$\sigma = \frac{\sigma_0}{2} f f' \quad (9)$$

where f is the fraction of zero phonon occurrences in the emitting lattice, f' is the fraction of such occurrences in the absorber lattice, and σ_0 is a basic cross section for electromagnetic absorption, given by

$$\sigma_0 = \frac{\lambda^2}{2\pi} \frac{2I_e + 1}{2I_g + 1} \frac{1}{1 + \alpha} \quad (10)$$

λ is the gamma photon wavelength, I_e is the nuclear spin in the excited state, I_g is the nuclear spin in the ground state, and α is the internal conversion coefficient (Ref 30:208, 4:222, 8:7). The above expressions assume an isotopic abundance of 100% of appropriate isotopes in the absorber lattice. If, for example, only 2% of

the atoms in the absorber lattice were the proper isotope for Mossbauer absorption, then the actual cross section per atom of the entire absorber would be only 2% of the value given by (9).

If the Mossbauer σ is plotted against energy as shown in Figure 3, a Breit-Wigner curve of width 2Γ results. The width is just twice the value given by the Heisenberg Uncertainty Principle for the nuclear transition energy and results from an overlap of the two curves shown in Figure 1 (Ref 14:6). It is precisely the narrow width of this curve that makes the Mossbauer Effect a delicate and sensitive tool for scientific experiments. Numerical examples will be discussed shortly, but first a short resume of the important experimental milestones in the history of the Mossbauer Effect is given.

Experiments

Prior to Mossbauer's discovery attempts were made (with some success) to observe nuclear resonance fluorescence by compensating for the recoil losses in various ways. Mossbauer himself was engaged in just such an attempt when he discovered the phenomena of recoilless emission and absorption. He was observing resonance fluorescence of the 129 - kev gamma ray from Ir^{191} . For this transition, R is 0.05 ev, and the broadening of the transition energy due to thermal motion at room temperature is about 0.1 ev. At room temperature the emission and absorption spectrum overlap

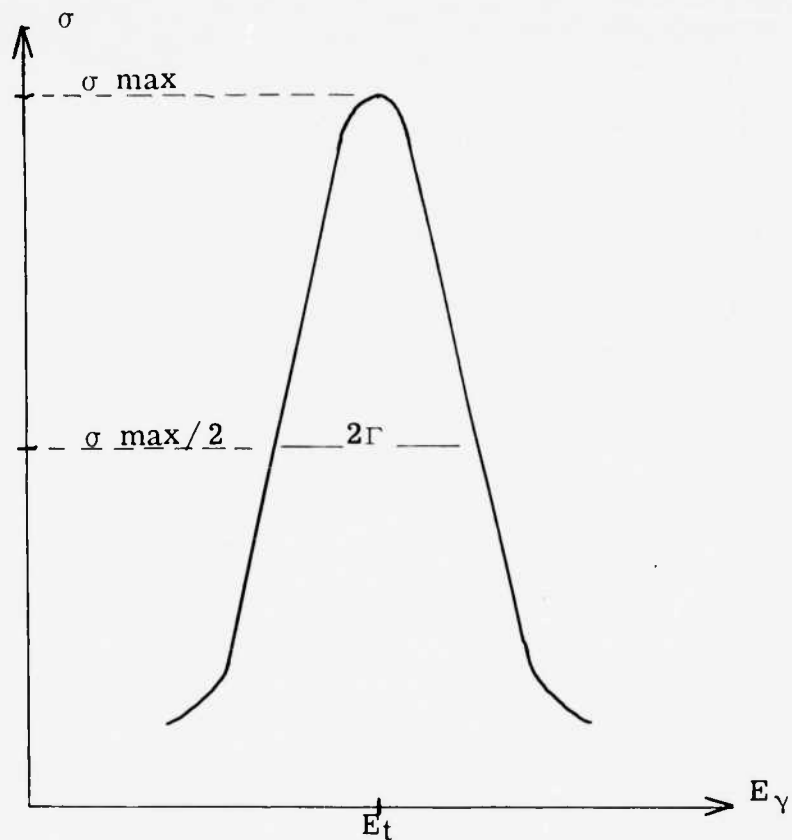


Figure 3

Mossbauer Cross Section vs Photon Energy

considerably, therefore resonance fluorescence can be observed. In order to reduce this scattering by reducing the overlap, Mossbauer cooled both source and absorber. He expected that the decrease in Doppler broadening of the transition energy would lead to a decrease in the resonance fluorescence. Instead, the very opposite happened! (Ref 8:11-12)

Mossbauer determined the nature of his result and recognized its significance, that by eliminating recoil losses he had removed the major cause of line broadening. He explained his observations by adapting a theory by W.E. Lamb, Jr. Lamb's theory was developed to explain the effect of lattice binding on the capture of slow neutrons (Ref 11:190). Mossbauer's experiments were repeated by other laboratories, particularly at Los Alamos and Argonne. All of these experiments were performed with Ir^{191} and were complicated by low-temperature requirements and by the low fraction of zero phonon processes that occur in Ir^{191} (about 5% at 0°K). (Ref 8:12).

The Mossbauer Effect assumed major importance for practical applications when the phenomena was observed in Fe^{57} . This discovery was made independently at Harvard, Harwell, the University of Illinois, and Argonne. Because of its high θ_d (about 450°K), Fe^{57} exhibits a large fraction of zero phonon processes at room temperature (about 70%). In addition, the linewidth, Γ ,

involved in the Fe^{57} Mossbauer Effect is very narrow, allowing very precise experiments (Ref 8:12-13).

A typical Mossbauer experiment is shown schematically in Figure 4. Photons from a source are passed through an opening in shielding and allowed to strike the absorber, usually a foil only a few millimeters thick. In the case of an absorption experiment the photons that pass through the foil are counted by a detector placed directly behind the foil. If conditions such as temperature and relative motion are properly controlled, Mossbauer absorption will take place and the counting rate will be a minimum. If conditions are changed such as to reduce the probability of Mossbauer absorption, the counting rate will increase. It is true that the absorbed photons will be reradiated, but the reradiation will take place isotropically. By proper arrangement of the geometry of the experiment this effect can be minimized. On the other hand, if the detector is placed as shown in Figure 4 for a scattering experiment, only the reradiated photons will be counted.

In order to fully realize the significance of the quantities involved in such an experiment, the properties of some important Mossbauer isotopes will be discussed.

Isotopes

A list of isotopes for which the Mossbauer Effect has been observed is given in Appendix B. Two of these isotopes will be

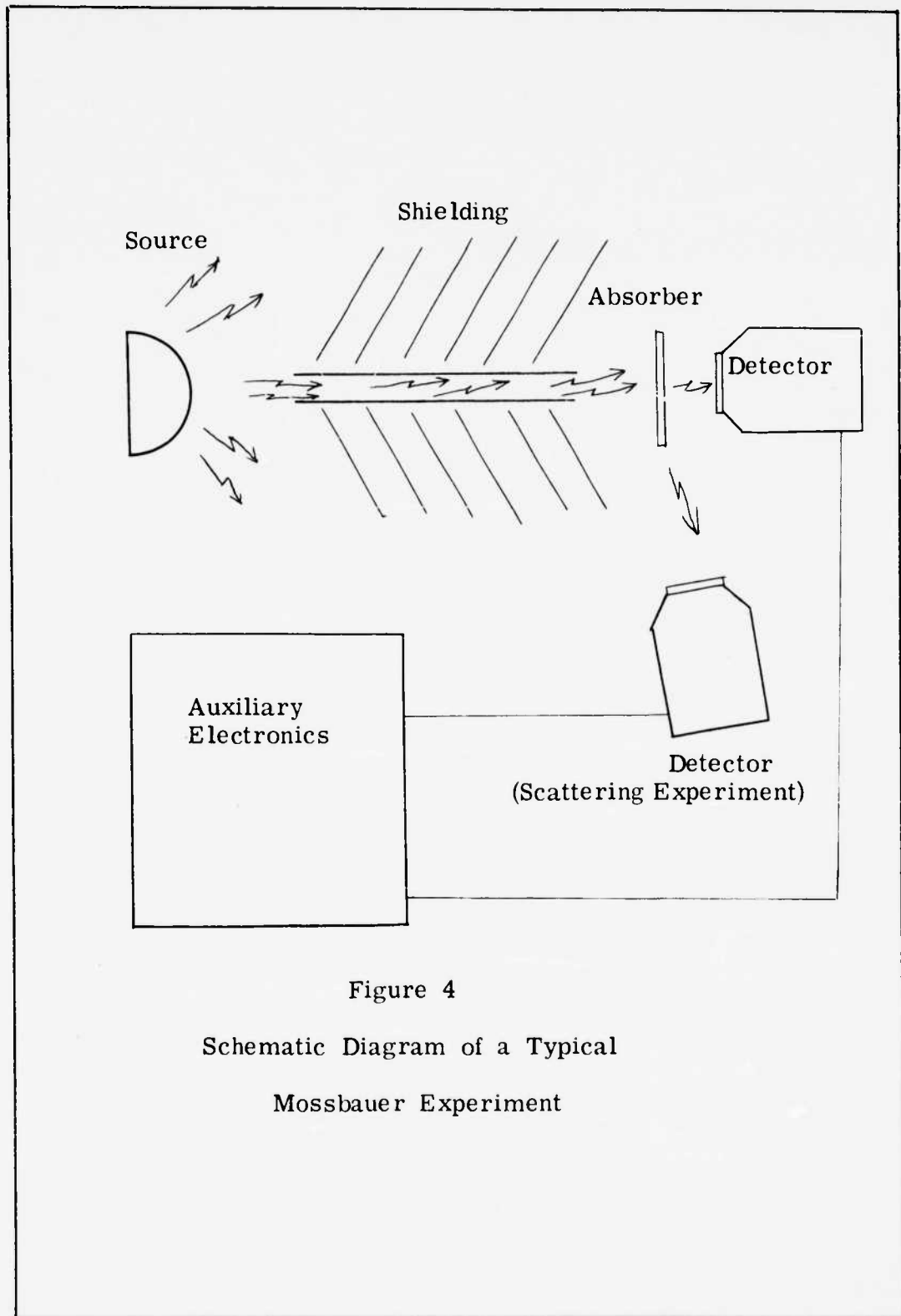


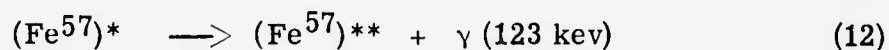
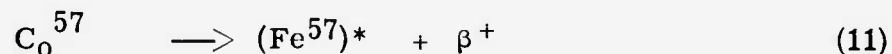
Figure 4

Schematic Diagram of a Typical

Mossbauer Experiment

discussed, Fe^{57} which has the best combination of desirable Mossbauer properties, and Zn^{67} which exhibits the smallest linewidth, Γ .

The following decay scheme is of interest for the Fe^{57} Mossbauer Effect.



$(\text{Fe}^{57})^*$ decays by (12) and (13) in 91% of the instances. The other 9% of the events result in a direct conversion to ground state by emission of a 137 kev gamma ray. The phenomenon of interest is (13), which happens without recoil in a very large fraction of the cases.

In addition to a high θ_d and relatively low value of R , Fe^{57} has the desirable property that its parent nucleus, C_0^{57} , decays with a relatively long half life of 270 days. Most important, the mean lifetime of the excited state in (13) is comparatively long for a Mossbauer transition. $T_m = 1.44 \times 10^{-7}$ sec. By (6), $\Gamma = 4.6 \times 10^{-9}$ ev. With the exception of Zn^{67} , Fe^{57} exhibits a smaller linewidth than any other Mossbauer isotope. Pound and Rebka, among others, have experimentally observed this small linewidth. Reasonable care in the preparation of source and absorber will yield a linewidth within

* or ** Excited states of the nucleus.

twice the value predicted by (6). (Ref 21:555).

A parameter, Q , can be defined which is a useful measure of the degree of resonance in a Mossbauer experiment.

$$Q = \frac{E_\gamma}{\Gamma} \quad (14)$$

For Fe^{57} , $Q \doteq 3 \times 10^{12}$. The interpretation is that any phenomena which changes the energy of the gamma photon by an order of one part in 10^{12} will significantly alter the resonant absorption. This is seen from the plot on σ versus E in Figure 3.

A comparison of the properties of Zn^{67} with those of Fe^{57} , as listed in Appendix B, will reveal that Fe^{57} is generally superior except for the fact that Zn^{67} has a longer mean lifetime of the excited nucleus, resulting in a very small Γ . It is difficult to perform a Mossbauer experiment with Zn^{67} . The low θ_d of this isotope, combined with a large value of R , results in a small fraction of zero phonon occurrences, even at temperatures near 0°K . However, the high value of Q (about 2×10^{15}) that can be obtained promises scientific applications of the utmost sensitivity. A group led by P.P. Craig has performed one of the few successful series of experiments with Zn^{67} . This group observed a linewidth about twice the value of Γ predicted by (6). (Ref 4:561-564)

The usefulness of a small linewidth and high Q value is made apparent by the following discussion of the effects of physical phenomena on Mossbauer experiments.

Physical Phenomena

Some of the major physical phenomena which can be observed through the Mossbauer Effect are relative velocity, acceleration, temperature effects, chemical shifts, and magnetic effects.

Relative Velocity. The first mentioned of these phenomena acts through the Doppler shift, whereby the frequency of a photon emitted by a moving source is seen by a stationary observer to be

$$\nu = \frac{\nu_0 C}{C - v} \quad (15)$$

The source is moving toward the observer with velocity v . ν is the observed frequency, and ν_0 is the emitted frequency (Ref 26:442-444). By subtracting ν_0 from both sides of the expression, the following can be obtained:

$$\Delta\nu = \nu - \nu_0 = \frac{\nu_0 v}{C - v} \quad (16)$$

If $v \ll C$, then

$$\Delta\nu = \nu_0 \frac{v}{C} \quad (17)$$

Since $E = h\nu$, this can be considered a shift, S , in photon energy, given by

$$S = \frac{v}{C} E_\gamma \quad (18)$$

The effect of this energy shift on the Mossbauer cross section can be closely approximated in the case $R \ll 2k\theta_d$, $S \ll k\theta_d$, by

$$\sigma = \frac{1}{2} \sigma_0 f f' \left(\frac{\Gamma^2}{S^2 + \Gamma^2} \right) \quad (19)$$

(Ref 30:208). By examining this expression it can be seen that any energy shift which is of the order of a linewidth will significantly affect absorption. Consider, for example, the relative velocity required to shift the energy by the value Γ , one linewidth, in an experiment involving Fe^{57} . By (18),

$$v = \frac{\Gamma}{E_\gamma} C = \frac{C}{Q} \doteq 10^{-2} \text{ cm/sec.}$$

If Zn^{67} is used, the corresponding velocity will be of the order of 10^{-5} cm/sec.

Acceleration. The experiment by Pound and Rebka (Ref 19) verified a simple approach to the effect of acceleration (or equally, gravity) on the Mossbauer Effect. A gamma photon can be considered to have an apparent mass, given by Einstein's relationship $E = MC^2$. In a gravity field this mass is acted upon by the force, weight; and movement of weight through the gravity field causes an energy shift in the photon.

$$S = Mg (h_2 - h_1) = \frac{E}{C^2} g (h_2 - h_1) \quad (20)$$

As before, S is the energy shift; g is the gravitational acceleration; and h_2 and h_1 are the initial and final height of the photon above the ground. For a nominal height, 100 meters, S is only 0.03Γ . Therefore, unless large distances or large accelerations are involved, the

effect of acceleration is small compared to the effect of relative velocity.

Temperature. In addition to the effect of the ambient temperature on the fraction of zero phonon events given in (7), there is a second effect. This effect is an energy shift in the gamma photons due to a difference in temperature between the source and the absorber. The energy shift arises from the second-order Doppler effect, which can be recognized in (16) as follows:

$$\Delta \nu = \nu_0 \nu (C - \nu)^{-1} \quad (16)$$

Expanding (16) in a binomial series,

$$\Delta \nu = \nu_0 \left(\frac{\nu}{C} + \frac{\nu^2}{C^2} \dots \right) . \quad (21)$$

The shift in energy arises from the second term in the parentheses. If the source and absorber are at different temperatures, then the squares of the velocities of the lattice vibrations will be different. The effect is measured by the expression

$$\frac{S}{E_\gamma} = \frac{C_1 \Delta T}{2A C^2} \quad (22)$$

where S is the energy shift, C_1 is the specific heat of the lattice, ΔT is the temperature difference between source and absorber, and A is the gram atomic weight of the isotope (Ref 22:275). For Fe^{57} , S is only 0.005Γ for a temperature difference of 1°K . Therefore, normal methods for keeping the source and absorber at the same temperature should be adequate for any but the most precise experi-

ments with Fe⁵⁷.

The question arises, why do not lattice vibrations induce a first order Doppler shift? These vibrations produce maximum velocities of the order 10^4 cm/sec (Ref 15:18), and, as previously discussed, the Mossbauer Effect is sensitive to relative velocities of the order 10^{-2} cm/sec. The answer is found in the fact that the period of the lattice vibrations is of the order 10^{-13} sec (Ref 5:57). The effects due to the vector, velocity, tend to cancel out over the "long" lifetime of the nuclear excited state which is of the order 10^{-7} seconds in Fe⁵⁷. On the other hand, the effects due to the scalar term, v^2 , are additive.

Chemical Shifts. If the source and absorber differ slightly in chemical or isotopic * composition, this will result in a difference in the nuclear transition energy involved in the Mossbauer Effect. As a consequence, no source-absorber pair will exhibit maximum Mossbauer absorption at exactly zero relative velocity, zero temperature difference, and zero acceleration. In addition, no two source-absorber pairs will have their absorption maxima displaced by exactly the same amount on an energy scale. However, if the source and absorber are carefully prepared, the chemical shift can be minimized to within a small fraction (10^{-2}) of Γ (Ref 19:339).

* The term "chemical shift" is used in the literature to denote both isotopic and chemical effects (Ref 8:53).

Magnetic Effects. Application of a magnetic field to either the source or absorber causes splitting of the energy levels in the nuclear excited state. The net result is that there will be several peaks in the plot of the Mossbauer cross section versus energy, instead of the one distinct peak shown in Figure 3. For Fe^{57} the magnitude of the magnetic field necessary to produce noticeable splitting is of the order 10^3 gauss (Ref 8:52).

III. Limitations on the Application of the Mossbauer Effect

In the previous Chapter the basic theory of the Mossbauer Effect was described, and relationships showing the effects of physical phenomena were presented. In this Chapter an analysis will be made to determine the order of accuracy that may be achieved in measuring physical phenomena by use of the Mossbauer Effect. This analysis will demonstrate why the measurement of the relative velocity shift is the most promising mechanism for practical applications, and why the choice of the isotope Fe^{57} is most clearly indicated for such applications. A brief discussion of other important limitations is included.

Counting

Chapter II contained an explanation of how a change in the Mossbauer cross section could be detected by counting the number of photons passed through an absorber. The questions that must be answered are how accurately can the change in cross section be measured, and how accurately can the change in cross section be related to the physical phenomenon causing the change. In order to answer the first question, it is necessary to describe the best method of counting gamma rays.

Gamma rays are most efficiently counted by a scintillation detector, shown schematically in Figure 5. The photon is absorbed in the scintillator by Compton scattering or the photoelectric process.

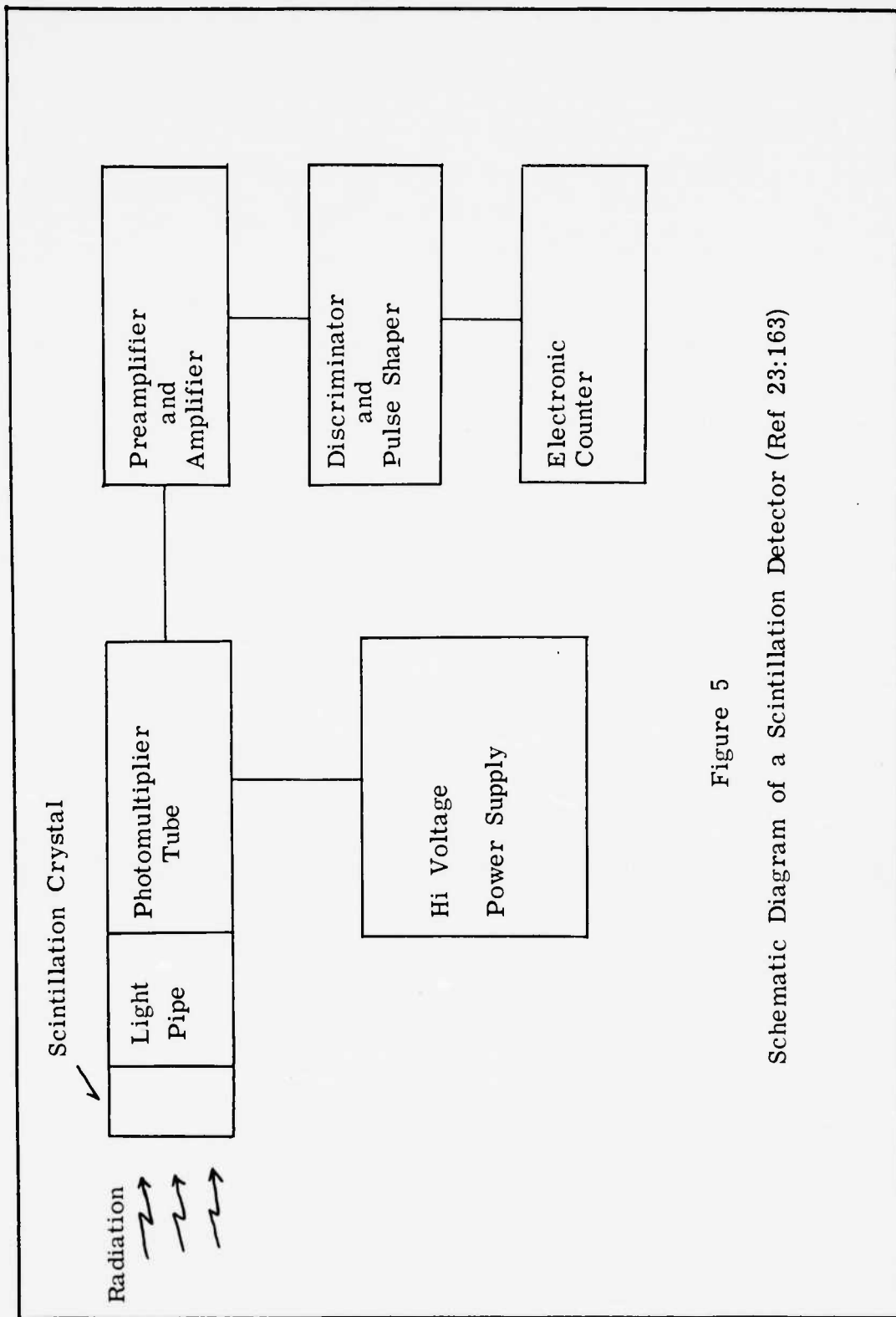


Figure 5

Schematic Diagram of a Scintillation Detector (Ref 23:163)

The light flashes from subsequent atomic transitions are converted to electronic pulses by the photo-multiplier tube, which also amplifies these pulses many times (Ref 23:163). The pulses can then be further amplified, measured, and counted by electronic devices. The use of a discriminator allows elimination of all pulses except those resulting from incident radiation in a particular energy range. In addition to high efficiency (number of photons counted divided by the number of photons incident on the detector), the use of the scintillation detector allows the highest possible counting rate (Ref 23:162).

The efficiency of the detector can be estimated from the number of photons absorbed by the scintillator. A sodium iodide, thallium activated - NaI(Tl) - crystal less than 0.1 cm thick will absorb nearly all the incident photons which are in the energy range of interest (10 - 100kev) for Mossbauer experiments (Ref 23:190, 191). The maximum counting rate is limited by the resolving time of the detector, which is the minimum time which must elapse between successive events if they are to be counted as two events. The resolving time is determined by the recovery time of the electronic scalar and by the decay time of the scintillator. A low value for resolving time is 10^{-7} seconds (Ref 23:198). This would allow a counting rate of 10^6 per second if a 10% correction is made for counts lost during the dead time of the detector (Ref 23:127).

The maximum counting rate determines a basic limitation in

the accuracy of a Mossbauer experiment. This results from the theory of statistics of detection systems. If n is the number of counts, then the standard deviation, σ_n , is given by

$$\sigma_n = \frac{100}{(n)^{1/2}} \% \quad (23)$$

(Ref 23:59). If the standard deviation is taken as an approximate criterion of accuracy, and if the maximum n is 10^6 , then the minimum detectable change in the counting rate is 0.1%. (For the purposes of this exercise, it is assumed that it is desirable to obtain a data point in one second; therefore, if the maximum counting rate is 10^6 / second, $n = 10^6$.)

Using this value, it is possible to estimate the minimum change in the Mossbauer cross section that can be detected. A parameter, F , is defined as the ratio of the flux, $I(x)$, of photons that reach the detector to the flux, I_0 , of photons incident upon the absorber.

$$F = \frac{I(x)}{I_0} = \exp(-N_0 \sigma x) \quad (24)$$

(Ref 13:344). N_0 is the number of atoms/cm³ in the absorber that are the proper isotope for Mossbauer absorption, and x is the absorber thickness. The equation assumes that the only absorption that takes place is due to the Mossbauer Effect, that the flux due to reradiation is negligible, and that all photons which pass through the absorber reach the detector. These conditions can be approximated by a suitable geometry, a thin absorber, and an isotope such as Fe⁵⁷

for which the Mossbauer cross section is much larger than the competing cross sections. If (24) is differentiated with respect to σ , and if $\Delta F \doteq dF$, then

$$\Delta F = - N_0 x \Delta \sigma \exp(-N_0 \sigma x) \quad (25)$$

In order to get as large a change in counting rate as possible, it is desirable to maximize ΔF . This can be done with respect to x , the absorber thickness. The result is an optimum thickness

$$x \text{ (optimum)} = \frac{1}{N_0 \sigma} \quad (26)$$

(An optimum iron absorber containing a natural abundance - 2% - of Fe^{57} would be about 0.05 cm thick.) If (26) is substituted in (25),

$$\Delta F = - .368 \frac{\Delta \sigma}{\sigma} \quad (27)$$

The conclusion can now be realized that if, by previous argument, the maximum change in counting rate that can be detected is 0.1%, then the fractional change in σ that can be detected is about 0.3%.

It is now a relatively simple matter to apply these results to a determination of the minimum detectable change in the energy of a physical phenomenon. The necessary equation is repeated:

$$\sigma = \frac{1}{2} \sigma_0 f f' \left(\frac{\Gamma^2}{S^2 + \Gamma^2} \right) \quad (19)$$

It is convenient to let $S = q\Gamma$. If (19) is differentiated with respect to q , and $\Delta \sigma \doteq d\sigma$,

$$\Delta \sigma = \frac{1}{2} \sigma_0 f f' \Delta q \left[\frac{-2q}{(1+q^2)^2} \right] \quad (28)$$

By substituting (19) and (28) in (27),

$$\Delta F = \Delta q \left[\frac{-2q}{(1+q^2)} \right] \quad (29)$$

(29) can be maximized with respect to q , giving $q = \pm 1$. Then, using $q = -1$,

$$\Delta F = \Delta q \quad (30)$$

This means that the minimum detectable change in the energy of a physical phenomenon, when expressed in units of Γ , is of the same order as the minimum detectable change in the counting rate. That is, $\Delta q = 0.1\%$, or $\Delta S = 10^{-3} \Gamma$.

Attention should be called to the fact that by letting $q = -1$, we imply that the emitted photons are biased in energy by one Γ away from the central maximum of the absorption spectrum, i.e. on that part of the σ vs E curve where the slope is greatest. An excellent example of this technique was provided by the Pound and Rebka experiment (Ref 19), in which the emitted photons were shifted in energy by a constant velocity drive in order to allow the small ΔS due to the gravity effect to result in the largest possible change in counting rate.

One further consideration is whether or not a scattering experiment offers an advantage in the matter of counting limitations. In an ideal scattering experiment, the detectors are completely shielded from the source, but detect all reradiation from the absorber.

Let the reradiated flux be I' , where

$$I' = I_0 - I(x) = I_0 \left[1 - \exp(-N_0 \sigma x) \right] \quad (31)$$

The quantity F' is defined,

$$F' = \frac{I'}{I_0} = 1 - \exp(-N_0 \sigma x) \quad (24a)$$

If (24a) is differentiated with respect to σ , an equation of the same form as (25) is obtained. Therefore, the analysis for the scattering experiment is the same as for the absorption experiment.

However, the scattering experiment poses certain practical difficulties since reradiation is isotropic.

Linewidth

In the section just presented, it was demonstrated that the statistics of counting limited the minimum detectable change in energy, ΔS , of a physical phenomenon to a certain fraction of the Mossbauer linewidth, Γ . The example given was for one second of counting at the high counting rate of 10^6 /second, in which case $\Delta S = 10^{-3} \Gamma$. The next questions of importance are, how small is Γ , and what does this mean in terms of the physical phenomena pertinent to the investigation.

As discussed in Chapter II, Zn^{67} exhibits the smallest linewidth. The difficulty in using Zn^{67} is that the Mossbauer cross section is so small as to prohibit the use of the optimum absorber thickness given by (26). If one uses the values from Appendix B, calculation

will show that even for an absorber enriched to 100% of the proper isotope, x (optimum) for Zn^{67} is about 13 cm. In order to penetrate an absorber this thick with photons of the 100 keV range, and still have 10^6 photons/sec for counting purposes, will require an incident flux of 10^{26} photons/second. (The calculation is made using a photoelectric cross section only.) To provide such an incident flux would require a source strength of at least 10^{16} curies, which is beyond the realm of reason for safety considerations alone. An absorber only 1 cm thick will mean in terms of the previous analysis that the ΔS which may be detected is just equal to Γ . This means that the best that can be done with Zn^{67} in a short counting time is to tell whether or not resonant absorption is present. As a consequence of this, and the additional difficulties that the use of Zn^{67} implies, (very low temperatures and equipment free from vibrations of 10^{-5} cm/sec) it is not deemed advisable to attempt a practical application with this isotope.

In contrast to Zn^{67} , Fe^{57} has a large enough Mossbauer cross section to permit the use of very thin absorbers. Since Fe^{57} also exhibits the second smallest Γ of any Mossbauer isotope, it is appropriate to use the Fe^{57} linewidth as the basis for calculations of what may possibly be done in the measurement of physical phenomena.

It was calculated in Chapter II that a relative velocity of 10^{-2} cm/sec will shift the Fe^{57} absorption spectrum by one line-width. If a shift of $10^{-3} \Gamma$ is measurable, then a relative velocity of 10^{-5} cm/sec may be detected.

The question of whether or not the Mossbauer Effect can be used directly to measure acceleration, A , may be answered by replacing g by A in (20). For a nominal distance, $h_2 - h_1$, equal to 1 meter, and basing calculation on Fe^{57} ,

$$\Delta S \doteq 3 \times 10^{-5} \Gamma \times A \quad (32)$$

where A is in meters/sec². Again using the minimum detectable $\Delta S = 10^{-3} \Gamma$, the measurable change in acceleration is 30 meters/sec², or about 3g! (In order to reduce this to 0.003g would require a source-absorber separation of 1000 meters or a counting time of 10^6 seconds.)

Similar calculations can be carried out for other physical phenomena by using the relationships from Chapter II. The effects of temperature difference, chemical shifts, and magnetic effects are small compared to the effect of relative velocity. Therefore, relative velocity is chosen as the mechanism for applying the Mossbauer Effect to accelerometers. Techniques to achieve this end are developed in Chapter IV and Chapter V.

Other Limitations

Problem areas in the application of the Mossbauer Effect can

be recognized from the effects of physical phenomena previously described. Temperature control must be employed, very strong magnetic fields must be avoided, and care must be used to prepare the source and absorber in order to minimize chemical shifts. There is no serious shielding problem in the use of Fe^{57} . A few millimeters of material will reduce the gamma flux to an acceptable level unless an extremely strong source is employed. One of the most serious problems is that of vibration. Small vibrations resulting in source-absorber relative velocities must be avoided. This could be a near impossible obstacle for a vehicular operation.

The best statement that can be made with regard to most of these limitations is that they have been overcome in the precise scientific experiments already mentioned. One would expect that as these experiments continue, techniques will be perfected and knowledge of the basic physics will increase. With regard to vehicular vibrations, rigid source-absorber coupling may minimize this difficulty. In space flights using low thrust propulsion devices the problem should be much less severe than for flights in the atmosphere. In the remainder of this study, the assumption will be made that these problems can be overcome by diligent research and engineering. The basic limitations of counting and the linewidth of Fe^{57} will be used to define the possibilities of the Mossbauer Effect. If future discoveries provide a better isotope, the results of the study may be easily extrapolated.

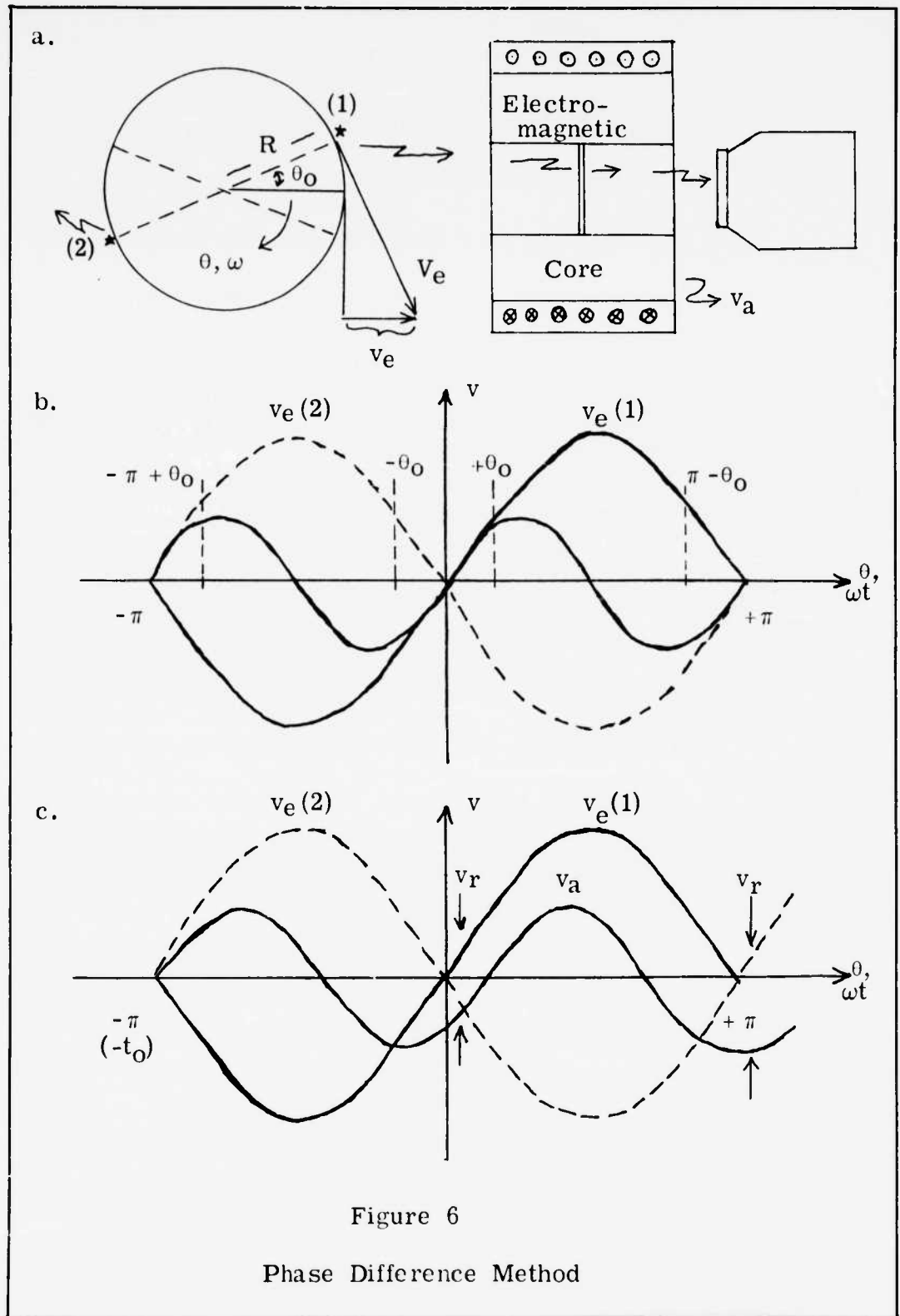
IV. Techniques for Measurement of Velocity and Position

As discussed in Chapter III, it is impractical to measure acceleration by the direct use of the Mossbauer Effect. On the other hand, the Mossbauer Effect is very sensitive to the first-order Doppler shift produced by a small relative velocity between the source and the absorber. In order to apply this phenomenon to accelerometers, it is necessary to find techniques by which the extreme sensitivity of the Mossbauer Effect to small velocities can be used as an instrument sensor or pickoff device. Two such techniques are presented. One technique allows measurement of velocity. Another provides for measurement of angular position. Chapter V contains an explanation of how these techniques can be applied to accelerometers.

Measurement of Velocity

Direct measurement of velocity by the use of the Mossbauer Effect is impractical in many instances because only small velocities can be measured. Two systems are presented which allow measurement of velocities of any magnitude. The first of these systems will be referred to as the phase difference method. The phase difference method is applicable to measurement of angular velocity.

Phase Difference Method. As shown in Figure 6a, emitter atoms are placed at discrete points 1 and 2 on the rim of a rotating wheel. When the wheel is rotating through the interval $-\theta_0$ to $+\theta_0$,



the emitted photons passing in a direct line to the absorber have a velocity component, v_e , along this line equal to $V_e \sin \theta = V_e \sin \omega t$. V_e is the tangential velocity of emitter atoms, ω is the angular velocity of the wheel. v_e is plotted for both emitter 1 and emitter 2 in Figure 6b. Motion is induced in the absorber through the vibration of an electromagnetic core. In practice, the absorber foil may be mounted in a very narrow slit in this core. The width of the slit will subtend and define the arc $2\theta_0$. Only gamma rays following the direct line normal to the absorber will be significant in the counting, since the photons striking the core or surrounding apparatus will be absorbed. The relative velocity of the absorber is assumed to be sinusoidal.

Let the absorber velocity, v_a , be $V_a \sin \omega t$ where V_a is the amplitude of the absorber velocity and ω is the angular frequency. We shall require initially that relative velocity, v_r , between the source and absorber be as small as possible in the interval $-\theta_0 < \omega t < +\theta_0$. Then, for this interval, let

$$v_r = 0 = V_e \sin \omega t - V_a \sin \omega t \quad (33)$$

or, if θ_0 is small, $V_e \omega = V_a \omega$.

One possible solution is: $V_e = V_a$, $\omega = \omega$.

However, since θ_0 is small, this will provide a minimum of counting time during one complete revolution of the wheel. When a second emitter is used, it is necessary to let $V_e = 2 V_a$, $\omega = \frac{\omega}{2}$. Thus, the

counting time available is doubled. As illustrated in Figure 6b, the absorber velocity is nearly coincident with the emitter velocity every half-cycle of revolution. Subject to limitations which will be presently examined, we may further increase counting time by placing more emitters on the wheel rim and by requiring, in general $V_e = n V_a$, $\omega = \frac{w}{n}$, where n is the number of emitters. If Fe^{57} is used as the emitter isotope, an appreciable Mossbauer absorption will take place as long as v_r is less than 10^{-2} cm/sec. The output of the instrument can be the electronic frequency, w , of the excitation voltage used to vibrate the electromagnetic core. The fact that w is larger than ω presents an advantage since in measuring cyclic events we may expect an accuracy of at least \pm one cycle. More generally, if we can measure a given part of one cycle, then the more cycles available, the more accurate will be our measurement.

In order to establish the sensitivity of this technique it is assumed that at some time, $-t_0$, the absorber velocity is in phase with the emitter velocity. Subsequently a small error in frequency of the absorber, Δw , results in a phase difference at time after $-t_0$. This is illustrated in Figure 6c. The resultant relative velocity may be expressed as follows for the interval $-\theta_0 < \theta < +\theta_0$.

$$v_r = V_e \sin \omega t - V_a \sin (n \omega t + \phi) \quad (34)$$

where ϕ is the phase difference.

$$\phi = n \Delta \omega (t + t_0) \quad (35)$$

If θ_0 is very small, say 10^{-2} radians, and ω is some nominal value, say 10 radians per second, then t is only 10^{-3} seconds.

On the other hand, t_0 should be on the order of one second if enough counts are to be obtained to establish whether there is a relative velocity between emitter and absorber. Therefore, the approximation is made, for $t \ll t_0$,

$$\phi = n \Delta \omega t_0 \quad (36)$$

Substituting (36) in (34),

$$v_r = V_e \sin \omega t - V_a \sin n (\omega t + \Delta \omega t_0) \quad (37)$$

or,

$$\begin{aligned} v_r = & V_e \sin \omega t - V_a \sin n \omega t \cos n \Delta \omega t_0 \\ & - V_a \sin n \Delta \omega t_0 \cos n \omega t \end{aligned} \quad (38)$$

If $V_a = \frac{V_e}{n}$, and if all the arguments are small angles,

$$v_r = - V_e \Delta \omega t_0 \quad (39)$$

The smallest increment of angular velocity that can be measured is therefore,

$$\Delta \omega = \frac{v_r}{V_e t_0} \quad (40)$$

where v_r is the relative velocity that can be detected by the Mossbauer Effect. If $v_r = 10^{-5}$ cm/sec, $t_0 = 1$ sec, and $V_e = R \omega = 100$ cm/sec, then an increment of 10^{-7} radians/sec will be measurable. An interesting feature of this technique is that the accuracy increases for large values of ω .

The above description was based on ideal conditions of operation. It is pertinent to examine some of these conditions in more detail in order to establish the limitations of the technique.

1. It was assumed that the amplitude of the absorber velocity is set at precisely the right value. The relative velocity due to amplitude deviation, ΔV_e , is

$$v_r = V_e \sin \omega t - \left(\frac{V_e + \Delta V_e}{n} \right) \sin n \omega t \quad (41)$$

This is at a maximum during the interval of counting when $\omega t = \theta_0$.

$$(\max) v_r = - \Delta V_e \theta_0$$

The relative velocity due to amplitude error should be less than the relative velocity due to phase difference in order for this technique to be effective. Therefore, it is required that

$$\Delta V_e \theta_0 < V_e \Delta \omega t_0 \quad (42)$$

If $t_0 = 1$ sec

$$\frac{\Delta V_e}{V_e} < \frac{\Delta \omega}{\theta_0} \quad (43)$$

If $\Delta \omega$ is to be measured to an accuracy of 10^{-5} radians /sec and $\theta_0 = 10^{-3}$ radians, then the amplitude of the absorber velocity must be held accurately to less than one part in 100. The relationship points out the advantage of making θ_0 as small as possible.

2. Another reason for making θ_0 small is to allow the use of as many emitters as possible, thus making the frequency multiplication factor, n , as large as possible. The question raised is, if

$$v_r = V_e \sin \omega t - \frac{V_e}{n} \sin n \omega t \quad (44)$$

or

$$v_r (\max) = V_e \sin \theta_0 - \frac{V_e}{n} \sin n \theta_0 \quad (45)$$

how small must θ_0 be to allow a given number of emitters, n , without causing the relative velocity from (45) to be larger than the relative velocity induced by the phase error? This problem may be analyzed by expanding both terms on the right hand side of (45) in a series, and combining the series term by term, yielding:

$$v_r (\max) = V_e \left[\frac{(n^2 - 1) \theta_0^3}{3!} - \frac{(n^4 - 1) \theta_0^5}{5!} + \frac{(n^6 - 1) \theta_0^7}{7!} - \dots \right] \quad (46)$$

It is required that the relative velocity due to phase error (39) be larger than the $v_r (\max)$ given by (46), or

$$\Delta \omega t_0 > \frac{(n^2 - 1) \theta_0^3}{3!} \quad (47)$$

Again it is assumed $t_0 = 1$ second, allowing computation of approximate values. $\Delta \omega$ must be numerically greater than the figures represented in the elements of the Table I. A typical result of the computation is that if a large number of emitters are to be used, then θ_0 must be on the order of 10^{-3} radians in order that a frequency error on the order of 10^{-5} radians / second can be measured.

3. There is a minimum below which θ_0 cannot be reduced. This limit is established by the physical dimensions of the wheel. The arc subtended on the wheel by the emitter must be less than the arc

Table I
Limiting Increments of Angular Velocity

θ_0	n = 2	n = 8	n = 16	n = 100
.1 rd	$.5 \times 10^{-3}$	1.0×10^{-2}	-----	-----
.01 rd	$.5 \times 10^{-6}$	1.05×10^{-5}	4.25×10^{-5}	1.6×10^{-3}
.001 rd	$.5 \times 10^{-9}$	1.05×10^{-8}	4.25×10^{-8}	1.67×10^{-6}

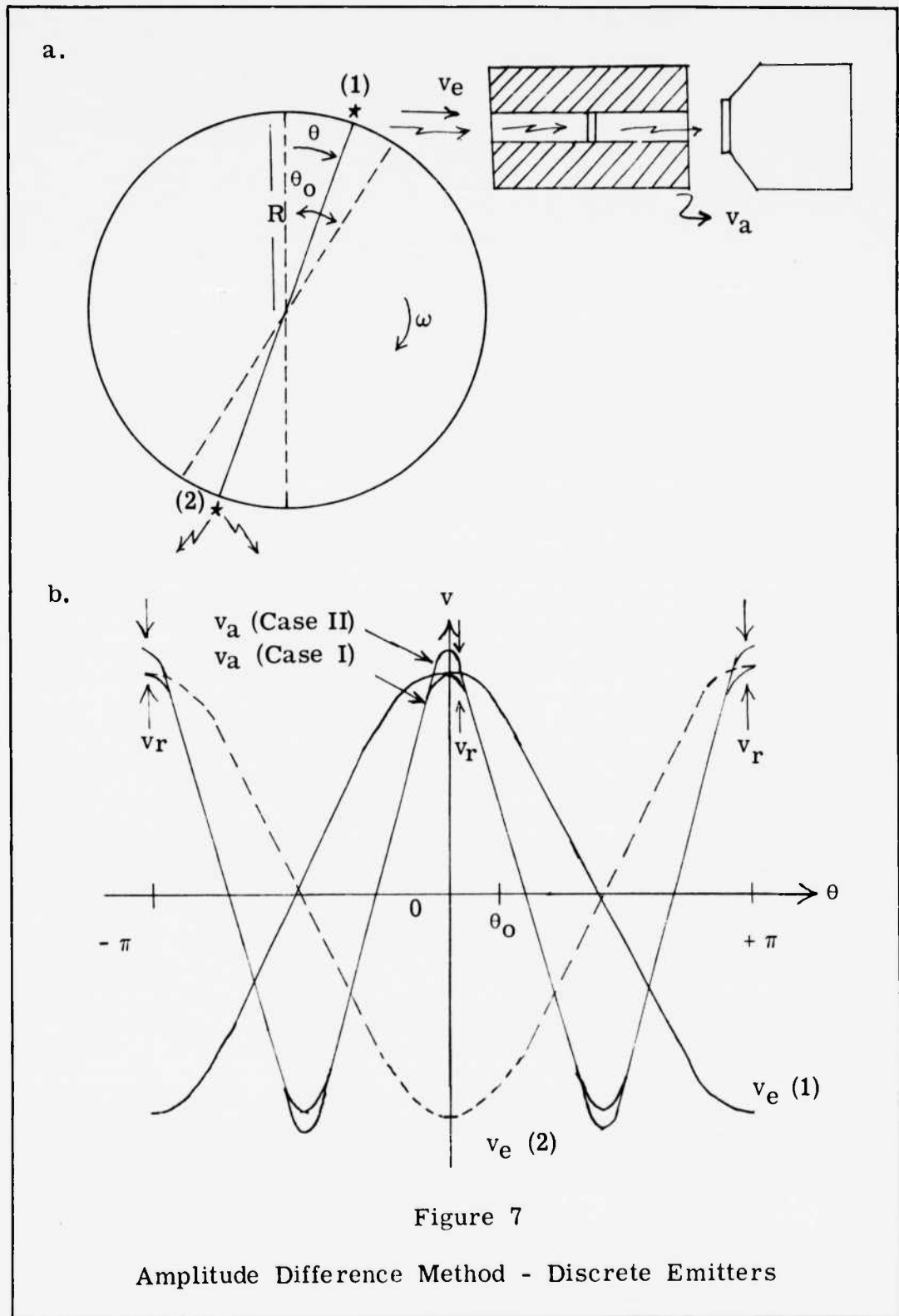
subtended by θ_0 in order for the foregoing calculations to have meaning (a point source was assumed). In the examples previously discussed, $R = 10$ cm. For $\theta_0 = 10^{-3}$ radians, the arc subtended is 10^{-2} cm. This would indicate that the arc subtended by the emitter should be 10^{-4} or 10^{-5} cm, or on the order of the wavelength of visible light. If a number of emitters are used, then the spacing between emitters should be accurate to 10^{-4} or 10^{-5} cms. This will require precise and expensive manufacturing techniques. However, the requirements should not be declared impossible in view of progress in solid state physics. For instance, superconductivity experiments have been carried out with films 10^{-7} cm in thickness (Ref 10:459).

4. It is evident from previous discussion that the length of time available for measurement is a critical limitation of this technique. Time is necessary to obtain statistical counting. Time (t_0) is necessary to allow a small frequency error to result in a phase difference between absorber and emitter. Time is necessary to count a sufficient number of cycles of the coil excitation voltage,

in order to read the angular velocity to the desired accuracy. Finally, since the phase, frequency, and amplitude of the absorber must be accurately adjusted, there may be a significant time necessary for even an electronic feedback system to establish "zero" relative velocity between source and absorber.

The four limitations just described establish guidelines for the use of the phase difference method for applying the Mossbauer Effect to measurement of angular velocity. A second velocity measurement technique will now be examined. This technique is referred to as the amplitude difference method. There are two variations of the system. The first variation employs discrete emitters and is applicable to measurement of angular velocity. The second variation employs continuous emission and is applicable to measurement of both angular velocity and linear velocity.

Amplitude Difference Methods. Figure 7a is a schematic representation of the amplitude difference method using discrete emitters. Point source emitters are placed at positions one and two on the rim of the wheel which is rotating through an angle θ , measured from a line parallel to the plane of the absorber. The absorber is mounted in a slit in an electromagnetic core similar to the device used in the phase difference method. (In practice, relative velocity of the absorber may be provided by a number of techniques; i.e., piezoelectric effect, small wheel driven by a synchronous



motor, tuning fork, etc.) The edge of the shielding below the absorber (bottom edge of the slit) defines one point on the arc θ_0 . The other point is defined by the tangent to the top of the wheel. θ_0 is the interval during which photons from the emitter are counted by the detector. If θ_0 is very small, only photons that pass on the line perpendicular to the absorber will be counted. In the interval of emission, $0 < \theta < \theta_0$, the relative velocity of the emitter is

$$v_e = V_e \cos \omega t \quad (48)$$

where $V_e = R\omega$, the tangential velocity of the rim of the wheel.

The velocity of the absorber is sinusoidal, given by

$$v_a = V_a \cos 2 \omega t \quad (49)$$

for the two emitters shown. In general, for n emitters, $v_a = V_a \cos n \omega t$. For case I plotted in Figure 7b, there is a relative velocity between emitter and absorber:

$$v_r = V_e \cos \omega t - V_a \cos 2 \omega t \quad (50)$$

The minimum v_r occurs if $V_e = V_a$. If this condition is fulfilled, the minimum relative velocity results in a maximum of Mossbauer absorption, and the angular velocity of the wheel may be measured by an electronic reading of the frequency or amplitude of the excitation current of the electromagnetic core. If this condition is not fulfilled, there will be a larger relative velocity difference as shown in Figure 7b, Case II. If the angles are small in (50),

$$v_r = V_e - V_a = \Delta V \quad (51)$$

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Since $\Delta V = R \Delta \omega$

$$\Delta \omega = \frac{v_r}{R} \quad (52)$$

For $v_r = 10^{-5}$ cm/sec, $R = 10$ cm, an angular velocity may be measured to 10^{-6} radians/second.

Much of the discussion regarding the limitations of the phase difference method is also applicable to the amplitude difference method. The major difference is that in seeking to measure the velocity resulting from a difference in amplitude of the velocity cycles, we incur an obligation to accurately control the amplitude of the absorber velocity. Further, if a frequency output is assumed to be desirable, it is necessary to accurately relate the frequency of the absorber vibration to the maximum absorber velocity. For a vibrating electromagnetic core this relationship is established through the length, l_0 , through which the core travels in one-half cycle; i. e., if

$$x = l_0 \sin \omega t \quad (53)$$

$$\frac{dx}{dt} = l_0 \omega \cos \omega t \quad (54)$$

For an electromagnetic core, l_0 , may be a variable due to effects of friction, hysteresis, electronic noise, and other causes. On the other hand, for a wheel ($l_0 = \text{radius}$), l_0 is a fixed quantity. Therefore, for the amplitude difference method it may be desirable to use small wheels driven by a synchronous motor, with absorbers

mounted as shown in Figure 8. The output quantity now becomes the frequency of the synchronous motor. If n emitters are used, then there is a frequency multiplication factor, n , between the angular frequency of the emitter wheel and the frequency of the smaller absorber wheels. The question may be raised, why not measure the velocity of the emitter wheel directly by use of a synchronous motor drive? Other than the frequency multiplication factor just mentioned, there is no reason why this cannot be done, if, in fact, it is permitted by the nature of the physical system involved. There are certain applications wherein it is necessary to allow a wheel to rotate as freely as possible. In such instances it is not permissible to drive the emitter wheel.

The number of emitters that may be employed in order to obtain a frequency multiplication factor is limited by the size of the angle θ_0 and the tangential velocity of the emitter, V_e . If

$$(\max) v_r = V_e \cos \theta_0 - V_a \cos n \theta_0 \quad (55)$$

and if, as required, $V_a = V_e$, then by expanding the cosine terms in series and subtracting term by term

$$(\max) v_r = V_e \left[\frac{(n^2 - 1) \theta_0^2}{2!} - \frac{(n^4 - 1) \theta_0^4}{4!} + \frac{(n^6 - 1) \theta_0^6}{6!} \dots \right] \quad (56)$$

It is imperative that $(\max) v_r$ given by (56) be much less than ΔV given by (51). It is required that

$$\frac{\Delta V_e}{V_e} > \left[\frac{(n^2 - 1) \theta_0^2}{2!} - \frac{(n^4 - 1) \theta_0^4}{4!} + \frac{(n^6 - 1) \theta_0^6}{6!} \dots \right] \quad (57)$$

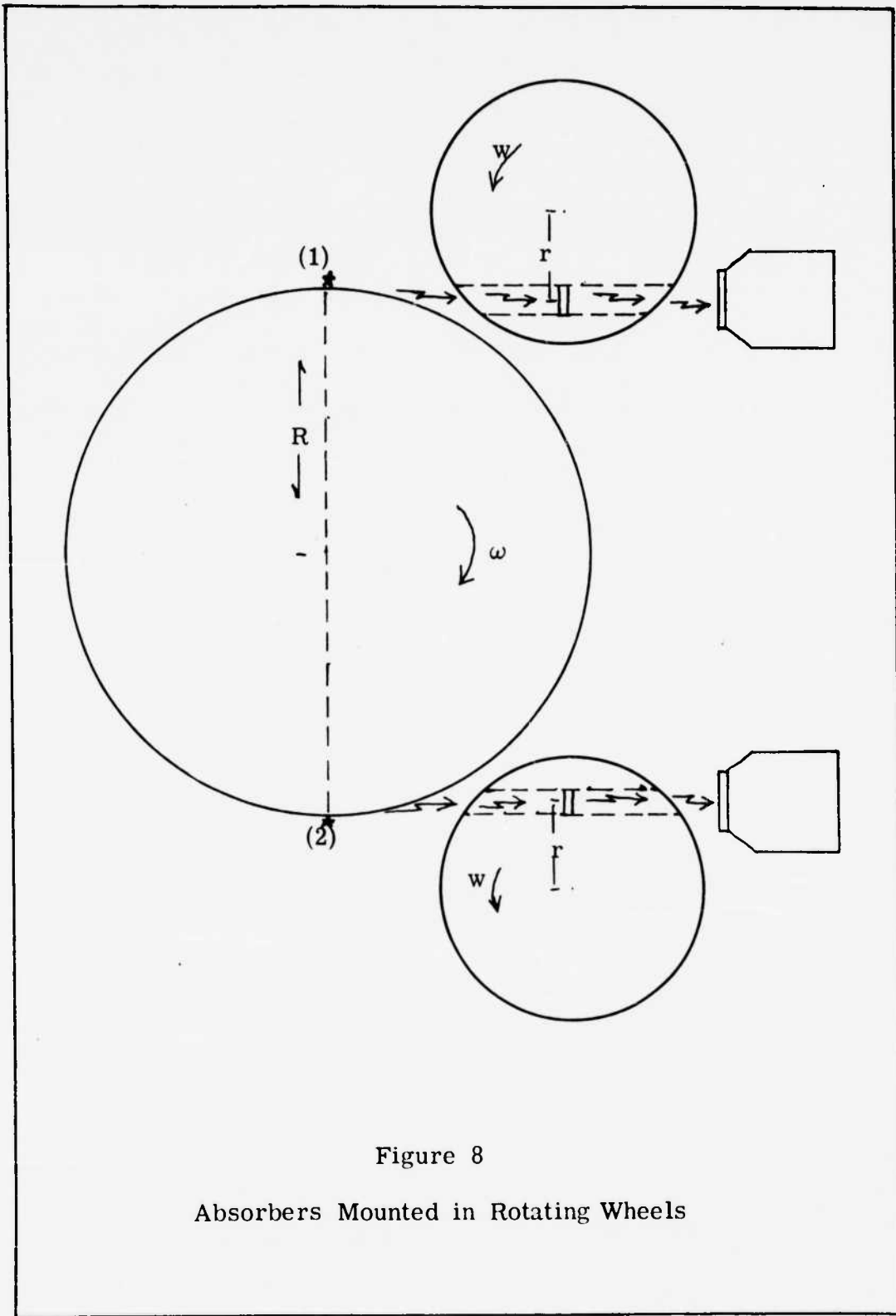


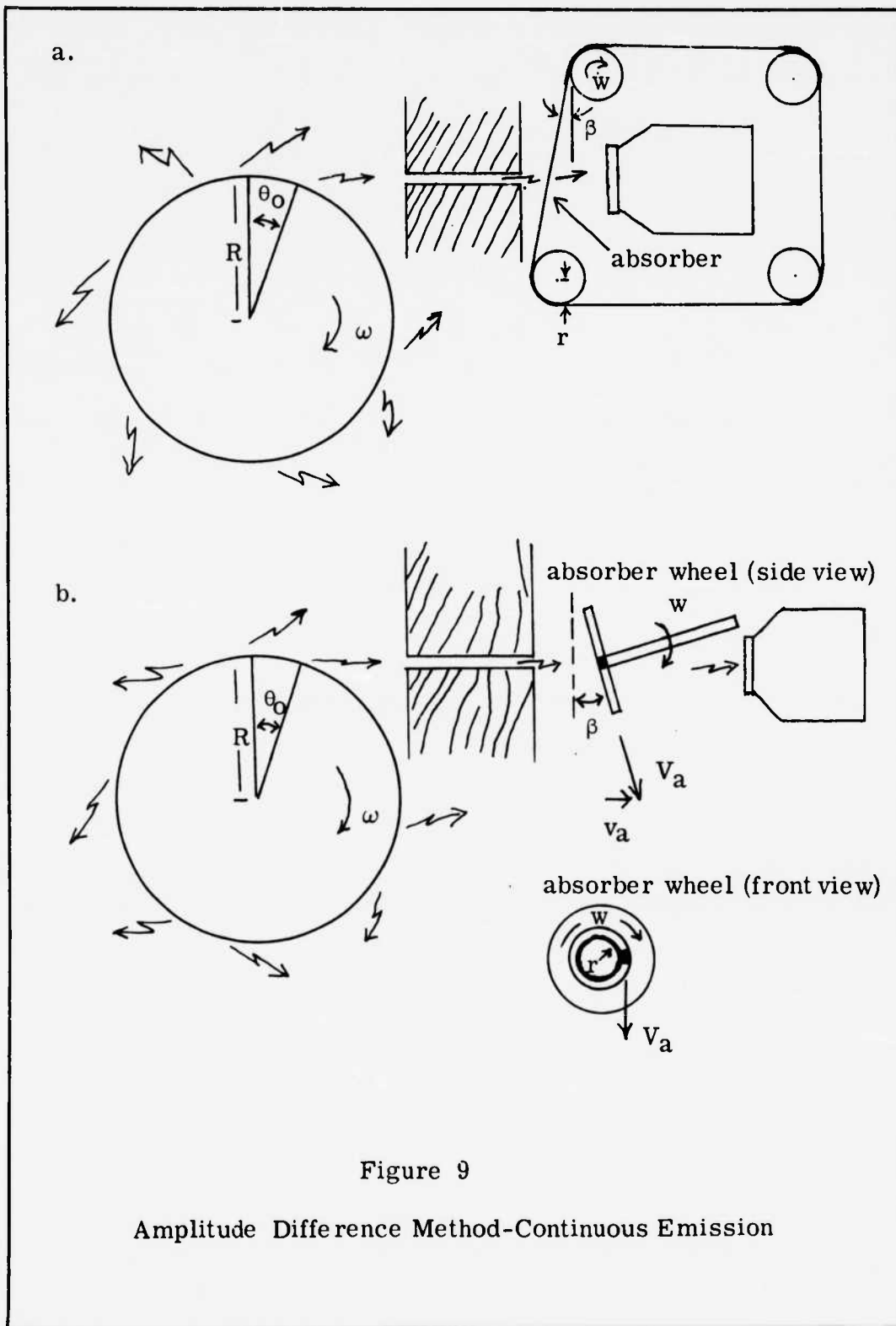
Figure 8

Absorbers Mounted in Rotating Wheels

This places a limit on the accuracy of the amplitude difference method, and in addition, places a limit on the magnitude of the velocity which may be measured. There was no such restriction involved in the phase difference method. On the other hand, it is possible by the amplitude difference method to place the emitter atoms along the entire arc θ_0 . Another advantage of this technique is that there is no requirement for a time interval in which a frequency error builds into a phase error.

In order to avoid the limitations inherent in a sinusoidally moving absorber, it is natural to question whether the amplitude difference method can be employed with an absorber moving with a constant, linear velocity. This question leads to a consideration of the second variation of the amplitude difference method. This variation employs continuous emission and is applicable to measurement of either angular or linear velocity. As applied to angular velocity, the method is illustrated in Figures 9a and 9b. Source atoms are placed around the entire rim of the emitter wheel. Only those photons which are emitted from the arc of the rim subtended by the angle θ_0 , and of these only those photons emitted in the direction parallel to the shielding material shown, are allowed to pass through the absorber and to the Detector. These photons have a relative velocity along their direction of propagation given by

$$v_c = R \omega \cos \theta \doteq R \omega \quad (58)$$



Relative velocity of the absorber may be provided by either of the methods shown in Figure 9. In Figure 9a this velocity is provided by a belt drawn by small wheels of radius, r . In Figure 9b the absorber velocity is provided by the rotation of a wheel tilted at an angle β to the direction of photon propagation. Only a small section of the wheel is used as an absorber. This section is indicated by the darkened portion in the face of the wheel shown in Figure 9b. In both instances, the absorber wheel, or wheels, are rotating at angular frequency w . This frequency may be controlled by a synchronous motor. The output of the system may be the frequency of this synchronous motor. In both cases, the velocity of the absorber is given by

$$v_a = r w \sin \beta \quad (59)$$

The relative velocity between source and absorber is approximately

$$v_r = R \omega - r w \sin \beta \quad (60)$$

By this technique a large frequency multiplication factor may be obtained, not only by making r smaller than R , but by making β a relatively small angle. This frequency multiplication factor, n , is given by the condition making v_r zero. Let $w = n \omega$, then by (60)

$$n = \frac{R}{r \sin \beta} \quad (61)$$

The accuracy of the system is primarily determined by the size of the emitter wheel. A deviation, $\Delta \omega$, in the angular frequency of

the emitter wheel will result in a relative velocity,

$$v_r = R\Delta\omega \text{ or } v_r = \Delta V_e \quad (62)$$

A limitation on the system accuracy is determined by the deviation of v_e due to the size of θ_0 . By (58), v_e will vary from $R\omega$ to $R\omega \cos\theta_0$. It should be noted that this limitation also applies to the amplitude difference method using discrete emitters, in the event the emitter atoms are placed along the entire arc θ_0 . The advantages of the amplitude difference method using continuous emission are the absence of any inherent error due to frequency multiplication and the absence of the requirement to adjust the phase of the absorber velocity. These advantages should make it a simpler and less expensive technique.

In summary, for the phase difference method;

$$\Delta\omega = \frac{v_r}{R\omega t_0} \quad (40)$$

and for the amplitude difference method;

$$\Delta\omega = \frac{v_r}{R} \quad (52)$$

These relationships are valid only within the limitations discussed.

A plot of (40) and (52) is shown in Figure 10. It is assumed in this plot that $t_0 = 1$ second, $R = 10$ cm.

Measurement of Position

A technique is possible for using the Mossbauer Effect as an accurate null sensor for angular position. As shown in Figure 11

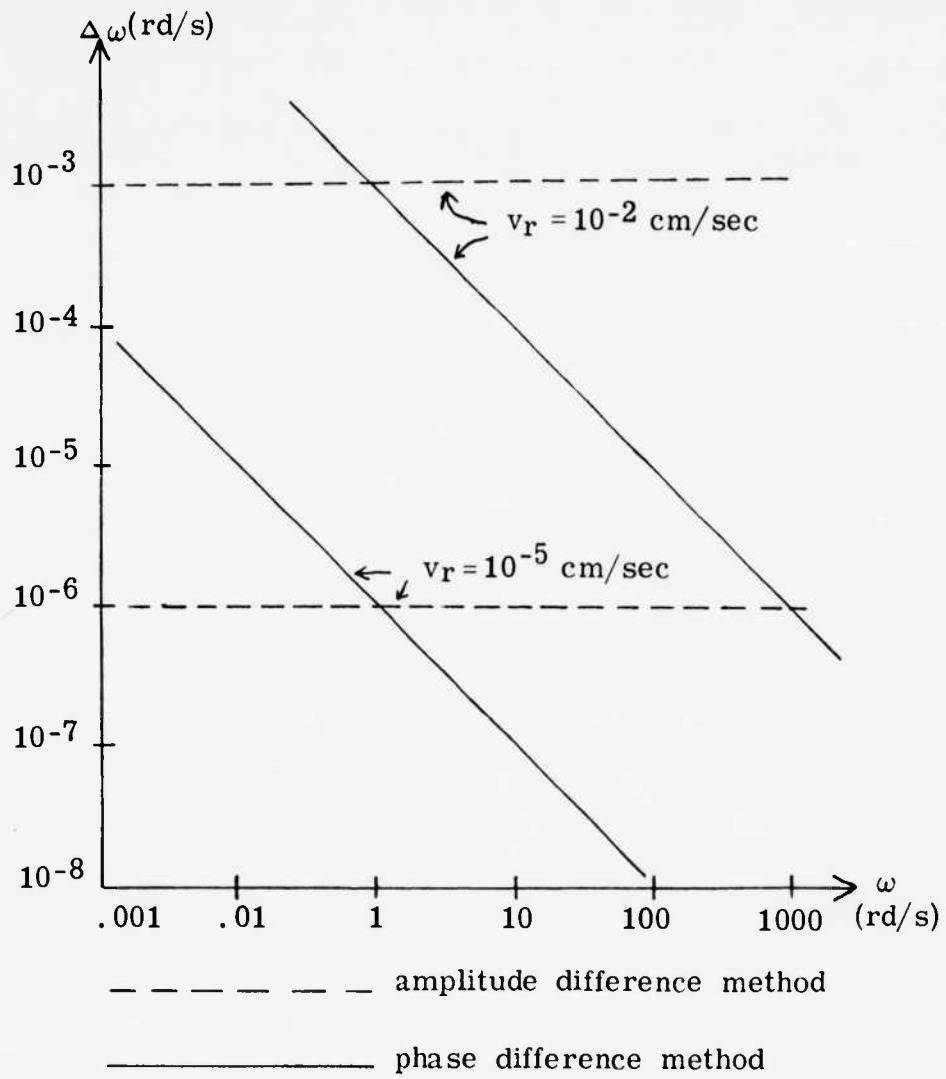


Figure 10

Accuracy of Angular Velocity Measurement

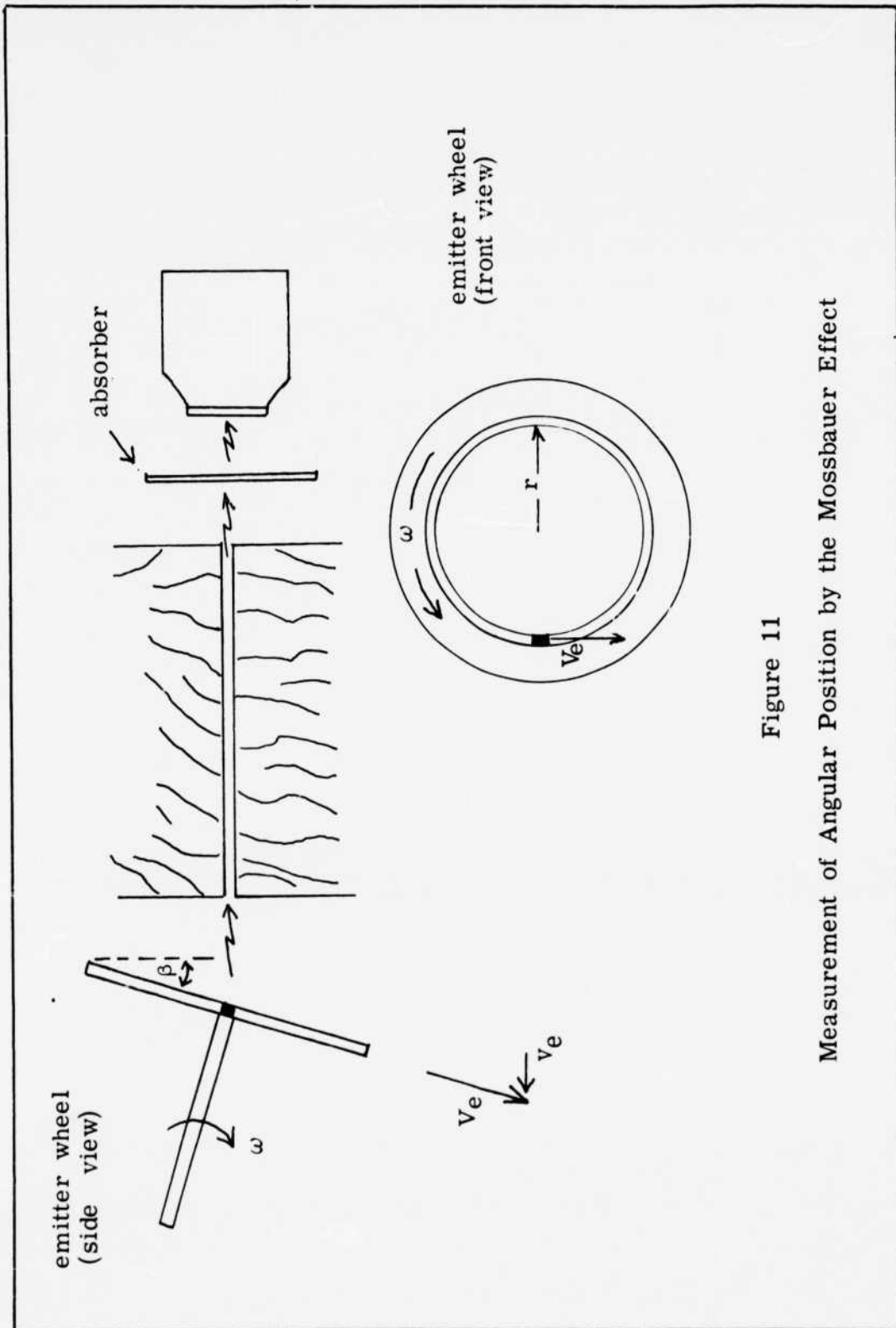


Figure 11
Measurement of Angular Position by the Mossbauer Effect

emitter atoms are placed in a concentric ring of radius, r , on the face of the emitter wheel. Shielding is employed such that only the atoms in the ring passing an aperture in the shielding produce photons that pass through the absorber and to the detector. This is indicated by the darkened portion on the face of the wheel. The photons so emitted have a velocity with respect to a stationary absorber, given by

$$v_e = r \omega \sin \beta \quad (63)$$

If, for a particular application, it is desired to maintain the wheel at a null position parallel to the plane of the absorber, then for this application we may take β as a small angle. The accuracy that may be obtained is given by

$$\beta = \frac{v_e}{r \omega} = \frac{v_r}{r \omega} \quad (64)$$

A limitation in this technique is the variation in the direction of photon propagation due to a need for a finite aperture in the shielding. An aperture only 10^{-4} cm wide will subtend an angle of 2" of arc at a distance of 10 cm. Therefore, the smallest angle that could be detected under these conditions would be greater than 2" of arc. An aperture of 10^{-4} cm is only 10,000 Angstrom Units and is equivalent to the spacing in a fine diffraction grating for light waves (Ref 10:86).

V. Application to Accelerometers

In Chapter IV techniques were presented by which the Mossbauer Effect could be used as a sensor to measure the velocity or position of an instrument component. In this Chapter these techniques are applied to accelerometers. The purpose of applying the techniques is to increase the resolution or to decrease the threshold sensitivity of accelerometers.

Resolution is defined as the reciprocal of the least fraction of acceleration which can be measured. That is, $R = A/\Delta A$, where R is resolution, A is the acceleration, and ΔA is the least increment of acceleration which can be measured. Threshold sensitivity is the smallest detectable unit of acceleration. Threshold sensitivity is distinguished from the term, sensitivity, which is the ratio of an output quantity to an input quantity for a given system or component.

In addition to resolution and threshold sensitivity, time response or frequency response is an important performance criterion. Time response indicates the highest rate of change of acceleration that can be measured. Frequency response is a more restricted statement of the same property. Frequency response gives the highest cyclical rate of acceleration that an accelerometer will read. Instrumentation using the Mossbauer Effect can generally be expected to consume one or more seconds to produce a single

data point because of the need for statistical counting. Therefore, there is little basis for improving the time response of conventional instruments, which may measure acceleration of 40 or more cycles/second (Ref 27:87-88). This property must be considered, however, in designing any accelerometer and the time response limitations of Mossbauer techniques should be appreciated.

In addition to resolution, threshold sensitivity, and time response, many other terms which denote the performance of accelerometers are found in the literature. These terms are usually measures of limitations rather than positive indications of performance. Examples are non linearity, hysteresis, friction, mass unbalance, and temperature effects.

In order to orient the study that follows, it is worthwhile to classify accelerometers in several major categories. Accelerometers from each category will then be analyzed from the standpoint of applications using the Mossbauer Effect.

Classification of Accelerometers

All accelerometers used for inertial navigation employ a mass, or inertial element, which is sensitive to acceleration through the principle of Newtonian mechanics, force equals mass times acceleration. The response of the inertial element may be measured and quantitatively related to acceleration by three basic methods. These methods serve to conveniently classify accelerometers:

1. Simply restrained devices: These accelerometers use the elastic property of a material or other medium to constrain the motion of the inertial element. Familiar examples are the strain gauge and market scale. The restraining force is proportional to an elastic constant, K , and the displacement, x , of the inertial element.

2. Force balance systems: Constraint is generated through a servomechanism which attempts to maintain the inertial element at a null position. The constraining device is usually an electric motor or other form of electromagnetic forcer. The magnitude of the constraint is proportional to the voltage and/or current supplying the forcer.

3. Unrestrained devices: The inertial element is allowed to move freely under the influence of acceleration. Output is a measurement of the displacement of the inertial element combined with a timing device. An example is the pendulum.

Other classifications often found in the literature include the following:

a. Linear or pendulous - A linear accelerometer constrains the inertial element to move along a line. A pendulous accelerometer constrains the sensitive mass to describe an arc of fixed radius.

b. Singly integrating or doubly integrating - indicates

an output of velocity or position, respectively.

c. Solid, liquid, or gas - may indicate the phase of the inertial element and/or the constraining device.

d. Other - such adjectives as electromagnetic, radioactive, gyroscopic, temperature-controlled, aniso-elastic compensated, etc. are often used by manufacturers to classify accelerometers. These adjectives are indicative of details in the construction of accelerometers.

Consideration will be given to an application of techniques using the Mossbauer Effect to several accelerometers from the three major classifications described.

Application to Simply Restrained Accelerometers

Simply restrained devices may be represented schematically as shown in Figure 12. The differential equations of motion are written as follows:

For Figure 12a, a linear device,

$$MA = M\ddot{x} + C\dot{x} + Kx \quad (65)$$

For Figure 12b, a pendulous device,

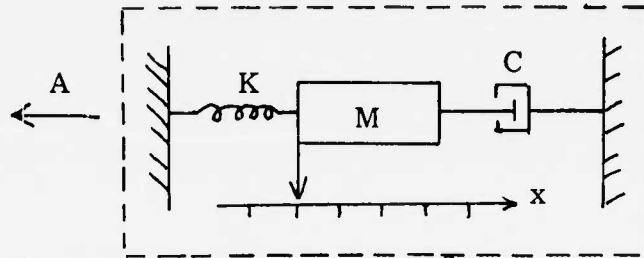
$$MrA = I\ddot{X} + C\dot{X} + KX \quad (66)$$

The steady-state solution of (65) is

$$x = \frac{M}{K} A \quad (67)$$

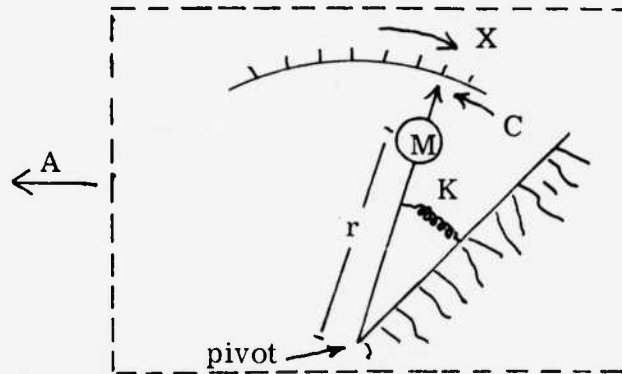
The resolution of a simply restrained device may be limited

a. Linear



- A - input acceleration, cms/sec^2
- K - spring constant, dynes/cm
- C - viscous friction force, $\text{dyne-sec}/\text{cm}$
- M - mass, gms
- x - displacement of M, cms .

b. Pendulous



- A - input acceleration, cms/sec^2
- K - elastic constant, $\text{dynes}/\text{radian}$
- C - viscous friction force, $\text{dyne-sec}/\text{radian}$
- M - mass, gms
- X - angular displacement of M, radians
- r - distance from pivot to center of M, cms
- I - (not shown) - moment of inertia of simple pendulous element (Mr^2)

Figure 12

Schematic Diagrams of Simply Restrained Accelerometers

by friction or by uncertainty in the value of K caused by non linearity and hysteresis (Ref 32:Ch. III, pp.72-81). An example is the Whittaker Z-125 Linear Accelerometer which is limited to end range resolution of 50 by non linearity (Ref 27:84-85). Another example is the Modified Seismic System by Research, Inc., which has a resolution of 10 at 1g input (Ref 27:62-63). In addition to being limited in resolution, simply restrained devices are limited in threshold sensitivity by static friction and hysteresis. It will be noted in (65) that it is necessary to have a significant value of C in order to damp out the transient solution. It is difficult to conceive of a means of obtaining high C without accompanying static friction.

A further difficulty arises in obtaining an output from a simply restrained device which is suitable for Mossbauer detection, that is, a relative velocity output. If (65) is solved for a step input in acceleration, A_0 , the following solutions in $\dot{x}(t)$ are obtained:

$$\dot{x}(t) = A_0 \left(\frac{M}{K}\right)^{1/2} \sin \left(\frac{K}{M}\right)^{1/2} t, \text{ for } C = 0 \quad (68)$$

$$\dot{x}(t) = 0, \text{ for } 0 < \frac{C^2}{4KM} < 1 \quad (69)$$

$$\dot{x}(t) = \frac{A_0 M}{C}, \text{ for } \frac{C^2}{4KM} < 1 \quad (70)$$

The condition, $C = 0$, in (68) is difficult to achieve in practice. In order to achieve high sensitivity in this system, M should be much

greater than K ; however, this results in a long period of vibration, $2\pi \left(\frac{M}{K}\right)^{1/2}$, thus limiting the time response of the system. Finally the resolution of the system will, in all probability, be defined by uncertainty in K and not by the pickoff mechanism. (69) yields no output. (70) presents certain practical difficulties, since for a large A_0 , x may rapidly exceed the physical dimensions of the instrument. The degree of resolution that may be attained is inherently limited by the condition of $C^2 > 4KM$, because the output is proportional to M/C . And again, performance is limited by uncertainty in the value of a material parameter, in this case, C .

In general, simply restrained devices are not widely accepted as the best approach to inertial quality accelerometers (Ref 7:99, 24:74). Since these devices are limited by uncertainties in restraint parameters such as C and K , there is no promise that simply restrained accelerometers can be improved in resolution or threshold sensitivity by the application of a sensitive pickoff system employing the Mossbauer Effect.

Application to Force Balance Systems

Force balance accelerometers offer certain advantages over simply restrained devices in that the inertial element is constrained to small displacements about a null position, thus minimizing the non linearity and hysteresis inherent in end range inputs to an elastic or friction constraint. This is illustrated by the Kearfott Pendulous

Accelerometer, Type No. F2401, which has a resolution of 10^4 over a range from 5×10^{-5} g to 20 g (Ref 27:76-77), and by the Litton Model A-200 Accelerometer which has a resolution of 10^4 over a range from 10^{-5} g to 60 g (Ref 27:92-93). To some extent performance of a force balance system may be limited by such factors as electromagnetic hysteresis and non linearities in the forcing device. A fair indication of the state of the art may be obtained by considering the pendulous integrating gyro accelerometer, abbreviated PIGA. The PIGA finds wide application as an inertial quality accelerometer (Ref 7:99, 24:77).

Pendulous Integrating Gyro Accelerometer. The essential elements of the PIGA are illustrated in Figure 13. The spinning wheel of a gyro creates an angular momentum vector \vec{H} , given by

$$\vec{H} = I\vec{\omega}\text{spin} \quad (71)$$

where I is the moment of inertia of the gyro wheel. The vector \vec{H} is constrained to move about two axes, indicated by an inner and outer gimbal. If a torque, \vec{T} applied, is applied to the inner gimbal by the action of system acceleration, \vec{A} , on the two masses, $M/2$, then the system will precess in the direction of $\vec{\omega}$. This is deduced from the law of conservation of angular momentum, expressed in this case as

$$\vec{T} \text{ applied} = \vec{\omega} \times \vec{H} \quad (72)$$

where

$$T \text{ applied} = MA l \cos \beta \quad (73)$$

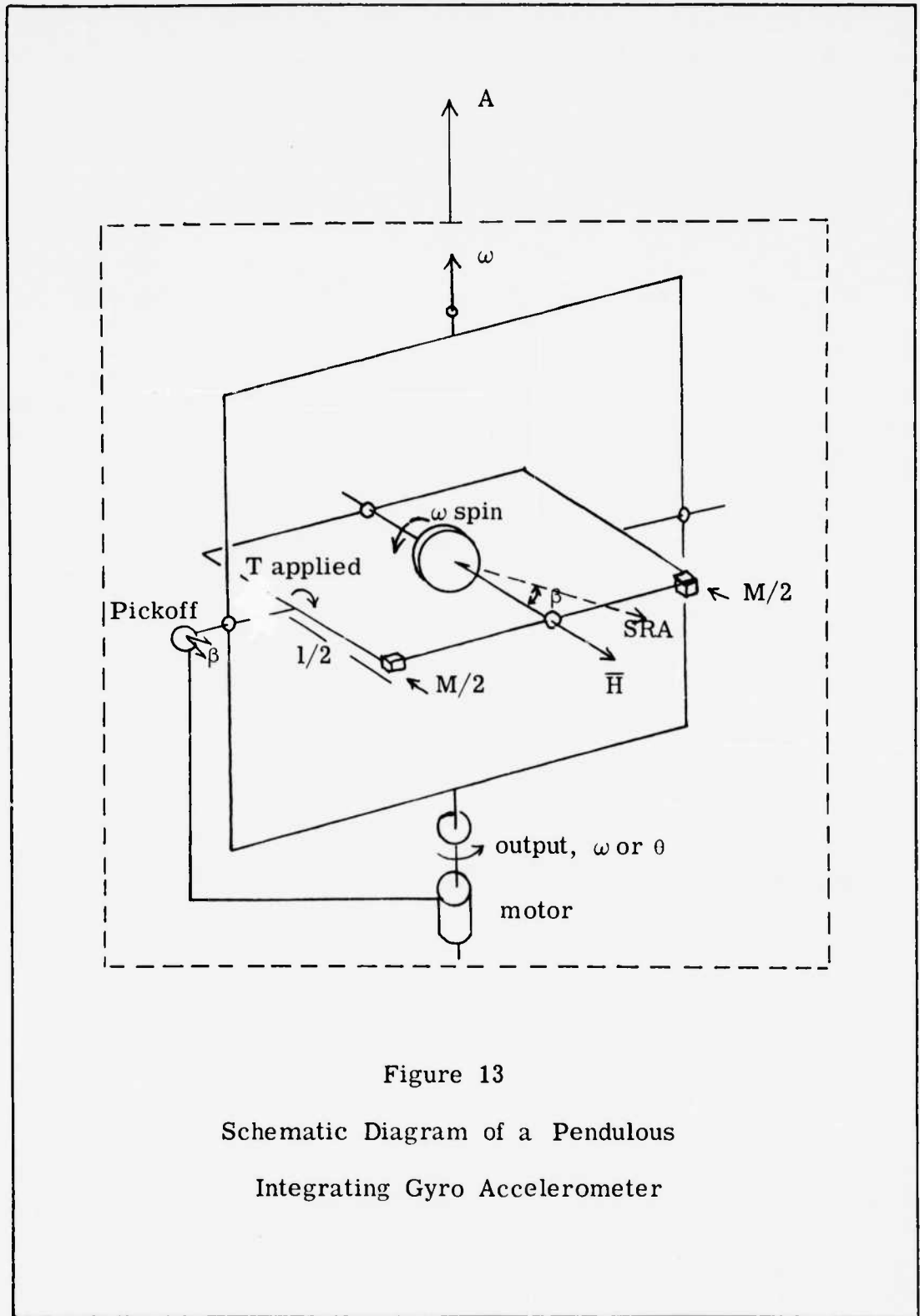


Figure 13

Schematic Diagram of a Pendulous
Integrating Gyro Accelerometer

β is the angle between the Spin Reference Axis (SRA), which is perpendicular to $\bar{\omega}$, and the direction of \bar{H} . In practice β is kept very small (to within a few seconds of arc) by the action of a servomechanism which turns the outer gimbal in the direction of $\bar{\omega}$. If β is nearly zero, then by (72) and (73)

$$\omega = \frac{M I A}{H} \quad (74)$$

The output ω is directly proportional to input acceleration, A , over a very wide range of performance. In practice, the output is usually taken as angular position, $\theta (= \omega t)$, which is directly proportional to the input velocity. Given the best possible design, the major uncertainty due to material properties is the uncertainty in mass unbalance (Ref 7:101). In the case of an inertial quality gyro the uncertainty would be about 15×10^{-8} cm, expressed as uncertainty in the arm length, l (Ref 7:103). Even for an arm length of only one centimeter, this represents a small inherent error of only about one part in 10^7 .

There are two possibilities for using the Mossbauer Effect in conjunction with the PIGA. The first way would be to use it as a position sensor for detecting β . The second way would be as a means of measuring ω , the output angular velocity.

The discussion of the Mossbauer Effect as a position sensor is contained in Chapter IV. The rotation of the gyro wheel is an

available means of establishing source absorber relative velocity. ω spin is usually high, of the order of 10^5 radians/sec (Ref 7:101, 102). For an absorber mounted just one centimeter from the center of rotation of the gyro wheel, and a source of photons aligned parallel to the SRA, by (64) an angle β of .02 seconds of arc would result in a relative velocity of 10^{-2} cm/sec. However, the limitation will not be the angular speed of rotation or the dimensions of the wheel but the size of the slit in the shielding that must be employed. The example in Chapter IV suggests a limitation of 2" of arc. A pickoff accuracy of 2 seconds of arc is not far (if at all) removed from the present state of the art. For instance, the M.I.T. Instrumentation Laboratory, Type H Integrating Gyro has a pickoff uncertainty of 4 seconds of arc (Ref 27:66,67).

The uncertainty in ω that would be caused by a mass unbalance of 15×10^{-8} cm is of the order of 10^{-7} radians/second (Ref 7:103). As mentioned before, present techniques call for measuring the angular displacement, θ , and thus determining the integral of acceleration, which is velocity. However, any uncertainty in ω will be integrated over time to yield an uncertainty in θ , thus an error in the velocity measurement. It is instructive to estimate whether, by using the Mossbauer Effect, it is possible to measure an angular velocity to an accuracy commensurate with the uncertainty in ω due to mass unbalance. Typical PIGA values are:

$$H = 10^6 \text{ gm cm}^2/\text{sec}$$

$$M = 500 \text{ gms}$$

(Ref 7:101-102). Using $l = 2 \text{ cm}$ in (74), $\omega = 10^{-3} \text{ A}$. If $\dot{A} = l\dot{g} = 10^3 \text{ cm/sec}^2$, then $\dot{\omega} = 1 \text{ rad/sec}$. From Figure 10, 10^{-6} rad/sec is the minimum discernable angular rate. This corresponds to an uncertainty in acceleration of 10^{-3} cm/sec^2 or 10^{-6} g , and a resolution at $l\dot{g}$ of 10^6 . This implies measurement of relative velocity of 10^{-5} cm/sec which requires a minimum counting time of 1 second. In other words, the system will have a comparatively slow response time. By the conventional method, measuring θ , just 100 seconds of system operation will result in an angular displacement error ($\Delta\theta = \Delta\omega t$) of 10^{-5} radians or 2 seconds of arc, (If $\Delta\omega = 10^{-7}$ radians/sec due to mass unbalance). This will be within the region of uncertainty of the pickoff and after any period of operation considerably longer than 100 seconds the system resolution will be limited by the inherent uncertainty in mass unbalance rather than by the uncertainty in pickoff accuracy. The conclusion is that there is no advantage to be gained by applying the Mossbauer Effect to measurement of the output angular rate of a PIGA.

It is hoped that the preceding discussion has revealed to some extent the orders of magnitude and degrees of tolerance involved in improving the performance of accelerometers used for inertial guidance. It is not inferred that the Mossbauer Effect cannot be used

to improve the performance of any type of force balance accelerometer. One of the difficulties in seeking to apply the Mossbauer Effect to the PIGA is the inherently low sensitivity of ω to A in such a device. This may be inferred from the relationship $\omega = 10^{-3}A$. In this case it is necessary to accurately measure very small values of ω in order to obtain a high resolution in A . One can conceive of a device which offers a more favorable relationship of ω to A . Such a device is illustrated schematically in Figure 14 and will be referred to as a centrifugal reaction accelerometer.

Centrifugal Reaction Accelerometer. Figure 14a is a diagram of the lumped mechanical components of a centrifugal reaction accelerometer. The accelerated system consists of an axis about which is suspended a mass, M , at a distance, l , by a weightless arm which makes an angle β with the axis. The mass, arm, and axis are all rotating at an angular rate ω . r is the distance from the mass perpendicular to the axis. The mass is acted on by two D'Alembert forces, due to the system acceleration and the centripetal acceleration. Taking moments about the pivot joining the arm and the axis,

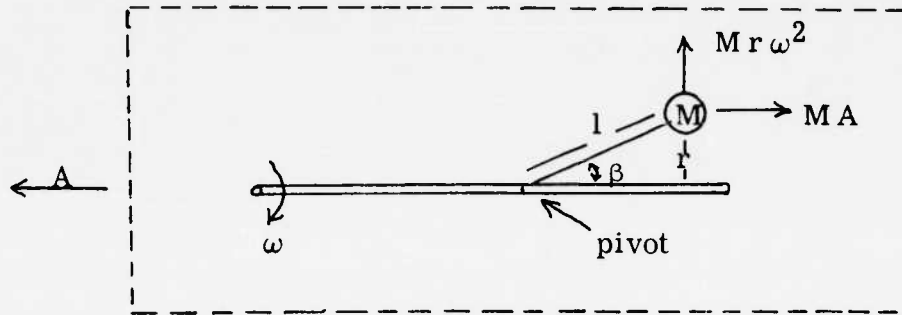
$$M r \omega^2 \cos \beta = M A l \sin \beta \quad (75)$$

Since $r = l \sin \beta$, it can be shown that

$$\omega^2 = A / l \cos \beta \quad (76)$$

Since l is a fixed quantity, when ω and β are known, A is determined.

a.



b.

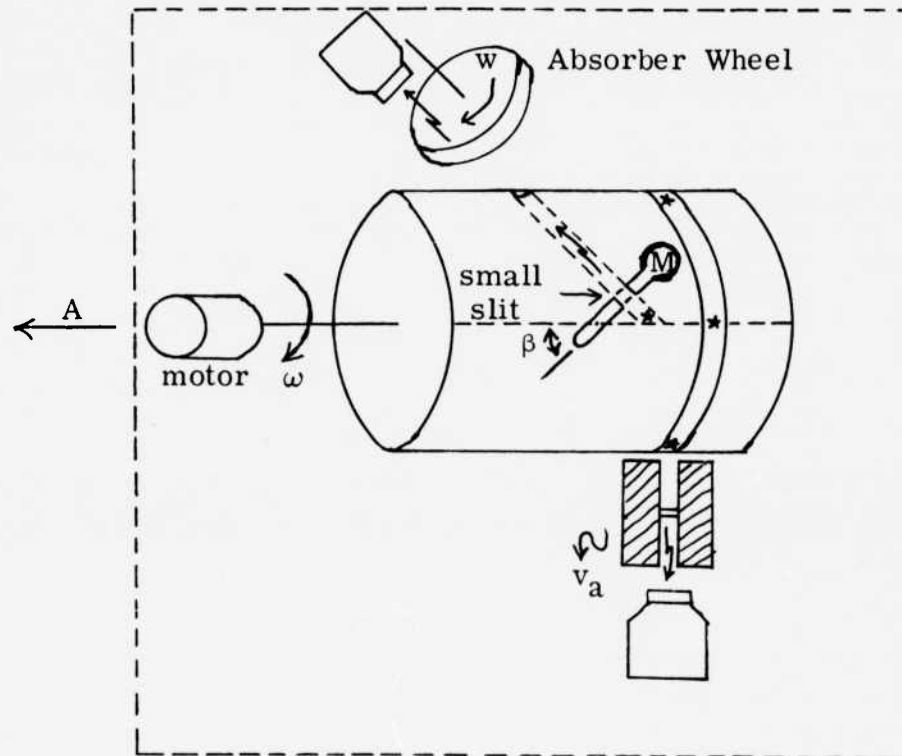


Figure 14

Schematic Diagrams of a Centrifugal Reaction Accelerometer

For nominal values take $l = 10$ cm, $\cos \beta = 1$. Then $\omega^2 = 0.1A$.

This indicates that the values of ω in the centrifugal reaction accelerometer will be correspondingly larger than the values of ω in the PIGA.

In practice, either β or ω may be held to a null value through the action of a servomechanism, or both β and ω may be measured. Figure 14b shows the Mossbauer Effect used to measure both β and ω . When β is at the prescribed value, photons from the source pass parallel to the axis of rotation of the absorber wheel. Thus there is zero relative velocity between source and absorber along the line of photon emission and a maximum of Mossbauer absorption takes place. When β is not at the proper value, the slit in the arm will not be parallel to the axis of rotation of the absorber wheel, and there will be relative velocity between source and absorber along the line of emission. ω may be measured by either the phase difference method or the amplitude difference method.

The resolution that may be obtained is estimated in the manner previously discussed. The size of the slit may limit resolution in β to about 10^{-5} radians or 2 seconds of arc. If, by (76)

$$A = \omega^2 l \cos \beta \quad (77)$$

then

$$\frac{dA}{d\beta} = -\omega^2 l \sin \beta \quad (78)$$

Dividing (78) by (77), and making the approximation $\Delta A = dA$,

$$\Delta A / A = -\tan \beta \Delta \beta \quad (79)$$

If $\tan \beta = 1$ and $\Delta \beta = 10^{-3}$ radians, then $A / \Delta A = 10^5$. This is not exceptional resolution as compared to the PIGA. It should be noted that by making $\tan \beta$, or β , very small, the resolution could be increased. However, if any lateral acceleration is present, making β small will lead to an unstable system. This is inferred by noting that as β becomes very small the moment arm for lateral acceleration becomes large compared to the moment arm for acceleration along the axis of rotation.

It is interesting to speculate what performance could be achieved using the Mossbauer Effect to measure angular velocity if unlimited resolution in β were achieved using some means other than the Mossbauer Effect. (77) is differentiated as follows:

$$2 \omega \frac{d\omega}{dA} = \frac{1}{I \cos \beta} \quad (80)$$

If $d\omega = \Delta\omega$, by (80 and (77)

$$\frac{\Delta\omega}{\omega} = \frac{1}{2} \frac{\Delta A}{A} \quad (81)$$

From Figure 10, if $\omega = 10$ rad/sec, $A / \Delta A = 5 \times 10^7$.

This is promising, however it must be accompanied by comparable resolution in β . A plot of resolution in acceleration vs resolution in β is shown in Figure 15. The plot assumes $\tan \beta = 1$ in (79). Unless a very large instrument is used, no known method

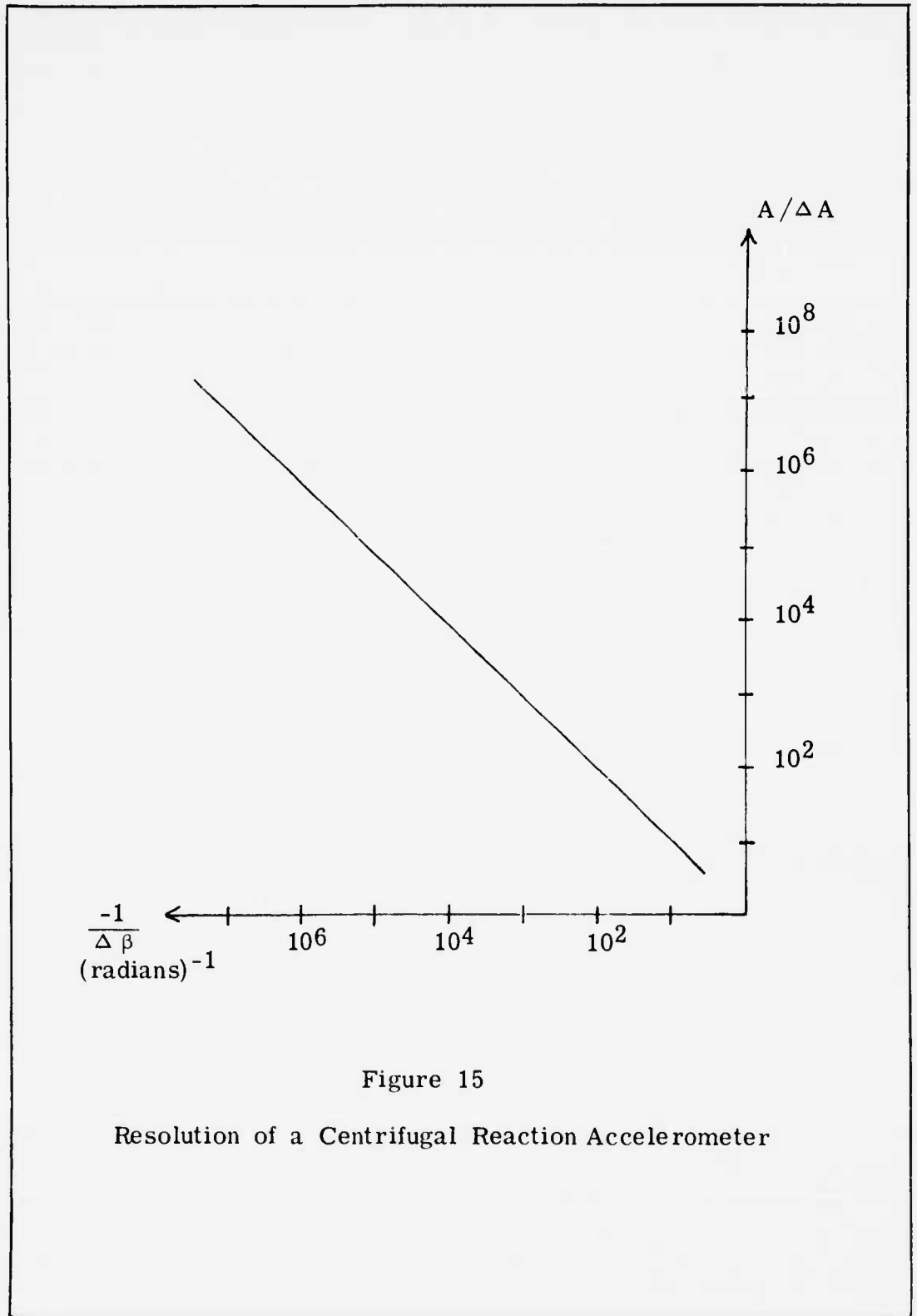


Figure 15

Resolution of a Centrifugal Reaction Accelerometer

will provide the accuracy of angular measurement required to allow a resolution higher than 10^5 with the centrifugal reaction accelerometer.

One further point is the question of whether the Mossbauer Effect can be used in a servo loop. When it is considered that a relative velocity of a little more than 10^{-2} cm/sec will destroy resonant absorption completely, and that with a maximum counting rate of 10^6 /sec one whole millisecond is required to obtain just 100 counts, it is difficult to justify the use of the Mossbauer Effect in a modern control system.

If the examples are indicative, then the application of the suggested techniques is not a promising method for improving the resolution of force balance accelerometers. However, the best that can be said is that the discussion emphasizes the fact that the resolution of such accelerometers is limited by a variety of factors. Even assuming a technique which promises extreme accuracy in the measurement of a particular parameter such as angular velocity, it is often useless, considering uncertainties in other important parameters such as mass unbalance and angular position, to apply this technique to improve the accuracy of the pickoff system.

In the light of these considerations, the accelerometer discussed in the following section is of particular significance.

Application to Unrestrained Devices

An interesting example of an unrestrained accelerometer is the rotating pendulum accelerometer, described by Samuel Schalkowsky and Henry F. Blazek (Ref 25:469-473). The system is illustrated schematically in Figure 16. A mass M is rigidly suspended from a pivot O at a distance r . The mass is rotated through an angle θ at a rate ω . The system is accelerated at the rate A in a direction measured from the y' y axis by the angle ϕ . The equation of motion is

$$I \frac{d^2\theta}{dt^2} + C \frac{d\theta}{dt} + MAr \sin(\theta + \phi) = T_d \quad (82)$$

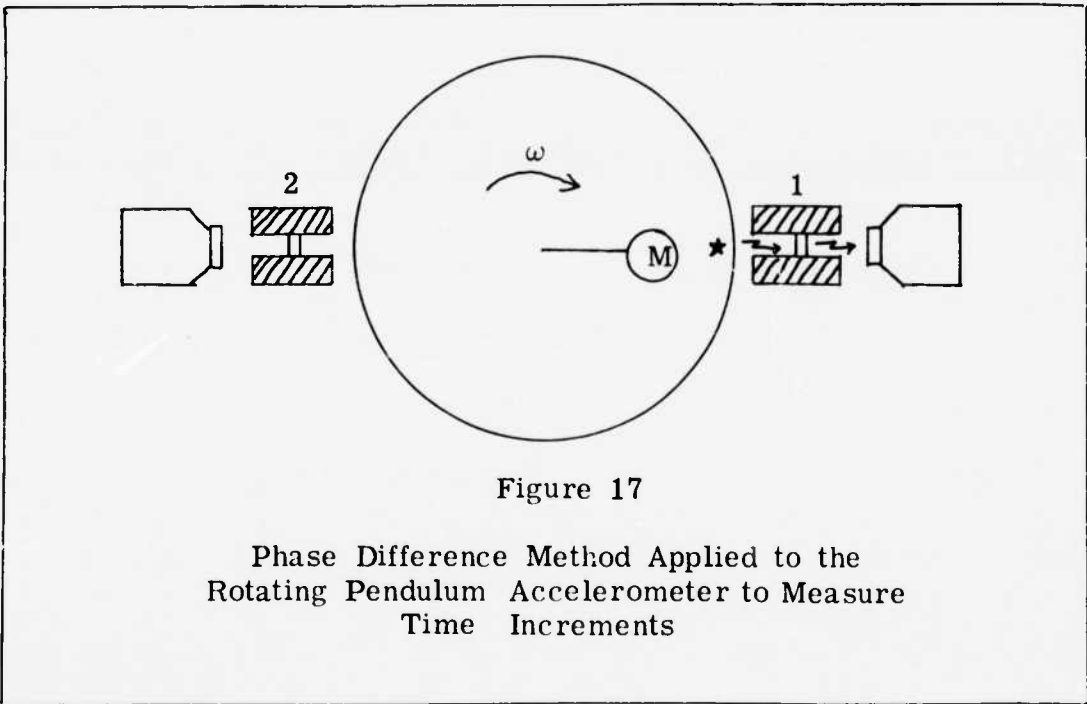
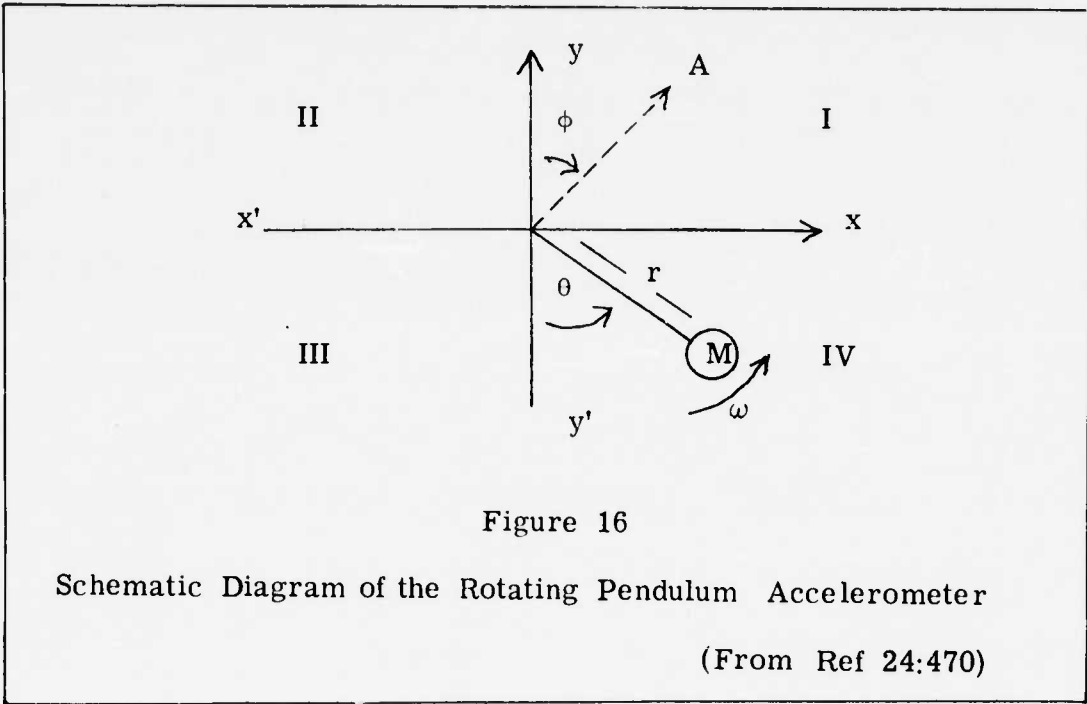
where I is the moment of inertia of the rotating pendulum, C is the coefficient of viscous friction, and T_d is the driving torque (Ref 25:471). The following approximations allow an analytic solution of the equation:

Let $\theta = \omega_a t$ in $\sin(\theta + \phi)$, assuming that the effect of the acceleration produces small distortion in a constant angular rate, ω_a .

Let $T_d = C\omega_a$, assuming that the driving torque is constant and just sufficient to overcome the effects of viscous friction.

The approximate solutions of the equation are:

$$\theta = \omega_a t + \frac{MrA}{I\omega_a^2 \left(\frac{1 + \frac{C^2}{I^2\omega_a^2}}{1 + \frac{C^2}{I^2\omega_a^2}} \right)^{1/2}} \times \sin \left(\omega_a t + \phi + \tan^{-1} \frac{C}{I\omega_a} \right) - \frac{MrA}{\omega_a C} \cos \phi \quad (83)$$



$$\dot{\theta} = \omega_a + \frac{MrA}{I\omega_a \left(1 + \frac{C^2}{I\omega_a^2}\right)^{1/2}} \times \cos \left(\omega_a t + \phi + \tan^{-1} \frac{C}{I\omega_a} \right) \quad (84)$$

(Ref 25:471). It is now noted that by making $C \ll I\omega_a$, the solution in $\dot{\theta}$ becomes independent of C ; that is,

$$\dot{\theta} = \omega_a + \frac{MrA}{I\omega_a} \cos (\omega_a t + \phi) \quad (85)$$

If the component of angular rate represented by the last term can be measured, then both the magnitude and direction of system acceleration can be determined: Let

$$\dot{\theta} = \omega_a + \Delta\omega_a \quad (86)$$

where

$$\Delta\omega_a = \frac{MrA}{I\omega_a} \cos (\omega_a t + \phi); \quad (87)$$

or, considering amplitude only

$$\Delta\omega_a = \frac{MrA}{I\omega_a} \quad (88)$$

If the entire rotating system consists of the mass, M , suspended at the distance r , then $I = Mr^2$, and

$$\Delta\omega_a = \frac{A}{r\omega_a} \quad (89)$$

It is desirable to make ω_a as small as possible in order to obtain a high sensitivity of $\Delta\omega_a$ to A . If $\omega_a = 1$ radian/sec, and $r = 10$ cm, and if $\Delta\omega_a$ is measured to 10^{-6} radians/sec using the Mossbauer Effect, then an acceleration of 10^{-5} cm/sec² or 10^{-8} g is the mini-

mum detectable acceleration, or threshold sensitivity. This may be reduced even further by reducing ω_a . The limiting condition is that $C \ll I\omega_a$.

It is possible to apply another sensing technique to the rotating pendulum accelerometer. By measuring the time taken by the pendulum to pass through two quadrants and comparing it to the time taken by the pendulum to pass through the remaining two quadrants, both the magnitude and direction of the incident acceleration may be measured. The expressions are

$$A = \frac{I\omega_a^3}{4Mr} (\Delta t_1^2 + \Delta t_2^2)^{1/2} \quad (90)$$

$$\phi = \tan^{-1} \Delta t_2 / \Delta t_1 \quad (91)$$

if $C \ll I\omega_a$ (Ref 25:472). Δt_1 is the time to traverse quadrants II and III minus the time to traverse quadrants I and IV. Δt_2 is the time to traverse quadrants I and II minus the time to traverse quadrants III and IV.

Figure 17 represents the phase difference method used to instrument the rotating pendulum accelerometer. Suppose that both absorbers 1 and 2 are kept in phase with the emitter which is located adjacent to the pendulous mass. Then the difference in phase between the excitation voltages supplying the coils 1 and 2 will represent a phase change due to Δt_1 or Δt_2 , depending on the orientation of the acceleration. For the case, $\Delta t_2 = 0$, a maximum phase difference

would be equal to $\omega_a \Delta t_1$. Then in analogy to (34), the expression for relative velocity between source and absorber becomes

$$v_r = V_e \sin \omega_a t - V_a \sin \omega_a (t + \Delta t_1) \quad (92)$$

This can be approximated (considering absolute values) by

$$v_r = V_e \omega_a \Delta t_1 = R \omega_a^2 \Delta t_1 \quad (93)$$

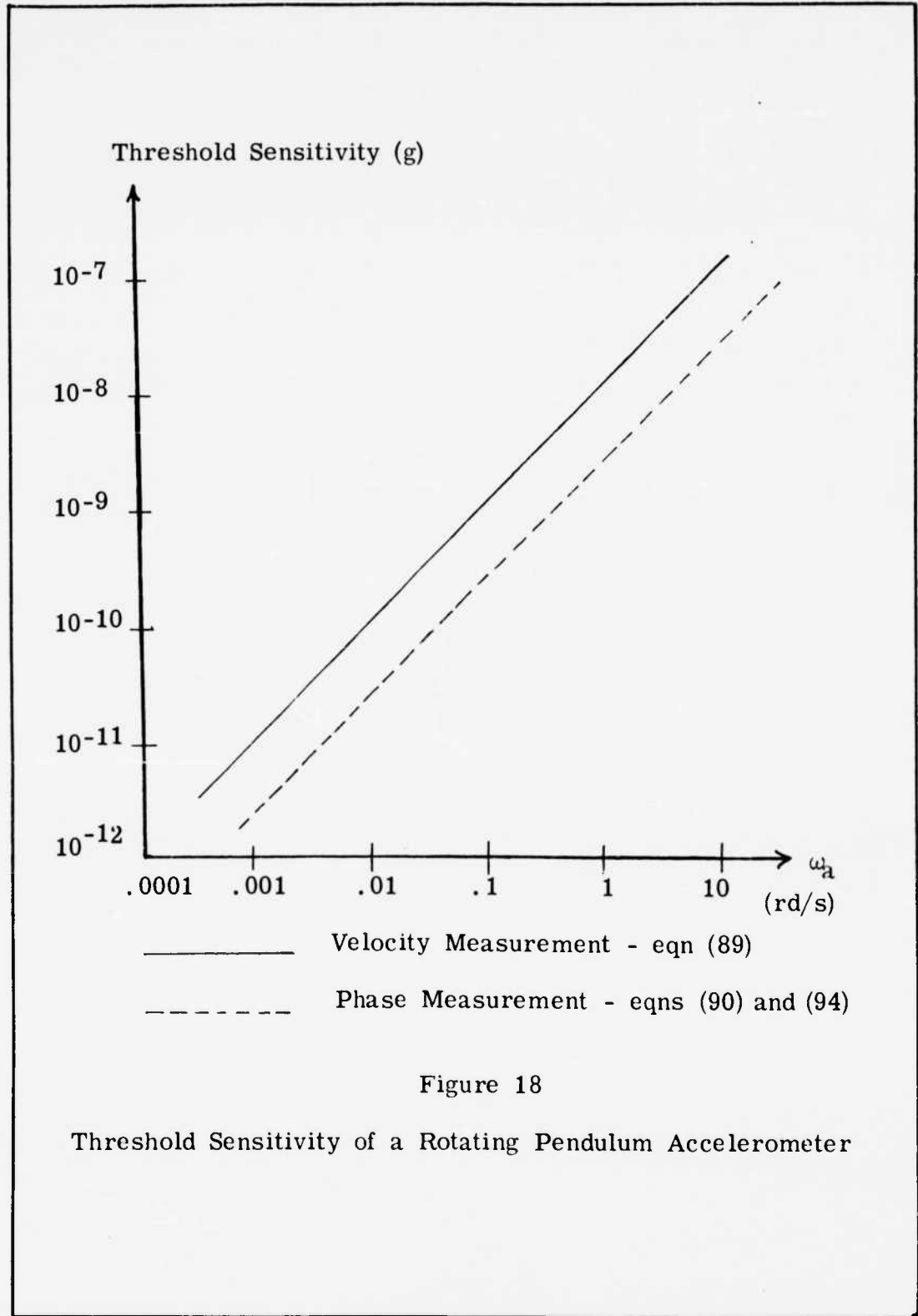
Or, equally

$$\Delta t_1 = \frac{v_r}{r \omega_a^2} \quad (94)$$

By using (90) and (94), an estimate can be made of the minimum threshold sensitivity that can be achieved with this method. The results are plotted in Figure 18, which also shows the equivalent results from (89). In both methods it is assumed that $I = Mr^2$, $v_r = 10^{-5}$ cm/sec, and $r = 10$ cm.

Figure 18 indicates that extremely low values of threshold sensitivity can be achieved for small values of ω_a . This implies that the time response of the system will be very slow. An application for such an instrument would probably be to instrument the gravity gradient techniques described in Appendix A, or to monitor the performance of a low thrust propulsion device.

The rotating pendulum accelerometer is unique in that by proper design the output can be made independent of troublesome parameters such as friction. It is better suited for an application of the Mossbauer Effect than most accelerometers because the slow



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period of rotation should afford ample time for counting. In a real instrument, of course, a number of emitters and absorbers would have to be used so that counting time would be maximized and so that a 360° profile of the pendulum velocity could be obtained.

VI. Conclusions and Recommendations

The purpose of this paper was to present the results of an investigation to determine whether the Mossbauer Effect can be applied to accelerometers used for inertial navigation. The investigation has not led to a final conclusion, but it is believed that the material presented in the Thesis supports the following statements.

Conclusions

1. The Mossbauer Effect is a practical means of eliminating recoil energy losses from the emission of gamma photons. When these recoil losses are eliminated the photon energy can be determined to an uncertainty limited only by the Heisenberg Uncertainty Principle. This allows experiments that are sensitive to energy differences of one part in 10^{12} - 10^{15} , depending on the precision of the experiment, the isotope used, and the time available for counting photons.

2. Major limitations on the application of the Mossbauer Effect are the time required to count photons and the energy linewidth of gamma photons from available isotopes. The use of the isotope Zn^{67} , which exhibits the smallest linewidth, is impractical because of the relatively small (1%) fraction of zero phonon events that occur in this isotope and the concomitant small cross section for resonant absorption. Fe^{57} , which has a Mossbauer cross section about 10^4 that of Zn^{67} , exhibits the second smallest energy

linewidth as well as the highest fraction of zero phonon events of any Mossbauer isotope. These factors, together with the desirable characteristic of a Debye Temperature of about 450°K which permits a large fraction of zero phonon events at high temperatures, make Fe^{57} the most suitable isotope for a practical application.

3. Of all the physical phenomena which alter Mossbauer absorption, a relative velocity between source and absorber produces the most pronounced effect. If 10^6 photons can be counted in one second, then a relative velocity of the order 10^{-5} cm/sec can be detected by the use of the isotope Fe^{57} . An acceleration of $3g$ would be required to produce the same effect as a relative velocity of 10^{-5} cm/sec. It is therefore desirable to attempt the use of the relative velocity effect rather than the direct measurement of acceleration for an application to accelerometers.

4. There are several possible techniques by which the Mossbauer Effect can be used to measure the velocity of an instrument component. It is conceivable that angular velocities may be measured to an accuracy of 10^{-6} radians/second or less. There is also a possibility that the Mossbauer Effect can be used to measure angular position, but accuracy at distances of 10 cm is limited to about $2''$ of arc by the requirement for collimating photons.

5. The application of the aforementioned techniques to accelerometers is limited primarily by uncertainties present in

accelerometers from causes other than the resolution of the pickoff system. Table II presents rough estimates of the resolution obtainable from various systems and the limiting factors.

Table II
Resolution of Various Types of Accelerometers

<u>Type Accelerometer</u>	<u>Maximum Resolution</u>	<u>Limitation</u>
Simply Restrained	$10^3 - 10^4$	Friction-Hysteresis
PIGA	10^7	Mass Unbalance
Centrifugal Reaction	10^5	Angular Measurement

It appears that none of the accelerometers discussed can be improved in resolution by application of the suggested techniques.

6. On the other hand, it should be possible to measure extremely small values of acceleration by applying the Mossbauer Effect to the measurement of angular velocity or phase difference from a rotating pendulum accelerometer. The threshold sensitivity that can be achieved in this manner is of the order $10^{-8}g$, if 2π seconds is the period of the rotating pendulum. For larger periods the threshold sensitivity decreases linearly.

Recommendations

Further study in the application of the Mossbauer Effect to accelerometers and other precision devices is indicated. It is certain that this paper has not "uncovered" all the possibilities. It would seem probable that an experienced engineer could contrive a useful device from a phenomenon as sensitive to small energy

changes as the Mossbauer Effect.

Experimental verification is required for the velocity and position measurement techniques described in Chapter IV. If proven feasible, the techniques should be inherently useful. The research involved in this study has not revealed any other techniques that purport to measure small angular velocities to the degree of accuracy indicated by Chapter IV.

It should be worthwhile to further study the application to the rotating pendulum accelerometer. If the small threshold sensitivity indicated by this study can be reached in practice, then this will provide a needed means of monitoring low thrust propulsion devices and a means of realizing the navigation techniques suggested in Appendix A.

Final Statement

The Mossbauer Effect is similar to many other discoveries of contemporary science in that it contributes knowledge and new techniques to nuclear and solid state physics. Just as other modern discoveries are leading to profound engineering applications such as the transistor, the LASER, and electric propulsion, it is probable that the Mossbauer Effect will eventually play a significant role in the development of now unknown devices. It is our belief that any effort spent studying the Mossbauer Effect from an engineering standpoint will not be wasted.

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Appendix A

Navigation by Gravity Gradient Techniques

This Appendix contains an example of how specific forces due to a gravity gradient may act on a space vehicle. The suggestion is made that, if an instrument capable of sensing these forces becomes available, then such an instrument can be used effectively as a means of navigation and control.

In Figure 19a a space vehicle is represented in free fall toward the center of attraction, G, of the earth. At the center of gravity (C.G.) of the vehicle, the kinematic acceleration of falling just balances g . However, the two masses, M_1 and M_2 , are subject to the horizontal component of g , $g \cos \beta$, and since there is no horizontal component of acceleration, the springs must supply a tensile force to restrain the masses. The magnitude of the force is small, since $\cos \beta$ is L / r where L is the distance of either mass from the C.G., and r is the distance from the earth center to the C.G. A nominal value of the specific force, for $L = 1$ foot, $r =$ earth radius, would be $5 \times 10^{-8} g$.

In Figure 19b another case of specific force due to a gravity gradient is represented. Suppose that M_1 and M_2 are oriented along the line of free fall as shown. Since M_1 and M_2 are constrained by the structure of the vehicle they must fall at the same rate as the C.G. However, the C.G. accelerates in response to g , whereas

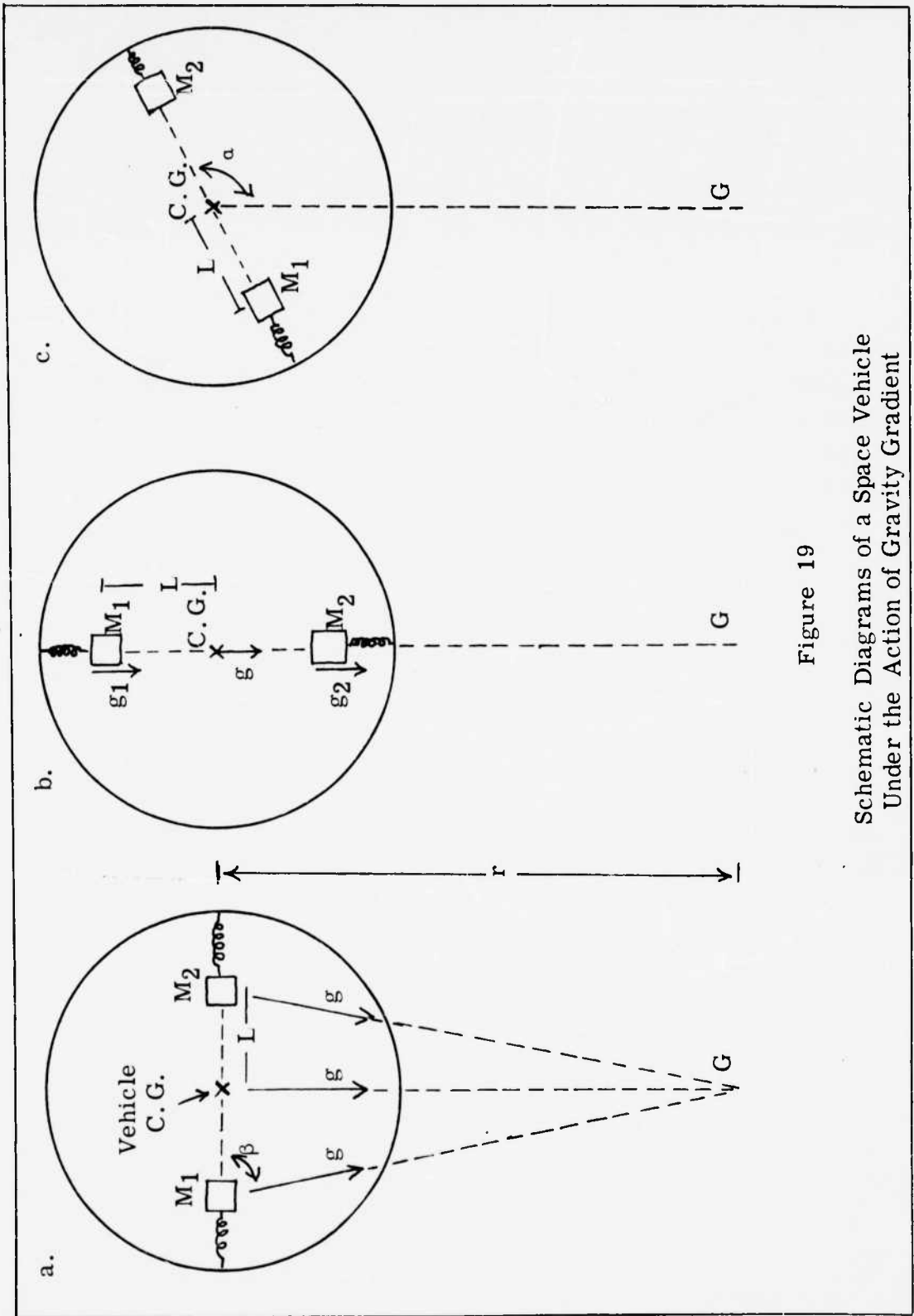


Figure 19
 Schematic Diagrams of a Space Vehicle
 Under the Action of Gravity Gradient

the specific forces acting on the masses are g_1 and g_2 , where, for example

$$g_1 = g \left(\frac{r}{r+L} \right)^2 \quad (95)$$

in accordance with the well known inverse square law of gravity. Therefore, the spring supporting M_1 must supply a compressive force of $M_1 (g - g_1)$. The nominal value of such a force is small. For $L = 1$ foot, $r =$ earth radius, the specific force would be about 10^{-7} g.

The two cases just discussed can be generalized for an arbitrary orientation of the vehicle. In Figure 19c α is the angle between the line joining Earth center and the C.G. and the line M_1 and M_2 . A good approximation for the specific force in the springs, Δg , is

$$\Delta g = - \left(2 g_s \frac{R^2}{r^2} \right) \left(\frac{L}{r} \right) (1 - 3 \cos^2 \alpha) \quad (96)$$

where g_s is the specific force of gravity at the Earth's surface, and R is the radius of the Earth. A tensile force is taken as positive (Ref 2:46). Using (96) one can determine that there is zero specific force for four vehicle orientations:

$$\alpha = 25^\circ 16', 54^\circ 44', 234^\circ 44', 305^\circ 16'.$$

Since specific force is equivalent to acceleration in its effect on a mass, it is apparent that very sensitive accelerometers could be used to determine the orientation of the vehicle with respect to

the attracting body. If the vehicle orientation is compared to an inertial reference established by gyroscopes, then the angular rate of the space vehicle about a planet can be determined. The magnitude of the specific forces will provide information about the distance of the vehicle from the central body. This is summarized by the following quotation from a more detailed discussion of the same topic presented here:

"First, it would be possible to slave an inertial platform in a satellite vehicle into continuous alignment with the equipotential surfaces of gravity and thus, by purely inertial means, to determine continuously the attitude of the vehicle with respect to the local vertical. Furthermore, this slaving could also be applied to the orbit plane, and, finally, the angular velocity of the satellite in its orbit can be directly derived.

"It follows that, if the gravitational constant of the attracting body is known, the radial distance from the center of attraction can be determined and, conversely, if the radial distance is known, then the gravitational constant can be determined."
(Ref 2:45)

Appendix B

List of Isotopes Exhibiting the Mossbauer Effect *

Stable Nucleus	a %	E _γ keV	T _m sec x 10 ⁷	α	R ev	θ _d OK	f at 0°K	σ ₀ Cm ² x10 ¹⁸	Parent Nucleus	T _{1/2}
Fe ⁵⁷	2.17	14.4	1.44	15	.002	420-467	0.9	1.5	Co ⁵⁷	270d
Zn ⁶⁷	4.11	93	134	(0.5 (.063	.069	(308 (235-320	(.004 (.01	.13	Cu ⁶⁷ / Ga ⁶⁷	(59h (80h
Sn ¹¹⁹	8.6	24	.27	6.3	.0025	(212 (140-260	0.78	1.1	Sb ¹¹⁹	39h
Dy ¹⁶¹	18.8	26	.39	unk	.0023	158-182	0.8	unk	Tb ¹⁶¹	6.88d
Er ¹⁶⁶	33.4	81	.024	1.9	.020	163-192	0.1	.64	Ho ¹⁶⁶ / Tm ¹⁶⁶	(27.3h (7.7h
Hf ¹⁷⁷	18.6	113	unk	0.75	.038	150-213	0.03	.14	Lu ¹⁷⁷ / Ta ¹⁷⁷	(6.75d (53h
Ta ¹⁸¹	100	136	.00082	1.2	.0545	225-245	0.02	.075	Hf ¹⁸¹ / W ¹⁸¹	44d (145d
W ¹⁸²	26.4	100	.019	4.4	.029	310-384	0.2	.23	Ta ¹⁸²	115d
Re ¹⁸⁷	62.9	134	unk	2.1	.051	300	0.05	.051	W ¹⁸⁷	24h
Ir ¹⁹¹	38.5	129	.0014	2.1	.047	285-289	0.06	.071	Os ¹⁹¹ / Pt ¹⁹¹	(16d (3d

Stable Nucleus	a %	E_γ keV	T_m sec x 10^7 a	R ev	θ_d °K	f at 0°K	σ_0 $\text{Cm}^2 \times 10^{18}$	Parent Nucleus	$T_{1/2}$
Ir ¹⁹³	61.5	73	.082 3	.014	285-289	0.4	unk	Os ¹⁹³	30.6h
Au ¹⁹⁷	100	77	.027 2.5	.016	165-180	0.2	.059	Pt ¹⁹⁷ / Hg ¹⁹⁷	18h 65h

* Extracted and compiled from Ref 15:61-68 and Ref 1:84-87.

The Mossbauer Effect has been observed for all the isotopes listed.

a = natural abundance of the isotope

$T_{1/2}$ = half life of the parent nucleus (d = days, h = hours)

Vita

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