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DESIGN ANALYSIS OF
A PARTICLE THERMAL RADIATOR
FOR SPACE VEHICLES

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THESIS

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Abstract

The design of a new type of thermal space radiator is extended. Micron-sized particles are heated and electrically charged on a high potential spherical electrode and ejected into space. A second high potential spherical electrode of opposite polarity attracts these particles and changes the polarity of their charge upon contact. The particles then return to the first electrode to complete the cycle. Along their trajectories the particles radiate thermal energy, giving a high area-to-mass ratio because of their small size. Equations are presented which permit calculation of power radiated-to-mass ratios for the entire system. Results of a computer study indicate that at power levels above one megawatt the new type of particle radiator will weigh approximately four to eight percent of the weight of its conventional tube-and-header counterpart. Limitations of the theory are investigated and discussed.

List of Symbols

A - Heat-transfer area of the heated electrode.

A_p - Surface area of each particle.

C_N - Specific heat of liquid sodium.

C_p - Specific heat of the particles.

d - Diameter of the particles in microns.

d' - Mean separation between the inner and outer surfaces of the heated electrode.

D - Separation distance between electrode centers.

e - Emissivity of the particles.

G - Weight flow rate of liquid sodium per unit area.

h - Heat-transfer coefficient.

I - Current due to charged particles flowing between the electrodes.

K_M - Thermal conductivity of molybdenum.

K_N - Thermal conductivity of liquid sodium.

K_p - Thermal conductivity of the particles.

m - Mass of each particle.

N_p - Number of particles in one layer on an electrode.

N_T - Total number of particles in the system.

q - Electric charge on each particle.

$\dot{q}$ - Rate of heat loss per particle.

$\dot{Q}$ - Rate of heat loss for the entire system.

r - Radius of particle in meters.

R - Radius of spherical electrodes.

t - Thickness of the outer sphere of the heated electrode.

t_c - Time of contact between particles and electrodes.

t_f - Representative time of flight from one electrode to the other.

t_h - Representative time required for a particle to heat from arrival temperature to the temperature of the electrode surface.

T_a - Temperature of the particles when they arrive on the heated electrode.

T_1 - Temperature of the particles as they leave the heated electrode.

ΔT - Temperature difference between the bulk temperature of the liquid sodium and the outer surface of the heated electrode.

U - Overall heat-transfer coefficient.

v - Velocity of the particle.

V - Potential difference required to produce an electric field on the surface of the electrode sufficient to cause charge transfer to a dielectric particle.

W_E - Weight of the electrical generator.

W_P - Weight of the particles.

W_S - Weight of the spherical electrodes and connecting structure.

W_T - Total weight of the radiator.

ρ_M - Density of molybdenum.

ρ_P - Density of the particle.

σ - Stephan Boltzmann constant.

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I. Introduction

The economical rejection of heat from space power plants has become an increasingly critical problem in space technology. For power plants operating above the one megawatt range, the conventional tube-and-header radiator is the heaviest single component. This weight is doubly critical because each extra pound makes vehicle launching and its subsequent maneuvering in space more difficult. Moreover, conventional radiators are vulnerable to meteoroid puncture. Extra weight in the form of protective thickness must be sustained to achieve the reliability needed for lengthy space missions. This study deals with a new type of variable-area thermal radiator which promises to have outstanding weight advantages over conventional radiators. In addition, its design permits protection from meteoroid damage with a very small weight penalty.

Modern technology compares radiators according to specific weight which is determined primarily by the area to volume ratio. For small spherical particles the area to volume ratio becomes

$$\frac{A}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r} \qquad (1)$$

It is tempting then to examine the use of particles of extremely small radius as radiators. Charles F. Neef has investigated the use of micron-sized charged particles confined in an electric field. For a representative radiator, his studies gave a performance figure of approximately 10^4 kw/kg for the particles alone, which did not include any weight for the source of the electrical system or for the external heat-transfer structures (Ref 8:26). This figure is three orders of magnitude higher than that of present day tube-and-header radiators.

The purpose of this study is to extend Neef's work to include the computation of the specific weight of the complete radiating system and, within certain limitations, to investigate the optimization of some of the design parameters. This study will include the following: (1) a general description of the radiator and the basic theory governing its operation, (2) the limitations of the theory including specific problem areas, (3) the analysis of the radiator weight and of the system reliability, and (4) the conclusions and recommendations of this study.

II. General Description and Theory

Description

The particle radiator depends on charge transfer from highly charged electrodes to small dielectric particles. The particles are placed on a dumbbell-shaped arrangement of two large spherical electrodes separated by a connecting cylinder as illustrated by Figure 1 on page 4. The electrodes are raised to a high potential of opposite polarity, and the associated electric field on the surface causes a charge transfer to the particles. One sphere is the heat source for the particles. The other sphere does no heating. Since the use of a liquid metal working fluid is contemplated, this arrangement simplifies the problem of electrical insulation of the two electrodes and allows greater separation without large heat-transfer mechanisms. As charge transfer occurs, the particles are repulsed by the like charge of the sphere. In the environment of the spacecraft the particles travel approximately along the lines of force between the two electrodes. The particles arrive at the second electrode and are held on the sphere by coulomb attraction. When charge transfer occurs, the particles return to the first electrode and the cycle repeats. As the particles travel, they individually radiate heat into space. Thus heat is removed from one sphere and radiated

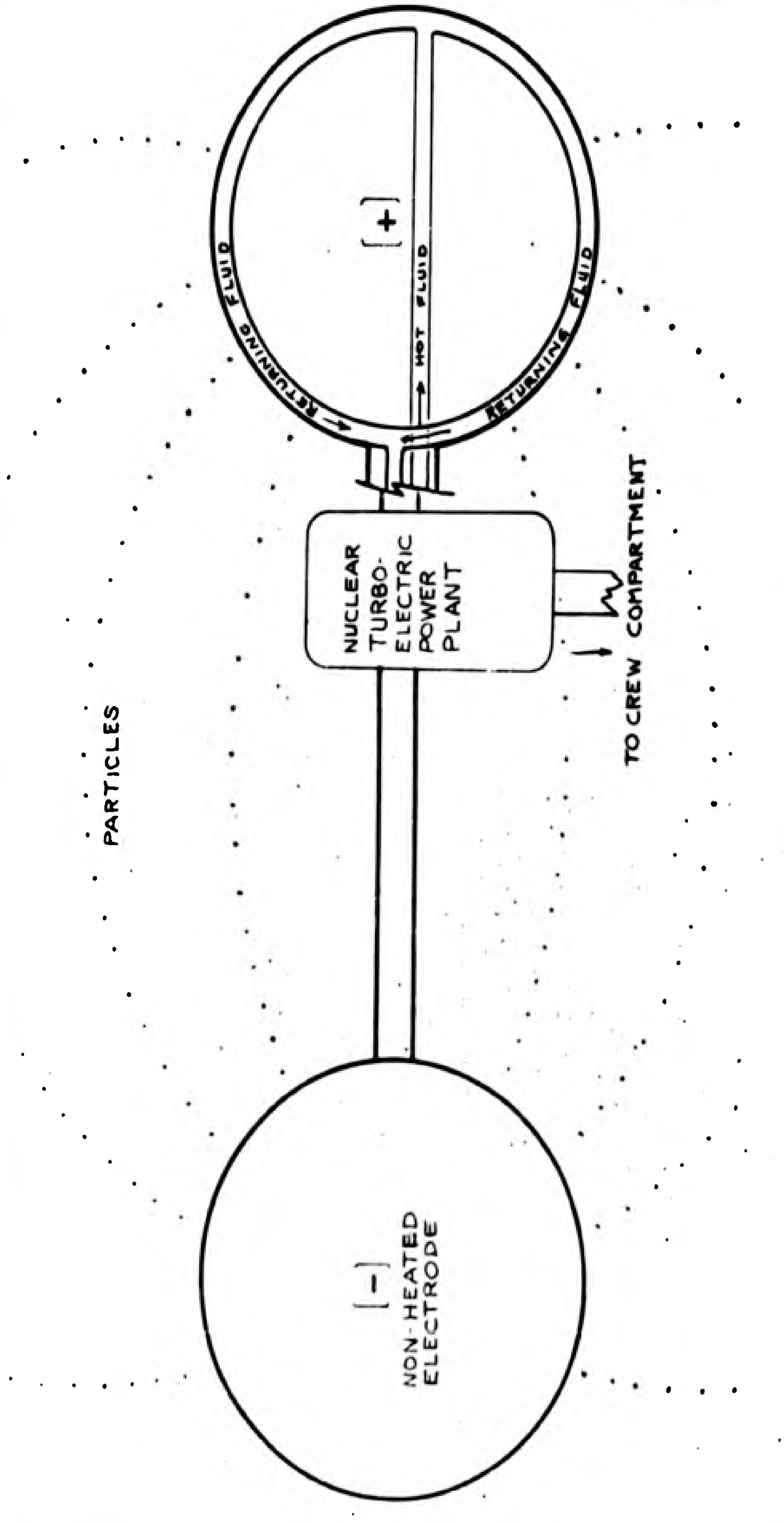

FIGURE 1 Basic Conceptual Design

into the space heat sink as the particles continue to circulate. This design differs from the one offered by Neef in that only one of the two electrodes transfers heat.

Particle Motion

To make some approximate calculations on the motion of the particles, one must consider several factors. The geometry of the electrodes and the physical properties of the particle material are important parameters. Also, the electric field, the operating voltage, and the charge carried by each particle must be found so that the time of flight may be determined. Knowing the time of flight will permit the calculation of the electric current carried by the particles and their specific weight.

Neef's study suggests that charge transfer does not occur until complete electrical breakdown of the dielectric (Ref 8:5); however, it appears that this may not be completely accurate. Several theories have been presented concerning the mechanisms which lead to electrical breakdown in solids. According to Stratton (Ref 11:235), the different theories of dielectric breakdown fall into two broad classifications, thermal and intrinsic. Intrinsic breakdown is further divided into three classes, collective, avalanche, and field emission breakdown. The field emission theory holds that electrons from the filled valance band of the dielectric can tunnel through the potential barrier, which corresponds to the gap of forbidden energy levels, into the conduction band. Once electrons are in the conduction band, charge transfer can occur. The collective and avalanche theories depend on different cumulative processes which precede actual breakdown. These three theories, in particular, lead one to believe that some charge transfer can occur in solid dielectrics prior to complete breakdown. Yet, due to the diversity of thought found in other theories and the lack of any single well-integrated theory on breakdown, experimental results appear to be the best source of information concerning charge transfer in dielectrics.

Shelton obtained a value of 1000 electrons when charging a 0.5 micron diameter particle with a high voltage probe (Ref 10:1243). Hewitt showed that for particles larger than 0.5 microns, the charge is proportional to the diameter squared (Ref 6:302). This finding correlates satisfactorily with the data compiled by Langer and Radnick which give charges of 1400, 500, and 120 electrons for dielectric particles of 0.9, 0.5, and 0.2 microns respectively (Ref 7:956). In the following analysis a charge of 1500 electrons for a one micron particle was adopted, and the charge on larger particles was assumed to vary in proportion to the diameter squared.

The electric field associated with the high potential spheres provides the force to move the particles through space and also charges the particles on the surface of the spheres. A representative path of motion between the two spheres was determined by dividing each sphere into two sectors each containing half of the particles resting on that sphere. The path was found by selecting a particle that departed the surface of the sphere at the dividing line. An averaging process on all paths was not attempted because of the great uncertainties in the problem as a whole.

For the system of two oppositely charged spheres, the lines of force lie on a series of spherical surfaces, each of which passes through the center of both charged electrodes. Methods exist whereby a more exact trajectory of a charged particle of finite mass can be calculated. For approximation purposes, however, it is assumed that in the zero gravity environment of space the particles will follow the lines of force between the electrodes. A two dimensional view of the spheres and the field lines is shown in Figure 2.

In view of the uncertainties of the charge and size of the particles and the complexity of the particle motion, only an order-of-magnitude calculation was made for the time of flight. A representative curvilinear path, ABC, was assumed and replaced by an equal length rectilinear path, AB'C, along which a constant acceleration was assumed to act. The dashed line in Figure 2 represents the curvilinear

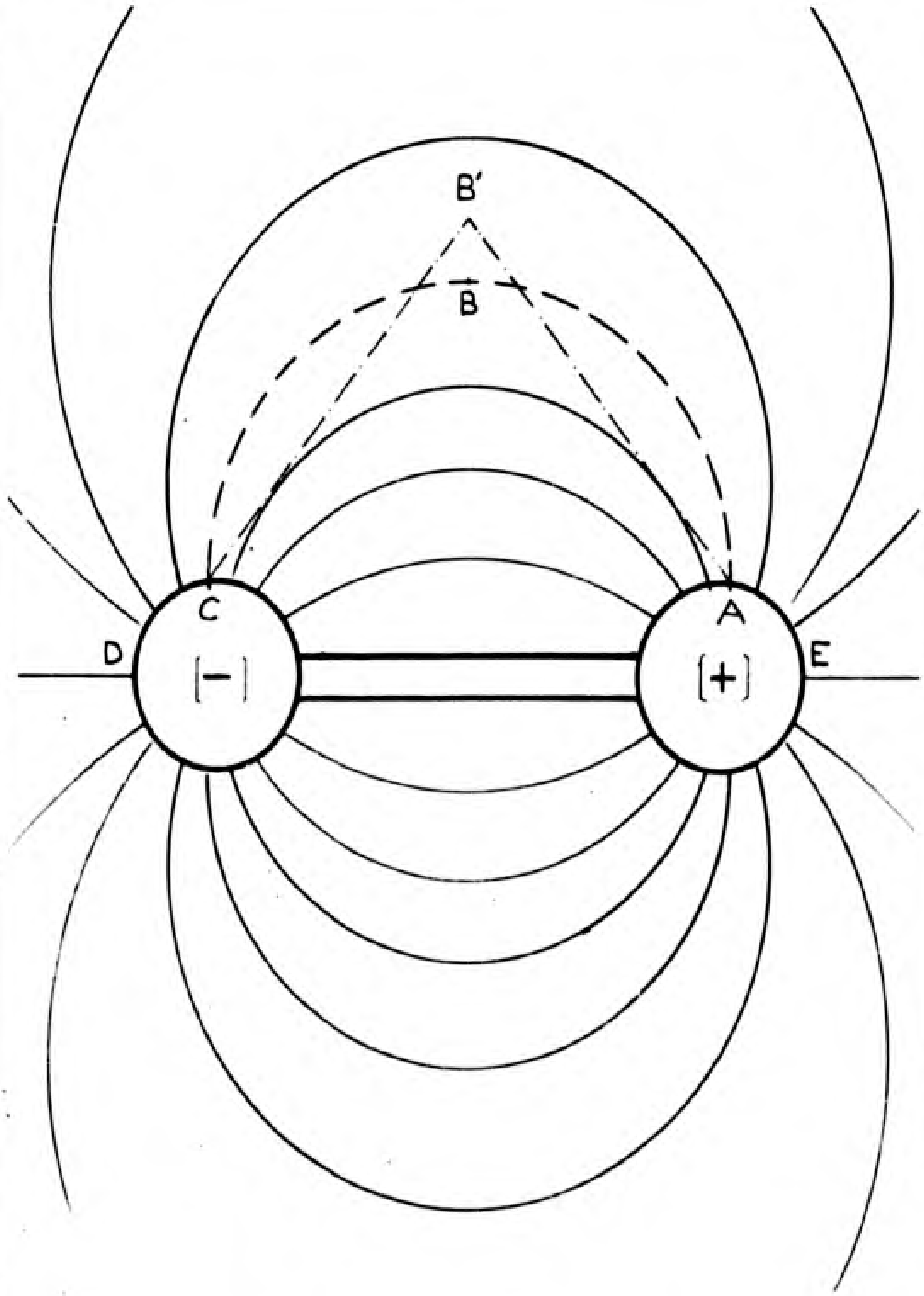

FIGURE 2. System Lines of Force

path for which the time of flight of a typical particle was calculated. Its length is $\frac{1}{2}\pi D$.

For an engineering calculation of the time of flight, specific selections of parameter values were made. In this study particles of Stained Ruby Mica were used throughout. Variations were made in the separation distance between the electrodes, D, the diameter of the particle, d, the operating voltage (the potential difference between electrodes), V, and, later, in the heat rejection temperature, T_1. The average velocity along each half of the straight line path was computed initially; the total time of flight was obtained by adding the time increments required for the particle to travel from A to B and from B to C. When any particle of mass (m) and charge (q) is accelerated from rest through a potential drop (V), its velocity (v) can be found from $v^2 = 2qV/m$.

For a particle at A initially at rest and acted upon by a constant voltage, the velocity of the particle at B is

$$v_B = \left[\frac{qV}{m}\right]^{\frac{1}{2}} \qquad (2)$$

where q is the charge on the particle, V is the potential difference between the two spheres and m is the mass of the particle. For a particle at B with initial velocity v_B, the velocity of C can be obtained from

$$v_C^2 = v_B^2 + \frac{qV}{m} = 2v_B^2 \qquad (3)$$

An approximate time of flight for a single representative particle is therefore obtained from

$$t_f = \frac{\frac{\pi}{4}D}{\frac{1}{2}v_B} + \frac{\frac{\pi}{4}D}{\frac{1}{2}\left[v_B + v_C\right]} = 2.21\left[\frac{m}{qV}\right]^{\frac{1}{2}} D \tag{4}$$

From the assumed charging conditions, q in coulombs is given by

$$q = 2.4\times10^{-16}\, d^2 \tag{5}$$

where d is the particle diameter in microns. For Stained Ruby Mica with a density of 3000 kg/m^3 the mass in kilograms is

$$m = 5\pi d^3 \times 10^{-16} \tag{6}$$

The time of flight in seconds is found by using Eqs (5) and (6) in Eq (4) which then becomes

$$t_f = 5.65\, D \left[\frac{d}{V}\right]^{\frac{1}{2}} \tag{7}$$

Variation of the size and charge of the particle, the operating voltage, and the separation distance gives the representative values shown in Table I.

Table I
Times of Flight

Particle Diameter (microns)	Operating Voltage (kilovolts)	Separation (meters)	Flight Time (seconds)
50	400	1.0	0.089
250	400	1.0	0.200
500	400	1.0	0.283
50	800	1.0	0.045
250	800	1.0	0.100
500	800	1.0	0.141
50	400	3.0	0.268
250	400	3.0	0.599
500	400	3.0	0.848

Heat Transfer

To be worthwhile the proposed radiator must reject heat efficiently into space. This is dependent on the working fluid transferring the excess heat to the electrode and on the particle removing this heat from the sphere and then radiating the energy during its flight between the spheres. Neef investigated heat transfer by the particle (Ref 8:18), and he determined that the arrival temperature, T_a , can be obtained from

$$\frac{1}{T_a^3} - \frac{1}{T_1^3} = \frac{3 A_p e \sigma t_f}{m c_p} \tag{8}$$

where m = mass of the particle

e = emissivity of the particle

A_p = surface area of the particle

C_p = particle specific heat

σ = Stephan Boltzmann constant

T_1 = temperature of the particle as it leaves the heated sphere

The arrival temperature in $^{\circ}K$ is found by inserting the physical properties of Stained Ruby Mica given in Appendix A into Eq (8) which becomes

$$T_a = \left[\frac{d T_1^3}{712 \times 10^{-7}\, t_f T_1^3 + d}\right]^{\frac{1}{3}} \tag{9}$$

In this study T_1 is assumed equal to the temperature at the surface of the heated electrode.

The heat loss rate per particle is given by

$$\dot{q} = m c_p \frac{\Delta T}{\Delta t} = m c_p \frac{T_1 - T_a}{2 t_f} \tag{10}$$

If $\dot{Q}$ is the total heat rejection rate, then the weight of the particles is given by

$$W_p = m \frac{\dot{Q}}{\dot{q}} \tag{11}$$

The specific weight of the Stained Ruby Mica particles in kg/kw is determined by substituting Eq (10) into Eq (11) which gives

$$\frac{W_p}{\dot{Q}} = \frac{2.32\, t_f}{T_1 - T_a} \tag{12}$$

The weight of the spheres and connecting structure, W_s, is primarily a function of the surface area of the rejecting sphere and the thickness necessary for meteoroid protection. For preliminary calculations a 95% probability of no puncture in one year was assumed. The heat-transfer electrode was considered to be a hollow molybdenum sphere in which an oblate sphere of smaller mean radius was fixed. The cross-section area along the flow direction was assumed constant. Liquid sodium was arbitrarily selected as the working fluid to flow between the spheres.

Preliminary calculations showed that the electrode surface area will be determined by the area required for heat transfer from within. An approximation was used to determine the heat-transfer coefficient for the sodium flow. Since the mean separation between the inner surface of the outer sphere and the outer surface of the inner sphere was small with respect to the radius of the electrode, a parallel plate assumption for heat flow was used. Such a correlation for liquid metal flow between parallel plates with heat flowing through only one plate is given by Brooks (Ref 1:278). The equation is

$$\frac{2hd'}{K_N} = 5.8 + 0.2\left(\frac{2d'c_N G}{K_N}\right)^{0.8} \qquad (13)$$

where h = heat-transfer coefficient

K_N = thermal conductivity of liquid sodium

d' = mean separation between inner and outer spheres

C_N = specific heat of liquid sodium

G = weight flow rate per unit area

The overall coefficient of heat transfer, U, is calculated from the following equation

$$\frac{1}{U} = \frac{1}{h} + \frac{t}{K_M} \qquad (14)$$

where t is the thickness of the outer sphere and K_M is the thermal conductivity of molybdenum. The required heat-transfer area, A, is given by

$$A = \frac{\dot{Q}}{U\,\Delta T} \tag{15}$$

where ΔT is the temperature difference between the bulk temperature of the working fluid and the outer surface of the heated electrode.

Only the outer surface of this electrode must be protected from meteoroid damage. Since very little data is available on meteoroid penetration into molybdenum, that given by Ross et al for steel was used (Ref 9:598). This choice is within the range of assumptions necessary in this study and should be on the conservative side in estimating the thicknesses required for meteoroid protection at differing heat rejection rates.

To obtain a minimum weight for the heat rejection sphere, the thickness which was assumed in the calculation of the rejection area was correlated with the minimum thickness required to meet the assumed criteria of the design. In the final analysis the total weight of the spheres and connecting structure is assumed to be 1.5 times the weight of the outer heated sphere alone. The weight of the spheres and supporting structure is then given by

$$W_S = 1.5 A t \rho_M = \frac{1.5 \rho_M \dot{Q} t}{U\,\Delta T} \tag{16}$$

where ρ_m is the density of molybdenum. Figure 3 gives the total weight of the electrodes and connecting structure at various heat rejection rates.

In Appendix B there is a sample calculation for a 100 megawatt radiating system. State-of-the-art representative values for system temperatures and flow rates were used. A heat rejection area of 40 square meters is required. This indicates that a particle radiator capable of rejecting 100 megawatts requires a heat-transfer electrode 3.6 meters in diameter.

Radiator Electrical System

An electrostatic generator is proposed to provide the electric field that charges the particles and forces them through space. Adequate voltages for electrostatic generators in the range from 200 kilovolts to one megavolt are now in development. A direct current, variable capacitance, dc-excited, machine has been chosen as the most attractive; it is described by Denholm et al (Ref 3:14). Its ceramic and metal construction lends itself to designs for high temperatures, and except for the flexibility of its stator and rotor disks, the generator is sturdy and reliable. Such electrostatic generators can be designed with a specific mass of 0.9 kg/kwe, but Gale estimates this figure can be reduced to 0.2 (Ref 4:168). The former specific mass was used for calculations in this study.

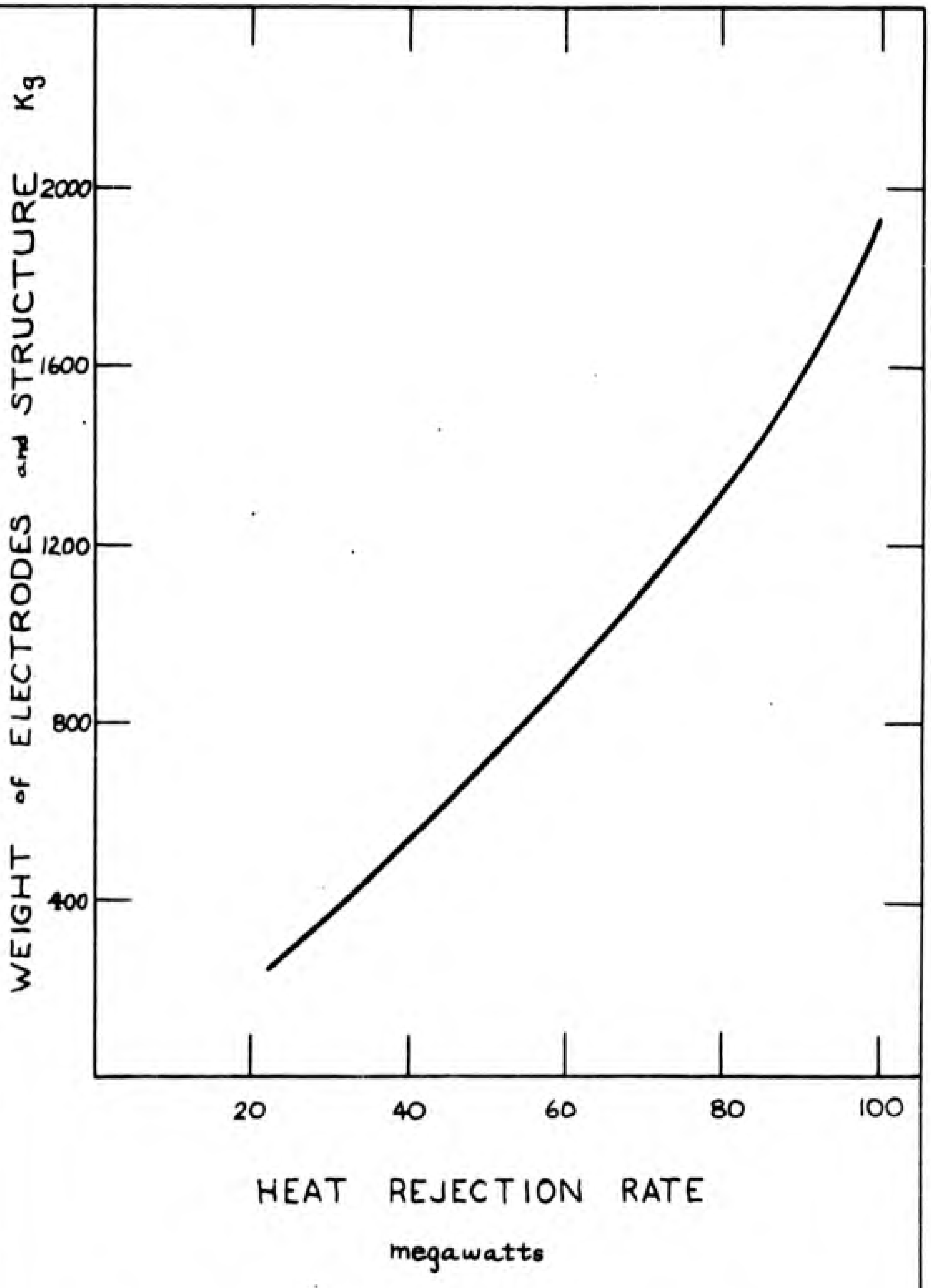

FIGURE 3. Weight of Electrodes and Structure

vs

Heat Rejection Rate

When the particles return to the heated electrode they will be at a temperature T_a. The rate of heat transfer to the particles will be a maximum at this time and decrease exponentially as the temperature reaches T_1. In the preliminary design (Ref 8:25) the time to heat the particles, t_h, was assumed equal to the time of contact, t_c. This time was determined by Neef to be much less than the time of flight. He proposed a design which utilized several layers of particles so that there would be particles on the electrodes at all times. If N_p is the number of particles on one layer of an electrode, then the maximum number of particles that can be contained in the system is approximated by

$$N_t = 2N_p \frac{t_f}{t_c} \tag{17}$$

The weight of the electrical system which is required to operate the radiator can be determined when the current caused by the motion of the charged particles is known. This current can be determined from

$$I = \frac{2N_p q}{t_c} \tag{18}$$

The weight of the electrical system, W_E, is given by

$$W_E = 0.9\,VI \tag{19}$$

where 0.9 is the assumed specific mass of the electrostatic generator. Since $N_t = W_p/m$, Eqs (17), (18), and (19) are combined to give the specific weight of the electrical system,

$$\frac{W'_E}{\dot{Q}} = \frac{0.9Vq}{t_f\, m}\,\frac{W_p}{\dot{Q}} \tag{20}$$

Eqs (5) and (6) are substituted into Eq (20) to give the specific weight of the electrical system in kg/kw. This weight is shown as

$$\frac{W_E}{\dot{Q}} = 1.39 \times 10^{-4}\,\frac{V}{t_f\, d}\left[\frac{W_p}{\dot{Q}}\right] \tag{21}$$

III. Limitations to the Theory

The particle radiator is not without limitations. Several of the potential problem areas can be easily recognized; however, little else can be done until a working model is built or a test device constructed to operate in a free fall vacuum condition. In addition to those problems briefly discussed by Neef (Ref 8:28), several others can be expected to arise. Some of these, with suggested solutions, are discussed below.

Loss of Particles

Even if it were possible to induce uniform curvilinear motion of the particles, it is apparent from two 180° changes of direction that inhomogenous flow will result. Because the particles do have mass, each will tend to travel outward from its initial line of force position. In addition, if particles impact elastically and depart the electrodes without losing the kinetic energy from the previous trip, their trajectories will be lengthened. The end result would be that particles would leave from the tip of the spherical electrode (point D or E in Figure 2) and never return. It may be possible to vary the configuration of the electrodes to prevent the buildup of significant radial velocity components. For example, this may be done with cavities or ridges which would mechanically direct the particles inward. Changing the overall electrode configuration and thus varying the electric field may also be a solution. To eliminate excess kinetic energy either the non-heated electrode or possibly both can be constructed of an absorbing rubber-like material.

Particles may also be lost from the system because of neutralization caused by oppositely charged particles colliding. An increase in electrode separation would decrease the collision rate because the particles would be further apart. This neutralization problem may not be too serious, however, since absolutely neutral particles would seldom result. Each dielectric bit will vary in size and charge, and although the coulomb attractive force may be decreased because of collision and partial neutralization, it will still be present to return the colliding particles. Forces of dielectrophoresis exist which tend to return those particles which become exactly neutral. Since it is probable that some will be lost, a spare supply of particles may be carried and introduced as required during the space mission.

Timing Considerations

Two essential changes must occur during the short interval that a particle is in contact with the heated electrode. Charge transfer must occur, and the particle must heat up sufficiently for effective operation. Neef gave as an approximate expression for the time to heat

$$t_h \cong \frac{2.6 \rho_p c_p r}{K_p} \qquad (22)$$

where r is the radius of the particle, ρ_p is the density of the particle, and K_p is the particle thermal conductivity. From Eq (22) the time to heat a representative particle of 100 micron diameter is approximately 2×10^{-3} seconds. From theoretical duration of impact equations developed by Goldsmith who considered a completely elastic collision between a spherical particle and a sphere, a lower limiting value for the time of contact was determined on the order of 10^{-5} seconds (Ref 5:90). With regard for the time required for charge transfer, Vorob'ev concluded that times on the order of 10^{-8} seconds are sufficient for the development of breakdown in solid dielectrics (Ref 12:228).

Since the collisions will not be completely elastic, it appears that the time of contact will be sufficiently lengthened to allow adequate heat transfer to the particle. A more serious problem exists when the time to heat is compared to the time required to charge the particle and force it away from the electrode. One solution may lie in the mechanism of breakdown. Vorob'ev at all times used voltages large enough to produce complete breakdown. It is possible that at voltages below the breakdown value, charge transfer alone will occur and that the process will be a slower one. More data is needed to prove this idea.

A more positive solution is in the design of the electrical system. A pulsing circuit could be introduced which would place an adequate charging voltage on the electrodes for periods long enough to give charge transfer and would space the intervals according to the time required to heat. Some residual voltage must always remain to provide the electric field between the electrodes. Such a pulsing technique may allow a smaller total voltage requirement for the generating unit and thereby decrease the overall weight of the radiator.

Electric Charging

Problems with regards to the electrical charging of dielectric particles are associated with the charging phenomena in a space environment. Neef based the particle radiator design partially on an experiment by H. A. Pohl in which solid dielectric particles were sprayed from a highly charged electrode of a Van der Graaff generator. According to Pohl* the spraying motion was produced not by complete breakdown but mainly by charge transfer as

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*Personal letter dated April 26, 1962, to the author from Dr. Herbert A. Pohl, Senior Resident Associate Lecturer and Supervisor of the Dielectric Section, School of Engineering, Princeton University.

suggested earlier in this paper. He says, however, that charge was transferred by an incipient corona effect which takes place in the gaseous medium surrounding the particles. In the vacuum conditions of space the action of the gaseous medium would be absent, and therefore, other effects would be needed.

Pohl also pointed out that particles must cover the entire electrode for continued action. This condition was necessary for Pohl's experiment, but at this stage of design it has not been determined if this is necessary for the particle radiator. If this is a requirement, the zero gravity condition may be an aid in providing complete coverage of the particle radiator electrodes. The use of additional particles will have to be anticipated, and, in addition, still other methods will be required to disperse the particles evenly.

Of all the problems considered, these associated with electrical charging in space are potentially the greatest. No precise solutions are offered here because more experimental data must be furnished first.

IV. Radiator Weight and System Analysis

The total weight of the particle radiator unit consists of the following elements:

$$W_T = W_P + W_S + W_E \qquad (23)$$

where Eqs (12), (16), and (21) can be used to obtain the contributions of each component. These elements are primarily a function of d, D, V, T_1, and the physical properties of the materials used in the design. For a first analysis the use of a molybdenum electrode containing a liquid sodium working fluid and particles of Stained Ruby Mica was assumed. This analysis was made in terms of the following design parameters: (1) particle diameter, (2) operating voltage, (3) heat rejection temperature, and (4) separation distance.

To aid in this analysis an IBM 1620 digital computer was used. The program and some representative data are described in Appendix C. The program input included incremental variations in d, D, V, and T_1 and utilized Eqs (7), (9), (12), and (21) to give the time of flight and specific weights as an output. The specific weight of the electrodes and connecting structure as computed for the 100 megawatt example was used as a constant in the computer analysis since the reduction of radiator weight at high power level is a primary concern.

The conclusions to be derived from a computer analysis such as this are only as good as the assumptions made. In this study many of the assumptions are obviously debatable, but the trends which are shown on the graphs in Figures 4, 5, 6, and 7 and discussed below are important whether or not the assumed values of some of the design parameters are exactly correct.

Optimum Particle Size

For each combination of d, D, V, and T_1 that was used for the computer input, an optimum particle size exists which minimizes the total specific weight. Figure 4 is a plot of total specific weight versus particle diameter for selected heat rejection temperatures. As the heat rejection temperature was increased, a broader minimum was obtained and less variation of specific weight is evident. Consequently, particle size becomes a less critical design parameter at higher temperatures. Other trends seen in the data include an increase in optimum particle size for higher operating voltages, an increase in optimum particle size for smaller electrode separation, and a slight increase in the optimum size for higher rejection temperatures. In the remainder of the weight analysis near optimum sized particles were used to determine the best operating voltage, separation distance, and rejection temperature.

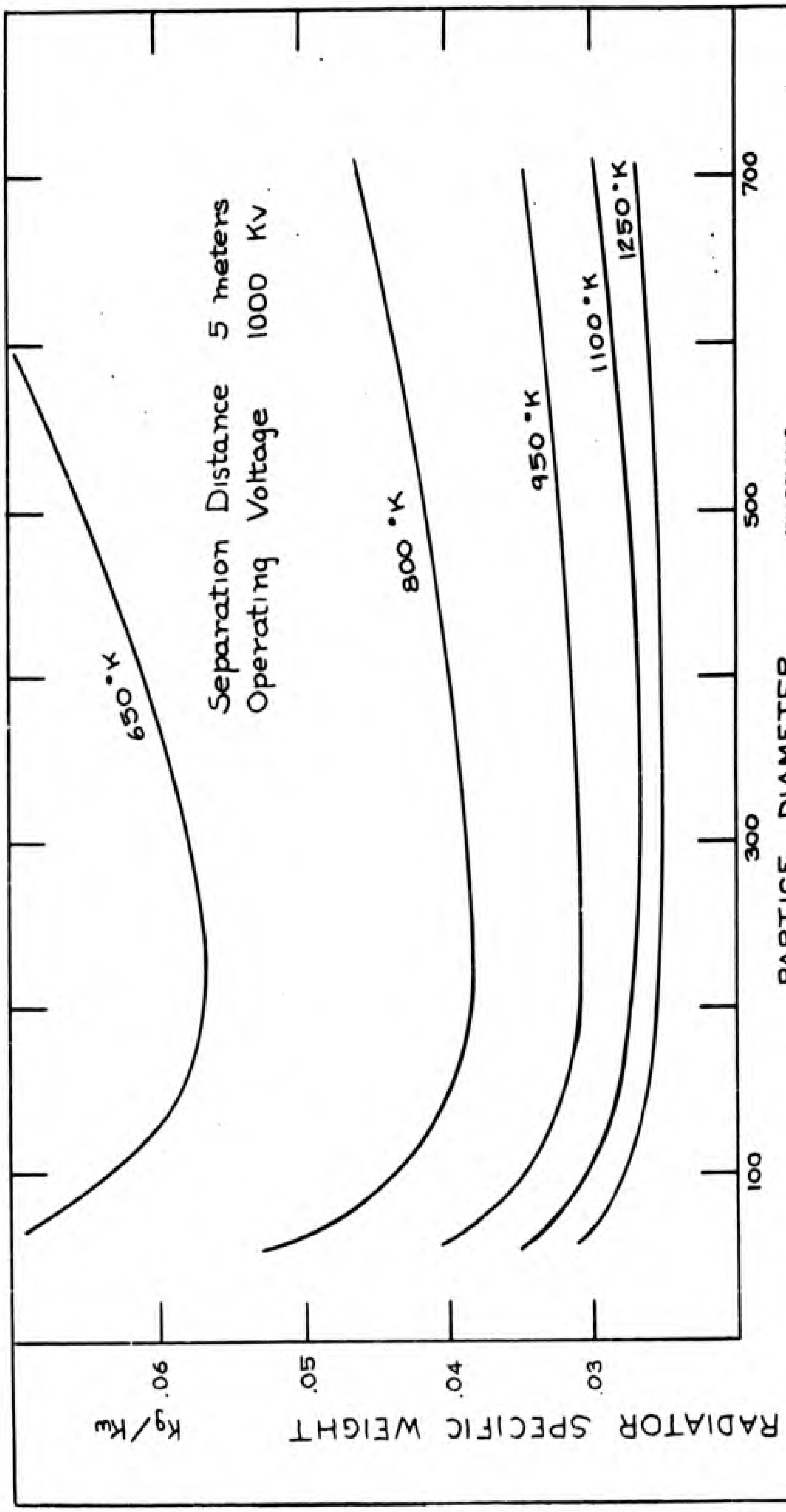

FIGURE 4 Radiator Specific Weight vs Particle Diameter

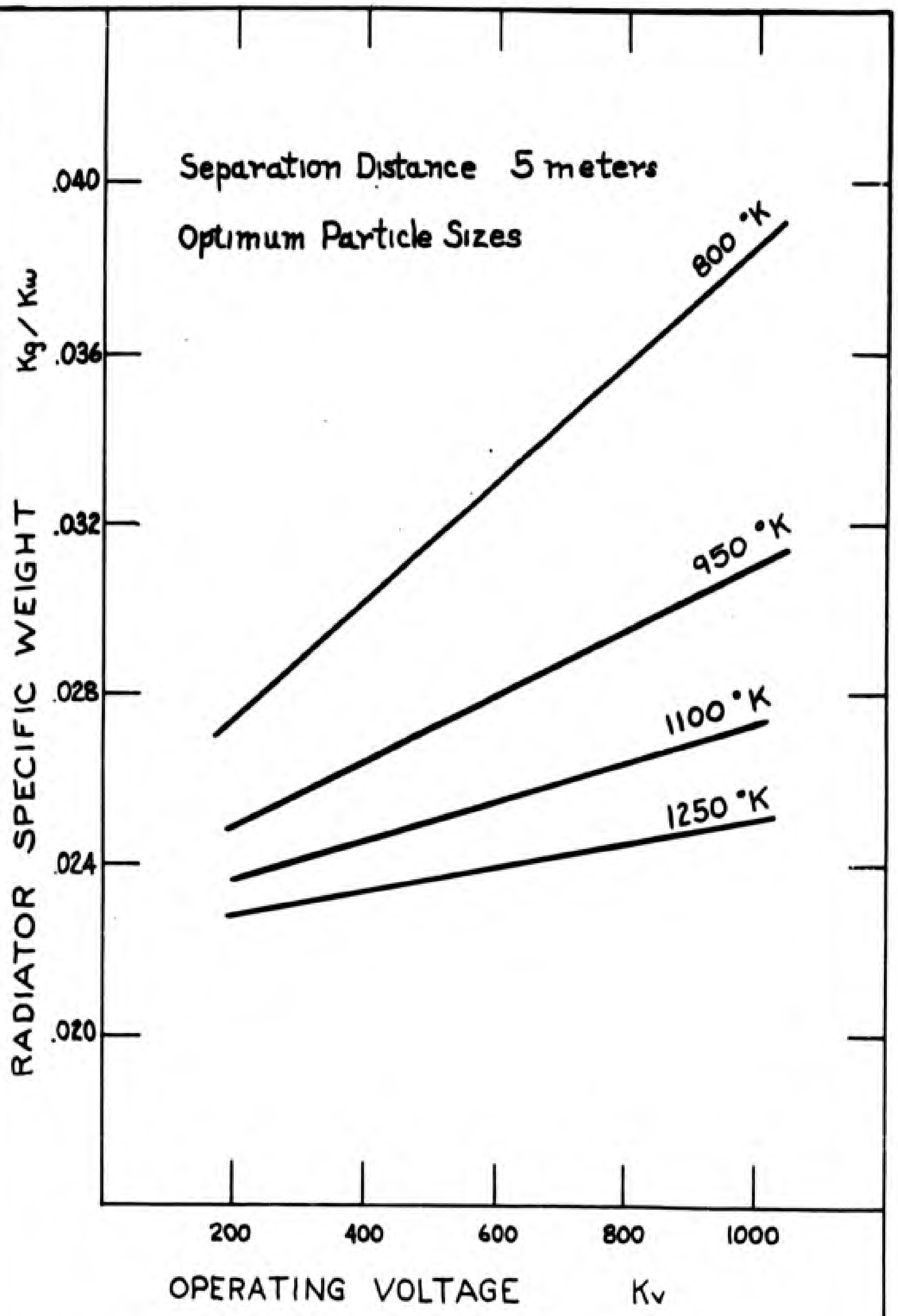

FIGURE 5. Radiator Specific Weight

vs

Operating Voltage

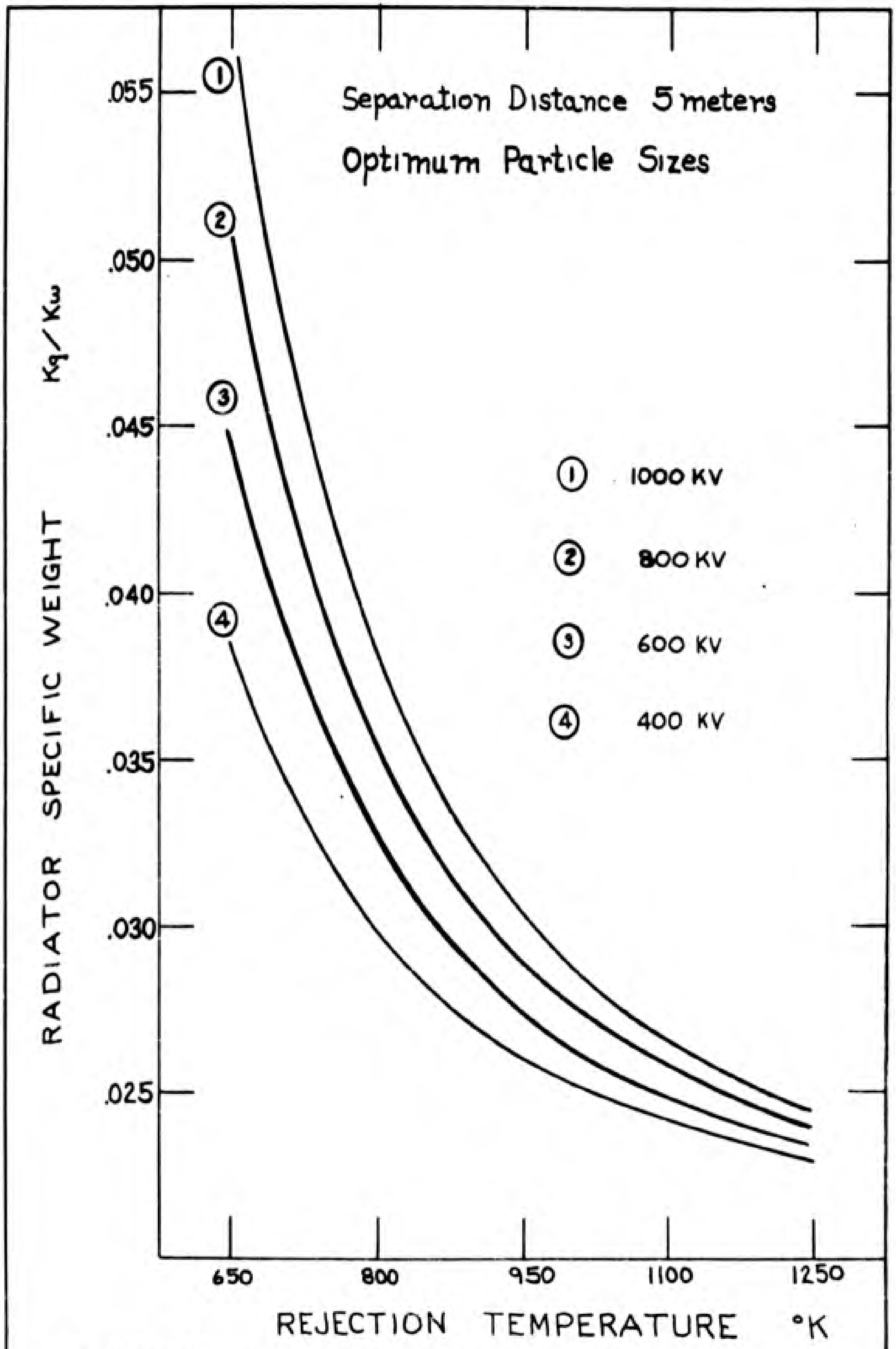

FIGURE 6. Radiator Specific Weight vs Rejection Temperature

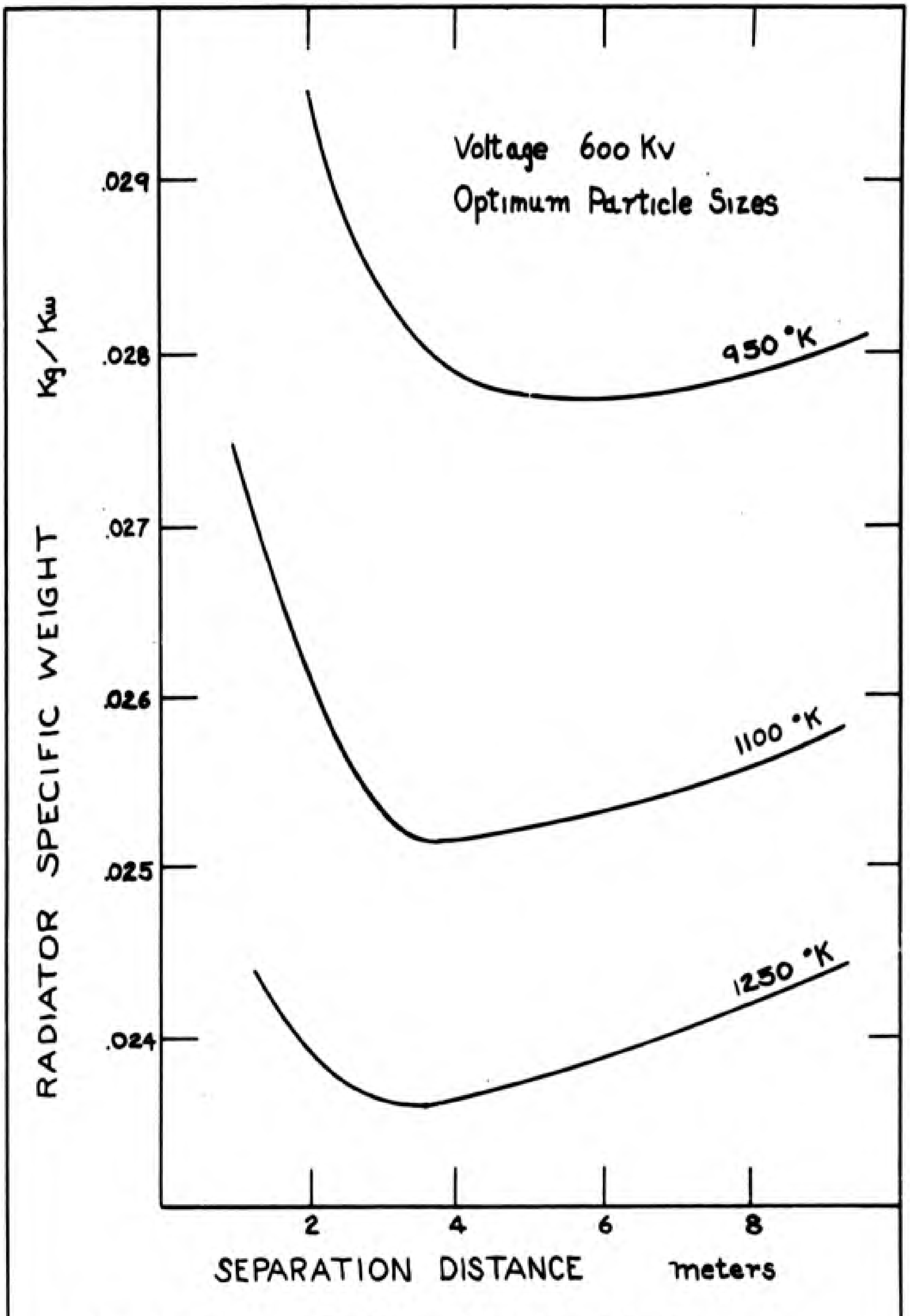

FIGURE 7. Radiator Specific Weight vs Separation Distance

Optimum Voltage and Temperature

Figures 5 and 6 on pages 27 and 28 are representative curves showing the variations of specific weight with V and T_1. Obviously, the minimum weight design should have an operating voltage as low as possible and a rejection temperature as high as possible. Yet, the voltage must still be large enough to provide adequate charge transfer and particle motion without causing breakdown in the rest of the system. The upper limit for the heat rejection temperature will be the point where structural strength markedly decreases due to high temperature effects. Figure 4 indicates a satisfactory compromise value for T_1 in the vicinity of 1100°K. Very little weight advantage is gained by operating at 1250°K and, in addition, refractory metals such as molybdenum and tungsten begin to lose strength near this temperature.

Optimum Separation

Curves of radiator specific weight versus separation distance for some of the heat rejection temperatures are shown in Figure 7. Each combination of design parameters has an optimum separation, but as D increased, no consideration was given in the analysis to the slight increase in W_s due to the attendant weight increase of the structure connecting the two electrodes. At this stage of the design, separation distance alone is not a primary consideration in determining radiator specific weight.

System Considerations

Up to this point the minimization of the radiator weight has been investigated. This consideration should be paramount whenever the radiator is the single heaviest component in the power plant. Presently the accepted design practice is to reject heat from the power generating equipment at a temperature higher than that giving a maximum power plant efficiency. This increases the weight of the power generating unit but decreases the ponderous radiator weight by allowing heat rejection at a higher temperature. When a particle radiator can be utilized in a system, it will prove beneficial to operate the power plant at somewhat lower heat rejection temperature. A lower T_1 will increase radiator weight but will permit smaller generator and turbine sizes. The best trade-off point was not analyzed quantitatively here and will have to be determined uniquely for any given set of cycle conditions.

Increased system reliability is another consideration worthy of investigation. If the reliabilities of the conventional and particle radiator are assumed equal, use of the particle radiator can increase the reliability of the entire power system in at least two ways. Initially, the operating temperatures can be reduced throughout the system, thus allowing simpler and easier construction. Since the particle system can operate very competitively at heat rejection temperatures lower than those essential for conventional radiators, operating temperatures at all stages of the power cycle can be reduced. This reduction in itself will increase system reliability and, furthermore, will permit design consideration for these lower temperatures. For example, refractory alloys which are difficult to fabricate, corrode easily, and react with liquid metals can be replaced by more reliable stainless steels.

System reliability can also be increased by using the simple, single-loop Brayton gas cycle rather than the mechanically complex, multi-looped Rankine vapor cycle. According to Zipkin and Schnetzer the Brayton cycle has several principal attractive features for space application, yet they do not use the gas cycle because of its required radiator size and weight (Ref 14:565). Table II is included from data by Corliss to give a better appreciation of the advantages to be gained by using the particle radiator in conjunction with the Brayton gas cycle for space power plants (Ref 2:102).

Optimization Results

An important limitation in the optimization study at this time is the fact that no variation has been made in the materials to be used. It would be best if an optimum particle could be selected from its physical properties alone.

Table II
Comparison of the Brayton and the Rankine Cycles

BRAYTON GAS CYCLE	RANKINE VAPOR CYCLE
Single loop has inherent simplicity.	Two or more loops increases mechanical complexity.
Inert gases of negligible mass are utilized.	Liquid metals add to the system weight.
Gases are not severely activated in a reactor.	Most fluids will be activated thus making extra shielding necessary.
Inert gases are corrosion-free and erosion-free.	Liquid metals corrode and erode.
Simple start, control, and restart procedures can be used.	More complicated procedures may require complex auxiliary systems.
Very large radiators are needed because of low effective radiator temperatures.	Smaller radiators can be used since condensation occurs at relatively high temperatures.
High pumping pressures are required.	Low pumping pressures can be used.
Closed cycles can be sealed easier.	Seals and liquid metal bearings are not well developed.

This may be possible when an adequate theory concerning charge transfer in dielectrics is developed. Until that time it appears that materials with certain essential properties must be tested with respect to charge transfer and also heat transfer to determine which is the best for the actual radiator. Similarly, operating voltages must be proved experimentally and for this study any V which is selected is assumed proper.

One way to show the results of the optimization analysis is to select a model radiator for a specific mission. This model was based on a heat rejection rate of 100 megawatts which would be suitable for the operation of a fairly large manned space vehicle. In view of the design analysis the following values were selected:

$$T_1 = 1100^\circ K$$
$$V = 400 \text{ kilovolts}$$
$$d = 100 \text{ microns}$$
$$D = 9.0 \text{ meters}$$

The 9 meter separation was used to further separate the electrodes which in the 100 megawatt system have diameters of 3.6 meters. With these operating conditions Appendix C shows that the time of flight of a particle is 0.80 seconds. The computer data furnishes the specific weight from which the weight of the electrostatic generator was computed to be 228 kg and that of the mica particles 330 kg. From Appendix B the weight of the electrodes and connecting structure was determined as 1940 kg. The entire radiating system would weigh approximately 2500 kg and have a power to mass ratio near optimum operation of 40 kw/kg.

A comparison of the particle radiator was made with a recently suggested belt radiator (Ref 13:55) and the conventional tube-and-header radiator. In this comparison which is shown in Table III, the power plant conditions were optimized for the conventional radiator and an overall power cycle efficiency of 0.727 was assumed for electric power outputs of 5, 10, and 20 megawatts. The particle radiator shows a marked weight advantage over both the belt and conventional radiator. By the use of the particle system, weight reduction of nearly 95% over the conventional radiator and more than 50% over the belt type appear feasible depending on operating voltage requirements.

Table III

Typical Radiators

Generator Power Output	5	10	20	mw
Powerplant Conditions				
Turbine Inlet Temperature	1250	1250	1250	°K
Heat Rejection Temperature	950	950	950	°K
Heat Rejection Rate	23.3	46.5	93.0	mw
Conventional Tube-and-Header Radiator				
Isotermal Radiation Temperature	950	950	950	°K
Required Radiating Area	565	1130	1695	m2
Tube Wall Thickness	.635	.635	.635	mm
Weight of Tubes	4420	8850	17700	kg
Total Radiator Weight - Steel	8250	16550	33000	kg
Belt Radiator				
Maximum Belt Temperature	950	950	950	°K
Required Radiating Area	895	1790	3580	m^2
Belt Width	6.8	9.5	13.4	m
Belt Length	68	95	134	m
Belt Thickness	0.127	0.177	0.254	mm
Weight of Belt	345	1230	3480	kg
Weight of Heat Transfer Mechanism	725	1030	1450	kg
Total Radiator Weight - Steel	1070	2260	4930	kg
Particle Radiator				
Maximum Heat Rejection Temperature	950	950	950	°K
Electrode Separation	5	7	9	m
Radius of Electrodes	0.9	1.2	1.7	m
Operating Voltage	400	600	800	kv
Weight of Particles	71	205	435	kg
Weight of Electrostatic Generator	89	185	465	kg
Weight of Structure and Electrodes	254	675	1650	kg
Total Radiator Weight - Molybdenum	314	1065	2550	kg

(Data taken in part from Ref 13)

V. Conclusions and Recommendations

As mentioned not all aspects of the particle radiator have been proved experimentally. In addition, this design has necessitated many assumptions because of the difficulty of simulating the space environment. Yet all assumed conditions were estimated conservatively; therefore, the results should be realistic.

The weight of the particle radiator is made up primarily of the component weights of the particles, of the radiator electrical system, and of the electrodes with their connecting structure. Of these, the latter appears to be dominant. Future design refinement in the internal heat-transfer mechanisms can do much to decrease this weight component. Improvements, such as those suggested by Gale (Ref 4:168), in the design of electrostatic generators can further decrease the total specific weight.

Although reliability, cost, and other factors concerning the radiator itself were not investigated, minimum weight is the paramount consideration in determining the type of radiator to be used in future space power plants. The potential advantages of the particle radiator are shown in Table III. In comparison with the weight of a conventional radiator, weight reductions larger than a full order of magnitude are possible. In addition, if the reliability of the particle radiator can be developed to equal that of the conventional radiator, the reliability of the overall system can be improved.

Equations similar to those in Section II can be used to optimize design parameters, and although still other parametric studies are required, the data obtained in this study can aid in designing an experimental model or other type of test device. Before any such apparatus is constructed, however, the charging of the dielectric particles in a space environment and the actual motion of the particles should be investigated. These appear to be the two greatest limitations of the theory and the problems areas in which the least data is available.

It can be appreciated from the preceding review that considerable development and study must be accomplished if the ultimate goal of utilization of the particle radiator in space power systems is to be met. Some of the most significant items which should receive early attention are:

1. Tests in vacuum conditions to determine voltages necessary for charge transfer in various dielectrics with various electrode materials.

2. Study to learn the relationships concerning heat transfer by conduction and radiation between the particles and large spheres.

3. Investigation to determine optimum material for the particles and the electrodes.

4. Experimentation to study motion of charged particles in space or a space-simulated environment.

An apparatus similar to that shown in Figure 8 is suggested for experiments in charge and heat transfer. Runs could be made with the electrode heated and not heated. Particles could be collected on a galvanometer or in a calorimeter; the entire study should be conducted in a vacuum with free falling particles to simulate as near as possible a space environment.

Only by persevering application in tests such as these and others which will undoubtedly come to light through continued development will it be possible for the particle radiator to assume its proper role in the science of space propulsion.

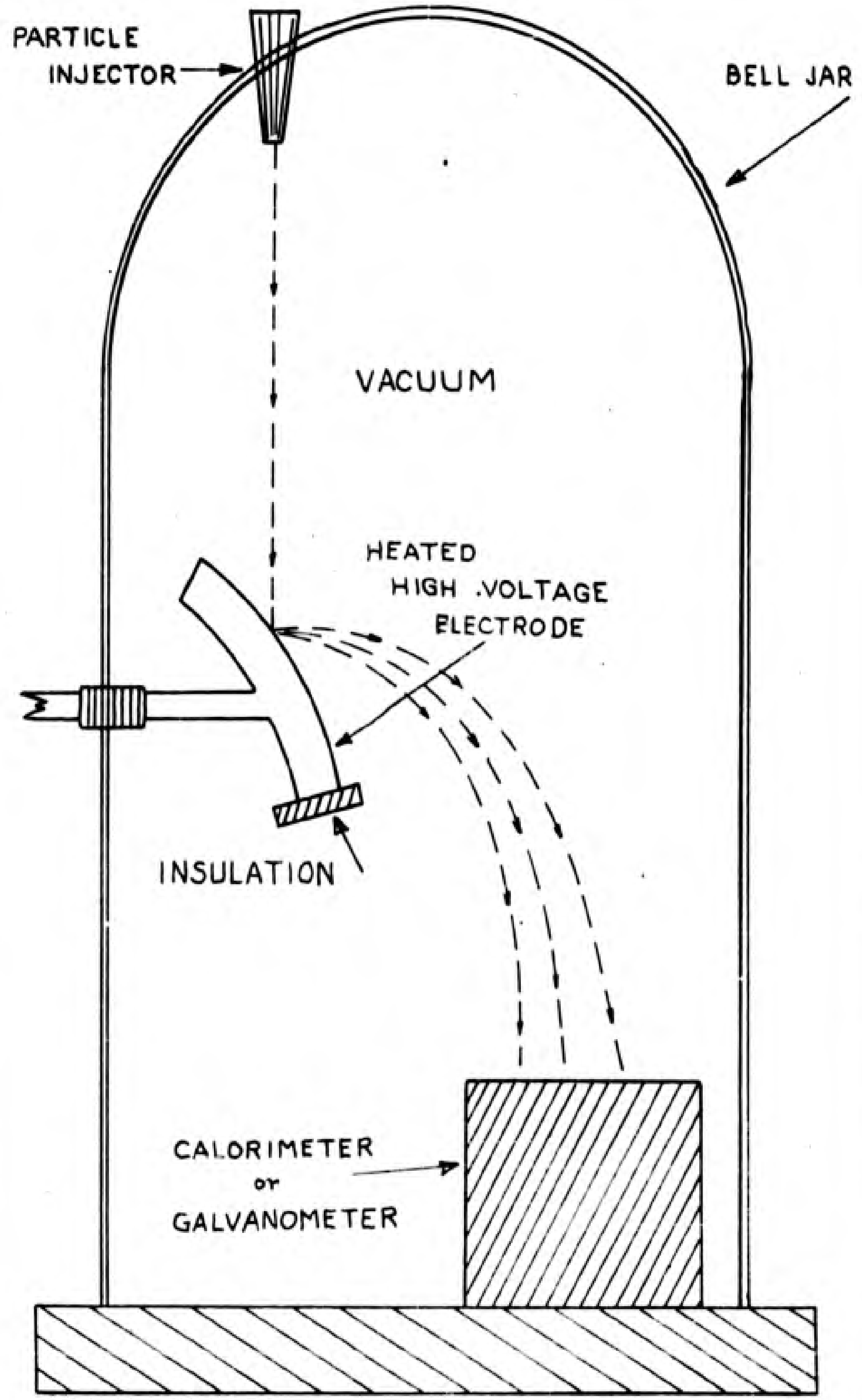

FIGURE 8. Proposed Study Apparatus

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APPENDIX A

SPECIFIC CONSTANTS

Appendix A

Specific Constants

General

σ	Stephan Boltzmann constant	5.67 x 10^{-8} joule/meter sec $^{o}K^{4}$

Stained Ruby Mica

ρ_P	density	3000 kilograms/meter^3
C_P	specific heat	863 joules/kilogram ^{o}K
e	emissivity	0.9 dimensionless
K_P	thermal conductivity	0.8 joules/meter sec ^{o}K

Molybdenum

ρ_M	density	10200 kilograms/meter^3
K_M	thermal conductivity	65 BTU/hour ft ^{o}R

Liquid Sodium

C_N	specific heat	0.305 BTU/lb ^{o}R
K_N	thermal conductivity	30 BTU/hour ft ^{o}R

APPENDIX B

SAMPLE CALCULATION OF ELECTRODE WEIGHT

Appendix B

Sample Calculation of Electrode Weight

Illustrated below are a set of sample calculations which show the method by which the weight of the electrodes and the connecting structure was calculated for a heat rejection rate of 100 megawatts.

<u>Film</u> <u>coefficient</u>. Equation (13) was solved for h.

$$h = \frac{K_N}{2d'}\left[5.8 + 0.02\left(\frac{2d'C_N G}{K_N}\right)^{0.8}\right]$$

where values for d' and G were taken as 0.0328 ft (1 cm) and 1.25×10^7 lbs/hr-ft^2 from state of the art designs. Physical property data is given in Appendix A.

$$h = \frac{30}{2(.0328)}\left\{58 + .02\left(\frac{2(.0328)(.305)\ 1.27 \times 10^{-7}}{30}\right)^{0.8}\right]$$

$$= 14{,}500 \ \text{BTU/hr ft}^2\ {}^\circ\text{R}$$

<u>Overall</u> <u>heat-transfer</u> <u>coefficient</u>. Under equilibrium conditions the heat flow rate through the molybdenum is equal to that through the liquid sodium.

$$\dot{Q} = hA(T_B - T_w)$$

$$= \frac{K_M}{t} A (T_w - T_1)$$

where T_B is the bulk temperature of the sodium and is the temperature of the inner electrode surface.

Adding Eqs (B2) and (B3) and rearranging

$$\dot{Q} = \left[\frac{1}{\frac{1}{h} + \frac{t}{K_M}}\right] A(T_B - T_1)$$

where U, the overall heat-transfer coefficient, is defined by

$$\frac{1}{U} = \frac{1}{h} + \frac{t}{K_M} \tag{B5}$$

A value of 125 mils was selected for the thickness of the molybdenum.

$$\frac{1}{U} = \frac{1}{14500} + \frac{.0104}{65}$$

$$U = 4400 \text{ BTU/hr ft}^2\,{}^\circ\text{R} = 25000 \text{ joules/sec m}^2\,{}^\circ\text{K}$$

Heat-transfer surface area. Eq (B5) was substituted into Eq (B4) to obtain

$$A = \frac{\dot{Q}}{U\,\Delta T} \tag{B6}$$

where $\Delta T = T_B - T_1$. For this calculation ΔT was assumed to be 100 °K.

$$A = \frac{10^8}{25000 \times 100} = 40\ \text{m}^2$$

From meteoroid penetration data given by Ross et al the minimum thickness required for a vulnerable area of 40 m^2 is approximately 125 mils; therefore, the assumed thickness was considered representative (Ref 9:598).

Weight. Equation (16) was solved to give the weight of the electrodes and connecting structure.

$$W_s = \frac{1.5 \rho_M \, t \, \dot{Q}}{U \, \Delta T}$$

$$= \frac{(1.5) \; 1.02 \times 10^4 \; (.00317) \; 10^5}{25000 \times 100}$$

$$= 1940 \text{ Kilograms}$$

The specific weight for the 100 megawatt case is 0.0194 kg/kw.

APPENDIX C

IBM 1620 COMPUTER DATA

Appendix C

IBM 1620 Computer Data

The printout of the program used on the IBM 1620 Digital Computer and some sample data are included in this Appendix. The computer program in Fortan machine language is reproduced on page 51. Pages 52 through 58 contain output data which is a sampling of the data used in the weight analysis portion of this study. Beginning on page 52 the first line of each block of output data contains, in the following order, the heat rejection temperature in ^{o}K, the separation distance of the electrodes in meters, and the operating voltage in volts. Each data block is grouped in six columns which, for the three variables listed above, give the following values:

Column 1: the particle diameter in microns for that particular line of calculations

Column 2: the representative time of flight in seconds for that sized particle

Column 3: the specific weight of the particles

Column 4: the specific weight of the radiator electrical system

Column 5: the specific weight of the particles and electrical system

Column 6: the total specific weight of the radiator. All specific weights have dimensions of kilograms per kilowatt.

IBM 1620 CØMPUTER PRØGRAM PRINTØUT

```
1       ACCEPT,DØ,DM,DD,VØ,VM,DV,TØ,TM,DT,SØ,SM,DS,WS
        T1=TØ
2       D=DØ
3       V=VØ
4       S=SØ
       PRINT 51,T1,D,V
5       TF=5.65*D*SQRT(S/V)
        TA=(S*T1**3/(7.12E-7*TF*T1**3+S))**.33333
        WP=2.32*TF/(T1-TA)
        WE=1.39E-4*V*WP/(TF*S)
        WEP=WE+WP
        WT=WEP+WS
       PRINT 53,S,TF
        PRINT 52,WP,WE,WEP,WT
        S=S+DS
        IF(SM-S)6,5,5
6        V=V+DV
        IF(VM-V)7,4,4
7       D=D+DD
        IF(DM-D)8,3,3
8       T1=T1+DT
       IF(TM-T1)1,2,2
51     FØRMAT (//F6.0,F12.0,F12.0)
52     FØRMAT (4F12.5)
53      FØRMAT (/F6.0,F10.5)
        END

END ØF CØMPILATIØN
```

```
650.        5.     1000000.
 50.   .19975      .00407      .05666      .06074      .08014
100.   .28250      .00739      .03636      .04376      .06316
150.   .34599      .01057      .02832      .03890      .05830
200.   .39951      .01369      .02381      .03751      .05691
250.   .44667      .01676      .02086      .03762      .05702
300.   .48930      .01980      .01875      .03855      .05795
350.   .52850      .02281      .01714      .03995      .05935
400.   .56500      .02580      .01587      .04168      .06108
450.   .59927      .02878      .01483      .04362      .06302
500.   .63168      .03175      .01397      .04572      .06512
550.   .66252      .03470      .01323      .04794      .06734
600.   .69198      .03765      .01260      .05025      .06965
650.   .72023      .04058      .01205      .05263      .07203
700.   .74742      .04351      .01156      .05507      .07447
750.   .77365      .04643      .01112      .05755      .07695

800.        5.     1000000.
 50.   .19975      .00223      .03113      .03337      .05277
100.   .28250      .00389      .01917      .02306      .04246
150.   .34599      .00544      .01459      .02004      .03944
200.   .39951      .00694      .01208      .01903      .03843
250.   .44667      .00841      .01046      .01887      .03827
300.   .48930      .00984      .00932      .01917      .03857
350.   .52850      .01126      .00846      .01972      .03912
400.   .56500      .01266      .00779      .02045      .03985
450.   .59927      .01405      .00724      .02130      .04070
500.   .63168      .01543      .00679      .02222      .04162
550.   .66252      .01680      .00641      .02321      .04261
600.   .69198      .01816      .00608      .02424      .04364
650.   .72023      .01952      .00579      .02531      .04471
700.   .74742      .02087      .00554      .02641      .04581
750.   .77365      .02221      .00532      .02753      .04693

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450.   .59927      .00813      .00419      .01232      .03172
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550.   .66252      .00963      .00367      .01331      .03271
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```

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 350.    .52850     .00434     .00326     .00761     .02701
 400.    .56500     .00483     .00297     .00780     .02720
 450.    .59927     .00530     .00273     .00803     .02743
 500.    .63168     .00577     .00253     .00831     .02771
 550.    .66252     .00623     .00237     .00860     .02800
 600.    .69198     .00668     .00223     .00892     .02832
 650.    .72023     .00713     .00211     .00925     .02865
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 750.    .77365     .00802     .00192     .00994     .02934

1250.        5.     1000000.
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 300.    .48930     .00278     .00263     .00542     .02482
 350.    .52850     .00312     .00234     .00546     .02486
 400.    .56500     .00345     .00212     .00557     .02497
 450.    .59927     .00377     .00194     .00571     .02511
 500.    .63168     .00408     .00179     .00588     .02528
 550.    .66252     .00439     .00167     .00607     .02547
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 700.    .74742     .00529     .00140     .00670     .02610
 750.    .77365     .00559     .00134     .00693     .02633
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250.	.99878	.02035	.00226	.02262	.04202
300.	1.09411	.02376	.00201	.02577	.04517
350.	1.18178	.02712	.00182	.02894	.04834
400.	1.26337	.03043	.00167	.03210	.05150
450.	1.34001	.03370	.00155	.03526	.05466
500.	1.41250	.03695	.00145	.03841	.05781
550.	1.48144	.04018	.00137	.04155	.06095
600.	1.54731	.04338	.00129	.04468	.06408

650.	5.	400000.			
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300.	.77365	.02169	.00519	.02688	.04628
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400.	.89334	.02801	.00435	.03237	.05177
450.	.94753	.03113	.00405	.03519	.05459
500.	.99878	.03423	.00381	.03804	.05744
550.	1.04753	.03731	.00360	.04091	.06031
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200.	.51577	.01446	.01169	.02615	.04555
250.	.57665	.01762	.01019	.02782	.04722
300.	.63168	.02075	.00913	.02988	.04928
350.	.68230	.02384	.00832	.03217	.05157
400.	.72941	.02691	.00769	.03461	.05401
450.	.77365	.02996	.00717	.03714	.05654
500.	.81550	.03300	.00675	.03975	.05915
550.	.85531	.03601	.00638	.04240	.06180
600.	.89334	.03902	.00607	.04509	.06449

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250.	.49939	.01711	.01524	.03236	.05176
300.	.54705	.02018	.01367	.03386	.05326
350.	.59089	.02323	.01249	.03572	.05512
400.	.63168	.02626	.01155	.03781	.05721
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500.	.70625	.03226	.01015	.04242	.06182
550.	.74072	.03524	.00961	.04486	.06426
600.	.77365	.03821	.00915	.04736	.06676

```
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 50.     .44667     .00235     .00293     .00528     .02468
100.     .63168     .00376     .00165     .00542     .02482
150.     .77365     .00500     .00119     .00620     .02560
200.     .89334     .00614     .00095     .00710     .02650
250.     .99878     .00722     .00080     .00803     .02743
300.    1.09411     .00826     .00069     .00896     .02836
350.    1.18178     .00926     .00062     .00989     .02929
400.    1.26337     .01024     .00056     .01081     .03021
450.    1.34001     .01120     .00051     .01172     .03112
500.    1.41250     .01214     .00047     .01262     .03202
550.    1.48144     .01306     .00044     .01351     .03291
600.    1.54731     .01397     .00041     .01439     .03379

950.          5.     400000.
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100.     .44667     .00307     .00382     .00689     .02629
150.     .54705     .00413     .00279     .00693     .02633
200.     .63168     .00512     .00225     .00737     .02677
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300.     .77365     .00698     .00167     .00866     .02806
350.     .83564     .00787     .00149     .00937     .02877
400.     .89334     .00875     .00136     .01011     .02951
450.     .94753     .00960     .00125     .01085     .03025
500.     .99878     .01044     .00116     .01161     .03101
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600.    1.09411     .01210     .00102     .01312     .03252

950.          5.     600000.
 50.     .25788     .00166     .01078     .01245     .03185
100.     .36470     .00275     .00629     .00905     .02845
150.     .44667     .00373     .00464     .00838     .02778
200.     .51577     .00465     .00376     .00842     .02782
250.     .57665     .00554     .00320     .00875     .02815
300.     .63168     .00640     .00281     .00922     .02862
350.     .68230     .00724     .00253     .00977     .02917
400.     .72941     .00806     .00230     .01037     .02977
450.     .77365     .00887     .00212     .01100     .03040
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550.     .85531     .01046     .00185     .01232     .03172
600.     .89334     .01125     .00175     .01300     .03240

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Vita

The author was born on [redacted] [redacted], the son of Albert and Aili [redacted] Smetana. After graduating from the [redacted] [redacted], in 1948, he attended Gogebic Junior College, Ironwood. In June 1950 he received his Associate Bachelor of Arts degree from that institution. He attended the University of Wisconsin at Madison, Wisconsin, until June 1951 at which time he entered the United States Military Academy at West Point, New York. He graduated in June 1955 with a degree of Bachelor of Science and a commission in the United States Air Force. His military assignment prior to his work at the Air Force Institute of Technology was as reconnaissance pilot with the Air Weather Service on Guam, Mariannas Islands.

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This thesis was typed by Mrs. Clarence Morefield.