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THEORETICAL STUDY OF GROUND MOTION IN UNIFORMLY VARYING AND LAYERED ELASTIC MEDIA

Final Report for Phase I

Propagation of Waves in Nonhomogeneous Elastic Media

TECHNICAL DOCUMENTARY REPORT NO. AFSWC-TDR-62-33

September 1962

Research Directorate
AIR FORCE SPECIAL WEAPONS CENTER
Air Force Systems Command
Kirtland Air Force Base
New Mexico

Project 1080, Task 108001

(Prepared under Contract AF 29(601)-4283 by F. C. Gair and R. C. Alverson of Stanford Research Institute, Menlo Park, California)

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AIR FORCE SPECIAL WEAPONS CENTER
Air Force Systems Command
Kirtland Air Force Base
New Mexico

FOREWORD

The two objectives of Contract AF 29(601)-4283, (SRI Project No. PU-3514) are:

(1) The development of uncoupled wave equations for a nonhomogeneous axisymmetric half-space, suitable for treatment by the method of characteristics.

(2) The theoretical study using the method of characteristics of stress, strains, and particle motions in layered and nonhomogeneous elastic half-spaces which are surface loaded by a nuclear airblast.

This contract, from January 1961 to June 1962, was a continuation of a previous study (Contract AF 29(601)-1948, SRI Project No. SU-2893).

The research effort is divided into five integral phases which will be reported separately. This is the final report for Phase I which is the development of uncoupled wave equations for a nonhomogeneous axisymmetric half-space. Phase II is the development of the numerical calculation method for a homogeneous and a layered half-space. Phase II includes coding and check-out of the resulting program. The final report on this phase will present detailed explanations of mathematical methods developed and sample calculations. Production runs and analysis of results for a layered half-space will constitute Phase III. This phase will establish the influence on ground motion of variations in independent problem parameters, e.g., the influence on the maximum stress field due to the difference in compressional wave speeds between layers, to depth of overburden, and to differences in Poisson's ratio. A complete description of the codes used, along with instructions to the user, will be presented at that time. Production runs and analysis of results for a nonhomogeneous half-space will constitute Phase IV. In Phase V theoretical results will be compared with field data. In this phase, the results of Phases III and IV will be compared with available test data and final conclusions presented as to the applicability and limitations of the theoretical results.

F.M. Sauer
Project Leader

ABSTRACT

Uncoupled wave equations are developed for a nonhomogeneous half-space for two cases, (1) where Poisson's ratio is constant, and (2) where Poisson's ratio increases with depth in a nonarbitrary but realistic manner. The resulting wave equations are numerically solvable by the method of characteristics since they are identical to those for a homogeneous medium.

PUBLICATION REVIEW

This report has been reviewed and is approved.

DONALD I. PRICKETT
Colonel USAF
Director, Research Directorate

JOHN J. DISHUCK
Colonel USAF
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CONTENTS

ILLUSTRATIONS

TABLES

1. INTRODUCTION

This report describes the work toward obtaining separable equations of motion which govern the propagation of elastic disturbances in nonhomogeneous media. Separability here means two independent equations of motion which may be subject to related boundary conditions. Attention has been confined to the case of axial symmetry where the Lamé parameters (λ, μ) and the density (ρ) are allowed to vary only as functions of the distance below the surface of the half-space.

This problem was first analyzed by Hook[1*] who was able to treat five special cases (Table I), all of which require a specific relationship between μ, ρ, and Poisson's ratio, σ. Hook's approach, however, suggests the possibility of a more general approach wherein such restrictions are not required. The practical importance of obtaining a generalized separation of the equations of motion is that this would allow direct application of the calculation procedures developed in Part I of this report to cases where there is a continuous (as opposed to a stepwise) variation of elastic properties with depth.

TABLE I

SUMMARY OF NONHOMOGENEOUS CASES TREATED IN REF. NO. 1

CASE	Variation with depth, z		
	Shear Modulus	Density	Poisson's Ratio
1	μ constant	$\rho \propto z^{-2}$	σ constant
2	$\mu \propto z^{n}$	$\rho \propto z^{n-2}$	$\sigma = 1/(2 + n)$
3	$\mu \propto z^{n}$	$\rho \propto z^{2(n-1)}$	$\sigma = (3 - 2n)/4$
4	$\mu \propto z^{n}$	$\rho \propto z^{n-1}$	$\sigma = 1/(n + 1)$
5	$\mu \propto z^{-1}$	ρ arbitrary	σ constant

Note that Poisson's ratio is an arbitrary constant (Cases 1 and 5 in Table I) or a constant which depends on the variation of shear modulus with depth, i.e., which depends on n. Hence, the compressional wave velocity, α, varies as the shear wave velocity, β. The loss in generality results from the unusual variation of either ρ or μ (Cases 1 and 5); or if ρ is constant, then $\sigma = 1/4$ (Cases 2 and 3) or $\sigma = 1/2$ (Case 4). For $\sigma = 1/2$, the shear contribution may be neglected initially, in which case the variation of compressional wave velocity with depth may be chosen arbitrarily. This leaves Cases 2 and 3 of practical importance, with ρ constant and $\sigma = 1/4$ (Case 2, n = 2; Case 3, n = 1).

The main mathematical result of this investigation is that separability may be obtained provided that λ, μ, and ρ are related by the ordinary differential equations

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* Numbered references are listed at the end of this report.

$$\frac{(\lambda + 2\mu)}{2} \frac{d^2\mu}{dz^2} = \left(\frac{d\mu}{dz}\right)^2$$

$$\frac{(\lambda + 2\mu)}{2\rho} \frac{d\rho}{dz} = \frac{d\mu}{dz}$$

Solutions of these equations lead to physically acceptable specifications for the variation of elastic parameters with depth.

The results of the investigation are examined in more detail in Section 2, while a method by which these results are obtained is given in Section 3.

2. RESULTS AND EXAMPLES

The differential equations which govern the propagation of elastic waves in nonhomogeneous as well as homogeneous media are the two equations of dynamic equilibrium together with the four equations given by the generalized Hooke's law

$$\frac{\partial\sigma_r}{\partial r} + \frac{\partial\sigma_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = \rho \frac{\partial^2 u}{\partial t^2} \qquad (1)$$

$$\frac{\partial\sigma_{rz}}{\partial r} + \frac{\partial\sigma_z}{\partial z} + \frac{\sigma_{rz}}{r} = \rho \frac{\partial^2 w}{\partial t^2} \qquad (2)$$

$$\sigma_r = (\lambda + 2\mu)\Delta - 2\mu\left(\frac{u}{r} + \frac{\partial w}{\partial z}\right) \qquad (3)$$

$$\sigma_\theta = (\lambda + 2\mu)\Delta - 2\mu\left(\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z}\right) \qquad (4)$$

$$\sigma_z = (\lambda + 2\mu)\Delta - 2\mu\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) \qquad (5)$$

$$\sigma_{rz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \qquad (6)$$

In these equations σ_r, σ_θ, σ_z, σ_{rz}, u, and w are the usual components of stress and displacement when conditions of axial symmetry prevail,* and Δ is the dilatation given by $\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}$.

* See, for example, Timoshenko, S., Theory of Elasticity, McGraw Hill Book Co., Inc., New York and London, 1934.

In a manner similar to that employed by Hook[1], introduction of the displacement potentials in the form

$$u = \phi_r - \theta_z - g\theta \tag{7}$$

$$w = \phi_z + \frac{1}{r}(r\theta)_r \tag{8}$$

yields two equations of motion which govern the propagation of waves where the medium is nonhomogeneous. The nonhomogeneity in this case is confined to a z variation of λ, μ, and ρ. Separability is achieved if $g = (\rho'/\rho)$. The resulting equations for the potential functions are

$$\nabla^2\phi + \frac{\rho'}{\rho}\phi_z - \frac{1}{\alpha^2}\phi_{tt} = 0 \tag{9}$$

$$\nabla^2\theta - \frac{\theta}{r^2} + \frac{\partial}{\partial z}\left(\frac{\rho'}{\rho}\theta\right) - \frac{1}{\beta^2}\theta_{tt} = 0 \tag{10}$$

In these equations a prime denotes differentiation with respect to z, while α^2 and β^2 have the usual definitions, $\alpha^2 = \frac{\lambda + 2\mu}{\rho}$: $\beta^2 = \frac{\mu}{\rho}$. In order that Equations (9) and (10) govern the motion, it is necessary that the Lamé parameters (λ, μ) and the density (ρ) satisfy the two ordinary differential equations

$$\frac{\rho'}{\rho} = \frac{2\mu'}{(\lambda + 2\mu)} \tag{11}$$

and

$$(\lambda + 2\mu)\mu'' = 2\mu'^2 \tag{12}$$

An informative consequence of Equations (11) and (12) is given by

$$\frac{\rho'}{\rho} = \frac{\mu''}{\mu'} . \tag{13}$$

It is to be noted that the restrictions imposed on λ, μ and ρ by Equations (11) and (12) do not imply that Poisson's ratio

$$\sigma = \frac{\lambda}{2(\lambda + \mu)} \tag{14}$$

is a constant. In this sense the present results are more general than those reported in Reference 1. However, when the specialization to σ = constant is made in (11) or (12) the present results do not contain all of those reported in Reference 1.

It is instructive to consider some examples in order to investigate to what extent the above results may be applicable in some possible and practical cases.

Example 1: Constant Poisson's Ratio

We define

$$\gamma = \frac{(\lambda + 2\mu)}{\mu}$$

Thus, in terms of Poisson's ratio,

$$\gamma = \frac{2(1 - \sigma)}{(1 - 2\sigma)} \text{ and } \sigma = \frac{1}{2}\frac{(2 - \gamma)}{(1 - \gamma)} .$$

It is evident that if $0 \le \sigma \le \frac{1}{2}$, then $2 \le \gamma \le \infty$. In terms of γ it is convenient to rewrite Equation (12) in the form

$$\left(\frac{\mu}{\mu'}\right)' = 1 - \frac{2}{\gamma} .$$

We consider first the case σ = constant. In this case Equation (12) is readily integrated to yield

$$\mu = B\left[\left(\frac{\sigma}{1 - \sigma}\right) z + A\right]^{(1-\sigma)/\sigma} \qquad (15)$$

where A and B are arbitrary constants. The density, ρ, is then found by integrating Equation (11) or (13) to give

$$\rho = C\left[\left(\frac{\sigma}{1 - \sigma}\right) z + A\right]^{(1-2\sigma)/\sigma} \qquad (16)$$

where C is an arbitrary constant. Since any numerical integrals of the equations of motion can at best be carried out for only a finite time and hence a finite range of z, Equations (15) and (16) can probably be made to fit many practical cases where only a finite depth of the medium is of interest.

Equations (15) and (16) may be put into a more convenient form for computation. Define $\rho(0)$ and $\mu(0)$ as the density and shear modulus at zero depth; then

$$\frac{\mu}{\mu(0)} = \left[(C_1 z) + 1\right]^{(1-\sigma)/\sigma} \qquad (15a)$$

$$\frac{\rho}{\rho(0)} = \left[C_2(C_1 z) + 1\right]^{(1-2\sigma)/\sigma} \qquad (16a)$$

where C_1 and C_2 are new arbitrary constants. From Equations (15a) and (16a) it may be seen that the density and shear modulus are power law variations with depth and that C_1 is a scale factor which adjusts the depth at which the shear modulus attains a given value. Typical variations with depth of density, shear modulus, and shear wave velocity which may be obtained using Equations (15a) and (16a) are illustrated in Fig. 1.

Example 2: Variable Poisson's Ratio

We consider next the case where Poisson's ratio varies from some arbitrary positive value less than 1/2 to 1/2 as $0 \leq z \leq \infty$. In particular we assume

$$\sigma = \frac{(z+b)^2 - a^2}{2(z+b)^2 - a^2} \quad \text{or} \quad \gamma = \frac{2(z+b)^2}{a^2} \tag{17}$$

The first integral of (12) gives

$$\frac{\mu}{\mu'} = z + \frac{a^2}{(z+b)} + C_1$$

where C_1 is an arbitrary constant of integration. Then with $C_1 = 2a + b$

$$\mu = C_2(z + a + b)\exp\frac{a}{z + a + b} \tag{18}$$

where C_2 is an arbitrary constant. The density, ρ, can then be found from (13) with the aid of (18) to give

$$\rho = C_3\frac{(z+b)}{(z+a+b)}\exp\frac{a}{(z+a+b)} \tag{19}$$

where C_3 is an arbitrary constant. It should be noted that μ as defined by (18) increases without limit, while the density increases to an asymptotic constant value with increasing depth. The variations of the speeds of the so-called dilatational and rotational waves with depth are easily written down from (17), (18) and (19) to give

$$\alpha^2 = 2\frac{C_2}{C_3}(z + a + b)\frac{2(z+b)}{a^2} \tag{20}$$

and

$$\beta^2 = \frac{C_2}{C_3}\frac{(z+a+b)^2}{(z+b)} \; . \tag{21}$$

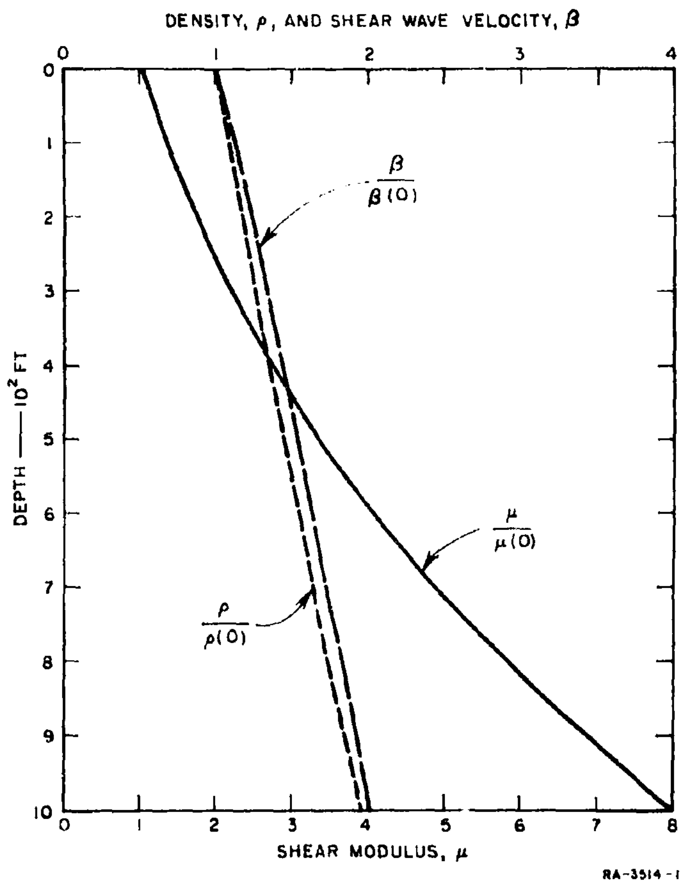

FIG. 1 TYPICAL DENSITY, SHEAR MODULUS, AND SHEAR WAVE VELOCITY PROFILES FOR CONSTANT POISSON'S RATIO,
σ = 1/4 [$\mu(0)$, $\rho(0)$, and $\beta(0)$ are values of parameters at zero depth]

Again, these variations can be made to fit many possible cases for finite depths. Equations (17), (19), (20) and (21) may be written in the following forms:

$$\sigma = \frac{(1 + b_1 z)^2 - a_1^2}{2(1 + b_1 z)^2 - a_1^2} \tag{17a}$$

$$\frac{\rho}{\rho(0)} = \frac{(1 + a_1)(1 + b_1 z)}{\exp\left(\frac{a_1}{1 + a_1}\right)(1 + a_1 + b_1 z)} \exp\left[\frac{a_1}{1 + a_1 + b_1 z}\right] \tag{19a}$$

$$\frac{\alpha}{\alpha(0)} = \frac{(1 + a_1 + b_1 z)}{1 + a_1} \sqrt{1 + b_1 z} \tag{20a}$$

$$\frac{\beta}{\beta(0)} = \frac{(1 + a_1 + b_1 z)}{1 + a_1} \frac{1}{\sqrt{1 + b_1 z}} \tag{21a}$$

where b_1 is an arbitrary constant serving as a depth scaling factor and a_1 is determined by the Poisson's ratio at zero depth, i.e.,

$$\sigma(0) = \frac{1 - a_1^2}{2 - a_1^2} \tag{17b}$$

or

$$a_1^2 = \frac{1 - 2\nu}{1 - \nu} \tag{17c}$$

Typical variations with depth obtained using Equations (17a), (19a), (20a), and (21a) are illustrated in Fig. 2 for $\sigma(0) = 0.265$.

The above results do not exhaust all possible cases for variable σ; however, each desired variations of σ would have to be substituted into Equations (11) and (12) and investigated separately.

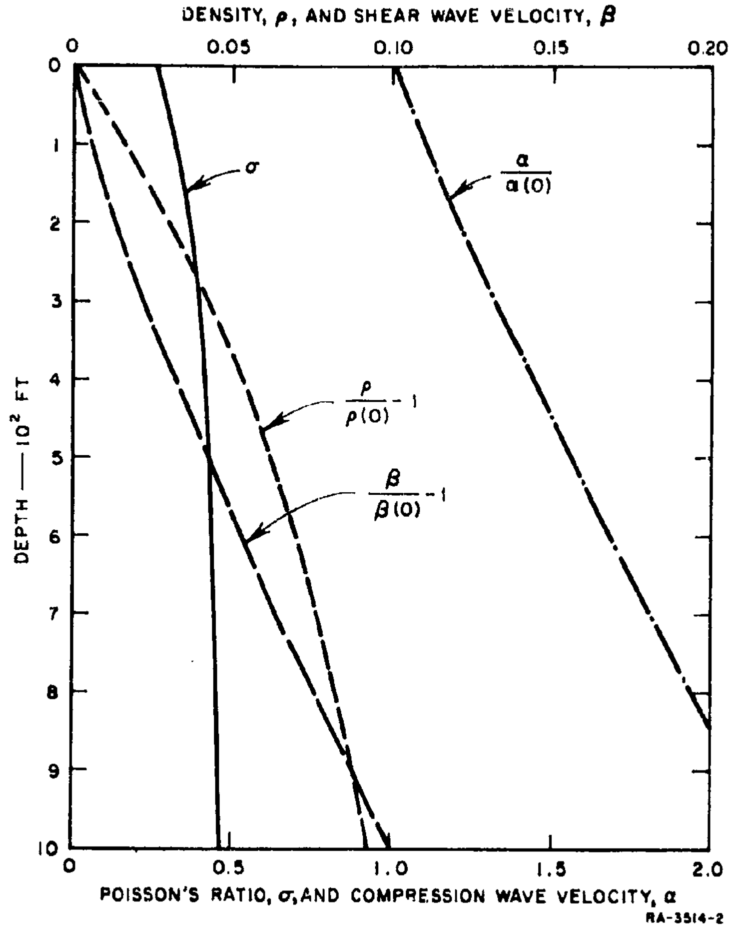

FIG. 2 TYPICAL DENSITY, POISSON'S RATIO, COMPRESSION WAVE VELOCITY, AND SHEAR WAVE VELOCITY PROFILES FOR VARIABLE POISSON'S RATIO [$\rho(0)$, $\alpha(0)$, and $\beta(0)$ are values of parameters at zero depth]

3. SEPARATION OF THE EQUATIONS OF MOTION

When Equations (3) to (6) are used to eliminate the stresses in Equations (1) and (2), the resulting equations which take into account the z dependence of λ, μ, and ρ are

$$\frac{\partial}{\partial r}\left\{(\lambda + 2\mu)\left[\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}\right]\right\} + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\right] + 2\mu'\frac{\partial w}{\partial r} = \rho\frac{\partial^2 u}{\partial t^2} \quad (22)$$

and

$$\frac{\partial}{\partial z}\left[(\lambda + 2\mu)\left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}\right)\right] + \frac{1}{r}\frac{\partial}{\partial r}r\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) - 2\frac{\partial}{\partial z}\mu\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = \rho\frac{\partial^2 w}{\partial t^2} \quad (23)$$

In a manner similar to that employed in Reference 1, the following substitutions are now made in (17) and (18);

$$\left.\begin{aligned} u &= \frac{\partial\phi}{\partial r} - \left(\frac{\partial\theta}{\partial z} + g\theta\right) \\ w &= \left(\frac{\partial\phi}{\partial z} + f\phi\right) + \frac{1}{r}\frac{\partial}{\partial r}(r\theta) \end{aligned}\right\} \quad (24)$$

where g and f are functions of z alone. Equations (22) and (23) then become

$$\begin{aligned} &\frac{\partial}{\partial r}\left\{(\lambda + 2\mu)\nabla^2\phi + [(\lambda + \mu)f + 2\mu']\frac{\partial\phi}{\partial z} + [(\lambda + \mu)f' + \mu' f]\phi - \rho\frac{\partial^2\phi}{\partial t^2}\right\} \\ &- \frac{\partial}{\partial z}\left\{\mu\left(\nabla^2\theta - \frac{\theta}{r^2}\right) + [2\mu' - (\lambda + \mu)g]\frac{\partial\theta}{\partial z}\right. \\ &\left. + [\mu g' + 2\mu' g - ((\lambda + 2\mu)g)']\theta - \rho\frac{\partial^2\theta}{\partial t^2}\right\} \\ &+ [2\mu' - (\lambda + 2\mu)g]\left(\nabla^2\theta - \frac{\theta}{r^2}\right) \\ &+ [2\mu'' + 2\mu' g - 2((\lambda + 2\mu)g)']\frac{\partial\theta}{\partial z} \\ &+ [2\mu'' g + 2\mu' g' - ((\lambda + 2\mu)g)'']\theta \\ &+ (\rho g - \rho')\frac{\partial^2\theta}{\partial t^2} = 0 \end{aligned} \quad (22a)$$

and

$$\frac{\partial}{\partial z}\left\{(\lambda + 2\mu)\nabla^2\phi + [(\lambda + \mu)f + 2\mu']\frac{\partial\phi}{\partial z} + [(\lambda + \mu)f' + \mu' f]\phi - \rho\frac{\partial^2\phi}{\partial t^2}\right\}$$

$$+ (\mu f - 2\mu')\nabla^2\phi - 2\mu''\frac{\partial\phi}{\partial z} + [\mu f'' - \mu'' f]\phi + (\rho' - \rho f)\frac{\partial^2\phi}{\partial t^2}$$

$$+ \frac{1}{r}\frac{\partial}{\partial r} r\left\{\mu\left(\nabla^2\theta - \frac{\theta}{r^2}\right) + (2\mu' - (\lambda + \mu)g)\frac{\partial\theta}{\partial z}\right.$$

$$\left. + [\mu g' + 2\mu' g - ((\lambda + 2\mu)g)']\theta - \rho\frac{\partial^2\theta}{\partial t^2}\right\} = 0 \qquad (23a)$$

Separability will have been achieved if f and g can be chosen so that Equations (22a) and (23a) can be put into the form

$$\left.\begin{aligned} \frac{\partial}{\partial r} K_1 + \left[\alpha(z) - \frac{\partial}{\partial z}\right] K_2 &= 0 \\ \left[\frac{\partial}{\partial z} + \beta(z)\right] K_1 + \frac{1}{r}\frac{\partial}{\partial r}K_2 &= 0 \end{aligned}\right\} \qquad (25)$$

where K_1 and K_2 are differential expressions involving ϕ and θ alone, respectively. If (22a) and (23a) are reduced to (25) then the solution $K_1 = 0$ and $K_2 = 0$ yields two separate partial differential equations for ϕ and θ.

One solution, and to date the only one which has been obtained, is $f = 0$, $g = \frac{\rho'}{\rho}$, where λ, μ, and ρ satisfy (11) and (12). The separated equations are then given by (9) and (10).

4. DISCUSSION

The work reported here, and especially the special case for variable Poisson's ratio given in Section 2, is capable of representing a physical situation which is very likely similar to the actual behavior of the elastic constants with depth in the earth. For instance, it is very likely that the earth becomes more incompressible with depth; this would indicate that Poisson's ratio increases with depth from some value less than 1/2 to 1/2 as it does in the example. Further, it is unlikely that the mass density increases without bound with depth; the real behavior is very probably a gradual increase with depth to an asymptotic constant value as given by Equation (19). The resistance of the earth to volume change increases more rapidly than resistance to shear as indicated by the resulting expressions for α^2 and β^2 in Section 2. The situation where Poisson's ratio is constant is a special case in the present work, but again it yields a space variation of the density which is much slower than the variation of either of the wave speeds.

The present work can be readily applied to the general three-dimensional case having medium property variation in one dimension with only slight modifications. It is the opinion of the authors that a more general solution can be found using the present methods. This more general solution should contain all of the results in Reference 2 as special cases.

Another investigation which has not been completed but which shows promise is directed toward obtaining more general differential equations of motion than those reported here or in Reference 1. It is to be noted that the differential equations (9) and (10) closely resemble those which govern the propagation of waves in a homogeneous medium. The more general equations which might be obtained are of the form

$$\phi_{rr} + a \frac{\phi_r}{r} + b\phi_{zz} + c\phi_z + d\phi - \frac{\phi_{tt}}{c^2} = 0 \qquad (26)$$

where b and $\frac{1}{c^2}$ are such that the characteristic equation

$$f_r^{\,2} + bf_z^{\,2} - \frac{1}{c^2} f_t^{\,2} = 0 \qquad (27)$$

admits real solutions. This type of generalization should permit more freedom in the space variation of λ, μ, and ρ.

REFERENCES

1. Hook, J. F., Separation of the Vector Wave Equation of Elasticity for Certain Types of Inhomogeneous, Isotropic Media, Journal of the Acoustical Society of America, 33, 3, 302-313 (March 1961)

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Unclassified Report

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2. Blast wave propagation
3. Differential equations
4. Ground motion
5. Kinematics
6. Poisson equation
7. Soils--effects of atomic explosions
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