ARL 62-373
Part II

STATISTICAL DESIGN OF COMPLEX EXPERIMENTAL PROGRAMS

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ROCKETDYNE
A DIVISION OF NORTH AMERICAN AVIATION
CANOGA PARK, CALIFORNIA

JULY 1962

AERONAUTICAL RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE

AF-WP-B-NOV 62 500

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STATISTICAL DESIGN OF COMPLEX EXPERIMENTAL PROGRAMS

II. THE DECISION THEORY OF APPROACH TO COMPLEX EXPERIMENTATION

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CONTRACT AF 33(616)-7372
PROJECT 7071
TASK 7071-01

AERONAUTICAL RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

This interim technical report was prepared by Rocketdyne, A Division of North American Aviation, Inc., Canoga Park, California, for the Aeronautical Research Laboratories, Office of Aerospace Research, on Contract AF33(616)-7372. The work reported herein was accomplished on Task 7071-01, "Research in Mathematical Statistics and Probability Theory" of Project 7071, "Mathematical Techniques of Aeromechanics" under the cognizance of Dr. H. Leon Harter of the Applied Mathematics Research Laboratory of ARL.

This work is divided into three distinct but related segments. Part I entitled "Optimum Experimental Designs Obtained by Minimizing a Loss Function" was written by Mr. Kenneth W. Last who also performed the major portion of the work of deriving and solving the problem as presented. The problem was conceived by Dr. Jack M. Zimmerman and Mr. David Rothman. The loss function on which the work was based was first stated by Mr. Rothman.

Part II entitled "The Decision Theory Approach to Complex Experimentation" is the work of Dr. Mitchell O. Locks, with consulting assistance from Dr. Albert Madansky of the Center for Advanced Study in the Behavioral Sciences. The programming was performed by Mrs. Bernyce J. Byars.

The third segment entitled "Tables of the Noncentral t Distribution" was conceived by Dr. Locks in connection with Part II. Mathematical analysis was performed by Dr. Madeline Alexander and Mr. Last. Dr. Alexander wrote the introductory material. Programming of the tables was performed by Mrs. Byars. This segment will be published as a separate ARL report.

ABSTRACT

The research reported upon is an application of decision theory to the analysis of the results of multifactorial experiments and the making of optimum redesign decisions on complex equipment during the period of development. Both the problem of acceptance testing for fixed configurations and that of testing and redesigning variable configurations are treated. A model ~~has been~~ developed and simulated on the IBM 7090 computer. A tentative utility function is presented which incorporates the gain of information from testing and losses due to fluctuations from a planned reliability schedule.

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CONCEPTS AND THEOREMS

INTRODUCTION

Among the many recent advances in statistics have been the development and growth of statistical decision theory, especially the Bayesian approach to decision theory, and its applications. The distinguishing feature of this new form of analysis is that it offers a consistent formal basis for combining prior information with that obtained from data-gathering procedures to make inferences regarding populations, and to help determine optimum courses of action in situations of great uncertainty.

Since the literature on the subject is already profuse, no attempt will be made here to give a review of it, or to discuss methodology or notation beyond the immediate needs of the research program at hand. Depending on his interests, the reader who wishes to pursue more extensive study is referred to review articles by Anscombe (Ref. 1) and Hirschleifer (Ref. 2), a book on applications by Schlaifer (Ref. 3), and a more technical book by L. J. Savage (Ref. 4) giving the mathematical and philosophical bases of decision theory.

The motivation for this particular research program is the application of decision theory to the development and acceptance testing of complex systems such as those used for advanced weapons and for outer-space vehicles. This appears to be a worthwhile application of decision theory, because the class of problems considered is that in which important decisions must be based upon limited information, a great deal of which is known or presumed to be known before the formal experimentation begins. A similar type of study of decisions under great uncertainty was made by Grayson (Ref. 5). That book, originally prepared as a Ph.D. dissertation at the Harvard Graduate School of Business Administration, is a case study developing the application of decision theory to the analysis of drilling decisions by independent oil and gas producers. In this context, both the current research program and Grayson's may be taken to be examples of attempts to use decision theory in a tradition which can be ascribed to a large extent to R. Schlaifer and H. Raiffa, who are currently very active in developing and teaching applications of decision theory. See Ref. 3 and 6 for examples of the above.

Certain innovations are claimed for the current study. The concepts involved may not all necessarily be brand new; however, they are further developed and refined beyond existing or known applications. These are:

1. The use of decision theory for analysis of the results of complex factorial experiments which are traditionally analyzed by means of linear statistical models.
2. The initiation of the analysis with a distribution on the sample space rather than the more conventional use of a prior distribution on the parameter space. This point can be clarified only by following the discussion.
3. A more extensive refinement and application of the action space employed in decision theory than has been discerned from

existing studies. That is, the incorporation into a model of the concept of an action space where the action taken affects the probability distribution for the parameter space. This development and refinement has led, for example, to an analysis in which we are able to follow the entire process of development of a complex system through a successive series of stages of modifications for reliability improvement.

4. A loss function governing the selection of an optimum hardware redesign decision which is directly related to the reliability growth schedule and which uses time for action explicitly as one of the criteria for decision. This loss function is still incomplete, and more analysis must be performed to determine its characteristics.

The work is divided into four major sections, as follows. The first section, or the current one, is a description of the elements of Bayesian decision theory. The relevant spaces are described and the theorems developed. The second section deals with an acceptance test procedure for a system having fixed configuration. The third section deals with a variable configuration model. The final section has to do with the loss function.

NOTATION AND CONCEPTS

The purpose of this section is to supply a brief description of Bayesian decision theory which is relevant to the models developed. It includes a discussion of the spaces; the probability theorems; and the formal similarity between decision theory, the theory of games, and sequential analysis. Discussion along similar lines may be found in Blackwell and Girshick (Ref. 7), Chernoff and Moses (Ref. 8) and Raiffa and Schlaifer (Ref. 6).

Spaces

We are concerned with real life situations which describe the relationships between the elements of certain sets. For notation purposes, a capital letter (e.g., A) will be used to denote the entire set, and a lower case letter of the same alphabetic designation (a) to denote an element of that set. Six sets or spaces are usually distinguished. These are,

Θ: the state or parameter space, or all relevant descriptions of the population or universe (i.e., system or component) upon whose members the testing is being performed and actions are being taken.

E: the experiment space, or class of experiments being performed to obtain information about Θ.

Z: the sample space, or all possible relevant results of the e's which yield information about Θ.

A: the action space, or class of actions taken by the experimenter as a result of the findings made in the tests about Θ.

$U(E,Z,A,\Theta)$: the utility, payoff, or negative loss function representing on some common scale of measurement the value of the consequences of an action taken by the experimenter in some given situation.

$P(Z,\Theta)$: the joint probability space of the sample result and the appropriate parameter.

Probabilities and Probability Theorems

The analysis proceeds formally from a description of the characteristics of the joint probability space $P(Z, \Theta)$. Since this is a two-dimensional joint probability distribution (p.d.) for the random variables Z and Θ, we may obtain from it two sets of marginal p.d.'s $P(Z)$ and $P(\Theta)$.

An element $p(z, \theta)$ $\left(z \in Z, \theta \in \Theta;\ p(z, \theta) \in P(z, \Theta) \text{ and } P(Z, \theta);\ P(Z, \theta) \text{ and } P(z, \Theta) \in P(Z, \Theta)\right)$ is obtained as the product of a marginal by a conditional probability. The marginal elements are labelled $p(z) \in P(Z)$ and $p(\theta) \in P(\Theta)$, and the conditional ones $p(z|\theta) \in P(Z|\theta) \in P(Z|\Theta)$, $\sum_{Z} p(z|\theta) = 1$; and $p(\theta|z) \in P(\Theta\, z) \in P(\Theta|Z)$, $\sum_{\Theta} p(\theta|z) = 1$. By the multiplication theorem of probability, the joint probability elements are computed either by Eq. 1 or Eq. 2.

$$p(z, \theta) = p(\theta).\ p(z|\theta) \qquad (1)$$

or

$$p(z, \theta) = p(z).\ p(\theta|z) \qquad (2)$$

Despite the similarity of these two equations, Eq. 1 is actually much better known than is Eq. 2. For example, with a known parameter [e.g., the p of a binomial (Bernoulli) process], it is frequently possible to determine conditional probability distributions for certain types of sample statistics (the number of successes, r, to be obtained from a sample of size n) by methods discussed in elementary textbooks. If θ

is not known with certainty, but is subject to a marginal probability distribution with element $p(\theta)$, the computation of the joint probability element by means of Eq. 1 is trivial. Equation 2 is justified by a similar line of reasoning.

In both cases, the joint probability element is the product of a marginal by a conditional element. The latter is assumed to be much better known and more objectively verifiable than is the former. The marginal element may be known only to the extent that prior data and the information obtained from testing and experimentation serve to verify what its value is.

Also employed is the concept of a prior marginal distribution which is updated by the information obtained from sampling. For example, Bayes and others have shown that given a prior marginal probability $p_0(\theta)$ and a conditional probability $p(z|\theta)$ (assumed to be known by some distribution such as binomial, Poisson, or normal for the given universe), a posterior conditional element $p_1(\theta|z)$ may be calculated by what is known as Bayes'Theorem, as follows:

$$p_1(\theta|z) = \frac{p_0(\theta)\, p(z|\theta)}{\sum_{\Theta}\left[p_0(\theta)\, p(z|\theta)\right]} = \frac{p_0(z,\theta)}{p_0(z)} \qquad (3)$$

The denominator of Eq. 3 is seen as the sum over Θ of the joint probability elements for the given z. Bayes' Theorem then states that the posterior conditional probability element is the quotient of a joint prior probability element by a marginal prior probability element which is itself a sum of joint probabilities. With respect to the entire set, Θ, Bayes' Theorem can be represented as follows:

$$P_1(\Theta|z) = \frac{P_o(z,\Theta)}{P_o(z)} \quad (4)$$

However, in terms of the entire (Z,Θ) space, the theorem becomes:

$$P_1(\Theta|Z) = \frac{P_o(Z,\Theta)}{P_o(Z)} \quad (5)$$

By the notation employed, it can be recognized that Eq. 3, 4 and 5 are variations of the same basic formula. However, Eq. 3 deals exclusively with elements, Eq. 4 with both sets and elements, and Eq. 5 exclusively with sets. The interrelationships between the three different versions of Bayes' Theorem are clarified as follows.

What are being calculated are the conditional probabilities for the parameter θ which is being treated as a random variable. Equation 3 represents an element of that probability distribution, Eq. 4 a set of these elements having the summation unity, and Eq. 5 the space of those sets.

On the right-hand side of the equality sign, the numerator terms of all three equations represent the joint probabilities which are the basic ingredients of all decision problems. In Eq. 3, that term is a joint probability element, in Eq. 4 it is a set of those elements and in Eq. 5 it is the complete joint probability space.

The denominator term represents the marginal or unconditional probability of the sample result, z. As can be seen from Eq. 3, an element of that space is the sum of a set of joint probabilities. The same element is considered in Eq. 4 as in Eq. 3 . However, in Eq. 5, the complete space of these elements is being treated.

It should be emphasized that what is being done in the application of Bayes' Theorem is the estimation of the probability elements. Thus, although Eq. 4 and 5 represent, in a type of shorthand, the estimation of these probabilities over a defined group or set, nevertheless, they represent repeated applications of Eq. 3.

It also should be noted that there is or may be a time separation between the marginal and prior $p_0(\theta)$ and the posterior and conditional $p_1(\theta|z)$. This implies a notion which has very strong intuitive appeal, i.e., that our opinions about the unknown parameter are modified by the results of the sample. Starting with the unconditional probability distribution $P_0(\Theta)$, the occurrence of z leads to the conditional probability distribution $P_1(\Theta|z)$, which becomes the unconditional probability distribution for Θ for purposes of the next stage of the sampling process. A number of illustrations of this technique are given in Schlaifer (Ref. 3).

Thus, the usefulness and application of Bayes' Theorem in multistage decision processes (cf. Bellman, Ref. 9), becomes direct and apparent. The occurrence of a given sample result leads to a revision of the expectations with respect to the parameter set. The revised set of opinions becomes an input to the next stage of analysis. Potentially, this concept can have important applications in connection with learning models, sequential analysis, and time-dependent programmed processes.

There is a corollary to Bayes' Theorem which deals with the probability of Z rather than of Θ. The reasoning is similar and it can be grasped by a symmetry argument. The relevant equations, similar to Eq. 3 , 4 , and 5 above, are

$$p_1(z|\theta) = \frac{p_0(z)\, p(\theta|z)}{\sum_Z \left[p_0(z)\, p(\theta|z)\right]} = \frac{p_0(z,\theta)}{p_0(\theta)} \tag{6}$$

$$P_1(Z|\theta) = \frac{P_0(Z,\theta)}{p_0(\theta)} \tag{7}$$

$$P_1(Z|\Theta) = \frac{P_0(Z,\Theta)}{P_0(\Theta)} \tag{8}$$

It should be noted that the joint probability element is computed by Eq. 2 rather than by Eq. 1. Also, by analogy to the discussion following Eq. 3, 4, and 5, it can be recognized that Eq. 6 deals exclusively with elements, Eq. 7 with both sets and elements, and Eq. 8 exclusively with sets. Likewise, Eq. 7 and 8 represent repeated applications of Eq. 6.

It is somewhat more difficult to explain the importance of the corollary than that of Bayes' Theorem. The reason is that in sampling situations we are likely to treat the z as the known condition of the problem rather than the θ. When θ is known [e.g., certain binomial (Bernoulli) processes such as the tossing of a coin], there are often much better ways of determining the conditional probability element $p(z|\theta)$ than by the corollary.

The practical importance of the corollary is that there are many classes of problems in which the uncertainty about the parameter is so great that it is easier to introspect with a prior distribution for Z than for Θ. With valid estimates of the conditional probabilities $P(\Theta | Z)$, the joint probabilities $P(Z, \Theta)$ can then be estimated by means of Eq. 2.

Raiffa and Schlaifer (Ref. 6) supply an example of the usefulness of the alternative formulation with respect to decision theory analysis of the results of drilling for oil. The uncertainty about the outcome of a drilling venture is so great, and the expense of drilling a well so high, that prospectors will frequently employ a test probe of the subsurface structure. Although this probe does not tell whether or not there is oil, experience establishes that certain structures are more favorable to the existence of oil than others. This experience, in effect, supplies a set of conditional probability distributions for oil, given the different possible results of a test probe. Thus, the analysis can proceed in two stages; a prior distribution for the results of the test probe, Z, and a conditional distribution for the oil, Θ.

A somewhat similar situation applies when testing is employed in connection with a research and development program for a new complex system. In this case, the parameter of interest is the reliability of the system. Until there is a certain amount of experience in the test program, it is difficult to ascertain the system reliability. However, enough may be known about the design, construction, and subsystem test conditions employed before the design is finished or the testing begun that it is possible to assume some kind of prior probability distribution for the results of the initial series of tests.

We have found usefulness for the corollary in the models constructed in this paper in the analysis of the results of complex experiments performed in connection with research and development programs for complex systems. The use is more fully explained and developed in subsequent sections.

DECISION THEORY AND GAME THEORY

Decision theory has been greatly influenced by Von Neumann and Morgenstern's Theory of Games (Ref. 10), and there is a great deal of formal similarity between the two systems of analysis. Usually an experimentation and decision situation is represented as a play of the game between the human decision maker and nature. The experimenter initiates the process by choosing and performing an e to obtain information about the unknown θ. Nature yields a result, z, which does not represent θ, but is related to it probabilistically. With limited information, and cognizant of his uncertainty, the decision maker commits himself to some action, $\hat{a}$, which is in some sense optimum, presumably in that it has a higher payoff $U(e,z,\hat{a},\Theta)$ than any other member of the A set.

The sets of couples (E,A) and (Z,Θ) represent the decision maker's and nature's strategies, respectively. The decision maker is a maximizing player who is consciously trying to maximize his payoff or utility in the light of the information generated by z and whatever prior information he had about Θ. In effect, he is using experimentation and sampling as a type of "spying" to find out the probability distribution for the opponent's strategies (Z,Θ).

On the other hand, nature does not play the game as a true opponent in the conventional sense of a two-person game. She does not have a utility function to maximize, nor is she trying to minimize the experimenter's. In effect, this appears to make it unnecessary for the experimenter to attempt to follow a mixed strategy to maximize his payoff. All he need do is select the strategy which maximizes his expected payoff.

The similarity between game theory and sequential statistical analysis has been pointed out by the authors mentioned elsewhere in the section. In the terminology employed, a sequential decision procedure is one in which the experimenter may decide to defer his action pending the outcome of a repetition of e. Presumably this will be done if the expected value of the additional information so obtained on some appropriate scale of measurement is greater than the combined cost of repeating the experiment and deferring action. In effect, E thus becomes a subset of A. In this context, sequential statistical analysis may be characterized as truncation at the third move followed by a repetition of the play.

A BAYESIAN MODEL OF AN ACCEPTANCE PROCEDURE FOR FIXED EQUIPMENT CONFIGURATION

PRELIMINARY

Having introduced the concepts of decision theory and the notation which will be employed throughout the paper, we are in a position to study applications. The problem will be treated as one of testing and redesigning a complex system or one of its components during the period of development.

Two models will be considered. The first of these is that of a sequential acceptance test for a fixed-equipment configuration and will be discussed in this section. The second model is more complex than the first and deals with testing to detect defects and redesigning for reliability improvement for a variable configuration. This model will be described in the next section. These are quite similar models, but the most important difference between them is that the action space assumes a great deal more importance in the second model than in the first. That is, the type of action taken in redesigning is presumed to have some effect on the relevant parameter. Both procedures are self-contained multistage models in the sense that the next action taken is based upon prior information, the previous action taken, and the additional information generated at the last stage of analysis or series of tests.

In the present section, a number of generalizations are made about the systems themselves and how and why they are tested. From this, the theory of the fixed-configuration model is developed. It may be described simply as a sequential acceptance testing procedure. That is, the completion of a test or test series supplies information to enable or justify the taking of one of three actions; accept the given configuration, repeat the test or test series, or reject (i.e., redesign).

As is true in all models, a certain amount of oversimplification must be employed, because it is necessary to concentrate upon the essential features of the process being considered which are relevant to the theory being exploited. The number of different considerations to be taken into account in any actual decision problem is so great that it would not be possible to manage the analysis if all of them were considered simultaneously. A certain amount of reality must be sacrificed for the sake of abstractness, and many important details are omitted. For further discussion on the theory and application of mathematical models, refer to Haavelmo (Ref. 11), Chapter I, and Karlin (Ref. 12), Chapter I.

SYSTEMS AND RELIABILITY GOALS

Considered in this study is a class of hardware systems of the type used for advanced weapons and for outer-space research. It would be worthwhile at the outset to give a generalized description of these mechanisms. The discussion below applies to this class as a whole, although it is not adequate for all members of the class.

Generally, these equipments can be characterized as having built-in intelligence properties. They are designed and built to respond automatically to certain input conditions and signals to produce responses or outputs. Because of highly specialized functions and rapid obsolescence due to constant changing of the relevant state of the art, they are unique, expensive, and must be produced and tested in limited quantities even after becoming operational.

In addition, they are usually used as building blocks for larger systems. For example, a cluster of rocket engine systems constitutes the propulsion system for a given stage of a missile. The missile system itself consists of a series of stages, a control system, a payload system, a propellant system, and a number of other subsystems, without any one of which it cannot operate. Thus, the performance of a given unit of hardware is influenced not only by its own characteristics, but also by the interdependence of and interaction among its own components and the other building blocks of the system.

In the discussion, the terms "component," "equipment," "subsystem," "hardware," "unit," and "system" will be used interchangeably to relieve the monotony of using the same word too many times. In either case, what is being referred to is a subsystem or system which itself is one of the building blocks in yet another larger system. The generality of the description is extended if we assume that this subsystem consists of components which are themselves "black boxes" or subsystems.

Every equipment in this class operates under the influence of certain variables which are the outputs of either some of its sister systems or of its own components. Likewise, it produces a response or responses which become the inputs to still other systems. These variables are either numerical quantities on a continuous scale (e.g., pressures, temperatures, and flowrates, as for thrust chambers and turbopumps), or switching functions and attributes (e.g., "go, no-go" as for the opening and closing of valves).

Since the system is built in units, and by necessity each component is developed by a different team of individuals and/or subcontractors, it is never quite known how well it will perform when assembled. What is done, in fact, is that design objectives are stated for each component in advance. These objectives represent to the best of the planner's knowledge that combination of values of the variables under which the system in the large will best be able to perform its desired function. The set of planned values of the input variables or environments, the output variables or responses, and the period of time over which these responses are to be maintained, is often known as the nominal operating point.

The fact that a complete assembly requires the mixing, matching, and intermingling of a number of subsystems made by different groups and having different electrical, mechanical, and thermal characteristics also influences their design. The tolerances or acceptable ranges of operating conditions cannot be so tight that the equipment will operate only at their nominal points. If any subsequent changes are made in specifications or design, different points may be required. It is desirable that performance be possible over a range of conditions near or about the nominal, including some severe environments. The wider this range of operating conditions which can be tolerated, the greater the probability of success. For this reason, and also because of other technical considerations having to do with variable operating conditions with sudden changes and wide ranges (e.g., start, steady-state, shutdown), equipment is designed with safety margins which make performance possible over a range of values for all of the variables (Ref. 13).

It is desired that the system be available at or about the scheduled delivery date with specified reliability. A definition commonly used for reliability is "probability of successful operation under nominal conditions for a defined or rated time period." For relatively large systems (e.g., rocket engines), these reliability goals are practically always contractual obligations. However, for smaller components, the reliability goals are often derived by mathematical formulations relating the reliability of a system to the reliabilities of its parts (Ref. 14, Chap. 10 and 11).

It is an empirical fact that the initial configuration adopted for a given type of unit of complex equipment will not work properly. There are always some deficiencies in conception, design, construction, or manufacture, most of which must be removed to enable performance. The successive removal of these defects as they become apparent hopefully results in ultimate achievement of the desired reliability goals.

Thus, in engineering, the concept of a reliability growth schedule is employed. Starting from an initial design and preliminary configuration, one obtains improvement by a series of modifications with each revised configuration more reliable than the preceding one. Verification that the planned growth schedule is being met may be obtained by testing and observing the frequency of breakdowns for each configuration adopted empirically.

The testing program should, therefore, have at least two important interrelated purposes if it is to serve the needs of enhancing development. These are as follows:

1. To verify that the equipment has the desired reliability in terms of scheduled growth.
2. To ascertain that the equipment is capable of performing in as wide a range of operating conditions as necessary, including regions of greater stress.

Because the tests are expensive and the time consumed in performing them delays development, only a few should be made. Likewise, the experiments should be informative. That is, with as few tests as necessary, it should be possible to ascertain which input variables or enviromental conditions exert the greatest effect upon performance.

Since 1935, it has been well known that there are certain preferred techniques (Ref. 15) for preplanning experiments so that they are efficient and informative. In particular, when a response or output is influenced by a number of different input conditions, a series of tests can be planned so that the effects of these variables, separately and/or in combination with each other, can be ascertained and measured. The family of testing plans is often known as "factorial experimentation," and the analysis techniques are associated with the terms "analysis of variance "and "regression."

Experiments of this type are performed as preplanned blocks or series of tests upon some homogeneous unit of material (e.g., a given configuration). All variables are systematically and deliberately varied so that every test point is different and the effects of all important variables and interactions between variables are represented. When it is known a priori, as is practically always the case, that certain variables and interactions are unimportant, the number of points in a block can be considerably reduced. A family of multifactorial techniques has been developed, which makes it possible to take advantage of this prior information and plan for the smallest number of tests that will yield the information required. This group includes fractional factorials via confounding; Latin Squares and other types of orthogonal designs (Ref. 15); rotatable designs (Ref. 16); hyperspherical designs (Ref. 17); and many other plans discussed in reference works on experimental design, such as Cochran and Cox (Ref. 18).

It is clear that the designed type of multifactor experiment as described can serve some very useful purposes in system development and testing. The most important of these is efficient exploration of the entire range of normal operating conditions by preselection of the smallest number of test points that will serve a useful purpose. Likewise, the fact that the tests are performed in series or blocks only on given configurations helps to solidify the basis for the estimation of reliability at each point on the growth schedule.

However the multiple purposes which these experiments must serve are partially conflicting goals. The same set of data must be used both to establish a statistical basis for reliability, which is defined in terms of a nominal operating point on a given configuration, and to determine performance under conditions more severe than nominal. Because the tests are expensive, only a few of them can be performed, and of these only a fraction can be nominal points. What seems to be needed is some proper extrapolation procedure which utilizes the information generated at the severe or off-nominal conditions, as well as that obtained at nominal, to estimate current reliability and subsequently influence redesign actions.

Decision theory offers a useful method of helping to solve this problem. Although the true reliability (θ) may never be actually known, it can be related to the results obtained (z) from a block of tests conducted upon the current equipment configuration including the overstress tests. The models described in the discussion below demonstrate by example such a set of techniques within the framework of decision theory.

EXPERIMENTAL TEST PLANS

What is being tested is a complex system whose operation is influenced by a multiple number of factors or variables. The experiment being performed is a predesigned and preplanned multifactorial block of n tests, of which only one is at the nominal point of operation, and the others all at different off-nominal conditions or severe environments.[1]

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[1] In certain types of designed experimental blocks (e.g., rotatable designs), a point in a block may be repeated several times. The repeated point may very well be the nominal point. However, this does not alter the basic argument that the number of nominal points in a block or series of replicated blocks is limited in a well-designed plan. Likewise, it should be noted that not all of the off-nominal points need be environments of greater stress than nominal.

The tests are always performed as complete blocks to obtain as much relevant information as possible from each point. Thus, the replication of a test either involves repeating the entire block as preplanned or an alternate block called the "image" in which the effects of the same variables and interactions are represented.[2]

The optimum or best experimental test plan, or the form of the block, is developed by collation of the engineering information requirements with the statistical know-how, taking account of the economic factors. In other words, a preliminary selection is made of the variables and interactions between variables presumed to be important enough that their effects should be ascertained. The block is designed so these effects can be observed with the minimum number of points, due consideration being given to the cost of a test. Refer, for example, to Part I of this report where the Loss Function Approach to the optimization of a test plan is discussed.

No attempt will be made in this section of the report to describe or develop techniques for designing the best tests or the best types of blocks. What we propose to do, instead, is to concentrate upon those aspects of the data which enable inferences about equipment reliability at nominal by observation and analysis of the results of a complete series of tests, most of which are at off-nominal points. In this respect, the term "reliability" is being used more or less as a catch-all

[2] In the terminology employed in experimental design, the image of a fractional factorial is obtained by using an alternate sign on one or more of the defining contrasts. For example, in an 2^{n-k} fractional factorial, an image can be obtained by interchanging the +'s and -'s for one or more of the factors in the original block. The use of this term for that purpose is not very well known; however, the practice of obtaining what is essentially the same block by alternating signs is frequently applied in replicating experiments.

to describe qualitatively rather than quantitatively all relevant aspects of the suitability of the equipment for its intended purposes. To the extent that the techniques employed help us formally to express uncertainty as to what the reliability actually is, we are enabled to take those actions with respect to redesign that appear to be optimum under the circumstances.

Spaces

As is true of decision problems generally, a series of partitioned spaces or groups of exhaustive lists of mutually exclusive groupings of variables, quantities, or objects must be defined. The definitions of the members of the E, Z, A, and Θ spaces directly follow from the description made of the fixed configuration model. Although each of these spaces will be defined as being exhaustive for purposes of the model at hand in order to keep the problem manageable, it will be recognized that each list is a scaled finite set which can be expanded, elaborated upon, or modified for any similar model.

E--The Experiment Space. e_1: A multifactorial block of n tests, or its image. At least one test is at nominal, and the others are all at different off-nominal or severe environments.

> The assumption as made above, that a sequential testing procedure for a unit of complex equipment during its development phases would be performed with only one type of block, is admittedly unrealistic. Every successive test or block of tests is, or should be, designed with the objective of enabling or increasing the probability of the discovery of operating problems. Frequently it is unnecessary to perform the complete block of tests to find out where the problems are.

However, an assumption that the E space has only one component greatly simplifies the analyses. In effect, it means no additional conditions need be stated with respect to the type of experiment performed in deciding on the optimum course of action. It should be recognized that in actual problems the E space is frequently a variable one.

Z--The Sample Space.

z_1: The system operates properly at all n points

z_2: The system fails at only one extreme point, but operates properly at nominal and at the other extreme points

z_3: Failure occurs at two of the extreme points, but not at nominal

z_4: Failure occurs at nominal

The Z space is difficult to define. The number of ways in which the sample results can be classified is always very large. Even if a very simple attribute classification scheme (e.g., failure does or does not occur at a particular point) is used, elementary combinatorial arithmetic will demonstrate that there are 2^n total possibilities that can be enumerated. If qualitative or scale variables (e.g., how bad the failure was) are used instead of attributes, the number of possibilities becomes infinite.

The best that one can hope to do under these circumstances is to use a modified and considerably abbreviated definition of the set so that all possibilities are included, and so that at the same time there is some scale on which the seriousness of the result can be evaluated.

An attempt was made to do that in partitioning the Z space above. A failure at the nominal point is more serious than a failure at one or even two off-nominal points because the equipment was supposed to have been designed to operate at nominal. Furthermore, its sister systems were designed to interact with it best when it was at nominal. Thus, the four components of the space are listed in what appears to be increasing order of seriousness. Nevertheless, it should be noted that this is a situation of great uncertainty and a single failure at a nominal point does not necessarily indicate that redesign action is called for.

A--The Action Space.

a_1: Accept the present design configuration

a_2: Repeat the block of tests (i.e., perform e_1)

a_3: Reject (i.e., redesign) the component

The above classification is identical to the traditional three choices of sequential sampling. However, certain additional technical considerations are worth mentioning.

The first of these is that the act of testing requires performance of e_1. Since e_1 is the entire E space, E in effect becomes a subset of A. The second consideration is that this paper deals with both fixed configuration and variable configuration models. The above partitioning of the A space applies only to the first of these, or the fixed configuration scheme. In the variable configuration model, yet to be discussed, the a_3 component is considerably modified to enable the taking into account of consequences of different types of redesigns or scales of effort.

Θ--The Parameter Space

θ_1: Reliability is 0.99 (i.e., acceptable)

θ_2: Reliability is 0.95 (almost acceptable)

θ_3: System reliability is unacceptable

As is true of the Z space, the Θ space is difficult to categorize and define. Reliability, a quantitative measure, was used as the basis for partitioning the elements of this space simply because it is the best known over-all measure of quality.

If the term "reliability" were taken literally in its generally accepted meaning as a probability of successful operation under nominal conditions, there are exact distributions (suchas binomial and Poisson) which can be used to estimate the probability relationships $P(Z|\Theta)$ and $P(\Theta|Z)$ (the latter by Bayes' Theorem). These exact distributions can then be used in a more or less conventional manner to estimate these probabilities if all tests were performed only at nominal (and similarly, if all elements of the Z space were defined in terms of the nominal point). Since only a few of the tests are being performed at nominal, the conception of the Θ space is a hypothetical one and should not be taken too seriously. The choice of reliability as the parameter is justified primarily because it supplies some basis for classification which is related to an index of merit.

The Utility (Loss) Space. The study of considerations involved in the choice and use of an appropriate utility function is a project for further research. To be able to proceed with application of the model, it is sufficient for the time being merely to assume that there is a utility function, and that it is a function of A and Θ only. A separate section of this paper deals with considerations involved in the choice and use of this function.

Procedure and Theorems

An acceptance test is being performed on a unit of complex equipment. The objective is to perform enough tests to justify a terminal decision either to accept or redesign the present planned configuration. The tests are performed sequentially as a series of replicated blocks of an appropriately designed multifactorial plan. At the completion of a block, the information obtained is collated with prior information (including that obtained from earlier tests) and the decision is made as to whether to accept or redesign, or to repeat the block.

The model described is a Bayesian procedure for performing this type of acceptance test plan. A numerical example is given based upon the problem as defined and the spaces delineated above. This example is supplemented by the attached flow chart (Fig. 1) which summarizes in a one-page description the sequence of events followed, and shows how the various equations (Eq. 1 through 8), including Bayes' Theorem and its corollary, are used.

The initial information given and that generated by the calculations are both summarized in a series of numbered tables. To facilitate the matching of the discussion to the flow chart, the number of each table in the illustration is given above a corresponding box in the flow chart. Likewise, the numbered equations developed in the text are given in the appropriate boxes. The subscripting adopted for the flow chart and illustration is somewhat modified from the text to facilitate the treatment of the problem by stage.

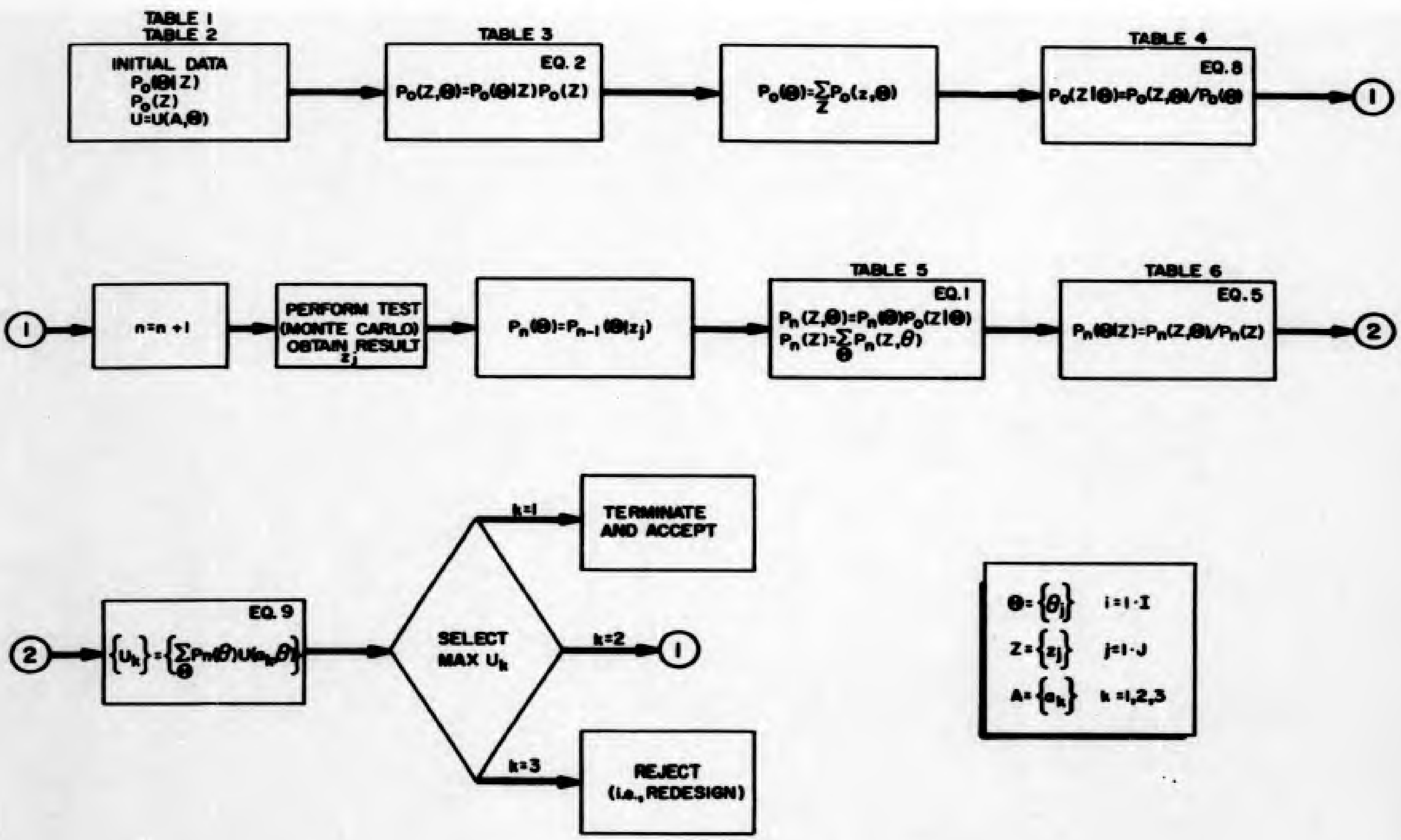

Figure 1. Flow Chart, Fixed Configuration Model

The prior information consists of a set of sets of conditional probabilities $P_0(\Theta|Z)$, and a set of marginal probabilities for Z $P_0(Z)$. These are given in Tables 1 and 2, respectively.

TABLE 1

$P_0(\Theta|Z)$

	z_1	z_2	z_3	z_4
θ_1	0.6	0.3	0.2	0.1
θ_2	0.3	0.4	0.3	0.2
θ_3	0.1	0.3	0.5	0.7
	1.0	1.0	1.0	1.0

TABLE 2

$P_0(Z)$

z_1	z_2	z_3	z_4
0.1	0.2	0.2	0.5

Then the joint prior distribution of Θ and Z is given by Table 3.

TABLE 3

$$P_0(Z, \Theta) = P_0(\Theta | Z) \cdot P_0(Z)$$

	z_1	z_2	z_3	z_4	$P_0(\Theta) = \sum_Z P_0(z, \Theta)$
θ_1	0.06	0.06	0.04	0.05	0.21
θ_2	0.03	0.08	0.06	0.10	0.27
θ_3	0.01	0.06	0.10	0.35	0.52
$P_0(Z) = \sum_{\Theta} P_0(Z, \theta)$	0.1	0.2	0.2	0.5	1.00

Then, by Eq. 8, $P_0(Z/\Theta)$ is calculated in Table 4.

TABLE 4

$$P_0(Z|\Theta) = \frac{P_0(Z, \Theta)}{P_0(\Theta)}$$

	z_1	z_2	z_3	z_4	Total
θ_1	0.286	0.286	0.190	0.238	1.000
θ_2	0.111	0.296	0.222	0.371	1.000
θ_3	0.019	0.116	0.192	0.673	1.000

NOTE: Certain figures are rounded to force the row total of 1.000.

It should be noted that the computations and analyses performed so far are based upon the use of the corollary (Eq. 8) rather than of Bayes' Theorem (Eq. 5). The preference of using the corollary at the outset over the better-known alternative is based upon heuristic considerations.

A system which is being developed and tested consists of a number of smaller building blocks, themselves "black boxes," and in many cases these have been developed, tested, and used for earlier hardware programs. A great deal more is usually known about such subsystems than about the system itself. This prior knowledge of the subsystem which is relevant to the operation of the complete system can be expressed in the form of a prior probability distribution $P_0(Z)$ on the results of a complete system test series.

Before beginning the test series, little can be said about the system reliability. However, if the spaces, Z and Θ, are defined in advance, it seems reasonable also to define a set of prior conditional probability distributions, $P_0(\Theta | Z)$, which in effect relate estimated reliability to the results of the tests.

Available prior information has not yet supplied the marginal probability distribution for Θ which is necessary to compute expected losses or utilities. The estimate is supplied only after the first test series is taken, and its results, z_j, used to justify the assertion that $P_1(\Theta) = P_0(\Theta | z_j)$. In other words, after the first test series, one of the columns of Table 1 becomes the new marginal distribution for Θ.

However, a complete Bayesian procedure requires that at every stage in the analysis there be some means of estimating the joint probability space, $P(Z, \Theta)$. This can be done only if there also is available a set of conditional probability distributions, $P(Z|\Theta)$. As has been pointed out elsewhere in this report, in problems having simpler definitions of the parameter space, these estimates can be obtained frequently by well-known procedures. Such estimates are not obtainable in any objectively verifiable fashion in this problem.

The usefulness of the corollary is that it supplies a method for $P_0(Z|\Theta)$ input, a probability distribution which is otherwise obtained by means of known functional relationships (e.g., binomial, Poisson, etc.). Furthermore, once these calculations have been made, they become the basis for all new calculations of the joint probability space as long as we continue testing the present equipment configuration. In other words, $P_0(Z|\Theta)$, as calculated by means of Table 4, is used together with the last $P_n(\Theta)$ to obtain the joint probability space every time a test is performed and its result observed. The way in which this repeated use will be made of Table 4 will become clearer from the illustration. The example proceeds as follows.

A block of tests is performed and yields the result z_4[1]. In other words, the equipment fails at the nominal point. This represents the worst case, or the highest presumption that the equipment is unsuitable for the purpose for which it was intended. Prior to testing, the probability distribution for Θ is given by $P_0(\Theta)$, or the last column of Table 3. For example, the prior probability that the equipment was unacceptable was 0.52. Given that it has failed at the nominal point

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[1] It is recognized, of course, that in the absence of an actual experiment such a result can be obtained by Monte Carlo. In this case, it was obtained by generating a random number and using the class limits of the cumulative probability distribution of $P_0(Z)$ to determine what the actual result was.

in the first block of tests performed, the appropriate probabilities for the Θ's are given by the fourth column of Table 1, or $P_0(\Theta|z_4)$. These become the new marginal probabilities, $P_0(\Theta)$. As a result, the probability that the equipment is unacceptable has now increased to 0.7.

The joint probabilities are now recomputed on the basis of the new marginal probabilities, as shown in Table 5.

Then the revised conditional probabilities, $P_1(\Theta|Z)$, can be computed by means of Bayes Theorem, which yields Table 6.

Table 6 is clearly of the same form as is Table 1. Each column of Table 6 provides a set of conditional probabilities for Θ, any one of which can become the new marginal probability dependent upon the outcome z_j of the next replication (if the test is to be repeated).

However, a replication of the test is not to be performed unless it is clearly indicated as being the optimum action under the circumstances; that is, unless the next series of tests has a higher expected utility than either accepting or rejecting the equipment configuration. At this state of the analysis, current available knowledge supplies the marginal probability distribution for Θ in the form of $P_1(\Theta)$. What is to be done at this point is to estimate the set of expected utilities

$$\left\{U_k\right\} = \left\{\sum_{\Theta} P_n(\theta)\ U(a_k, \theta)\right\} \quad (9)$$

and to take that optimum action, $\hat{k}$, which has the highest expected utility. If, then, $\hat{k} = 2$ and the decision is to replicate and, for example, z_4 were observed again on the next test, $P_2(\Theta)$ then becomes, respectively, 0.0418, 0.1304, and 0.8278. A table similar in form to Table 5 is recomputed by $P_2(Z,\Theta) = P_2(\Theta) \cdot P_0(Z|\Theta)$. The marginal probabilities

TABLE 5

$$P_1(Z,\Theta) = P_1(\Theta)\ P_o(Z|\Theta)$$

	z_1	z_2	z_3	z_4	$P_1(\Theta) = P_o(\Theta \mid z_4)$
θ_1	0.0286	0.0286	0.0190	0.0238	0.1
θ_2	0.0222	0.0592	0.0444	0.0742	0.2
θ_3	0.0133	0.0812	0.1344	0.4711	0.7
$P_1(Z) = \sum_{\Theta} P_1(Z, \theta)$	0.0641	0.1690	0.1978	0.5691	1.00

TABLE 6

$$P_1(\Theta|Z) = P_1(Z,\Theta)/P_1(Z)$$

	z_1	z_2	z_3	z_4
θ_1	0.4462	0.1692	0.0960	0.0418
θ_2	0.3463	0.3503	0.2245	0.1304
θ_3	0.2075	0.4805	0.6795	0.8278
Totals	1.0000	1.0000	1.0000	1.0000

$P_2(Z)$ are then utilized for another Bayes' theorem calculation as in Table 6 to obtain $P_2(\Theta|Z) = P_2(Z,\Theta)/P_2(Z)$.

The pattern of the recursion scheme for recomputing probabilities after each replication then becomes apparent. Given that z_j is observed on a test series at stage n

$$P_n(\Theta) = P_{n-1}(\Theta|z_j)$$
$$P_n(Z,\Theta) = P_n(\Theta) \cdot P_o(Z|\Theta)$$
$$P_n(Z) = \sum_{\Theta} P_n(Z, \theta)$$

and by Bayes' Theorem

$$P_n(\Theta|Z) = P_n(Z, \Theta)/P_n(Z)$$

The complete details may be followed from the flow chart (Fig. 1).

A BAYESIAN MODEL OF A PROCEDURE FOR SEQUENTIALLY REDESIGNING AND TESTING VARIABLE EQUIPMENT CONFIGURATIONS

PRELIMINARY

In the previous section, a Bayesian model was developed for pursuing an optimum course of action with respect to sequentially testing and redesigning, or accepting a fixed configuration of complex equipment during the period of research and development. Henceforth, that model will be called FC (for "fixed configuration"). In this section, an extended model is constructed applying the same group of concepts to analysis of variable configurations as well. The latter has been given the descriptive acronym of FIXIT (Fully Informative eXperimentation to Improve Things).

FIXIT differs from FC in several respects. Instead of there being the simple action, "redesign", corresponding to rejection of the current configuration, there is a family of different redesign actions. The procedure provides a means for prediction of the effect upon the parameter for each selected action. This means that the probability distribution (p.d.) of the reliability is affected not only by the results of past tests, but also by past redesigns. Furthermore, a prediction is obtainable for the effect of any future or contemplated redesign upon the reliability p.d. for the next configuration.

To apply the FIXIT concept, it was necessary to develop a body of theory which deals with a p.d. for Θ, the parameter space, dependent not only upon Z, the sample space, but also A, the action space, or the group of actions any one of which can be performed prior to performing the experiment and obtaining Z. Certain modifications are made to Bayes' Theorem and its corollary to permit the mathematical manipulation of probability expressions having the additional condition on A. These modifications are similar in form to certain probability axioms provided by I.J. Good (Ref. 19).

A complete procedure has been devised leading up to ultimate acceptance at the required reliability levels, and based upon a sequence of assumed events, some of which are tests and some redesign actions.[1] The logic of the procedure is given by a flow chart (Fig. 2) and a programmed computer simulation based upon it. A number of hypothetical data cases have been run on the 7090 computer.

AXIOMS AND THEOREMS

The underlying probability structure of FIXIT is provided by Bayes' Theorem and its corollary. In the form required, they may be derived from certain well known theorems in probability, with an additional condition given on the probability. For example, the equivalentsof the addition and multiplication theorems can be stated as follows.

Addition Theorem:

If $$p(\theta_1, \theta_2 | a) = 0 \tag{9}$$

(i.e., if θ_1, and θ_2 are mutually exclusive given any particular a,) then

$$p \left\{ (\theta_1 \vee \theta_2 | a) \right\} = p(\theta_1 | a) + p(\theta_2 | a) \tag{10}$$

Multiplication Theorem:

$$p(z, \theta | a) = p(z | a) . \ p(\theta | z | a) \tag{11}$$

[1]Whether the sequence of redesign actions results in an acceptable system during the scheduled development period is a matter for conjecture and something which cannot be discussed rigorously at the moment. Considerations of type of loss function and time taken for redesign are all important. Future research will be directed at studying the convergence properties of these functions.

By Eq. 12, we also have

$$p(z, \theta | a) = p(\theta | a). \; p(z | \theta | a) \tag{12}$$

Eq. 13 can be obtained from Eq. 11.

$$p(\theta | z | a) = \frac{p(z, \theta | a)}{p(z | a)} \tag{13}$$

and by substituting Eq. 12 in the numerator

$$p(\theta | z | a) = \frac{p(\theta | a). \; p(z | \theta | a)}{p(\theta | a)} \tag{14}$$

It can be recognized that Eq. 14 is simply Bayes' Theorem, similar to Eq. 3 , but unsubscripted and with the additional condition, a, imposed upon each term. To phrase the result in a more conventional form to facilitate computation, it remains to be demonstrated that

$$p(z | a) = \sum_{\Theta} p(z, \theta | a)$$

This can be done by means of an inductive proof applying Eq. 10 because Θ is defined as a set of exhaustive and mutually exclusive elements.

When z is taken as the random variable instead of θ, by a similar procedure Eq. 15 is obtained

$$p(z | \theta | a) = \frac{p(z | a). \; p(\theta | z | a)}{p(\theta | a)} = \frac{p(z, \theta | a)}{p(\theta | a)} \tag{15}$$

The similarity to Eq. 6 is obvious.

With θ as the random variable, Bayes' Theorem which relates the prior $p_0(\theta | a)$ to the posterior $p_1(\theta | z | a)$ is then restated as follows.

As an element

$$p_1(\theta|z|a) = \frac{p_0(\theta|a).\ p(z|\theta|a)}{\sum_{\Theta}[p_0(\theta|a).\ p(z|\theta|a)]} = \frac{p_0(z,\theta|a)}{p_0(z|a)} \qquad (16a)$$

The p.d. for θ is obtained with fixed z and a

$$P_1(\Theta|z|a) = \frac{P_0(z,\Theta|a)}{P_0(z|a)} \qquad (16b)$$

The denominator is the marginal probability

$$p_0(z|a) = \sum_{\Theta} p_0(z,\theta|a)$$

For the Z space and fixed a, a set of p.d.'s for θ

$$P_1(\Theta|Z|a) = \frac{P_0(Z,\Theta|a)}{P_0(Z|a)} \qquad (16c)$$

For both the Z and A spaces, a set of sets of p.d.'s for θ

$$P_1(\Theta|Z|A) = \frac{P_0(Z,\Theta|A)}{P_0(Z|A)} \qquad (16d)$$

With z as the random variable, the corollary is restated. As an element

$$p_1(z|\theta|a) = \frac{p_0(z|a)\ p(\theta|z|a)}{\sum_{Z}[p_0(z|a).\ p(\theta|z|a)]} = \frac{p_0(z,\theta|a)}{p_0(\theta|a)} \qquad (17a)$$

The p.d. for z is obtained with fixed θ and a

$$P_1(Z|\theta|a) = \frac{P_0(Z,\theta|a)}{p_0(\theta|a)} \qquad (17b)$$

The denominator is a marginal probability

$$P_0(\theta \mid a) = \sum_{Z} P_0(z, \theta \mid a)$$

For both the Θ spaces with fixed a, a set of p.d.'s for z

$$P_1(Z \mid \Theta \mid a) = \frac{P_0(Z, \Theta \mid a)}{P_0(\Theta \mid a)} \tag{17c}$$

For both the Θ and A spaces, a set of sets of p.d.'s for z

$$P_1(Z \mid \Theta \mid A) = \frac{P_0(Z, \Theta \mid A)}{P_0(\Theta \mid A)} \tag{17d}$$

The central importance of these theorems is that they provide a probabilistic tool for evaluating the consequences of an action. That is, instead of treating θ as a predetermined random variable whose value is not influenced by the actions of the experimenter, we may handle it as a random variable representing nature's move in response to some hypothetical action yet to be taken. If, in addition, given some action, there are also available some means of evaluating the utility of each different value of the random variable, then the expected utility of the action itself can be stated as follows.

$$U(a_k) = \sum_{\Theta} \left[P(\theta \mid a_k) \cdot U(a_k, \theta) \right] \tag{18}$$

Given a set of actions and the appropriate sets of probability and utility functions, it then becomes possible to select from among the family of actions that one having the highest expected utility.

THE FIXIT MODEL

Descriptive

A unit of complex equipment is being developed. The configuration is being revised from time to time to enhance reliability. Information on which to base these redesign decisions is obtained from component and system tests performed as blocks of multifactorial experiments. The rationale has been discussed extensively in the preceding section.

Spaces

The spaces used (i.e., Z, A, Θ, U, P) are identical with those employed in FC. Except for the A and U spaces, the same elements can be used in any given space for either model. The lists of names of elements for each space given below are only illustrative and can easily be expanded or modified without in any way affecting the correctness of the theory employed.

E- The Experiment Space (Identical with E in FC).

e_1: A Multifactorial Block of n Tests or its Image. One test is at nominal, and the others are all at different off-nominal or severe environments.

The limitation noted in the previous section, with respect to the assumption that the E space has only one component, applies with even greater strength and force to the FIXIT model than it does to the FC model. It is highly unlikely that an experimental test plan could operate with only one type of block from beginning to end. This is particularly true when we are dealing with a variable equipment configuration. The justification for making this assumption, as before, is only its simplicity and the fact that it obviates the necessity for requiring additional conditions on the probabilities.

Z - The Sample Space (Identical with Z in FC).

z_1: The system operates properly at all n points

z_2: The system fails at only one extreme point, but operates properly at nominal and at the other extreme points

z_3: Failure occurs at two extreme points, but not at nominal

z_4: Failure occurs at nominal

A - The Action Space.

$\{a_k\}$ $k = 1 \ldots K \ldots L$

a_1: Accept the present design configuration (identical with a_1 in FC)

a_2: Repeat the block of tests (i.e., perform e_1) (identical with a_2 in FC)

$a_3 \ldots a_K$: An ordered set of redesign actions. The higher the index of the subscript, the more effective the action is *a priori* in improving reliability

K: The maximum number of ordered actions (including a_1 and a_2) which will be considered simultaneously at any given stage in the analysis

FIXIT differs from FC primarily in that the "rejection" (a_3) of FC has been partitioned into $L - 2$ different redesign actions of which $K - 2$ are considered at a time. This treatment is believed to be quite realistic. At the time of initial design, it is known that some changes will be necessary. Some of these changes are suspected in advance in the sense that it is known that one or more of them will have to be made. Other changes to be made are not known in advance, but will be considered only after the equipment has improved sufficiently so that the deficiencies which make them necessary become apparent.[1]

[1] The discussion above is related to the concept of the "masking" effect of a defect which is used in reliability engineering. One defect which is more obvious "hides" another which is less apparent. When the obvious one is fixed, the other one becomes detectable. In other words, the designer never runs out of deficiencies to correct, but must always decide from among a relatively small group of contenders at a given time.

Θ - The Parameter Space (Identical with Θ in FC).

θ_1: Reliability is 0.99 (acceptable)

θ_2: Reliability is 0.95 (almost acceptable)

θ_3: Reliability is unacceptable

The p.d. of the random variable representing the parameter of the current configuration is given by

$$P(\Theta|a_1) = P(\Theta|a_2)$$

Since A is an ordered set with actions of higher index representing improved configurations a priori it will be assumed that the initial probabilities are

$$P_o(\theta_1|a_i) \geq P_o(\theta_1|a_j)$$

and

$$P_o(z_1|a_i) \geq P_o(z_1|a_j)$$

for $i > j$, and $i \leq L$, $j < L$. In other words a separate p.d. for the test results is assigned to each of the $L - 2$ redesign actions. The higher the index, the better the result expected.

U - The Utility Space. As noted elsewhere, the concept of a utility space is discussed more fully in the next section, and requires more work. To be able to proceed with solution and analysis of the model, the utility space is being treated simply as a fixed table of utility values independent of time

$$U(a_k, \theta_i) \quad k = 1...K, \quad i = 1...I$$

The Basic Model

The basic model of FIXIT is given in the flow diagram (Fig. 2) which supplies in detail the exact sequence of events including testing and redesign actions leading up to final acceptance. Since it is the same

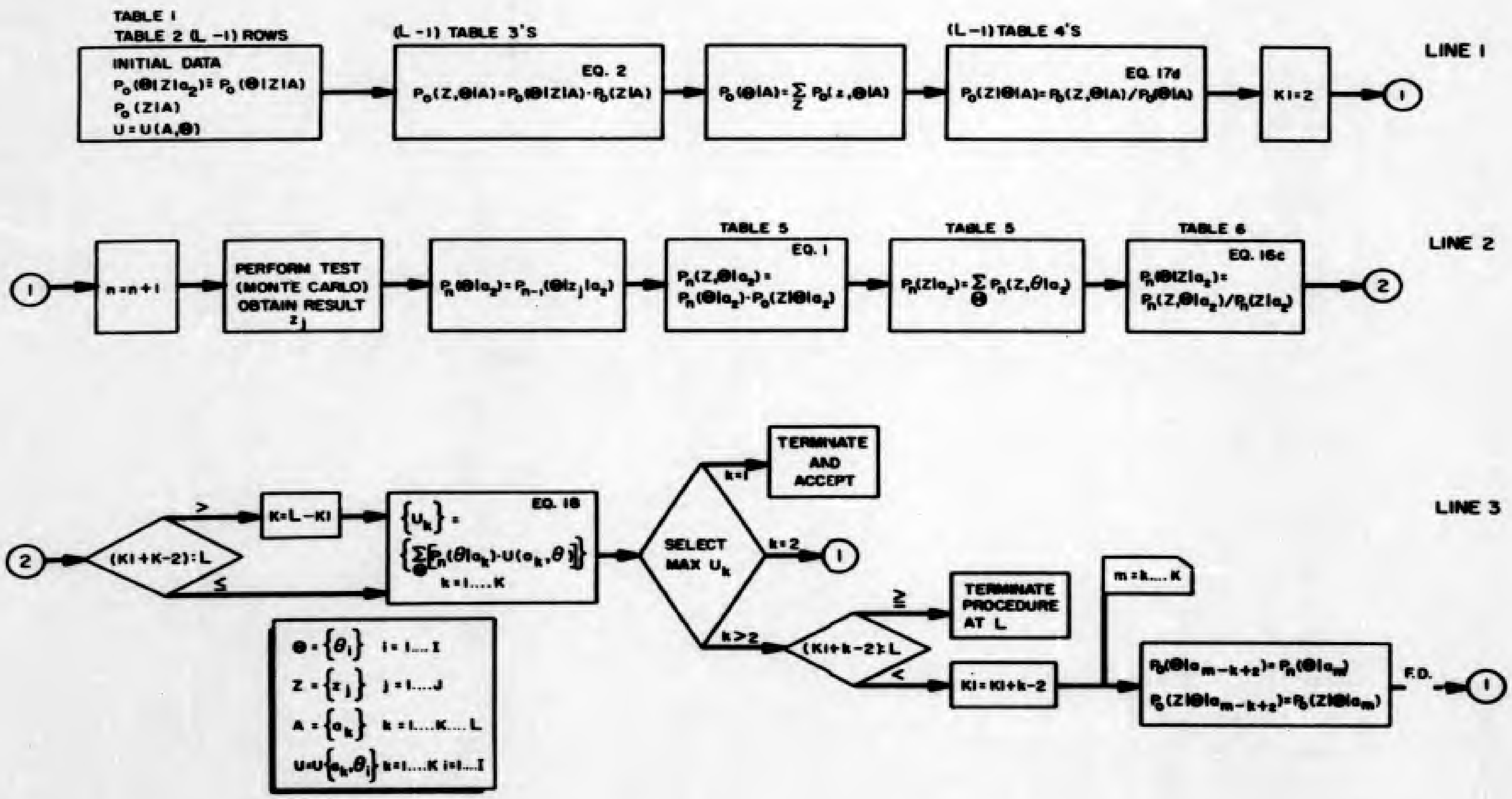

Figure 2. Flow Chart; Variab Configuration Mode

flow chart from which the computer simulation program was prepared, it is written in the terminology and symbolism of computer programming. The square boxes represent operations; the diamond-shaped boxes represent decisions; the flag-shaped indicator is an assertion (as to the value or range of values for an index so as to indicate how many times a given repetitive process will be performed), and the symbol F.D. (for "Flag Down") indicates end of assertion.

Structurally, the flow chart is similar to that of FC as given in Fig. 1. The primary difference is in the effects of enrichment of the A space so that after a redesign is performed a test is made, and the next action taken is based upon the results of the test. The increased complexity of A requires some additional program "housekeeping" instructions, most of which are on the first part of Line 3. However, the important similarities and differences can be ascertained with some additional study and/or explanation. As in Fig. 1, the number of the equation from the text being used is given inside the appropriate operation box and the corresponding table number (see illustrative example from the previous section) above the box.

Line 1 of the flow chart represents the use of prior information, with respect to Z, and the application of Bayes' theorem corollary to estimate the set of sets of prior p.d.'s $P_0(Z|\Theta|A)$. It differs from Fig. 1 in that this estimation is performed for all $L - 2$ redesign configurations as well as the current one, whereas in FC, $P_0(Z|\Theta|a_2)$ (i.e., the set of prior p.d.'s for the current configuration) only is estimated.

Line 2 is identical to the second line of Fig. 1. A test is performed on the current configuration; result z_j is obtained;

$$P_n(\Theta|a_2) = P_{n-1}(\Theta|z_j|a_2)$$

and by Bayes' Theorem $P_n(\Theta|Z|a_2)$ is obtained. The only formal difference is the use of the additional condition a_2. However, since a_2 means "retest the current configuration," no basic substantive change is involved.

Line 3 represents evaluation of utility for K actions (including a_1 and a_2), and the taking of that action with the highest utility. If the optimum action is a redesign (lower right-hand corner), that action is taken; and the $P_o(Z|\Theta|..)$ for that action as calculated on Line 1 becomes $P_o(Z|\Theta|a_2)$ for purposes of the next series of tests.

When any given redesign action is performed, a new group of actions comes into focus as contenders. The plan for moving these previously unused actions into place is a simple index-decrementing operation known as "constant differences." The way it is done is given by the operation box in the lower right hand corner of the flow chart. However, it can be illustrated with an example.

Suppose

$$K = 5 \quad \text{(i.e., three redesign actions are considered at a time)}$$

no redesigns have been performed, and

$$\hat{a}_k = a_4$$

Then, redesign action a_4 is performed;

$$P_o'(\Theta|a_2) = P_o(\Theta|a_4)$$

$$P_o'(Z|\Theta|a_2) = P_o(Z|\Theta|a_4)$$

$$P_o'(\Theta|a_k) = P_o(\Theta|a_{k+2}), \; k = 3...(L-2)$$

$$P_o'(Z|\Theta|a_k) = P_o(Z|\Theta|a_k + 2), \; k = 3...(L-2)$$

In other words, all action indexes are decremented by two and the old $P_0(\Theta \mid a_4)$ and $P_0(Z \mid \Theta \mid a_4)$ become $P_0(\Theta \mid a_2)$ and $P_0(Z \mid \Theta \mid a_2)$ respectively. Furthermore, the redesign actions that will be considered after the next series of tests are the old a_5, a_6, and a_7 which have now been numbered, respectively, a_3, a_4, and a_5. The primes (') are dropped in the flow chart; however, they are implied.

The significance of this evaluation, when it is followed to its logical conclusion, is that whenever a given redesign action is performed, a marginal p.d., $P_0(\Theta \mid a_2)$, and set of conditional p.d.'s, $P_0(Z \mid \Theta \mid a_2)$, are also automatically provided for the new configuration. With these, the reliability p.d. for the new configuration can be updated after a test by employing Bayes Theorem.

Furthermore, in evaluating the utilities of the K-2 redesign actions that might be considered, the marginal p.d.'s $P(\Theta \mid a_3) \ldots P(\Theta \mid a_K)$ are also needed, and are provided by the program. Thus, as was originally proposed, a complete sequence of redesign and testing actions can be followed to the point of ultimate acceptance (provided no more than L-2 redesign actions need be considered altogether).

AN ILLUSTRATIVE DATA CASE

The following example is given of a case that was run on the IBM 7090 computer as an operation of the FIXIT Model.

TABLE 7

$$P_0(\Theta \mid Z \mid a_2) \equiv P_0(\Theta \mid Z \mid A)$$

	z_1	z_2	z_3	z_4
θ_1	0.6	0.3	0.2	0.1
θ_2	0.3	0.4	0.3	0.2
θ_3	0.1	0.3	0.5	0.7
Total	1.0	1.0	1.0	1.0

TABLE 8

$P_o\ (Z|A)$

	z_1	z_2	z_3	z_4	Totals
a_2	0.1	0.2	0.2	0.5	1.0
a_3	0.1	0.2	0.2	0.5	1.0
a_4	0.1	0.3	0.2	0.4	1.0
a_5	0.1	0.3	0.3	0.3	1.0
a_6	0.2	0.3	0.3	0.2	1.0
a_7	0.2	0.4	0.3	0.1	1.0
a_8	0.2	0.4	0.4	--	1.0
a_9	0.3	0.3	0.4	--	1.0
a_{10}	0.3	0.4	0.3	--	1.0
a_{11}	0.3	0.5	0.2	--	1.0
a_{12}	0.3	0.6	0.1	--	1.0
a_{13}	0.4	0.5	0.1	--	1.0
a_{14}	0.4	0.6	--	--	1.0
a_{15}	0.5	0.5	--	--	1.0
a_{16}	0.6	0.4	--	--	1.0
a_{17}	0.7	0.3	--	--	1.0
a_{18}	0.8	0.2	--	--	1.0

$P_o(Z, \Theta|A) = P_o(\Theta|Z|A) \cdot P_o(Z\ A)$, is a set of 17 3 x 4 tables, one for each a. The first of these, $P_o(Z, \Theta|a_2)$, is identical to Table 3 in the second section. The remainder of them need not be reproduced; however, for illustration, the table for $a = a_5$ will be reproduced.

TABLE 9

$$P_o(Z, \Theta|a_5) = P_o(\Theta|Z|a_5) \cdot P_o(Z|a_5)$$

	z_1	z_2	z_3	z_4	$P_o(\Theta|a_5) = \sum_Z P_o(z, \Theta|a_5)$
	0.06	0.09	0.06	0.03	0.24
	0.03	0.12	0.09	0.06	0.30
	0.01	0.09	0.15	0.21	0.46
$P_o(Z|a_5) = \sum_{\Theta} P_o(Z, \theta|a_5)$	0.1	0.3	0.3	0.3	1.00

$P_o(Z, \Theta|a_2)$ is identical to Table 4 of the second section. By example, again, Table 10 for $a = a_5$ is reproduced.

TABLE 10

$$P_o (Z|\Theta|a_5) = \frac{P_o (Z, \Theta|a_5)}{P_o (\Theta|a_5)}$$

	z_1	z_2	z_3	z_4	Total
θ_1	0.2500	0.3750	0.2500	0.1250	1.0000
θ_2	0.1000	0.4000	0.3000	0.2000	1.0000
θ_3	0.0217	0.1957	0.3261	0.4565	1.0000

A fixed utility Table $U(A, \Theta)$ is employed

TABLE 11

$U (A, \Theta)$

	a_1	a_2	a_3	a_4	a_5
θ_1	--	-2.0	-7.0	-9.0	-11.0
θ_2	-25.0	-10.0	-8.0	-10.0	-12.0
θ_3	-65.0	-15.0	-21.0	-17.0	-13.0

The numbers in this table are arbitrary, but were made up with the idea of having a reasonable correspondence to reality. They are all negative numbers, so they are in fact "losses." For example, the "loss" of having a system with appropriate reliability (θ_1) and accepting it (a_1) is zero. But the consequences of accepting (a_1) when the system is defective (θ_3) is a great loss (65).

Based upon the these data, the following sequence of events took place.

Stage(s)	Action Taken
0	a_2 (test)
1	a_5 followed by test
2	a_2 (test)
3	a_5 followed by test
4-28	a_2 (test)
29	a_1 (accept)

29 blocks of tests

2 redesign actions

1 acceptance

These results are not conclusive. Future research will be directed towards study of the convergence properties of the procedure.

UTILITY FUNCTIONS FOR VARIABLE CONFIGURATION MODELS

The concepts developed in the earlier sections show that within the framework of decision theory, the problem of sequentially testing and redesigning variable configuration hardware to final acceptance can be treated almost as simply as that of merely testing fixed configurations. By means of the FIXIT model which offers procedures both for translating the results of complex environmental tests into reliability p.d.'s for the current configuration, and for predicting the reliability p.d.'s of different contemplated redesign actions, the designer is supplied with information of a probabilistic nature on which to base the next decision.

For a model of this type to be used as a consistent formal guide to optimum redesign and/or testing actions, it must be employed in connection with an objective function which is maximized by the action taken at each stage. This function should not only be well-defined, theoretically justifiable, and conform to empirical reality, but should also be comprehensive enough to cover any situation which might occur. Only the last of these criteria (comprehensiveness) can be satisfied by the fixed utility space $U(A, \Theta)$ which is used currently with each data case of the FIXIT model.

The remainder of this section and part is devoted to a discussion of the choice and form of a measurable utility function which is not only comprehensive, but meaningful. The principal components are the information obtained from experiments, and the fluctuations from the planned reliability growth schedule. A specific form of this type is given as an example. The treatment of utility is not rigorous; however, it is anticipated that further development will enable a more axiomatic treatment.

The concept of "utility," discussed extensively in the literature of economics for about two centuries, has received a great deal of impetus lately particularly as a result of the publication of Von Neumann and Morgenstern's Theory of Games (Ref. 10). That book and the research that has been based upon it have opened up important vistas in terms of the measurement of the value of an action by estimation of the expected values of its consequences.

Briefly, if an action has more than one possible consequence, the viewpoint is now general that the expected utility of each consequence can be measured and stated in arbitrarily defined, but numerically additive units which have frequently been called "utiles." The value of the action is the sum of the expected values of all the consequences. Mathematically, the utility of the action is a linear combination of functions of the consequences.

In developing and testing complex equipment the most important consideration is the enhancement of reliability. Because reliability growth to a large extent is scheduled, the most important component of the designer's utility function is the anticipated fluctuation from this growth schedule. Without losing any generality in the description, we may speak of being ahead of schedule as a situation of positive utility, and being behind schedule as one of negative utility or loss; moreover the utility function is concave.

The designer always has a number of different ways of redesigning to improve performance. The more major the improvement undertaken, the better the reliability is likely to be afterwards, but the more time is required. Because of schedule limitations, too many major improvements cannot be attempted, although some are necessary. Throughout the entire development period the designer is constantly trading off relatively small growth in short spurts against larger growth which requires more time.

For this concept of "fluctuation from the growth schedule" to be usable for purposes of an objective function, it must be expressible in terms of a set of units. However, it should be kept in mind that as the development program advances and the system reliability more closely approaches unity, improvement by any given order of numerical magnitude becomes more difficult and expensive. What this suggests is that it is appropriate to use a transformation which in some sense corresponds to the increased difficulty of continued improvement rather than the reliability (R) itself. One example is the arcsin transformation. If one uses the arcsin transformation, the following function is suggested:

$$F_{kt} = \left\{ \arcsin \bar{R}_{kt} - \arcsin \hat{R}_{t+\tau_k} \right\} \tag{19}$$

where

t is current time

τ_k is the time required for action k (including test)

$\bar{R}_{kt} = E_{t+\tau_k}(\theta \mid a_k) = \sum_{\Theta} \theta \cdot p(\theta \mid a)$

(i.e., expected reliability at $t + \tau_k$ due to action k

$\hat{R}_{t+\tau_k}$ = scheduled reliability at $t + \tau_k$.)

In other words, the principal component of the utility function of t for action k is the difference between the arcsin of the expected reliability at $t + \tau_k$ if that action is taken and the arcsin of the scheduled reliability. However, any convex function of reliability or log reliability could serve as well.

The second major consideration is the information from testing. Tests are performed after redesign, and if warranted are repeated on the same configuration before another redesign is attempted. However, testing, too, requires time during which reliability improvement is not being performed. Then a proper utility function clearly should include and be affected by the value of the information obtained in the tests so that this information can be traded off against the time required for testing to enable or justify the optimum decision.

In the engineering sense, the most important information emanating from a test is the localization of a design fault. Admittedly, it would be very difficult statistically to assign a value to information of this complexity. However, it can be approximated if we keep in mind the fact that there is available both a p.d. on the current reliability, and a separate predicted p.d. for every action which may be taken. When such a situation exists, the information content of the test resulting from that action can easily be measured by the Kullback-Leibler information numbers (Ref. 20, p 4-9).

The following derived information numbers seem to be consistent both with Kullback's theory and notation and the theory of the FIXIT model.

$$I_{kt}\,(\theta : \Theta | z) = \log \left\{ p_{t+\tau_k}\,(\theta | z | a_k) / p_t\,(\theta) \right\}$$

$$I_{kt}\,(\Theta | z) = \sum_{\Theta} I_{kt}(\theta : \Theta | z)$$

$$I_{kt} = \sum_{Z} I_{kt}\,(\Theta | z) = \sum_{Z}\sum_{\Theta} \log \left\{ p_{t+\tau_k}\,(\theta | z | A_k) / p_t\,(\theta) \right\} \qquad (20)$$

The predicted information content of the test performed after action a_k $(k = 3..K)$ at t is the sum of the information supplied by all z with respect to all θ.

A form of the utility function which now seems to be appropriate is the following:

$$U_{kt} = C_1 F_{kt} + C_2 I_{kt} \quad (21)$$

or the total utility is a weighted sum of both components.

Other specific forms of the F and I functions may be feasible and usable; however, the basic concept that $U = g(F, I)$ is not affected thereby.

What has been accomplished so far is a tentative specification of a utility function, both as to components and as to form. What is needed, however, is a completely general utility function which is not only comprehensive, meaningful, and justifiable, but also (i.e., in the case of the FIXIT model) a determination of whether or not it always results in final acceptance at required reliability levels.

The simulations that have been performed on FIXIT suggest that, even in extreme cases, the process always converges to acceptance although not necessarily on schedule. However, it remains to be demonstrated mathematically that this convergence always occurs (i.e., except in situations of dominance). This line of attack is the principal objective for the future, in addition to a verification of the form of the utility function specified.

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