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REPORT NO. RR-TR-62-6

ON THE DIVISION OF THE ATMOSPHERE INTO LAYERS FOR DENSITY CALCULATIONS IN ATMOSPHERIC BALLISTICS

15 August 1962

U S ARMY MISSILE COMMAND

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ON THE DIVISION OF THE ATMOSPHERE INTO LAYERS FOR DENSITY CALCULATIONS IN ATMOSPHERIC BALLISTICS

by

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ABSTRACT

Subdivisions of the altitude range from surface to 25 km into four to eight layers are presented. These layer combinations have been chosen to provide optimum average fit of the corresponding step functions to a combined set of individual air density profiles from the Northern Hemisphere. Separation into zones such as tropical, temperate and arctic would render optimum layers different from the presented Northern Hemisphere combination.

The investigation has been based on evaluation of samples from the radiosonde stations Guam, Wiesbaden and Fairbanks. The samples have been selected to approximate closely the climatological frequency of occurrence of density profiles.

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ON THE DIVISION OF THE ATMOSPHERE INTO LAYERS FOR DENSITY CALCULATIONS IN ATMOSPHERIC BALLISTICS

I. INTRODUCTION

For certain applications in problems of artillery and missile ballistics it is desired to restrict the number of layers used to describe the vertical profile of air density. This can be accomplished by combining the deviations for several altitude levels to a single layer* average. The totality of all such layers (or steps) spanning the total altitude range of a profile will be called "stair-step function".

Naturally, the approximation of the original profile by a step function introduces an inaccuracy. For a given number of steps, this inaccuracy can generally be reduced if layers of different, instead of equal, thickness are chosen, In the present study several groups of layer combinations or "models" have been investigated which consist of four, five, six, seven or eight steps each. After defining a rather simple and preliminary measure of the inaccuracy of fit, one model from each group is presented which yields the smallest inaccuracy of fit when applied to a sample of 108 individual cases selected from the stations Guam, Wiesbaden and Fairbanks. An attempt has been made to compose this sample in such a way that it is somewhat representative for the population of all density profiles likely to be encountered over the Northern Hemisphere.

II. ESTABLISHMENT OF GLOBAL STEP FUNCTION MODELS

A. Selection of a Sample of 108 Individual Density Profiles

In order to find a sample of density profiles which is approximately representative for the profiles of the Northern Hemisphere, individual profiles from one station each in the Tropical, the Temperate,

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*Frequently the expression "zone" is used instead of the term "layer" which is preferred here.

and the Arctic Zones have been selected and used jointly in this study. The three stations are listed in Table I.

Table I.

LIST OF STATIONS USED IN THIS REPORT

Region	Station	Geographical Latitude	Geographical Longitude
Tropics	Guam (Andersen AFB)	13° 34' N	144° 55' E
Temperate Zone	Wiesbaden, Germany	50° 03' N	8° 20' E
Arctic	Fairbanks, Alaska	64° 49' N	147° 52' W

From these stations individual density values at constant altitude intervals were available which had been computed from the temperature and pressure observations of routine meteorological radiosonde balloon ascents. The computation procedures have been described by Alfuth and Smith (Reference 1). From the profiles which reached at least to 25 km altitude, 36 cases were selected per station. This small number was made necessary by technical restrictions. For the station at Guam a random selection of three profiles from each month yielded values of the sample mean and standard deviation at all observed altitude levels that were sufficiently close to the corresponding climatological values of the observation period 1951 through 1957. For the other two stations, however, the cases have been chosen to provide examples of winter and summer profiles exhibiting large negative and average positive deviations from the monthly average density at the approximate altitude of maximum temporal density variation (around 14 km). The classification into low, average and high density values at this altitude has been based on the monthly frequency distributions of density for the 14 km level (References 2, 3, 4) in the following manner:

low:	below the 16% cumulative percentage frequency level (CPF)
average:	between 16% and 84% CPF
high:	above 84% CPF

The mixtures of low, average, and high profiles of which the samples have been composed are shown in Table II. The actual dates of the selected ascents are given in Table III.

Table II

COMPOSITION OF THE SAMPLE OF SELECTED DENSITY PROFILES

(Number of Cases)

Month	Fairbanks			Wiesbaden			Guam
	Low	Avg	High	Low	Avg	High	Random
Jan	3	4	3	3	4	3	3
Feb	-	-	-	-	-	-	3
Mar	-	-	-	-	-	-	3
Apr	-	8	-	-	8	-	3
May	-	-	-	-	-	-	3
Jun	-	-	-	-	-	-	3
Jul	3	4	3	3	4	3	3
Aug	-	-	-	-	-	-	3
Sep	-	-	-	-	-	-	3
Oct	-	8	-	-	8	-	3
Nov	-	-	-	-	-	-	3
Dec	-	-	-	-	-	-	3
Sum	36			36			36

Table III

COMPILATION OF THE DATES OF THE SELECTED RADIOSONDE ASCENTS

Station Fairbanks	Station Wiesbaden	Station Guam
27 Apr 51, (15)* a*	2 Jul 52, (15) h	15 Apr 51, (03) r
9 Jul 51, (04) h	28 Jul 52, (15) *l*	15 Jun 51, (03) r
21 Oct 51, (15) a	8 Oct 52, (15) a	20 Aug 51, (03) r
28 Jan 52, (03) a	17 Apr 53, (15) a	15 Jan 52, (03) r
6 Jul 52, (03) a	26 Apr 53, (03) a	15 Feb 52, (03) r
10 Oct 52, (15) a	30 Apr 53, (15) a	19 May 52, (03) r
17 Oct 52, (15) a	26 Oct 53, (06) a	17 Jul 52, (03) r
30 Oct 52, (15) a	24 Jun 54, (15) *l*	13 Sep 52, (03) r
12 Jan 53, (15) *l*	5 Apr 54, (16) a	18 Dec 52, (03) r
19 Jan 53, (15) *l*	24 Apr 54, (15) a	15 Mar 53, (03) r
14 Apr 53, (03) a	29 Apr 54, (15) a	15 May 53, (15) r
26 Apr 53, (03) a	1 Jul 54, (15) a	14 Jun 53, (15) r
5 Jul 53, (15) a	7 Jul 54, (03) *l*	15 Aug 53, (05) r
18 Jul 53, (15) *l*	17 Jul 54, (15) a	15 Jan 54, (03) r
24 Jul 53, (15) h	6 Oct 54, (15) a	14 Feb 54, (15) r
7 Oct 53, (15) a	8 Oct 54, (15) a	15 Apr 54, (03) r
12 Jan 54, (15) a	26 Oct 54, (03) a	12 May 54, (03) r
26 Apr 54, (03) a	12 Apr 55, (03) a	15 Jul 54, (04) r
22 Jul 54, (15) *l*	18 Apr 55, (15) a	16 Sep 54, (03) r
9 Oct 54, (15) a	2 Jul 55, (15) a	12 Oct 54, (15) r

*Figures in parentheses show balloon release time in hours Greenwich Central Time; last symbol indicates mode of selection:

a - average density at 14 km
h - high density at 14 km
l - low density at 14 km
r - ascent selected arbitrarily (at random)

Table III. Concluded

Station Fairbanks	Station Wiesbaden	Station Guam
3 Jan 55, (15) *l*	17 Jul 55, (15) h	8 Nov 54, (03) r
23 Jan 55, (15) a	4 Oct 55, (15) a	13 Dec 54, (03) r
16 Apr 55, (15) a	16 Oct 55, (03) a	15 Aug 55, (03) r
30 Apr 55, (15) a	5 Jan 56, (15) h	15 Oct 55, (03) r
20 Jul 55, (15) a	9 Jan 56, (14) *l*	15 Jan 56, (03) r
2 Oct 55, (15) a	15 Jan 56, (14) *l*	15 Feb 56, (03) r
13 Jan 56, (15) *l*	18 Jan 56, (14) a	15 Mar 56, (03) r
22 Apr 56, (03) a	13 Jul 56, (02) a	15 Jun 56, (15) r
27 Jul 56, (15) h	16 Jul 56, (02) *l*	15 Jul 56, (03) r
26 Oct 56, (03) a	9 Oct 56, (02) a	15 Sep 56, (03) r
1 Jan 57, (03) a	5 Jan 57, (14) h	13 Oct 56, (03) r
11 Jan 57, (15) h	8 Jan 57, (14) h	16 Nov 56, (03) r
15 Jan 57, (03) h	9 Jan 57, (14) a	15 Dec 56, (15) r
21 Apr 57, (03) a	12 Jan 57, (14) a	15 Mar 57, (03) r
1 Jul 57, (12) a	23 Jan 57, (02) a	15 Apr 57, (03) r
26 Jul 57, (00) *l*	5 Jul 57, (00) h	14 Nov 57, (00) r

Such a nonrandom selection provides a better approximation to the statistical population of air density profiles for Wiesbaden and Fairbanks than could have been achieved by choosing a sample of 36 cases at random.

Tables IV and V give numerical values to enable a comparison between the selected sample and the average values for 1951 - 1957 for the three stations Guam, Wiesbaden and Fairbanks at five altitude levels. The two quantities which are compared are the mean of the relative density, $\frac{\rho - \rho_{ARDC}}{\rho_{ARDC}}$, (Table IV) and its standard deviation σ (Table V) at the five altitude levels. Unfortunately the arithmetic means and the corresponding standard deviations were not available in the Climatological Ringbook Data. The following substitutes had to be taken:

median value (50% CPF) instead of arithmetic mean and
approximate σ = (value at 84.1% CPF minus value at 15.9% CPF)/2
instead of the standard deviation.

Meteorological frequency distributions can exhibit appreciable deviations from normality. The difference between the available median values and the desired arithmetic means of the climatological density values are probably nonnegligible in a large sample. Comparison of the mean values of the selected 36 profiles against the sample from the period 1951 through 1957 therefore introduces a small bias. However, since deviations between median and arithmetic mean cannot be proven significantly on smaller samples, the presented differences as shown in Table IV can be explained as sampling effect without indicating a significant bias in the data selection.

Likewise, Table V displays no significant bias between the selected 36 profiles and the complete sample from 1951 through 1957.

The frequency distributions in Figure 1 further indicate the good agreement between 36 cases and complete data from 1951 through 1957. Although the graphs exhibit some discrepancy between selected and complete sample for Wiesbaden, 15 km, and Guam, 15 km, application of the χ^2 test gives a probability value of .30 to .10, which is within the range of statistical insignificance.

It should be stressed that larger samples retaining the same features may prove significant; however, larger samples are not very likely to show the same tendency of deviation. The hypothesis that the selected sample of 36 cases is a fair selection for the intended purpose of this report cannot be rejected by statistical significance.

Table IV

COMPARISON OF THE ARITHMETIC MEANS OF THE SELECTED SAMPLES WITH MEAN VALUES 1951 - 1957 AT FIVE ALTITUDE LEVELS

Mean Value of Relative Density $\frac{\rho - \rho_{ARDC}}{\rho_{ARDC}}$ (%)

Station		Altitude				
		1 km	5 km	10 km	15 km	20 km
Guam	a	- 4.34	- 3.23	1.51	17.88	7.04
	b	- 4.41	- 3.33	1.42	17.84	6.83
Wiesbaden	a	0.71	- 0.27	1.42	- 0.75	- 0.22
	b	.61	.21	0.53	- .59	- .68
Fairbanks	a	2.70	.15	- 2.71	- 5.39	- 3.45
	b	2.38	.53	- 3.20	- 6.78	- 3.52

a - arithmetic mean for the sample of 36 cases

b - annual 50% (median) value for the observation period 1951 through 1957

Table V

COMPARISON OF THE STANDARD DEVIATIONS OF THE SELECTED SAMPLES WITH STANDARD DEVIATIONS FROM 1951 - 1957 AT FIVE ALTITUDE LEVELS

Standard Deviation of Relative Density $\frac{\rho - \rho_{ARDC}}{\rho_{ARDC}}$ (%)

Station		Altitude				
		1 km	5 km	10 km	15 km	20 km
Guam	a	0.59	0.36	0.52	1.29	2.21
	b	.53	.40	.54	0.88	1.65
Wiesbaden	a	2.68	1.60	3.01	3.53	3.30
	b	2.69	1.46	2.73	4.02	3.41
Fairbanks	a	3.75	1.48	4.49	4.21	3.46
	b	4.63	1.73	5.16	4.40	3.96

a - standard deviation σ for the sample of 36 cases

b - approximate annual standard deviation for the observation period 1951 through 1957

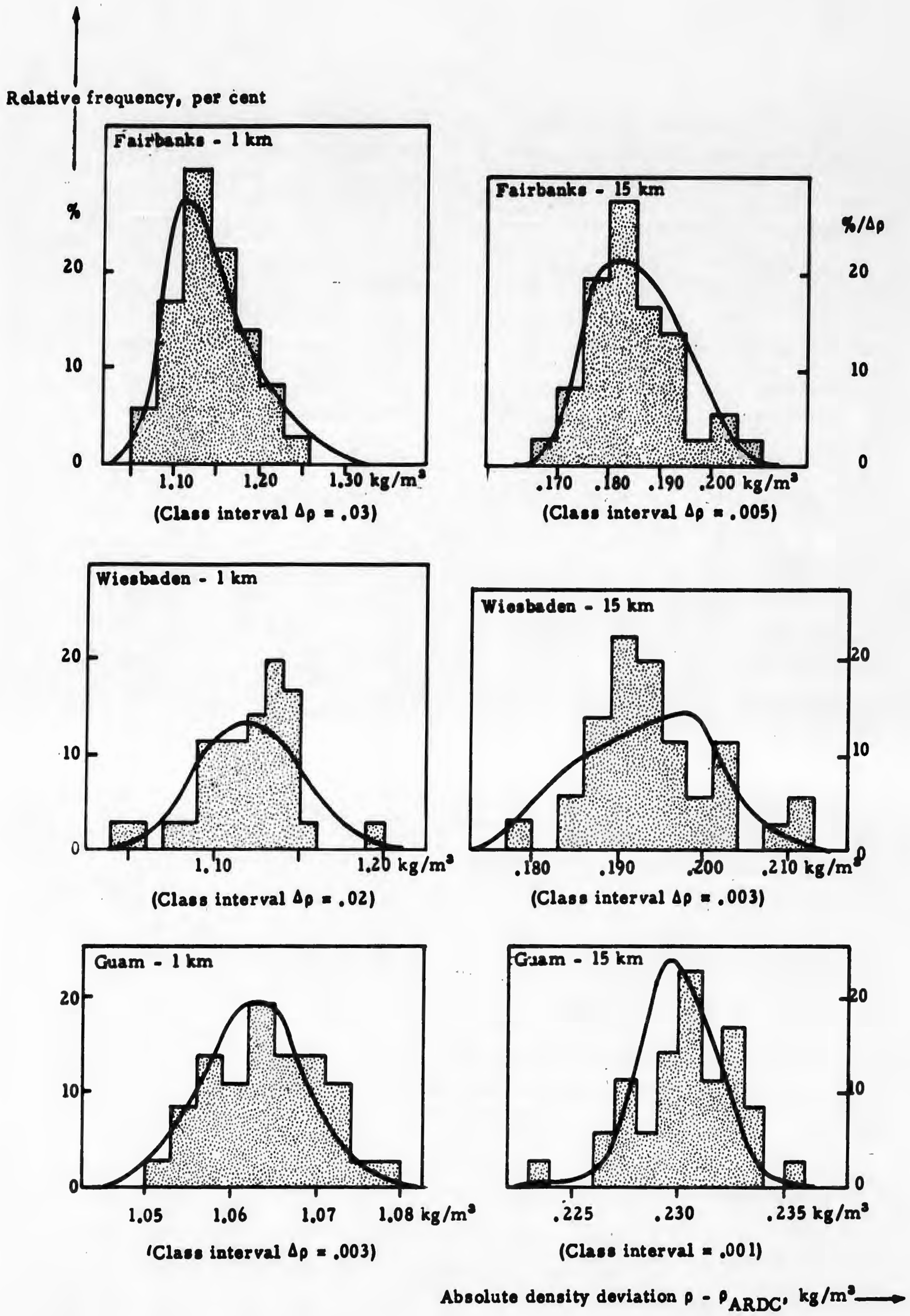

Figure 1. Comparison of the frequency distribution of density at two altitude levels between the selected sample of 36 cases per station and the values from the period 1951 through 1957.

Histograms (shaded): Frequency distribution of the selected sample.

Smooth curve: Period 1951 through 1957.

B. Evaluation Procedure

1. Conversion from ρ to ρ'. Density data were available in the form of absolute densities (expressed in kg/m^3) at the surface and at one kilometer intervals starting at 1 km above sea level up to 25 km altitude. As a first step these values have been used to find the deviations ρ'_j from the corresponding ARDC model atmosphere density:

$$\rho'_j = \rho_j - \rho_{j|ARDC} \quad (kg/m^3) \tag{1}$$

with

j = 0, 1 through 25 (zero = surface),

ρ_j ≙ density at level j,

$\rho_{j|ARDC}$ ≙ ARDC model density at level j.

All further calculations involved only these departures from the ARDC standard. The ρ' - values served as input data for the computer calculations.

2. Computation of the Step Function $\tilde{\rho}'$. For each profile the arithmetic mean over layers of 2 through 10 km thickness was calculated and used as a step function. For the layers starting at the level j = a and having the thickness k, the step function ρ' is defined by

$$\tilde{\rho}'_{ak} = \frac{1}{k+1} \sum_{j=a}^{j=a+k} \rho'_j \quad (kg/m^3) \tag{2}$$

Note, the number of levels from a to a + k is k + 1.

3. Computation of the Departures of the Actual Profile from the Step Function. For each altitude level j the differences were computed between the values ρ'_j and all those step functions ρ'_{jk} which include the level j. The absolute values of this difference divided by the standard value of the sea level density ($\rho_{o|ARDC} = 1.225\ kg/m^3$) will be expressed here as

$$d_{ajk} = \frac{|\rho'_j - \tilde{\rho}'_{ak}|}{\rho_{o|ARDC}} \quad (\%), \tag{3}$$

where a denotes the starting level, j the level, and k the thickness of the layer.

For each of the profiles under study the numerical values of d were calculated. The investigation could now have proceeded by establishing the frequency distributions of the d values of all individual profiles. Then it would have been possible, e.g., to study the individual extreme d values which do not exceed a specified probability of occurrence in a given level j and for a given layer thickness k. Due to the time and work limitation involved it has not been possible to provide answers expressed in relation to extreme values for this report. Instead, the d values of the 36 profiles of each station have been averaged separately for each thickness value k and for each kilometer level j. Averages over 36 cases will be denoted by a bar. These averages have been obtained by computing

$$\overline{d}_{ajk} = \frac{1}{36}\sum_{i=1}^{36} d_{(ajk)i} \quad (\%), \tag{4}$$

where i stands for the single ascent of the sample of 36 cases. The $\overline{d}$ values were printed out and served as a base for the establishment of recommended models as described below.

4. <u>Establishment of Recommended Models</u>. The altitude range between surface and 25 km altitude can be divided into a certain small number of steps, e.g., four, in a number of different ways. Among these is one ordering of steps, consisting, generally, of a combination of layers of different thicknesses, that will yield the smallest mean departures of the individual profiles from the approximating step function.

As an expression for the accuracy of the fit of a proposed layer step, the largest d value (average deviation) which occurs at the levels of this layer at any of the three stations is used here. The definition of this value is

$$\begin{aligned} D_{ak} = \max(&d_{s_1 a}, d_{s_1 a+1}, d_{s_1 a+2}, \ldots, d_{s_1 a+k}, \\ &d_{s_2 a}, d_{s_2 a+1}, d_{s_2 a+2}, \ldots, d_{s_2 a+k}, \\ &d_{s_3 a}, d_{s_3 a+1}, d_{s_3 a+2}, \ldots, d_{s_3 a+k}), \end{aligned} \tag{5}$$

s ≙ station index (Fairbanks, Wiesbaden, Guam)

Small D values indicate a good approximation and vice versa. A recommended model is defined as that combination of a given number of layers which yields the smallest maximum average value of D (the smallest maximum of the maxima).

The profiles of the quantity ρ' show maximum curvature below 3 km, and between 9 km and 15 km altitude. Fitting of density profiles by a step function of layers of equal thickness would be inaccurate in these altitudes. Selection of models of acceptable departures started, therefore, by finding acceptable layers of relatively small thickness encompassing these two critical altitude regions. As models consisting of steps of variable thickness are established, the number of layers needed to divide the remaining noncritical altitude regions can be kept small while still retaining the accuracy as determined by fitting of the critical zones. By dividing the altitude regions from 3 km to 9 km and above 15 km by the smallest permissible number of layers the assembling of layers to a model was then completed.

C. Presentation of Selected Models

Table VI gives the most recommendable arrangement of layers for models of four, five, six, seven or eight layers. For each of the proposed layers the largest average deviation is shown as found in any level of the particular layer at any of the three stations. The recommended models thus established are to be considered as a compromise to fit the profiles from all three stations at the same time. They will be designated here as "models of optimum global fit". If profiles from one station only are to be fitted, the global models will probably not result in optimum fit. Also, regional models with layer division tailored specifically for a particular geographical region would yield better average fit than the models given here.

From the investigated sample of profiles it can be concluded that the thickness of the uppermost layer can be chosen to be larger than 5 km which means that all levels from 20 km or from even lower levels on upward are suited for combination into a single layer step. Further examination of the top layers has shown that the inaccuracy of fit is already determined by the first or first two levels of that layer. The proposed models are therefore likely to retain their validity when they are used to fit radiosonde profiles with ceilings at 20 km or at any higher altitude.

Any regrouping of levels into layers different from the arrangements given in Table VI may result in a slightly better (i.e., smaller) D value at a specific layer. At another layer, however, a larger D value than the one given in Table VI will appear. As an illustration, the models exhibiting the second smallest max (D) for the sample of 108 cases are listed in Table VII as alternate models or models of second-best global fit.

If the selected layer has a thickness of only 1 km, i.e., if it is represented by one level only, the deviation d is zero because ρ'_j and $\tilde{\rho}'_{jk}$ are then identical.

Table VI

RECOMMENDED GLOBAL MODELS

Proposed Layers	D value (in per cent of $\rho_{o,\mathrm{ARDC}}$)	Occurring at	
		Station	Level
Recommended Global Four-Layer Model			
(1) Surface	0.00%	---	---
(2) 1 - 3 km	1.66	Wiesbaden	2 km
(3) 4 - 10 km	1.52	Guam	4 km
(4) 11 - 25 km	1.58	Guam	15 km
Recommended Global Five-Layer Model			
(1) Surface	0.00	---	---
(2) 1 - 2 km	1.42	Wiesbaden	1, 2 km
(3) 3 - 6 km	0.80	Guam	3 km
(4) 7 - 11 km	1.14	Fairbanks	11 km
(5) 12 - 25 km	1.55	Guam	15 km
Recommended Global Six-Level Model			
(1) Surface	0.00	---	---
(2) 1 km	0.00	---	---
(3) 2 - 3 km	1.30	Wiesbaden	2, 3 km
(4) 4 - 9 km	1.27	Guam	4 km
(5) 10 - 20 km*	1.24	Guam	10 km
(6) 21 - 25 km*	.19	Guam	21 km

*In the recommended global six-layer model, the boundary between layers (5) and (6) can be shifted to the next lower level without changing the corresponding D values appreciably:

Layer (5') 10 - 19 km. D = 1.38%, occurring at Guam, 10 km

Layer (6') 20 - 25 km. D = 0.33%, occurring at Guam, 20 km

Table VI. Concluded

Proposed Layers	D value (in per cent of $\rho_{o\,ARDC}$)	Occurring at	
		Station	Level
Recommended Global Seven-Layer Model			
(1) Surface	0.00	---	---
(2) 1 km	0.00	---	---
(3) 2 km	0.00	---	---
(4) 3 - 7 km	1.04	Guam	3 km
(5) 8 - 11 km	0.94	Fairbanks	11 km
(6) 12 - 18 km	.84	Fairbanks	12 km
(7) 19 - 25 km	.63	Guam	19 km
Recommended Global Eight-Layer Model			
(1) Surface	0.00	---	---
(2) 1 km	0.00	---	---
(3) 2 km	0.00	---	---
(4) 3 - 5 km**	0.55	Guam	5 km
(5) 6 - 8 km**	.46	Guam	8 km
(6) 9 - 11 km	.67	Fairbanks	11 km
(7) 12 - 18 km	.84	Fairbanks	12 km
(8) 19 - 25 km	.63	Guam	19 km

The underlined figures are the largest D values of each model

** In the recommended global eight-layer model, the boundary between layers (4) and (5) can be shifted to the next lower level without changing the corresponding D values appreciably:

Layer (4') 3 - 4 km: .29% occurring at Fairbanks

Layer (5') 5 - 8 km: .71% occurring at Guam, 5 km

Table VII

ALTERNATE GLOBAL MODELS

Proposed Layers	D value (in per cent of $\rho_{o,ARDC}$)	Occurring at	
		Station	Level
Alternate Global Four-Layer Model			
Not Established			
Alternate Global Five-Layer Model			
(1) Surface	0.00%	---	---
(2) 1 km	0.00	---	---
(3) 2 - 3 km	1.30	Wiesbaden	2,3 km
(4) 4 - 10 km	1.52	Guam	4 km
(5) 11 - 25 km	1.58	Guam	15 km
Alternate Global Six-Layer Model			
(1) Surface	0.00	---	---
(2) 1 - 2 km	1.42	Wiesbaden	1,2 km
(3) 3 - 8 km	1.29	Guam	3 km
(4) 9 - 12 km	1.06	Guam	12 km
(5) 13 - 18 km	0.91	Guam	18 km
(6) 19 - 25 km	.63	Guam	19 km
Alternate Global Seven-Layer Model			
(1) Surface	0.00	---	---
(2) 1 km	0.00	---	---
(3) 2 - 3 km	1.30	Wiesbaden	2,3 km
(4) 4 - 7 km	0.79	Guam	4 km
(5) 8 - 11 km	.94	Fairbanks	11 km
(6) 12 - 18 km	.84	Fairbanks	12 km
(7) 19 - 25 km	.63	Guam	19 km

Table VII. Concluded

Proposed Layers	D value (in per cent of $\rho_{0,ARDC}$)	Occurring at Station	Level
Alternate Global Eight-Layer Model			
(1) Surface	0.00	---	---
(2) 1 km	0.00	---	---
(3) 2 km	0.00	---	---
(4) 3 - 6 km	0.80	Guam	3 km
(5) 7 - 10 km	.78	Fairbanks	10 km
(6) 11 - 13 km	.89	Guam	11 km
(7) 14 - 18 km	.50	Guam	14 km
(8) 19 - 25 km	.63	Guam	19 km

The underlined figures are the largest D value of each model

III. BRIEF DISCUSSION OF THE RECOMMENDED MODELS

The accuracy of the various models is summarized in Table VIII which is based on the underlined D values from the list of recommended models (Table VI). As explained above, the largest of the D values of all layers of one model is used here as a measure of the goodness of fit for this model. The smaller the largest occurring D value, the better is the rating of the model.

Table VIII

SUMMARY OF ACCURACY OF RECOMMENDED GLOBAL MODELS

Recommended Model	Largest D value
Four-layer	< 1.7%
Five-layer	< 1.6%
Six-layer	< 1.4%
Seven-layer	< 1.1%
Eight-layer	< 0.9%

This table shows that an increase in the number of layers from four to five results in an insignificant reduction of the D value only. If the four-layer model is considered too crude, then the next global model to be recommended is the six-layer model. When going from six to seven layers, the inaccuracy can be further decreased by 0.3%. With eight layers an additional reduction by 0.2% is possible.

If a maximum inaccuracy of max (D) = 1.0% is not to be exceeded then the seven-layer global model can practically be considered sufficient.

IV. CONCLUDING REMARKS

Although a thorough study has been made with the available data, time limitations have somewhat restricted the generalization of the presented results. These may be summarized as follows:

1. All conclusions are based on the evaluation of a sample of 108 individual density profiles. Although it has been attempted to arrange this selection so that it closely resembles the total population of density profiles, repetition of the study with a larger number of cases may increase the reliability of the conclusions presented here.

2. The proposed global models of step functions are intended to be applied to a collective set of profiles composed of approximately equal number of cases from the Tropical, the Temperate and the Arctic Zones. Separate models for each region have not been included here. They are likely to differ from the global models.

3. Utilization of frequency distributions of the $\bar{d}$ values permits refinement of the optimum layer division. It remains to be investigated whether the results would be altered appreciably.

As a second part of this report, it is planned to investigate the extent of these limitations in a follow-on study which will also consider (1) the usefulness of deriving step functions for mean profiles, (2) models with approximately equal inaccuracy of each layer step, and (3) models adaptable for ascents with varying ceiling.

Pending the outcome of these additional analyses the models presented here should be considered as preliminary. Especially, not too much weight should be given to the exactness of the presented numerical values of the quantity D used to describe the departures between step function and original function.

Nevertheless, the essential features of division into layers will not undergo drastic changes in a refined investigation and thus the models should be useful in the interim in applications where more precision is not needed.

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Army Msl Cmd RR-TR-62-6, 15 Aug 62, 23 pp - illus. Unclassified Report

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