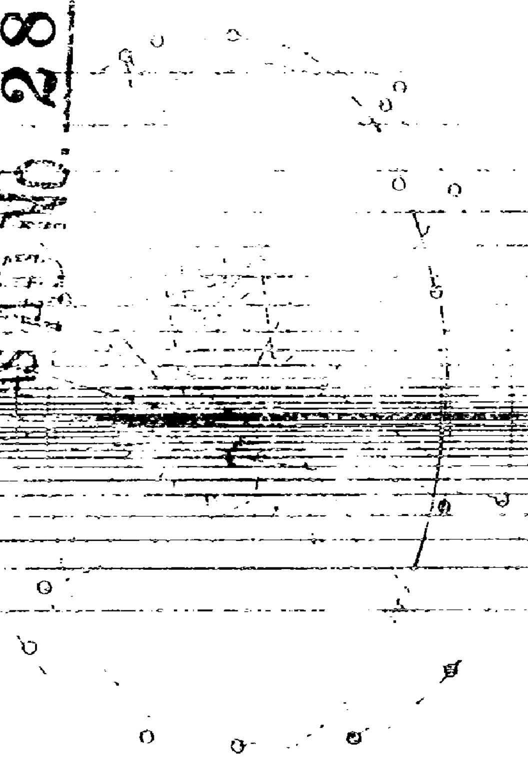
TECHNICAL DOCUMENTARY REPORT

WDL - TR1906

20 SEPTEMBER 1962



AN ACCURACY STUDY OF THE INITIAL CONDITIONS GENERATOR USING VERLORT DATA

BY M. GROSSBERG

PREPARED FOR

AIR FORCE SPACE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
INGLEWOOD, CALIFORNIA

CONTRACT NO. AF04 (695) - 113

PHILCO WESTERN DEVELOPMENT LABORATORIES
PALO ALTO, CALIFORNIA

WDL-TR1906

TECHNICAL DOCUMENTARY REPORT

AN ACCURACY STUDY
OF THE INITIAL CONDITIONS GENERATOR
USING VERLORT DATA

Prepared by

PHILCO CORPORATION
Western Development Laboratories
Palo Alto, California

Contract AF04(695)-113
AFBM Exhibit 58-1, Paragraph 4.2 1
AFSSD Exhibit 61-27A

Prepared for

SPACE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
Inglewood, California

ABSTRACT

PHILCO WDL-TR1906 UNCLASSIFIED
AN ACCURACY STUDY OF
THE INITIAL CONDITIONS
GENERATOR USING VERLORT DATA
M. Grossberg/Math. Analysis Dept. 12 pp incl. illus.
20 September 1962 Contract AF04(695)-113

In this report, the propagation errors of the initial condition generator are studied in two ways: (1) by comparing observational data on one pass with predictions from initial conditions generated on a previous pass of the same satellite, and (2) by statistically comparing the elements generated by different passes of the satellite. These two methods are found to agree. Excluding one set of poor data, it is found that the propagation error has a position standard deviation of about ten nautical miles perpendicular to the orbit plane, and of about fifty nautical miles in this plane. The period error is found to have a standard deviation of about seventy-five to one hundred seconds. A means of reacquiring the satellite on the basis of this analysis is presented and linearized equations for the antenna program are listed.

THIS UNCLASSIFIED ABSTRACT IS DESIGNED FOR RETENTION IN A STANDARD 3-BY-5 CARD-SIZE FILE, IF DESIRED. WHERE THE ABSTRACT COVERS MORE THAN ONE SIDE OF THE CARD, THE ENTIRE RECTANGLE MAY BE CUT OUT AND FOLDED AT THE DOTTED CENTER LINE. (IF THE ABSTRACT IS CLASSIFIED, HOWEVER, IT MUST NOT BE REMOVED FROM THE DOCUMENT IN WHICH IT IS INCLUDED.)

FOREWORD

This Technical Documentary Report was prepared under Definitive Contract AF04(695)-113 and is submitted in accordance with Exhibit "A" of that contract and Paragraph 4.2.1 of AFBM Exhibit 58-1, "Contractor Reports Exhibit," dated 1 October 1959, as revised and amended.

The report was prepared by the Philco WDL Mathematical Analysis Department in fulfilling the requirements of Section II, Tab A, Paragraph 1.2.1.2 and Section II, Tab J, Paragraph 13.1 of AFSSD Exhibit 61-27A, "Satellite Control Subsystem Work Statement," dated 15 February 1962, as revised and amended.

This interim report was prepared to partially satisfy the requirements of the High and Low Altitude Tracking Simulation Studies.

TABLE OF CONTENTS

LIST OF TABLES

AN ACCURACY STUDY OF THE INITIAL CONDITIONS GENERATOR USING VERLORT DATA

I. INTRODUCTION

The "Initial Conditions Generator" is a Philco 2000 program which estimates the six elliptic elements describing the trajectory of an artificial satellite from a set of observations of the satellite on a single pass over a single station. The observations may be either Verlort or angle plus doppler. The program is described in the reference.

The estimated elements may be used as a starting point for differential correction. In this case, the elements will be of succificent accuracy if the differential correction converges. Since the differential correction usually converges after a few iterations, the initial conditions generator is, indeed, adequate.

However, the estimated elements may also be used to predict an ephemeris directly. This elimination of the differential correction procedure involves a significant saving of computer time at the expense of prediction accuracy. This note describes a numerical study of the accuracy of predictions made from elements estimated on the basis of Verlort tracking data.

II. DESCRIPTION OF METHOD:

Several passes of a Program 461 and a Program 162 satellite over various Verlort stations have been used. Some characteristics of these passes are listed in Table I. Each of these passes was run through the initial conditions generator program to obtain elements. Each element set corresponding to the Program 461 satellite was then run through the "General Predicitions" program; the Program 162 data was treated in a different manner described in section V. The general predictions program numerically integrated the equations of motion; tested whether the satellite was in the field of view of one or more observing stations; and, if so, printed the range, azimuth, and elevation of the satellite at each time step, for as long as the satellite remained observable. By comparing these printed quantities with the observations actually recorded on successive passes of the satellite, a measure of the propagated error of the initial conditions generator may be obtained. This is possible because the propagation error is much larger than either the integration error of general predictions, or the random error in the observations.

III. WILD POINT REMOVAL:

Besides the occasional wild observations usually recorded by electronic sensors, some of the passes used in this study contain calibration data. On those passes in Table I, for which the column marked "CAL?" is checked, the measurements taken in the course of calibrating the Verlort are transmitted in a format indistinguishable from that of the real observations. Since these calibration measurements are wildly inconsistent with the orbit, a screening procedure is necessary.

Fortunately, the initial condition generator includes an optional screening procedure. This scheme consists of calculating the average linear distance between the observed points and the plane of best fit; rejecting those points which are farther from the plane than a constant, alpha, times this average distance; and then refitting a plane to the remaining points. After some experimentation, it was found that this procedure will remove both the calibration and the isolated wild points. For the 461 data, alpha was set at 3, and for the 162 data at 1.6. In both cases, the rejection was repeated a second time. The reason for this second rejection is that the average distance calculated the first time is quite large, so only the most horrible points are rejected.

IV. PERIOD ERROR:

It is a well-known but rarely-published fact that the bulk of the propagation error arising from a single pass orbit estimation is due to the error in the period. That is, if the period is estimated exactly, and the other elements are subject to small errors; the propagated error will, to a first-order theory, be a periodic function of time. On the other hand, a small error in the period will produce a propagation error whose magnitude is roughly proportional to prediction time. An equivalent interpretation is that the predicted position and velocity co-ordinates are roughly correct, but that the associated time is in error by the amount, Δt.

$$[1] \quad \Delta t = \frac{\Delta P}{P} (t_P - t_I) ,$$

where P is the period; ΔP, the period error; t_P, the present time; t_I, the time of the initial conditions.

The interpretation just given to the inertial co-ordinates cannot be directly extended to the radar co-ordinates - - range, azimuth, and elevation. The reason is that, although the predicted position at time, t_P, is roughly equal to the true position at time, $t_P + \Delta t$; the position of the observing station is different at these two times, due to the rotation of the earth. Thus, the predicted radar co-ordinates at time, t_P, are to be compared with the following:

$$[2] \quad \begin{aligned} S_P (t_P) &\approx S_0 (t_P + \Delta t) + \Delta S \\ A_P (t_P) &\approx A_0 (t_P + \Delta t) + \Delta A \\ E_P (t_P) &\approx E_0 (t_P + \Delta t) + \Delta E \quad , \end{aligned}$$

where a 'P' subscript indicates prediction, and an 'O' subscript indicates observation. The deltas may be approximated as linear functions of Δt. After carrying out the necessary differentiation

$$[3] \quad \Delta S \approx -\Omega_E \Delta t \, R \cos E \sin A \cos L$$

$$\Delta A \approx \Omega_E \Delta t \, (\sin L - \cos A \cos L \sec E \, (\sin E + R/S))$$

$$\Delta E \approx \Omega_E \Delta t \sin A \cos L \, (1 + R \sin E/S),$$

where Ω_E is the rotation rate of the earth (expressed in radians per second for ΔS, and in degrees per second for the angles); L, "the" latitude of the station; and R, "the" radius of the earth. Then equations [2] and [3] may be solved simultaneously for Δt. Indeed three values of Δt may be derived and averaged at each prediction time. Finally, from Δt, t_P, t_I, and P, ΔP may be calculated.

Equation [3] may also be used in the field as an antenna directing program as suggested in the conclusion.

V. DIRECT ANALYSIS OF THE ELEMENTS

Instead of running the elements estimated by the initial conditions generator, through general predictions, it is possible to analyze them directly. Of the six elements - one, η, is a rapidly changing function of time. The other five elements, suffer only perturbative changes. Thus, if the perturbations are sufficiently small, the element sets estimated for different passes of the same satellite may be treated as random samples. In particular, an estimate of the period, may be derived from the other elements as follows:

[4]
$$e = (V^2 + W^2)^{1/2}$$
$$A = \left\{L(1 - e^2)\right\}^{-1}$$
$$P = 2\pi\,\mu^{-1/2}\,A^{-3/2}$$

where μ is the product of earth's mass and the universal gravitation constant. For the 461 data, the estimates of 'P' calculated by equation [4] , and the estimates of 'ΔP' calculated by the procedure of section IV have been obtained. The quantity, '$P - \Delta P$' was then computed for each pass. This quantity is an estimate of the true period; its value should be approximately the same for each pass. In table II, the values of 'P', 'ΔP' and '$P - \Delta P$' are listed, along with their means and standard deviations. It should be noted that the standard deviation of $P - \Delta P$ is much smaller than that of the other two - - thus, the procedure is self-consistent.

Because of this self-consistency, it was deemed unnecessary to apply the general prediction program to the 162 data. The standard deviation of the period error was obtained from the dispersion of the estimated periods calculated by equation [4] .

In tables III and IV, the estimates of five elements - - semi-major axis, A; eccentricity, e; right ascension of ascending node, Ω ; inclination, I; and period, P -- together with the means and standard deviations are listed for the 461 and 162 satellites, respectively. 'A', 'e', and 'P' are calculated from equations [4] ; 'I' is taken directly from the initial conditions generator; 'Ω' is also taken from the initial conditions generator, but the values are corrected by a linear function of time to remove the well-known secular perturbation caused by the first harmonic of the earth's potential.

VI: CONCLUSIONS:

A glance at tables II and III shows that serial number 544 contains much poorer data than the other 461 passes. An examination of the data listing reveals that active tracking was lost while the satellite was still approaching the station. Aside from this peculiarity, the observations look reasonably smooth. If we eliminate this one bad pass from the analysis, the results are greatly improved. Table V shows the means and standard deviations of the estimated 461 elements as calculated from the four good passes - - 551, 552, 553, 561. It should be noted that these standard deviations are in good agreement with those for 162 (Table IV) - - an in-plane position error of magnitude 40 to 50 n.m., and an out-of-lane position error of less than 10 n.m. Noting that a chord of 50 n.m. length subtends an angle of less than 2° at 1500 n.m., we see that the orbit estimated from the initial conditions generator may be of sufficient accuracy to reacquire a satellite after one or more revolutions if the antenna is programmed to remain fixed in inertial space for several minutes. This program might be accomplished by exact computation; or a linear approximation may be employed. To instrument the linear approximation, a single acquisition time t_a, and radar co-ordinates S_a, A_a, E_a, are computed. Then at time, $t_a + \Delta t$, the radar is directed to the co-ordinates $S_a + \Delta S$, $A_a + \Delta A$, $E_a + \Delta E$, where the deltas are given by equation [3].

__Reference:__ Midas Simulation and Evaluation Study Report. WDL - TR1597. 27 Dec. 1961, Appendix E.

TABLE I

CHARACTERISTICS OF 461 and 162 DATA

SAT.	SERIAL	STATION	DATE	TIME (sec.)	MIN. RANGE (N.M.)	MAX ELEV (DEG)	NO. OF GOOD POINTS	CAL ?
461	344	NEWB	10/23	81380	2200	50.7	97	✓
"	551	VAFB	10/24	15620	3230	16.4	131	
"	552	VAFB	10/23	45330	2380	47.0	66	
"	553	VAFB	10/24	85470	2390	42.0	168	
"	561	VAFB	10/25	8810	2140	55.0	70	
162	476	NEWB	4/9	15860	490	36.8	125	✓
"	500	NEWB	4/9	21480	750	18.8	131	✓
"	505	VAFB	4/9	27030	550	32.6	98	
"	513	HAWI	4/9	38110	360	66.0	76	✓
"	516	KODI	4/9	38610	300	78.0	47	✓
"	523	VAFB	4/9	32610	830	16.4	77	

TABLE IA.

STATION LOCATIONS

STATION	NORTH LATITUDE (DEG.)	EAST LONGITUDE (DEG.)
NEWB	42.95	288.37
VAFB	34.79	239.50
HAWI	21.57	201.74
KODI	57.60	207.83

TABLE II

ESTIMATED PERIOD AND PERIOD ERRORS FOR 461 RUNS

SERIAL	P (SEC.)	ΔP (SEC.)	P - ΔP (SEC.)
544	10519	+520	9999
551	10002	+31	9971
552	9862	-95	9957
553	10054	+67	9987
561	10112	+180	9932
MEAN	10109.8	140.6	9969.2
STD. DEV.	246.8	233.7	26.2

TABLE III

ESTIMATED ELEMENTS FOR 461 RUNS

SERIAL	A(N.M.)	e	I(DEG)	Ω (DEG)	P (SEC)
544	5598.99	.048837	95.99089	265.61233	10519
551	5418.93	.013331	95.88709	265.77391	10002
552	5363.55	.014089	95.77398	265.65816	9862
553	5432.86	.018370	95.88710	265.61156	10054
561	5453.80	.022394	95.85065	265.58401	10112
MEAN	5453.626	.0234042	99.877942	265.647994	10109.8
STD. DEV.	87.86	.01468	.07823	.07525	246.8

TABLE IV

ESTIMATED ELEMENTS FOR 162 RUNS

SERIAL	A (N.M.)	e	I (DEG.)	Ω (DEG.)	P (SEC.)
476	3736.51	.01795	82.40315	-2.14883	5734.6
500	3793.19	.02164	82.42089	-2.17686	5865.6
505	3702.16	.02212	82.28449	-2.25360	5655.7
513	3740.33	.01612	82.24016	-2.29336	5743.4
516	3768.80	.02265	82.36774	-2.22675	5809.1
523	3769.50	.01613	82.34294	-2.28153	5810.7
MEAN	3751.748	.019435	82.345228	-2.230155	5769.85
STD. DEV.	32.08	.00305	.0697	.0577	73.96

TABLE V

FOUR PASS STATISTICS OF 461 ELEMENTS

	A (N.M.)	e	I(DEG)	Ω (DEG.)	P(SEC)	P-ΔP(SEC)
MEAN	5417.285	.017046	99.849705	265.65691	10007.5	9961.75
STD. DEV.	38.58	.00420	.0333	.0838	106.9	23.3

DISTRIBUTION LIST

No. of Copies	Address
	Commander Space Systems Division Air Force Systems Command Air Force Unit Post Office Los Angeles 45, California
10	Attn: Technical Data Center
2	Attn: Lt. Binsak, AFSSD/SSOCE
1	USAF Contract Support Detachment No. 3 Philco Corporation Western Development Laboratories Palo Alto, California
49	Philco Corporation Western Development Laboratories Palo Alto, California
62	

UNCLASSIFIED