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LIFETESTING TIME REQUIREMENTS TO ASSURE REQUIRED RELIABLE LIFE*

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Henry P. Goode

Department of Industrial Engineering and Administration
Cornell University
Ithaca, New York

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LIFETESTING TIME REQUIREMENTS TO ASSURE REQUIRED RELIABLE LIFE

SUMMARY

This paper presents a procedure and associated tables of factors for use in determining the minimum lifetesting time required for sample items to provide assurance at a confidence level of 95% that the items in the lot or population from which the sample was taken meet the life requirements specified. The life quality of the lot or population is evaluated in terms of reliable life which is the life beyond which some specified proportion of the items will survive. Tables of factors are provided for three proportions or reliability indices; .99, .90, and .50 (median life). In addition, another table of factors is provided for lot evaluation in terms of mean item life. The Weibull distribution is used as a failure model with the exponential distribution being included as a special case of the Weibull. Inspection of sample items is by attributes with the acceptance criterion being simply that no more than some specified number of items fail before the end of the minimum test time as computed by the use of the factors.

INTRODUCTION

Sampling inspection procedures and tables of conventional form have recently been made available for evaluation in terms of reliable life and of mean life using the Weibull distribution as a statistical model.[1][2][3] Procedures and tables have also been prepared for hazard rate evaluation.[4] Included in these publications are tables of basic conversion factors or relationships from which plans of other desired forms could be constructed. The tables of sampling plans that are included were designed for applications for which some sample lifetesting time had been specified; the tables

accordingly tabulate the sample size and acceptance number combination required to give the desired operating characteristics.

For many reliability or lifelength evaluation applications, however, it is apparently most useful to employ an acceptance procedure of different form. This is to simply determine the minimum test time required (for some specified or selected small acceptance number -- which may be zero) to provide a high degree of confidence that the items in the lot or population meet the specified lifelength requirements. Such a procedure would seem to be particularly helpful in the many cases currently encountered for which item lifelength is relatively quite long and at the same time for which the sample lifetesting time must be kept relatively quite short for lot evaluation to be economically and chronologically feasible. For this reason the following tables of factors for easily determining the minimum lifetesting times required have been compiled. Factors have been determined for each acceptance number from zero through five. The work has been done through use of the conversion factors mentioned above.

Tables 1, 2, and 3 list in terms of multiples of the specified reliable life the minimum lifetesting times required to assure lot compliance with specifications for accepted lots. Reliable life is defined as the life beyond which some specified proportion of the items in the lot can be expected to survive. Table 1 provides values for cases for which the proportion expected to survive is .50 or more. This measure, commonly known as the median life for the population, is one frequently employed in many areas of biological assaying. Table 2 provides values for cases for which the desired proportion surviving (also known as the "reliability index") is .90; Table 3 provides values for which the proportion or reliability index required is .99. These two latter proportions or indices should be of particular usefulness in fatigue failure evaluation of mechanical components. Also, they offer a useful alternative form of evaluation for general use

with all forms of materials and components when item lifelength and reliability is the quality of concern.

In Table 4 are tabulated factors for use to make an evaluation in terms of mean item life. The values given are multiples of the required or specified mean item life that must be employed as sample lifetesting times to assure lot or population compliance. Mean life is another alternative measure that may be useful in wide variety of applications.

The level of assurance or confidence provided by all the tables of factors is 95%. If the number of sample items failing within the computed minimum lifetesting time is equal to or less than the acceptance number, the probability is .05 or less that the reliable life (or mean life) for the population is less than the value specified. One final point to note is that lifelength may be measured in any appropriate unit -- hours, cycles of operation, or revolutions, for example.

THE WEIBULL DISTRIBUTION

As a part of the papers previously referenced the Weibull distribution is described, the underlying assumptions are outlined, and the relationships between it and the exponential distribution are discussed. In addition, those interested in more details will find an extensive discussion of the Weibull distribution as a statistical model for lifelength and reliability in a paper by Kao[5]. Since this and other material dealing with this distribution is widely available, a general discussion will not be repeated here. A few comments relating to the application of the procedures under discussion may be of use, however.

One should recall that the Weibull model has three parameters. One is a scale or characteristic life parameter. Another is a shape parameter. The third is a threshold or location parameter. The first of these, the scale parameter, is not of interest in the application of the factors and techniques described in this paper; the procedure is independent of its

magnitude. However, the other two parameters are of importance and their magnitude for the population or lot being evaluated must be known, estimated, or assumed.

Of these two, the one of most frequent concern is the shape parameter. (This parameter is commonly symbolized by the letter β.) The lifetesting time that will be required depends very much upon its magnitude. For this reason separate factors have been determined and tabulated for a representative range of β, or shape parameter, values. A suitable estimate of the value to use in an application can in many cases be obtained by an analysis of past experimental and inspection measurements for the product in question. (Methods for estimating the Weibull parameters, including a simple graphical procedure, are outlined in the paper by Kao.[5]) In other applications an estimate may have to be made based on experience with similar items. In the application of any form of sampling procedure for lifelength, some assumption must be made regarding the shape of the distribution. Even when assuming the exponential distribution applies one is assuming, in effect, a Weibull distribution with a β value of unity.

It should be noted that for products for which the rate of failure is relatively high in early life and for which the rate decreases with the passage of time, the value for β, the shape parameter, will be less than 1. The greater the rate of decrease in the hazard rate with the passage of time, the smaller the value of β. As mentioned previously, the case for which $\beta = 1$ is the exponential case for which the failure rate is constant and does not change with the passage of time. For populations for which the hazard rate is relatively low early in life and for which the rate increases with the passage of time, the value for β will be greater than 1. The greater the rate of increase in the hazard rate, the greater will be its magnitude. Experience has indicated that values for β of less than 1 are associated with many electronic components, components which seem to be

characterized by considerable "infant mortality" or early failure. On the other hand, for products for which failure is due predominantly to fatigue and wearout, β values greater than 1 will apply. Such is the case, for example, for ball bearings. It may be a matter of interest to also note that for $\beta = 3\frac{1}{4}$, the Weibull approximates closely the normal distribution, a distribution commonly used as a lifelength model for wearout failures.

The other parameter of concern in the application of the procedure under discussion is the location or threshold parameter. (This parameter is usually symbolized by the letter γ.) This parameter is the measure of service time or age for the lot or population at which some risk of item failure will initially be experienced. For perhaps a majority of applications the value for this parameter will be zero; there will be no initial period of item life completely free of risk of failure. In the direct use of the values tabulated in the tables this value -- zero -- is assumed for the location parameter. However, if it is known for the product under evaluation that γ has some value other than zero, a simple modification in the procedure to allow for this is available. Details of this modification will be discussed later and will be illustrated in one of the examples that follow. Just as for the shape parameter, the value to assume for the location parameter must be determined from past failure experience with the product under evaluation or with similar products. Methods for estimation of this parameter value from lifelength data are included in the paper by Kao referenced earlier.[5] In another paper a method of joint estimation of the parameters is provided.[6]

THE TABLES AND THEIR USE

As previously mentioned, tables of minimum testing time values have been prepared for three proportions or reliability indices. (The letter r will be used to symbolize this index or proportion.) These are r = .50, r = .90, and r = .99. In addition there is a table of values for use in

evaluation in terms of a specified minimum mean life. Within each of these four tables, values will be found for seven different magnitudes for the shape parameter: $\beta = \frac{1}{2}$, $\beta = \frac{3}{4}$, $\beta = 1$ (which is the exponential case), $\beta = 1\frac{1}{3}$, $\beta = 1\frac{2}{3}$, $\beta = 2$, and $\beta = 2\frac{1}{2}$. For each β value, testing time values have been tabulated for acceptance numbers, c, of 0, 1, 2, 3, 4, and 5. For each possible pair of c and β values, testing times are listed for sample sizes, n, of 10, 25, 50, 100, 250, 500, and 1000 items. It is expected that these ranges of values for r, β, c, and n will encompass those values most commonly required in reliability and lifetesting practice. For cases for which values specifically required for n or β are not listed but are within the range covered by the tables, interpolation may be employed to find the required minimum testing time provided one understands that only an approximation to the specified level of confidence (.95) will be obtained.

The values tabulated in the body of the table are multiples of the specified reliable life or minimum mean life that must be used as a testing time for sample items to assure lot compliance with a confidence level of .95. If no more than the specified number of items, c, fail before the end of this testing time, it may be assumed, with this confidence level, that the reliable life (or, alternatively, the mean item life) for the lot or population is equal to or greater than the required or specified value. The meaning and use of these tables of values can perhaps be better described through the several simple examples that follow.

Example (1)

A sampling inspection plan for acceptance is required for a certain electronic component purchased in some quantity from time to time. Past experimental and inspection data indicates the Weibull distribution clearly applies as a lifelength model and that a value for the shape parameter, β, of approximately $\frac{3}{4}$ and for the location parameter, γ of 0 can be assumed.

Each lot is to be evaluated in terms of a required reliable life of 2,000 hours, with reliable life defined as the life beyond which 90% of the items in the lot will live. That is, r must equal .90. Testing facilities are available for testing 100 items at a time. The lot size and the costs of inspection per item are such that a sample of this size can be economically justified. A decision on each lot should be reached as quickly as possible and for this reason the duration of the lifetesting time should be kept as short as possible.

The necessary minimum lifetesting time is found by reference to Table 2 which lists values for r, the proportion that must survive beyond the reliable life, of .90. Under the section of this table for $\beta = \frac{3}{4}$, it is found that for an acceptance number, c, of 0, the time must be .18 times the required reliable life. Since this has been specified as 2,000 hours, the testing time must be .18 x 2,000 or 360 hours. The acceptance number, c, of 0 is used since the duration of the lifetesting time must be kept short; use of larger acceptance numbers will require longer testing times.

Thus if 100 items are drawn at random from a lot and put under life test, and if no items fail before the end of 360 hours, the lot may be accepted at that time as meeting the reliable life specification. One may be 95% confident that 90% or more of the items in the lot will have a life of at least 2,000 hours. A slightly different way of expressing this is that one may be 95% confident that the life beyond which 90% of the items will survive is 2,000 hours or more.

Example (2)

A lifelength evaluation is to be made for new source of supply for a product. A value of $1\frac{1}{3}$ can be assumed for β, the shape parameter, and a value of 0 for γ, the location parameter. For the supply of product to be suitable for use, the mean item life must be at least 400 hours. Because of the unit cost of the item, the sample size should be kept small --

preferably not over 25 items. However, relatively long test times can be tolerated. For this reason an acceptance number of 5 with the comparatively long test time it will require will be used. It is expected that the more extensive test experience thus accumulated will provide a better overall evaluation of the product.

Examination of Table 4 which lists values for use in mean life evaluation indicates for $\beta = 1\frac{1}{3}$, $n = 25$, and $c = 5$ that the lifetesting time must be .62 times the required minimum mean life. Hence for the source of supply to be acceptable, no more than 5 items must fail before the end of .62 x 400 or 248 hours.

NON-ZERO THRESHOLD PARAMETERS

For cases for which the location or threshold parameter, γ, -- the lifetime below which there is no risk of item failure -- is greater than zero, the following procedure may be used: (a) subtract the value for γ from the required reliable or mean life to get a converted value in terms of $\gamma = 0$, (b) multiply this converted value by the factor selected from the table (in the usual way) to get a lifetesting time in converted terms, and (c) add the value for γ to this testing time to get a required testing time in absolute terms. The following example will illustrate this simple variation in technique.

Example (3)

Consider an application for which the Weibull distribution with a value of 2 for β, the shape parameter, may be assumed. Past experience with the item in question indicates a value for the threshold parameter, γ, of 3,000 cycles should be expected. The sample size, n, is to be 50 items; the acceptance number, c, is to be 1 item. Lot quality is to be measured in terms of reliable life. The minimum reliable life that can be tolerated is 5,000 cycles. The reliability index, r, specified is .99. The minimum

lifetesting time for sample items to assure lot compliance with 95% confidence is required.

Subtraction of the value for the location parameter from the required reliable life gives 5,000 - 3,000 or 2,000 cycles as a converted value for required reliable life. From Table 3 in which time values for $r = .99$ are tabulated, a value of 3.0 is found for $\beta = 2$, $n = 50$, and $c = 1$. The required testing time in converted terms is thus 3.0 x 2,000 or 6,000 cycles. Addition of the value for γ, 3,000 cycles, to this converted value gives 6,000 + 3,000 or 9,000 as the minimum number of cycles required in absolute or real terms.

CHOICE OF SAMPLE SIZE

The size of sample that will be most suitable will depend on a number of factors. One is the unit value of an item. If this is high and the usefulness of the item is impaired by testing, the sample size will have to be relatively small. A related factor is the size of the lot. If it is small and items are made useless by testing, the sample size must be kept small for practical and economic reasons. Another factor in making a choice is the amount of lifetesting facilities available. The sample size may have to be limited to the testing positions available. A fourth and important factor is the period of time available for conducting the life tests. If a decision must be reached quickly because the items are urgently needed, the required test time may be minimized by employing a relatively large sample size. For many components currently in use, the required reliable or mean life is many hundreds of thousands of hours. For these products sample sizes must of necessity be quite large; if not, the testing period required may be many months or years. Another and related factor is the unit-hour cost of lifetesting; this may be high because costly test facilities are required. In such cases the total unit-hours of testing must be minimized by suitable choice of sample size and test duration. In

addition to the factors just listed, many other minor related factors may have to be considered such as the current availability of existing test facilities.

No systematic method for determining the most economical or most satisfactory sample is available at this time. However, a somewhat reasonable decision may be made in most applications of the procedures described here by the use of some judgment together with some rough inspection cost estimates for the various alternatives. One may first examine the test time values tabulated in the tables for each of the sample sizes listed. Then by consideration of the costs -- both those reducible to money terms and those not reducible -- associated with each item in the sample and the costs associated with each unit of required test time, one may determine, for example, that a sample as small as 10 or 25 items is clearly too small and that one of 500 or 1,000 items is clearly too large and that one of 100 items is perhaps reasonably close to the optimum size.

It should be noted at this point that nothing will be gained if one uses large sample sizes in an attempt to obtain sharp discrimination between good and bad lots. Sharp discrimination for this Weibull (and exponential) procedure is obtained by the employment of large acceptance numbers; the size of the sample is of little importance. This point will be discussed further in the following section.

One should note particularly (by examination of the tabulated values) that for small values for β the choice of sample size is quite critical. Any increase in sample size, given some value for c, allows a very considerable decrease in the required lifetesting time -- a decrease far out of proportion to the increase in sample size. Doubling the sample size may reduce the required test time to one-fourth or less of its former value, for example. This is not true, it may be noted, for large values for β. A general rule to follow if one wishes to minimize item hours of testing is

to use relatively large sample sizes when the value for β is small and to use relatively small sample sizes when the value for β is large. Plans have been made to study this sample-size and test-duration-time relationship and to find procedures for determining a value for n that will minimize the item hours required in any given application.

CHOICE OF ACCEPTANCE NUMBER

If in the application of these procedures the only costs of concern are those associated with the lifetesting of sample items and with the acceptance, by chance, of unsatisfactory lots, then a low acceptance number -- preferably 0 -- should be used. This practice will minimize the sample sizes and lifetesting times required. At the same time one will obtain the specified confidence, 95%, that accepted lots or populations comply with reliable life or mean life requirements.

With this practice, however, only the consumer's risk is considered. The extent of the producer's risk -- which is the risk of acceptable lots being rejected by chance -- is ignored. Actually, the use of low acceptance numbers maximizes this risk. The reliable life or mean life for a lot may have to be many times the life specified if there is to be a reasonable probability of acceptance. This will be true even if quite large sample sizes are employed. For Weibull sampling-inspection procedures of the form used here, the ability of a selected plan to discriminate sharply between acceptable and unacceptable lots depends almost entirely on the magnitude of the acceptance numbers; the size of the sample will make little, if any, difference. The larger the acceptance number the better the ability to discriminate -- the steeper the slope of the operating-characteristic curve. This is contrary to what can be expected from inspection plans of the usual attribute form for which the slope of the operating-characteristic curve depends primarily on the sample size and relatively little on the magnitude of the acceptance number.

Thus if there is concern for the producer's risk and it is important to avoid rejecting an undue number of acceptable lots, larger acceptance numbers must be used. Longer lifetesting times than for smaller numbers will be required. However, good discrimination between acceptable and unacceptable lots can be obtained only through an adequate amount of inspection and it must be obtained in this way. The use of larger acceptance numbers is particularly important for small values of β. It becomes relatively less important for large β values.

To provide some guidance in the selection of acceptance numbers, a table of ratios, Table 5, has been provided. Each value tabulated is the ratio between the reliable life or mean life for which the probability of acceptance is .95 (symbolized by $L_{.95}$) and the reliable life or mean life for which the probability of acceptance is .05 (symbolized by $L_{.05}$). This latter life, $L_{.05}$, one should note, is the "specified reliable life" or the "specified minimum mean life" used in determining the minimum lifetesting times through the procedures and tables presented in this report. (The selection of a plan that provides a confidence level of 95% that the life requirement for the lot or population has been met provides, in effect, a consumer's risk of .05. That is, if the reliable life or mean life is precisely at the specified value, the probability of acceptance is .05.) To find the value for reliable life or mean life necessary to assure a high probability of acceptance, one need simply to multiply the specified life, $L_{.05}$, by the appropriate ratio from Table 5. This use of the ratios will be illustrated in the example that follows. It should be pointed out that the ratios in this table apply for all sample sizes, for all values of the reliability index, r, and for minimum mean life applications.

<u>Example (4)</u>

A plan is required for the acceptance inspection of a mechanical component. A value for β of $1\frac{2}{3}$ and for γ of 0 can be assumed. Assurance is

required that the reliable life for each lot will be at least 100 hours with the proportion surviving, r, being .90. A sample size of 250 items has been specified. It is also desirable that the probability of acceptance be high for lots whose reliable life is around 400 hours.

The ratio $L_{.95}/L_{.05}$ is 400/100 or 4. Reference to Table 5 indicates for $\beta = 1\frac{2}{5}$ a ratio of 4.7 for c = 1 and of 3.4 for c = 2. With the use of 1 as the acceptance number the lifetesting time must be .36 x 100 or 36 hours (the factor .36 is obtained in the usual way from Table 2 which gives values for r = .90). The reliable life for a lot must be 4.7 x 100 or 470 hours or more if the probability of acceptance is to be .95 or more. With the use of 2 as the acceptance number, the testing time must be .43 x 100 or 430 hours; the reliable life must be .34 x 100 or 340 hours to provide a probability of acceptance of .95. A choice can accordingly be made between these two alternatives for c.

MATHEMATICAL BASIS FOR THE TABULATED VALUES

The following are the equations expressing the Weibull form of frequency distribution:

$$f(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}}, \qquad x \geqq \gamma,\ \beta,\ \eta > 0, \text{ and}$$

$$F(x) = 1 - e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}}, \qquad x \geqq \gamma,\ \beta,\ \eta > 0 .$$

In these expressions, β = shape parameter
η = scale or characteristic life parameter
γ = location or threshold parameter.

Since the times tabulated in the tables are based on an assumption of $\gamma = 0$, the above expressions may be simplified to the following forms:

$$f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} e^{-\left(\frac{x}{\eta}\right)^{\beta}}, \quad \text{and}$$

$$F(x) = 1 - e^{-\left(\frac{x}{\eta}\right)^{\beta}} .$$

If ρ_r is used to represent reliable life with r representing the proportion of items that will survive to or beyond this life, then

$$\rho_r = \eta(-\ln r)^{1/\beta}$$

(with $\gamma = 0$.)

If one lets t represent the time at which life testing is terminated and p' represent the probability of an item failing prior to time t, combining the equation above with the equation for F(x) gives,

$$p' = F(t) = 1-e^{-[t(-\ln r)^{1/\beta}/\rho_r]^{\beta}} .$$

This can be simplified to,

$$p' = 1-e^{[\ln(r)(t/\rho_r)^{\beta}]} .$$

Solving for t gives,

$$t = \rho_r[\ln (1-p')/\ln(r)]^{1/\beta} .$$

For the mean life case, use is made of the equation for the mean, μ, of the Weibull distribution which is,

$$\mu = \eta\, \Gamma\, (1/\beta + 1) .$$

Combining this equation for the equation for F(x) gives,

$$p' = F(t) = 1-e^{-[(t/\mu)\Gamma(1/\beta + 1)]^{\beta}} .$$

Solving for t gives,

$$t = \mu[-\ln(1-p')]^{1/\beta}/\Gamma(1/\beta + 1) \quad .$$

Since the criterion for acceptance in the procedure described here is of the usual attribute form, a first step in finding required lifetesting times is to determine appropriate values for p'. This was done for each combination of sample size and acceptance number. Values of p' were found which would provide a probability of acceptance of .05 (which assures lot compliance with 95% confidence). The binomial distribution was used for sample sizes 10, 25, 50 and 100. The Poisson approximation was used for sample sizes 250, 500, and 1000.

These p' values were then used to obtain values for testing time t. To make the values available for general use, they were determined as multiples of ρ_r and of μ rather than in absolute terms. Actually, for the tables of values presented here, direct computations were not made. The relationship between (t/μ) and p' had been established for a wide range of p' values as a foundation for the work in the report listed as Reference 1; the relationship between (t/ρ_r) and p' had been found over a wide range as part of the underlying work for Reference 3. Hence use was made of these established relationships.

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Table I

Minimum Lifetesting Times to Assure Lot Compliance
with 95% Confidence

In Multiples of Specified Reliable Life -- r = .50

	c	Sample Size - n						
		10	25	50	100	250	500	1000
$\beta=\frac{1}{2}$	0	.18	.030	.0076	.0019	.00030	.000076	.000019
	1	.51	.080	.019	.0047	.00080	.00019	.000047
	2	1.0	.14	.034	.0084	.0014	.00034	.000083
	3	1.7	.23	.054	.013	.0021	.00050	.00012
	4	2.7	.33	.077	.018	.0029	.00071	.00017
	5	4.4	.46	.097	.024	.0039	.00093	.00023
$\beta=\frac{3}{4}$	0	.32	.095	.038	.015	.0046	.0018	.00071
	1	.66	.18	.070	.028	.0085	.0033	.0013
	2	1.0	.27	.10	.041	.012	.0049	.0019
	3	1.4	.37	.14	.054	.016	.0065	.0025
	4	2.0	.48	.18	.068	.020	.0081	.0031
	5	2.8	.60	.21	.083	.025	.0097	.0038
$\beta=1$ (Exp.)	0	.42	.17	.085	.043	.017	.0087	.0043
	1	.72	.27	.13	.068	.028	.014	.0069
	2	1.0	.38	.18	.090	.036	.018	.0090
	3	1.3	.47	.23	.11	.045	.022	.011
	4	1.7	.57	.27	.13	.053	.026	.013
	5	2.1	.67	.31	.15	.061	.030	.015
$\beta=1\frac{1}{3}$	0	.53	.27	.15	.095	.048	.028	.017
	1	.78	.38	.22	.13	.067	.040	.023
	2	1.0	.48	.28	.16	.083	.049	.029
	3	1.2	.57	.33	.19	.098	.058	.034
	4	1.4	.66	.38	.22	.11	.065	.038
	5	1.7	.75	.41	.24	.12	.070	.043
$\beta=1\frac{2}{3}$	0	.59	.35	.23	.15	.088	.057	.038
	1	.81	.46	.30	.20	.11	.075	.050
	2	1.0	.55	.36	.24	.13	.090	.059
	3	1.2	.64	.41	.27	.15	.10	.067
	4	1.3	.71	.45	.30	.17	.11	.074
	5	1.5	.78	.49	.32	.18	.12	.080
$\beta=2$	0	.65	.41	.29	.21	.13	.093	.066
	1	.85	.52	.36	.26	.16	.11	.082
	2	1.0	.61	.42	.30	.19	.13	.095
	3	1.1	.69	.47	.33	.21	.15	.10
	4	1.3	.76	.52	.36	.23	.16	.11
	5	1.4	.82	.55	.39	.24	.17	.12
$\beta=2\frac{1}{2}$	0	.70	.50	.37	.28	.19	.15	.11
	1	.88	.59	.45	.34	.23	.18	.13
	2	1.0	.67	.50	.38	.26	.20	.15
	3	1.1	.73	.55	.41	.28	.22	.16
	4	1.2	.79	.59	.44	.30	.23	.17
	5	1.3	.85	.62	.47	.32	.24	.18

Table 2

Minimum Lifetesting Times to Assure Lot Compliance
with 95% Confidence

In Multiples of Specified Reliable Life -- r = .90

	c	Sample Size - n						
		10	25	50	100	250	500	1000
$\beta=\frac{1}{2}$	0	8.2	1.3	.32	.081	.013	.0032	.00080
	1	22	3.3	.82	.20	.033	.0080	.0020
	2	43	6.2	1.5	.36	.059	.014	.0035
	3	72	10	2.3	.55	.090	.022	.0053
	4	110	15	3.3	.78	.12	.031	.0074
	5	160	20	4.1	1.0	.16	.040	.0099
$\beta=\frac{3}{4}$	0	4.0	1.2	.46	.18	.056	.022	.0087
	1	8.0	2.2	.87	.34	.10	.040	.016
	2	12	3.4	1.3	.50	.15	.060	.023
	3	18	4.6	1.7	.67	.20	.079	.031
	4	24	5.9	2.2	.85	.25	.099	.038
	5	34	7.3	2.6	1.0	.30	.12	.047
$\beta=1$ (Exp.)	0	2.8	1.1	.57	.28	.11	.057	.028
	1	4.7	1.8	.90	.45	.18	.091	.045
	2	6.5	2.5	1.2	.60	.24	.12	.060
	3	8.7	3.1	1.5	.75	.30	.15	.074
	4	11	3.7	1.8	.88	.35	.17	.088
	5	14	4.4	2.0	1.0	.41	.20	.10
$\beta=1\frac{1}{3}$	0	2.2	1.1	.65	.38	.20	.11	.068
	1	3.2	1.5	.92	.55	.28	.16	.097
	2	4.1	2.0	1.2	.67	.34	.20	.12
	3	5.1	2.3	1.3	.80	.40	.24	.14
	4	6.2	2.7	1.5	.91	.45	.27	.16
	5	7.2	3.1	1.7	1.0	.51	.30	.18
$\beta=1\frac{2}{3}$	0	1.8	1.0	.71	.47	.27	.18	.12
	1	2.5	1.4	.93	.62	.36	.24	.15
	2	3.1	1.7	1.1	.73	.43	.28	.18
	3	3.7	2.0	1.2	.83	.48	.32	.21
	4	4.3	2.2	1.4	.92	.53	.35	.23
	5	4.9	2.4	1.5	1.0	.58	.38	.25
$\beta=2$	0	1.6	1.0	.76	.53	.34	.24	.17
	1	2.1	1.3	.95	.67	.43	.30	.21
	2	2.6	1.5	1.1	.78	.50	.35	.24
	3	3.0	1.7	1.2	.85	.55	.38	.27
	4	3.4	1.9	1.3	.95	.60	.42	.29
	5	3.8	2.1	1.4	1.0	.64	.45	.32
$\beta=2\frac{1}{2}$	0	1.5	1.0	.79	.60	.42	.32	.24
	1	1.8	1.2	.96	.72	.50	.38	.29
	2	2.1	1.4	1.1	.81	.57	.43	.32
	3	2.4	1.5	1.2	.88	.62	.47	.35
	4	2.6	1.7	1.2	.96	.66	.50	.37
	5	2.9	1.8	1.3	1.0	.70	.53	.39

Table 3

Minimum Lifetesting Times to Assure Lot Compliance
with 95% Confidence

In Multiples of Specified Reliable Life -- r = .99

	c	Sample Size - n						
		10	25	50	100	250	500	1000
$\beta=\frac{1}{2}$	0	850	140	35	9.0	1.4	.35	.090
	1	2400	360	90	23	3.6	.90	.22
	2	4700	670	160	39	6.3	1.5	.39
	3	8500	1000	250	60	9.8	2.4	.60
	4	13000	1500	360	86	13	3.4	.83
	5	22000	2100	450	110	19	4.4	1.1
$\beta=\frac{3}{4}$	0	91	27	10	4.2	1.3	.50	.20
	1	180	52	20	7.9	2.3	.94	.37
	2	290	77	30	11	3.4	1.3	.53
	3	410	100	40	15	4.6	1.8	.70
	4	560	130	50	19	5.7	2.2	.87
	5	770	170	59	23	6.9	2.7	1.0
$\beta=1$ (Exp.)	0	29	12	5.9	3.0	1.2	.59	.29
	1	51	19	9.5	4.8	1.9	.95	.47
	2	70	26	12	6.3	2.5	1.2	.63
	3	92	33	16	7.8	3.1	1.5	.77
	4	110	39	19	9.3	3.6	1.8	.90
	5	150	47	21	10	4.3	2.1	1.0
$\beta=1\frac{1}{3}$	0	12	6.3	3.7	2.2	1.1	.67	.40
	1	18	9.1	5.4	3.2	1.6	.96	.56
	2	24	11	6.7	3.9	2.0	1.2	.70
	3	29	13	7.8	4.6	2.3	1.4	.81
	4	34	15	9.0	5.3	2.6	1.6	.92
	5	41	17	9.8	5.9	2.9	1.7	1.0
$\beta=1\frac{2}{3}$	0	7.5	4.4	2.9	1.9	1.1	.73	.48
	1	10	5.8	3.8	2.5	1.5	.96	.64
	2	12	7.0	4.6	3.0	1.7	1.1	.75
	3	15	7.9	5.2	3.4	2.0	1.3	.85
	4	17	9.0	5.7	3.8	2.2	1.4	.94
	5	20	10	6.2	4.1	2.4	1.5	1.0
$\beta=2$	0	5.5	3.4	2.4	1.7	1.1	.77	.54
	1	7.1	4.3	3.0	2.1	1.3	.96	.67
	2	8.4	5.1	3.6	2.5	1.6	1.1	.78
	3	9.6	5.8	4.0	2.8	1.7	1.2	.87
	4	11	6.3	4.3	3.0	1.9	1.3	.95
	5	12	6.9	4.6	3.2	2.0	1.4	1.0
$\beta=2\frac{1}{2}$	0	3.8	2.7	2.0	1.5	1.0	.80	.61
	1	4.7	3.2	2.4	1.8	1.3	.97	.73
	2	5.4	3.6	2.7	2.0	1.4	1.1	.83
	3	6.0	4.0	3.0	2.2	1.5	1.2	.89
	4	6.7	4.3	3.2	2.4	1.6	1.2	.95
	5	7.3	4.6	3.4	2.6	1.7	1.3	1.0

Table 4

Minimum Lifetesting Times to Assure Lot Compliance
with 95% Confidence

In Multiples of Specified Minimum Mean Life

	c	Sample Size - n						
		10	25	50	100	250	500	1000
$\beta=\frac{1}{2}$	0	.045	.0071	.0018	.00045	.000076	.000018	.000004
	1	.12	.018	.0045	.0011	.00018	.000047	.000011
	2	.25	.034	.0080	.0020	.00033	.000084	.000020
	3	.44	.055	.012	.0031	.00050	.00012	.000030
	4	.73	.080	.018	.0043	.00069	.00017	.000041
	5	1.20	.11	.023	.0058	.00095	.00022	.000057
$\beta=\frac{3}{4}$	0	.17	.049	.019	.0078	.0023	.00091	.00037
	1	.34	.092	.037	.014	.0042	.0017	.00067
	2	.53	.14	.054	.021	.0063	.0025	.00098
	3	.77	.19	.073	.027	.0083	.0032	.0013
	4	1.00	.25	.091	.035	.010	.0040	.0016
	5	1.30	.31	.11	.041	.012	.0049	.0019
$\beta=1$ (Exp.)	0	.29	.12	.060	.030	.012	.0060	.0030
	1	.50	.19	.095	.048	.019	.0097	.0048
	2	.70	.26	.13	.063	.025	.013	.0064
	3	.94	.33	.16	.078	.031	.016	.0077
	4	1.20	.40	.19	.093	.037	.018	.0091
	5	1.50	.47	.21	.11	.043	.021	.010
$\beta=1\frac{1}{3}$	0	.44	.22	.13	.077	.040	.023	.014
	1	.65	.32	.18	.11	.056	.033	.019
	2	.83	.40	.23	.13	.069	.041	.024
	3	1.00	.48	.27	.16	.080	.048	.028
	4	1.20	.55	.31	.18	.091	.054	.032
	5	1.50	.62	.34	.20	.10	.060	.036
$\beta=1\frac{2}{3}$	0	.54	.31	.20	.13	.079	.052	.034
	1	.75	.41	.27	.18	.10	.068	.045
	2	.90	.50	.32	.21	.12	.082	.053
	3	1.00	.58	.37	.24	.14	.093	.060
	4	1.20	.65	.41	.27	.15	.10	.067
	5	1.50	.71	.44	.29	.17	.11	.073
$\beta=2$	0	.62	.39	.27	.19	.12	.087	.062
	1	.80	.49	.35	.24	.15	.11	.078
	2	.95	.58	.40	.28	.18	.13	.089
	3	1.00	.65	.45	.31	.20	.14	.10
	4	1.20	.72	.49	.34	.21	.15	.11
	5	1.40	.78	.52	.37	.23	.16	.12
$\beta=2\frac{1}{2}$	0	.69	.48	.36	.27	.19	.15	.11
	1	.85	.58	.43	.33	.23	.17	.13
	2	.98	.66	.49	.37	.26	.19	.15
	3	1.10	.72	.53	.40	.28	.21	.16
	4	1.20	.77	.57	.43	.30	.23	.17
	5	1.30	.82	.60	.46	.32	.24	.18

Table 5

Values of $L_{.95}/L_{.05}$

c	$\beta = 1/2$	$\beta = 3/4$	$\beta = 1$	$\beta = 1\ 1/3$	$\beta = 1\ 2/3$	$\beta = 2$	$\beta = 2\ 1/2$
0	3,300	220	58	21	11	7.6	5.0
1	170	31	13	7.0	4.7	3.6	2.8
2	59	15	7.7	4.6	3.4	2.8	2.3
3	32	10	5.7	3.6	2.8	2.4	2.0
4	22	7.6	4.6	3.1	2.5	2.2	1.8
5	16	6.2	4.0	2.8	2.3	2.0	1.7

$L_{.95} = \rho_r$ or μ for which $P(A) = .95$

$L_{.05} = \rho_r$ or μ for which $P(A) = .05$