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A THIN AIRFOIL WITH A MINIMUM AVERAGE COEFFICIENT OF HEAT TRANSFER FOR A GIVEN LIFT

Translated from the Russian by Joy B. Gazley

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TRANSLATOR'S PREFACE

This Memorandum is a translation of the Russian paper, "Tonkii Profil' Krylas Minimal'nym Srednim Koeffitsientom Teplootdachi Pri Zadannoi Pod''emnoi Sile," (A Thin Airfoil with a Minimum Average Coefficient of Heat Transfer for a Given Lift), by E. V. Bulygina, M. B. Poliakov, and Ia. S. Shcherbak, published in Izvestiia, Vysshikh Uchebnykh Zavedenii, Aviatsionnaia Tekhnika, 2, 1961, pp. 17-25.

This is one of a series of translations on the general subject of heat transfer.

A THIN AIRFOIL WITH A MINIMUM AVERAGE COEFFICIENT OF HEAT TRANSFER FOR A GIVEN LIFT

E. V. Bulygina, M. B. Poliakov, Ia. S. Shcherbak

The transition to high supersonic velocities of flight leads to significant kinetic heating of the wing of an aircraft. The value of the local heat flow q_i from the boundary layer into the wing is determined by the product of the local heat-transfer coefficient h_i and the difference between the local boundary-layer temperatures T_{awi} and the wall temperatures, T_{wi}:

$$q_i = h_i \left(T_{awi} - T_{wi}\right) \qquad (1)$$

It is clear that a decrease in the heat-transfer coefficient on the wing leads to a decrease of thermal flows into the wing. The coefficient of heat transfer depends, basically, on the local flow parameters at the outer edge of the boundary layer. The angle of attack and the profile shape, which determine in addition the lift coefficient Cy, affect the local flow parameters.

The problem set is to determine some airfoil shape, which, at a given Cy, provides for a minimum average heat-transfer coefficient at the profile. The problem raised is solved with the following simplifying assumptions.

1. A thin profile is assumed (the angles of inclination of its upper and the lower surface are the same).

2. The lower and the upper surfaces of the profile are heat insulated.

3. The boundary layer is either completely laminar or completely turbulent.

Let the equation of the mean line of the profile be:

$$y = y(x) \qquad (2)$$

We will designate the angle of inclination of the surface of an infinitely thin airfoil:

$$\theta = y'(x)$$

Since the profile is thin, we determine, in the first approximation, the pressure coefficient, $\bar{p}$, by the linear theory:

$$\bar{p} = \pm \frac{2\theta}{\sqrt{M_\infty^2 - 2}}, \qquad (3)$$

where M_∞ is the Mach number of the external flow.

The lift coefficient of the profile is determined by the integral of the local angle of the mean line of the profile:

$$Cy = \frac{4}{b\sqrt{M_\infty^2 - 1}} \int_0^b \theta\,(x)dx, \qquad (4)$$

where b is the chord of the profile (meters).

The mean heat-transfer coefficient is obtained by integrating the local heat-transfer coefficients for the upper and lower surfaces of the profile:

$$h_{cp} = \frac{1}{2b} \int_0^b \left[h^+ (x,\theta) + h - (x,\theta)\right] dx, \qquad (5)$$

where $h(x,\theta)$ is the local heat-transfer coefficient (k cal/m^2 hr ^{o}C) and x is the distance along the chord from the leading edge of the profile (meters). Here, and below, the superscript "+" indicates a heat-transfer coefficient for values of x and θ on the lower surface of the profile and the superscript "-" for values on the upper surface of the profile.

There is a series of equations for the determination of local heat-transfer co-efficients on a plate under a zero angle of attack.[1,2,3] For the calculation of local heat-transfer coefficients on a thin profile under a small angle of attack it is possible, in the first approximation, to use the dependencies obtained for plates under a zero angle of attack if one substitutes in these dependencies the local-flow parameters at the outer edge of the profile boundary layer. As is shown in (4), this method of calculating heat-transfer coefficients for not very thick profiles with a pressure gradient for a laminar boundary layer gives results which agree satisfactorily with calculations by more accurate methods.

Reference (5) points out the possibility of using this method for the turbulent boundary layer. In the present article the heat-transfer co-efficient is determined according to the equations of Ref. (1), which, using the local values of the flow parameters at the thin profile in flow, may be transformed to:

$$h = Af_H(H)\ f_x\ (x)\ f_{\bar{p}}\ (M_\infty,\ M_1), \qquad (6)$$

where

$$f_H\ (H) = C_{pH}\ T_H^{\frac{1-n}{2}} \cdot \mu_H^n\ \rho_H^{1-n},$$

$$f_x\ (x) = x^{-n},$$

$$f_{\bar{p}}\ (M_\infty,\ M_1) = (1 + \frac{\kappa-1}{2} M_\infty^2)^\beta \frac{M_1^n}{(1 + \frac{\kappa-1}{2} M_1^2)^\beta} (\frac{T_1}{T_{w1}})^\alpha,$$

μ_H, T_H, ρ_H = the coefficients of viscosity, temperature, and density of air at an altitude H at standard atmospheric conditions,

C_{pH} = specific heat of air at an altitude H(k cal/kg°C) (in the calculations, C_{pH} = 0.24),

κ = adiabatic index for air (κ = 1.4),

T_{wi} = the local temperature of the surface (^{o}K),

T_i, M_i = the local temperature and Mach number of flow at the outer edge of the profile boundary layer.

For the turbulent boundary layer

$$A = 14550,\ n = 0.2,\ \beta = 2.56,\ \alpha = 0.44;$$

for the laminar boundary layer

$$A = 64460,\ n = 0.5,\ \beta = 1.9,\ \alpha = 0.1.$$

The temperature of the wall surface, T_{wi}, may change from the temperature of the external flow, T_i, to the temperature of the heat-insulated wall:

$$T_{awi} = T_i \left(1 + \frac{\kappa-1}{2} r M_i^2\right). \qquad (7)$$

In this equation r is the temperature recovery factor. For the turbulent boundary layer, r = 0.9, for the laminar, $r \simeq 0.85$.

It is possible to isolate two extreme cases of temperature change of the wall:

1. $T_{wi} = T_{awi}$ corresponds to a continuous steady flight without taking into account heat-insulation,

2. $T_{wi} = T_i$ corresponds to instantaneous climb to altitude H and a Mach number, M_∞, of flight ("instantaneous start").

For ease of calculation, it is possible to combine these cases if the temperature recovery factor on the surface is assumed approximately equal to one:

$$f_{\bar{p}}(M_\infty, M_i) = \left(1 + \frac{\kappa-1}{2} M_\infty^2\right)^{\beta} \frac{M_i^{1-n}}{\left(1 + \frac{\kappa-1}{2} M_i^2\right)^{\gamma}}, \qquad (8)$$

where

$$\gamma = \beta + \alpha$$

For the turbulent boundary layer

$\gamma = 3$ for $T_{wi} = T_{awi}$ ($\alpha = 0.44$),

$\gamma = 2.56$ for $T_{wi} = T_i$ ($\alpha = 0$).

For the laminar boundary layer

$\gamma = 2$ for $T_{wi} = T_{awi}$ ($\alpha = 0.1$),

$\gamma = 1.9$ for $T_{wi} = T_i$ ($\alpha = 0$).

The local-pressure coefficient and local Mach number, M, with their small changes are combined in the equation:

$$M_i = \sqrt{\frac{2}{x-1}}\sqrt{\left(1 + \frac{x-1}{2} M_\infty^2\right)\left(1 + \frac{x}{2} M_\infty^2 \bar{p}_i\right)^{\frac{1-}{x}} - 1} \qquad (9)$$

Thus the problem of finding the configuration of a thin profile having, at a given flight regime, a minimum mean-heat-transfer coefficient for given values of the lift coefficient leads to the finding of some function y(x) which gives the extreme functional equation (5) for the given values of the functional equation (4).

For the solution of this single boundary-variation problem we take advantage of the LaGrange method of multiples according to which the function sought must satisfy the Euler equation:

$$\frac{\partial F}{\partial y} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y'}\right) = 0 \qquad (10)$$

where

$$F = h^+(x,\theta) + h^-(x,\theta) + \lambda_1 \cdot \theta(x),$$

λ_1 being the constant of the LaGrange multiple. Since F apparently does not depend on y, Euler's equation is reduced and then its integral is found:

$$A f_H f_x \left[\left(\frac{\partial f_{\bar{p}}}{\partial M_i} \cdot \frac{\partial M_i}{\partial \bar{p}_i} \cdot \frac{\partial \bar{p}_i}{\partial \theta}\right)^+ + \left(\frac{\partial f_{\bar{p}}}{\partial M_i} \cdot \frac{\partial M_i}{\partial \bar{p}_i} \cdot \frac{\partial \bar{p}_i}{\partial \theta}\right)^-\right] + \lambda_1 = C \qquad (11)$$

The values $\frac{\partial f_{\bar{p}}}{\partial M_i}$, which are calculated on the basis of Eq. (8), enter Eq. (11):

$$\frac{\partial f_{\bar{p}}}{\partial M_i} = \left[\frac{(1-n)M_i^{-n}}{\left(1+\frac{x-1}{2}M_i^2\right)^{\gamma}} - \frac{(x-1)rM_i^{2-n}}{\left(1+\frac{x-1}{2}M_i^2\right)^{\gamma+1}}\right]\left(1+\frac{x-1}{2}M_\infty^2\right)^{\beta}. \qquad (12)$$

On the basis of Eq. (9) we obtain

$$\frac{\partial M_i}{\partial \bar{p}_i} = -\frac{1}{4}\sqrt{2(x-1)}\;\frac{1\,M_\infty^2\left(1+\frac{x-1}{2}M_\infty^2\right)\left(1+\frac{x}{2}M_\infty^2\,\bar{p}_i\right)^{\frac{1-2x}{x}}}{\sqrt{\left(1+\frac{x-1}{2}M_\infty^2\right)\left(1+\frac{x}{2}M_\infty^2\,\bar{p}_i\right)^{\frac{1-x}{x}}-1}}\,. \qquad (13)$$

The derivative $\frac{\partial \bar{p}}{\partial \theta}$ for the upper surface will have the value:

$$\left(\frac{\partial \bar{p}}{\partial \theta}\right)^{-} = -\frac{2}{\sqrt{M_\infty^2 - 1}}, \qquad (14a)$$

for the lower surface:

$$\left(\frac{\partial \bar{p}}{\partial \theta}\right)^{+} = \frac{2}{\sqrt{M_\infty^2 - 1}}. \qquad (14b)$$

The relations

$$\left(\frac{\partial f_{\bar{p}}}{\partial M_i}\right)^{+},\ \left(\frac{\partial M_i}{\partial \bar{p}_i}\right)^{+} \text{ and } \left(\frac{\partial f_{\bar{p}}}{\partial M_i}\right)^{-},\ \left(\frac{\partial M_i}{\partial \bar{p}_i}\right)^{-}$$

for the upper and lower surfaces have one and the same form (12), (13) and they differ only in the fact that in the first case the pressure coefficient is calculated as positive, in the second as negative.

The equation which combines the pressure $\bar{p}$ and the coordinate of point x takes the form:

$$x^{-n}\left\{\left[\frac{\left(1+\frac{x}{2}M_\infty^2\bar{p}\right)^{\frac{1-2x}{x}}}{\sqrt{\left(1+\frac{x-1}{2}M_\infty^2\right)\left(1+\frac{x}{2}M_\infty^2\bar{p}\right)^{\frac{1-x}{x}}-1}}\;\frac{(1-n)\,M_1^{-n}+(x-1)\left(\frac{1-n}{2}-\gamma\right)M_1^{2-n}}{\left(1+\frac{x-1}{2}M_1^2\right)^{\gamma+1}}\right]^{-}\right.$$

$$\left.-\left[\frac{\left(1+\frac{x}{2}M_\infty^2\bar{p}\right)^{\frac{1-2x}{x}}}{\sqrt{\left(1+\frac{x-1}{2}M_\infty^2\right)\left(1+\frac{x}{2}M_\infty^2\bar{p}\right)^{\frac{1-x}{x}}-1}}\;\frac{(1-n)\,M_1^{-n}+(x-1)\left(\frac{1-n}{2}-\gamma\right)M_1^{2-n}}{\left(1+\frac{x-1}{2}M_1^2\right)^{\gamma+1}}\right]^{+}\right\}=C_1. \qquad (15)$$

Here the values n, γ are the same as in Eq. (8),

$$C_1=\frac{2(C-\lambda_1)\sqrt{M_\infty^2-1}}{\sqrt{2(x-1)}\cdot Af_H\,M_\infty^2\left(1+\frac{x-1}{2}M_\infty^2\right)^{\beta+1}}.$$

This equation defines $\bar{p}$ as an implicit function of x:

$$\phi(\bar{p})=\phi\left(\frac{2\theta}{\sqrt{M_\infty^2-1}}\right)=x^n\lambda^n, \qquad (16)$$

where $\lambda = C_1^{\frac{1}{n}}$ is the new derivative constant.

It isn't difficult to see from Eq. (15), for $\bar{p}=0$ (or $\theta=0$), that the function $\phi(\bar{p})$ reduces to zero and, consequently, for any value of the derivative constant C_1, the angle of inclination, equal to zero, corresponds to the origin of the coordinates. Eq. (15) permits the easy determination of x as a function of ϕ.

In the nature of an example, in Figs. 1 and 2 are introduced graphs of this dependency calculated for conditions of "instantaneous start" and subsequent flight at an altitude H = 20 km with a Mach number $M_\infty = 3$ for the laminar and the turbulent boundary layers.

The lift coefficient Cy is obtained by numerous integrations of the curve $\bar{p}$ (x) (or $\theta(x)$):

$$Cy = \frac{4}{\sqrt{M_\infty^2 - 1}} \frac{1}{b\lambda} \int_0^{b\lambda} \theta(x\lambda)\, d(x\lambda)$$

The coordinate of the profile y is determined by the integration of the angle of inclination of the mean line of the profile:

$$\lambda y = \int_0^{x\lambda} \theta(x\lambda)\, d(x\lambda) .$$

The lift coefficient Cy and the final ordinate of the profile $\bar{y}_{fin.}$ are combined by the simple dependency:

$$Cy = \frac{4}{\sqrt{M_\infty^2 - 1}} \bar{y}_{fin.}$$

A specific value of the chord bλ corresponds to each value of Cy.

The dependence of the rate bλ on Cy for $M_\infty = 3$ and H = 20 km is represented for the turbulent and the laminar boundary layers in Figs. 3 and 4.

The curves of the corresponding profiles at a given value of Cy = 0.178 are shown in Fig. 5.

As is evident from this figure, the profiles essentially differ from the plate to the adjacent leading surface where the profile obtained has a zero angle of inclination.

It must be noted that the value of the heat-transfer coefficient, as in the case of the turbulent, so also in the case of the laminar boundary layer, depends, essentially, on the temperature factor.

But since the form of the profile for the case of two edges follows from the calculations and from Fig. 5, $T_{wi} = T_i$ and $T_{wi} = T_{awi}$ are obtained as the same thing; i.e., the form of the optimal profile depends slightly on the temperature factor. Thus, the profile found will be optimal also at all intermediate wall temperatures.

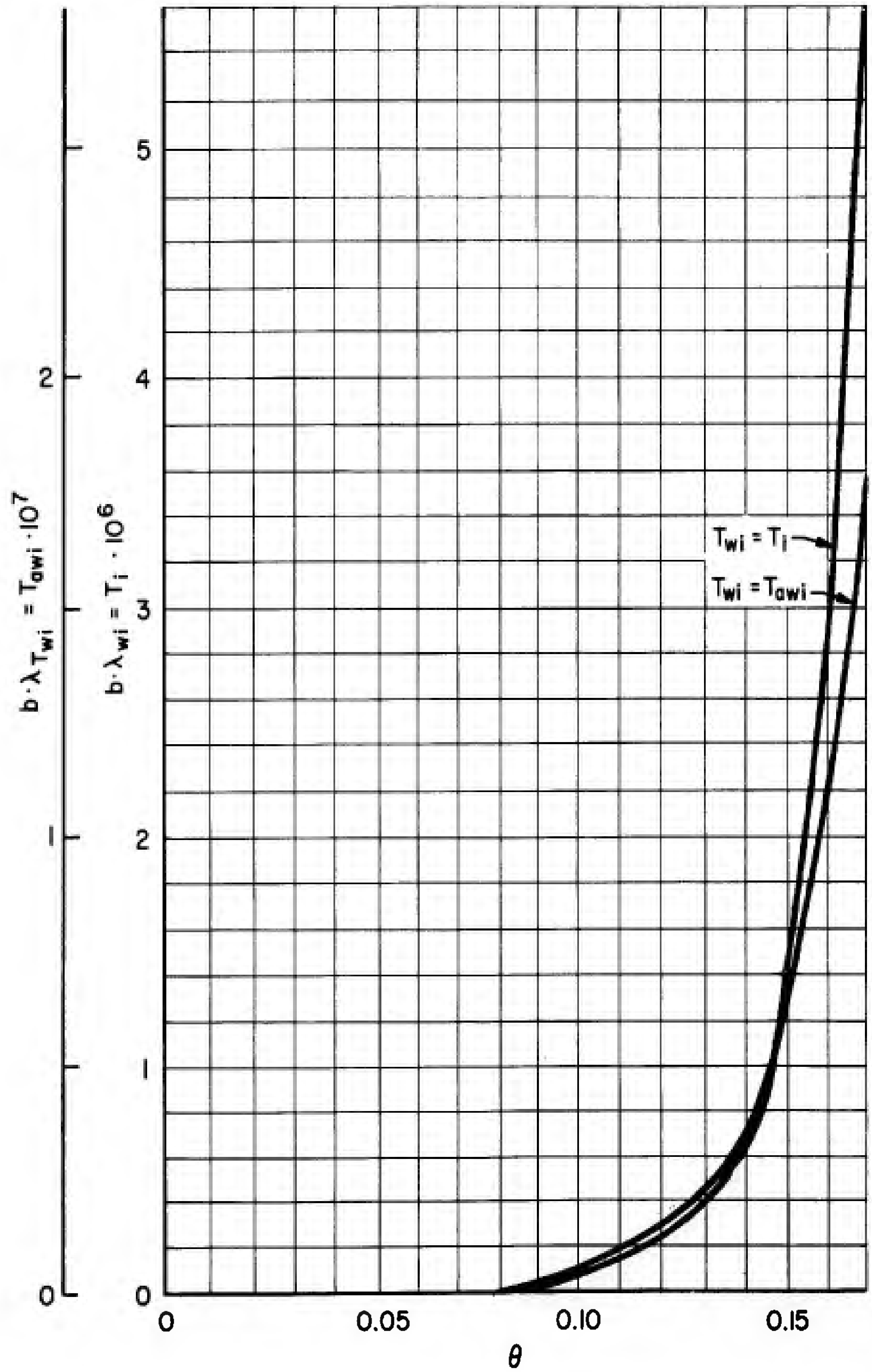


Fig. 1. The Turbulent Boundary Layer

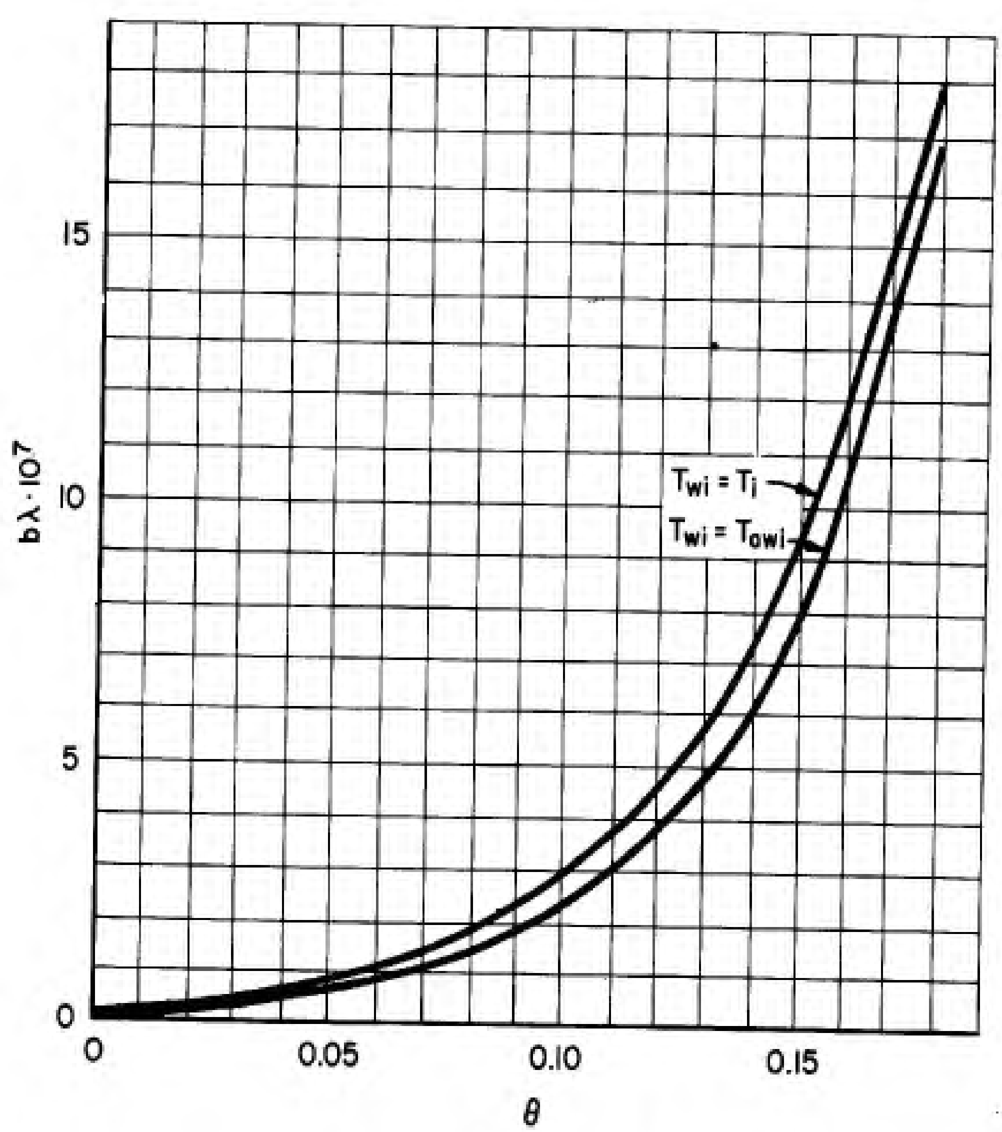


Fig. 2. The Laminar Boundary Layer

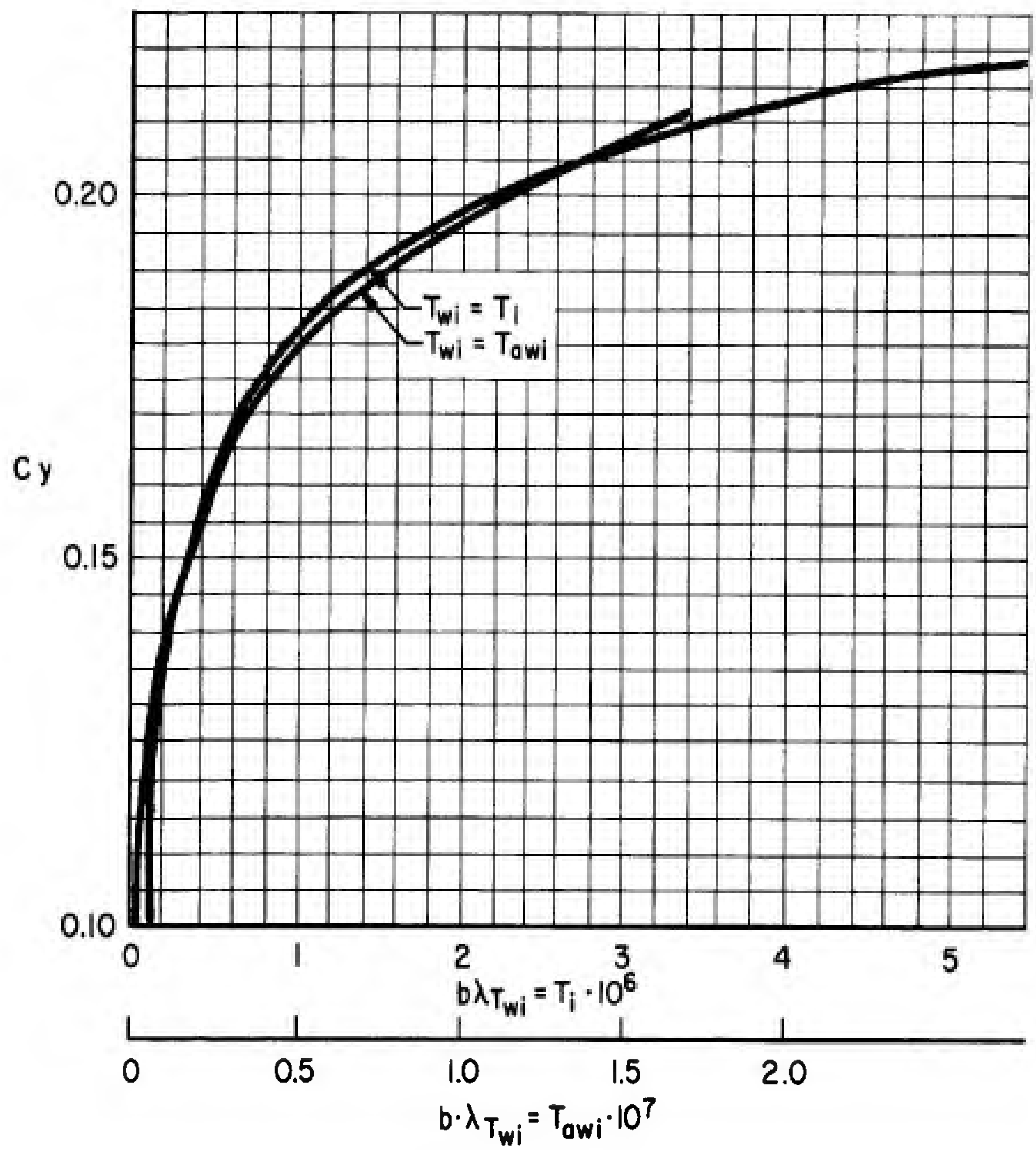


Fig. 3. The Turbulent Boundary Layer

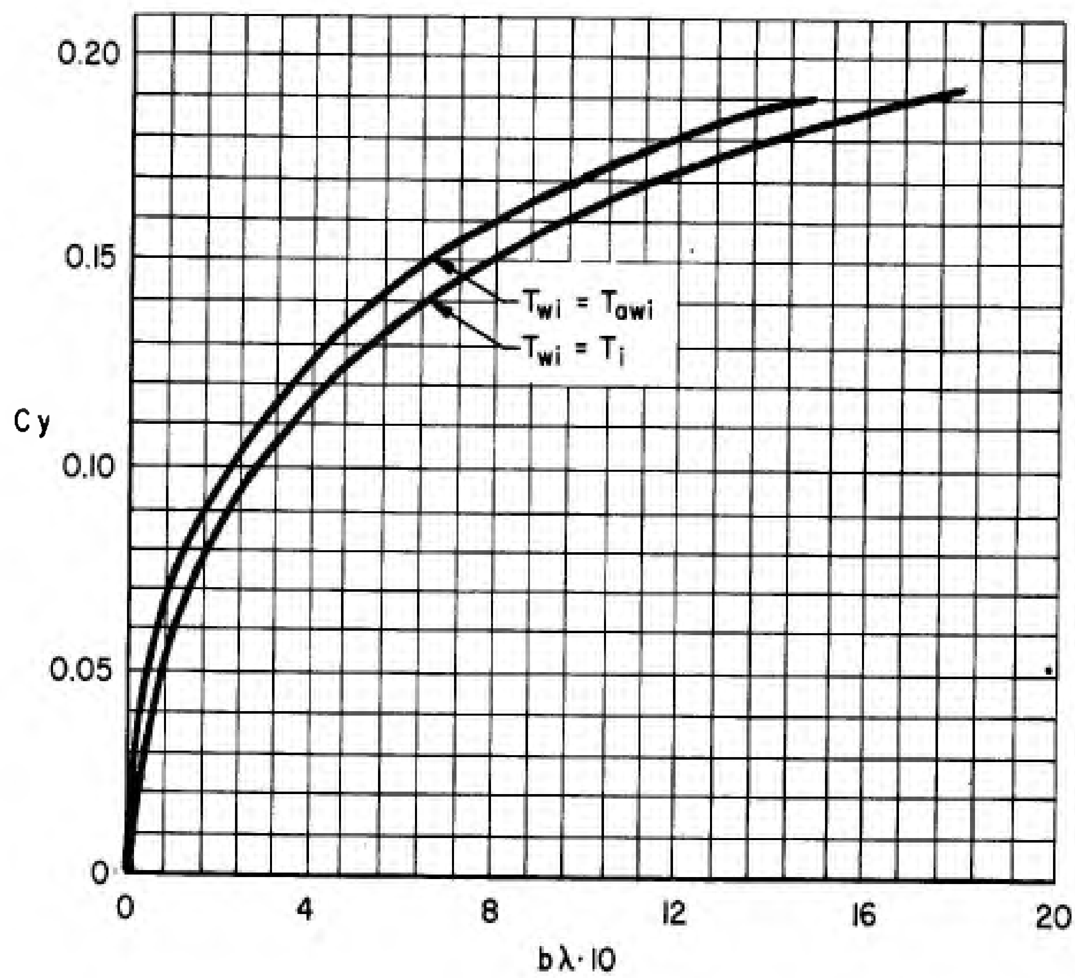
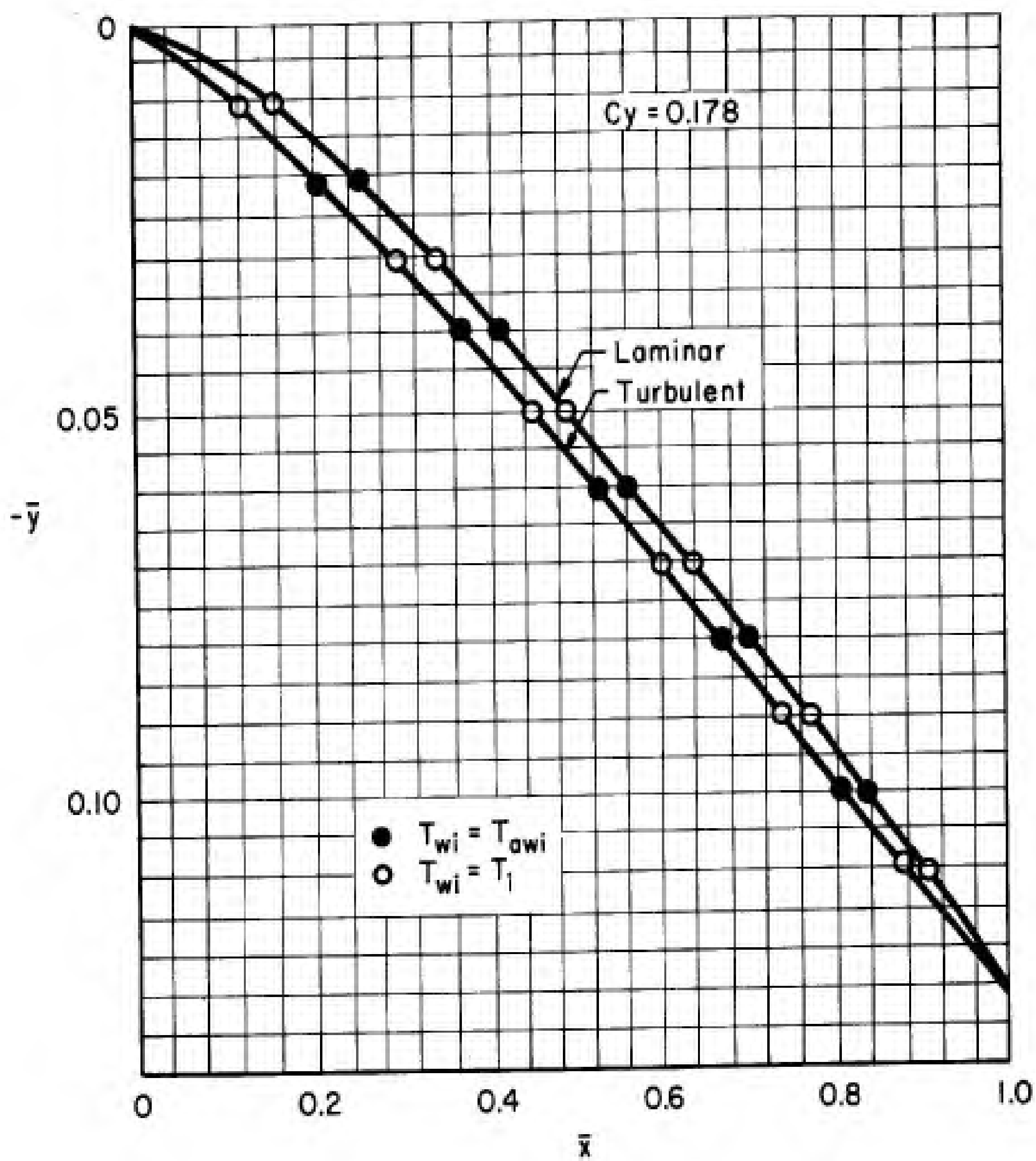


Fig. 4. The Laminar Boundary Layer



0
0.05
0.10
-ȳ
Cy = 0.178
Laminar
Turbulent
$T_{wi} = T_{awi}$
$T_{wi} = T_1$
0
0.2
0.4
0.6
0.8
1.0
x̄

Fig. 5.

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