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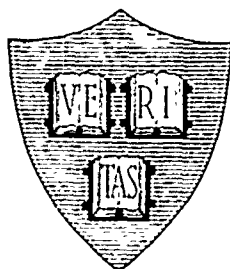
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SCATTERING BY A NARROW
PERFECTLY CONDUCTING INFINITE STRIP
IN A GYROTROPIC MEDIUM



By

S. R. Seshadri

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Technical Report No. 380

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Cruft Laboratory
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Scattering by a Narrow Perfectly Conducting Infinite Strip in a Gyrotropic Medium

by

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Abstract

The scattering of a plane electromagnetic wave of wave-number k by a perfectly conducting infinite strip of width $2a$ is investigated for the case in which the surrounding medium is gyrotropic. The gyrotropic axis is taken parallel to the edges of the strip. The problem is formulated in terms of an integral equation whose solution is obtained in the form of a series in powers of ka . Expressions for the far-zone fields and the first two terms in the series for the total scattering cross section are obtained.

The scattering of electromagnetic waves by obstacles embedded in an anisotropic medium is of current interest. It is known that wave propagation in an unbounded anisotropic medium is more complicated than in isotropic space. As a consequence, the scattering of electromagnetic waves by obstacles immersed in an anisotropic medium is a difficult problem. However, there are two general categories of problems which turn out to be very similar to the corresponding problems in isotropic space. The scattering of obstacles in a uniaxially anisotropic medium belongs to the first category which Felsen⁽¹⁾ has investigated systematically. The scattering in a gyrotropic medium by perfectly conducting cylindrical obstacles belongs to the second category. The generators of the cylinder are parallel to the gyrotropic axis but perpendicular to the direction of propagation of the incident wave. In this report, a simple problem of the second category is studied.

Consider a homogeneous plasma of infinite extent. Let a static magnetic field be assumed to be impressed uniformly throughout the plasma. Under certain simplifying approximations⁽²⁾, the plasma becomes equivalent to a dielectric medium characterized by a tensor dielectric constant. A perfectly conducting infinite strip of width $2a$ is assumed to be embedded in such a medium and oriented so that its edges are parallel to the direction of the external magnetic field. The scattering by the infinite strip of a plane electromagnetic wave of wave-number k is investigated. Of the two polarizations, that with the magnetic vector parallel to the edges of the strip is the one which is different from the corresponding problem in isotropic space; hence only the

H-polarization is treated. A solution is obtained when the incident wavelength is much larger compared to the width of the strip i. e., $ka \ll 1$. The problem is formulated in terms of an integral equation which specifies the current induced on the strip. The integral equation is solved for $ka \ll 1$ and explicit expressions for the diffraction pattern in the far-field and the first two terms in the series for the total scattering cross section are obtained.

Formulation of the Problem:

A perfectly conducting infinite strip occupies the region $|x| < a$, $-\infty < y < \infty$ and $z = 0$, where x , y and z form a right-handed rectangular coordinate system. The medium surrounding the infinite strip is filled with a homogeneous plasma which is threaded uniformly by a static magnetic field in the y direction. Only the two-dimensional problem in which all the field quantities are independent of y is considered. Also, as was pointed out earlier, the treatment is given only to the case of the E-mode for which the nonvanishing components of the electric and magnetic fields are $E_x(x, z)$, $E_z(x, z)$ and $H_y(x, z)$. A harmonic time dependence $e^{-i\omega t}$ is implied for all of the field components. For the E-mode it may be shown⁽²⁾ that $H_y(x, z)$, the only component of the magnetic field, satisfies the following wave-equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2 \right] H_y(x, z) = 0 \quad (1)$$

in the region exterior to the strip. In (1)

$$k^2 = k_0^2 \frac{(\epsilon_1^2 - \epsilon_2^2)}{\epsilon_1} = k_0^2 \frac{\epsilon}{\epsilon_1} \quad (2)$$

where

$$\epsilon_1 = 1 - \left(\frac{\omega_p}{\omega} \right)^2 \left[1 - \frac{\omega_c}{\omega} \right]^{-1} \quad (3)$$

$$\epsilon_2 = \left(\frac{\omega_p}{\omega} \right)^2 \left[\frac{\omega}{\omega_c} - \frac{\omega_c}{\omega} \right]^{-1} \quad (4)$$

The gyromagnetic and plasma frequencies are denoted by ω_c and ω_p respectively.

The wave-number pertaining to vacuum is k_0 . Also, the components of the electric field $E_x(x, z)$ and $E_z(x, z)$ are obtained in terms of $H_y(x, z)$ as follows:

$$E_x(x, z) = -\frac{i\epsilon_1}{\omega\epsilon_0\epsilon} \frac{\partial}{\partial z} H_y(x, z) - \frac{\epsilon_2}{\omega\epsilon_0\epsilon} \frac{\partial}{\partial x} H_y(x, z) \quad (5)$$

$$E_z(x, z) = \frac{i\epsilon_1}{\omega\epsilon_0\epsilon} \frac{\partial}{\partial x} H_y(x, z) - \frac{\epsilon_2}{\omega\epsilon_0\epsilon} \frac{\partial}{\partial z} H_y(x, z) \quad (6)$$

It is obvious from (1) and (2) that the E-mode can propagate only for certain frequency ranges for which k^2 and hence $\frac{\epsilon}{\epsilon_1}$ are positive. It is assumed that the frequency is restricted to the range for which k is real and positive.

In view of (1), it is reasonable to assume that the incident field is given by

$$H_y^i(x, z) = e^{ik(Lx + nz)} \quad (7a)$$

where

$$l = \cos \theta_i, \quad n = \sin \theta_i \quad (7b)$$

The application of Green's theorem to the volume bounded by the two sides of the strip and a cylindrical surface at infinity yields

$$\begin{aligned} H_y(x, z) &= H_y^i(x, z) + H_y^s(x, z) \\ &= e^{ik(lx + nz)} + \frac{i}{4} \int_{-a}^a [I_2(x') + I_1(x') \frac{\partial}{\partial z}] H_0^{(1)}[k\sqrt{(x-x')^2 + z^2}] dx' \end{aligned} \quad (8)$$

where

$$I_1(x) = H_y(x, 0^-) - H_y(x, 0^+) \quad (9a)$$

and

$$I_2(x) = \frac{\partial}{\partial z} H_y(x, 0^-) - \frac{\partial}{\partial z} H_y(x, 0^+) \quad (9b)$$

The integral in (8) evidently gives the scattered field $H_y^s(x, z)$. On the surface of the strip, since it is perfectly conducting, the tangential component of the electric field vanishes and hence, the following boundary condition

$$E_x(x, 0^\pm) = 0 \quad \text{for } |x| < a \quad (10)$$

holds. With the use of (9) and (10) in (5), it follows that

$$i\varepsilon_1 I_2(x) + \varepsilon_2 \frac{\partial}{\partial x} I_1(x) = 0 \quad (11)$$

The value of $I_2(x)$, as obtained from (11), is substituted in the integral in (8) and an integration by parts is carried out. The surface current $I_1(x)$ on the strip, since it is normal to the edge, is zero for $x = 0$. Hence,

$$H_y(x, z) = e^{ik(bx+nz)} + \frac{1}{4} \int_{-a}^a \left[\frac{\partial}{\partial z} + \frac{\epsilon_2}{\epsilon_1} \frac{\partial}{\partial x} \right] I_1(x') H_0^{(1)} [k\sqrt{(x-x')^2 + z^2}] dx' \quad (12)$$

The substitution of (12) in (5), together with the boundary condition (10), yields the following integral equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{k^2 \epsilon_1^2}{\epsilon} \right) \int_{-a}^a I_1(x') H_0^{(1)} [k|x-x'|] dx' = \frac{ik\epsilon_1^2}{\epsilon} \left(n - \frac{i\epsilon_2}{\epsilon_1} \right) e^{ikbx} \quad \text{for } |x| < a \quad (13)$$

Solution of the Integral Equation for $ka \ll 1$

The solution of the differential equation (13) for the integral yields

$$\begin{aligned} & \int_{-a}^a k \left(n + i\ell - \frac{\epsilon_2}{\epsilon_1} \right) I_1(x') H_0^{(1)} [k|x-x'|] dx' \\ &= 4e^{ikbx} + Ce^{\frac{ik\epsilon_1}{\sqrt{\epsilon}}x} + De^{\frac{-ik\epsilon_1}{\sqrt{\epsilon}}x} \quad \text{for } |x| < a. \end{aligned} \quad (14)$$

The arbitrary constants C and D are determined by the requirement that $I_1(\pm a) = 0$. It is noticed that the integral equation (14) is similar to the one obtained in the treatment of the problem of scattering by a narrow, unidirectionally-conducting infinite strip⁽³⁾. Therefore, the solution of (14)

can be written down by making use of the solution obtained in [3]. For that purpose let

$$x' = a \cos v \quad (15)$$

$$(n + i\ell \frac{\epsilon_2}{\epsilon_1}) I_1 (a \cos v) = \sum_{n=c}^{n=\infty} f_n (ka)^{n+1} \quad (16)$$

The solution of (14) is then given in the form

$$f_{2n} = \sum_{r=0}^{r=n} a_{2n,r} \sin (2r + 1) v$$

$$f_{2n+1} = \sum_{r=0}^{r=n} a_{2n+1,r} \sin (2r + 2) v \quad n=0, 1, 2, \dots \quad (17)$$

where

$$a_{0,0} = \frac{2i\beta_0}{\pi}$$

$$a_{1,0} = -\frac{3\beta_1}{2\pi}$$

$$a_{2,0} = -\frac{2i\beta_0 \gamma_2 p}{\pi} + \frac{i\beta_0 \gamma_2}{\pi} + \frac{i\beta_0 p}{2\pi} + \frac{i\beta_0}{8\pi} - \frac{3i\beta_2}{\pi}$$

$$a_{2,1} = \frac{i\beta_0}{24\pi} - \frac{i\beta_2}{\pi}$$

$$a_{3,0} = \frac{\beta_1}{8\pi} - \frac{3\beta_1 \gamma_2}{4\pi} + \frac{5\beta_3}{2\pi}$$

$$a_{3,1} = \frac{5\beta_3}{8\pi} - \frac{\beta_1}{64\pi}$$

$$\begin{aligned}
a_{4,0} = & \frac{15i\beta_4}{4\pi} - \frac{3i\beta_2 p}{4\pi} - \frac{i\beta_2}{4\pi} + \frac{3i\beta_2 \gamma_2 p}{\pi} - \frac{3i\beta_2 \gamma_2}{4\pi} \\
& + \frac{13i\beta_0}{384\pi} + \frac{3i\beta_0 \gamma_2 p}{8\pi} - \frac{i\beta_0 p}{32\pi} - \frac{i\beta_0 \gamma_2 p^2}{\pi} - \frac{2i\beta_0 \gamma_2^2 p}{\pi} \\
& + \frac{2i\beta_0 \gamma_2^2 p^2}{\pi} + \frac{i\beta_0 \gamma_2^2}{2\pi} + \frac{i\beta_0 \gamma_2}{16\pi} + \frac{i\beta_0 p^2}{8\pi} + \frac{3i\beta_0 \gamma_4 p}{\pi} - \frac{3i\beta_0 \gamma_4}{2\pi} \\
a_{4,1} = & \frac{15i\beta_4}{8\pi} - \frac{i\beta_2}{32\pi} + \frac{i\beta_0 \gamma_4 p}{\pi} - \frac{i\beta_0 \gamma_4}{2\pi} \\
& - \frac{i\beta_0 \gamma_2 p}{24\pi} + \frac{i\beta_0 \gamma_2}{48\pi} - \frac{i\beta_0 p}{192\pi} - \frac{i\beta_0}{512\pi} \\
a_{4,2} = & \frac{3i\beta_4}{8\pi} - \frac{i\beta_2}{160\pi} - \frac{i\beta_0}{7680\pi} \quad (18)
\end{aligned}$$

and

$$\beta_{2n} = \frac{2\pi}{(2n+2)!} \left[\epsilon_1^{2n+2} - \epsilon_1^{2n+2} \epsilon^{-(n+1)} \right]$$

$$\beta_{2n+1} = \frac{\epsilon_1}{(2n+2)} \beta_{2n}$$

$$\gamma_{2n} = \frac{\epsilon_1^{2n} \epsilon^{-n}}{2n!}$$

$$p = \log \frac{\gamma ka}{4} - \frac{\pi i}{2}$$

$$\log \gamma = 0.5772157 \text{ is Euler's constant} \quad (19)$$

Total Scattering Cross Section

The total scattering cross section σ per unit length of the strip is given by $\frac{p^s}{s^i}$, where p^s is the total power scattered per unit length of the strip and s^i is the incident power flow through unit area normal to the direction of the incident wave. The integration of the real part of $\nabla \cdot \vec{E}^{s*} \times \vec{H}^i$ throughout the volume bounded by the two sides of the strip and a cylindrical surface at infinity leads to

$$p^s = \frac{1}{2} \operatorname{Re} \int_{-a}^a E_x^{i*}(x', 0) I_1(x') dx' \quad (20)$$

Also,

$$S^i = (\hat{x} + n\hat{z}) \cdot \frac{1}{2} \operatorname{Re} [\hat{x} E_x^{i*} + \hat{z} E_z^{i*}] \times \hat{y} H_y^i \quad (21)$$

The use of (5) and (7a) in (20) and (21) yields for the normalized scattering cross section

$$\frac{\sigma}{2a} = \frac{p^s}{2a s^i} = \frac{1}{2a} \operatorname{Re} \int_{-a}^a (n + i\ell \frac{\epsilon_2}{\epsilon_1}) e^{-ikfx'} I_1(x') dx' \quad (22)$$

The use of (15) - (17), in (22) gives the first few terms in the power series of $\frac{\sigma}{2a}$ to be

$$\frac{\sigma}{2a} = \frac{\pi}{4} \operatorname{Re} \sum_{n=0}^{\infty} t_{2n+1} (ka)^{2n+1} \quad (23)$$

where

$$\begin{aligned}
 t_1 &= a_{0,0} \\
 t_3 &= a_{2,0} - \frac{ila_{1,0}}{2} - \frac{l^2 a_{0,0}}{8} \\
 t_5 &= a_{4,0} - \frac{ila_{3,0}}{2} - \frac{l^2}{8} (a_{2,0} + a_{2,1}) + \frac{if^3 a_{1,0}}{24} + \frac{l^4 a_{0,0}}{192} \quad (24)
 \end{aligned}$$

The use of (18), (19) and (24) in (23) gives the first two terms in the total scattering cross section. These are

$$\begin{aligned}
 \frac{\sigma}{2a} &= -\frac{\pi(ka)^3}{4} \beta_0 \left(\gamma_2 - \frac{1}{4} \right) + \frac{\pi}{4} (ka)^5 \left[\frac{\beta_0 l^2}{8} \left(\gamma_2 - \frac{1}{4} \right) \right. \\
 &\quad + 2\beta_0 d \left(\gamma_2 - \frac{1}{4} \right)^2 - \frac{3\beta_0}{4} \left(\gamma_2^2 - \frac{\gamma_2}{4} + \frac{1}{48} \right) \\
 &\quad \left. + \frac{3\beta_2}{2} \left(\gamma_2 - \frac{1}{4} \right) \right] \quad (25)
 \end{aligned}$$

where

$$d = \log \frac{\gamma ka}{4} \quad (26)$$

With the use of β_0 and γ_2 from (19), the leading term in (25) for normal incidence becomes

$$\frac{\sigma}{2a} = \frac{\pi^2}{16} (k_0 a)^3 \left(\frac{\epsilon_1}{\epsilon_1^2 - \epsilon_2^2} \right)^{\frac{1}{2}} (\epsilon_1^2 + \epsilon_2^2) \quad (27)$$

From (27), it is obvious that for normal incidence the total scattering cross section of a narrow, infinite strip in a plasma medium is enhanced when the

plasma is rendered gyrotropic by the application of a uniform, static magnetic field parallel to the edges of the strip.

Diffraction Pattern in the Far-Zone

It is desired to find the expression for the scattered field $H_y^s(x, z)$ in the far-zone. With $x = \rho \cos \theta$, $z = \rho \sin \theta$ in (12) and with only the leading term in the asymptotic expansion of the Hankel function for large ρ , it is found that

$$H_y^s(\rho, \theta) = -\frac{k}{4} \sqrt{\frac{2}{\pi k \rho}} e^{i(k\rho - \frac{\pi}{4})} \int_{-a}^a [\sin \theta + i \frac{\epsilon_2}{\epsilon_1} \cos \theta] I_1(x') e^{-ikx' \cos \theta} dx' \quad (28)$$

The use of (15) - (17) in (28) gives

$$H_y^s(\rho, \theta) = -\frac{1}{4} \sqrt{\frac{\pi}{2k\rho}} e^{i(k\rho - \frac{\pi}{4})} \sum_0^\infty T_{2n+1} (ka)^{2n+2} \quad (29)$$

where T_{2n+1} is the same as t_{2n+1} with l replaced by $\cos \theta$. In particular, the leading term in (29) is obtained with the use of (24), (18) and (19). It is

$$H_y^s(\rho, \theta) = \sqrt{\frac{\pi}{8k\rho}} e^{i(k\rho - \frac{3\pi}{4})} (k_0 a)^2 \frac{(\epsilon_1^2 - \epsilon_2^2)}{\epsilon_1} \left[\cos^2 \theta - \frac{\epsilon_1^2}{\epsilon_1^2 - \epsilon_2^2} \right] \quad (30)$$

When the plasma is isotropic, it is easily seen from (30) that $H_y^s(\rho, \theta)$ has a null for $\theta = 0, \pi$. The result is in accordance with the general law that the

tangential component of the magnetic field, in an isotropic medium, is undisturbed even in the presence of the strip, in its plane but exterior to it. However, when the medium is rendered gyrotropic by the application of an external magnetic field parallel to the edges of the strip, $H_y^s(\rho, \theta)$ as given in (30), has no null since $\frac{\epsilon_1^2}{\epsilon_1^2 - \epsilon_2^2}$ is greater than unity.

It has been stated earlier that the results of this scattering problem are valid only for the range of frequencies for which $\frac{\epsilon}{\epsilon_1}$ is positive. With the help of (2) - (4), it is possible to show that $\frac{\epsilon}{\epsilon_1}$ is positive only for the frequency ranges given by

$$\omega_p^2 + \frac{\omega_c^2}{2} - |\omega_c| \sqrt{\omega_p^2 + \frac{\omega_c^2}{4}} < \omega^2 < \omega_p^2 + \omega_c^2 \quad (31a)$$

and

$$\omega_p^2 + \frac{\omega_c^2}{2} + |\omega_c| \sqrt{\omega_p^2 + \frac{\omega_c^2}{4}} < \omega^2 < \infty \quad (31b)$$

Hence, the results are valid only for the frequency ranges given by (31). Also, it can be shown that k becomes very large near $\omega = \sqrt{\omega_p^2 + \omega_c^2}$. Therefore, the analysis presented in this report is not valid near $\omega = \sqrt{\omega_p^2 + \omega_c^2}$.

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