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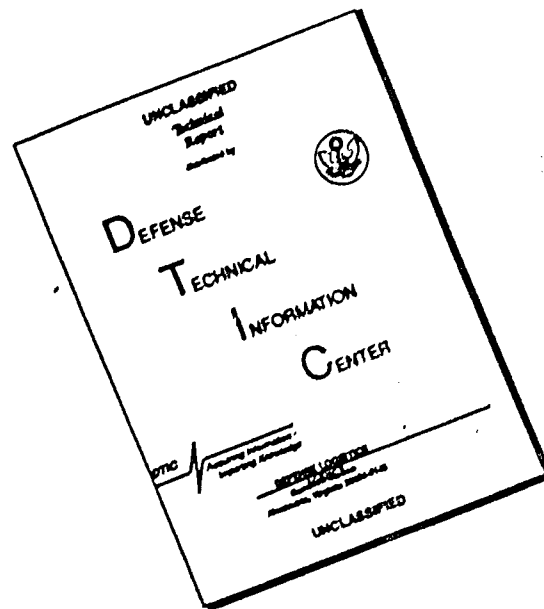
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**AIR DISPERSION OF
ROCKET EXHAUST**

TECHNICAL DOCUMENTARY REPORT NO. SSD-TDR-62-136
October 1962

6593rd Test Group (Development)
Space Systems Division
Air Force Systems Command
Edwards Air Force Base, California



Prepared under Contract AF01(611)7540
by Systems Engineering Division
DANIEL MANN, JOHNSON, & MENDENHALL
Los Angeles, California

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FOREWORD

This report was prepared by the Systems Engineering Division of Daniel, Mann, Johnson, & Mendenhall under Contract AF 04(611)-7540. This contract authorizes studies to be made of test systems for toxic propellant rockets. This volume presents the meteorological aspects of the studies as found during the contractual period covering 1 August 1961 through 31 October 1962, and is part of the final report of the contract.

Conclusions as to the specific use of the results of the "Air Dispersion of Rocket Exhaust" are presented in Report SSD-TDR-62-137, entitled "Design of a Toxic Rocket Test System".

PUBLICATION REVIEW

This technical documentary report has been reviewed and is approved.


MILBURN R. RALEIGH, Chief
Research & Development Section

ABSTRACT

An analysis of the ability of the atmosphere to dissipate toxic exhaust products from a rocket test system was performed. The analysis covered dispersion stack height, total integrated dosage, inversion layer penetration, and sampling and weather data systems. Methods for determining toxicity levels downwind of a toxic emission were developed. The conditions for safely releasing various quantities of toxic materials were defined for various types of rocket firings.

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LIST OF SYMBOLS

A_s	Internal stack area, feet squared
C	Generalized diffusion coefficient, (meters) ^{$\frac{N}{2}$}
C_1	Sutton diffusion coefficient
C_2	Bosanquet dilution coefficient, dimensionless
C_x, C_y, C_z	Diffusion coefficients in x, y, and z planes respectively
C_p	Specific heat of air at constant pressure, BTU per pound degree F
C_{ps}	Specific heat of stack gas at constant pressure, BTU per pound degree F
d	Internal exit diameter of stack top, feet
d_{max}	Distance to point of maximum concentration
$d_{max \text{ dosage}}$	Distance to point of maximum integrated dosage
e	2.7183 (base of Natural Logarithms)
g	Acceleration due to gravity, feet per second squared or centimeters per second squared.
h	$h_o + \Delta h$
h_o	Physical stack height, meters unless stated as feet
Δh	Plume rise, feet or meters
Δh_v	Plume rise due to upward momentum, meters unless stated as feet.
Δh_b	Plume rise due to buoyancy, meters unless stated as feet
H	Height to inversion or thickness of stable layer
M	Mass of propellants, pounds

M_o	Mass of flared gases, pounds
n	General stability parameter associated with stability (dimensionless)
n_y	Lateral stability parameter
n_z	Vertical stability parameter
p	Vertical diffusion coefficient
Q	Source strength (gm/sec) continuous emission or (μ gm/sec) where μ gm = 10^{-6} gm; or (gm) for instantaneous emission
Q_h	Heat emission rate of stack gas relative to ambient atmosphere, BTU per second or calories per second
Q_{v_1}	Volume emission rate of stack gas at temperature T_1 , cubic feet per second
Q_{vs}	Volume emission rate of stack gas at temperature T_s , cubic feet per second
q	Lateral diffusion coefficient
R	Radius of stack, feet
r	Radius of spherical cloud, meters
T	Absolute temperature of ambient atmosphere, degrees Rankine or degrees Kelvin (stated as used)
T_1	Absolute temperature at which density of flared gas would be equal to that of the ambient atmosphere
T_s	Absolute temperature of gas at stack top (initial)
T_s'	Absolute temperature of flared gas (initial)
ΔT	$T_s' - T$

ΔT	$T_s - T_i$
$\Delta T'$	Temperature difference between 54' and 6' elevation above ground level ($T_{54'} - T_{6'}$), (F)
TID	Total integrated dosage, $\mu\text{gm} \cdot \frac{\text{sec}}{\text{m}^3}$
TID_{max}	Total integrated dosage at point of maximum effect
t	Time, seconds
$\bar{u}$	Wind speed, feet per second or meters per second
V_s	Stack gas ejection speed, feet per second or meters per second
x, y, z	Downwind, crosswind, and vertical coordinates measured from ground point beneath a continuous source and from the center of the moving cloud in the instantaneous case, meters
$\bar{x}^2, \bar{y}^2, \bar{z}^2$	Variance of wind velocity fluctuation
θ'	Potential temperature of ambient atmosphere, or T in a neutral (adiabatic) atmosphere
$\Delta\theta'$	Difference between potential temperature of plume and ambient atmosphere
$d\theta'/dz$	Potential temperature gradient of ambient atmosphere
π	3.1416
ρ	Density of ambient atmosphere, pounds per cubic foot or grams per cubic centimeter
ρ_s	Density of stack gas
$\sigma(A)$	Mean deviation in wind direction around the mean, degrees azimuth

σ_y	Dispersion coefficient along the y-axis
σ_z	Dispersion coefficient along the z-axis
χ, C_p'	Concentration in (weight/m ³) usually (gm/m ³) or (μ gm/m ³)
$\chi_{(x, y, z)}$	Maximum concentration at ground level downwind of elevated source
ψ	Angle between plume axis and the horizontal

Other symbols will be defined as they appear.

LIST OF UNITS

ft.	feet
m	meter
cm	centimeter
fps	feet per second
mps	meters per second
mph	miles per hour
fps ²	feet per second squared
sec	second
BTU	British Thermal Unit
lb.	pound
F	degree Fahrenheit
R	degree Rankine
K	degree Kelvin
C	degree Centigrade

INTRODUCTION

Large increases in payload or range are available with missiles and spacecraft from relatively small increases in the specific impulse of the propellants. Thus, there is a continuous search for propellants and methods to increase specific impulse levels. This search for rocket fuels which contain higher energy has led researchers to many propellant combinations which are extremely toxic. Testing these highly toxic rockets at the Edwards Rocket Site presents a problem in toxicity which may involve the health and lives of people living in surrounding communities as well as personnel at the base.

Since it is imperative to our national interests to perform further propellant research, it is of the highest importance that toxic rockets are tested and developed. It follows that a means of safely testing such rockets must be evolved. The 6593rd Test Group at the Edwards AFB Rocket Site has been assigned Air Force responsibility for this mission, and the purpose of this report is to present the theory and application of meteorology to assist in the accomplishment of this goal.

The study covered in this report presents a means of determining the extent of ground level contamination caused by the rocket effluent discharged into the atmosphere. The primary purpose for this meteorological study is to provide a basis for facility design for testing toxic rocket propellants. Trade offs in cost and time may be accomplished between the two extremes of complete filtration or decontamination of the rocket effluent and the direct discharge of effluent into the atmosphere. An elaborate and extensive meteorological network and accurate weather prediction may be required to permit safe open-air firing. The trade-offs in design must at all times take into consideration the maintenance of a high degree of safety.

Section I of this report covers the theoretical and empirical derivations of the techniques for analysis of movement of clouds of toxic products in the atmosphere. The study includes an analysis of both the equations and the coefficients according to present state-of-the-art. Atmospheric dispersion phenomena, effective stack height, inversion penetration, and the total integrated dosage theories were evaluated and are presented.

Section II of this report covers the application of the theories to hardware and support system designs and to the parameters which must be known for these designs.

SUMMARY

The theories and empirical findings concerning the capacity of the atmosphere to dilute man-made air pollutants were evaluated from the standpoint of dispersing rocket exhaust. The primary problem found in this evaluation centered around the physical size of the rocket engines under consideration. The larger size (5,000 to 100,000 pounds thrust) rockets require meteorological data of a magnitude higher than micro-meteorology but a magnitude lower than found in atomic explosions.

The general considerations covered in this study were the various dispersion processes encountered in diffusion, effective stack height, inversion penetration and in total integrated dosages of toxic materials. The rate and type of diffusion are influenced by wind speed, time variation in wind direction, gustiness, vertical stability or lapse rate, and ground surface influences. Dispersion theories and formulae investigated were Sutton's, Bosanquet and Pearson's, Holland's Gaussian form, and the Ocean Breeze derivation. Continuous point source dispersion formulae of all above types are essentially similar and can be equated to each other. The reliability and the procedure for application of these formulae are presented. Any of the formulae may be used with equal reliability, depending on the particular type analysis being conducted.

In addition to the dispersion process, the effective height to which a rocket exhaust plume will rise must be known for the determination of downwind contamination. The factors influencing this plume rise are: rocket test size, propellant characteristics, rocket firing orientation, and atmospheric conditions. A number of equations are available for the determination of plume rise. Each formula has been discussed, and optimum relations have been chosen.

The equation which will be used for stack height determination depends on the type of emission. A hot rocket exhaust pointed upward will require the use of an equation for momentum rise and an equation for buoyancy rise. The directing of the exhaust horizontally will omit the need of the momentum rise equation and, therefore, reduce the overall effective stack height.

The above relations evaluated for stack height determination are for the continuous emission of an effluent. The possibility of a detonation occurring during a rocket test firing is always present and must be provided for. Therefore, the buoyant rise of explosion clouds was considered. The U.S. Atomic Energy Commission, in extensive weapons tests, has accumulated a large quantity of data on the rise of nuclear clouds as a function of weapon energy yield. This information has been compiled into graphical form for the rapid estimation of cloud rise from explosion sources.

As a result of the effective stack height studies, an empirical expression was formulated to cover the range of conditions expected in rocket test work. In addition, an equation was derived by dimensional analysis which includes all factors affecting cloud rise. The empirical expression may be utilized (with appropriate safety factors) for immediate applications, and the broader equation derived by dimensional analysis will provide a higher degree of reliability when directly applicable data become available.

Inversions, conditions of negative lapse rate in which air tends to mix at much lower rates than normal, have been studied. The indications are that the energy available from rocket tests is sufficient to penetrate inversions when the thrust level is above a minimum value.

Applications to the testing of toxic rocket engines have been examined. The thrust levels covered by the equations presented are within the range of 6,000 to 100,000 pounds thrust for solid propellant grains of 100 to 11,000 pounds weight. The formulae presented for diffusion are applicable to any combination of pollutants, whether they be gaseous, liquid vapor, or solid phases. This means that the presentation covers the normal testing of liquid or solid propellants with solid additives, test stand explosions, and spillage of liquids. In addition, the diffusion of effluents from nuclear explosions or reactors can be calculated by using the equations presented in this report.

In many cases, the cost of toxic rocket test systems may be appreciably reduced by taking advantage of meteorological phenomena. Due to the absence of directly available experimental data, methods of improving the dispersion and plume rise formulae are necessary to enable the maximum utilization of meteorology in test systems for a toxic, but otherwise desirable, propellant. This improvement can be accomplished with a suitable micrometeorological network including a sampling system and data reduction equipment.

SECTION I

ANALYTICAL CONSIDERATIONS

This section of the report covers the analytical and theoretical treatments of the action of a cloud of pollutants in the atmosphere. Extensive literature research was conducted on the meteorological aspect of toxic rocket testing. Properties of meteorology which have been covered include the general considerations affecting atmospheric dispersion. To this end, a study of the reflection of a cloud as it reaches ground level, the effects of stack heights, and the penetration of inversions was performed. Dispersion is governed by properties which include:

- A. Wind
- B. Daily and Periodic Weather Changes
- C. Gaseous Diffusion

The concentration of a pollutant downwind of its source is dependent upon these influences, as well as the interactions between them.

Many different formulae for the calculation of effective stack height have been covered, and definite choices have been made as to the optimum relations. Inversion penetration is basically a part of the stack height analysis, but the atmospheric conditions are sufficiently different to cause it to be a distinct problem in itself.

The variability of parameters used in the diffusion equations is presented in addition to the above analyses. The direction in which the toxic concentration varies is a complex function of many variables, all of which should be known to a high degree of accuracy to attain the optimum design of an actual facility as formulated upon meteorological considerations.

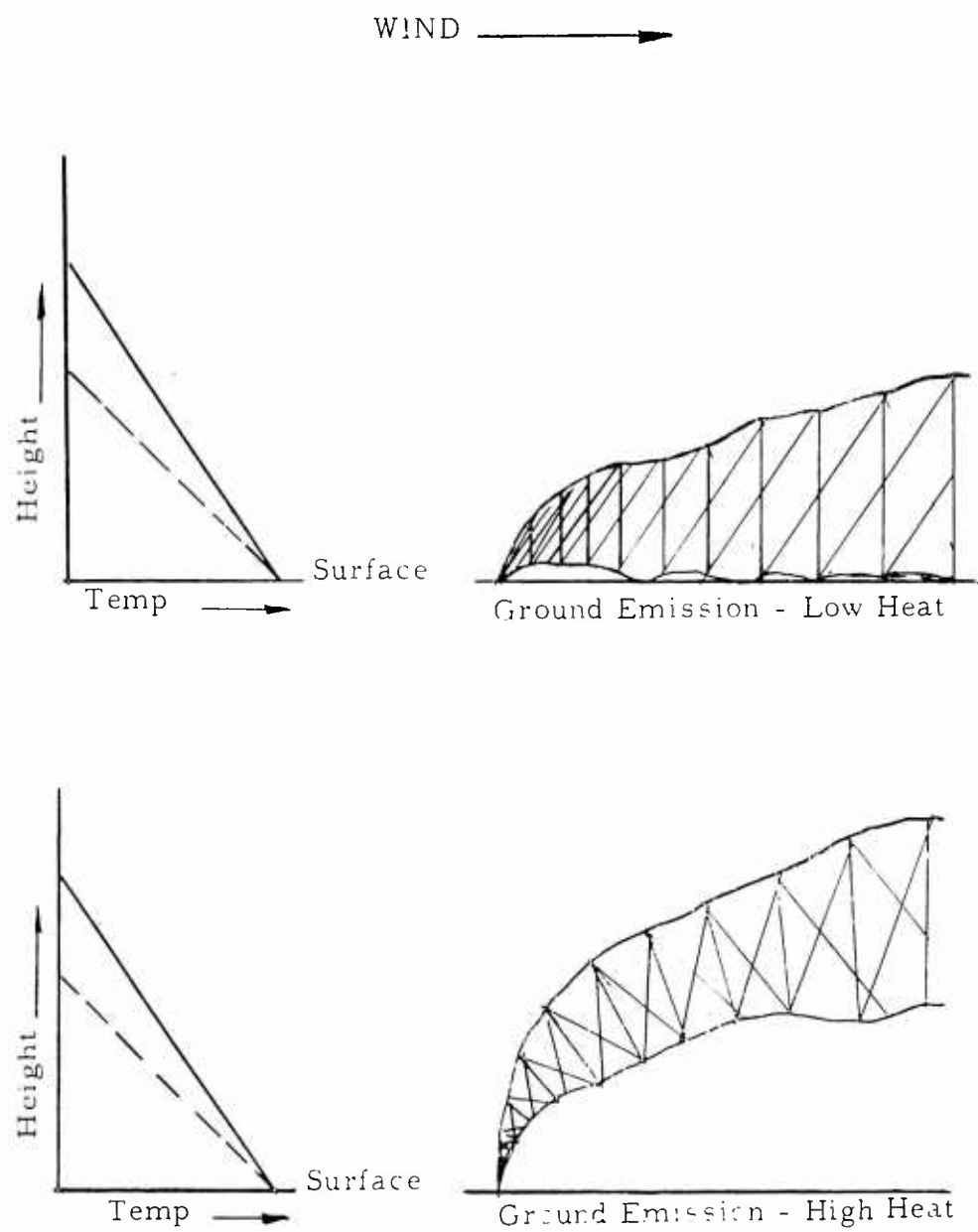
GENERAL METEOROLOGICAL CONSIDERATIONS

The concentration of a pollutant downwind of its release into the atmosphere is dependent upon many variables. The pollutant is transported with the air mass and is acted upon by the dynamic forces of the air. Instability of large air masses is caused by differential surface heating, rotation of the earth, surface friction, mechanical mixing of moving air masses by surface irregularity, temperature variation with altitude, and a host of other influences as well as interactions between those various influences.

The surface air behavior can only be determined by measurement of its performance over a period of time. By application of the data, together with an understanding of fluid dynamics as it is applied to meteorology and micrometeorological phenomena, reasonable predictions of atmospheric behavior can be made. With this background of accumulated data, weather knowledge, and current weather information, much can be predicted about the behavior of pollutants which might be released into the atmosphere.

Micrometeorology is normally concerned with the layer of air within the first hundred meters above the earth's surface and for a horizontal distance of a thousand or so meters. When a pollutant is very toxic, the areas of interest become greater and may extend to the stratosphere and cover the hemisphere as is the case with atomic explosions. When applied to rocket engine test and missile launch, micrometeorology may involve large areas or may be very limited depending upon the toxicity of propellants or exhaust products and the quantities concerned. As rocket technology is advanced, the use of toxic propellants is increased since higher reactive fuels and oxidizers tend to be toxic. Even so-called nontoxic fluids handled in large quantities can smother human life by displacement of air and the oxygen needed for support of life. Other propellants can be lethal at levels as low as one part per million parts of air or even one part per billion may be in excess of tolerable levels in inhabited areas. Meteorological information for extended areas must be known where very toxic substances are released into the atmosphere, whether by accident or of necessity.

Information about the potential source of atmospheric contamination includes toxicity levels, nature of the toxic reaction, quantities involved, relative density between pollutant and air, and particle size of pollutants where solids are involved. The toxicity level of the pollutant determines the area of involvement and extent of meteorological details required to predict safe atmospheric dispersion.



NOTE: - - - - Dry adiabatic lapse rate.

Figure 1. Various Plume Types

Atmospheric dispersion processes include differential movement between parcels of air which results in mixing of adjoining parcels. This causes dispersion and dilution of suspended vapors and solids contained in the air parcels. The mixing is primarily microscopic in that intermingling of air masses results in mixing of a portion of one air mass with another where individual portions slowly lose their identity. Air masses of different density retain their identity with the more dense masses remaining in contact with the surface of the earth. Resistance to intermingling expresses itself in the formation of weather fronts. A cold-front is formed when a cold air mass "under runs" a warm air mass forcing the warm air upward and creating very turbulent conditions. Rainfall formed from such a front is of an intermittent "downpour" type because of the rapid rise of warm air by the relatively steep cold air wedge. Warm-fronts are produced when warm air blows over the top of the trailing edge of a cold air mass. The trailing edge is of small slope being stretched out by friction between the cold air mass and the ground surface. Rainfall produced along a warm-front is continuous but normally light. Smaller parcels of air mixed together tend to retain their identity for a period of time but always eventually lose their identity.

Diffusion processes are influenced by wind speed, variation in wind direction and velocity (gustiness), and vertical stability as measured by the vertical temperature profile (lapse rate), and ground surface influences. Measurable wind speeds normally result in turbulent flow where there is a tumbling action rather than streamlined flow. Gustiness results in mixing of air at a rate significantly greater than that produced by molecular motion and resulting Brownian movement. Because of Brownian motion, diffusion of gases, vapors, and suspended solids will always be greater than zero. Diffusion by Brownian motion results from random motion of the molecules. The net transport of a property in a given direction is proportional to the gradient in that direction. Eddy currents in the air result in an apparent random motion which tends to distribute a property uniformly by reducing its gradient. The molecular diffusion equation is extended by analogy to turbulent diffusion, but the rate of diffusion must be determined from field observations. Exponential or Gaussian distribution of a property is assumed for turbulent diffusion and is fairly well confirmed by field tests where known quantities of tracer substances are released in the atmosphere and samples collected using relatively long sample periods.

The temperature lapse rate is defined as the decrease of temperature with increase in height. When the lapse rate is a negative value, where temperature increases with height, the condition is called an inversion. The atmospheric lapse rate is an indication of atmospheric stability which affects vertical diffusion. The higher the lapse rate, the greater the vertical diffusion. A negative lapse rate (inversion conditions) results in very stable air layers and reduced vertical dispersion. The precise

relationship between dispersion rates and atmospheric lapse rate is not established, but an approximate relationship is incorporated in dispersion equations. One difficulty is the change of lapse rate with altitude as well as with time, humidity conditions, surface heating, wind speeds, and mechanical mixing by surface features. Inversion layers are often formed near the earth's surface at night when loss of heat by radiation to the ground results in a transfer of heat from the lower air layer to the ground and the temperature of the lower air layer drops below that of the air above. A cloud cover reradiates heat back to the ground surface, reduces the loss of heat by the surface, and minimizes the formation of an inversion at the surface. However, the cloud cover may be caused by a weather front producing a high level inversion.

A parcel of air displaced upward will expand and cool adiabatically assuming no transfer of heat between the parcel and the surrounding air. The dry adiabatic lapse rate is the drop in temperature with change in height of dry air when displaced upward. The dry adiabatic lapse rate is 5.5 F per 1000 feet. A dry parcel of air displaced vertically in atmospheric air with an adiabatic lapse rate will have neutral buoyancy since its temperature, pressure, and, therefore, density will change at the same rate as the surrounding air.

Stability of the atmosphere affects dispersion of a pollutant not only by reducing vertical dispersion, but during stable conditions, diffusion transverse to the plume axis is usually reduced. Where the pollutant is buoyant due to low molecular weight or reduced density because of high temperature, the effluent will rise above the actual stack and produce an effective stack height greater than the physical stack. When the atmosphere is very stable due to isothermal conditions or a negative lapse, the plume buoyancy rapidly decreases with increased height since the atmospheric density decreases faster than the plume which cools adiabatically.

For treatment of dispersion problems, atmospheric stability is divided into several types from very unstable (superadiabatic lapse rate) to very stable (strong inversion). Unstable atmospheric conditions can cause looping of an exhaust plume. Looping occurs with a high degree of turbulence, especially with convective turbulence, and is typical of a daytime condition with intense solar heating of the earth's surface. Coning plumes occur under more nearly neutral thermal conditions when mechanical mixing of small scale predominates. Coning is more likely to occur when a cloud cover reduces thermal effects either by reducing incoming solar radiation at daytime or outgoing terrestrial radiation at night. The half-angle of the coning plume is on the order of 10° , from center line to edge. Fanning plumes occur under stable conditions when mechanical turbulence is suppressed. The vertical component is suppressed more than the horizontal and the plume is wider horizontally than vertically. Fanning is most likely to occur at night when the earth's surface is cooled by outgoing

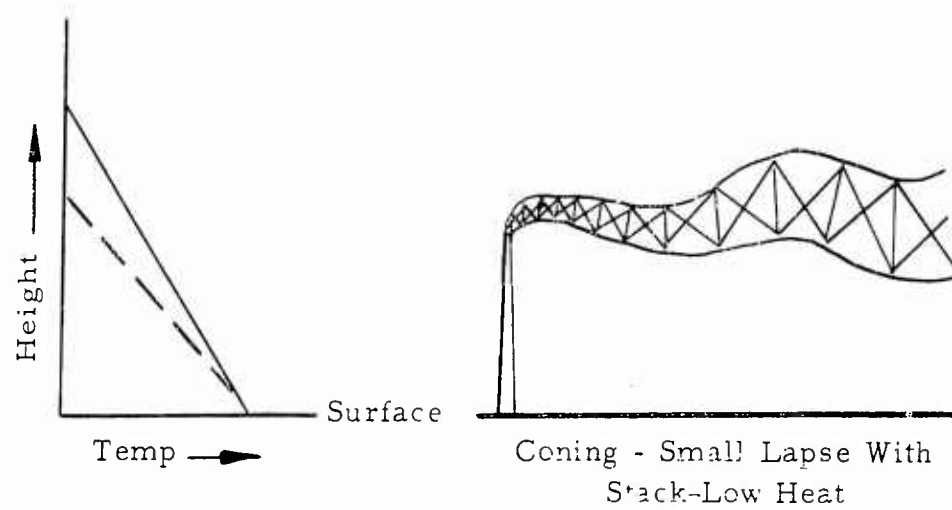
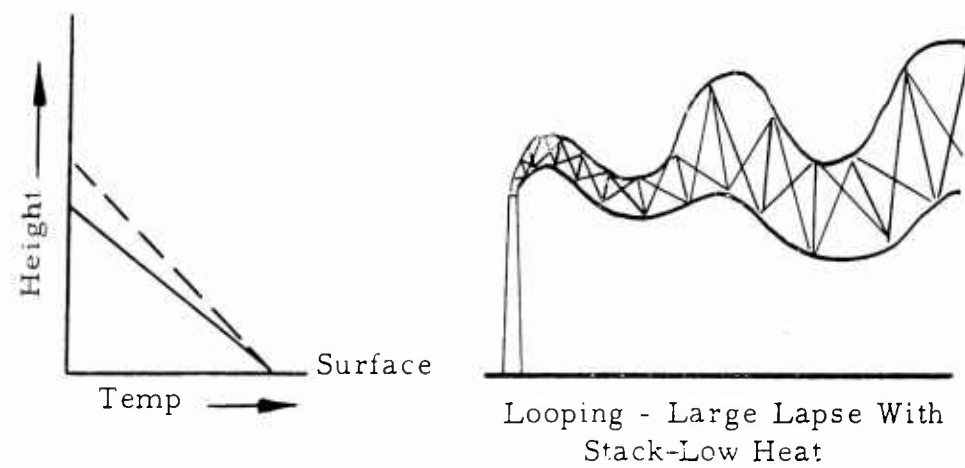


Figure 2. Various Plume Types

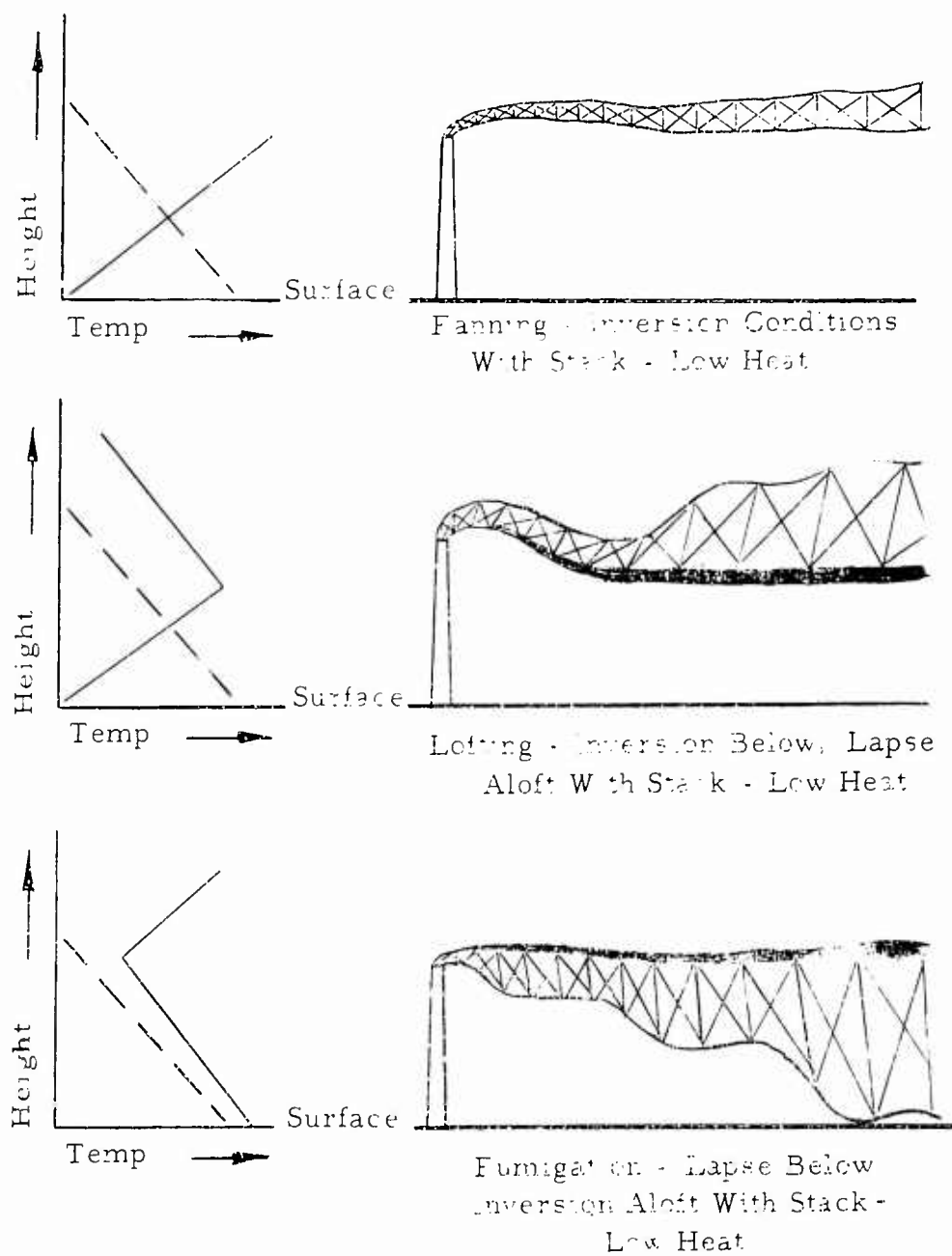


Figure 3. Various Plume Types

radiation. Fumigation is caused by unstable air at and below the plume with stable air above. Dispersion of the plume downwind brings the exhaust to ground level in a relatively short distance before dispersion can become effective. Lofting is caused by stable air below the plume and turbulence above. The turbulence above results in effective dispersion yielding very low ground surface concentrations. Lofting will occur when the plume is released above or penetrates an inversion. On the other hand, fumigation will result if the inversion is not penetrated and ground turbulence is present due to thermal effects from ground surface warm-up and inversion break-up. The described plumes are samples of conditions which may be found during actual operation of a facility, but the capacity to test must be determined at the given test site and at the time of test.

Stable conditions will keep a plume compact and prevent contact with the ground for a considerable downwind distance if its density is not greater than that of the surrounding atmosphere. Contact of the plume with the ground and the point of maximum concentration occur closer to the point of release with increasing atmospheric instability. For extreme instability at low wind speeds, portions of a looping plume may reach the ground within a distance of about one stack height. High wind speeds increase mechanical mixing caused by ground features and reduce thermal effects. Plume features which arise from thermal effects are, consequently, more likely to occur at low wind speeds.

The following discussion on diffusion, dispersion, effective stack heights, and inversion layer penetration includes the mathematical techniques utilized in explaining these phenomena.

ATMOSPHERIC DIFFUSION FORMULAE

Diffusion is the motion of fumes or dust particles in the atmosphere. The principles of diffusion or dispersion are well established. Many theoretical analyses on this phenomena have been prepared. In addition, a vast amount of experimental data has been accumulated.

Diffusion is sometimes limited to fluids in suspension and dispersion to the activity of particulate matter in suspension. Diffusion due to molecular activity is small compared with mixing and other effects due to atmospheric turbulence. A smoke plume released in the atmosphere will spread outward by the energy within the plume and the activity of the atmosphere. The concentration of smoke will be greatest near the center, decrease toward the edges, and diminish along the axis of the plume with time or with distance from the source. The degree of stability, wind velocity, and gustiness affect the rate of dispersion of the plume in the air. The buoyancy of the plume also affects the rate of dispersion, but the most important effect of buoyancy is that of raising the plume above the lower air layers. By the time the edge of the plume touches the ground, diffusion will have been effective in considerably reducing the concentration of the cloud.

This discussion is intended to present the formulae required for use with toxic rocket testing. Emphasis will be placed on problems involved with rockets using toxic propellants and those yielding toxic exhaust products in order to provide maximum usefulness for rocket test system design and operation. Spills tests have been made on liquid propellants in which toxic liquids have been allowed to evaporate with and without combustion either by air or in conjunction with other oxidizers (for example, UDMH+NTO). For liquid propellant spills accompanied by detonation, the source strength (Q) will be instantaneous and, therefore, the downwind toxic concentration (X) can be easily found. This also holds true of solid propellant detonations. When a liquid spill simply evaporates or burns for a period of time, variables of spill volume, surface area exposed to the atmosphere, liquid temperature and vapor pressure or flame temperature and heat of combustion, and wind velocity must be known to determine the continuous source strength.

Atmospheric diffusion has been expressed as a differential equation which takes into account the rate of diffusion in the three dimensions of space. The major problem is the determination of dispersion rates in the air. Fick's law of molecular diffusion (Reference 1) may be stated as follows: "Diffusion of material is in the direction of decreasing concentration and is proportional to the concentration gradient." Atmospheric diffusion is primarily caused by turbulence or gustiness of the atmosphere and is influenced by vertical stability as indicated by the lapse rate. The influence of atmospheric turbulence in changing the properties of a smoke cloud or plume can be compared to that of molecular activity where the time interval is sufficient to average out random variations. By fitting atmospheric

observations to basic differential equations the resulting diffusion values will vary from about $0.2 \text{ cm}^2/\text{sec.}$ for molecular diffusion to $10^{10} \text{ cm}^2/\text{sec.}$ for diffusion due to large-scale cyclonic storms in the atmosphere. The pronounced influence of conditions of the atmosphere on diffusion has resulted in the development by Sutton and Frenkiel (Reference 2) of the statistical theories of turbulent diffusion, following Taylor's statistical theory of turbulence (Reference 1). Sutton's statistical theory seems to apply to intermediate range atmospheric dispersion problems where hundreds of meters to kilometers are involved.

A pronounced shear effect on the mean wind is exhibited for layers of air near the ground. This is due to the wind's increasing from zero at ground level to relatively high velocities at several thousand feet in elevation. The shearing action affects dispersion by elongating the upper portion of a smoke cloud or plume. The ground and structural features of the ground influence dispersion by mechanical mixing of the air. Diffusion equations have been derived based on this shearing action but are limited in application to the lower air levels. Statistical determination of dispersion parameters appears to be more reliable than the theoretical expressions and is easily determined at or near the earth's surface. There is confirmation that concentrations in a diffusing cloud are distributed according to a three-dimensional Gaussian law (Reference 1). Frenkiel advanced the following equation for the concentration distribution from an instantaneous point source in an infinite region

$$x(x, y, z, t) = \frac{Q}{(2\pi \bar{y}^2)^{3/2}} e^{-\frac{r^2}{2\bar{y}^2}}$$

The coordinate axes are imagined to be affixed to the center of the cloud and are translated with the mean wind value. For nonisotropic diffusion the expression may be written as follows

$$x = \frac{Q}{(2\pi)^{3/2} (\bar{x}^2 \bar{y}^2 \bar{z}^2)^{1/2}} e^{-\frac{1}{2} \left(\frac{x^2}{\bar{x}^2} + \frac{y^2}{\bar{y}^2} + \frac{z^2}{\bar{z}^2} \right)}$$

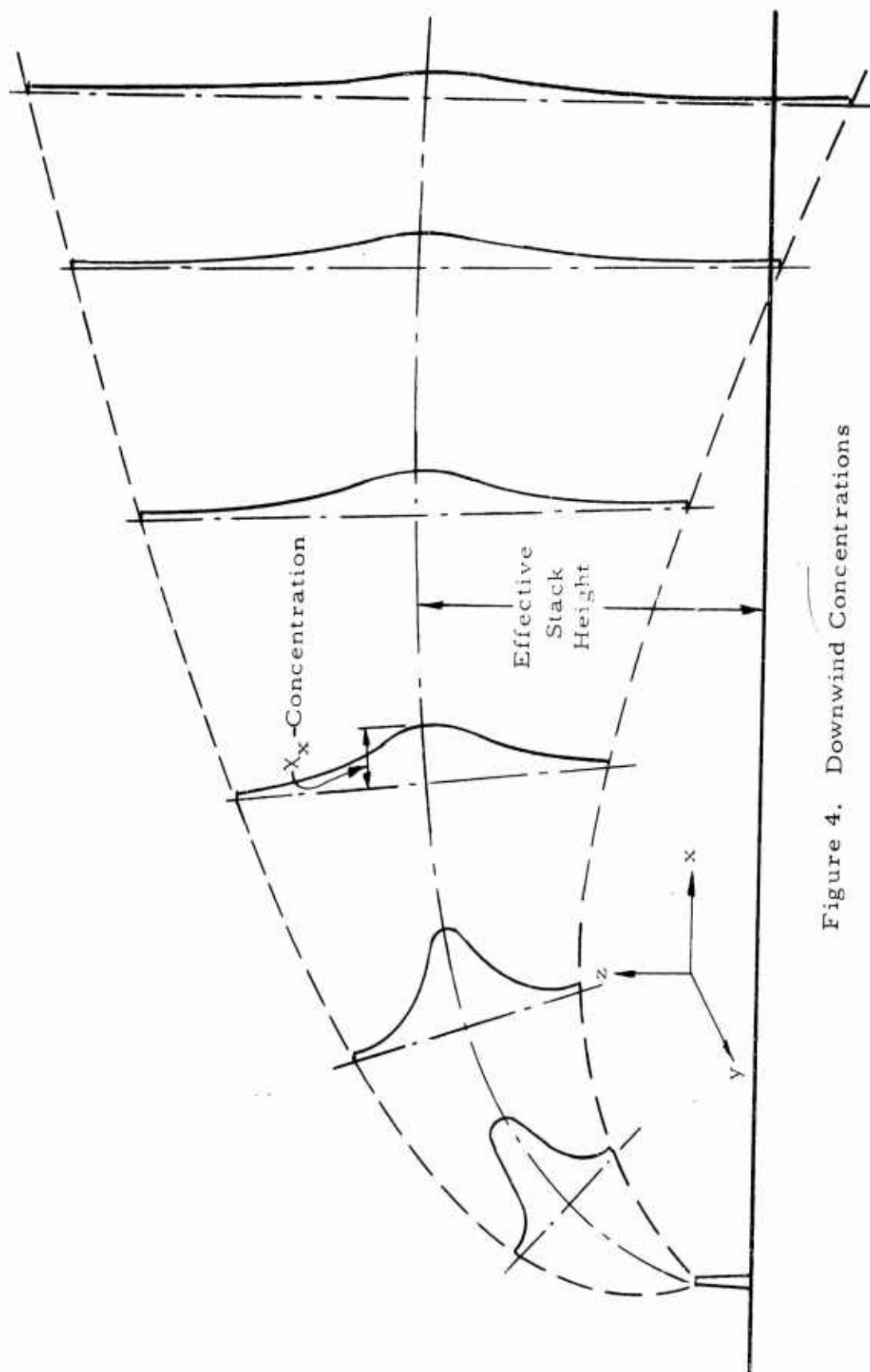


Figure 4. Downwind Concentrations

Sutton's instantaneous point source equation for the nonisotropic case is:

$$\chi(x, y, z, t) = \frac{Q}{\pi^{3/2} C_x C_y C_z (\bar{u} t)^3 (\epsilon - n)^{1/2}} \cdot e^{-\frac{(\bar{u} t)^{n-2}}{C_x^2} \left(\frac{x^2}{C_x^2} + \frac{y^2}{C_y^2} + \frac{z^2}{C_z^2} \right)}$$

Sutton considered that the ground could act as a perfect reflector of diffusing particles and accounted for it by using the "method of images." By doubling the right hand side of the expressions, this reflection influence can be accounted for when determining ground level concentrations. This reflection effect can be expected for gases that do not react chemically with the ground or vegetation and small particulate matter. The effect of thermal stability on atmospheric dispersion in Sutton's equation is represented by a stability factor "n" which ranges from almost zero for extremely turbulent conditions to a value of 1 as a limit for extremely stable conditions. Sutton's formula has been successfully utilized to predict diffusion over distances up to several kilometers under certain meteorological conditions, and this formula and modifications thereof have been fairly widely accepted.

Bosanquet and Pearson (Reference 3) derived the following formula for ground level concentration distribution from a continuous elevated point source near the ground:

$$\chi = \frac{Q}{(2\pi)^{1/2} p q \bar{u} x^2} \cdot e^{-\left(\frac{h}{p x} + \frac{y^2}{2 q^2 x^2} \right)}$$

Where p and q are vertical and lateral diffusion coefficients. The equation is very similar to Sutton's continuous point source formula where extremely turbulent conditions exist (n = 0).

Various point source formulae may be integrated with respect to time to give equations for concentration distribution downwind from a continuous point source. Sutton's equation for continuous elevated point source yields the following expression when integrated:

$$\chi(x, y) = \frac{2 Q}{\pi C_y C_z \bar{u} x^{2-n}} \cdot e^{-x^{n-2} \left(\frac{y^2}{C_y^2} + \frac{z^2}{C_z^2} \right)}$$

Or for isotropic diffusion:

$$\chi_{(x,y)} = \frac{2Q}{\pi C^2 \bar{u}} \frac{1}{x^{2-n}} \cdot e^{\left(-\frac{y^2 + h^2}{C^2 x^{2-n}}\right)}$$

Continuous line source formulae can be derived from point source formulae. Sutton's continuous infinite elevated crosswind line source for nonisotropic diffusion is as follows:

$$\chi_{(x)} = \frac{2Q}{\pi^{1/2} C_z \bar{u}} \frac{1}{x^{(2-n)/2}} \cdot e^{\left(-\frac{h^2}{C_z^2 x^{2-n}}\right)}$$

This formula may be applied to a row of stacks or facility roof-level emission. For rocket testing, where intermittent firings and a limited number of test cells are involved, the crosswind line source would seldom apply.

All of the above equations have been developed based on gaseous diffusion. If a smoke plume has any solid or liquid particles within it the values calculated for elevated sources will be in error by a function of the settling rate of the particles. The settling rate of particles varies with the particle diameter and density and may be determined by use of Stoke's law, by other suitable formulae, or by observation. In general, the simplest solution is to define an average particle size and determine the settling velocity by Stoke's law. This particle settling will effectively tilt the axis of the plume (from a continuous point source) downward so that the ground will be intersected at an angle (θ). An expression for this angle can be written as a function of settling rate (v) and wind velocity ($\bar{u}$).

$$\tan \theta = \frac{v}{\bar{u}}$$

This angle will generally be rather small. The elevation of the plume (z') may then be approximated at any distance (x) downwind from the maximum effective stack height.

$$z' = h - \frac{v x}{\bar{u}}$$

The continuous elevated point source formula, by the substitution of z' for h in the power of e , becomes:

$$\chi = \frac{Q}{\pi C^2 \bar{u} x^2} \cdot e^{-\frac{\left(h - \frac{v x}{\bar{u}}\right)^2}{C^2 x^2 - n}}$$

The factor of 2 has been dropped since complete deposition is now assumed, rather than partial reflection.

Holland derived the following formula for an instantaneous volume source using a Gaussian concentration distribution, in terms of Sutton's theory.

$$\chi = \frac{2 Q}{\pi^{3/2} C_0^3 (x_0 + \bar{u} t)^3 (2 - n)^{1/2}} \cdot e^{-\frac{h^2}{C^2 (x_0 + \bar{u} t)^2 - n}}$$

The term x_0 is the distance upwind from the stack to the virtual source. For distances of many miles it may be neglected in computing χ .

The Holland derivation for an instantaneous volume source can also be modified for particle diffusion from an elevated source. By analogy with the continuous elevated point source formula it will become:

$$\chi = \frac{Q}{\pi^{3/2} C^3 (x_c + \bar{u} t)^3 (2 - n)^{1/2}} \cdot e^{-\frac{\left(h - \frac{v x}{\bar{u}}\right)^2}{C^2 (x_0 + \bar{u} t)^2 - n}}$$

The factor of 2 has been dropped since complete deposition is now assumed, rather than any reflection.

In the present use of these equations for predicting the fallout from rocket testing, the particle size is in the range of less than one micron up to about 20 microns in diameter. Generally the particles will be about 1 micron for normal tests and less than twenty for malfunctions. If the assumption is made that their diameter is 10 microns, the settling velocity in air will be about 0.02 feet per second at a particle density of 2.0. This will mean that the particles will drop about seven feet per mile traveled downwind. Any error introduced into a calculation of χ by neglecting the settling velocity of particles less than 10 microns will be much less than the probable error of the calculation induced by other variables. Therefore, in the calculation of toxic concentrations where

the particles are sufficiently small the gaseous diffusion equations may be used. This can be shown by the following example.

If a detonation occurs in which 1650 # of toxic material is released up to an effective altitude of 12,400 ft. and a calculation is made of χ versus distance, for the assumptions of a gaseous release and also for particles of ten microns diameter the resulting curves are found (Figure 5). The maximum difference between the two curves is about forty percent, and this error will decrease on either side of the maxima for χ . If the particles are less than 10 μ the particle curve will approach the gas curve. The significance of this is that the toxic concentration can be calculated based on the diffusion of gases instead of the sedimentation of solids. For small particle sizes, the error introduced is small using this method.

Maximum ground concentration resulting from various sources of toxic gas or particulates in the atmosphere is usually of primary concern. By differentiating the above expressions and solving for maximum concentration at ground level, the following expressions result:

Instantaneous point source equation from Sutton's equation for the nonisotropic case:

$$d_{\max} = \left(\frac{2 h^2}{3 C^2} \right)^{1/(2-n)}$$

$$\chi_{\max} = \frac{2 Q}{\left(\frac{2}{3} e \pi \right)^{3/2} h^3}$$

Continuous point source equation from Sutton's equation for an elevated point source:

$$d_{\max} = \left(\frac{h^2}{C^2} \right)^{1/(2-n)}$$

$$\chi_{\max} = \frac{2 Q}{e \pi \bar{u} h^2} \cdot \frac{C_z}{C_y}$$

For isotropic conditions: $\frac{C_z}{C_y} = 1$

Continuous, infinite crosswind line source equation from Sutton's equation for the nonisotropic case:

$$d_{\max} = \left(\frac{2 h^2}{C^2} \right)^{1/(2-n)}$$

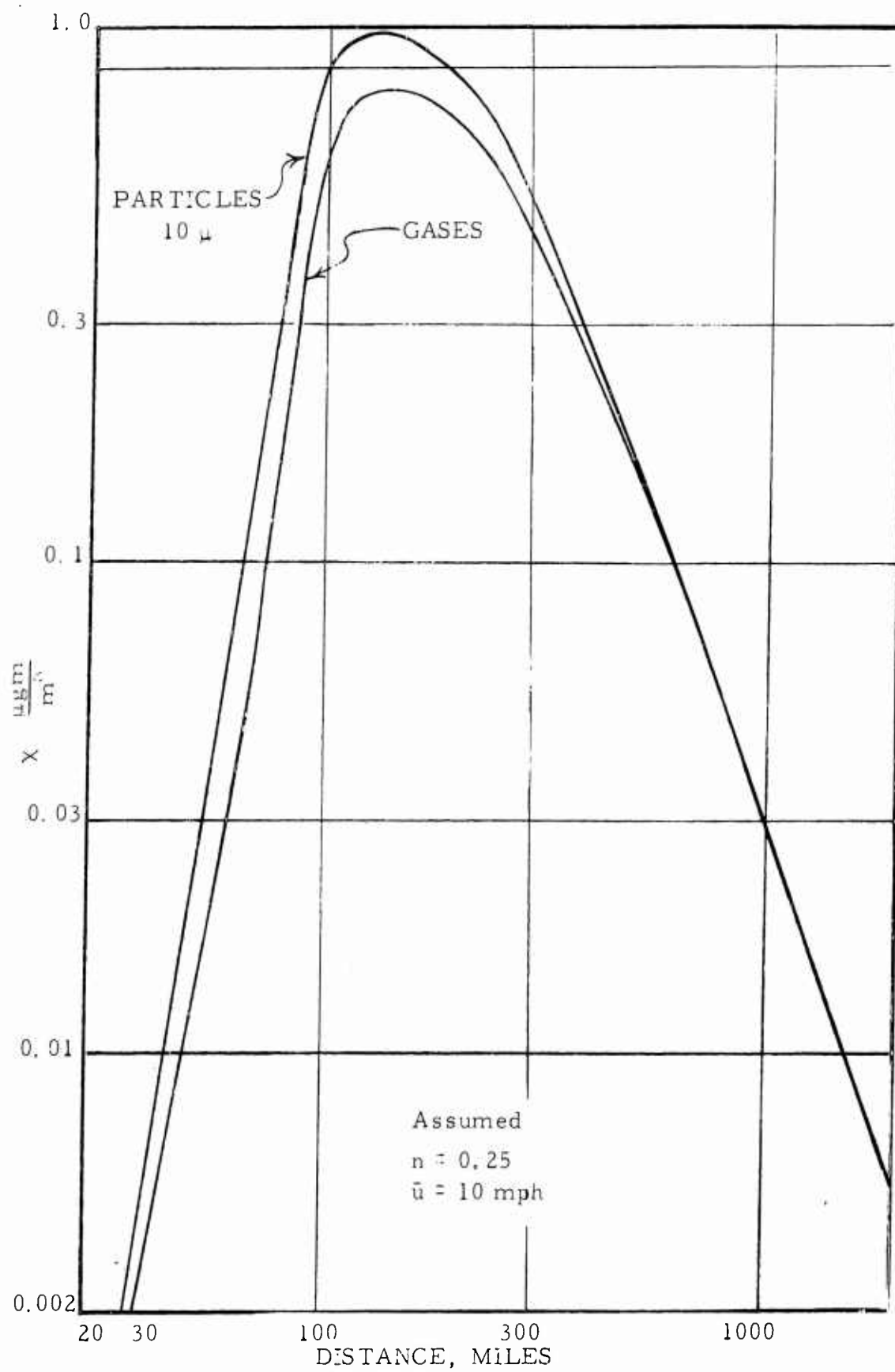


Figure 5. Particulate and Gaseous Concentrations at Various Distances

$$\chi_{\max} = \frac{2 Q}{(2 e \pi)^{1/2} h \bar{u}}$$

These equations may be utilized for calculating the distance from the source to the point of maximum concentration ($d_{\max}$) and for the calculation of maximum concentration ($\chi_{\max}$).

If the Gaussian concentration distribution is integrated with respect to time, the resulting expression (Reference 4) describes the total integrated dosage (TID):

$$\text{TID} = \frac{2 Q}{\pi C^2 \bar{u} (\bar{u} t)^{2-n}} \cdot e^{\left(\frac{-h^2}{C^2 (\bar{u} t)^{2-n}} \right)}$$

$$\text{TID}_{\max} = \frac{2 Q}{\pi e \bar{u} h^2}$$

$$d_{\max \text{ dosage}} = \left(\frac{h^2}{C^2} \right)^{1/(2-n)}$$

The maximum total integrated dosage ($\text{TID}_{\max}$) and the distance from the source to the point of maximum dosage ($d_{\max \text{ dosage}}$) as described by these equations are useful tools for determining toxicity problems downwind of a test facility.

So-called fumigation conditions may develop when the accumulation of material in very stable air layers is brought to ground levels by surface turbulence during the initial stages of stable layer break-up or by strong turbulence below relatively stable layers. When effluent is limited in vertical dispersion by a stable atmosphere or inversion condition, the center of the plume which contains the highest concentration may be forced down near ground level. The following formula is used to express the fumigation concentration when the material is confined within a height, H .

$$\chi_{(\text{downwind})} = \frac{Q}{\pi^{1/2} C_y \bar{u} H x^{(2-n)/2}}$$

The equations presented provide fairly accurate results provided the proper diffusion coefficient can be determined for the particular situation. Additional experimentation and observation is needed to define the conditions possible for an infinite number of surface features and climatic

conditions. Precise predictions can probably never be attained, but a confidence level of predictability may be built up through years of experimentation. Dispersion coefficients are discussed in a separate section of this report along with a discussion of the reliability of the coefficients and their effect on the over-all reliability of predicted concentrations.

Continuous point source dispersion formulae of the Gaussian distribution type and the Sutton form are essentially equivalent. Sutton's dispersion coefficient can be expressed as a simple function of distribution probabilities. Recent work accomplished for study of toxic propellant vapor diffusion under the name of "Ocean Breeze" (Reference 5) has resulted in statistical determination of a purely empirical formula for continuous ground source diffusion with essentially zero buoyancy. The "Ocean Breeze" composite formula is expressed as follows:

$$\log \frac{C_p'}{Q} = 0.1309 - 1.855 \log x + 2.273 \log (\Delta T' + 5) - 0.7261 \log \sigma(A)$$

Or:

$$C_p' = \frac{1.352 Q (\Delta T' + 5)^{2.273}}{x^{1.855} \sigma(A)^{0.7261}}$$

Where:

C_p' = Maximum concentration along plume axis, gm/m³ at distance x, meters

Q = Source strength, gm/sec

$\sigma(A)$ = Standard deviation of wind direction, degrees

Sutton's equation for continuous point source axial concentration with reflection from ground surface is:

$$\chi = \frac{2 Q}{\pi C_y C_z \bar{u} x^{2-n}}$$

C_p' and χ have the same meaning and can be expressed in the same terms.

Therefore, the Sutton and "Ocean Breeze" equations can be equated as follows:

$$\frac{2 Q}{\pi C_y C_z \bar{u} x^{2-n}} = \frac{1.352 (\Delta T' + 5)^{2.273} Q}{x^{1.855} \sigma(A)^{0.7261}}$$

The value of n in Sutton's equation can vary between 0 and 1 and normally falls within a range of 0.10 to 0.50. Where $n = 0.145$, $x^{2-n} = x^{1.855}$. The values $4 \sigma(A)^{0.7261}$ and C_y are representative of horizontal diffusion perpendicular to the plume axis in the "Ocean Breeze" and Sutton equation respectively.

Then the above equation reduces to expressions of vertical dispersion.

$$\frac{1}{2 \pi C_z \bar{u}} = 1.352 (\Delta T' + 5)^{2.273}$$

Or:

$$C_z = \frac{1}{2 \pi \bar{u} (1.352) (\Delta T' + 5)^{2.273}}$$

To convert from the "Ocean Breeze" diffusion formula to Sutton's diffusion formula, continuous point source, the following values may be equated:

$$X^{2-n} = X^{1.855}$$

$$C_y = 4 \sigma(A)^{0.7261}$$

$$C_z = \frac{1}{2.704 \pi \bar{u} (\Delta T' + 5)^{2.273}}$$

This analysis should help to illustrate the compatibility of the two equations and suggests a way to apply diffusion parameters and variations of one expression with respect to the other. By analogy, the "Ocean Breeze" equation can be modified for continuous elevated point source applications. The following expression results:

$$C_{p'} = \frac{1.352 Q (\Delta T' + 5)^{2.273}}{x^{1.855} \sigma(A)^{0.7261}} e^{\left(-\frac{y^2 + h^2}{C^2 x^{2-n}} \right)}$$

But:

$$C^2 = C_y \cdot C_z = \frac{2 \sigma(A)^{0.7261}}{1.352 \pi \bar{u} (\Delta T' + 5)^{2.273}}$$

And:

$$x^{2-n} = x^{1.855}$$

Therefore, the "Ocean Breeze" formula with stack height effect added becomes:

$$\frac{C_p}{Q} = \frac{1.352 (\Delta T' + 5)^{2.273}}{1.855 \sigma(A)^{0.7261}} \left(\frac{(y^2 + h^2) 1.352 \pi \bar{u} (\Delta T' + 5)^{2.273}}{2 \sigma(A)^{0.7261}} \right)^{-1/1.855}$$

For maximum ground level concentration for instantaneous elevated point source using terms available under the "Ocean Breeze" experiments:

$$X_{\max} = C_p(\max) = \frac{2 Q}{\left(\frac{2}{3} e \pi \right)^{1/2} h^3}$$

and

$$d_{\max} = \frac{2 h^2}{3 C^2} \left(\frac{1.416 h^2 \bar{u} (\Delta T' + 5)^{2.273}}{\sigma(A)^{0.7261}} \right)^{1/1.855}$$

For total integrated dosage at the point of maximum concentration downwind of an instantaneous elevated point source, using terms available under the "Ocean Breeze" experiments.

$$TID_{\max} = \frac{2 Q}{\pi e \bar{u} h^2}$$

$$d_{\max \text{ dosage}} = \left(\frac{h^2}{C^2} \right)^{1/2} \left(\frac{1.352 \pi \bar{u} h^2 (\Delta T' + 5)^{2.273}}{2 \sigma(A)^{0.7261}} \right)^{1/1.855}$$

These formulae should provide a suitable method for predicting downwind maximum concentrations and distances for relatively low stack height, where the diffusion constants are not greatly different from those determined by instruments on network meteorological towers. The maximum point of total integrated dosage and the distance to this point are expressed in standard diffusion and "Ocean Breeze" parameters.

The foregoing analytical treatments of diffusion theory are equivalent to the extent of availability of experimental data. The chart on the following page (Figure 6) illustrates the differences in results utilizing two of the equations. As more appropriate experimental data become available, the curves will more nearly coincide. These data are for a continuous emission source.

Due to the extremely short duration of rocket tests, continuous emission equations are not reliable. Rocket test durations are more nearly

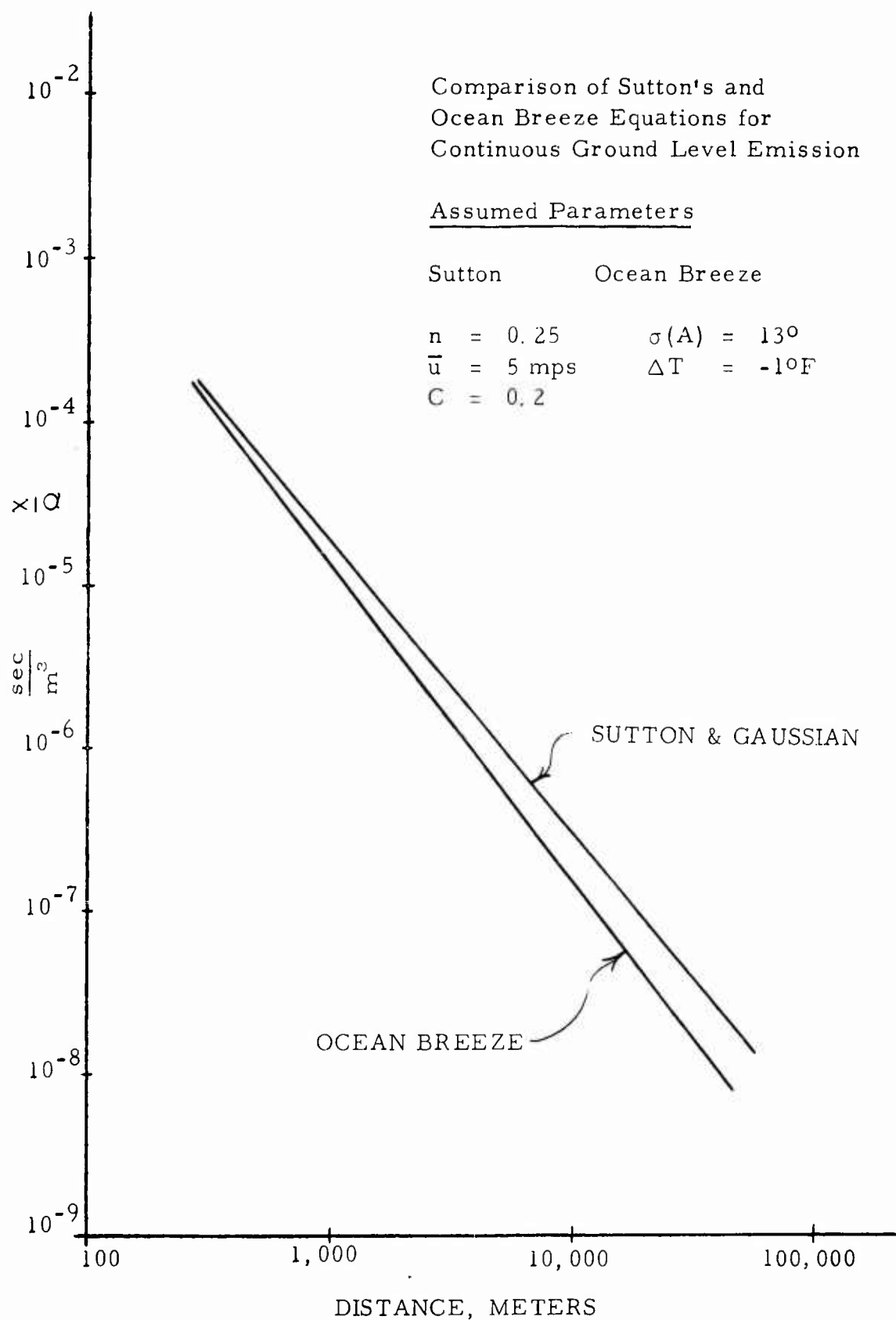


Figure 6. Atmospheric Diffusion

comparable to instantaneous point source emissions. The chart on the following page illustrates this difference (Figure 7).

Also, on the same chart, a curve of concentration at various downwind distances of an assumed source with a stack height of 100 meters is shown. It may be seen that stacks will prove highly beneficial in the analysis of meteorology problems.

1. 10 mph Wind
2. $C_x = C_y = C = 0.22$
3. $n = 0.25$
4. $Q = 0.5 \text{ #toxic/sec for 10 sec (Rocket Engine)}$

The continuous source curve assumes 0.5#/sec continuously, while the instantaneous source curves assume a source strength of 5 pounds released instantaneously.

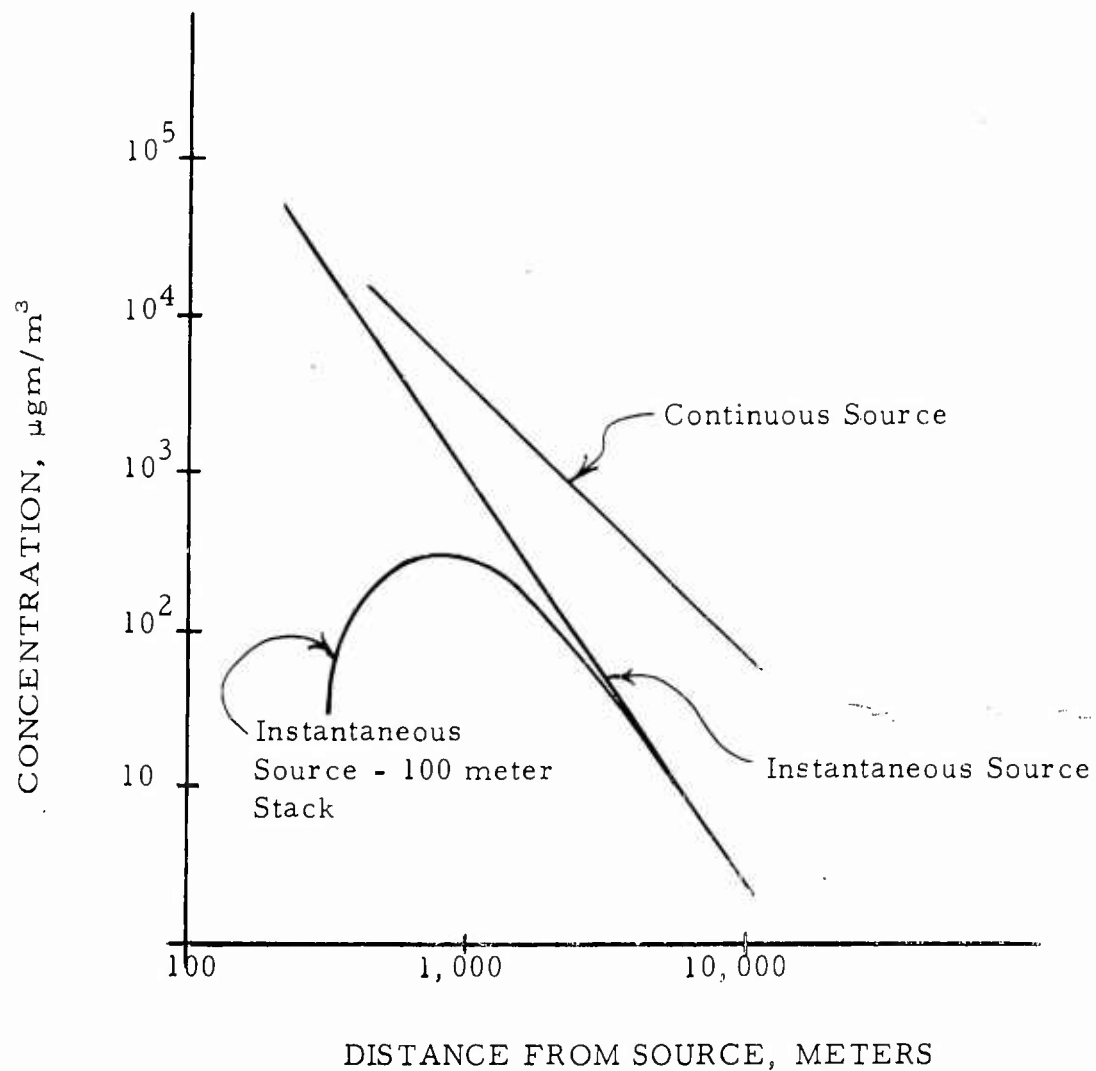


Figure 7. Atmospheric Diffusion Sutton's Equation

VARIABLE PARAMETERS IN ATMOSPHERIC DIFFUSION THEORIES

The frictional drag of the earth's surface on moving air results in a decrease of the wind velocity at the surface. Energy from the moving wind aloft is rapidly brought to the earth by eddy currents. This process results in a change of location of material within the air mass or in a mixing of clean air with effluent material or both. The eddy motion produces diffusion of the material in the air mass and is caused by mechanical disturbance, convection or thermal currents, or both. Diffusion in the polluting cloud will begin almost immediately after material is released into the atmosphere due to these eddy currents.

The diffusive capacity of an air mass is influenced also by its vertical thermal structure. The temperature profile, together with atmospheric buoyancy effects, is a necessary item for consideration when calculating downwind pollution concentrations. (The rate of decrease of temperature with altitude of an atmospheric condition is referred to as its lapse rate.) Also to be considered is the degree of windiness, since high winds favor rapid diffusion.

In the most widely used calculations, Sutton (Reference 1) and modified Sutton type equations, the diffusion values are broken into three distinct groupings which are dependent upon a thermal stability parameter, n . The diffusion coefficients are taken as C_x , C_y , and C_z , which are measured in the horizontal downwind, horizontal crosswind, and vertical directions respectively. The dimensionless number, n , is described as approaching zero under extreme turbulence and tends to approach unity as the turbulence decreases. For low altitudes, less than 100 meters, the empirical values of the diffusion coefficients and stability parameters have been used under certain meteorological conditions to give successful predictions of downwind concentrations for distances of a few kilometers. Any diffusion due to varying wind velocities in the atmosphere giving rise to shear forces is ignored. Sutton states that the diffusion coefficients, C_y and C_z , depend on the value of n and the magnitude of surface gustiness. The diffusion coefficients decrease with increased height due to a normal steady decrease of turbulence and tend to approach each other as the mechanical turbulence decreases with increased height.

The Sutton form for ground level concentration for a continuous point source emission is given as:

$$\chi = \frac{2 Q}{\pi C_y C_z \bar{u} x^{2-n}} \exp\left(-\frac{1}{x^{2-n}} \left[-\frac{y^2}{C_y^2} + \frac{h^2}{C_z^2} \right]\right)$$

Where:

χ = Downwind concentration

$\bar{u}$ = Mean wind velocity

x = Distance downwind

Q = Weight emission rate per unit time

y = Lateral distance from vertical plane through source
and parallel to mean wind direction

h = Stack or cloud height, meters

The evaluation of shell bursts in the air has been used by Sutton to obtain the decrease in diffusion coefficients with gain in height. The decrease in value has been found to follow the empirical form (Reference 1):

$$C = C(o) - 0.075 \log_{10} Z$$

where $C(o)$ is the ground level diffusion coefficient and Z is the height measured in meters.

The values of C_y and C_z are assumed to be equal for heights above 25 meters and $C_y = C_z = C$. The value of n is obtained through its relationship to the mean wind velocity profile in the equation (Reference 1).

$$\bar{u} = \bar{u}_1 \left(\frac{Z}{Z_1} \right)^{\frac{n}{2-n}}$$

Where:

$\bar{u}$ = Average wind velocity at elevation Z

$\bar{u}_1$ = Average wind velocity at reference level Z_1

The diffusion coefficients can be obtained from the relationship (Reference 1).

$$C_y = \frac{4 \nu^n}{(1-n)(2-n) U_n} \left(\frac{\bar{V}^2}{\bar{V}_1^2} \right)^{1-n}$$

Where:

ν = Kinematic viscosity

$\bar{V}$ = Average wind variation

n = Stability parameter

$\bar{u}$ = Average wind velocity

In the case of rough surfaces, the kinematic viscosity is replaced by macroviscosity, N .

Experimental determinations by Holland have shown the value of C_y for various stability parameters to be greater than postulated by Sutton, and the values of n to generally be larger than Sutton's value for approximately the same defined stability conditions. Holland also obtained data showing that the diffusion coefficients decrease with increasing wind speed. The discrepancies between measured and theoretical values are attributed to time differences in sampling, wind speeds, and terrain. The values of Sutton's parameters and the experimentally obtained values by Holland are shown in the following table :

Values from Sutton for C_y and C_z ($m^{\frac{N}{2}}$) follow:

Stability Condition	Elevation (Meters)			
	<u>25</u>	<u>50</u>	<u>75</u>	<u>100</u>
Large Lapse $n = .20$	0.21	0.17	0.16	0.12
Small Temperature Gradient $n = .25$	0.12	0.10	0.09	0.07
Moderate Inversion $n = .33$	0.08	0.06	0.05	0.04
Large Inversion $n = .50$	0.06	0.05	0.04	0.04

Oak Ridge Experimental Diffusion Parameters by Holland are presented below:

Stability Condition	Parameter	Elevation (Meters)			
		<u>5.5</u>	<u>16.5</u>	<u>47</u>	<u>24.4</u>
Daytime (Superadiabatic Lapse Rate) n = 15 to 20	$C_y (m \frac{n}{2})$	0.34	0.28	0.22	0.21
	$C_z (m \frac{n}{2})$	0.23			
	$\bar{u} (m/sec)$	1.5	1.8	2.4	2.7
Average (Neutral Stability) n = 25	$C_y (m \frac{n}{2})$	0.20	0.17	0.15	0.13
	$C_z (m \frac{n}{2})$	0.21			
	$\bar{u} (m/sec)$	0.09	1.2	1.8	2.7
Night (Moderate Inversion) n = 30 to 40	$C_y (m \frac{n}{2})$	0.14	0.11	0.08	0.08
	$C_z (m \frac{n}{2})$	0.14			
	$\bar{u} (m/sec)$	0.3	0.6	1.5	2.7

On the basis of the Round Hill (Reference 21) and Project Prairie Grass (References 22 and 23) experiments, a different type diffusion equation has been developed. The values of C_y and C_z are replaced by standard deviations of gas concentration distributions at distance x . The form of the equation is:

$$\chi = \frac{Q}{2 \pi \sigma_y \sigma_z \bar{u}} \exp \left[-\frac{1}{2} \left(\frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2} \right) \right]$$

Where χ , Q , $\bar{u}$, and y are the same quantities defined in Sutton's equation, and z is the distance measured from a horizontal plane through the source and the various diffusion parameters are:

$$\sqrt{2 \sigma_y} = C_y x^{1 - \left(\frac{ny}{2} \right)}$$

$$\sqrt{2 \sigma_z} = C_z x^{1 - \left(\frac{nz}{2} \right)}$$

It is well to note that in the Gaussian dispersion equation the value of n is replaced by n_y and n_z respectively, since recent work indicates a variation does exist for lateral and vertical diffusion at low altitudes. At high altitudes the assumption that $n_y = n_z = n$ seems reasonable but cannot be supported due to lack of experimental data.

From experimental data obtained at Edwards Air Force Base, the Stanford Research Institute computed a series of diffusion coefficients for that particular high desert area (Ref. 24). Using the instantaneous point source equation postulated by Sutton, the Stanford Research Institute back calculated the coefficients by obtaining concentrations of tracer materials downwind of a tracer source. The Sutton equation for point source and ground level emission is:

$$X = \frac{Q}{\pi^{3/2} C_x C_y C_z x^{3(n-1)/2}}$$

and the values computed by Stanford Research Institute are:

<u>Stability Condition</u>	<u>n</u>	<u>C_x</u>	<u>C_y</u>	<u>C_z</u>
Midday (Turbulent)	0.20	0.23	0.51	0.23
Heavily Overcast	0.25	0.16	0.38	0.16
Inversion	0.33	0.07	0.10	0.07

Since the tracer was released at a relatively low altitude, the diffusion coefficients approximately coincide with Sutton's values.

The value of the diffusion coefficient, C_y , as obtained by the Stanford Research Institute at Edwards Rocket Site, is given as 0.51 for midday lapse rates. The value was obtained by substituting into the Sutton formula for plume width:

$$2 y_0 = 2 \left(\ln \frac{100}{p} \right)^{\frac{1}{2}} C_y x^{2-n/2}$$

When:

y_0 = Plume width from centerline, meters

p = Per cent of peak concentration at distance y_0 from centerline

C_y = Lateral diffusion coefficient

x = Distance to point of determination, meters

n = Stability parameter

Using the information collected from a series of diffusion tests, an average plume width was obtained and the above equation solved for C_y .

As may be surmised, the conditions under which the average plume distribution curve concentrations were obtained varied for each test. Accurate measurements of the distance to the arc of sampling stations were not taken but were between six and seven miles.

From the graphical data presented in the Stanford Research Institute report, a more accurate determination of y_0 versus p can be obtained, and this will give a revised value of C_y .

The new value of y_0 is equal to 2900 meters rather than 3220 meters; a new value of C_y is found to be 0.46. This value is approximately what is found by interpolation of Sutton's values to one foot above ground level. Interpolation to the height of 3 feet by Sutton's values gives 0.395 for C_y which is the approximate height at which the samples were collected. The difference between the C_y values can be attributed to terrain roughness, lack of accurate measurement of the sampling arc, and plume width measurements.

The values for C_y recalculated on the basis of six and seven miles in lieu of 6.5 miles give 0.49 and 0.43 respectively, or a possible error of 6 per cent on this variable alone. It would then be assumed that Sutton's values of C_y are in good agreement for the Rocket Site.

Logarithmic plots of Sutton's values have been extrapolated to heights of over 1,000 feet, as shown in Figure 8, in an attempt to obtain approximate diffusion coefficients at high altitudes, but the values obtained have not been checked with actual experimental operating conditions.

The discrepancies between Sutton's diffusion coefficients and Holland's experimentally derived values are pointed out in Figures 9 and 10 for low and medium altitudes. Extrapolation of the values given on Page 29 to great heights was used to illustrate the converging characteristics of the data. The Sutton values were modified where applicable by the recommended correction factor given by Sutton for $n = 0.20$ and corrected by the Wanta equation for $n = 0.25$.

Figure 11 shows dispersion coefficients and stability parameters advanced by Sutton and determined by Holland (Ref. 2). By taking the average values, fairly accurate predictions can be made. Data taken at the test site will be helpful in further improvement of the values and will lead

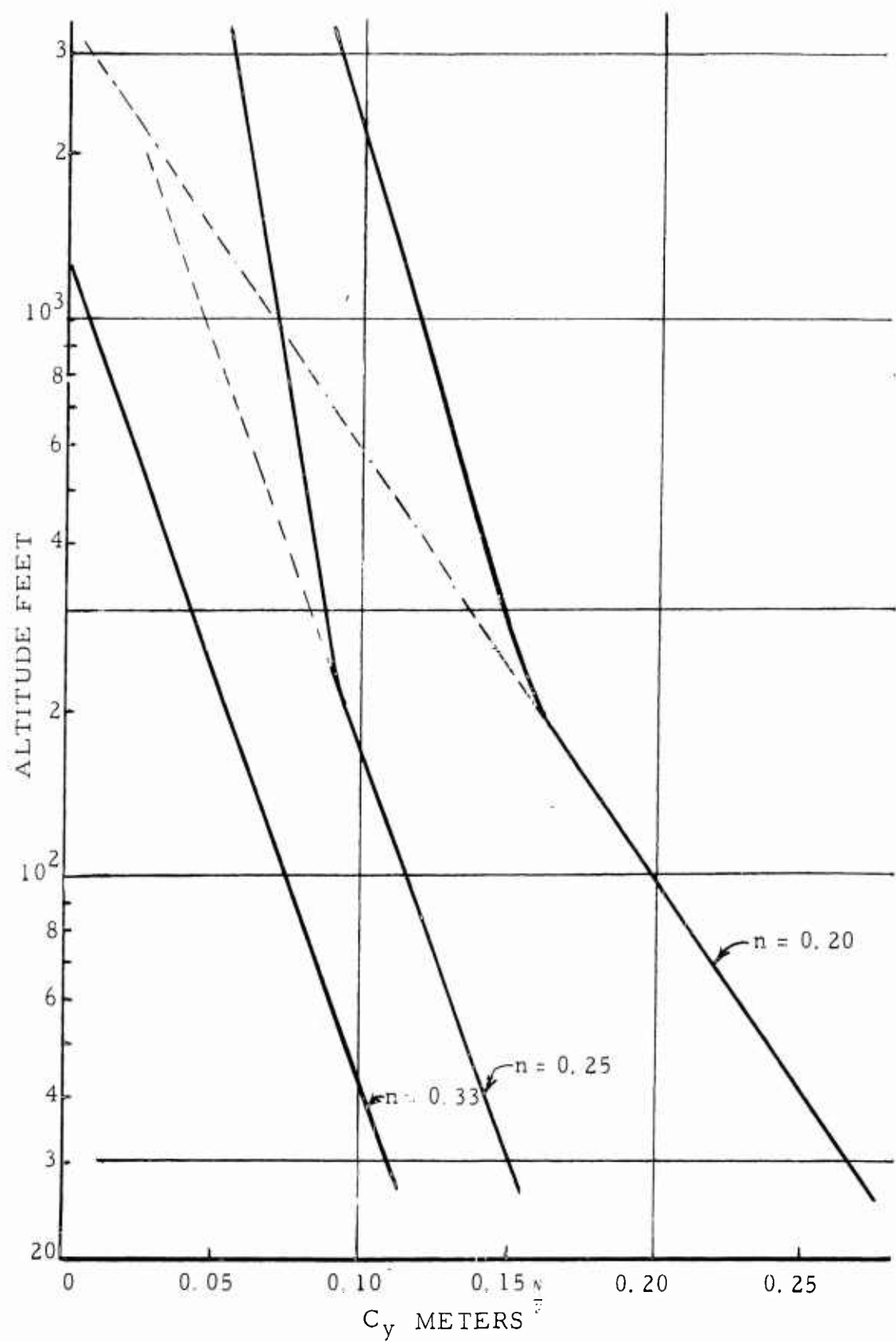


Figure 8. Sutton Diffusion Coefficient vs. Altitude

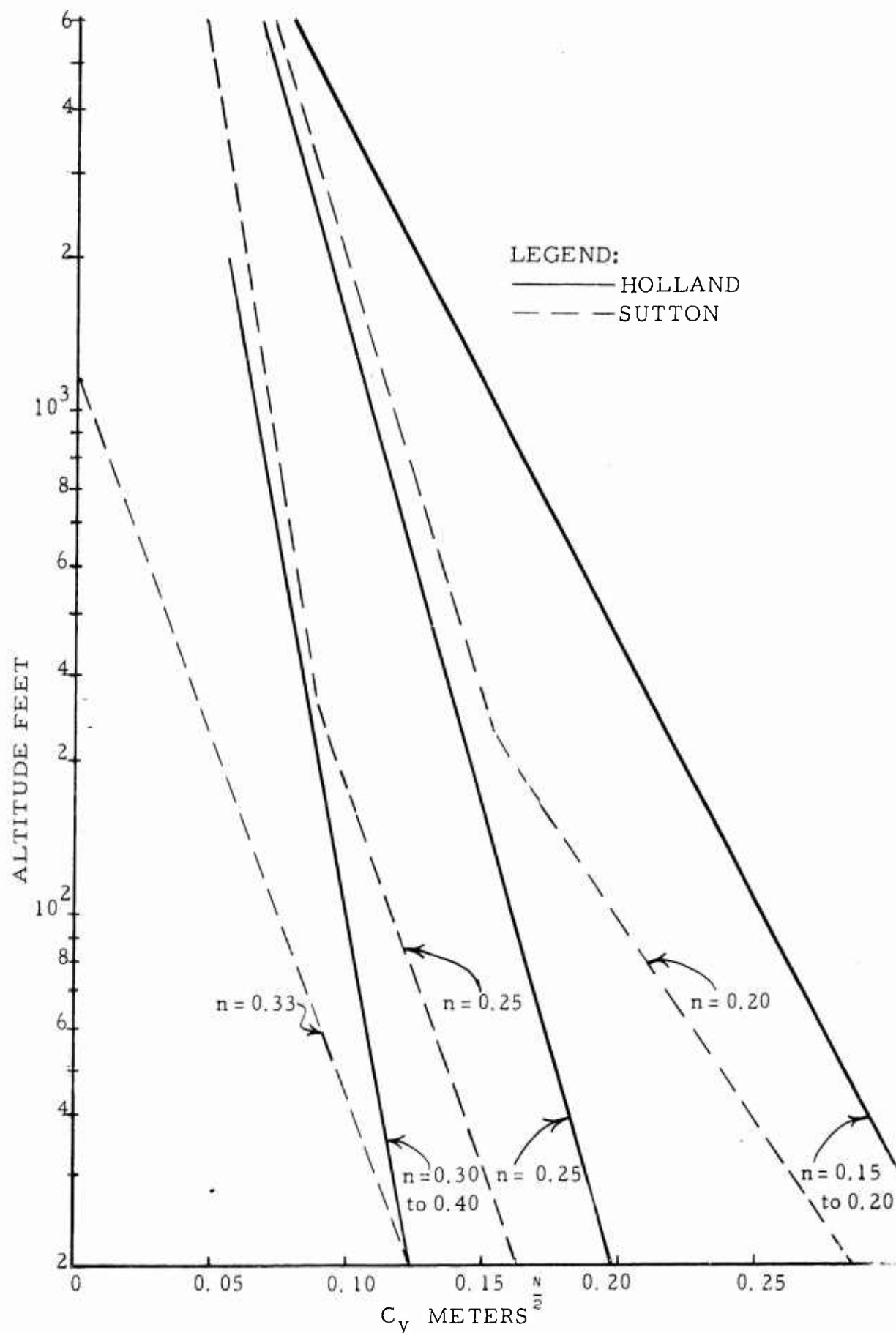


Figure 9. Holland & Sutton C_y Values vs. Altitude

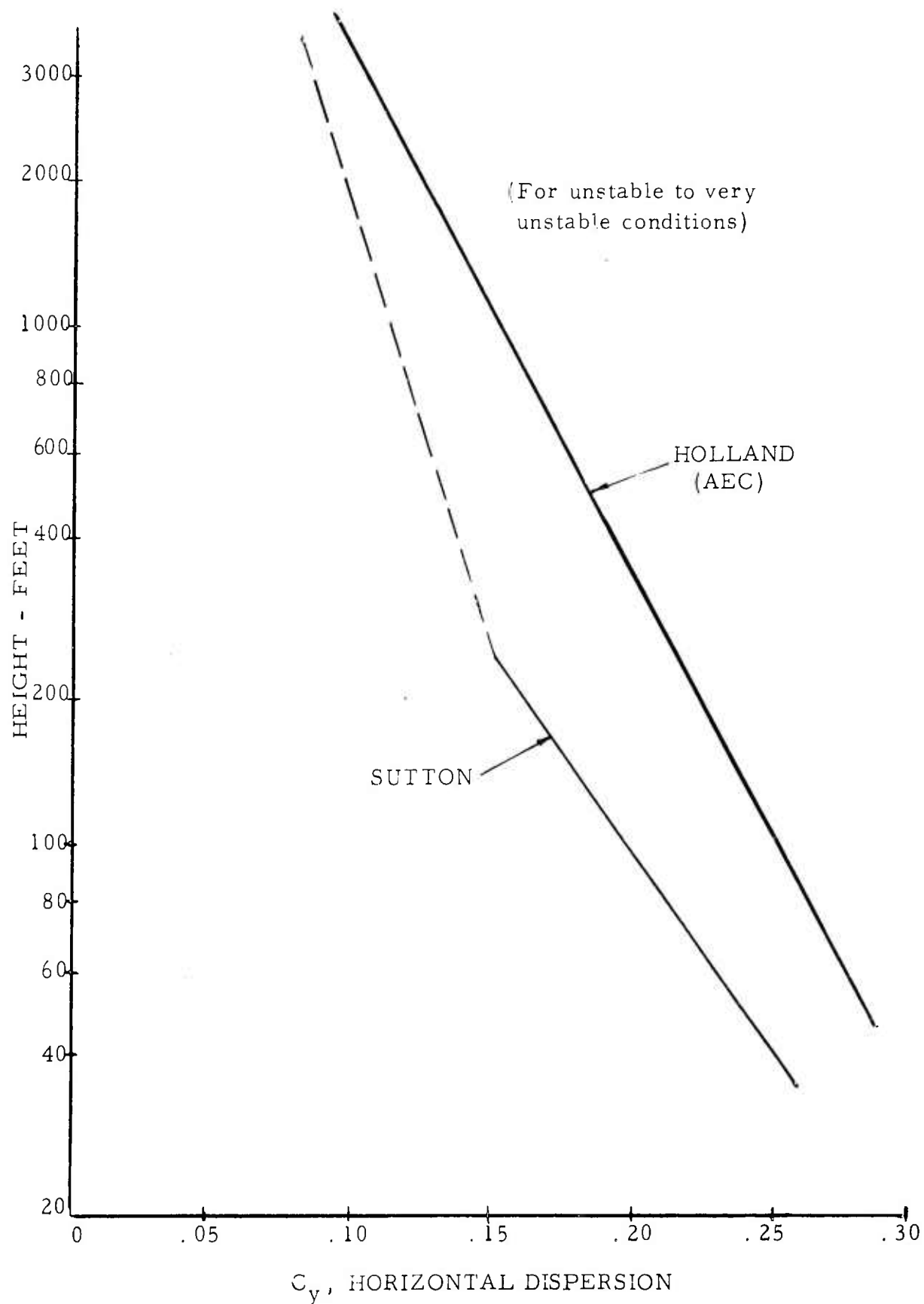


Figure 10. Comparison of Dispersion Coefficients

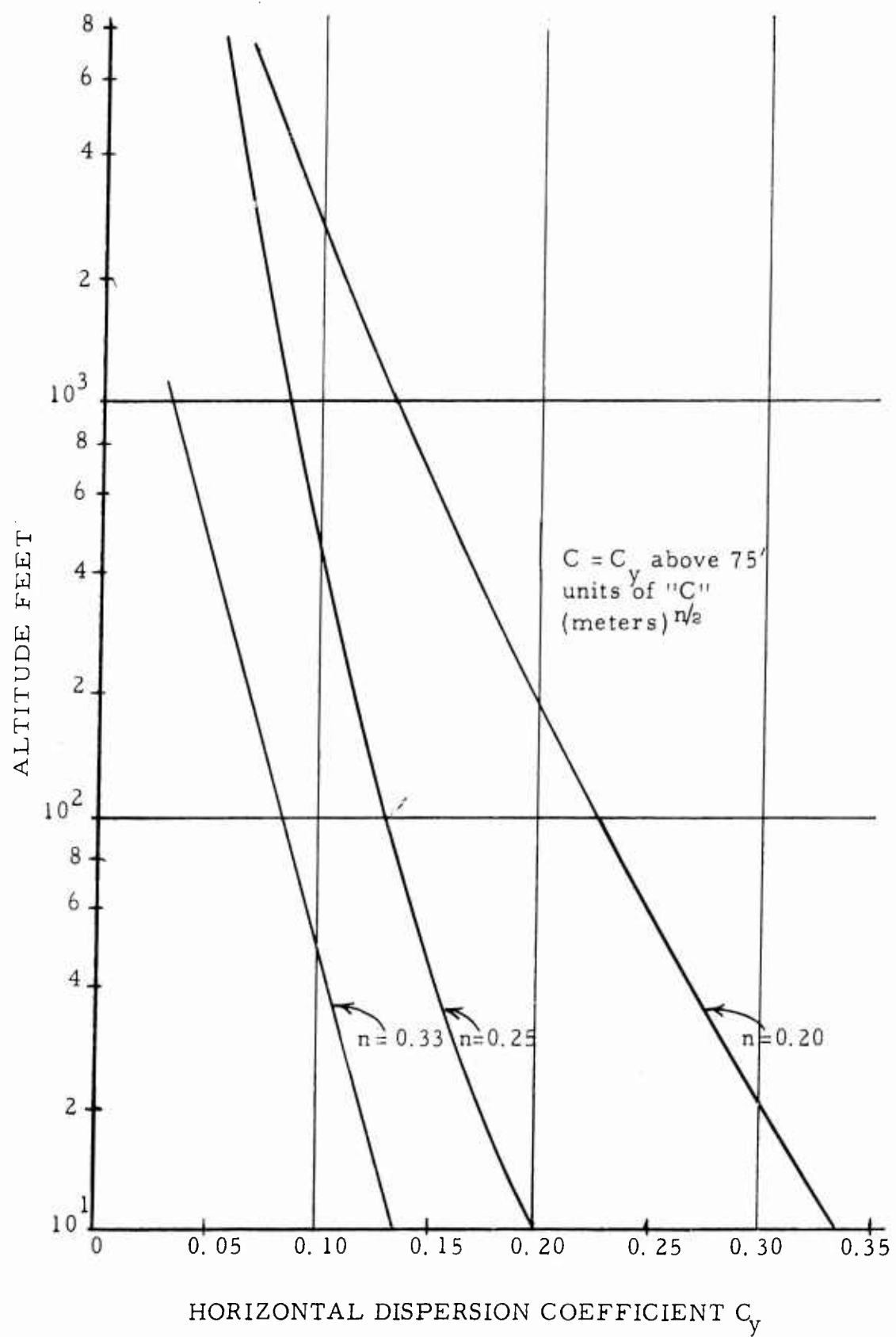


Figure 11. Recommended Diffusion Coefficients
for Sutton's Equations

to values applicable to Edwards. Surface diffusion coefficients determined for the Edwards Rocket Site (Ref. 24) agree reasonably well with surface values indicated by this graph. The decrease in dispersion coefficients with altitude has been applied to elevated source and stack height effects throughout this report. The reduced values result in increased distances to maximum points of concentration and TID. Use of these dispersion values is recommended until such time as improvement can be made through observations and interpretation of experimental releases of non-toxic plumes and those from actual toxic tests.

Variances in relating Ocean Breeze-type equations to peak concentrations downwind of an emitting source are a definite factor to consider. A major factor which must be taken into consideration when applying Ocean Breeze-type equations for obtaining downwind pollutant concentration is plume meandering. A correction factor of this type does not seem to appear in the original report. From wind trace photos and actual plume photos of meandering smoke plumes, a pattern strongly suggesting a sinusoidal wave appears. The Ocean Breeze report mentions normalized peak concentration, $\frac{\text{actual concentration}}{\text{source strength rate}}$, which was obtained by experimentation as the concentration downwind for varying parameters. It is assumed that the concentrations obtained during the experiment are actually average concentrations for the period of tracer emission; hence the actual peak concentration has a somewhat greater value than that obtained by Ocean Breeze experiments.

If the assumption is correct that the pattern of the plume cross section is sinusoidal, the peak value would be $1/0.636$ times the average peak concentration, i. e., 1.57 times average.

Comparison of the values obtained using the Ocean Breeze equation and Sutton's equation for a continuous emitting source and multiplying a correction factor, 1.57, shows the results to be quite close for a point 5,000 meters downwind. Of course, through judicious choice of Sutton's diffusion coefficients, it should be possible to obtain coinciding values for the two equations. A mathematical model, which would be based on probability curves, could be set up to take this meandering effect into account. A probability function is necessary in view of the nearly infinite combination of variables which affect the plume dispersion, such as diffusion coefficients, wind velocity, rate of direction change, and height of emitting source. Based on these assumptions, the peak concentration of contaminant downwind of a long time emitting source, such as a rocket test lasting for several minutes, will be greater than calculated by Ocean Breeze equations.

TOTAL INTEGRATED DOSAGE

Integration of ground level concentration for instantaneous point source releases, with respect to time, produces an equation that can be used to calculate the concentration-time exposure for atmospheric release of toxic products downwind of the source. Differentiation with respect to distance from the source yields an expression from which the maximum dosage can be found and distance determined. The equations (Ref. 2) have been presented under the discussion of diffusion formulae and are repeated here for convenience.

$$\begin{aligned} \text{TID} &= \frac{2 Q}{\pi C^2 \bar{u} (\bar{u} t)^{2-n}} \cdot e^{\left(-\frac{h^2}{C^2 (\bar{u} t)^{2-n}} \right)} \\ \text{TID}_{\max} &= \frac{2 Q}{\pi e \bar{u} h^2} \\ d_{\max} \text{ dosage} &= \left(\frac{h^2}{C^2} \right)^{1/(2-n)} \end{aligned}$$

Using these equations, the downwind dosage can be plotted with respect to distance. The $\text{TID}_{\max}$ expression defines the maximum point on the TID curve at distance $d_{\max}$ dosage.

The two most significant toxic emission parameters are the maximum concentration and the total time of exposure. With some toxic products, the limits are expressed only as maximum allowable, but with others the integrated dosage is apparently of prime significance.

Integrated dosage will be important when the maximum allowable concentration (MAC) varies with the interval of exposure. This occurs when the allowable exposure for personnel is greater per eight-hour work day than a continuous exposure for the general public. The computation of TID is done by multiplying the period allowable concentration by the time period to obtain the dimensions of mass time per volume for TID. The allowable TID varies between inhabited areas and in-plant short time, daily, or weekly doses. Therefore, TID would have to be applied to the specific occurrence as required. The allowable TID for a definite period would be found by multiplying the MAC for the period by the duration of the period. The units obtained could be gram seconds per cubic meter.

The standard methods of reducing TID are followed for toxic rockets; that is, reduction of toxic effluent rate by scrubbing or filtration, increasing the stack height via afterburning and/or flaring, and scheduling for optimum atmospheric conditions.

EFFECTIVE STACK HEIGHT FORMULAE

A key factor in considering the fallout from a toxic rocket test is the maximum altitude reached by the exhaust plume. This is because the dispersion of the toxic material effectively begins at the exhaust plume maximum altitude, and the toxic material disperses in three dimensions. Therefore, a relatively small portion reaches the ground.

The factors governing the maximum altitude or effective stack height are a combination of the following parameters:

- A. Test rocket size
 - 1. Engine thrust
 - 2. Duration
- B. Propellant characteristics
 - 1. Exhaust temperature
 - 2. Mixture ratio
 - 3. Average molecular weight of exhaust
- C. Rocket orientation
 - 1. Vertical upward
 - 2. Horizontal
- D. Atmospheric conditions
 - 1. Wind
 - a. Velocity
 - b. Direction
 - c. Time variations
 - d. Altitude variations
 - 2. Diurnal changes
 - a. Ambient temperature

b. Temperature variation with altitude (lapse rate)

(1) Adiabatic lapse condition

(2) Inversion (negative lapse rate)

c. Convective circulation

From these many parameters, a large number of distinct formulae have been derived, utilizing data from wind tunnel tests of smoke models, full scale power plant observations, mathematical derivations, and actual rocket firings. Very few have a common resemblance, as they differ in specific variables treated, general form, and method of approach. This will not produce any contradictory information in the ultimate results if the proper equation is used for the particular application. Therefore, the best of the available equations for the present situation may be selected on its particular applicability or by a process of elimination. An alternate method would be the derivation of a completely new formula. The new formula would be designed to cover the range of stack heights required for rocket usage. Derivations of two new formulae are presented. One formula is intended for use in the near future. The other formula depends on an accurate determination of constants and coefficients; however, the use of this equation will eventually provide highly accurate results.

The following survey of the existing stack height formulae is presented. The sources, accuracy, and general applications are presented in a later section of the report.

1. Bryant - Davidson formula

$$\Delta h = d \left(\frac{V_s}{\bar{u}} \right)^{1.4} \left(1 + \frac{\Delta T}{T_s} \right)$$

This formula (Ref. 2, 3) was derived from wind tunnel experiments and gives the height to where the centerline of the plume is near horizontal. This is not to be misconstrued as the maximum stack height. Factors which are not considered are the stability of the atmosphere (i. e., lapse rate) and the velocity scaling ratio $\frac{V_s}{\bar{u}}$, which varies due to compressibility effects as the stack gas ejection speed (V_s) becomes large. These factors would indicate optimistic plume rise and a high degree of plume dilution as the velocity ratio increases. As the above equation is presented, there is no limit to the effect that a large velocity ratio gives to the calculated height. This equation also does not include the effect of energy dissipation at high rocket gas exhaust temperatures which would decrease the temperature ratio, $\frac{\Delta T}{T_s}$. Therefore, this equation is not recommended for use in

analysis of rocket test systems.

2. Bosanquet, Carey, and Halton equation

The equation (Ref. 6) given by Bosanquet, et al., for the maximum plume rise in a stable atmosphere is:

$$\Delta h_{\max} = \frac{4.77}{1 + \frac{0.43 u}{V_s}} \left(\frac{Q_{v1} V_s}{u} \right)^{\frac{1}{2}} + 6.37 g \frac{Q_{v1} \Delta T_1}{u^3 T_1} \left(\log_e J^2 + \frac{2}{J} - 2 \right)$$

Where:

$$J = \frac{\frac{u^2}{(Q_{v1} V_s)^{\frac{1}{2}}}}{\left\{ 0.43 \left(\frac{T_1}{g} \frac{d\theta}{dz} \right)^{\frac{1}{2}} - 0.28 \frac{V_s}{g} \cdot \frac{T_1}{\Delta T_1} \right\} + 1}$$

The Bosanquet, et al. equation is based on theoretical developments and employs some fundamental experimental constants of diffusion.

This equation has been tested using chimneys and similar low heat sources. A report by Moses and Strom (discussed below under the Scorer formula), stated that a value of stack height calculated from this equation gave a minimum average difference between the calculated and observed heights. It would seem that this equation would cover most of the parameters, but certain important ones are left out. These are entrainment rate, which would modify the velocity rate at something other than a constant rate, and the effect of extreme exhaust temperatures on the temperature ratio and buoyancy of the gases. In order to be useful for the purpose of rocket firings, this equation would require modification and testing.

3. Oak Ridge formula

$$\Delta h = \frac{1.5 V_s d + 0.000134 Q_h}{u}$$

Where:

Q_h is heat emission rate of stack gas relative to ambient air, calories per second

V_s and u are in meters per second

Δh and d are in feet

The Oak Ridge formula (Ref. 3) was derived empirically from observations of hot plume rise from steam generating plants, assuming the velocity effect as derived by Rupp, et al. The purpose of the Oak Ridge equation was to improve agreement between predicted and observed results. This formula does not give maximum plume height, since it was derived from data taken only a few stack heights downwind.

The experimental data used for deriving this empirical equation did not take into consideration variations in lapse rate. Holland suggests from 10 to 25 % should be added to Δh for large lapse rates and a like amount subtracted for small lapse conditions. Various investigators found that the Oak Ridge formula gives consistently low values for stack height. Therefore, a safe assumption may be made that any equation which gives a lower numerical answer than the above equation should be disregarded for the application at EAFB rocket site.

The effluent from the X-10 reactor cooling air exhaust at Oak Ridge and two other hot exhaust systems were used to empirically derive the buoyancy term of this equation. The disappointing factor is that the observed value of the stack height is not the maximum since observations were made only a short distance downwind from the stack. This equation is, therefore, not recommended since it does not give the maximum stack height.

4. Thomas formula

$$\Delta h = \frac{1.5 V_s d + 0.000268 Q_h}{\dots}$$

The units in this equation are the same as the Oak Ridge formula. The Thomas equation (Ref. 7) is used to derive the maximum plume rise. The possible variation due to changes in stability is ± 10 to 25 % as mentioned in the Oak Ridge formula. This method is considered by Stern (Ref. 3) to give a fairly accurate value for stack height. Whether it can be extrapolated to the degree required for large rocket firings must be determined by experiment.

5. Sutton formula

$$\Delta h = 1.5 \left(\frac{v_1}{u} \right)^3 \left[\cot \psi \operatorname{cosec} \psi - \log_e (\cot \psi \operatorname{cosec} \psi) \right]$$

$$\text{Where: } v_1^3 = \frac{7 Q_h g}{3 \pi C_p c T C_s}$$

$$\text{And: } C_p c = C_{ps} c_s$$

The distance along the plume path (s) is:

$$s = \left(\frac{v_1}{u} \right)^3 \cot^2 \psi$$

When ψ is less than 10° , the effective stack height approaches the maximum, and when the wind speed is high, s equals the downwind distance from the stack (x).

Sutton's formula only takes buoyancy into consideration and does not include momentum rise (Ref. 1). One should refer to his book prior to using his equations, since the value of C_1 is not clear as to the units which should be used. However, for dimensional continuity, C_1 must be dimensionless. In the determination of the effective stack height, the value of ψ chosen is quite important since, as the angle between the plume axis and horizontal (ψ) approaches zero, the function of ψ goes through a maximum and then becomes discontinuous as a limit of zero degrees is reached.

The results from this method are generally too high to be of use, and are considered to be too high in most of the referenced material.

6. Bryant and Cowdrey experiment

This procedure (Ref. 8) is based on wind tunnel tests and on observations of stack effluents. A series of graphs has been formulated but has to be greatly extrapolated and/or extended to obtain a result for the purpose of this application.

One factor apparent from the graphs is the decrease in effective stack height with increase in turbulence. Another important factor is that of condensation from the cooling plume. As the gas rises and cools, any water formed from the combustion process will condense when the temperature decreases below the dew point. Therefore, if the rising plume does not mix with the surrounding air, the water will tend to rain out and carry any solids or water soluble gases with it. However, under actual field tests, Bryant and Cowdrey state that effluent dilutions of the order of magnitude of several hundreds to one occur. This then would be sufficient to unsaturate any rocket exhaust plume.

In conclusion, it is believed that this procedure is of the correct general type, but the conversion to rocket testing would involve extrapolations of such magnitudes that the accuracy of information would be invalidated.

7. Priestley procedure

Priestley gives a two-phase solution to stack height (Ref. 9). The first is that of a vertical jet moving through a medium. The motion of the jet causes resistance, spreading, and dilution due to eddy turbulence. As the jet is decelerated by the action of the air, the intensity of this jet energy becomes less than that of normal air turbulence. At this point, the second phase of dilution begins; one that is dependent on atmospheric properties.

By this method, Priestley derived the following relations:

$$\Delta h_c = \text{Maximum plume rise}$$

$$\Delta h_c = \Delta h_1 + \frac{w_{a1}}{k} \frac{g \Delta \theta_1'}{k^2 \theta_1'}$$

In these equations, subscripts of "o" refer to the chimney top, and subscripts of "1" refer to conditions at transition from phase 1 to phase 2. In phase 1, the axial speed of the plume is given as w_a .

$$w_a^3 = \frac{3 A_1 g}{2 \theta_1' c^2} \left(\frac{1}{\Delta h_v} - \frac{\Delta h_{vo}^2}{\Delta h_v^3} \right) + \frac{w_{ao}^3 \Delta h_{vo}^3}{\Delta h_v^3}$$

Values of w_a between "o" and "1" may be found, w_{a1} being the necessary one. Other relations are given below:

$$A_1 = \frac{Q_h}{\pi c_s^2 C_{ps}}$$

This is a buoyancy term. Δh_v is measured from a point located a distance Δh_{vo} below the chimney top where the lines of plume expansion form a virtual point of origin. Radius R of the plume cross section is related to Δh_v .

$$R = c \Delta h_v$$

Also at point "o" this relation hold true.

$$R_o = c \Delta h_{vo}$$

Priestly suggested that an alternate form for the determination of R_o should be used.

$$R_o^2 = \frac{A_1}{w_{ao} \Delta \theta_o'}$$

The value of the spreading coefficient (c) in the above equations is based on smoke plume photographs in wind tunnel experiments.

$$c = 0.75 \left(\frac{\bar{u}}{2.44} \right)^{0.5}$$

Where u is wind speed in meters per second and c is dimensionless.
The value of Δh_{v1} is then defined as :

$$\left(\Delta h_{v1}\right)^6 - \frac{3 A_1 g}{2 \theta' c^2 k^3} \Delta h_{v1}^2 - \frac{1}{k^3} \left(w_{a0}^3 \Delta h_{v0}^3 - \frac{3 A_1 g \Delta h_{v0}^2}{2 \theta' c^2} \right) = 0$$

The mixing rate (k) is given as proportional to wind speed and is in units of inverse seconds when u is in meters per second.

$$k = 0.0197 u$$

The value of Δh_1 may be replaced with $(\Delta h_{v1} - \Delta h_{v0})$ when solving for Δh_c . In the procedure, the time to reach the transition point can also be found. This is given below:

$$t = \frac{\theta' c^2}{2 A_1 g} \left(\left[\frac{3 A_1 g}{2 \theta' c^2} (\Delta h_v^2 - \Delta h_{v0}^2) + w_{a0}^3 \Delta h_{v0}^3 \right]^{\frac{2}{3}} - \left[w_{a0}^3 \Delta h_{v0}^3 \right]^{\frac{2}{3}} \right)$$

The value of w_{a1} is now found, and then the potential temperature is found at "1".

$$\Delta \theta_1' = \frac{A_1}{c^2 \Delta h_{v1}^2 w_{a1}}$$

Finally, the value of Δh_c can be found. This method gives the lowest calculated stack height of any of the formulae. It is of a form that should give "order of magnitude" results but apparently does not. A simpler empirical relation is more to be desired.

8. Bosanquet formulae

$$\Delta h = A_2 u \left[f_1(a) + f_{11}(a_0) - \frac{0.615 a_0^{\frac{1}{2}}}{\left[\left(\frac{V_s}{\bar{u}} \right)^2 + 0.57 \right]^{\frac{1}{2}}} \right]$$

Where:

$$A_2 = \frac{g Q_{v1} \Delta T_1}{2 \pi C_p^2 T_1 u^4}$$

$$t_0 = \frac{4 V_s T_1}{3 g \Delta T_1}$$

$$a_0 = \frac{t_0}{A_2}$$

$$a = \frac{t + t_0}{A_2} = \frac{t}{A_2} + a_0$$

$$t = \frac{x}{u}$$

The above procedure (Ref. 3, 10, 11) is evaluated using consistent units, such as the English ones used in previous equations; t is time and $f_1(a)$ and $f_{11}(a_0)$ are graphically solved on page 142 (Ref. 3). For values outside the range of the graphs, the following apply:

Where (a) is very large:

$$f_1(a) = \log_e a - 0.12$$

When (a) is very small:

$$f_1(a) = 1.054 a^{\frac{3}{4}}$$

When (a_0) is very large:

$$f_{11}(a_0) = 1.311 a_0^{\frac{1}{2}} - \frac{\log_e a_0}{2} - 1$$

When (a_0) is very small:

$$f_{11}(a_0) = -0.527 a_0^{\frac{3}{4}}$$

The value of C_2 is 0.13 and $t + t_0 < 200$ sec. For momentum rise only:

$$\Delta h_{\max} = \frac{\left(2 Q_{v1} \frac{V_s}{3}\right)^{\frac{1}{2}}}{C_2 \pi^{\frac{1}{2}} u} \left\{ 1.311 - \frac{0.615}{\left[\left(\frac{V_s}{u}\right)^2 + 0.57\right]^{\frac{1}{2}}} \right\}$$

When $V_s / u > 0.5$

For small values of V_s / u

$$\Delta h_{\max} = \frac{\left(2 Q_{v1} \frac{V_s}{3}\right)^{\frac{1}{2}}}{C_2 \pi^{\frac{1}{2}} u} \times 0.9 \left(\frac{V_s}{u}\right)^{\frac{1}{2}}$$

The two equations for $\Delta h_{\max}$ give the same value at $V_s / u = 0.48$.

The maximum rise of a buoyant plume in a stable atmosphere is found for $t + t_0$ of 200 sec., or the following, whichever is smaller:

$$t + t_0 = 1.527 \left(\frac{2 T}{g \frac{d\theta'}{dz}} \right)^{\frac{1}{2}}$$

The accuracy of this method has been stated by Bosanquet to be about $\pm 21\%$.

Bosanquet derived this procedure by assuming that when a cloud of hot gas is rising, the total heat content and total upward momentum are unaffected by dilution with atmospheric air. It is also assumed that the upward momentum increases at a rate proportional to the heat content. The rate of dilution is assumed to be proportional to the surface area of the cloud multiplied by a function of the wind velocity and the velocity of the cloud relative to the surrounding atmosphere. This procedure gives a low value for stack height when compared with other methods.

It can be noted that the calculation of the time or rise is a function of lapse rate. Since the lapse rate varies with altitude, any calculation of time must weigh this factor.

The statement by Bosanquet that the time of plume rise is equal to or less than 200 seconds seems to imply low powered systems since time periods greater than this have been observed for large explosions. For example, up to ten minutes for atomic bombs has been reported by the AEC (Reference 17).

The velocity ratio (V_g/u) and the buoyancy or temperature ratio ($\Delta T_1/T_1$) have been considered by Bosanquet, but perhaps not enough for the purpose of this report. This is because the compressibility effect of high velocity exhaust streams becomes noticeable and the large buoyancies due to temperature differences also become important.

The dilution coefficient (C_2) has been evaluated at 0.13, which provides an adequate fit to the calculated stack heights. Other values have been offered, 0.09 and 0.17 for example, which give less accurate answers.

The Bosanquet formula is not recommended due to the consistent low values of stack height obtained by this method. The equation is not recommended until further trials at stack heights more pertinent to rocket tests have been made.

9. Scorer formula

Maximum rise of a nonbuoyant plume (Ref. 3).

$$\Delta h_{c \max} = 2.5 R_0 \left[\frac{V_s}{u} \left(1 - \lambda^{-\frac{1}{2}} - \mu^{-\frac{1}{2}} + 2 \mu^{-1} \right) - 2 \right]$$

Where the units are:

R_0 = radius of gas plume cross section at stack top, feet

λ = 0.1 for neutral stability, dimensionless

μ = 1.0 for neutral stability, dimensionless

Maximum rise of a buoyant plume.

$$\Delta h_{c \max} = \frac{0.58 F}{\lambda^2 u^2} + 2.5 R_0 \left[\frac{V_s}{u} \left(1 - \lambda^{-\frac{1}{2}} + 2 \mu^{-1} \right) - 2 \right]$$

When,

$$g \left(\frac{\rho - \rho_s}{\rho} \right) > \frac{1.4 \lambda^{\frac{1}{2}} u^2}{R_0}$$

$$\frac{\rho - \rho_s}{\rho} \sim \frac{\Delta T_s}{T_s}$$

$$F = \pi g V_s R_0^2 \left(\frac{\rho - \rho_s}{\rho} \right)$$

For plumes of large buoyancy when,

$$g \left(\frac{\rho - \rho_s}{\rho} \right) > \frac{0.7 \lambda^{\frac{1}{2}} u^2}{R_0}$$

Plume rise is given by,

$$\Delta h_{c \max} = \left[\frac{0.58}{\lambda^2} + \frac{2.25}{\mu^{1/2}} + \frac{3.8}{\mu^3} \right] \frac{F}{u^3} + 5 R_0$$

Scorer assigns three phases to the rise of a plume. Phase one has the plume nearly vertical and the upward velocity exceeds the horizontal wind velocity. This phase ends when the vertical velocity approximately equals the horizontal wind velocity. Phase two is a transition stage in which the plume bends over and become horizontal. This phase ends when the vertical velocity equals the turbulent eddy velocity of the atmosphere. This occurs when the upward velocity is about 10% of the wind velocity, according to Scorer. Phase three is the regime of diffusion at the effective stack height. This is accomplished by the Sutton and other diffusion formulae. Scorer's method is derived from the observations of chimney plumes. Upon application, it appears as accurate as other methods. Moses and Strom (Ref. 12) have compared observed plume rises with values obtained from various formulae.

They calculated the stack height from (1) Holland (Oak Ridge), (2) Bryant and Davidson, (3) Sutton, (4) Scorer, (5) Bosanquet, Carey, and Halton, and (6) Bosanquet, and compared the results. Their conclusions state, "There is no one formula which is outstanding in all respects." They do not recommend the Sutton formula, but consider the others about equally valid for the low energy level conditions tested. Whether the Scorer method can be extrapolated to rocket testing is not known, but it does seem to give far too optimistic results. Scorer's assumptions for the values of λ and μ are not clearly explained, and these constants may vary with the system being considered.

10. Machta formula

$$\Delta h = \frac{1}{\frac{1}{M} \cdot \frac{\partial M}{\partial z}} \cdot \log_e \left(\frac{\frac{1}{M} \cdot \frac{\partial M}{\partial z}}{\frac{\partial \theta'}{\partial z}} \left[(\Delta \theta')_0 + \frac{\frac{\partial \theta'}{\partial z}}{\frac{1}{M} \cdot \frac{\partial M}{\partial z}} \right] \right)$$

M = mass of cloud, kilograms

z = upward pointing coordinate, meters

θ' = temperature, degrees Kelvin

$(\Delta \theta')_0$ = potential temperature difference between cloud and environment, degrees Kelvin

$$\frac{1}{M} \cdot \frac{\partial M}{\partial z} = 0.5 \cdot 10^{-5} \text{ cm}^{-1} \text{ (cumulus clouds)}$$

This equation (Ref. 2, 13) is for an instantaneous source, an atomic bomb, and gives the height at which a cloud is no longer warmer than its surroundings. Close agreement was obtained upon comparison with actual bombs.

The factor which tends to give optimistic heights from this formula is the entrainment rate. Cloud size is an important factor, the larger the cloud the smaller the rate, with the given value being for cumulus clouds. In the case of rocket testing, an order of magnitude increase is given as a function of cloud radius to rocket cloud radius. Any further improvement in this method would be subject to actual tests. This equation could possibly be used after an explosive malfunction of a rocket engine.

11. Empirical formula for small solid rockets

$$\Delta h = 3.4 (1.2 - n)^2 \left(\frac{M_o T_s'}{u^2} \right)^{0.5}$$

Where:

n is a stability parameter equal to Sutton's n so that $n = 0.25$
for adiabatic lapse rate.

M_o = propellant weight, pounds

T_s' = gas effluent temperature, degrees Centigrade

u = wind velocity, miles per hour

Δh = cloud height, feet

This equation (Ref 14) is for small solid rockets fired horizontally, and does not contain upward momentum. It was developed empirically during actual testing and is not necessarily valid upon extrapolation to high thrust levels. It is not known if the calculated height from this equation is a maximum.

12. Jet Penetration Equation

A wind tunnel investigation was made under NACA direction into the relationship the momentum of gases from an orifice or stack has to the altitude achieved by the gases. (Ref. 26)

Based on experimental data, an empirical formula was derived for the penetration of an air stream by a jet discharging through an orifice perpendicular to the air stream. The equation for the penetration was obtained in terms of the jet diameter, distance downstream from the jet, the ratios of the jet, and air stream velocities and densities:

$$\left(\frac{\Delta h_M}{D_s} \right)^{1.65} = 2.91 \frac{\rho_s V_s}{\rho_a V_a} \sqrt{\frac{x}{D_s}}$$

where Δh_M = depth of jet penetration (stack height due to momentum)

x = distance downstream from orifice centerline

D_s = orifice diameter (stack diameter)

ρ_s = mass density of jet air at vena contracta

ρ_a = mass density of free air

V_s = velocity of jet at vena contracta

V_a = velocity of free stream air

Use of the above equation, with proper substitution of terms related to the exhaust gas stream, gives a stack height of 120 meters, for a 333 # /sec engine of 40 % gaseous exhaust at 4000 F and 13.2 psia atmospheric pressure with a molecular weight of 29 discharging through a 2 meter stack at 150 meters per second initial velocity, at a distance of 6.2 miles. As the distance downstream is increased to 62 miles the height due to velocity is 240 meters and at 400 miles is only 425 meters. Although no term appears for different lapse rates, it is believed that only a very small error is introduced by the omission since the height due strictly to exhaust velocity is small in comparison to the buoyancy effect of the hot gas.

This formula has merit for obtaining stack heights, since it is based on actual, physical, controlled experimental data.

A further simplification of the formula is:

$$\Delta h_w = 1.92 D^{0.70} \left(\frac{V_s}{V_a} \right)^{0.606} \left(\frac{\rho_s}{\rho_a} \right)^{0.606} \times 0.30$$

A major drawback to this formula is that the information was gathered at extremely large free stream air velocities, 260 and 360 feet per second. However, smooth curves are shown in the original report for the Penetration Coefficient, $\frac{h}{D_s}$, versus the density-velocity ratio, $\frac{\rho_s V_s}{\rho_a V_a}$; from these

data, the assumption can be made that the ratio should hold for small free stream air velocities. The variation in penetration coefficient was found to be negligible for Reynolds numbers between 60,000 and 500,000 for the jet and for viscosity ratios $\frac{\mu_s}{\mu_a}$ of 1.5 to 1.9.

13. Buoyancy of Explosion Clouds

The buoyancy term for stack height has been shown to be a function of stack diameters, ratio of the kinematic viscosities, lapse rate, temperature difference, flow rate, wind speed, and stack velocity. This term is presented below as the buoyancy equation derived for stack height due to buoyancy from a continuous source. The buoyancy equation was derived for normal runs. For explosions a somewhat different view must be taken. Instead of stack diameter, flow rate, and stack velocity, the mass of explosive must be considered as well as the other terms mentioned above. Some of these terms will generate a relatively large change in height of the plume with a small change in their magnitude. The exact degree to which this change will be important is not presently known, but the total function has an experimentally determined maximum from nuclear explosions.

The eventual height reached by a nuclear explosion cloud depends on the heat energy of the bomb and on the temperature gradient (lapse rate) and density of the surrounding air. The greater the explosive yield, and therefore the heat released, the greater will be the upward thrust due to buoyancy and therefore a higher stack height. Factors which influence the cloud height other than those given above are of importance in considering rocket test explosions as well as nuclear explosions. These factors include the height of the explosive device at the instant of detonation since any large distance between the fireball and the ground will be added to the total height reached by the cloud whereas if the fireball touches the ground a large amount of debris will be vaporized and picked up with the other buoyant material and cause a net loss in cloud height.

The maximum height achieved by the nuclear cloud is strongly influenced by the tropopause, the boundary between the troposphere and the stratosphere, assuming the cloud reaches the height of the tropopause. The tropopause is between the troposphere and the relatively stable air of the isothermal stratosphere. It varies in altitude with seasons and latitude, ranging from 25,000 feet near the poles to about 55,000 feet in equatorial regions. When a nuclear cloud reaches the tropopause, there is a tendency to spread laterally. This effect is caused by the relative magnitude of the diffusion coefficients changing at the altitude, the horizontal ones being greater than the vertical ones. A portion of the cloud will penetrate the tropopause, and it will ascend into the more stable air of the stratosphere provided sufficient energy remains in the cloud at this height.

The nuclear detonations carried out by the United States have been heavily instrumented for scientific purposes. The properties of interest for this report are the explosive yield of a test, the height of the burst, type of burst (air drop, tower, surface, or underground), the mean height of the cloud, and the height of the tropopause. From this information for many

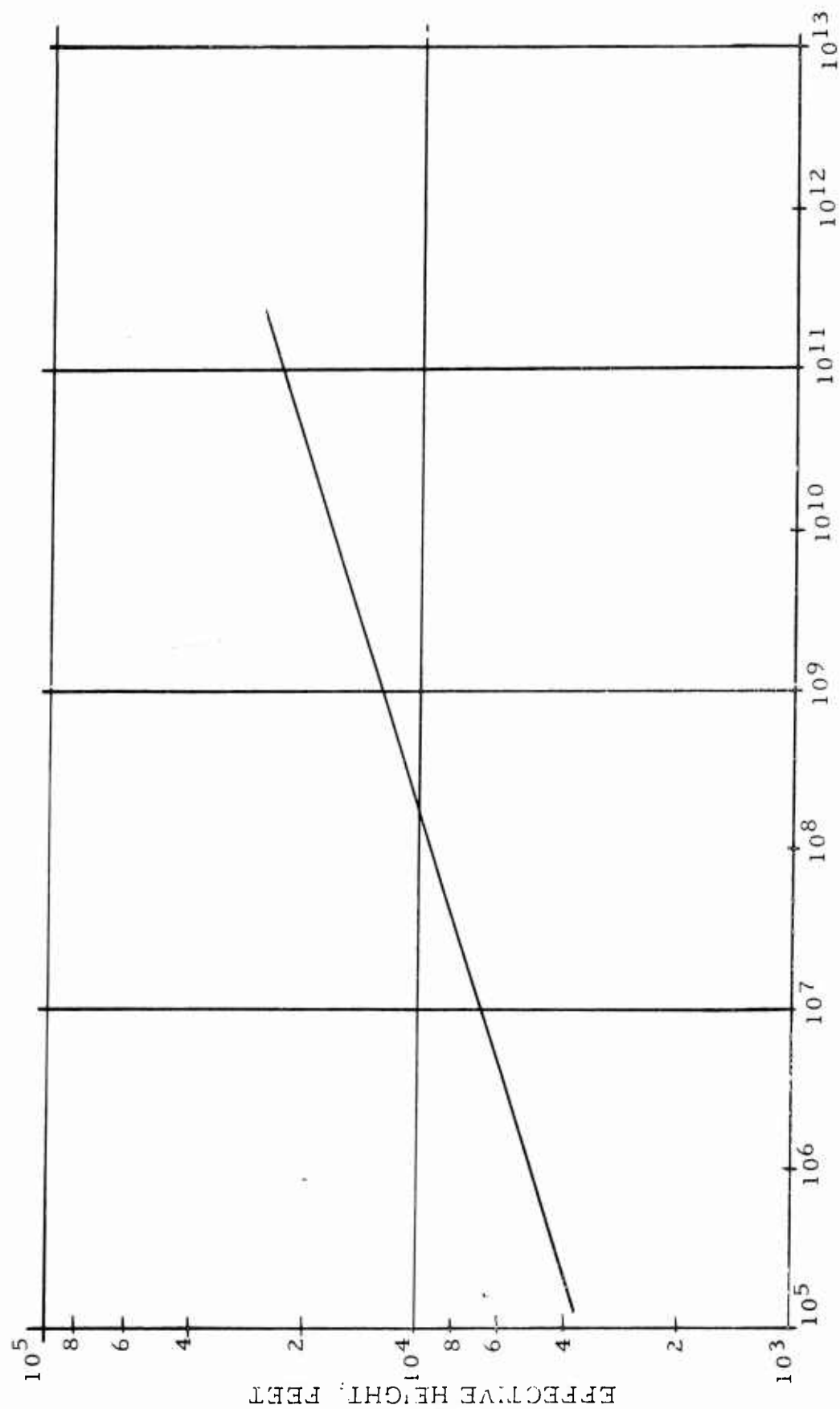
different magnitude explosions, a graph has been constructed of the thermal yield versus the effective height of the cloud in feet. The US Atomic Energy Commission has stated that the equivalent of one pound of TNT is 900 B T U. Therefore, if the heat release of an explosion is known, the effective height of the resulting cloud can be determined.

From the following graph (Figure 12) the height that the explosion cloud will reach is obtained. A complete detonation of a 600 # solid propellant grain will give an effective height of 4,700 feet (assuming 50% of the total heat is available).

Also, a complete detonation of a 11,300 # grain will give an effective height of 8,500 feet (assuming 50% of the total heat is available). The basis for this height calculation is that the total enthalpy of the solid propellant is 4346. calories per gram.

The probability of a detonation of such large solid propellant grains is low but it definitely is present. A test stand designed for toxic rockets must provide for such an eventuality. This can be done by completely containing any explosion, or by creating an open structure which will give the maximum upward impetus to the toxic materials. The open structure would be constructed with sloping sides (upward and outward) so that the maximum fraction of the explosive forces will be directed upward. Any vaporizing or powdering of the structure should be avoided since the explosive energy would be attenuated by this action.

A final recommendation for the use of the graph would be to calculate the explosive yield and the minimum safe effective stack height for the rocket to be tested. If the height given by the graph is too low, then either the test should be forbidden or a means of augmenting the explosive stack height should be found. One possible, but not too feasible, way to accomplish this would be to place a sufficient quantity of shock sensitive non-toxic explosive adjacent to the toxic rocket.



EXPLOSIVE YIELD, BTU

Figure 12. Buoyancy Rise of Explosion Clouds

14. Dimensional Analysis Formula for Effective Stack Heights

The effective stack height is a function of many variables which are both dependent and independent. By the use of dimensional analysis these variables may be related to each other and to the effective stack height. Dimensional analysis is useful since it assists in the determination of convenient arrangements of variables in a physical relation and helps plan systematic experiments. The first step is to list all the variables involved. This list is a result of experience and judgment as to the variables of importance. After the list has been made, the formal procedure of analysis is used to relate the variables.

The stack height equation can be written symbolically as:

$$\Delta h = f \left(G, V_s, u, \frac{d\theta'}{dz}, \frac{dM}{dz}, T, \Delta T, \right. \\ \left. g, Q_{v1}, C_p, d, \rho, \overline{M}, u \right)$$

The value of Δh will be computed in two distinct parts; those of upward momentum and of buoyancy. That is:

$$\Delta h = \Delta h_m + \Delta h_b$$

$$\Delta h_m = f(\text{Momentum})$$

$$\Delta h_b = f(\text{Buoyancy})$$

The assumption is made that all of the momentum is dissipated before buoyancy is added.

By use of the Buckingham π Theorem, a versatile mathematical tool, the two expressions for the plume rise were obtained. The momentum equation is:

$$\Delta h_m = \left\{ K' d^2 q \left(\frac{V_s}{u} \right)^a (Re)^b (Fr)^c - mg \right\} \frac{t_1^2}{M}$$

This expression is applicable to jet flow systems and also possibly to instantaneous upward releases or puffs. For a continuous flow the mass (M) would equal $G t_1$.

where:

$$q = \frac{1}{2} \rho V_s^2 = \text{Dynamic pressure of stream}$$

$$Re = \frac{d V_s \rho}{\mu} = \text{Reynolds' Number of stream}$$

$$Fr = \frac{V_s^2}{d g} = \text{Froude's Number} = \frac{\text{Inertia Force}}{\text{Gravity Force}}$$

The plume is assumed to lose no buoyancy during the time t_1 . Therefore, an expression for Δh_b would start from Δh_m and continues up to Δh_{Total}

Therefore:

$$\Delta h_b = K'' g^f d^k \Delta v^m \left(\frac{d\theta}{dz} \right)^{a'} Q_{v1}^{-c'} u^{-d'} V_s^{-e'}$$

where

$$\nu = \frac{\mu}{\rho} = \text{kinematic viscosity}$$

Therefore, it is shown that the buoyancy rise is a function of the force of gravity, stack diameter, gas viscosity, lapse rate, gas temperature, volume flow rate, wind speed, and gas velocity. Finally, from the above derivations, the equation for effective stack height is written by combining the two equations, one of momentum and one of buoyancy. The equation for effective stack height must be evaluated in terms of real numbers for K' , K'' , and the various exponents before it can be of any practical use. A program to accomplish this is possible. The parameters to be measured are part of the equation. A series of tests in which the various known quantities are changed would allow an evaluation of the constants and exponents. Such a series, at low energy levels, has been conducted.

Using the given data, a series of equations were set up to find the unknown constants and exponents. As is shown in the Appendix, the evaluation of the problem leaves an equation with twelve unknowns.

For a complete evaluation of the exponents and constants, this equation must be evaluated a total of twelve (12) times using twelve different runs. **One factor** which will assist in evaluating the buoyancy term is the necessity of maintaining dimensional consistency. The unit equation is:

$$(m)^{+1} = \left(\frac{m}{\text{sec}^2} \right)^f (m)^k (m^{-1})^{a'} \left(\frac{m^3}{\text{sec}} \right)^{-c'} \left(\frac{m}{\text{sec}} \right)^{-d'} \left(\frac{m}{\text{sec}} \right)^{-e'}$$

Therefore these relations must hold true:

$$1 = f + k - (a') + 3(-c') + (-d') + (-e')$$

and $0 = 2f + (-c') + (-d') + (-e')$

The above analysis indicates the degree of complexity involved in the calculation of effective stack heights. This holds true even when all constants and exponents are known, and when there are twelve unknowns the problem becomes one for a computer program.

15. Buoyancy Equation for Continuous Source

An expression for the rise due to buoyancy has been derived, based on the basic properties of the atmosphere and the laws of thermodynamics. The final form consists of a relation between buoyancy rise, heat emission rate, atmospheric stability parameters, and wind velocity. The equation is presented below; the entire analysis is in the Appendix.

$$\Delta h_b = \frac{0.050 Q_h^{0.864}}{(\bar{u})^{0.463}} \text{ for standard lapse (n = 0.25)}$$

The above expression assumes 100 per cent thermodynamic efficiency and is based on standard lapse conditions. The lapse rate will affect the work done against gravity in lifting the diffused air. For dry adiabatic lapse rates, the work done would be zero. However, a strong lapse will be limited in height, or infinite buoyancy would result. The strong lapse will result in an appreciable increase in buoyancy rise over isothermal or inversion conditions. The overall lapse rate effect from limited plume rise data is shown to be the following power of Q_h to provide lapse rate adjustment. Assuming 80 per cent thermal efficiency, the equation takes the following form:

$$\Delta h_b = 0.04 (Q_h)^{\frac{2-n}{2}} (\bar{u})^{-\frac{1}{2}}$$

Where Δh_b = maximum rise due to buoyancy, meters

Q_h = heat release above atmospheric, BTU/sec

$\bar{u}$ = mean wind velocity to maximum rise, meters/sec

n = stability parameter of Sutton where:

n = 0.20, strong lapse

n = 0.25, standard lapse

n = 0.33, inversion

n = 0.50, strong inversion

The above formula is the buoyancy effect rise of a hot plume from a continuous point source. This formula will at least give a theoretical maximum buoyancy height. When any other factors which might influence the buoyancy rise, are included in the expression, they might lower the calculated height. All indications are that the height will not be greater than the value obtained from this expression for standard lapse conditions. When strong lapse or inversion conditions are present, the equation will only give approximations, the answers being directed in the proper direction. As is recommended with any theoretical expression, this equation should be tested under a wide range of conditions before complete confidence is placed in it.

Considering only buoyancy due to heat, a comparison of several formulae has been made (Figure 13). The formulae shown are those considered to be the most useful for rocket engine testing. The graph plots plume rise in meters versus heat release rate. The Thomas formula is considered conservative but is based primarily on a heat release rate of 3500 BTU. Only the buoyancy portion of the Thomas formula is plotted on the graph.

The thermally derived equation based on published data provides a reasonable fit for much of the published data on observed plumes where the molecular weight of the cloud is essentially that of air. It is recommended that the buoyancy equation for continuous source be used until proven or modified by test observation. The wind velocities assumed on Figure 13 are estimated averages for the various lapse conditions. Also shown is the altitude attained by the horizontal firing of small rockets from the empirical equation. It is well to note that no consideration is given to the rate at which the propellant burns. For this reason, the use of this formula is not recommended for application to larger engines where high rates of heat release are obtained. Also since some energy must be expended in diffusion in the horizontal plume with further loss due to the momentum energy expended in the horizontal direction, much lower plume heights would be expected than with conventional formulae.

A presentation of the most useful inversion penetration equation is also shown to indicate how easily formulae break down at low heat release rates.

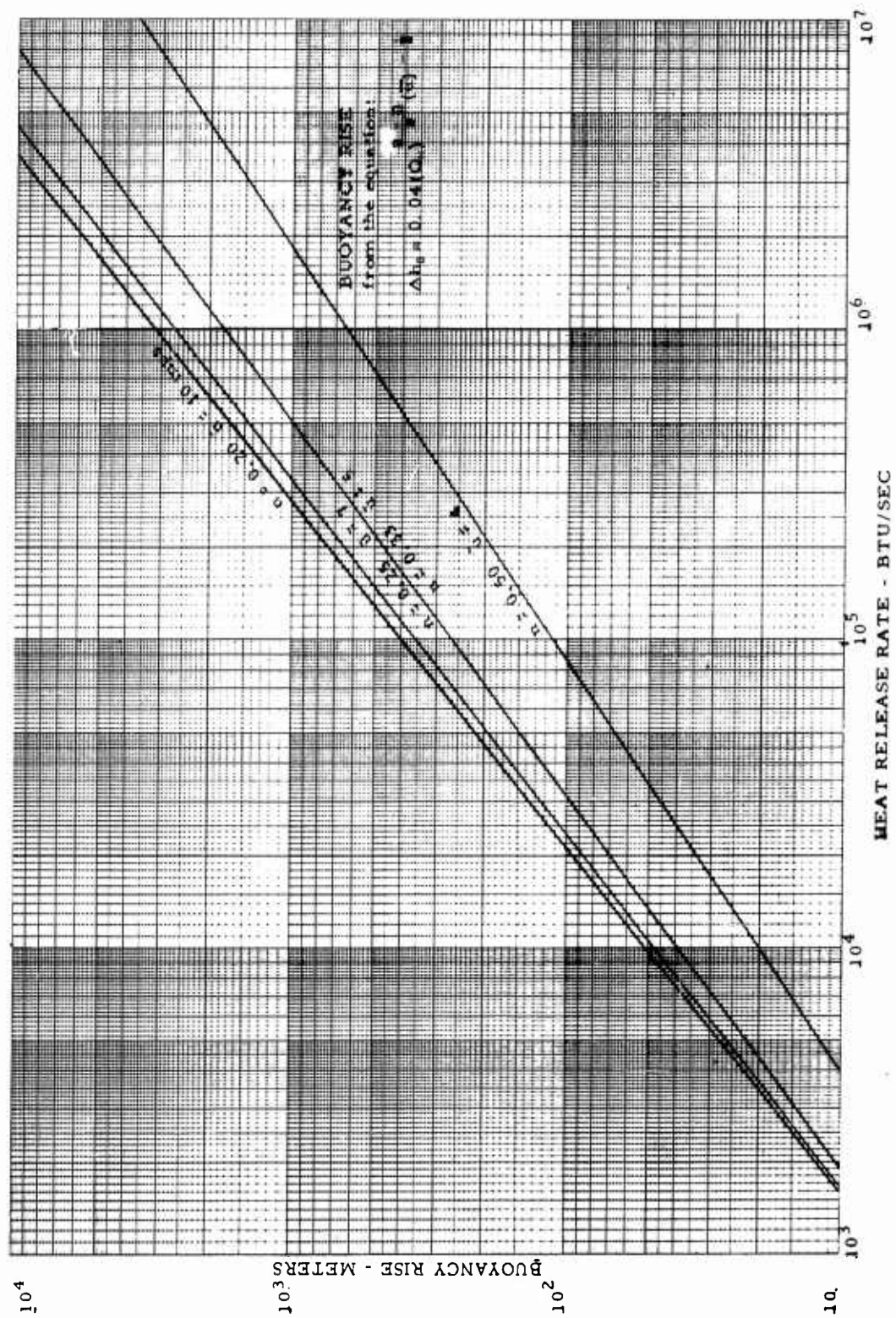


Figure 3. Buoyancy Rise from Continuous Sources of Hot Gas from Recommended Equation

General Discussion of Stack Formulae

From the above descriptions of the available formulae, the conclusion is that there is no perfect equation for the prediction of stack height over an infinite range. This is true whether the reference is to rocket firing, steam plants, atomic reactors, or industrial waste discharges. There is no equation that will give complete accuracy outside the region from whence it was developed. Various investigators have attempted to show the regions of application for certain formulae. Their findings have been that each equation will provide results with a certain magnitude of error. Part of this error is due to defects in the formulae and part is due to inherent properties of the atmosphere.

The equations of Sutton, Bosanquet, et al., and the Oak Ridge formula were studied by A. C. Best (Ref.15) who calculated the stack heights obtained under various wind velocities, and strength of heat source. His conclusions were ". . . that if the stack designer is concerned with the maximum ground level gas (or toxic) concentrations ever likely to be experienced, he will arrive at similar answers, probably within the limits of the variability of human reaction to specified concentrations, whichever formula he chooses." Best states that Sutton's equation gives only approximate answers. Bosanquet, et al., has been shown to give reasonable agreement with some full-scale observations, and the Oak Ridge gives "a reasonable approximation to the results given by the other two." How Best can come to these conclusions when he has calculated differences of over ten times for some of the examples of stack heights is not known.

Upon discussion of the article by Best before the Institute of Fuel in London, various dissident voices were raised. Dr. R.S. Scorer (Equation 9) stated that many phenomena of great importance were completely ignored in Best's presentation. The most obvious were the results of variations of height with changes in the stability of the atmosphere. Also, a Mr. Lucas stated that he thought Sutton's formula underestimated the plume rise, and that the Oak Ridge formula underestimated it by such a large factor that he considered it to be practically worthless.

J. E. Hawkins and G. Nonhebel, in their article on "Chimneys and the Dispersal of Smoke" (Ref.16), state that the Oak Ridge formula cannot be extrapolated over large ranges, but the Thomas equation is much better. Upon comparison of the Bosanquet formula with actual plume rise, an accuracy of $\pm 25\%$ was found in the chimneys observed.

As was mentioned above, under the Scorer formula, the work of Moses and Strom indicates that there is no one formula which is outstanding in all respects.

Some of the formulae are of an extremely simple form (i.e., The Oak Ridge formula) and probably do not cover all the necessary parameters to insure accuracy. In order to insure a reasonable approximation, they must include these parameters in empirically determined constants. These constants will apply more closely in regions near which they were determined.

The decision as to which stack height equation to use depends upon the type of effluent system present. In the case of explosive detonations the graph of explosive heat release versus effective height represents the most advanced information available. This graph is therefore recommended for use with detonations.

Where the rocket is oriented to fire horizontally, and a flame deflector is not used to direct the exhaust vertically, the equation for buoyant gases applies. This equation was derived from laws of thermodynamics and is estimated to give answers to 1 $\frac{1}{2}$ or 2 places of accuracy.

The directing of the exhaust out of a vertical stack will provide a momentum rise component to the gases. This rise is considered additive to the buoyancy rise and thus a summation of the two is made to obtain the total effective stack height. The momentum rise equation chosen is based on jet penetration studies in wind tunnels and is believed to give reasonable results.

The effects of atmospheric variations are important. They will introduce scatter in observed versus calculated heights and scatter will increase with distance downwind from the source of emission. Atmospheric parameters of most consequence are changes in lapse rate with altitude, wind velocity with altitude, and wind direction. These changes will cause eddies in any direct path that a plume might follow when unhindered. A forceful example of the effects produced by atmospheric variation on man-made clouds would be in the differences in height of nuclear weapons tests. A graph of mean cloud height versus explosive yield will show a scatter of points for any one nominal yield (Ref 17). For example, in the one hundred-ton range, the observed heights vary between seven and fifteen thousand feet above mean sea level. Also, in the twenty thousand-ton range, the heights vary between 25 and 35 thousand feet. Factors causing these variations include the atmospheric stability, moisture content, and the height of the tropopause (30,000 - 55,000 ft.)

All of the stack height equations contain a wind velocity parameter. This parameter is considered fairly constant during the calculation but, in actuality, has considerable variation with altitude. At ground level the wind velocity may be about ten to twenty feet per second but as altitude increases, the velocity increases. The mean wind velocity at the altitude of maximum winds is generally between 80 and 180 fps (Ref. 18) and these

upper winds may have velocities of 200 to 300 fps one per cent of the time. The altitude at which this occurs is between 30,000 and 40,000 feet, or approximately that of the tropopause.

The mean wind direction of the upper air is from the west by southwest, at the latitude of Edwards Air Force Base, at an elevation of about 35,000 feet. The steadiness parameter (mean vector wind/mean scalar wind) is approximately 0.8 for the mean wind at 35,000 feet. This will enable a prediction of fallout to be made with a greater average confidence level.

From the above discussion, it is shown that the exact height attained by a plume cannot at present be predicted with a high degree of accuracy. However, a minimum height and a probable maximum height are possible to calculate.

The maximum heights would be based on explosion heights for an instantaneous source, and the buoyancy plus momentum equations for continuous sources.

The maximum concentration of contaminants at ground level can be found if the minimum height is known. The energy of a plume may be augmented by flaring if the contamination is excessive, but in order to reduce the cost per test caused by flaring, the stack height should be known. A definite program for the determination of empirical formulae might be implemented. This program would include the static testing of rocket engines of different thrust levels.

During test firings, information should be recorded as to lapse rates, wind vectors, exhaust temperature and composition, engine thrust, test duration, and plume height as a function of distance downwind from the test stand. From these data, the dimensional analysis equation can be further developed to reduce the probable error of calculated stack heights from other methods. The probable error of the refined data would also be available for use, and from this combination a method of stack height prediction would be possible.

In the absence of precise data and experience with stack heights for large rockets at the Edwards Rocket Site, it is recommended that the following equations be used:

1. For detonation the relation between energy release versus stack height curve, Fig 12.
2. For horizontal firing the buoyancy equation, equation 15.
3. For vertical firing the buoyancy equation plus the jet penetration equation, equation 12.

4. After sufficient data are accumulated, a final solution of the dimensional analysis equation may be used.
5. During inversion conditions, a combination of equations could be used for cross-checking of predicted results. That is, the buoyancy and momentum equation's (15 and 13) answers would be compared to the answer from the inversion penetration formula presented below.

INVERSION PENETRATION

The atmospheric temperature has a daily variation with height. This variation is caused by solar radiation, absorption and emission of heat by clouds and the ground, and mixing of layers of air by wind forces. When the temperature of the air decreases with an increase in altitude, this is called a lapse condition and the rate of change is the lapse rate. The U.S. Standard Atmosphere has a lapse rate of 3.5 F/1000 feet. If the lapse rate is zero, an isothermal condition exists. When the air temperature varies directly with altitude, a "negative" lapse rate or an inversion condition exists. An inversion is caused by the rapid cooling of surface air. The nocturnal radiation of the ground without interference by clouds often results in inversion conditions.

An inversion is a critical condition during a firing of a toxic rocket. The hot exhaust gases will rise until they reach an equilibrium height within the inversion layer and will remain at this level until the inversion is dissipated by outside forces such as those produced by sunrise. This dissipation of the inversion will distribute the pollutant vertically due to the turbulent mixing which occurs during the formation of normal lapse conditions.

If the rocket exhaust gases have sufficient potential energy, they will penetrate an inversion. This energy is supplied by upward gas velocity, gas buoyancy, and the large amount of heat produced in the rocket combustion chamber. The inversion height which can be penetrated is mathematically definable as:

$$Z = f(g, Q_h, T_a, \rho, C_p, \Gamma)$$

Where:

Z = Vertical distance to top of inversion

g = Gravitation constant

Q = Heat potential of exhaust gases

ρ = Air density

C_p = Specific heat of air

Γ = Average lapse rate from ground to top of inversion

The expression becomes:

$$Z = 4.9 \left(\frac{g}{Ta} \Gamma \right)^{-\frac{3}{8}} \left(\frac{g Q_h}{Ta \pi \rho C_p} \right)^{\frac{1}{4}} \quad (\text{cgs units})$$

$$\text{or: } Z = K \left(\Gamma \right)^{-\frac{3}{8}} Q_h^{\frac{1}{4}}$$

$$\text{where } K = 4.9 \left(\frac{g}{Ta} \right)^{-\frac{3}{8}} \left(\frac{g}{Ta \pi \rho C_p} \right)^{\frac{1}{4}}$$

This expression has previously been used in the prediction of power plant exhaust plume paths (Reference 19 and 20). In that function, its form was:

$$Z = 784 \left(\Gamma \right)^{-\frac{3}{8}} Q_h^{\frac{1}{4}}$$

Γ is in F per 1000 feet.

Z is in feet.

Q is the megawatt rating of power plant based on the assumption that one third of the power goes out the stack.

Converting Q to BTU/sec available from a rocket exhaust, the expression becomes:

$$Z = K' \left(\Gamma \right)^{-\frac{3}{8}} Q_h^{\frac{1}{4}}$$

And:

$$K' = 784 \left(\frac{3 \text{ times the power}}{(948 \text{ BTU/sec) per megawatt}} \right)^{\frac{1}{4}}$$

$$K' = 186.$$

Then:

$$Z = 186 \left(\Gamma \right)^{-\frac{3}{8}} Q_h^{\frac{1}{4}}$$

This will provide a formula that will allow the scheduling of a toxic rocket test during an inversion. Given the parameters of inversion intensity and height, the minimum thrust level can be calculated. Conversely, the strongest inversion that a given thrust level may penetrate can be found.

As an example:

Given: 100K toxic solid rocket engine

Total heat available with afterburner (approx) = 1.5×10^6
BTU/sec

Strong inversion where $\Gamma \sim 20$ F/1000 feet

Solving for Z:

$$Z = (186)(20)^{-\frac{3}{8}} (1.5 \times 10^6)^{\frac{1}{4}} \text{ Feet}$$

$$Z = 2120 \text{ Feet.}$$

Since diurnal inversions of both this strength and height are extremely rare, the 100K engine could pierce almost any inversion.

There are other available means of overcoming inversions. The simplest is to avoid them by proper scheduling of tests for periods of minimum inversion probability. This would mean firing during late morning to early afternoon and avoiding the early evening to early morning inversion. Due to normal working hours and check out procedures, this should not be an inconvenience to test personnel.

Additional methods of overcoming inversions include the use of tall stacks, heat implementation of the rocket source, and proper upward velocity. A tall stack is the device generally used at power generating plants to achieve the dispersion of hot exhaust gases. These stacks are quite high, ranging to 700 feet (Reference 3) and, therefore, expensive. It is, of course, evident that any physical stack will help in normal testing to increase the effective stack height. The most likely stack height is one higher than all other adjacent structures, perhaps one of 50 to 100 feet.

Normally, the exhaust from the rocket will by itself provide an after-burning effect due to its fuel rich nature, since a large portion could consist of gaseous hydrogen and carbon monoxide. These gases will be emitted above their ignition temperature and will spontaneously ignite. Heat implementation would consist of injection of some inexpensive fuel into the high temperature fuel rich exhaust in the stack. After mixing with the toxic exhaust, the combined stream would be ignited by a pilot flame and the resulting torch would penetrate the inversion.

The velocity of the rocket exhaust gases emitting from the stack has an effect on the entrainment rate of the plume in the atmosphere. Excessive air turbulence could be generated if the gas velocity is too great, which

would cause dissipation of energy, preventing penetration of the inversion layer. The same applies to a gas velocity which is too low. A velocity of less than half that of sound and perhaps closer to a few hundred miles per hour would be optimum.

As has been stated above, inversions present a resistance to penetration. This holds true in both directions. That is, once the inversion has been penetrated, any subsequent fallout of toxic material will be inhibited by the top of the inversion. This condition, called lofting, will cause the plume to diffuse upward and sideways and very little downward. This is a most desirable situation and perhaps one that might be planned for, since the inversion impedes the toxic material from reaching the ground. At the same time, the toxic material is rapidly diluted by the normal lapse conditions above the inversion. It should be noted that an inversion need not disappear with the sunrise, but could persist for several days. Atmospheric conditions are not constant, and transient conditions may occur at any location and at any time. The probability of these changes taking place at any particular time can be predicted for the test site, but for large distances downwind, no rigid prediction can be made unless data are taken at these points. General weather forecasts, location of fronts and storms, and the time of day will assist in prediction of diffusion changes. When changes occur, averaging of diffusion calculations may be made to estimate the hazard of a change from normal lapse to inversion or washout conditions.

For proper test scheduling, it would be advisable to determine the strength and height of inversions at the test site. The standard method is the use of a radiosonde, a balloon-borne device which transmits temperature and pressure to a ground station. The data transmitted to the ground are then reduced to a plot of temperature versus altitude, since altitude is a function of air pressure. The lapse rate is then directly obtainable, and the existence and strength of any inversion will be known. Whether the inversion can be penetrated can then be calculated and a test firing scheduled accordingly. The existence and strength of the inversion layers need to be monitored before, during and after tests to predict hazards. A computer is the only means of accomplishing this. The confidence in the data is dependent directly on past experience and the immediate availability of applicable data.

SECTION II

APPLICATION OF METEOROLOGICAL CRITERIA

The value of meteorology in the efficient design and operation of a toxic rocket test stand will be found upon consideration of the influencing factors pertinent to rocket propulsion testing. These factors involve dispersion, attainment of stack heights, the penetration of inversions, and the effects of total integrated dosage upon test system design. To attain the degree of reliability required for an operational system, a knowledge of specific toxicity levels, improvement of predictabilities, meteorological data systems and sampling systems is required.

The following section of this report presents discussions and analyses on the various toxicity considerations and the effect of a finite emission time versus a continuous emission as it would pertain to limitations in test durations. Also covered are the variability and reliability of toxic concentrations as predicted by equations and as actually determined by instrumentation. The degree of prediction error is calculated in order to establish a basis for improving the ability to predict toxicity levels.

The instrumentation requirements for the determination of meteorological and toxicity parameters are presented. Weather stations and the data reduction system required for a complete dispersion analysis are introduced. The toxic sampling network for determination of hazardous conditions is also related to data reduction.

It has been found that specific toxic rockets may be safely tested in the proper environment. A complete analysis of the various test stand configurations and operational descriptions will be found in the report, "Design of a Toxic Rocket Test System", SSD-TDR-62-137, (Reference 30) which was published simultaneously.

TOXICITY CONSIDERATIONS

The necessity for any study of the dispersion of rocket exhaust products is based on the observation that certain substances are injurious to human life. Their effects are observed in both chronic and acute diseases as well as systemic poisons or annoyances. That the use of these toxic chemicals in rocket technology is becoming more common is apparent. The justification for this increased use of toxic chemicals is found in their higher energy content, improved storability, or other properties. These improvements in propellants have the inherent advantage to improve vehicle performance either in the increase in range or payload.

Propellants are generally divided into two types: liquid and solid, although a third type is also possible (hybrid or tripropellant) in which both liquid and solid phases occur. The presence of two varieties of propellants and three possible forms, solid, liquid, and gas for rocket exhaust products increases the complexities of the toxicity problem.

The effects of a toxic substance upon animal or human life depend, in general, on its physical state. A liquid may be absorbed through the skin, swallowed, entrained in air and breathed, or the vapors may be breathed. Typical effects of a toxic fuel spill such as UDMH or hydrazine, are choking, difficult breathing, nausea, and upon extreme exposure, convulsive seizures and death.

Of course, these symptoms would not occur unless the concentration of vapors are of dangerous proportions, and any concentrations of that magnitude would exceed allowable dosages.

There are generally three limits of concentration specified for a chemical; an environmental one for the general public, an eight-hour or working day concentration, and an evacuation limit for everyone who might be exposed to the chemical. Depending on the chemical, the eight-hour limit might vary from 5,000 ppm for a mildly toxic substance like carbon dioxide to 0.01 ppm for an extremely toxic substance like pentaborane.

The method of poisoning can come from a number of ways but, in general, is limited to inhalation of vapors or cutaneous action for corrosive propellants. The chances of personnel dosages by oral, intramuscular, intravenous, or subcutaneous means are low with regard to rocket propellants at test stands or launch sites. Before any usage of toxic chemical at a facility, the noxious properties must be defined and suitable safeguards provided for personnel. These safeguards include safety clothing and breathing apparatus, toxic detection equipment, and decontamination provisions.

The liquid propellants can be introduced into the body by many different routes, but solid toxic substances have only three ways to enter the body; through cuts, ingestion and by inhalation.

A brief presentation of the respiratory system and its action on solid phase particulate matter is included to aid in the understanding of deposition of toxic matter in the lungs. With the point of initial intake being either the nose or the mouth, air passes in succession through the trachea, bronchi, bronchioles, alveolar ducts, and finally into the alveolar sacs. The over-all structure consists of a series of branching passages decreasing in size and increasing in number as the distance from the point of intake increases. The hairs and small bones of the nose provide an efficient filtering system for large particles, while the hair-like cilia, lining the respiratory ducts above the bronchioles, trap and transport to the mouth any insoluble particles trapped in this region. The very fine particles penetrate to the deeper recesses of the lungs and are deposited there to varying degrees depending on their size. Particles deposited in the alveoli and alveolar sacs are ingested by migratory cells and transported, with a few passing into the bloodstream, through the lung tissue to collect in the lymphatic system. Eventually the particles are carried to the bronchioles, then to the ciliated regions and from here eliminated. Analysis of particles size distributions in the inhaled and expired air of human subjects and examination of lung deposits of deceased workers has been used to study the mechanism of deposition. The result of this analysis is reproduced and shown in Figure 14, after Green and Lane (Reference 25). If the characteristics of the toxic rocket motor are similar to an aluminum motor, one would expect the solid exhaust products to be in the range of maximum retention, approximately 0.8 to 1.6 microns in diameter.

The possibility of poisoning from toxic solid propellants is presently less than from liquid propellants because they are not yet in general usage. This condition promises to be only a temporary one since extensive thought and research is being expended for various toxic solid compounds.

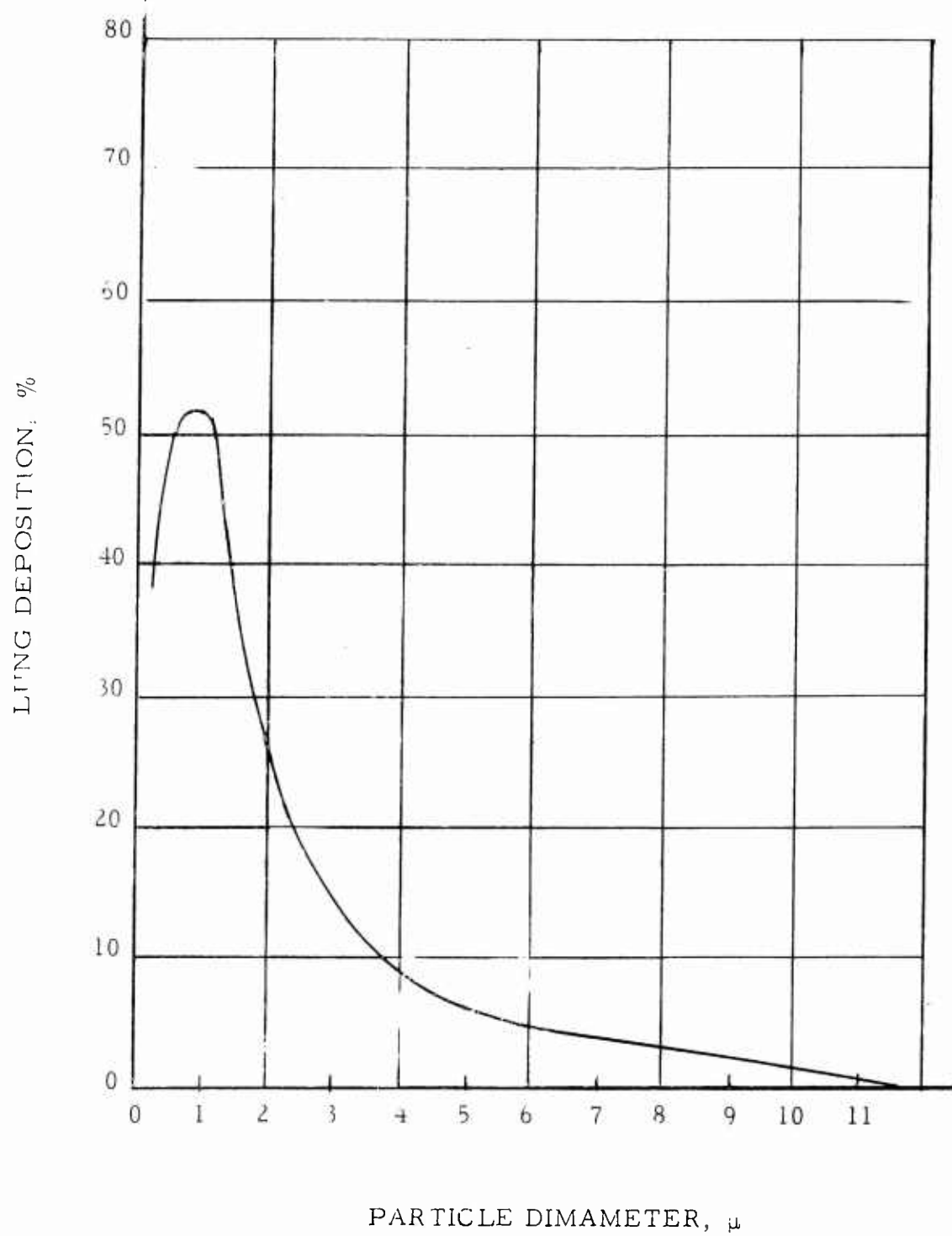


Figure 14. Particle Deposition Rate

LIMITED TEST DURATION

During rocket motor test, the emission is normally short time firing rather than instantaneous or continuous. A study of meteorology will reveal that short time firings or finite emissions will never exceed the downwind concentration found with a continuous point source of the same emission rate. When the test duration is long, the downwind concentration from a finite emission approaches the continuous point source concentration as a limit. When the plume reaches the point of interest downwind before the emission is discontinued, the concentration will be very nearly that for a continuous source. When the plume reaches about a quarter of the way toward the point of interest before the emission is stopped, the peak concentration will be about three quarters of that from a continuous point source.

The downwind concentration from a finite source will never exceed the concentration from an instantaneous point source if the quantities of toxic products emitted are identical. The total amount of substance emitted is compared in this instance, while the rate of emission was used in comparing finite with continuous emissions. For short duration and finite release, the instantaneous point source plume is approximated and so are the downwind concentrations. In fact, with increased distance, the clouds are practically identical in shape and concentration distribution. For a release time as long as 500 seconds in a turbulent atmosphere with a ten mile per hour wind, the maximum concentration for the finite release is about 90% of the concentration from an instantaneous release ten miles downwind. At greater distances the concentrations become essentially identical.

For short duration testing of rocket motors up to several minutes running time, the instantaneous point source emission equations are appropriate for estimating downwind concentrations. The total integrated dosage for instantaneous point source release can be applied to finite release to determine dosages downwind.

The relationship between center of plume concentrations for continuous point source emission and instantaneous emission can be expressed as follows:

$$\frac{\chi_c}{\chi_i} = \frac{\pi^{\frac{1}{2}} C x^{(2-n)/2}}{u \theta}$$

Where: χ_c = concentration downwind of continuous point source

- χ_1 = concentration downwind of instantaneous point source
 C = diffusion coefficient
 n = Sutton's stability parameter
 x = distance downwind of point source
 u = wind velocity, meters/sec
 θ = time in seconds required to emit from the continuous point source the amount of material issued instantaneously from the instantaneous point source

The duration of integration for determining concentrations along the axis of the plume is given as follows: (References 4 and 27)

Duration of integration:

$$\theta_I = \frac{5.48 C d'^{(2-n)/2}}{u}$$

Where d' = distance downwind or is equal to x and other symbols have the usual meaning

By dividing the duration time for a finite emission by the duration of integration, a fraction is obtained from which the ratio of concentration for a finite release compared with a continuous emission can be determined. A chart in Reference 27 can be used to determine the relationship. The curve is a straight line on log-log plot to the point of 60 % finite release concentration. The straight line portion of the curve can be expressed as follows:

$$\log \frac{\chi_f}{\chi_c} = 0.9813 \log \frac{\theta_f}{\theta_I} + 0.4362$$

Where χ_f = concentration downwind of a finite point source release for time θ_f

From these expressions, a table can be developed to show the relationship between concentrations downwind of continuous, finite, and instantaneous point source releases. This table is based on unstable atmospheric conditions, but the relationships hold true for other atmospheric conditions with increased downwind distances. The first column of the chart gives distance. The second column (χ_c / χ_1) presents the ratio of downwind concentrations for continuous as compared with instantaneous releases.

The period of time required for the continuous point source to evolve the same amount of effluent establishes the relationship between the source strengths and are shown for periods of 5, 30, and 500 seconds.

The third column of the chart shows the relationship between χ_f/χ_c for various finite release times where the rates of release are identical. The ratios are shown to increase with increased release time and to decrease with downwind distance. The last column shows the product of column (2) times column (3), which shows χ_f/χ_i . For example, where a five-second finite firing is compared with an instantaneous source releasing the same amount of effluent in a very short period of time, the downwind concentrations are the same within 3% at one mile distance.

RELATIVE DOWNWIND CONCENTRATIONS FROM CONTINUOUS, FINITE, AND INSTANTANEOUS POINT SOURCES

Distance	$\frac{\chi \text{ continuous}}{\chi \text{ instantaneous}}$			$\frac{\chi \text{ finite}}{\chi \text{ continuous}}$			$\frac{\chi \text{ finite}}{\chi \text{ instantaneous}}$		
	Finite Firing Time for Equal Total Emissions								
Miles	5 sec*	30 sec*	500 sec*	5 sec*	30 sec*	500 sec*	5 sec	30 sec	500 sec
1	24.2	4.03	0.242	0.04	0.23	1.0	0.968	0.927	0.242
5	104.0	17.33	1.04	0.0095	0.056	0.77	0.988	0.972	0.802
10	194.0	3.23	1.94	0.0051	0.0305	0.46	0.990	0.985	0.893

* Where Q_c , continuous emission rate, is such that an amount equal to Q_i is liberated every 5, 30, and 500 seconds, respectively.

* Duration of finite emission at rate Q_c .

In summary, instantaneous point source concentration equations may be utilized for finite releases for rocket test systems.

CONCENTRATION VARIABILITY AND RELIABILITY

Dispersion coefficients for diffusion formulae have been determined for a number of locations. The process of diffusion is influenced by many conditions of the atmosphere and topographic features. However, as shown by Ocean Breeze experiments run in the Atlantic and Pacific Missile Ranges, prediction of ground level diffusion can be made within a factor of two with a 99% confidence level. Ground level diffusion varies more with atmospheric conditions and surface influence than does elevated diffusion, but data are more accessible near the ground than at points above. With sufficient upper air measurements, diffusion in the upper air layers will be as predictable as that at the surface. A factor of between 2 to 1/2 the predicted concentration should be possible given the height of the diffusion process and atmospheric condition along the predicted path.

The height at which the supposed diffusion process takes place is not easily predicted. For ground level concentrations, the actual vertical profile of the smoke plume is less important than the ultimate height to which the plume rises and from where the greater part of diffusion can be assumed to take place. Various effective stack height formulae have been derived to predict the plume path and/or maximum height of plume rise. The effective stack height includes not only the physical stack height when used as the source of emission, but also exit velocity effect and cloud buoyancy.

Atmospheric influence on effective stack height is very complex, and most stack height formulae are derived for limited conditions and are overly simplified. Some formulae do not consider effluent exit velocity, others consider plume rise within a short distance of the stack, and still others neglect the effect of atmospheric lapse rate on plume path. The Thomas effective stack height formula has been derived empirically to predict actual smoke plumes issuing from steam generating plants. It is intended to yield maximum rise of effluent rather than the short distance downwind used in the Oak Ridge formula, which is admittedly conservative. The energy release of power generating plants is more nearly equivalent to third stage ICBM motors and is probably most reliable for this size motor. Smoke plumes from smaller engines tend to lose buoyancy by rapid diffusion and larger plumes are rendered less buoyant by stable atmospheric lapse rates. Detonation of explosives equivalent to second stage Minutemen have produced smoke clouds with center-line elevations in excess of 2000 meters. Explosion of 400-pound charges have produced smoke clouds with top-of-plume elevations of 1500 meters. The Thomas stack height formula appears to be fairly accurate and possibly conservative for small upper stage motors, but possibly optimistic by a factor of 2 for very small propellant charges and for second stage ICBM solid motors.

Assuming dispersion and stack height errors are cumulative, the resulting error in predicted concentration can be off as much as eight fold. Until data have been collected from dispersion and stack height measurements, a safety factor of 10 is recommended for initial rocket testing.

The maximum downwind concentration from an elevated, instantaneous point source varies inversely as the cube of the stack height. The distance to this point is influenced by the ratio of stack height over dispersion coefficient raised to a power of 1.1 to 1.3, depending on atmospheric stability. However, the total integrated dosage, which may be considered of more significance than maximum concentration, varies inversely as the square of the effective stack height. This means an estimated stack height twice actual will result in a total integrated dosage of 4 times predicted. The maximum total integrated dosage is not affected by dispersion coefficients assuming constant atmospheric conditions and the absence of weather fronts, rainfall and other major disturbances between the source and point of maximum contamination. The difficulty of monitoring for toxic products at various locations and at great distances downwind suggests a safety factor of up to 10-fold for larger engines. Small engines, which are more easily monitored and where area control can be maintained might soon be fired using smaller safety factors than 10-fold as the confidence level is improved. Even the dispersion and stability parameters for major types of atmospheric conditions may vary as much as 50% from one type to the next. Without an accurate statement of atmospheric conditions, prediction of concentration of toxic products cannot be precise.

Actual prediction of concentration of toxic products is not necessary. The intent is to establish safe limits and avoid overexposure of any inhabited areas.

In general, toxic limits as established by the Manufacturing Chemists' Association and the Atomic Energy Commission are apparently conservative. Another safety factor of 10 appears overly conservative. However, human lives are at stake and the results of overexposure are conclusive. Once confidence levels are established within a narrower range for three dimensional dispersion, a safety factor of 10 may not be required below allowable concentrations recommended.

IMPROVEMENT OF PREDICTABILITY

The amount of error inherent in any one case of dispersion can be reduced to a minimum when more complete data, recorded from previous tests, have been added to the prediction equations. These increased data will include topographical features of the test site, characteristics of the tested propellant systems, and atmospheric conditions. The topographical features comprise the mechanical eddies generated by terrain and structures in the immediate vicinity of the stack, and by the stack itself. Also included in this category are the terrain features downwind of the stack, since these features influence the surface boundary layer at the distance of maximum concentration (when dispersed from a stack).

Necessary, therefore, to a comprehensive test program is a thorough survey of any proposed test sites to determine the optimum location. A trade-off will be involved in which access to the site, load bearing ability of soil, and topography related to meteorology are all considered.

The characteristics of the propellant system consist of the toxic ingredient(s) and their quantities, the thrust level, and orientation of the exhaust. These are easily determined parameters which are of great importance in predicting stack heights.

Observations of maximum plume rise can be equated to rocket parameters. Standard formulae for plume rise include the parameters of gas buoyancy, upward velocity, and atmospheric lapse rates.

Atmospheric conditions affecting dispersion are: wind velocity and gustiness, wind variation with altitude, lapse rate and variation in lapse rate with altitude, vertical air density structure, atmospheric turbulence, and wind shear.

If any of the above sets of parameters is not properly considered in the prediction of dispersion of toxic effluents the resulting calculations will not agree with the actual observed concentration of contaminants.

Assuming all variables of importance are known and determined, the accuracy to which they are known is of next degree of importance to the determination of the predictability of the final result.

The probable error of a calculated function can be derived. The simplest example in the present study is the calculation of the maximum toxic concentration, $X_{\max}$ for a continuous point source. It is written as

$$X_{\max} = \frac{2 Q}{e \pi \bar{u} h^2} \frac{C_z}{C_y}$$

where $C_z/C_y = 1$ for isotropic conditions. To determine the probable error of $\chi_{\max}$, the probable errors of Q (emission rate), $\bar{u}$ (wind velocity), and h (effective stack height) need to be known. Error theory can be applied to functions of several variables, the best value of the derived quantity being obtained by substituting the mean values of the several variables in the function. Defining the probable error, Q_e , of the best value, p , of the function:

$$p = f(S_1, S_2, S_3, \dots, S_N)$$

and relating it to the probable errors, $Q_1, Q_2, Q_3, \dots, Q_N$, of the mean values $m_1, m_2, m_3, \dots, m_N$, of the several independently measured quantities, $S_1, S_2, S_3, \dots, S_N$ by the following equation:

$$Q_e = \sqrt{\sum_j \left(\frac{\partial p}{\partial S_j} \right)^2 Q_j^2} \quad (\text{Ref. 29})$$

If this expression is expanded for the present case, it becomes:

$$Q_e = \sqrt{\left(\frac{\partial \chi}{\partial Q} \right)^2 Q_Q^2 + \left(\frac{\partial \chi}{\partial \bar{u}} \right)^2 Q_{\bar{u}}^2 + \left(\frac{\partial \chi}{\partial h} \right)^2 Q_h^2}$$

An example problem is presented:

If $\chi_{\max}$ is desired,

$$\text{and } Q = 1000 \pm 10 \text{ gm/sec}$$

$$\bar{u} = 10 \pm 0.5 \text{ meters/sec}$$

$$h = 50 \pm 5 \text{ meters}$$

$$\begin{aligned} \text{Then: } \chi_{\max} &= \frac{2(1000)}{e \pi (10)(50)^2} \text{ gm/m}^3 \\ &= 0.009368 \text{ gm/m}^3 \end{aligned}$$

$$\chi_{\max} = 9.368 \times 10^{-3} \text{ gm/m}^3$$

$$\text{or } \chi_{\max} = 9.368 \text{ milligram/meter}^3$$

The probable error is then computed:

$$\frac{\partial \lambda}{\partial Q} = \frac{2}{e \pi \bar{u} h^2} = 9.368 \times 10^{-6}$$

$$\frac{\partial \lambda}{\partial \bar{u}} = \frac{2 Q}{e \pi h^2 (-\bar{u}^2)} = -9.368 \times 10^{-4}$$

$$\frac{\partial \lambda}{\partial h} = \frac{2 Q}{e \pi \bar{u} (-2 h^3)} = -9.368 \times 10^{-6}$$

Then:

$$\begin{aligned} Q_e &= \sqrt{(9.368 \cdot 10^{-6})^2 (10)^2 + (-9.368 \cdot 10^{-4})^2 (0.5)^2 + (-9.368 \cdot 10^{-6})^2 (5)^2} \\ &= \sqrt{87.76 \cdot 10^{-10} + 87.76 \cdot 10^{-8} (0.25) + 87.76 \cdot 10^{-10} (25)} \\ Q_e &= \sqrt{44.76 \times 10^{-8}} = 6.69 \times 10^{-4} \text{ gm/m}^3 \end{aligned}$$

Therefore, $\lambda_{\max} = 9.368 \pm 0.669$ milligram/meter³

The number of significant figures present in the above values of $\lambda_{\max}$ is too great. A value more in keeping with the given values of Q , $\bar{u}$, and h is:

$$\lambda_{\max} = 9.4 \pm 0.7 \text{ milligram/meter}^3$$

and the value is correct to about 7%. From this example it is clear that all of the variables in an equation are important when considering the probable error of the dependent variable.

DATA SYSTEMS

Micrometeorological Stations

Data stations are a necessary part of the toxic test facility. Since such data stations exist and are well within the state-of-the-art, a thorough description of these stations will not be presented. For specific test programs in which the particular propellant characteristics are known, the rate of heat release vs. the buoyancy rise is given (Figure 13). The heights which are attainable in advanced development programs are greater than those covered in previous diffusion studies. Towers and other equipment are required to determine the prediction capability for plume rises above normal inversion layers and in excess of 3000 feet altitude.

Vandenberg Air Force Base has a satisfactory meteorological station network for diffusion studies (Reference 28) and most of their hardware can be duplicated for such stations as required by the Rocket Site.

It is recommended that three towers 150 to 200 feet high are provided for the purposes of testing and monitoring the required meteorological conditions at the Rocket Site. These towers would allow the obtaining of data pertaining to lapse rates and wind velocity gradients, which were not necessary at PMR because non-buoyant releases were the only types covered. The use of towers would reduce the dependency on radiosonde or wiresonde equipment which would otherwise be required. The data obtained from towers are continuously available and will reduce pretest complications by permitting an uninterrupted meteorological forecast to be made. A computer is the optimum means for achieving accurate and continuous forecasts due to its data retention and short time readout capabilities.

These towers will monitor at specified heights such parameters as wind velocity, wind direction, temperatures, temperature differences between various heights, and heat radiation. The data measured at these towers should be transmitted to a data reduction center for rapid analysis. A computer is recommended for data correlation at this point so that information can be transmitted as rapidly as possible to cognizant personnel conducting the tests.

A helium filled radiosonde or wiresonde capable of 500 feet altitude with future expansion capabilities to 2500 feet is also desirable in order to determine lapse rate. The information transmitted from the radiosonde should also be transmitted to the central data reduction center for rapid data reduction. Radiosondes and wiresondes have inherent economic and accuracy drawbacks which tend to reduce their use to a necessary minimum. High low-level winds tend to reduce their usefulness since launching is more difficult and a vertical data profile cannot be obtained.

Cameras are required in addition to the other meteorological instruments. The cameras will be used to record height, size, and velocities of plumes. The recordings will be made of nontoxic plumes for meteorological studies as well as for actual test conditions when the plume will contain toxic material. The use of smoke plume photography is a relatively recent innovation, but is particularly appropriate for air pollution studies, since direct measurement of the dispersion caused by wind turbulence may be obtained. The method has a number of advantages but fails when the visibility is low. Analogous techniques to the photography of plumes is the use of infra-red cinetheodolites, radar tracking by use of radar reflective additives, or the addition of small balloons to the effluent and optically or visually following them. Cameras have been used by NASA at the Langley Research Center, Langley, Virginia. Also, an angle-of-attack sensor attached to the nose of a Scout vehicle was used at Marshall Space Flight Center.

In an implementation program, it is recommended that the three towers and the low altitude measuring devices be installed first. As the interest in larger toxic rockets increases, the high altitude and camera instrumentation should be added.

Sampling System

A rocket test stand configuration employing toxic propellants must be designed to safely handle both the propellants and the exhaust products. These chemicals will be emitted wholly or in part to the atmosphere where they will be dispersed by the macroscopic turbulent available energy. Due to the possibility of a large variation in thrust levels between tests and the change in quantity or type of toxic exhaust product, no single device could monitor all possible contaminants. Some possible toxic materials present during rocket testing are: (before combustion) hydrazine and its derivatives, nitrogen tetroxide, fluorine, methyl alcohol, nitric acid, ammonia, pentaborane, diborane, borohydrides, ozone, benzene, and others.

Upon combustion, many of the above chemicals become relatively harmless, but many exhaust products are dangerous. Poisonous compounds of boron, fluorine, chlorine, as well as carbon monoxide, are sometimes present and must be considered.

Because of the extremely low exposure limits for the more toxic compounds, the concentration must be monitored to determine hazards at the test stand and in the surrounding area.

Monitoring techniques have been developed for use with toxic liquid propellants, such as hydrazine, UDMH, and nitrogen tetroxide. These chemicals, for use with the TITAN II and other weapons systems, must be carefully handled and every precaution must be exercised to prevent spills. When spillage does occur, the presence and spread of gaseous vapors must be detected quickly. Examples of excellent existing systems for this purpose are found in those developed for the Atlantic and Pacific Missile Ranges (Reference 5). Here, a controlled emission of zinc sulfide aerosol was allowed to disperse, and upwards of 500 air samplers were used to detect its concentration at locations downwind of the source. The samplers were positioned in two and in one case three arcs in excess of 90° spread in the prevailing wind directions and in a pattern near the aerosol generator. The data collected by this system were used to derive highly reliable dispersion coefficients for use in calculating diffusion rates for spills. In addition, a series of equations for the prediction of aerosol concentration with distance was developed from these experiments.

A similar program was performed by the Stanford Research Institute (Reference 24), at Edwards Air Force Base, to determine dispersion coefficients. The coefficients may be used in the Sutton diffusion equations to calculate downwind concentrations. The Stanford Research Institute study was performed to determine the hazards of a hydrogen and fluorine test stand at the Edwards Rocket Site. Ground level dispersion was the only factor investigated in this study.

Because of a high level of toxicity possible with the more exotic fuels, a program to evaluate dispersion from elevated sources is required and would use a more elaborate data gathering system than used heretofore at Edwards. Sampling stations are required both at the Rocket Site and outside of the boundaries of Edwards Air Force Base. Appreciable concentrations of toxics could exist at even hundreds of miles downwind of a high thrust firing and the hazard at ground level must be known. It has been shown that sampling equipment up to at least 150 miles downwind of the Rocket Site would be required for direct release into the atmosphere. A total of approximately 75 units would be needed. Three central data gathering units in 45° arcs would be the minimum for safety.

Some meteorological systems use more than 75 sampling units. At Vandenberg Air Force Base (PMR) hundreds of zinc sulfide samplers were used in the development of the "Ocean Breeze" formulae. Also at Vandenberg samplers are required for monitoring propellant spills of hydrazine, UDMH, and nitrogen tetroxide.

A small toxic test stand would require only a few samplers if the propellant toxicity is of a low order whereas the larger systems require greater sampling complexity for safe operation. The rate of toxic emission and the toxic properties of the propellant will determine the sampling system.

The actual equipment needed to accomplish the detection of toxic propellants or exhaust products may vary widely in form, operation, and cost. For the volatile chemicals a number of methods are possible; gas chromatographs or sensitive chemical detectors being but two ways to detect toxic gases. Toxic solid materials require different methods of analysis. First particles of finite size have to be collected and then must be analyzed. This could be done by air samplers which would gather the toxic solids close to the level at which they would be inhaled. The sample may be collected on a filter paper for analyzing. The detection of radioactive waste material will probably become necessary in relation to propulsion systems. Therefore, the means of detecting a specific toxic effluent will be chosen as rocket test schedules require since new propellant combinations are continually being discovered.

Data Handling System

A data handling system is an important auxiliary to a toxic test facility and will be used to assist in the prediction of meteorological conditions and indicate whether a test could be safely initiated. Input into the system would include data from radiosonde balloons, local weather stations, general weather reports, and facility-based instruments. In addition, the type, orientation, and thrust level of the toxic propellant rocket would be added to the general equation of the computer. The safe firing time would be obtained from this input. In addition, a plot of predicted downwind concentrations as a function of real time, distance, and changing weather would be possible to derive.

Implementation of such a system may follow a number of paths. The least expensive would be a manually operated calculator with the calculations done only as required. The most complex system would be a computer providing continuous plots of test conditions. The chosen system for the requirements presented will be the optimum of these two extremes. It would give the go - no-go prediction based on short time inputs. The time lag between the taking of meteorological and other data and the go - no-go prediction would be a dominant factor in the choice of data handling system complexity.

TYPICAL APPLICATIONS TO ROCKET TEST SYSTEMS

Design of a test system depends on the development of design criteria. Motor sizes and characteristics are of highest importance in design considerations since every effort must be made to obtain data of the actual test item operating conditions. Where highly toxic propellants and propellant exhaust products are involved, measures must be taken to prevent overexposure of personnel to toxic products. The test system must be designed to provide the protection needed. Anticipated test frequency for various size motors must be established to prevent excessive set-up time and undue scheduling of test runs. The frequency of testing is important in determining allowable release of toxic products both from the standpoint of total dosage at inhabited areas downwind and the probability of suitable weather conditions for safe dispersion of toxic products.

Once the test program has been established, meteorological principles can be employed to estimate the toxicity hazards that can result from such a test program. A study of meteorological phenomena will suggest ways of using atmospheric dispersion to the greatest advantage. Emission of effluent from one large stack is recommended over many small stacks. The ideal stack for maximum dispersion of effluent is tall, large in diameter, and with high exit velocity and buoyancy of effluent. The stack should be high enough to prevent downwash due to turbulence caused by structural and topographical features near the stack. A stack at least twice as high as obstructions should be employed.

Current meteorological information will be required for the test site to determine go-no go conditions where climatic influences can result in hazardous conditions from a test or malfunction. Sufficient meteorological measuring instruments must be provided to distinguish between safe and unsafe firing conditions. Data reduction equipment with the capacity to make current predictions of safe conditions must be provided. Sophistication can range from simple pen type recorders of wind velocity, direction, variability and temperature lapse rate to an elaborate network of instruments feeding information into a computer for continuous prediction of toxicity limitations. The more complex the meteorological station, the greater the initial cost. Maintenance and operating costs will not necessarily increase with the complexity of the meteorological system. For highly toxic exhaust products where great distances may be involved for adequate dispersion, the weather network and toxicity monitoring system may become so extensive that greater economy can be realized through application of safety measures and devices at the source. Without sufficient monitoring devices for weather and for toxicity determinations to improve confidence levels for dispersion determinations, reliance on atmospheric dispersion is dependent upon local

dispersion determinations and effective stack height measurements. Sufficient area meteorological information must be available to determine rainfall and frontal conditions between the test facility and downwind areas.

An economic balance between test system, meteorological network, and operating costs should be established. Further considerations include the value of meteorological data accumulated for use on other programs and estimated savings that may be realized in future facility design. The meteorological station will be useful and possibly necessary for scheduling toxic liquid propellant handling operations anticipated in the near future. A highly sophisticated micrometeorological network is needed for spill tests, future toxic liquid propellant transfer facilities, small toxic solid motor testing, and local monitoring of large toxic solid motor test system.

One method of simplifying a test system design and capital investment would be the addition of a flare to the rocket or stack exhaust. This is well within the state-of-the art.

The graph shown on Figure 15 is a plot of diffusion from a point source with no stack height for ground emission. The source strength is doubled to account for reflection of particulates or gas reflection from the ground. Along the axis of an elevated plume, the concentration will be half these values. The ratio of χ / Q is plotted with respect to distance to show dilution ratios for maximum convenience in use. The dilution ratios required for selected motors are marked for concentrations equal to one microgram per cubic meter. The distance required to these dilutions points out the necessity for elevated release and/or buoyant plumes to yield adequate dilution at ground level. The slightly diverging series of lines represents various atmospheric stabilities from extremely turbulent to strong inversion. Dilution ratios at a downwind location vary a thousand-fold between climatic extremes. However, from an elevated point source, the maximum ground level concentration remains essentially the same, but the distance to the maximum is increased ten-fold when climatic conditions change from very unstable to very stable.

The graph shown on Figure 16 is a plot of the empirical formula derived from ground-level emission under Project Ocean Breeze (Ref. 5). The preliminary report under this project presented four formulae, one for each major atmospheric stability category. However, it was found that changing from one formula for one lapse rate to the formula for the adjoining lapse rate category, discontinuities resulted of greater magnitude than the deviation resulting from one all-inclusive formula including a lapse term. A high confidence level has been established for this new, pre-release formula. A 99% probability of predicting within one half or double the measured concentration has resulted from the use of this formula for ground level point source emission. For ground emissions, such as toxic liquid spills, the Ocean Breeze formula is very promising

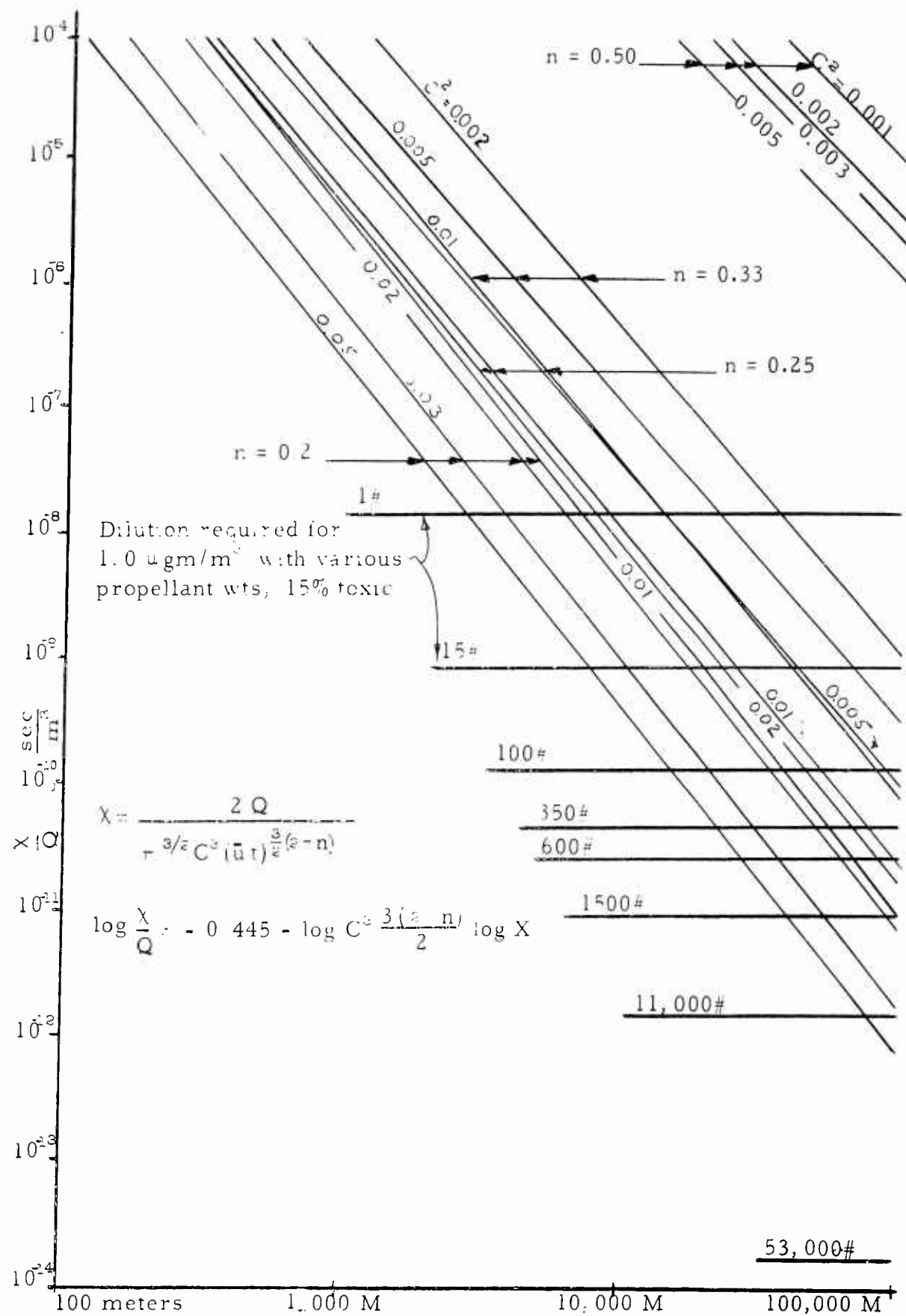


Figure 15. Instantaneous Point Source-Distance Dilution Ratio

$$\log \frac{cP}{Q} = 0.1309 - 1.855 \log x + 2.273 \log (\Delta T' + 5) - 0.7261 \log \sigma (A)$$

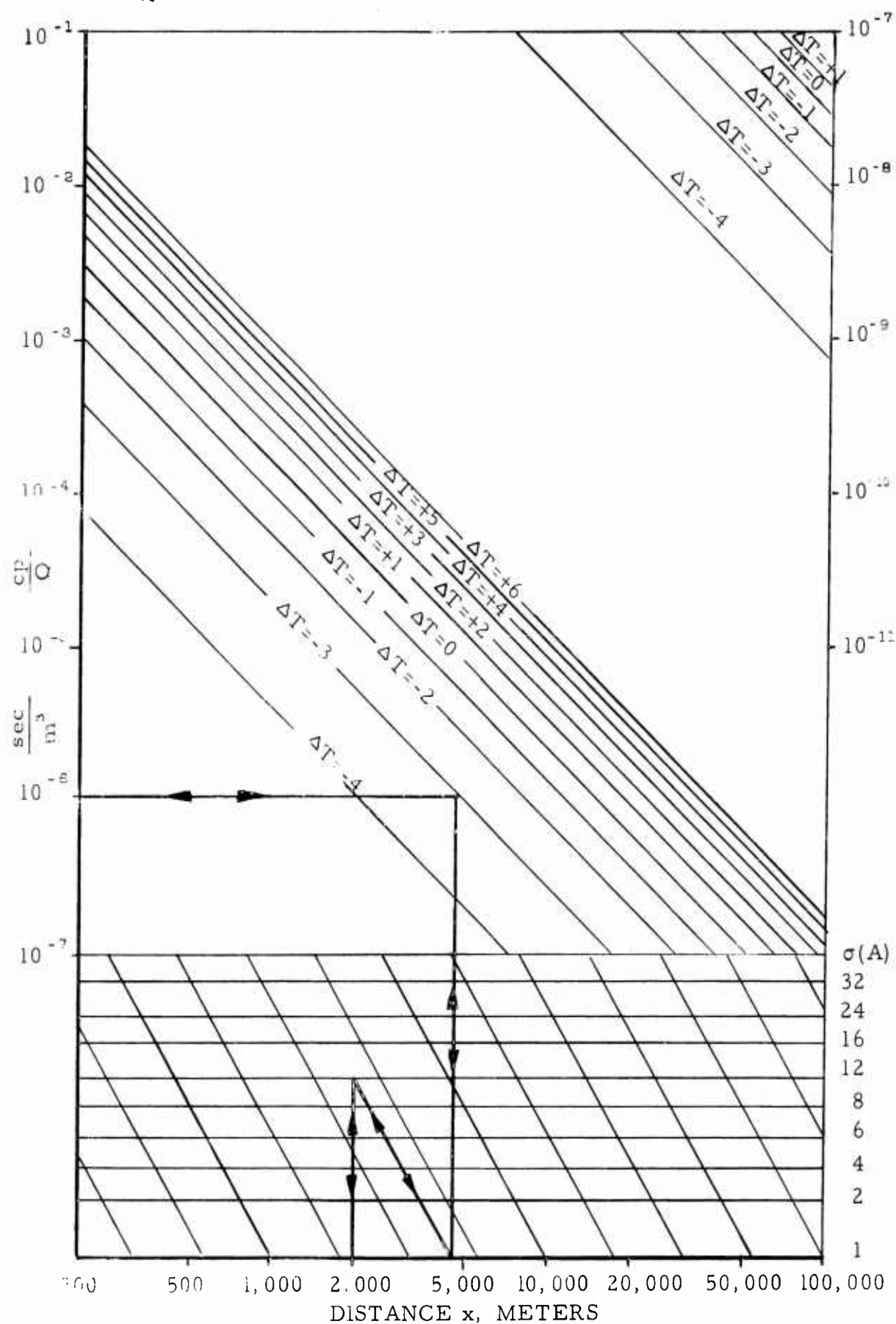


Figure 16 Ocean Breeze Point Source Diffusion

CONCLUSIONS AND RECOMMENDATIONS

The study of the effect that meteorology has on toxic rocket systems has resulted in certain basic conclusions. These conclusions must be rather general until a specific propellant system is chosen, after which more exact calculations may be made from definite parameters of the propellant toxicity and the associated thrust levels. The conclusions are:

1. Toxic rockets may be fired directly into the atmosphere with the following qualifications.
 - a. The test stand location must be a safe distance from inhabited areas.
 - b. Any limitations on thrust levels, and toxic emission rates, must be maintained.
 - c. The effluent should be directed in a vertical direction for maximum natural stack height.
 - d. The weather conditions must be suitable or artificial equipment must be used. This could include flaring of the exhaust gases for greater buoyancy of the plume.
 - e. The weather conditions should be predicted to be constant and near ideal for the duration of the test.
 - f. Air sampling must be conducted to insure that toxic limits are not exceeded.
2. Toxic rocket tests of large size may be conducted, but tests should not be made where the maximum allowable concentrations (MAC) might be exceeded beyond the borders of a test site.
3. In addition to the atmosphere diffusion of toxic rocket exhaust, a filtration or scrubber system could be added to allow greater toxic emission rates prior to filtration. Trade-offs would have to be made between the two extremes of complete filtration to complete emission of the toxic components. From these trade-offs the facility could be optimized for:
 - a. Maximum personnel safety and confidence in system performance.

- b. Optimum cost where open air firing would require elaborate and costly meteorological networks and experimentation, and where complete filtration would require elaborate and costly blast containment and filtration networks.
 - c. Maximum facility utilization based on setup and time percentages.
4. A health and safety group must be formed to establish and enforce regulations, establish sampling stations for background readings, and obtain and monitor physical examinations of test personnel. This group's activities would increase in proportion to test size and frequency.

Based on these conclusions and the study of applicable meteorological analytical techniques, the following recommendations have been developed:

1. Atmospheric diffusion for highly toxic rockets of small size can utilize point source ground level emission equations. Either Sutton's form or the "Ocean Breeze" form of point source ground level emission formula can be used. Where the downwind distance to the control area limit is over five miles, these equations have led to the conclusion that small rockets may be tested during turbulent atmospheric conditions. The maximum safe size would have to be calculated for the specific propellant combination.
2. Large toxic rockets may require a control area greater than five miles downwind to prevent exceeding the weekly allowable dosage. It is important to strive for maximum effective stack heights and use Sutton's elevated instantaneous point source formula. "Ocean Breeze" formulae are for non-buoyant emissions but could be modified for elevated plumes. Dispersion coefficients appropriate for the effective elevation of the source should be determined in either case.
3. Dispersion coefficients have been established for various atmospheric conditions and the variation with altitude delineated. However, experimentation in this area should be accomplished to improve reliability for long distance predictions where very toxic exhaust products are found. Reliable dispersion coefficients for elevations of several thousand feet will be required for large toxic engines.

4. The effective stack height must be determined from the relation specific to the emission type. That is, the recommended procedure for an explosion is to use the graphical relation between heat release and cloud height. A horizontal release will use the buoyancy formula, and a vertical firing will use the buoyancy formula plus the jet penetration equation; the buoyancy equation was developed in this study. The effective stack height can be augmented by flaring the fuel rich exhaust gases or even by adding additional fuel to the exhaust prior to flaring to provide a greater buoyancy to the gases.
5. Inversion penetration has been discussed and is shown to be possible with large engines. Where the high stability associated with an inversion results in low dispersion and high downwind concentrations, piercing the inversion can result in the lowest downwind concentrations. Experimental work with non toxic charges or fuel flaring should prove the reliability of inversion penetration. Release and flaring of ordinary fuel could be used prior to a toxic test to prove that piercing can be accomplished for the particular test climate. Large motors should be capable of piercing inversions, and smaller motors may be augmented to do so.
6. Air sampling stations in inhabited areas must be set up prior to and during test phases to monitor background concentrations and extent of contamination. A few of the more obvious areas where these stations should be located are at the rocket test stand, the support facilities, guard houses, nearby public highways, and downwind communities. Other air sampling stations both in-plant and out-plant will be evolved from the type of test facility system used and at the direction of cognizant health safety personnel.

Based on the analysis of meteorological phenomena, a conceptual design may be prepared for specific toxic test systems.

REFERENCES

1. Sutton, O. G., Micrometeorology, McGraw-Hill, New York (1953).
2. Meteorology and Atomic Energy, AECU 3066. Published by USAEC, Washington, D.C. (July 1955)
3. Stern, A. C. Air Pollution, Academic Press, New York (1962),
Volume I
4. Toxic Hazards of Rocket Propellants Aeronautronic Division, Ford Motor Company, Technical Report No. U 108 99 (30 November 1960)
Astia No. AD 253236
5. Haugen, D. A., Myers, R. F. and Taylor, Major J. H., Preliminary Report on Project Ocean Breeze and the Wind System for AMR, GRD Air Force Cambridge Research Laboratories, Bedford, Massachusetts (March 1962)
6. Bosanquet, Carey, and Halton, "Dust Deposition from Chimney Stacks," Proc. Inst. Mechanical Engineers, CLXII, 1950, 355.
7. Thomas, F. W., Journal of the Air Pollution Control Association, IV (1954) 7
8. Bryant and Cowdrey, "Effects of Velocity and Temperature of Discharge on the Shape of Smoke Plumes from a Funnel or Chimney; Experiments in a Wind Tunnel," Proc. Inst. Mechanical Engineers (London) CLXIX (1955) 371
9. Priestley, C. H. B., "A Working Theory of the Bent-Over Plume of Hot Gas," Quarterly Journal of the Royal Meteorological Society, LXXXII (1956) 165.

10. Bosanquet, C. H., "The Rise of a Hot Waste Gas Plume," The Journal of the Institute of Fuel, XXX, (1957) 322.
11. Thring, M. W., Air Pollution, Butterworth's Scientific Publications, London (1957).
12. Moses, H. and Strom, G. H., "A Comparison of Observed Plume Rises with Values Obtained from Well Known Formulas," Journal of the Air Pollution Control Association, XI No. 10 (October 1961) 455-66.
13. Machta, Bulletin of the American Meteorological Society, XXXI, (June 1950) 6.
14. Private Communication with Captain Owen Kiddlestad, USAF, Edwards Air Force Base, California.
15. Best, A. C., "Maximum Gas Concentration at Ground Level from Industrial Chimneys," Journal of the Institute of Fuel, XXX (June 1957) 197.
16. Hawkins, J. E., and Nonhebel, G., "Chimneys and the Dispersal of Smoke," Journal of the Institute of Fuel, XXVIII (November 1955) 178.
17. Glasstone, S., The Effects of Nuclear Weapons. Published by USAEC, Washington D.C. April 1962).
18. Astronautics, VII, No. 6 (June 1962) 68-69.
19. Spurr, G., "The Penetration of Atmospheric Inversions by Hot Plumes," Journal of Meteorology, XVI, (February 1959) 30-37.

20. Estoque, M. A. Venting of Hot Gases Through Temperature Inversions,
GRD Research Note No. 3 (December 1958) Astia No. AD 160756.
21. Cramer, Record and Vaughan. The Study of the Diffusion of Gases or
Aerosols in the Lower Atmosphere. M.T. Department of Meteorology
January 1959) Astia No. AD 210482
22. Barad, M. L. Project Prairie Grass: A Field Program in Diffusion.
GRD Research Note No. 59 (July 1958) Astia No. AD 152572.
23. Barad, M. L. and Hauger, D. A. "A Preliminary Evaluation of
Sutton's Hypothesis for Diffusion from a Continuous Point Source,"
Journal of Meteorology, XVI, No. 1, February 1959) 12-20.
24. Robinsor, E. Micrometeorological and Diffusion Study for High
Energy Test Facility, Stanford Research Institute Project No. SU
1964. Contract AF 04-6111-2315. August 30, 1957.
25. Green, H. L. and Lane, W. R. Particulate Clouds, Dusts, Smokes,
and Mists, D. Van Nostrand, London, 1957).
26. Callaghan, E. E. and Ruggeri, R. S. Investigation of the Penetration
of an Air Jet Directed Perpendicularly to an Airstream. NACA Tech.
Note 1615. Astia No. 26921. June 1948.
27. Launch Siting Criteria for High Thrust Vehicles. Aeronutronic Division,
Ford Motor Company. Technical Report No. U-108:118 (March 31, 1961).
28. Private communication between Walter W. Attini of Daniel, Mann,
Johnson, & Mendenhall staff and Captain Gordon Tucker, USAF,
Vandenberg Air Force Base, California.

29. Livingston, R., Physico Chemical Experiments, Macmillan Co., (1956).
30. Systems Engineering Division. Daniel, Mann, Johnson, & Mendenhall,
Design of a Toxic Rocket Test System, Contract AF 04(611)-7540,
SSD-TDR-62-137 (November 1962).

BIBLIOGRAPHY

Church, P. E., "Dilution of Waste Stack Gases in the Atmosphere," Industrial and Engineering Chemistry, XL1, No. 12 (December 1949) 2753.

Culkowski, W. M., Time Exposure Photography of Smoke Plumes, ORO-350, U.S. Weather Bureau Research Station Oak Ridge, Tennessee (April 1961).

Hodgman, M. S., Handbook of Chemistry and Physics, 41st Edition (1959-1960) Chemical Rubber Publishing Co.

Lucas, D. H., Spurr, G., Williams, F., "The Use of Ballons in Atmospheric Pollution Research," Quarterly Journal of the Royal Meteorological Society, LXXXIII (October 1957) 508-516

Marks, L. S., Mechanical Engineer's Handbook, 5th Edition (1951) McGraw-Hill.

Olson, J. S., Ecological Sampling and Meteorological Calculation of Fallout on Forests Near Oak Ridge, ORNL-3181 Oak Ridge National Laboratory, Oak Ridge, Tennessee

Perry, J. H., Chemical Engineer's Handbook, 3rd Edition (1950) McGraw-Hill.

Sherlock and Stalker, "The Control of Gases in the Wake of Smokestacks," Mechanical Engineering, LXII (June 1940) 455.

Stewart, N. G., Gale, H. J., and Crooks, R. N., "The Atmospheric Diffusion of Gases Discharged from the Chimney of the Harwell Reactor BEPO," International Journal of Air Pollution, 1 (1958) 87-102.

Von Hohenleiten and Wolf, "Wind-Tunnel Tests to Establish Stack Height for Riverside Generating Station," Trans. ASME, LXIV (1942) 671.

ABSTRACTS

The following abstracts of reports and books are presented in alphabetical order. These documents cover meteorology, toxicity, and rocket technology applicable to this study.

Barad, M. L., Project Prairie Grass, A Field Program in Diffusion, GRD Research Note No. 59, (July 1958) Astia No. AD 152572.

A field program was designed to provide experimental data on the diffusion of a tracer gas over a range of 800 meters. Seventy tests were conducted in which the gases were continuously emitted for ten-minute intervals. The releases were made over flat Nebraska prairie under a variety of meteorological conditions during July and August of 1956. Tabulations of the diffusion data and the meteorological data collected during the gas releases are also presented.

Barad, M. L. and Haugen, D.A., "A Preliminary Evaluation of Sutton's Hypothesis for Diffusion from a Continuous Point Source," Journal of Meteorology, XVI, No. 1, (February 1959) 12-20.

In Sutton's equation the observed concentration distribution is predicted only if there are two values of n , a n_y , and a n_z . Statistical tests indicate that n_y and n_z are invariant with distance between 100 and 800 meters of the source, but that the values of n_y and n_z appropriate for these distances exceed the values within 100 meters of the source. It is also shown that neither n_y nor n_z can be specified by n_w , the value of n found from a power-law fit to the wind profile in the lowest 8 meters.

These conclusions were obtained from data from Project Prairie Grass. The values of h were 50 centimeters and 1.5 meters over level countryside covered with grass of a height of about 5 centimeters. These conclusions probably do not present valid indications for macroscopic rocket tests where h is much greater than 1.5 meters and where n_y is approximately equal to n_z .

Beers, Norman R., "Stack Meteorology and Atmospheric Disposal of Radioactive Waste," Nucleonics, IV, No. 4, (April 1949) 28-38.

Discusses the problem of air cooling of piles since argon in air becomes radioactive with a half-life of 110 minutes. Article covers Sutton's equations for a continuous point source and gives the oil-fog smoke test method. Concludes that for the high reactor stacks at Brookhaven there is no danger of radioactivity and indications are that this is true.

Best, A. C., "Maximum Gas Concentration at Ground Level from Industrial Chimneys " Journal of the Institute of Fuel, XXX, (June 1957) 197.

The maximum ground level concentration of toxic effluent and the distance to the maximum depend upon the stack and meteorological characteristics. Covers several formulae for computing stack height, giving results of calculations and attempts to show that for the purpose of computing the maximum ground-level concentration the three are in reasonable agreement.

Bosanquet, C. H., "The Rise of a Hot Waste Gas Plume," Journal of the Institute of Fuel XXX (June 1957) 322.

It is assumed when a cloud of hot gas is rising, the total heat content and total upward momentum are unaffected by dilution with air. Also assumed that upward momentum increases at a rate proportional to the heat content. A set of equations for stack height is derived.

Bosanquet, Carey, and Halton, "Dust Deposition from Chimney Stacks," Proc. Inst. of Mech. Engrs. CLXII (1950) 355.

A general description of the interaction of stacks and diffusion is given. Derives formulae for plume rise and diffusion of dust with rates of deposition of dust. Numerous nomographs are given to assist in working method.

Cramer, Record, and Vaughan, The Study of the Diffusion of Gases or Aerosols in the Lower Atmosphere, Mass. Inst. of Technology, Dept. of Meteorology (January 1959), Astia No. AD210482.

Diffusion measurements were taken at Round Hill, Mass., of a similar nature to Project Prairie Grass. These experiments were for the determination of instantaneous diffusion coefficients at ground level. Emission times were of 30 second and three-minute intervals and are compared to concentration levels obtained at 10-minute intervals. Average concentrations were 2.5 and 1.5 times larger respectively than for 10-minute periods.

DeMarrais, G. A., Workbook in Atmospheric Diffusion Calculations, U. S. Weather Bureau, Idaho Falls, Idaho, (February 1959).

This workbook was prepared as a practical supplement to Meteorology and Atomic Energy. Included are the behavior of stack effluents, the determination of n and the coefficients of diffusion, the application of the basic equations, and nomographic solutions of the equations.

Estoque, M. A., Venting of Hot Gases Through Temperature Inversions, GRD Research Note No. 3 (December 1958) Astia No. AD 160756.

The penetration of temperature inversions in the lower atmosphere by plumes of hot air is investigated as part of a safety analysis in the nuclear aircraft program. This program is studied with the aid of existing theoretical as well as experimental work. A nomogram showing the relationship between the maximum height attained by a hot plume of a given heat source intensity and the temperature gradient of the environment is presented.

Gifford, F. A. Jr., The Problem of Forecasting Dispersion in the Lower Atmosphere, Weather Bureau Research Station, Oak Ridge, Tennessee (July 1961).

As the nuclear power industry develops, weather forecasters will be called on more and more to make estimates of dispersion from isolated sources in the lower atmosphere. The various equations and parameter values needed for this purpose are given. The generalized Gaussian plume model is presented in some detail, and diffusion parameters based on recent observational and theoretical studies are discussed.

Glasstone, S., The Effects of Nuclear Weapons, Published by USAEC, Washington, D. C., April 1962.

The subjects covered are the general principles and descriptions of nuclear explosions, blast effects, the effects on personnel, and principles of protection. Tabular data of interest included descriptions of announced nuclear detonations in which the name of the shot, date and time of ignition, location, height and type of burst, yield, and the altitude of the cloud top, cloud base, and tropopause.

Green, H. L. and Lane, W. R., Particulate Clouds: Dusts, Smokes, and Mists, D. Van Nostrand, London, 1957.

The process by which dust is passed into the lungs or trapped by the cilia and other parts of the respiratory tract is covered. A curve for deposition in the lung of particles of unit density is presented. The maximum rate of deposition is for one micron diameter particles and over 12 microns the rate is nil.

Greenfield, Harvey, Theoretical Derivations and Nomographic Methods for Sutton's Diffusion Equations, Research Directorate, Dugway Proving Ground, Dugway, Utah (September 10, 1956) Astia No. AD 113307.

A survey is made of diffusion equations including hypotheses underlying derivations and limits to applicability. Nomograms are derived, and their use is demonstrated for Sutton's diffusion equations. The use of nomograms is shown to simplify and expedite diffusion calculations. Six computed nomograms are included with derivations and demonstrations of their use.

Haugen, D. A., Barad, M. L. and Antanaitis, P., "Values of Parameters Appearing in Sutton's Diffusion Models," Journal of Meteorology, XVII, No. 3 (June 1961), 368-372.

Statistical summaries are presented of computed values of parameters appearing in a modification of Sutton's diffusion equations. Experimental data used for the computations are those obtained during Project Prairie Grass at O'Neil, Nebraska and those obtained at Round Hill, Massachusetts. Stratification of the parameters n_y and n_z according to stability class reveals systematic variations of class median values of n_y and n_z with stability class and a large range of n_y and n_z within any given stability class. No difference in the results was found between the two sets of experiments for these parameters.

The parameter C_y has a mean value of about 0.4 for the Prairie Grass experiments and about 0.9 for the Round Hill experiments. No relationship between C_y and stability was found for the Prairie Grass experiments, but a correlation coefficient of 0.22 between C_y and stability ratio was found for the Round Hill experiments.

The parameter C_z , computed only for the Prairie Grass experiments, has a mean value of about 0.07 and is independent of stability for those experiments permitting a statistically stable estimate of C_z .

Hawkins, J. E. and Nonhebel, G., "Chimneys and the Dispersal of Smoke," Journal of the Institute of Fuel, XXVII, (November 1955) 178.

The authors discuss effects of disturbance of a plume by passage of wind past the emitting chimney. The smoke is drawn down to ground level by trailing vortices. A plot of emission velocity vs. wind speed at which downwash will not occur is given. Contains discussion of Oak Ridge, Thomas and Bosanquet formulae.

Hill, Thomas and Abersold, "Dispersion of Gases from Tall Stacks," Industrial and Engineering Chemistry, XL, No. 11 (November 1959) 2409.

The Theoretical equations of Bosanquet and Pearson and Sutton for the dispersion of smoke from factory chimneys have been solved in terms of the conventional units of the smelting industry. The

theoretical curves and confirming data illustrate forcefully the beneficial effects of the use of tall stacks in dispersing air contaminants from factories. Increases in stack temperature increase effective stack height and improve dispersion.

Launch Siting Criteria for High-Thrust Vehicles. Aeronutronic Division, Ford Motor Company, Technical Report No. U-108:118, (March 31, 1961).

Launch hazards are described which include acoustical, explosion, and toxicity hazards. Rocket engine noise and its propagation and effects at various sound power levels are covered. Propellant explosions are described along with the resulting fragmentation and overpressure and a table of distances based on available data, is derived. In the toxicity hazard section the diffusion of poisonous propellants in the atmosphere is presented based on Sutton's equations.

Magill, Holden, and Ackley, Air Pollution Handbook, McGraw-Hill, New York (1956).

Presented are various facets of pollution. The section on calculation of concentration of pollutants gives the variation in the generalized eddy-diffusion coefficients with height and with lapse rate. The statement is made that Sutton's values for maximum concentration from a source are order of magnitude correct.

Meteorology and Atomic Energy, AECU 3066. Published by USAEC, Washington, D. C., July 1955.

This is considered to be one of the best volumes on meteorology as applied to pollution by air-borne wastes, especially radioactive. Presented are outlines of diffusion theories, behavior of stack effluents and explosion debris clouds, fall-out, wash-out, and rain-out from air-borne clouds and radioactive cloud dosage calculations. Also covered are graphical solutions to atmospheric diffusion problems, reactor hazard analyses, and a selection of recommended equations, parameters, and conversion factors.

Moses, H. and Strom, G. H. "A Comparison of Observed Plume Rises with Values Obtained from Well-Known Formulas." Journal of the APCA, XI, No. 10, (October 1961) 455-66.

Actual observations were made of over one hundred different smoke runs, and the measured values of stack height were compared with calculated ones from six sources: 1) Holland (Oak Ridge), (2) Bryant and Davidson, (3) Sutton, (4) Scorer, (5) Bosanquet, Carey, and Haltor, and (6) Bosanquet. Concluded that no one formula is outstanding in all respects and the nature of the problem would influence the selection of the one used.

Priestley, C.H.B., "A Working Theory of the Bent-Over Plume of Hot Gas," Quarterly Journal of the Royal Meteorological Society, LXXXII, 1956, 165-76

An ascending plume of hot gas is initially diffused by the turbulence which is induced by its own motion and later, but before all upward motion and buoyancy are lost, by the natural turbulence of the environment. A working solution is presented for the first phase in terms of spreading coefficient C and for the second in terms of a mixing rate K , and it is shown that the transition from one phase to the other should be quite well marked

Robinson, E., Micrometeorological and Diffusion Study for High Energy Test Facility, Stanford Research Institute Project No. SU-1964, Contract No. AF 04(611)-2315, August 30, 1957,

Detailed studies were made of the air pollution problem posed by a planned high energy test facility at Edwards Air Force Base. Both theoretical and experimental phases of the problem were included in this study. An experimental field program was carried out for the determination of diffusion coefficients and stability parameters at Edwards AFB.

Scorer, R. S., "Plumes from Tall Chimneys," Weather, X, (April, 1955) 106-109

A general discussion of the problem. The conclusions are that the gaseous products of combustion are only harmful when they are at the ground. The objective should be, therefore, to get them as high into the air as possible by means of a few tall wide chimneys and have a certain amount of buoyancy. The solid particles have a harmful effect at all levels, and it is the removal of these that should claim the engineer's attention.

Spurr, G., "The Penetration of Atmospheric Inversions by Hot Plumes," Journal of Meteorology, X, February 1959, 30-37

The article covers a derivation of the magnitude of heat sources to penetrate inversions. Equation for height is a function of lapse rate and heat release rate. Predicts that the worst inversion on record can be penetrated by a power plant of 100 megawatts.

Stern, A. C., Air Pollution, Academic Press, New York (1962), volume 1.

This book covers air pollution and its dispersion, the effects of air pollution, measuring and monitoring air pollution. A

discussion of stack height formulae and dispersion formulae is very thoroughly covered. The effects of air pollution on humans and animals are also presented.

Stokinger, H. E., "Air Pollution and the Particle Size - Toxicity Problem, " Nucleonics Magazine, (December 1949)

Included is a general discussion of natural air pollution such as dust storms, and then man-made pollution, which is divided into two types: (1) community contamination, such as smog, and (2) those occurring within the plant itself. The toxic substances covered are: sulfur dioxide, fluorine, and dusts of beryllium compounds. The methods of investigation for finding beryllium air contamination are given. The effect of hydrogen fluoride on beryllium poisoning is presented. The toxic effects of beryllium sulfate mist were found to have been doubled by the addition of concentrations of hydrogen fluoride. The same concentration of hydrogen fluoride was absolutely without effect in the control animals when inhaled alone.

Sutton, O. G., "Dispersion of Hot Gases in the Atmosphere, " Journal of Meteorology, VII, No. 5 (October 1950) 307-312.

The problem of the disposition of a stream of hot gas from a point is considered. It is shown that a plausible assumption concerning the mechanism of entrainment of air by the jet leads to simple expressions for the mean temperature and mean velocity of a jet of hot air rising in a calm atmosphere of uniform potential temperature. The theoretical expressions are shown to agree with the laboratory measurements of Schmidt, and it is found that the coefficient of diffusion for these conditions is of the same order of magnitude as that which is derived from the large-scale spreading of cold smoke in the atmosphere. Finally, an approximate solution is given for the shape of the plume from a hot source in a horizontal wind; and it is demonstrated that the reduction of maximum concentration at ground level, caused by adding heat to the effluent from a stack, is directly proportional to the strength of the heat source and inversely proportional to the height of the chimney and the cube of the horizontal wind speed.

Thring, M. W., Air Pollution, Butterworth's Scientific Publications, London (1957)

The chapter on the "Flow of Chimney Gases" is by C. H. B. Bosanquet. He derives the stack height formula covered in the text above.

Toxic Hazards of Rocket Propellants, Aeronutronic Division, Ford Motor Company, Technical Report No. U-108:99 (November 30, 1960) Astia No. AD253236.

This report discusses the general subject of toxicity and the method for obtaining and the philosophy for establishing the maximum allowable concentration (MAC). Eighteen liquid rocket propellants in particular are discussed along with their toxic hazards and operating procedures now used to minimize the dangers of each to personnel. Atmospheric diffusion was covered with Sutton's equation and modifications thereof used. Conditions of fallout and rainout with ground deposition and integrated concentrations are included. The sources are classified as continuous point, line, and volume sources. The special hazards of total instantaneous washout as caused by a rain storm, and fumigation as caused by a temperature inversion are included with typical dispersion problems.

Turbulent Diffusion in the Atmosphere, Technical Note No. 24, World Meteorological Organization, WMO. No. 77, TP-31, Geneva, Switzerland, (1958)

The early theories of Taylor, Schmidt, and Richardson as well as Sutton are included and Sutton's equations are given. The diffusion from a stack source with formulae for stack height and maximum ground level concentration is given.

The Use of Effective Stack Height for Dispersal of Toxic Rocket Exhaust Gases, Marquardt Corporation, Report FE-223-2, Contract No. AF 04(611)-4304 (April 20, 1961,

The feasibility and safety of atmospheric dispersal of rocket exhaust gases at Edwards AFB are studied. Conclusions are that under suitable conditions large quantities of toxic gases can be safely discharged directly into the atmosphere. Included in the study are rockets in the 50,000 to 800,000 lb. thrust range, using fluorine and chlorine trifluoride oxidizers and hydrogen and hydrazine fuels. Stack height is shown to be the most important single parameter influencing the safe dispersal of the gases in the atmosphere.

APPENDICES

The meteorological aspects of toxic rocket testing have been investigated, and the areas requiring further mathematical analysis have been covered. The synthesizing of an equation relating stack height, stability constant (n), wind velocity, and diffusion coefficient has been done to facilitate the development of an equation for buoyant plume rise. They will be found in calculation sets A and B. The momentum rise of a stack emission was developed empirically by wind tunnel testing and related to the system of units used in this report in calculation set C. Dimensional analysis was used to develop a new equation for the momentum and buoyancy rise of a plume, and is presented in set D.

The degree to which errors in parameter measurement cause the calculated functions of error theory to functions of several variables in set E. A comparison of ground level concentration versus distance for gaseous and particulate emissions is done in set F.

EQUATION FOR OBTAINING VALUES OF C

Since the basic derived equation for plume rise due to buoyancy includes values of "C", an equation is required which is based on some value of h to solve for "C". This is due to the included mass of a cloud increasing because of diffusion effects. Using the extended values of "C" plotted in the "Meteorology and Atomic Energy" nomogram as functions of h, n and $\bar{u}$, an equation was derived which included these values plus a constant. An alternate solution would have been to solve the plume rise equation for various heights and selecting values of "C" by successive approximations until close fit values are obtained for each calculation. The equation of the nomogram was solved assuming the curves were plotted on an equation of the type:

$$y = CX_1^A X_2^B X_3^D$$

where y = "C" = Diffusion coefficient

C = Constant

X_1 = h = Plume height

X_2 = n = Lapse

X_3 = $\bar{u}$ = Mean wind velocity

Values for "C" were obtained from the nomogram for various h, n and $\bar{u}$ values and the basic equation solved algebraically to yield "C" = $(3.67 \times 10^{-3}) (h^{-1.47}) (n^{-3.11}) (\bar{u}^{-1.59})$. With arbitrarily chosen values of n and $\bar{u}$, the plume rise can now be expressed as a function of the heat released. A check of the equation versus nomogram derived values of "C" showed very good agreement. However, due caution must be exercised in the use of this equation; it is doubtful if values obtained at $h < 10$ meters or $h > 1000$ meters will be of sufficient accuracy to justify the use of the equation under these conditions.

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Equation for determining values of
 "C" given height, lapse rate & wind
 velocity

JOB NO. 507-1-1

SHEET NO. A-1

DESIGNED BY WLL

DATE

APPROVED

GIVEN: NOMOGRAM APPEARING IN "METEOROLOGY AND
 ATOMIC ENERGY" WITH EXTENDED HEIGHTS.

SOLUTION: ASSUMING THAT SINCE 3 VARIABLES ARE GIVEN
 ON THE NOMOGRAM TO FIND A FOURTH VALUE,
 THE EQUATION IS OF THE TYPE

$$y = C x_1^a x_2^b x_3^d$$

WHERE: x_1 = HEIGHT IN METERS, h

x_2 = LAPSE, n

x_3 = WIND VELOCITY IN METERS/SECOND, $\bar{u}$

C = ARBITRARY CONSTANT

y = DIFFUSION COEFFICIENT AS (METERS²/SEC) C

STEP I.

READING FROM THE NOMOGRAM AT THE FOLLOWING
 POINTS, HOLDING n AND $\bar{u}$ CONSTANT

	$\frac{C}{\bar{u}}$	$\frac{h}{\bar{u}}$	$\frac{n}{\bar{u}}$	$\frac{\bar{u}}{\bar{u}}$
A	0.336	10	0.20	1.0
B	0.251	100	0.20	1.0
C	0.162	1000	0.20	1.0

SUBSTITUTING THESE VALUES INTO THE EQUATION TYPE
 FORM

$$A-1 \quad 0.336 = C (10^a) (0.20^b) (1.0^d)$$

$$B-1 \quad 0.251 = C (100^a) (0.20^b) (1.0^d)$$

$$C-1 \quad 0.162 = C (1000^a) (0.20^b) (1.0^d)$$

TAKING LOGARITHMS

$$A-2 \quad \log 0.336 = \log C + a \log 10 + b \log 0.20 + d \log 1.0$$

$$B-2 \quad \log 0.251 = \log C + a \log 100 + b \log 0.20 + d \log 1.0$$

$$C-2 \quad \log 0.162 = \log C + a \log 1000 + b \log 0.20 + d \log 1.0$$

	JOB NO.	SHEET No. A-2
	DESIGNED BY WLL	DATE
	APPROVED	

OBTAINING AND SUBSTITUTING THE PROPER VALUES

$$\begin{aligned} \text{A-3 } -0.474 &= \log C + a(1) + b(-0.699) + d(0) \\ \text{OR } -0.474 &= \log C + a - 0.699b \end{aligned}$$

$$\begin{aligned} \text{B-3 } -0.600 &= \log C + a(2) + b(-0.699) + d(0) \\ \text{OR } -0.600 &= \log C + 2a - 0.699b \end{aligned}$$

$$\begin{aligned} \text{C-3 } -0.790 &= \log C + a(3) + b(-0.699) + d(0) \\ \text{OR } -0.790 &= \log C + 3a - 0.699b \end{aligned}$$

SOLVING EQUATION A-3 FOR a

$$\text{A-4 } a = 0.699b - \log C - 0.474$$

SUBSTITUTING A-4 INTO B-3 AND SOLVING FOR b

$$\begin{aligned} \text{B-4 } -0.600 &= \log C + 2(0.699b - \log C - 0.474) - 0.699b \\ -0.600 &= 0.699b - \log C - 0.948 \\ 0.699b &= 0.348 + \log C \end{aligned}$$

SUBSTITUTING B-4 INTO C-3 AND SOLVING FOR a

$$\begin{aligned} \text{C-4 } -0.790 &= \log C + 3a - (0.348 + \log C) \\ -0.790 &= 3a - 0.348 \\ -0.442 &= 3a \\ \therefore a &= -0.147 \end{aligned}$$

THEN EQUATION TO THIS POINT IS:

$$y = C X_1^{-0.147} X_2^b X_3^d$$

STEP II

READING FROM THE NOMOGRAM FOR VARIOUS VALUES OF η , HOLDING h AND $\bar{u}$ CONSTANT

	C	h	η	$\bar{u}$
D	0.251	100	0.20	1.0
E	0.125	100	0.25	1.0
F	0.059	100	0.33	1.0

SUBSTITUTING INTO EQUATION FORM AS BEFORE

$$\text{D-1 } 0.251 = C (100^{-0.147}) (0.20^b) (1.0^d)$$

$$\text{E-1 } 0.125 = C (100^{-0.147}) (0.25^b) (1.0^d)$$

$$\text{F-1 } 0.059 = C (100^{-0.147}) (0.33^b) (1.0^d)$$

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	JOB No.	SHEET No. A-3
	DESIGNED BY WLL	DATE
	APPROVED	

TAKING LOGARITHMS

$$\underline{D-2} \quad \log 0.251 = \log C + (-0.147 \log 100) + b \log 0.20 + d \log 1.0$$

$$\underline{E-2} \quad \log 0.125 = \log C + (-0.147 \log 100) + b \log 0.25 + d \log 1.0$$

$$\underline{F-2} \quad \log 0.059 = \log C + (-0.147 \log 100) + b \log 0.33 + d \log 1.0$$

SUBSTITUTING IN PROPER VALUES

$$\underline{D-3} \quad -0.600 = \log C + (-0.147 \times 2) + b(-0.699) + d(0)$$

$$\text{OR } -0.600 = \log C - 0.294 - 0.699b$$

$$\underline{E-3} \quad -0.902 = \log C + (-0.147 \times 2) + b(-0.602) + d(0)$$

$$\text{OR } -0.902 = \log C - 0.294 - 0.602b$$

$$\underline{F-3} \quad -1.230 = \log C + (-0.147 \times 2) + b(-0.482) + d(0)$$

$$\text{OR } -1.230 = \log C - 0.294 - 0.482b$$

SOLVING EQUATION D-3 FOR LOG C

$$\underline{D-4} \quad \log C = -0.306 + 0.699b$$

SUBSTITUTING D-4 INTO E-3 AND SOLVING FOR b

$$\underline{E-4} \quad -0.902 = -0.306 + 0.699b - 0.294 - 0.602b$$

$$-0.302 = 0.097b$$

$$\therefore b = -3.11$$

SUBSTITUTING VALUES INTO EQUATION F-3 TO FIND C

$$\underline{F-4} \quad -1.230 = \log C - 0.294 - 0.482(-3.11)$$

$$\log C = -1.230 + 0.294 - 1.499$$

$$\log C = -2.435$$

$$C = 3.67 \times 10^{-3}$$

EQUATION IS THEN WRITTEN AS

$$y = (3.67 \times 10^{-3}) (X_1^{-0.147}) (X_2^{-3.11}) (X_3^d)$$

STEP III

READING FROM NOMOGRAM

	C	h	n	u
G	0.094	100	0.25	10

SUBSTITUTING INTO THE EQUATION FORM

$$\underline{G-1} \quad 0.094 = (3.67 \times 10^{-3}) (100^{-0.147}) (0.25^{-3.11}) (10^d)$$

	JOB NO.	SHEET NO. A-4
	DESIGNED BY WLL	DATE
	APPROVED	

TAKING LOGARITHMS

$$G-2 \quad \log 0.094 = \log 3.67 \times 10^{-3} + (-0.147)(\log 10) + (-3.11)(\log 0.25) + d \log 10$$

SUBSTITUTING IN VALUES

$$G-3 \quad -1.026 = 2.435 - 0.274 + 1.872 + d$$

SOLVING FOR d

$$G-4 \quad d = -0.169$$

THEN HAVING SOLVED ALL POWERS & CONSTANTS

$$C = (3.67 \times 10^{-3})(h^{-0.147})(\bar{u}^{-3.11})(n^{-0.169})$$

CHECK OF EQUATION

1. $h = 50$ METERS, $\bar{u} = 5$ METERS/SECOND, $n = 0.33$

$$\begin{aligned} \log C &= -2.435 + (-0.147)(1.699) + (-3.11)(-0.482) + (-0.169)(0.677) \\ &= -2.435 - 0.250 + 1.496 - 0.118 \\ &= -1.307 \\ C &= 0.0492 \quad \text{NOMOGRAM} = 0.049 \end{aligned}$$

2. $h = 320$ METERS, $\bar{u} = 3$ METERS/SECOND, $n = 0.25$

$$\begin{aligned} \log C &= -2.435 + (-0.147)(2.505) + (-3.11)(-0.602) + (-0.169)(0.477) \\ &= -2.435 - 0.369 + 1.872 - 0.081 \\ &= -1.013 \\ C &= 0.097 \quad \text{NOMOGRAM} = 0.091 \end{aligned}$$

3. $h = 800$ METERS, $\bar{u} = 15$ METERS/SECOND, $n = 0.20$

$$\begin{aligned} \log C &= -2.435 + (-0.147)(2.903) + (-3.11)(-0.699) + (-0.169)(1.176) \\ &= -2.435 - 0.426 + 2.178 - 0.199 \\ &= -0.882 \\ C &= 0.131 \quad \text{NOMOGRAM} = 0.13 \end{aligned}$$

4. $h = 400$ METERS, $\bar{u} = 2$ METERS/SECOND, $n = 0.33$

$$\begin{aligned} \log C &= -2.435 + (-0.147)(2.602) + (-3.11)(0.482) + (-0.169)(0.301) \\ &= -2.435 - 0.383 + 1.500 - 0.051 \\ &= -1.369 \\ C &= 0.0427 \quad \text{NOMOGRAM} = 0.0425 \end{aligned}$$

CONTINUOUS POINT SOURCE PLUME RISE DUE TO BUOYANCY BECAUSE OF HEAT CONTENT ABOVE AMBIENT AIR.

THE WIDTH OF A SMOKE PLUME FROM A CONTINUOUS POINT SOURCE IS DEFINED BY THE FOLLOWING EQUATION.

$$2Y_0 = 2 \left(\log_e \frac{100}{P} \right)^{\frac{1}{2}} C x^{\frac{2-n}{2}} \quad (1)$$

WHERE $P = \%$ OF CENTERLINE CONCENTRATION AT EDGE OF CLOUD. 10% IS FREQUENTLY USED AND APPROXIMATES OBSERVABLE LIMITS.

$$\therefore 2Y_0 = 2(2.3)^{\frac{1}{2}} C x^{\frac{2-n}{2}} = 3.034 C x^{\frac{2-n}{2}}$$

THE PRODUCT OF A PARAMETER CONTAINED WITHIN THE CLOUD LIMIT OF $P=10\%$ WILL BE LESS THAN 100% OF THE TOTAL. HEAT IN THE CLOUD IS THE PARAMETER BEING CONSIDERED.

CONSIDERING A HORIZONTAL CYLINDRICAL SEGMENT OF THE CLOUD, THE TOTAL HEAT CONTAINED WITHIN THE SEGMENT OF UNIT LENGTH CAN BE FOUND BY INTEGRATING THE FOLLOWING EXPRESSION:

$$Q = \int_0^{\infty} 2\pi X y dy \quad (2)$$

$$\text{BUT } X = \frac{Q}{\pi C^2 \bar{U} x^{2-n}} e^{-\left(\frac{y^2 + h^2}{C^2 x^{2-n}}\right)} \quad (3)$$

(SEE TEXT FOR DEFINITION OF TERMS)

WHERE $h=0$

$$X = \frac{Q}{\pi C^2 \bar{U} x^{2-n}} e^{-\left(\frac{y^2}{C^2 x^{2-n}}\right)}$$

$$\text{BUT } C^2 x^{2-n} = \frac{Y_0^2}{\log_e \frac{100}{P}} \quad (4) \text{ FROM (1)}$$

$$\text{HENCE } Q_0 = \int_0^{Y_0} \frac{2\pi Q y dy}{\pi \bar{U} \frac{Y_0^2}{\log_e \frac{100}{P}}} e^{-\left(\frac{y^2 \log_e \frac{100}{P}}{Y_0^2}\right)} \quad (5)$$

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Y_0 IS DEFINED AS THE POINT WHERE THE CONCENTRATION HAS DROPPED TO 10% OF THE CENTER LINE CONCENTRATION OR $P=10$

$$\text{AND } Q_0 = \frac{\bar{u} Q}{\pi \bar{u}} \frac{2\pi \log_e \frac{100}{P}}{Y_0^2} \int_0^{Y_0} y dy e^{-\frac{Y^2 \ln \frac{100}{P}}{Y_0^2}}$$

$$Q_0 = Q \left[1 - e^{-\log_e \frac{100}{10}} \right] = 0.90 Q \quad (6)$$

OR THE 10% CONCENTRATION EDGE CONTAINS 90% OF THE TOTAL PARAMETER (HEAT).

ATMOSPHERIC PRESSURE AND DENSITY FOR STANDARD CONDITIONS VARIES APPROXIMATELY ACCORDING TO THE FOLLOWING EXPRESSIONS

$$P_2 = P_1 (1 - 2.255 \times 10^{-5} h)^{5.256} \quad (7)$$

$$\rho_2 = \rho_1 (1 - 2.255 \times 10^{-5} h)^{4.256} \quad (8)$$

THE PLUME ISSUES HORIZONTALLY DUE TO THE WIND WITH ZERO BUOYANCY NEGLECTING EXIT VELOCITY. AN EXPRESSION FOR PLUME RISE AS A FUNCTION OF HORIZONTAL DISTANCE, x , IS REQUIRED FOR A BUOYANT PLUME.

$$h = f(x)(Q)(u)$$

FROM EXPERIMENTS PERFORMED BY BRYANT AND COWDREY (REF 8)

$$h = k Q_h \left(\frac{x}{u} \right)^{\frac{1}{2}} \quad \text{AS BEST APPROXIMATION OF DATA.}$$

THE OAKRIDGE FORMULA (REF 3) WAS DERIVED FOR HEAT RELEASES OF ABOUT 3500 BTU/SEC. AND 300 METERS DOWNWIND. THE OAKRIDGE FORMULA FOR EFFECTIVE STACK HEIGHT BUOYANCY TERM ONLY IS -

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$$\Delta h = \frac{0.000134 Q_h}{U}$$

Δh = BUOYANCY RISE FEET
 Q_h = HEAT RELEASE GRAM CAL/SEC.
 U = WIND VELOCITY METERS/SEC.

CONVERTING UNITS Q_h , (BTU/SEC); h , (METERS).

$$h = \frac{0.0103 Q_h}{U}$$

h = PLUME RISE DUE TO
 BUOYANCY AT 300 METERS
 Q_h = HEAT RELEASE BTU/SEC.
 U = MEAN WIND VELOCITY

WHERE $Q_h = 3500$ BTU/SEC. AND $U = 4.47$ METERS/SEC.

$$h = \frac{0.0103 \times 3500}{4.47} = 8.06 \text{ METERS}$$

BEING DERIVED FROM MEASUREMENTS TAKEN AT 300 METERS AND FROM A FAIR RELEASE OF HEAT, THIS EQUATION IS THE BEST AVAILABLE SOURCE OF INFORMATION FOR DETERMINING THE PROPORTIONALITY CONSTANT FOR PLUME TRAJECTORY. THE OAKRIDGE FORMULA IS DERIVED FOR NEUTRAL STABILITY WITH A 90% FACTOR FOR STANDARD LAPSE. THEREFORE $h = 7.25$ METERS.

$$k = \frac{h}{Q_h} \left(\frac{U}{x} \right)^{\frac{1}{2}} = \frac{7.25 (4.47)^{\frac{1}{2}}}{3500 (300)^{\frac{1}{2}}} = 2.53 \times 10^{-4}$$

THEREFORE $h = 2.53 \times 10^{-4} Q_h \left(\frac{x}{U} \right)^{\frac{1}{2}}$

AND $x = \frac{h^2 U}{(2.53 \times 10^{-4} Q_h)^2} = \frac{h^2 U}{6.40 \times 10^{-8} Q_h^2}$

$$x^{1.75} = \frac{h^{3.5} (U)^{1.75} \times 10^{13}}{2.56 Q_h^{3.5}} \quad (9)$$

FROM THESE EQUATIONS THE CLOUD VOLUME DUE TO DIFFUSION AND ADIABATIC EXPANSION, THE CLOUD WEIGHT AND DISPLACED AIR CAN BE DETERMINED.

VOLUME OF PLUME DUE TO DIFFUSION CONSIDERING ONE SECOND EMISSION FROM CONTINUOUS POINT SOURCE

$$V_d = \pi r^2 U \quad \text{BUT } r = Y_{10} = 1.517 C x^{\frac{2-11}{2}} \quad (10)$$

THE DISPERSION COEFFICIENT, C , WILL DECREASE WITH ALTITUDE ACCORDING TO THE FOLLOWING EXPRESSION:

$$C = 3.67 \times 10^{-3} (h^{-0.147} n^{-3.11}) (\bar{U}^{-0.169}) \quad (11)$$

BY SUBSTITUTION

$$V_d = \pi U \left[1.517 \times 3.67 \times 10^{-3} (h^{-0.147} (0.25)^{-3.11}) (\bar{U}^{-0.169}) x^{\frac{2-11}{2}} \right]^2$$

WHERE $n = 0.25$ FOR STANDARD LAPSE

$$V_d = \frac{\pi U \times 1.517^2 \times 3.67^2 \times h^{-0.294} \bar{U}^{-0.338} x^{11.75}}{(0.25)^{6.22} \times 10^6}$$

$$\text{but } x^{11.75} = \frac{h^{-3.5} \bar{U}^{1.75} \times 10^{13}}{2.56 Q_h^{3.5}}$$

$$V_d = \frac{\pi U^{2.412} \times 1.517^2 \times 3.67^2 \times 10^7 h^{3.206}}{(0.25)^{6.22} \times 2.56 \times Q_h^{3.5}}$$

$$V_d = \frac{2.114 \times 10^{12} U^{2.412} h^{3.206}}{Q_h^{3.5}} \quad (12)$$

VOLUME OF CLOUD FROM 1 SECOND EMISSION INCLUDING DIFFUSION AND EXPANSION

$$V_c = V_d + \int \frac{dV}{dh} dh \quad (13)$$

$$\frac{V}{V_1} = \left(\frac{P_1}{P} \right)^{\frac{1}{k}} \quad \text{BUT } P = P_1 (1 - 2.255 \times 10^{-5} h)^{5.256} \quad (14)$$

$$\text{AND } \frac{V}{V_1} = \frac{P_1^{\frac{1}{k}}}{P_1^{\frac{1}{k}} (1 - 2.255 \times 10^{-5} h)^{3.754}} = (1 - 2.255 \times 10^{-5} h)^{-3.754}$$

$$\frac{dV}{V_1} = -3.754 (-2.255 \times 10^{-5}) (1 - 2.255 \times 10^{-5} h)^{-4.754} dh$$

$$\frac{dV}{dh} = V_1 (3.754) (2.255 \times 10^{-5}) (1 - 2.255 \times 10^{-5} h)^{-4.754} \quad (15)$$

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BUT $V_i = V_d$ AND $V_d = 2.114 \times 10^{12} U^{2.412} h^{3.206}$

$$\text{AND } V_c = V_d + \int \frac{dV}{dh} dh \quad (16)$$

THEREFORE VOLUME OF CLOUD BY DIFFUSION AND EXPANSION IS -

$$V_c = \frac{2.114 \times 10^{12} U^{2.412} h^{3.206}}{Q_h^{3.5}} + \frac{2.114 \times 10^{12} \times U^{2.412}}{Q_h^{3.5}} \left(-3.754 \right) \left(-2.255 \times 10^{-5} \right) \int_0^h \left(1 - 2.255 \times 10^{-5} h \right)^{-4.754} h^{3.206} dh \quad (17)$$

WEIGHT OF CLOUD FROM 1 SECOND EMISSION FROM CONTINUOUS POINT SOURCE AS AMBIENT AIR IS DIFFUSED INTO THE CLOUD

$$W_c = \int \rho_a dV_d$$

WHERE ρ_a = DENSITY OF AIR AT HEIGHT h

V_d = VOLUME OF CLOUD DUE TO DIFFUSION

ρ_i = DENSITY OF AIR AT $h=0$ (METERS)

h = HEIGHT ABOVE GROUND (SEA LEVEL) METERS

U = WIND VELOCITY METERS / SECOND

$$\rho_a = \rho_i (1 - 2.255 \times 10^{-5} h)^{4.256}$$

$$V_d = \frac{2.114 \times 10^{12} U^{2.412} h^{3.206}}{Q_h^{3.5}}$$

$$dV_d = \frac{3.206 \times 2.114 \times 10^{12} U^{2.412} h^{2.206} dh}{Q_h^{3.5}}$$

THEREFORE WEIGHT OF CLOUD IS

$$W_c = \frac{\rho_i \times 6.777 \times 10^{12} U^{2.412}}{Q_h^{3.5}} \int_0^h \left(1 - 2.255 \times 10^{-5} h \right)^{-4.754} h^{2.206} dh \quad (18)$$

WORK DONE AGAINST GRAVITY BY CLOUD AS IT RISES, ASSUMING HEAT SOURCE IS USED FOR ACCOMPLISHING WORK OF OTHERWISE NON-BUOYANT PLUME, $WORK = W$.

$$W = \int_0^h (-V_c \rho_a + W_c) dh \quad (19)$$

$$\text{BUT } \rho_a = \rho_i (1 - 2.255 \times 10^{-5} h)^{4.256}$$

FOR STANDARD ATMOSPHERE TO 36,000 FT. (5)

THEREFORE THE EXPRESSION FOR WORK BECOMES

$$W = \int (-V_c \rho_a + W_c) dh \quad (19)$$

$$\begin{aligned} W = & \int \left[\frac{-2.114 \times 10^{12} U^{2.412} h^{3.206}}{Q_h^{3.5}} \right] \rho_1 (1 - 2.255 \times 10^{-5} h)^{4.256} dh \\ & - \frac{2.114 \times 10^{12} U^{2.412}}{Q_h^{3.5}} (-3.754) (-2.255 \times 10^{-5}) \int_0^h \int_0^h \left[(1 - 2.255 \times 10^{-5} h)^{-4.754} h^{3.206} \right] \\ & \times (1 - 2.255 \times 10^{-5} h)^{4.256} dh \\ & + \int_0^h \left\{ \frac{\rho_1 \times 6.777 \times 10^{12} U^{2.412}}{Q_h^{3.5}} \int_0^h (1 - 2.255 \times 10^{-5} h)^{4.256} h^{2.206} dh \right\} dh \quad (20) \end{aligned}$$

$\rho_1 = 2.64 \text{ LB/METER}^3$ AT SEA LEVEL, W IS IN LB-METERS BUT BY CLOUD BOUNDARY DEFINITION 90% OF TOTAL HEAT RELEASED WILL BE CONTAINED IN CLOUD AND $237 \text{ LB-METERS} = 1 \text{ BTU}$, THEREFORE $W = 213 Q_h$

THEREFORE BUOYANCY EQUATION FOR 1 SECOND RELEASE OF PLUME IS:

$$\begin{aligned} 213 Q_h = & \frac{U^{2.412}}{Q_h^{3.5}} \left\{ -2.114 \times 10^{12} \times 2.64 \int_0^h (1 - 2.255 \times 10^{-5} h)^{4.256} h^{3.206} dh \right. \\ & - 2.114 \times 10^{12} (-3.754) (-2.255 \times 10^{-5}) \int_0^h \int_0^h \left[(1 - 2.255 \times 10^{-5} h)^{-4.754} h^{3.206} \right] \\ & \times (1 - 2.255 \times 10^{-5} h)^{4.256} dh \\ & \left. + 2.64 \times 6.777 \times 10^{12} \int_0^h \int_0^h \left[(1 - 2.255 \times 10^{-5} h)^{4.256} h^{2.206} \right] dh \right\} \quad (21) \end{aligned}$$

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EXPANDING BINOMIALS TO FOUR TERMS & FACTORING

$$\begin{aligned}
 \frac{213 Q_h^{4.5}}{U^{2.412}} = & -5.581 \times 10^{12} \left\{ \int_0^h h^{3.206} - 4.256 (2.255 \times 10^{-5}) h^{4.206} \right. \\
 & + \frac{4.256 (3.256) (2.255 \times 10^{-5})^2}{2} h^{5.206} \\
 & \left. - \frac{4.256 (3.256) (2.256) (2.255 \times 10^{-5})^3}{6} h^{6.206} \right\} dh \\
 & - 2.114 \times 10^{12} (-3.754) (-2.255 \times 10^{-5}) \left\{ \int_0^h h^{3.206} - (-4.754) (2.255 \times 10^{-5}) h^{4.206} \right. \\
 & + \frac{(-4.754) (-5.754) (2.255 \times 10^{-5})^2}{2} h^{5.206} \\
 & \left. - \frac{(-4.754) (-5.754) (-6.754) (2.255 \times 10^{-5})^3}{6} h^{6.206} \right\} dh (1 - 2.255 \times 10^{-5} h)^{4.256} \\
 & + 2.64 \times 6.777 \times 10^{12} \left\{ \int_0^h h^{2.206} - 4.256 (2.255 \times 10^{-5}) h^{3.206} \right. \\
 & + \frac{4.256 (3.256) (2.255 \times 10^{-5})^2}{2} h^{4.206} \\
 & \left. - \frac{4.256 (3.256) (2.256) (2.255 \times 10^{-5})^3}{6} h^{5.206} \right\} dh, dh
 \end{aligned}$$

INTEGRATING THE FIRST TIME

$$\begin{aligned}
 \frac{213 Q_h^{4.5}}{U^{2.412}} = & -5.581 \times 10^{12} \left\{ \frac{h^{4.206}}{4.206} - \frac{4.256 (2.255 \times 10^{-5}) h^{5.206}}{5.206} \right. \\
 & + \frac{4.256 (3.256) (2.255 \times 10^{-5})^2 h^{6.206}}{2 \times 6.206} - \frac{4.256 (3.256) (2.256) (2.255 \times 10^{-5})^3 h^{7.206}}{6 \times 7.206} \left. \right\} \\
 & - 2.114 \times 10^{12} (-3.754) (-2.255 \times 10^{-5}) \left\{ \frac{h^{4.206}}{4.206} - \frac{(-4.754) (2.255 \times 10^{-5}) h^{5.206}}{5.206} \right. \\
 & + \frac{(-4.754) (-5.754) (2.255 \times 10^{-5})^2 h^{6.206}}{2 \times 6.206} - \frac{(-4.754) (-5.754) (-6.754) (2.255 \times 10^{-5})^3 h^{7.206}}{6 \times 7.206} \left. \right\} \\
 & \times [(1 - 2.255 \times 10^{-5} h)^{4.256}] dh + 2.64 \times 6.777 \times 10^{12} \left\{ \frac{h^{3.206}}{3.206} - \frac{4.256 (2.255 \times 10^{-5}) h^{4.206}}{4.206} \right. \\
 & + \frac{4.256 (3.256) (2.255 \times 10^{-5})^2 h^{5.206}}{2 \times 5.206} - \frac{4.256 (3.256) (2.256) (2.255 \times 10^{-5})^3 h^{6.206}}{6 \times 6.206} \left. \right\} dh
 \end{aligned}$$

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INTEGRATING THE SECOND TIME

$$\begin{aligned}
\frac{213 Q_1^{4.5}}{U^{2.412}} &= -5.581 \times 10^{12} h^{4.206} \left\{ \frac{1}{4.206} - \frac{4.256(2.255 \times 10^{-5})}{5.206} h \right. \\
&\quad + \frac{4.256(3.256)(2.255 \times 10^{-5})^2}{2 \times 6.206} h^2 - \frac{4.256(3.256)(2.256)(2.255 \times 10^{-5})^3}{6 \times 7.206} h^3 \Big\} \\
&\quad - 2.114 \times 10^{12} (-3.754)(2.255 \times 10^{-5}) \left\{ \frac{h^{5.206}}{4.206 \times 5.206} - \frac{(4.256)(2.255 \times 10^{-5}) h^{6.206}}{4.206 \times 6.206} \right. \\
&\quad + \frac{4.256(3.256)(2.255 \times 10^{-5})^2}{2 \times 4.206 \times 7.206} h^{7.206} - \frac{4.256(3.256)(2.256)(2.255 \times 10^{-5})^3}{6 \times 4.206 \times 8.206} h^{8.206} \\
&\quad + \frac{4.754(2.255 \times 10^{-5}) h^{6.206}}{5.206 \times 6.206} - \frac{4.754(4.256)(2.255 \times 10^{-5})^2}{5.206 \times 7.206} h^{7.206} \\
&\quad + \frac{4.754(4.256)(3.256)(2.255 \times 10^{-5})^3}{5.206 \times 2 \times 8.206} h^{8.206} - \frac{4.754(4.256)(3.256)(2.256)(2.255 \times 10^{-5})^4}{5.206 \times 6 \times 9.206} h^{9.206} \\
&\quad + \frac{(4.754)(5.754)(2.255 \times 10^{-5})^2}{2 \times 6.206 \times 7.206} h^{7.206} - \frac{(4.256)(4.754)(5.754)(2.255 \times 10^{-5})^3}{2 \times 6.206 \times 8.206} h^{8.206} \\
&\quad + \frac{(4.256)(3.256)(4.754)(5.754)(2.255 \times 10^{-5})^4}{2 \times 6.206 \times 2 \times 9.206} h^{9.206} \\
&\quad - \frac{(4.256)(3.256)(2.256)(4.754)(5.754)(2.255 \times 10^{-5})^5}{2 \times 6.206 \times 6 \times 10.206} h^{10.206} \\
&\quad + \frac{(4.754)(5.754)(6.754)(2.255 \times 10^{-5})^3}{6 \times 7.206} \left[\frac{h^{8.206}}{8.206} - \frac{4.256(2.255 \times 10^{-5}) h^{9.206}}{9.206} \right. \\
&\quad + \frac{(4.256)(3.256)(2.255 \times 10^{-5})^2}{2 \times 10.206} h^{10.206} - \frac{(4.256)(3.256)(2.256)(2.255 \times 10^{-5})^3}{6 \times 11.206} h^{11.206} \Big\} \\
&\quad + 2.64 \times 6.777 \times 10^{12} \left\{ \frac{h^{4.206}}{3.206 \times 4.206} - \frac{4.256(2.255 \times 10^{-5}) h^{5.206}}{4.206 \times 5.206} \right. \\
&\quad + \frac{4.256(3.256)(2.255 \times 10^{-5})^2}{2 \times 5.206 \times 6.206} h^{6.206} - \frac{4.256(3.256)(2.256)(2.255 \times 10^{-5})^3}{6 \times 6.206 \times 7.206} h^{7.206} \Big\}
\end{aligned}$$

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REDUCING TERMS:

$$\begin{aligned} \frac{213 Q_h^{4.5}}{U^{2.412}} &= -5.581 \times 10^{12} h^{4.206} \left\{ 0.2378 - 1.844 \times 10^{-5} h + 5.677 \times 10^{-10} h^2 - 8.29 \times 10^{-15} h^3 \right\} \\ &\quad - 1.790 \times 10^8 h^{5.206} \left\{ 0.04567 - 0.3677 \times 10^{-5} h + 1.162 \times 10^{-10} h^2 - 17.31 \times 10^{-15} h^3 \right\} \\ &\quad + 0.3318 \times 10^{-5} h - 2.742 \times 10^{-10} h^2 + 8.841 \times 10^{-15} h^3 - 13.36 \times 10^{-20} h^4 \\ &\quad + 1.443 \times 10^{-10} h^2 - 13.11 \times 10^{-15} h^3 + 42.89 \times 10^{-20} h^4 - 65.59 \times 10^{-25} h^5 \\ &\quad + 49.00 \times 10^{-15} \left[0.1219 h^3 - 1.0426 \times 10^{-5} h^4 + 3.452 \times 10^{-10} h^5 - 5.331 \times 10^{-15} h^6 \right] \\ &\quad + 17.89 \times 10^{12} h^{4.206} \left\{ 0.07416 - 0.4383 \times 10^{-5} h + 1.090 \times 10^{-10} h^2 - 1.336 \times 10^{-15} h^3 \right\} \end{aligned}$$

$$\begin{aligned} \frac{213 Q_h^{4.5}}{U^{2.412}} &= 10^7 h^{5.206} \left\{ + 2.457 - 12.18 \times 10^{-5} h + 22.37 \times 10^{-10} h^2 \right. \\ &\quad - 0.8175 + 0.6426 \times 10^{-5} h + 2.452 \times 10^{-10} h^2 + 279.4 \times 10^{-15} h^3 \\ &\quad \left. + 385.9 \times 10^{-20} h^4 - 1854 \times 10^{-25} h^5 + 4676 \times 10^{-30} h^6 \right\} \end{aligned}$$

$$\begin{aligned} \frac{213 Q_h^{4.5}}{U^{2.412}} &= 10^7 h^{5.206} \left\{ + 1.633 - 12.18 \times 10^{-5} h + 24.82 \times 10^{-10} h^2 \right. \\ &\quad \left. + 279.4 \times 10^{-15} h^3 + 385.9 \times 10^{-20} h^4 - 1854 \times 10^{-25} h^5 + 4676 \times 10^{-30} h^6 \right\} \end{aligned}$$

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$$\text{EVALUATION OF TERM } \left\{ +1.633 - 12.18 \times 10^{-5} h + 24.82 \times 10^{-10} h^2 + 279.4 \times 10^{-15} h^3 + 385.9 \times 10^{-20} h^4 - 1854 \times 10^{-25} h^5 + 4676 \times 10^{-30} h^6 \right\}$$

WHERE $h = 10$ METERS

$$\left\{ \right\} = \left\{ -1.633 - 0.0012 + 0.0000002 \right\} = 1.632$$

WHERE $h = 100$ METERS

$$\left\{ \right\} = \left\{ 1.633 - 0.012 + 0.000002 \right\} = 1.621$$

WHERE $h = 1000$ METERS

$$\left\{ \right\} = \left\{ 1.633 - 0.122 + 0.0025 + 0.00028 \right\} = 1.514$$

WHERE $h = 10,000$ METERS

$$\left\{ \right\} = \left\{ 1.633 - 1.218 + 0.2482 + 0.2794 + 0.0386 - 0.0185 + 0.0047 \right\} = 0.967$$

SOLVING FOR h :

$$h = \left[\frac{213 Q_h^{4.5}}{10^7 U^{2.412} \left\{ \right\}} \right]^{\frac{1}{5.206}} = \frac{0.0523 Q_h^{0.864}}{U^{0.463} \left\{ \right\}^{\frac{1}{5.206}}} \quad (22)$$

EVALUATING $\left\{ \right\}$ FOR VARIOUS HEIGHTS

$$10 \text{ METERS } \left\{ 1.632 \right\}^{0.19209} = 1.099$$

$$100 \text{ METERS } \left\{ 1.621 \right\}^{0.19209} = 1.097$$

$$1000 \text{ METERS } \left\{ 1.514 \right\}^{0.19209} = 1.083$$

$$10000 \text{ METERS } \left\{ 0.967 \right\}^{0.19209} = 0.9936$$

USE 1.05 FOR TWO SIGNIFICANT FIGURE FOR FLUME RISE

THEREFORE MAXIMUM CONTINUOUS POINT SOURCE PLUME RISE TO TWO SIGNIFICANT FIGURES BECOMES:

$$h = \frac{0.050 Q_h^{0.864}}{U^{0.463}} \quad (23)$$

ASSUMING 80% EFFICIENCY DUE TO TURBULENCE CAUSED BY HIGH EXIT VELOCITY AND IRREVERSIBILITY BECAUSE OF HIGH EXIT TEMPERATURE, THE FORMULA THEN BECOMES —

$$h = \frac{0.040 Q_h^{0.864}}{U^{0.463}} \quad \text{OR} \quad \cong \frac{0.040 Q_h^{0.864}}{U^{1/2}} \quad (24)$$

FOR LAPSE RATES OTHER THAN STANDARD,

$n=0.25$, THE POWER OF Q VARIES SOMEWHAT

$$\text{FOR } n = 0.20 \quad h = k Q_h^{\frac{4.6}{5.206}} = Q_h^{0.883}$$

$$n = 0.25 \quad h = k Q_h^{\frac{4.5}{5.206}} = Q_h^{0.875}$$

$$n = 0.33 \quad h = k Q_h^{\frac{4.34}{5.206}} = Q_h^{0.835}$$

$$n = 0.50 \quad h = k Q_h^{\frac{4.00}{5.206}} = Q_h^{0.768}$$

TO ADJUST THE FORMULA FOR OTHER LAPSE RATE, THE POWER OF Q APPROXIMATES $(2-n)/2$, WHICH IS COMPARED WITH ABOVE POWER OF Q_h :

WHERE	n	POWER OF Q	$(2-n)/2$
	0.20	0.883	0.90
	0.25	0.864	0.875
	0.33	0.834	0.835
	0.50	0.768	0.75

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THE PLUME RISE FORMULA FOR CONTINUOUS
POINT SOURCE THEN TAKES THE FORM:

$$h = \frac{0.040 Q_h^{\frac{2-y}{2}}}{(\bar{U})^{1/2}} \quad (25)$$

WHERE

h = BUOYANCY RISE
DUE TO HEAT FOR
EXHAUST WITH MOLE
WEIGHT EQUAL
TO THAT FOR AIR
(METERS)

Q_h = HEAT RELEASE
RATE (BTU/SEC)

y = STABILITY PARAMETER
(SUTTON'S)

$\bar{U}$ = MEAN WIND
VELOCITY TO
HEIGHT OF RISE
(METERS/SEC)

DANIEL MANN, JOHNSON, & MENDENHALL

Penetration of Air Jet
 Designation: Penetration To Air
 Air Jet

JOB NO. 507-1-1

SHEET NO. C-1

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TYPICAL EXAMPLE OF JET PENETRATION

$$\left(\frac{L}{D_j}\right)^{1.65} = 2.91 \frac{\rho_j V_j \sqrt{S}}{\rho_o V_o}$$

WHERE:

L = DEPTH OF JET PENETRATION

D_j = ORIFICE DIAMETER

ρ_j = MASS DENSITY OF JET AIR AT VENA CONTRACTA

ρ_o = MASS DENSITY OF FREE STREAM AIR

V_j = VELOCITY OF JET AT VENA CONTRACTA

V_o = VELOCITY OF FREE STREAM AIR

S = DISTANCE DOWNSTREAM FROM ORIFICE CENTERLINE

OR, CONVERTING TO COMMON USL METEOROLOGY UNITS

$$\left(\frac{h}{D_{JET}}\right)^{1.65} = 2.91 \frac{\rho_{GAS} U_{STACK} \sqrt{X}}{\rho_{AIR} U_{AIR} D_{STACK}}$$

ASSUME: 333 ft/sec EMISSION, 40% GAS (HCl SCRUBBED OUT, REMAINING GAS

$\rho_{AIR, STD} = \rho_{GAS, STD}$ $\therefore \left(\frac{\rho_{GAS}}{\rho_{AIR}}\right) = \left(\frac{T_{AIR}}{T_{GAS}}\right)$ (HEATED)

$T_{AIR} = 100^\circ F$

$T_{GAS} = 4000^\circ F$

M.W.GAS = 29

$U_{AIR} = 4.47 \text{ Meters/sec}$

$D_{JET} = 2 \text{ METERS}$

$U_{STACK} = \frac{VOLUME}{AREA_{STACK}} = \left(\frac{333 \text{ ft}^3}{29 \times 21} \times \frac{0.40}{1} \times \frac{1540}{1} \times \frac{4910^\circ R}{1}\right) \left(\frac{M^3}{35.3 \text{ ft}^3} \times \frac{1}{\pi \frac{2^2}{4}}\right) = 150 \frac{\text{METERS}}{\text{SEC}}$

$X = 1000 \text{ METERS}$ (13.2 x 144)

HENCE:

$$h = \left[2.91 D^{1.15} \left(\frac{T_{AIR} U_{GAS}}{T_{GAS} U_{AIR}} \right) \left(X^{\frac{1}{2}} \right) \right]^{1.65} = \left[2.91 (2)^{1.15} \left(\frac{560 \times 150}{4460 \times 4.47} \right) (1000^{\frac{1}{2}}) \right]^{1.65}$$

$$h = \left[2.91 (2.24) (4.2 \times 100) \right]^{0.606} \text{ METERS}$$

$$h = [2740]^{0.606} \text{ METERS}$$

$$h = 120 \text{ METERS AT 1000 METERS}$$

DANIEL MANN, JOHNSON, & MENDENHALL

By THOMAS D. JOHNSON
DESIGNED PERPENDICULAR TO AIR
STREAM

JOB NO.

SHEET No. C-2

DESIGNED BY WLL

DATE

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At 6.2 MILES = 100,000 METERS

$$h = [(27.4)(100,000^{\frac{1}{2}})]^{0.606}$$

$$h = [(27.4)(316)]^{0.606}$$

$$h = [8650]^{0.606}$$

$$h = 240 \text{ METERS}$$

At 400 MILES = 645,000 METERS

$$h = [(27.4)(645,000^{\frac{1}{2}})]^{0.606}$$

$$h = [(27.4)(804)]^{0.606}$$

$$h = [22,000]^{0.606}$$

$$h = 425 \text{ METERS}$$

The basic equation was taken from "Investigation Of The Penetration
 Of An Air Jet Directed Perpendicularly To An Air Stream" By

Edmund E. Callaghan & Robert S. Ruggeri, N.A.C.A. Tech. Note 1615

ASTIA #26921

Dimensional Analysis Formula
For Effective Stack Heights

DESIGNED BY D. Stai

DATE Aug 14 '62

APPROVED

The effective stack height is a function of many variables which are both dependant and independent. By the use of dimensional analysis these variables may be related to each other and to the effective stack height. Dimensional analysis is useful since it assists in the determination of convenient arrangements of variables in a typical relation and helps plan systematic experiments. The first step is to list all the variables involved. This list is a result of experience and judgment as to the variables of importance. After the list has been made the formal procedure of analysis is used to relate the variables. The dimensions of interest and their symbols are: Length (L), mass (M), time (T), temperature (θ), and thermal energy (H).

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The stack height equation can be written symbolically as:

$$\Delta h = f \left(G, V_s, u, \frac{d\theta'}{dz}, \frac{dM}{dz}, T, \Delta T, g, Q_v, C_p, d, \rho, \bar{M}, \mu \right)$$

where:

Parameter	Definition	Units
Δh	Effective stack height	L
G	Stack mass flow	$\frac{M}{T}$
V_s	Stack gas vert. velocity	$\frac{L}{T}$
u	Horizontal wind velocity	$\frac{L}{T}$
$\frac{d\theta'}{dz}$	Lapse rate of atmosphere	$\frac{\theta}{L}$
$\frac{dM}{dz}$	Entrainment rate of cloud	$\frac{M}{L}$
T	Abs. ambient temperature	θ
ΔT	Temp. difference, stack gas minus ambient	θ
g	Gravitation constant	$\frac{L}{T^2}$
Q_v	Volume emission rate of stack gas	$\frac{L^3}{T}$
C_p	Specific heat of a gas	$\frac{H}{M\theta}$
d	Stack diameter	L
ρ	Density of a gas	$\frac{M}{L^3}$

and:	Parameter	Definition	Units
	$\bar{M}$	Momentum of gas	$\frac{ML}{T}$
	μ	Viscosity of gas	$\frac{M}{LT}$

The value of Δh will be computed in two distinct parts; those of upward momentum and of buoyancy. That is:

$$\Delta h = \Delta h_m + \Delta h_b$$

$$\Delta h_m = f (\text{Momentum})$$

$$\Delta h_b = f (\text{Buoyancy})$$

The assumption is made that all of the momentum is dissipated before buoyancy is added. Let $\bar{M}_0$ be the stack gas exit (or initial) momentum, and $\bar{M}_1$ be the stack gas momentum at Δh_m . Then:

$$\bar{M}_1 = 0$$

$$\bar{M}_1 - \bar{M}_0 = F t_1$$

$$\bar{M}_1 + F t_1 = 0$$

"F" is defined as the deceleration force provided by wind, gravity, and

entrainment with the viscous gas-air interface, and t , is the time that "F" acts. The force can then be written:

$$F = f(V_s, d, g, \mu, \rho, u)$$

or
$$F = K(V_s^a d^b g^c \mu^d \rho^e u^f)$$

and a unit equation can be written conforming to this expression:

$$\frac{ML}{T^2} = K \left(\left(\frac{L}{T} \right)^a (L)^b \left(\frac{L}{T^2} \right)^c \left(\frac{M}{LT} \right)^d \left(\frac{M}{L^3} \right)^e \left(\frac{L}{T} \right)^f \right)$$

In order to relate variables they are arranged in dimensionless groups or combinations by means of the Buckingham π Theorem. In the equation for F there are seven variables and three fundamental units. The physical equation has four dimensionless ratios which can be written as π_1, π_2, π_3 , and π_4 .

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These ratios can also be expressed as:

$$\pi_1 = d^{x_1} V_s^{y_1} \rho^{z_1} \mu$$

$$\pi_2 = d^{x_2} V_s^{y_2} \rho^{z_2} \mu$$

$$\pi_3 = d^{x_3} V_s^{y_3} \rho^{z_3} q$$

$$\pi_4 = d^{x_4} V_s^{y_4} \rho^{z_4} F$$

The twelve exponents can be set so that each π function is dimensionless:

$$\pi_1 = (L)^0 \left(\frac{L}{T}\right)^{+1} \left(\frac{M}{L^3}\right)^0 \left(\frac{L}{T}\right)^{-1}$$

$$\pi_1 = \frac{V_s}{\mu}$$

$$\pi_2 = (L)^{+1} \left(\frac{L}{T}\right)^{+1} \left(\frac{M}{L^3}\right)^{+1} \left(\frac{M}{L \cdot T}\right)^{-1}$$

$$\pi_2 = \frac{d V_s \rho}{\mu}$$

$$\pi_3 = (L)^{-1} \left(\frac{L}{T}\right)^{+2} \left(\frac{M}{L^3}\right)^0 \left(\frac{L}{T^2}\right)^{-1}$$

$$\pi_3 = \frac{V_s^2}{d q}$$

$$\pi_4 = (L)^{+2} \left(\frac{L}{T}\right)^{+2} \left(\frac{M}{L^3}\right)^{+1} \left(\frac{M L}{T^2}\right)^{-1}$$

$$\pi_4 = \frac{d^2 V_s^2 \rho}{F}$$

The result may be written:

$$\pi_4 = f(\pi_1, \pi_2, \pi_3)$$

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Then:

$$\frac{d^2 V_s^2 \rho}{F} = f\left(\frac{V_s}{u}, \frac{d V_s \rho}{\mu}, \frac{V_s^2}{d g}\right)$$

Also:

$$F = K d^2 V_s^2 \rho \left(\frac{V_s}{u}\right)^a \left(\frac{d V_s \rho}{\mu}\right)^b \left(\frac{V_s^2}{d g}\right)^c$$

where the coefficients K , a , b , and c can be found from experimental data. The equation can be rewritten upon inspection as:

$$F = K' d^2 g \left(\frac{V_s}{u}\right)^a (Re)^b (Fr)^c$$

where:

$$g = \frac{1}{2} \rho V_s^2 = \text{Dynamic pressure of stream}$$

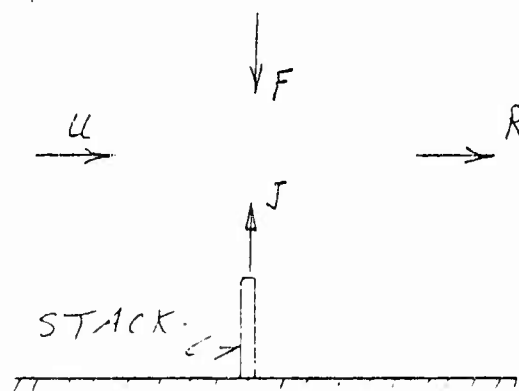
$$Re = \frac{d V_s \rho}{\mu} = \text{Reynold's Number of stream}$$

$$Fr = \frac{V_s^2}{d g} = \text{Froude's Number} = \frac{\text{Inertia Force}}{\text{Gravity Force}}$$

$$K' = 2K$$

Therefore, it is shown that the deceleration force on a jet is a function of jet pressure, diameter of stack, velocity ratio, Reynold's Number of jet, and jet kinetic energy.

A force diagram can now be drawn for the vertically directed jet or stack gas.



Assume that the wind vector (u) does not lower the stack height, but that the accompanying turbulence that is associated with the wind will influence the stack height. Therefore:

$$R = u$$

R = Resultant Vector of Gases

$F = -J$, where J is the upward force of the jet.

Let

$$J = \frac{d(mV_s)}{dt} = \text{Change of jet momentum with respect to time.}$$

$$J = V_s \frac{dm}{dt} + m \frac{dV_s}{dt}$$

Then

$$F = -V_s \frac{dm}{dt} - m \frac{dV_s}{dt}$$

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The definition of V_s is $\frac{dH}{dt}$ and then

$$F = - \frac{dH}{dt^2} \frac{dm}{dt} - mg$$

where

$g = \frac{dV_s}{dt}$ = acceleration due to gravity

and mg = force exerted by column of gas if the density differs from air.

$$\frac{dH}{dt^2} \frac{dm}{dt} = -F - mg$$

$$dH \frac{dm}{dt} = (-F - mg) dt$$

$$\frac{dm}{dt} \int_0^{\Delta h_m} dH = (-F - mg) \int_0^{t_m} dt$$

$$\Delta h_m = (-F - mg) t_m \left(\frac{dm}{dt} \right)^{-1}$$

or

$$\Delta h_m = \left(\frac{dm}{dt} \right)^{-1} \left\{ K' d^2 g \left(\frac{V_s}{u} \right)^a (Re)^b (Fr)^c - mg \right\} t,$$

Then regrouping a new expression is obtained:

$$\Delta h_m \frac{dm}{dt} = \left\{ K' \dots \right\} t, dt$$

Integrating both sides:

$$\Delta h_m \int_0^M dm = \left\{ K' \dots \right\} t, \int_0^{t_1} dt.$$

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Then:

$$\Delta h_m M = \left\{ K' \dots \right\} t_1^2$$

and

$$\Delta h_m = \left\{ K' d^2 g \left(\frac{V_s}{u} \right)^a (Re)^b (Fr)^c - mg \right\} \frac{t_1^2}{M}$$

This expression is applicable to jet flow systems and also possibly to instantaneous upward releases or puffs. For a continuous flow the mass (M) would equal $G t_1$. Hence for a continuous flow:

$$\Delta h_m = \left\{ K' d^2 g \left(\frac{V_s}{u} \right)^a (Re)^b (Fr)^c - mg \right\} \frac{t_1}{G}$$

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The plume is assumed to lose no buoyancy during the time t . Therefore, an expression for Δh_b would start from Δh_m and continue up to Δh_{Total} . Writing the expression:

$$\Delta h_b = f(\text{Buoyancy})$$

$$\Delta h_b = f\left(\rho_a, \rho_s, T_a, T_s, \frac{d\theta}{dz}, \frac{dM}{dz}, c_{pa}, c_{ps}, Q_{vi}, u, v_s, g, d, \mu_a, \mu_s\right)$$

This expression can be simplified by substituting:

$$\nu = \frac{\mu}{\rho} = \text{kinematic viscosity}$$

Therefore:

$$\nu_a = \frac{\mu_a}{\rho_a} \quad \text{and} \quad \nu_s = \frac{\mu_s}{\rho_s}$$

Also, by the use of delta quantities, more of the complexities of the equation are removed:

$$\Delta T = T_s - T_a$$

$$\Delta \nu = \nu_s - \nu_a$$

$$\Delta C_p = C_{ps} - C_{pa}$$

Then:

$$\Delta h_b = f\left(\Delta T, \frac{d\theta}{dz}, \frac{dM}{dz}, \Delta C_p, \Delta \nu, Q_{vi}, u, v_s, g, d\right)$$

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This is also written:

$$\Delta h_0 = K'' \left((\Delta T)^a \left(\frac{d\Theta}{dz} \right)^b \left(\frac{dM}{dz} \right)^c (\Delta C_p)^d (\Delta V)^e (Q_{vi})^f (u)^g (V_s)^h (g)^i (d)^j \right)$$

and the unit equation is:

$$L = K'' \left(\Theta^a \left(\frac{\Theta}{L} \right)^b \left(\frac{M}{L} \right)^c \left(\frac{H}{Me} \right)^d \left(\frac{L^2}{T} \right)^e \left(\frac{L^3}{T} \right)^f \left(\frac{L}{T} \right)^g \left(\frac{L}{T^2} \right)^h \left(L \right)^i \right)$$

By use of the Buckingham π Theorem it is known that there are eleven variables and five fundamental units, and the physical equation has six dimensionless ratios (π_1 through π_6). That is:

$$\pi_1 = \Delta T^{R_1} \Delta C_p^{S_1} \Delta V^{T_1} g^{U_1} d^{V_1} \frac{d\Theta}{dz}$$

$$\pi_2 = \Delta T^{R_2} \Delta C_p^{S_2} \Delta V^{T_2} g^{U_2} d^{V_2} \frac{dM}{dz}$$

$$\pi_3 = \Delta T^{R_3} \Delta C_p^{S_3} \Delta V^{T_3} g^{U_3} d^{V_3} Q_{vi}$$

$$\pi_4 = \Delta T^{R_4} \Delta C_p^{S_4} \Delta V^{T_4} g^{U_4} d^{V_4} u$$

$$\pi_5 = \Delta T^{R_5} \Delta C_p^{S_5} \Delta V^{T_5} g^{U_5} d^{V_5} V_s$$

$$\pi_6 = \Delta T^{R_6} \Delta C_p^{S_6} \Delta V^{T_6} g^{U_6} d^{V_6} \Delta h_0$$

The 30 exponents can be set so that each π function is dimensionless.

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$$\pi_1 = (\Theta)^{-1} \left(\frac{H}{M\Theta} \right)^0 \left(\frac{L^2}{T} \right)^{-2} \left(\frac{L}{T^2} \right)^{+1} (L)^{+4} \left(\frac{\Theta}{L} \right)^{+1}$$

$$\pi_1 = \frac{\rho d^4 \frac{d\Theta'}{dz}}{\Delta T \Delta V^2}$$

$$\pi_2 = (\Theta)^0 \left(\frac{H}{M\Theta} \right)^0 \left(\frac{L^2}{T} \right)^{+2} \left(\frac{L}{T^2} \right)^{-1} (L)^{-3} \left(\frac{M}{L} \right)^0$$

$$\pi_2 = \frac{\Delta V^2}{\rho d^3}$$

$$\pi_3 = (\Theta)^0 \left(\frac{H}{M\Theta} \right)^0 \left(\frac{L^2}{T} \right)^{-1} \left(\frac{L}{T^2} \right)^{+1} (L)^{+4} \left(\frac{L^3}{T} \right)^{-1}$$

$$\pi_3 = \frac{\rho d^4}{\Delta V Q_{v1}}$$

$$\pi_4 = (\Theta)^0 \left(\frac{H}{M\Theta} \right)^0 \left(\frac{L^2}{T} \right)^{-1} \left(\frac{L}{T^2} \right)^{+1} (L)^{+2} \left(\frac{L}{T} \right)^{-1}$$

$$\pi_4 = \frac{\rho d^2}{\Delta V \mu}$$

$$\pi_5 = (\Theta)^0 \left(\frac{H}{M\Theta} \right)^0 \left(\frac{L^2}{T} \right)^{-1} \left(\frac{L}{T^2} \right)^{+1} (L)^{+2} \left(\frac{L}{T} \right)^{-1}$$

$$\pi_5 = \frac{\rho d^2}{\Delta V V_s}$$

$$\pi_6 = (\Theta)^0 \left(\frac{H}{M\Theta} \right)^0 \left(\frac{L^2}{T} \right)^{-2} \left(\frac{L}{T^2} \right)^{+1} (L)^{+2} (L)^{+1}$$

$$\pi_6 = \frac{\rho d^2 \Delta h_b}{\Delta V^2}$$

The result may be written:

$$\pi_6 = f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$$

or

$$\pi_6 = K''(\pi_1^{a'} \pi_2^{b'} \pi_3^{c'} \pi_4^{d'} \pi_5^{e'})$$

or

$$\Delta h_b = \frac{K'' \Delta V^2}{g d^2} \left(\frac{g d^4 \frac{d\theta'}{dz}}{\Delta T \Delta V^2} \right)^{a'} \left(\frac{\Delta V^2}{g d^3} \right)^{b'}$$

$$\left(\frac{g d^4}{\Delta V Q_{v1}} \right)^{c'} \left(\frac{g d^2}{\Delta V u} \right)^{d'} \left(\frac{g d^2}{\Delta V Y_s} \right)^{e'}$$

Combining terms:

$$\Delta h_b = K'' (g^{-1+a'-b'+c'+d'+e'}) (d^{-2+4a'-3b'+4c'+2d'+2e'})$$

$$(\Delta V^{2-2a'+2b'-c'-d'-e'}) \left(\frac{d\theta'}{dz} \right)^{a'} (\Delta T^{-a'})$$

$$(Q_{v1}^{-c'}) (u^{-d'}) (Y_s^{-e'})$$

Simplifying exponents:

$$g^{-1+a'-b'+c'+d'+e'} = g^f$$

$$d^{-2+4a'-3b'+4c'+2d'+2e'} = d^k$$

$$\Delta V^{2-2a'+2b'-c'-d'-e'} = \Delta V^m$$

The equation then becomes:

$$\Delta h_b = K'' g^f d^{1/2} \Delta V^m \left(\frac{\frac{d\theta'}{dz}}{\Delta T} \right)^{a'} Q_{v1}^{-c'} u^{-d'} V_s^{-e'}$$

Therefore, it is shown that the buoyancy rise is a function of the force of gravity, stack diameter, gas viscosity, lapse rate, gas temperature, volume flow rate, wind speed, and gas velocity.

Finally, from the above derivations, the equation for effective stack height is written:

$$\Delta h = \Delta h_m + \Delta h_b$$

$$\Delta h = \left\{ K' d^2 g \left(\frac{V_s}{u} \right)^a (Re)^b (Fr)^c - mg \right\} \frac{T_1}{G} + K'' g^f$$

$$d^{1/2} \Delta V^m \left(\frac{\frac{d\theta'}{dz}}{\Delta T} \right)^{a'} Q_{v1}^{-c'} u^{-d'} V_s^{-e'}$$

Where

$$g = \frac{1}{2} \rho V_s^2$$

$$Re = \frac{d V_s \rho}{\mu}$$

$$Fr = \frac{V_s^2}{d g}$$

$$V = \frac{\mu}{\rho}$$

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The equation for effective stack height must be evaluated in terms of real numbers for K' , K'' , and the various exponents before it can be of any practical use. A program to accomplish this is possible. The parameters to be measured are part of the equation. A series of tests in which the various known quantities are changed would allow an evaluation of the constants and exponents. Such a series, at low energy levels, was conducted and published under the title: "A Comparison of Observed Plume Rises with Values Obtained from Well-Known Formulas", by Moses and Strom, Journal of the APCA Vol 11, #10, Pages 455-66 (Oct. 1961). Using the data presented, which is complete enough for our purposes, a series of equations may be set up to find the unknown constants and exponents.

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The data presented in the article was obtained at the Argonne National Laboratory, Argonne, Ill. The air pressure was not recorded and so is estimated to be that for an elevation of 600 FT above sea level, the elevation of Chicago.

$$\begin{aligned}
 P_{600} &= P_0 (1 - 0.000006875 h)^{5.2561} \\
 &= 760 (1 - 0.004125)^{5.2561} \text{ mm Hg} \\
 &= 760 (0.995875)^{5.2561} \\
 &= 760 (0.9785)
 \end{aligned}$$

$$P_{600} = 743.66 \text{ mm Hg}$$

Since the actual pressures could have varied widely, the value for P_{600} is chosen to be 740 mm Hg to give two place accuracy. The diameter of the stack was not stated. Therefore, a back-calculation was necessary to obtain it. An average value was $d = 0.446$ meters. The density of the gas and air are taken to be equal, and $\rho = 1.27 \text{ gram/liter} = 1.27 \cdot 10^{-3} \text{ gm/m}^3$ at an elevation of 600 ft. The viscosity of air under these conditions is $\mu = 0.018 \text{ cp}$. or $\mu = 1.8 \cdot 10^{-2} \frac{\text{gm}}{\text{m} \cdot \text{sec}}$. The various values, the known quantities can now be written for each run of interest.

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Run 104

$$\Delta h = 9 \text{ meters}$$

$$d = 0.446 \text{ meters}$$

$$g = \frac{1}{2} \rho V_s^2 = \frac{1}{2} \cdot 1.27 \cdot 10^{-3} \cdot 4.87^2 \frac{\text{gram}}{\text{meter sec}^2}$$

$$g = 1.505 \cdot 10^{-2} \frac{\text{gram}}{\text{meter sec}^2}$$

$$\frac{V_s}{u} = \frac{4.87}{6.17} = 0.79$$

$$Re = \frac{d V_s \rho}{\mu} = \frac{0.446 \cdot 4.87 \cdot 1.27 \cdot 10^{-3}}{1.8 \cdot 10^{-2}} = 15.3$$

$$Fr = \frac{4.87^2}{0.446 \cdot 9.80} = 5.43$$

$$G = \rho A_s V_s$$

$$A_s = \frac{\pi}{4} d^2$$

$$G = 1.27 \cdot 10^{-3} \cdot \frac{\pi}{4} \cdot 0.446^2 \cdot (4.87) \frac{\text{gm}}{\text{sec}}$$

$$G = 0.965 \cdot 10^{-3} \frac{\text{gm}}{\text{sec}}$$

$$T_1 = \frac{x}{u} = \frac{\text{Distance Downwind at } \Delta h}{\text{Wind Velocity}}$$

$$T_1 = \frac{100 \text{ m}}{6.2 \text{ m/sec}} = 16.1 \text{ sec}$$

$$g = 9.80 \text{ m/sec}^2$$

$$\Delta V = \rightarrow 0$$

$$\frac{d\theta'}{dz} = \frac{1.4^\circ \text{C}}{100 \text{ m}} = 1.4 \cdot 10^{-2}^\circ \text{C/m}$$

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$$\Delta T = T_s - T_a$$

$$\Delta T = 327.4^\circ K - 291.5^\circ K$$

$$\Delta T = 35.9^\circ C$$

$$Q_{vi} = A_s V_s$$

$$Q_{vi} = \frac{\pi}{4} \cdot 0.446^2 \cdot 4.87 \frac{m^3}{sec}$$

$$Q_{vi} = 0.76 \frac{m^3}{sec}$$

Having evaluated the major parameters the equation can now be written with the remaining unknowns showing:

$$g_m = \left\{ K' (0.446)^2 m^2 (1.505 \cdot 10^{-2} \frac{gm}{m \cdot sec^2}) (0.79)^2 (15.3)^6 (5.43)^5 \right. \\ \left. - m \cdot 9.80 \frac{m}{sec^2} \right\} \frac{16.1 sec}{0.965 \cdot 10^{-3} \frac{gm}{sec}} + K'' (9.80 \frac{m}{sec^2})^f \\ (0.446)^k (\sim 0)^m \left(\frac{1.4 \cdot 10^{-2} ^\circ C / m}{35.9^\circ C} \right)^{a'} (0.76 \frac{m^3}{sec})^{-c'} (6.2 \frac{m}{sec})^{-d'} (4.87 \frac{m}{sec})^{-e'}$$

Simplifying: (Where m is a function of relative Mol. Wts. of stream to air)

$$g_m = \left\{ K' (2.99 \cdot 10^{-3} \frac{gm \cdot m}{sec^2}) (0.79)^a (15.3)^b (5.43)^c - 0 \right\}_{m \rightarrow 0} \\ 16.7 \cdot 10^3 \frac{sec^2}{gm} + K'' (9.80 \frac{m}{sec^2})^f (0.446 m)^k (3.8 \cdot 10^{-4} m^{-1})^{a'} (0.76 \frac{m^3}{sec})^{-c'} \\ (6.2 \frac{m}{sec})^{-d'} (4.87 \frac{m}{sec})^{-e'}$$

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This leaves an equation with eleven unknowns plus one term which dropped out, but for which the exponent must then approach zero (ΔV^m).

For a complete evaluation of the exponents and constants, this equation must be evaluated a total of twelve (12) times using twelve different runs. One factor which will assist in evaluating the buoyancy term is the necessity of maintaining dimensional consistency. The unit equation is:

$$M^1 = \left(\frac{M}{\text{sec}^2}\right)^f (M)^k (M^{-1})^{a'} \left(\frac{M^3}{\text{sec}}\right)^{-c'} \left(\frac{M}{\text{sec}}\right)^{-d'} \left(\frac{M}{\text{sec}}\right)^{-e'}$$

Therefore these relations must hold true:

$$1 = f + k - (a') + 3(-c') + (-d') + (-e')$$

and

$$0 = 2f + (-c') + (-d') + (-e')$$

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The above analysis indicates the degree of complexity involved in the calculation of effective stack heights. This holds true even when all constants and exponents are known, and when there are twelve unknowns the problem becomes one for a computer program.

A Calculation of Error Theory to Relations of Several Variables	JOB No. 507-1-1	SHEET No. E-1
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Probable error of a calculated function. The simplest example in the present study is the calculation of the maximum toxic concentration, X_{max} , for a continuous point source. It is written as

$$X_{max} = \frac{2Q}{e\pi \bar{u} h^2} \cdot \frac{C_z}{C_y}$$

where $C_z/C_y = 1$ for isotropic conditions.

To determine the probable error of X_{max} , the probable errors of Q (emission rate), $\bar{u}$ (wind velocity), and h (effective stack height) need to be known. Error theory can be applied to functions of several variables, the best value of the derived quantity being obtained by substituting the mean values of the several variables in the function.

Defining the probable error, Q_e , of the best value, P , of the function:

$$P = f(S_1, S_2, S_3, \dots, S_n)$$

and relating it to the probable errors, $Q_1, Q_2, Q_3, \dots, Q_n$ of the mean values, $m_1, m_2, m_3, \dots, m_n$ of the several

independently measured quantities, $S_1, S_2, S_3, \dots, S_n$, by the following equation:

$$Q_e = \sqrt{\sum_j \left(\frac{\partial p}{\partial S_j} \right)^2 Q_j^2} \quad (\text{Ref A})$$

If this expression is expanded for the present case, it becomes:

$$Q_e = \sqrt{\left(\frac{\partial X}{\partial Q} \right)^2 Q_Q^2 + \left(\frac{\partial X}{\partial \bar{u}} \right)^2 Q_{\bar{u}}^2 + \left(\frac{\partial X}{\partial h} \right)^2 Q_h^2}$$

An Example Problem is presented:

$$\text{If } X_{\max} = ?$$

$$\text{and } Q = 1000 \pm 10 \text{ gm/sec}$$

$$\bar{u} = 10 \pm 0.5 \text{ meters/sec}$$

$$h = 50 \pm 5 \text{ meters}$$

(Ref. A) Livingston, R. Physico Chemical Experiments
Macmillan Co., (1956)

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Then:

$$X_{max} = \frac{2(1000)}{e \pi (10)(50)^2} \text{ gm/m}^3$$

$$= 9.368 \times 10^{-6} \text{ gm/m}^3$$

$$X_{max} = 9.368 \times 10^{-3} \text{ gm/m}^3$$

or

$$X_{max} = 9.368 \text{ milligram/meter}^3$$

The probable error is then computed.

$$\frac{\partial X}{\partial Q} = \frac{2}{e \pi r h^2} = 9.368 \times 10^{-6}$$

$$\frac{\partial X}{\partial \bar{u}} = \frac{2Q}{e \pi h^2 (-\bar{u}^2)} = -9.368 \times 10^{-4}$$

$$\frac{\partial X}{\partial h} = \frac{2Q}{e \pi \bar{u} (-2h^3)} = -9.368 \times 10^{-5}$$

Then:

$$R_e = \sqrt{(9.368 \cdot 10^{-6})^2 (10)^2 + (+9.368 \cdot 10^{-4})^2 (0.5)^2 + (-9.368 \cdot 10^{-5})^2 (5)^2}$$

$$= \sqrt{87.75 \cdot 10^{-10} + 87.76 \cdot 10^{-8} (0.25) + 87.76 \cdot 10^{-10} (25)}$$

$$R_e = \sqrt{14.76 \cdot 10^{-8}} = 6.69 \cdot 10^{-4} \text{ gm/m}^3$$

Therefore:

$$X_{max} = 9.368 \pm 0.669 \text{ milligram/meter}^3$$

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The number of significant figures present in the above value of $X_{\max}$ are too great. A value more in keeping with the given values of Q , U , and h is:

$$X_{\max} = 9.4 \pm 0.7 \text{ milligram/meter}^3$$

and the value is correct to about $7\frac{1}{2}\%$. From this example it is clear that all of the variables in an equation are important when considering the probable error of the dependent variable.

A Comparison of X vs X for Gaseous and Particulate Emissions	JOB NO. 507-1-1	SHEET NO. F-1
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The necessity of using the elevated source equation is obvious, but the effect of particulate matter on the X vs X relation must be determined for the emission of solids from a stack. The assumption will be a detonation with the subsequent release of toxic solids into the air. The source will be a solid propellant grain with a weight of 11,000 #, 15 wt % being toxic. The effective stack height is $h = 3790$ meters. Calculate X vs X from $X=0$ to 100 miles for gaseous effluent and for a particle diameter of 10μ (micron). The modified instantaneous elevated source equation is used:

$$X = \frac{Q}{\pi^{3/2} C^2 (x_0 + \bar{u} t)^{3(2-h)/2}} \cdot e^{-\frac{(h - \frac{v_x}{\bar{u}})^2}{C^2 (x_0 + \bar{u} t)^{2-h}}}$$

and:

$$Q = (0.15)(11,000 \#) = 1650 \# = 750,000 \text{ grams} = 7.5 \times 10^{11} \mu\text{gm}$$

$$h = 3,790 \text{ meters} \sim 12,400 \text{ feet}$$

$$x_0 = 0, \text{ assumed for large values of } t$$

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The value of "C" is a function of h.
It is written

$$C = 3.67 \times 10^{-3} (h^{-0.147})(n^{-3.11})(\bar{u}^{-0.169})$$

The assumption is made that a standard lapse rate is present. Therefore, $n = 0.25$. Also assumed is a wind speed of $\bar{u} = 10$ mph. The particle settling velocity (V) is a function of particle size. Therefore, a gas will have $V_g = 0$. The settling rate of 10μ particles, with a density = 2.0 is 0.02 feet/sec. or 0.0061 meters/sec. Solving for C.

$$C = 3.67 \times 10^{-3} (3790.^{-0.147})(0.25^{-3.11})(4.47^{-0.169})$$

$$C = 3.67 \times 10^{-3} \left(\frac{1}{3365} \right) (75.0) (0.776)$$

$$C = 6.33 \times 10^{-2} = 0.06 \text{ meters}^{\frac{n}{2}}$$

Therefore, the value of X vs x may be found, since $\bar{u} t = x$, where t is time since emission.

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For gases:

$$X_g = \frac{7.5 \cdot 10^{11} \mu_{gm}}{\pi^{3/2} (0.06)^3 X^{2.63}} e^{-\frac{(3790-0)^2}{(0.06)^2 X^{1.75}}}$$

$$X = \frac{6.22 \cdot 10^{14}}{X^{2.63}} e^{-\frac{4.0 \cdot 10^9}{X^{1.75}}} = \frac{10^{14.794}}{X^{2.63}} \cdot e^{-\frac{10^{9.602}}{X^{1.75}}}$$

Solving for X

Let X meters	$X^{2.63}$	$X^{1.75}$	n	e^{-10^n}	X μ_{gm}/m^3
100	$10^{5.26}$	$10^{3.5}$	6.162	$\rightarrow 0$	$\rightarrow 0$
1000 = 0.328 m	$10^{7.7}$	$10^{5.25}$	4.352	$\rightarrow 0$	$\rightarrow 0$
10,000 = 3.28 m	$10^{10.53}$	$10^{7.00}$	2.602	$\rightarrow 0$	$\rightarrow 0$
50,000 = 31.7 m	$10^{12.36}$	$10^{8.22}$	1.38	3.7×10^{-11}	1.26×10^{-8}
100,000 = 62.1 m	$10^{12.50}$	$10^{8.750}$	0.852	$2.1 \cdot 10^{-4}$	$3.54 \cdot 10^{-2}$
200,000	$10^{12.93}$	$10^{9.275}$	0.327	0.119	0.87
500,000	$10^{14.98}$	$10^{9.97}$	-0.27	0.65	0.423
10^6	$10^{15.77}$	$10^{10.5}$	-0.90	0.88	0.293
150,000	$10^{13.63}$	$10^{9.06}$	0.542	0.0305	0.445
200,000	$10^{14.42}$	$10^{9.58}$	+0.02	0.35	0.83

For 10μ Particles

$$X = \frac{10^{14.794}}{X^{2.63}} \cdot e^{-\frac{277.5(3790 - 1.36 \cdot 10^{-3} X)^2}{X^{1.75}}}$$

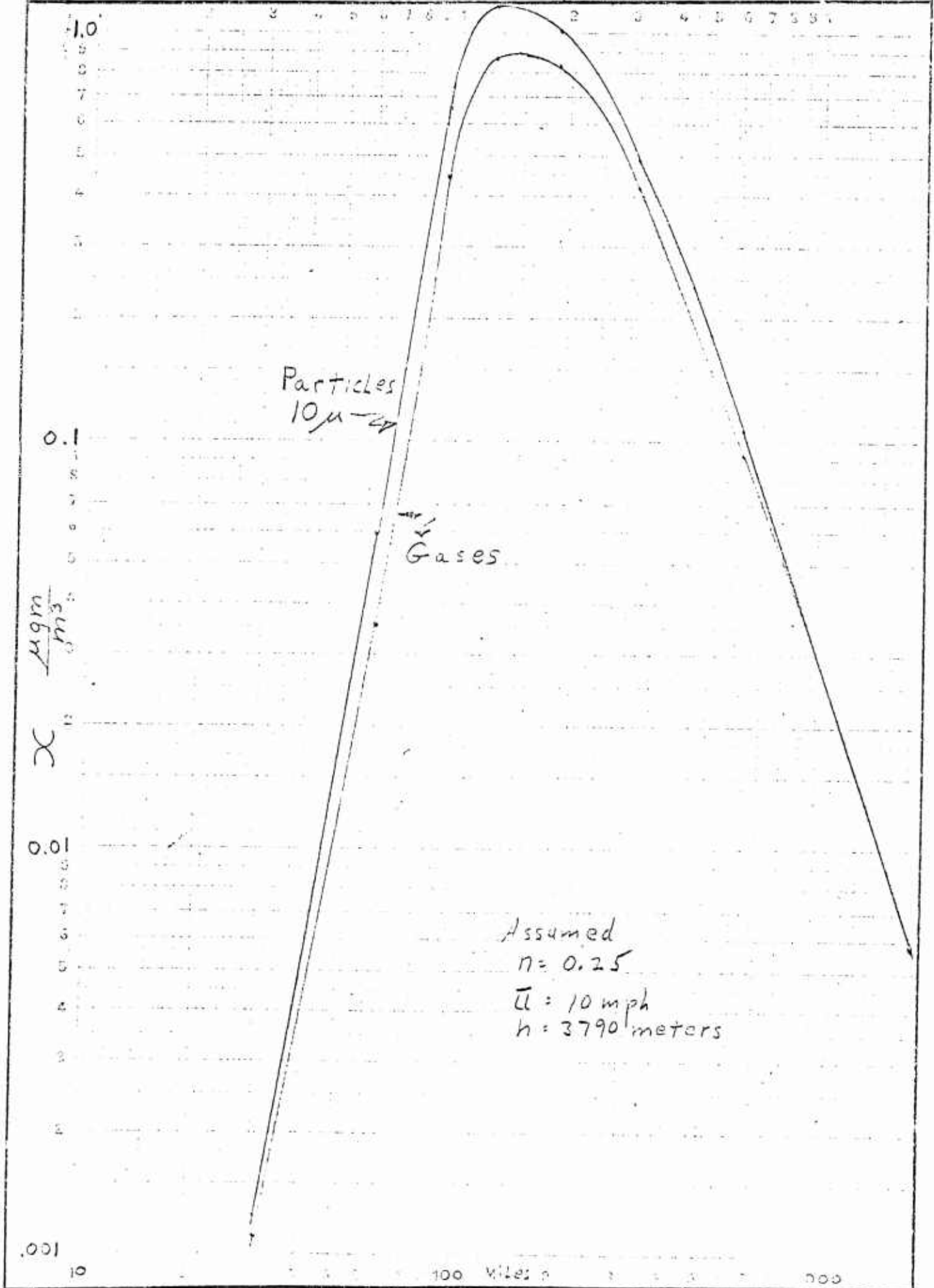
Let X meters	$X^{2.63}$	$X^{1.75}$	n	e^{-10^n}	X $\mu\text{gm}/\text{m}^3$
10000	$10^{10.52}$	$10^{7.00}$	2.598	$\rightarrow 0$	~ 0
50000	$10^{12.36}$	$10^{8.22}$	1.36	1.27×10^{-10}	4.32×10^{-8}
100000	$10^{12.150}$	$10^{8.750}$	0.819	0.0136	$5.94 \cdot 10^{-2}$
200000	$10^{13.92}$	$10^{9.275}$	0.260	0.160	1.17
500000	$10^{14.98}$	$10^{9.97}$	-0.54	0.75	0.488
10^6	$10^{15.77}$	$10^{10.5}$	-1.29	.95	0.0996
150000	$10^{13.67}$	$10^{9.06}$	0.49	0.045	0.657
300000	$10^{14.42}$	$10^{9.58}$	-0.078	0.43	1.02

Converting from meters to miles and placing the results in a chart:

X meters	X miles	X gases	X $\mu\text{gm}/\text{m}^3$ 10μ particles
$5 \cdot 10^4$	31.	1.26×10^{-8}	4.32×10^{-8}
10^5	62.	.0354	0.0594
$1.5 \cdot 10^5$	93.	.445	0.657
$2.0 \cdot 10^5$	124.	.87	1.17
$2.0 \cdot 10^5$	186.	.83	1.02
$5.0 \cdot 10^5$	310.	.42	0.49
10^6	620.	.093	0.0996
10^7	6200.	.00024	0.00024
$3.0 \cdot 10^6$	1860.	.0058	0.0058

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