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GENERAL MOTORS CORPORATION

TECHNICAL REPORT
ON

SCATTERING OF ELECTROMAGNETIC WAVES BY A PLASMA CYLINDER

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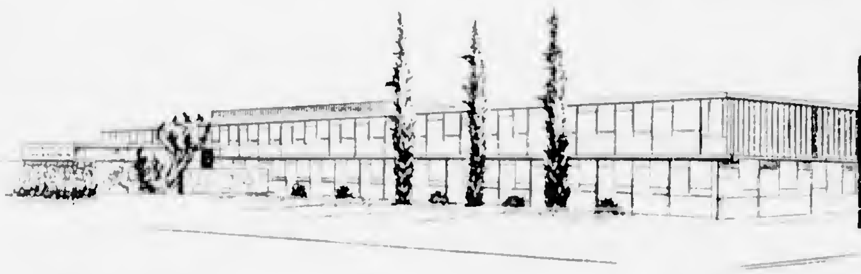
HYPERVELOCITY RANGE RESEARCH PROGRAM

DEFENSE RESEARCH LABORATORIES
SANTA BARBARA, CALIFORNIA

AEROSPACE OPERATIONS DEPARTMENT



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PREPARED BY J. A. FEJER

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ABSTRACT*

The scattering of an electromagnetic wave by a plasma cylinder is investigated. The problem of the homogeneous cylinder is solved analytically. Numerical solutions are obtained for two selected examples in which the plasma is cylindrically symmetrical but inhomogeneous. In the calculations the thermal motion of the electrons is taken into account approximately by a scalar pressure term.

The results of the calculations show that resonant scattering takes place at several frequencies. At the lowest resonant frequency the thermal motion of the electrons plays little part, and the plasma behaves essentially as a dielectric. Thermal motion plays a prominent part in the resonances which occur at the higher frequencies and which are caused by standing "plasma waves" (electron acoustic waves). Results of the present analysis and numerical calculations show that these higher resonant frequencies are rather closely spaced (starting from just above the plasma frequency), if the plasma cylinder is homogeneous and its diameter is much greater than the Debye length. The separation between the resonant frequencies increase, however, if the electron density is taken to be substantially greater on the axis of the cylinder than at the boundary. The results of the present computations for such an inhomogeneous cylinder appear to be in qualitative agreement with data obtained from experiments on the resonant scattering of microwaves by a gas discharge column [1) 2)].

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SECTION I
INTRODUCTION

Theoretical studies by Herlofson [3] show that a uniform plasma cylinder, in which the number of electrons per unit length along the axis is well below a certain critical value, has a natural mode of oscillation whose frequency is $2^{-1/2}f_p$, where f_p is the plasma frequency. At this frequency an electromagnetic wave, whose electric field is perpendicular to the axis, should be resonantly scattered by the cylinder according to Herlofson's theory.

Measurements of microwave scattering from a low pressure cylindrical discharge tube [1] [2] [4] [5] [6] show the presence of several additional, higher, resonant frequencies besides the predicted one. A satisfactory explanation of these additional resonances has yet been given, although many attempts have been made. One of the latest of these is that of Astrom[7] who also gives a summary of all previous work.

Herlofson[3] treats the plasma as a dielectric medium. Under the conditions assumed by him the scattering problem is then mathematically equivalent to the electrostatic problem of a homogeneous dielectric cylinder in a uniform external electric field. The solution of that problem (and the more general problem of the ellipsoid) is well known [8]; the perturbation field caused by the cylinder becomes infinitely large if the dielectric constant of the cylinder has the value -1. This corresponds to Herlofson's resonance condition. (A more correct treatment would reveal that radiation damping limits the perturbation field to finite values.)

In the solution of the "electrostatic" problem for an inhomogeneous cylinder a difficulty is caused by the singularity at the radial distance where the "dielectric constant" passes through the value zero.

At this singularity, the electric field becomes infinitely large in the absence of losses and there is a finite dissipation even if the collision frequency is assumed infinitely small. In the vicinity of such a singularity, linear theory is of doubtful validity. If it is nevertheless applied to a cylindrically symmetrical plasma whose density is almost constant within a certain radial distance. Beyond this distance, the density falls to zero steeply, but with a finite gradient. The resonance predicted by Herlofson is shown to be damped to an extent that is inversely proportional to the steepness of the density gradient. It thus appears that the Herlofson resonance is somehow associated with the abrupt change in density at the edge.

If a cylindrically symmetrical electron density distribution with several steps is assumed, then the number of natural modes of oscillation may be shown to be equal to the number of steps [9]. In this way an arbitrary number of resonant frequencies may be obtained, but these do not provide a likely explanation for the experimentally observed resonances.

The treatment of a plasma as a dielectric is known to be of limited validity [10]. Gould [11] has tried to take the effect of thermal motions approximately into account in the calculation of microwave scattering by a plasma cylinder, by the inclusion of a scalar pressure term in the basic plasma equations. The pressure term often leads to physically significant results. For example, in the theory of electron acoustic waves in a plasma, such an approximation leads to the correct wave velocity [12] although it does not show the presence of damping [13].

Gould's[11] treatment of a homogeneous plasma cylinder leads to several characteristic oscillation frequencies but their distribution is rather different from those observed experimentally. Gould suggests that the treatment of an inhomogeneous cylinder by similar methods may lead to a better agreement with the experiments.

The present paper is an extension of Gould's treatment.

The differential equations for an inhomogeneous, cylindrically symmetrical plasma are derived in Section II. In Section III these equations are solved analytically for the special case of a homogeneous cylinder. The expressions obtained for the scattered field are then discussed in considerable detail.

In Section IV the general differential equations are solved numerically for two selected electron density distributions that were thought to represent typical experimental conditions.

SECTION II
THE DIFFERENTIAL EQUATIONS

It is convenient to assume for the purposes of the present calculation that only the motion of electrons is of interest. The positive ions are assumed to be infinitely heavy and their motion is neglected. Their presence approximately compensates for the space charge of the free electrons. A small residual space charge, necessary for the establishment of a static electric field that can maintain the assumed distribution of free electrons, remains.

For $r < R$ the unperturbed number density $N(r)$ of electrons is assumed to be a continuous function of the distance r from the cylinder axis. At $r = R$, the number density is assumed to change discontinuously from $N(r)$ (for values of r that are slightly less than R) to zero (for $r > R$). Such an abrupt change in electron density requires the existence of a perfectly reflecting potential barrier at $r = R$; the mean radial component of the electron velocities must therefore vanish at $r = R$.

For $r < R$ we may write the linearized equations of motion for the electron gas in the form:

$$m(N+n)\frac{\partial \vec{v}}{\partial t} = (N+n)e\left(\vec{\nabla}\phi + \vec{\nabla}\varphi\right) - KT\vec{\nabla}N - \gamma KT\vec{\nabla}n \qquad (1)$$

where m is the mass, e is the charge (without the sign) of an electron, n is the perturbation in the electron density, $\vec{v}$ is the velocity of the electron gas, ϕ is the unperturbed potential, φ is the perturbation potential, k is Boltzmann's constant, T is the temperature and γ is the ratio of specific heats which, in agreement with Spitzer's [13] reasoning, is set equal to 3 in the present calculations. The assumption that the perturbation electric field may be derived from a potential φ is justified later.

Without the last two terms on the right, that represent the gradient of electron pressure, equation (1) would have the appearance of an equation of motion for individual electrons. The unperturbed electron pressure is taken to be KTN; the perturbation in the pressure is taken as γ KTn.

In the unperturbed case, equation (1) assumes the form:

$$Ne\vec{\nabla}\phi - KT\vec{\nabla}N = 0 \qquad (2)$$

and determines the potential ϕ (r) required to maintain the assumed number density N(r).

A combination of equations (1) and (2), and neglect of second order terms, leads to:

$$mN\frac{\partial\vec{v}}{\partial t} = Ne\vec{\nabla}\varphi + nKT\frac{\vec{\nabla}N}{N} - \gamma KT\nabla n \qquad (3)$$

A combination of the divergence of this equation with the equation of continuity:

$$\vec{\nabla}.(N\vec{v}) = -\frac{\partial n}{\partial t} \tag{4}$$

yeilds:

$$-m\frac{\partial^2 n}{\partial t^2} = Ne\vec{\nabla}^2\varphi + e\vec{\nabla}\varphi.\vec{\nabla}N + KT_n\frac{\vec{\nabla}N}{N} - \gamma KT\vec{\nabla}^2 n \tag{5}$$

and Poisson's equation gives:

$$\varepsilon_0\vec{\nabla}^2\varphi = en \tag{6}$$

All perturbations are assumed to be harmonic functions of the time. Replacement of $\partial^2 / \partial t^2$ by $-\omega^2$ and the substitution of (6) into (5) then leads, after some rearrangement, to the fourth order differential equation:

$$(P-S)\vec{\nabla}^2\varphi - \vec{\nabla}\varphi.\vec{\nabla}S + \vec{\nabla}^4\varphi - \gamma^{-1}\vec{\nabla}(\vec{\nabla}S.\vec{\nabla}\varphi/S) = 0 \tag{7}$$

where the quantity S defined by:

$$S(r) = \frac{e^2}{\varepsilon_0\gamma KT}N(r) \tag{8}$$

is proportional to the unperturbed electron density and is independent of the frequency.

The quantity P defined by:

$$P = \frac{m}{\gamma K T} \omega^2 \tag{9}$$

is independent of position (if the temperature is independent of position) and is proportional to the square of the frequency.

If the number of free electrons per unit length of the cylinder is much smaller than the critical line density of about 10^{12} electrons (familiar from the theory of scattering by meteor trails [14)]), then for the frequencies of present interest the wave length of the incident wave may be shown to be much larger than the radius of the cylinder. The external electric field may then be regarded as uniform in the vicinity of the cylinder and the above quasi-electrostatic treatment may then be justified. The potential φ must be of the form:

$$\varphi = a(r) \cos\theta \tag{10}$$

in order to satisfy the boundary conditions at infinity, where the potential must be approximately given by $E_0 \, r \cos\theta$, if E_0 is the magnitude of the uniform external field. Cylindrical coordinates r, θ, Z are used in the above expressions.

With the substitution (10), equation (7) may be transformed into a fourth order linear differential equation for a(r). For numerical integration it is more convenient, however, to convert equation (7) into four first order linear differential equations. The conversion may be started with the definition of a variable u(r) by:

$$u = a'' + r^{-1} a' - r^{-2} a \tag{11}$$

where the prime denotes differentiation with respect to r.

The physical significance of u may be seen from the equation:

$$h = \frac{\varepsilon_0}{e} u \cos\theta \tag{12}$$

that is obtained from equations (6), (10) and (11). Equation (7) may then be written in the form:

$$(p-s)u - s'a' + u'' + r^{-1}u' - r^{-2}u - \gamma^{-1}r^{-1}(rs'u/s)' = 0 \tag{13}$$

Equations (11) and (13) are two second order differential equations in a and u. After the introduction of the new variables:

$$x = ar - a'r^2 \tag{14}$$

$$y = ur - u'r^2 \tag{15}$$

$$v = a/r \tag{16}$$

$$w = u/r \tag{17}$$

equations (11) and (13) may be transformed into the four first order linear differential equations:

$$x' = -r^3 w \tag{18}$$

$$y' = \left[r^3(p-s) - \frac{2r^2 s'}{\gamma s} + \frac{r^3(s')^2}{\gamma s^2} - \frac{r^3 s''}{\gamma s}\right]w + s'x - r^2 s'v + \frac{s'}{\gamma s}y \tag{19}$$

$$v' = -r^{-3}x \tag{20}$$

$$w' = -r^{-3}y \tag{21}$$

These equations are well suited for solution by numerical integration, if S is a given function of r. It is, however, convenient to consider first the special case of a homogeneous plasma cylinder (S = constant) that is solved analytically in the following section.

SECTION III
THE HOMOGENEOUS PLASMA CYLINDER

The dependent variables a and u in the two second order differential equations (11) and (13) are related to the potential φ and perturbation number density n by equations (10) and (12). Since φ and n must be single valued functions of position, the variables a(r) and u(r) must both vanish for r = O. It may be shown by substitution into (11) and (13) that two linearly independent solutions, that satisfy the above conditions, are:

$$a_1(r) = r \quad , \quad u_1(r) = 0 \tag{22}$$

and:

$$a_2(r) = I_1\left[(S-P)^{\frac{1}{2}} r\right] \quad , \quad u_2(r) = (S-P)\, I_1\left[(S-P)^{\frac{1}{2}} r\right] \tag{23}$$

where $S > P$ is assumed and I_1 is the usual notation for the modified Bessel function of first order.

If $S < P$ is assumed, then equation (23) is replaced by:

$$a_2(r) = J_1\left[(P-S)^{\frac{1}{2}} r\right] \quad , \quad u_2(r) = -(P-S)\, J_1\left[(P-S)^{\frac{1}{2}} r\right] \tag{24}$$

where J_1 is the Bessel function of first order. In the following section these solutions will be required in terms of the four variables x, y, v, w. If $S > P$, then equations (14) - (17) yield:

$$x_1 = 0, \quad y_1 = 0, \quad v_1 = 1 \quad , \quad w_1 = 0 \tag{25}$$

$$
\left.\begin{aligned}
x_2 &= 2rI_1\left[(S-P)^{\frac{1}{2}}r\right] - r^2(S-P)^{\frac{1}{2}}I_0\left[(S-P)^{\frac{1}{2}}r\right], & v_2 &= r^{-1}I_1\left[(S-P)^{\frac{1}{2}}r\right] \\
y_2 &= 2(S-P)rI_1\left[(S-P)^{\frac{1}{2}}r\right] - r^2(S-P)^{\frac{3}{2}}I_0\left[(S-P)^{\frac{1}{2}}r\right], & w_2 &= (S-P)r^{-1}I_1\left[(S-P)^{\frac{1}{2}}r\right]
\end{aligned}\right\} \tag{26}
$$

If $S < P$, then equations (26) must be replaced by:

$$
\begin{aligned}
x_2 &= 2rJ_1\left[(P-S)^{\frac{1}{2}}r\right] - r^2(P-S)^{\frac{1}{2}}J_0\left[(P-S)^{\frac{1}{2}}r\right], & v_2 &= r^{-1}J_1\left[(P-S)^{\frac{1}{2}}r\right] \\
y_2 &= -2(P-S)rJ_1\left[(P-S)^{\frac{1}{2}}r\right] + (P-S)^{\frac{3}{2}}r^2J_0\left[(P-S)^{\frac{1}{2}}r\right], & w_2 &= -(P-S)r^{-1}J_1\left[(P-S)^{\frac{1}{2}}r\right]
\end{aligned} \tag{27}
$$

Any linear combination of the two solutions, x_1, y_1, v_1, w_1, and x_2, y_2, v_2, w_2 satisfies the differential equations (18) - (21). The solution to the problem must, however, also satisfy the boundary condition at $r = R$ where the radial component of the velocity $\vec{v}$ must vanish or, what is equivalent, the radial component of the vector quantity on the right hand side of equation (3) must vanish. This yields for $r = R$ the condition:

$$
\left(Sa' + \frac{S'}{\gamma S}u - u'\right)_{r=R} = 0 \tag{28}
$$

or expressed in terms of the variables, x, y, v, w, of equations (14) - (17):

$$
\left[S\left(v - \frac{x}{r^2}\right) + \frac{rS'}{\gamma S}w - \left(w - \frac{y}{r^2}\right)\right]_{r=R} = 0 \tag{29}
$$

Equations (28) and (29) have been kept more general than required by the needs of the present section, in which $S' = 0$ will be assumed.

If the linear combinations:

$$x=\alpha_1 x_1+\alpha_2 x_2,\quad y=\alpha_1 y_1+\alpha_2 y_2,\quad v=\alpha_1 v_1+\alpha_2 v_2,\quad w=\alpha_1 w_1+\alpha_2 w_2 \tag{30}$$

are substituted into equation (29) (which is only valid at $r = R$) then the values of α_1 and α_2 may be determined apart from an arbitrary constant factor. Possible values are:

$$\alpha_1=\left[S\left(v_2-\frac{x_2}{r^2}\right)-\left(1-\frac{rS'}{8S}\right)w_2+\frac{1}{r^2}y_2\right]_{r=R} \tag{31}$$

$$\alpha_2=\left[-S\left(v_1-\frac{x_1}{r^2}\right)+\left(1-\frac{rS'}{8S}\right)w_1-\frac{1}{r^2}y_1\right]_{r=R} \tag{32}$$

The solutions of the differential equation are then given, apart from an arbitrary constant factor, by equations (30), if the values of α_1 and α_2 are substituted from (31) and (32).

The above solutions are valid inside the cylinder where $r < R$. For $r > R$ (outside the cylinder) the potential must satisfy Laplace's equation and therefore $a(r)$ for $r > R$ must have the form:

$$a(r) = A/r + E_0 r \tag{33}$$

where E_0 is the uniform external (alternating) electric field and A/r represents the potential due to the plasma cylinder. If the glass wall around the plasma is assumed infinitely thin then both a and a' must be continuous at $r = R$ and therefore:

$$\frac{A}{E_0}=\frac{a(R)-Ra'(R)}{a(R)+Ra'(R)}=\left(2R^2\frac{v(R)}{x(R)}-1\right)^{-1} \tag{34}$$

and with the aid of equations (30), (31) and (32) it may be shown that:

$$\frac{R^2 v(R)}{\chi(R)} = \left[\frac{S(x_1 v_2 - x_2 v_1) + r^2\left(1 - \frac{rS}{\gamma S}\right)(w_1 v_2 - w_2 v_1) + (v_1 y_2 - v_2 y_1)}{S(x_1 v_2 - x_2 v_1) + \left(1 - \frac{rS'}{\gamma S}\right)(w_1 x_2 - w_2 x_1) + r^{-2}(x_1 y_2 - x_2 y_1)}\right]_{r=R} \tag{35}$$

Equations (34) and (35) give A/E_0 in terms of the values of two particular solutions x_1, y_1, v_1, w_1 and x_2, y_2, v_2, w_2 at $r = R$. In the next section these equations will be applied to solutions other than those given by equations (25) - (27). In the present section equations (34) and (35) are applied to a homogeneous cylinder, for which equations (25) - (27) are appropriate. The result, for $S > P$, is:

$$\frac{A}{E_0} = \frac{SR^2(S-P)^{\frac{1}{2}} I_0\left[(S-P)^{\frac{1}{2}}R\right] - 2SRI_1\left[(S-P)^{\frac{1}{2}}R\right]}{(2P-S)R^2(S-P)^{\frac{1}{2}} I_0\left[(S-P)^{\frac{1}{2}}R\right] - 2PRI_1\left[(S-P)^{\frac{1}{2}}R\right]} \tag{36}$$

and for $S < P$:

$$\frac{A}{E_0} = \frac{SR^2(P-S)^{\frac{1}{2}} J_0\left[(P-S)^{\frac{1}{2}}R\right] - 2SRJ_1\left[(P-S)^{\frac{1}{2}}R\right]}{(2P-S)R^2(P-S)^{\frac{1}{2}} J\left[(P-S)^{\frac{1}{2}}R\right] - 2PRJ_1\left[(P-S)^{\frac{1}{2}}R\right]} \tag{37}$$

In Rommel's [1] and Dattner's [2] experiments the radius R was much greater than the Debye length and therefore $S^{\frac{1}{2}}R \gg 1$. For frequencies smaller than and not too close to the plasma frequency for which $(S-P)^{\frac{1}{2}}R \gg 1$ is satisfied, equation (36) is then well approximated by:

$$\frac{A}{E_0} = \frac{S}{2P-S} \tag{38}$$

which is the value obtained if the plasma is treated as a dielectric. The resonance predicted by Herlofson [3)] occurs when $2P = S$ or $\omega = 2^{-\frac{1}{2}}\omega_p$ where $\omega_p = (Ne^2/\mathcal{E}_0 m)$ is the plasma frequency.

Similarly for frequencies greater than and not too close to the plasma frequency, for which $(P-S)^{\frac{1}{2}}R \gg 1$ is satisfied, equation (37) is well approximated by:

$$\frac{A}{E_0} = \frac{S}{2P-S} = \frac{1 - 2\tan\left[(P-S)^{\frac{1}{2}}R - \frac{\pi}{4}\right]\Big/(P-S)^{\frac{1}{2}}R}{1 - [2P/2P-S]\tan\left[(P-S)^{\frac{1}{2}}R - \frac{\pi}{4}\right]\Big/(P-S)^{\frac{1}{2}}R} \tag{39}$$

where the Bessel functions are approximated by trigonometric functions. The limit of the right hand side of (32) is then still approximately $S/(2P-S)$ except in the immediate vicinity of the frequencies for which the denominator on the right hand side of (39) vanishes and for which resonances occur. These frequencies are given approximately by:

$$(P-S)^{\frac{1}{2}}R \sim 3\pi/4 + n\pi \tag{40}$$

where n is an integer. More accurately the resonances occur where the denominator on the right hand side of equation (37) vanishes, or:

$$(P-S)^{\frac{1}{2}}R \sim 5.14,\ 8.41,\ 11.6,\ 14.8\ \ldots \tag{41}$$

which for large n are very near to the values given by (40). For typical experimental conditions $S^{\frac{1}{2}}R$ is about 60 and equation (41) then leads to resonances at $\omega = 1.0033,\ 1.0088,\ 1.0167,\ 1.0274\ \ldots$ besides the resonance at $\omega = 0.707\,\omega_p$.

It should be noted that the quantity A/E_0, in addition to passing through infinity at each resonant frequency, passes through zero and changes its sign in the vicinity of each of those resonant frequencies that are above plasma frequency.

The above theory predicts an isolated resonant frequency at $\omega = 0.707\omega p$ and a cluster of closely spaced resonant frequencies above ωp. The experimental data shows a much wider spacing of the higher resonant frequencies and therefore do not agree with the predictions of the theory for the homogeneous plasma cylinder.

It should be mentioned here that the resonant frequencies of a homogeneous plasma cylinder have been previously derived by Gould [11], who, however, does not give the present equations (36) and (37) for the intensity of the scattered field.

SECTION IV
THE INHOMOGENEOUS PLASMA CYLINDER

In the present treatment of scattering by an inhomogeneous plasma cylinder, similar assumptions are made to those of the previous section. The electron density is again assumed to drop abruptly from a finite value inside the boundary $(r > R)$ to zero outside the boundary $(r > R)$. The radial component of the velocity of the electron gas is again set equal to zero at the boundary.

It is assumed that the unperturbed electron density is constant (the plasma in homogeneous) for $O < r < R_1$ and that it decreases linearly for $R_1 < r < R$. For $O < r < R_1$ the solutions (25) and (26) or (27) are valid. The values of the dependent variables of these two solutions x_1, y_1, v_1, w_1, and x_2, y_2, v_2, w_2, at $r = R_1$ were then taken as the initial conditions within the range $R_1 < r < R$ where the continuation of those two solutions was found by numerical integration (with the aid of the Runge - Kutta method). The solution was carried in this manner up to $r = R$ and the values at $R = R$ were then used to calculate A/E_0 with the aid of equations (34) and (35). The procedure was carried out for different values P and in this manner A/E_0 was obtained as a function of P.

Figure 1a shows the results of such calculations for $R_1 = 0.1$ cm, $R = 0.2$ cm, $S(R_1) = S(0) = 10^5 cm^{-2}$, $S(R) = 3.10^4 cm^{-2}$; in Figure 1b the value of S(R) was taken as $5.10^4 cm^{-2}$, with the values of the other parameters unchanged. The numerical values for these two models were chosen on account of their closeness to the values of the same parameters used in Dattner's experiments.

In Figures 1a and 1b the absolute value of A/E_0 is shown as a function of P; the sign of A/E_0 is indicated by using a solid line for the graph where the sign is positive and an interrupted line where the sign is negative. Figures 1a and 1b show that A/E_0 passes through infinity at several values of P (a quantity that according to equation 9 is proportional to the square of the frequency). These values of P correspond to the resonant frequencies of the plasma cylinder. The main resonance, that occurs at the lowest value of P (the lowest frequency), is of the type predicted by Herlofson [3], suitably modified by the inhomogeneous nature of the plasma. The secondary resonances that occur for higher values of P, are due to standing electron acoustic waves that can exist between the boundary at $r = R$ and the radial distance for which $P = S$ or $\omega = \omega_p$.

These secondary resonant frequencies are far more widely spaced in Figures 1a and 1b than they were in the numerical example of the homogeneous plasma cylinder given in the previous section for the same value of $RS^{\frac{1}{2}}(o)$. The spacing is closer in Figure 1b than in Figure 1a, since the model represented by Figure 1b is closer to the homogeneous cylinder.

Figure 1a may be taken to show the field strength of the scattered wave (assuming a constant field strength of the incident wave) as a function of the square of the frequency. Consideration of Figure 1a then indicates at least a qualitative agreement with the results of Rommel's and Dattner's experiments. A quantitative agreement could hardly be expected in view of the approximate nature of the present theory and the arbitrary choice of the electron density profile on which Figure 1a is based.

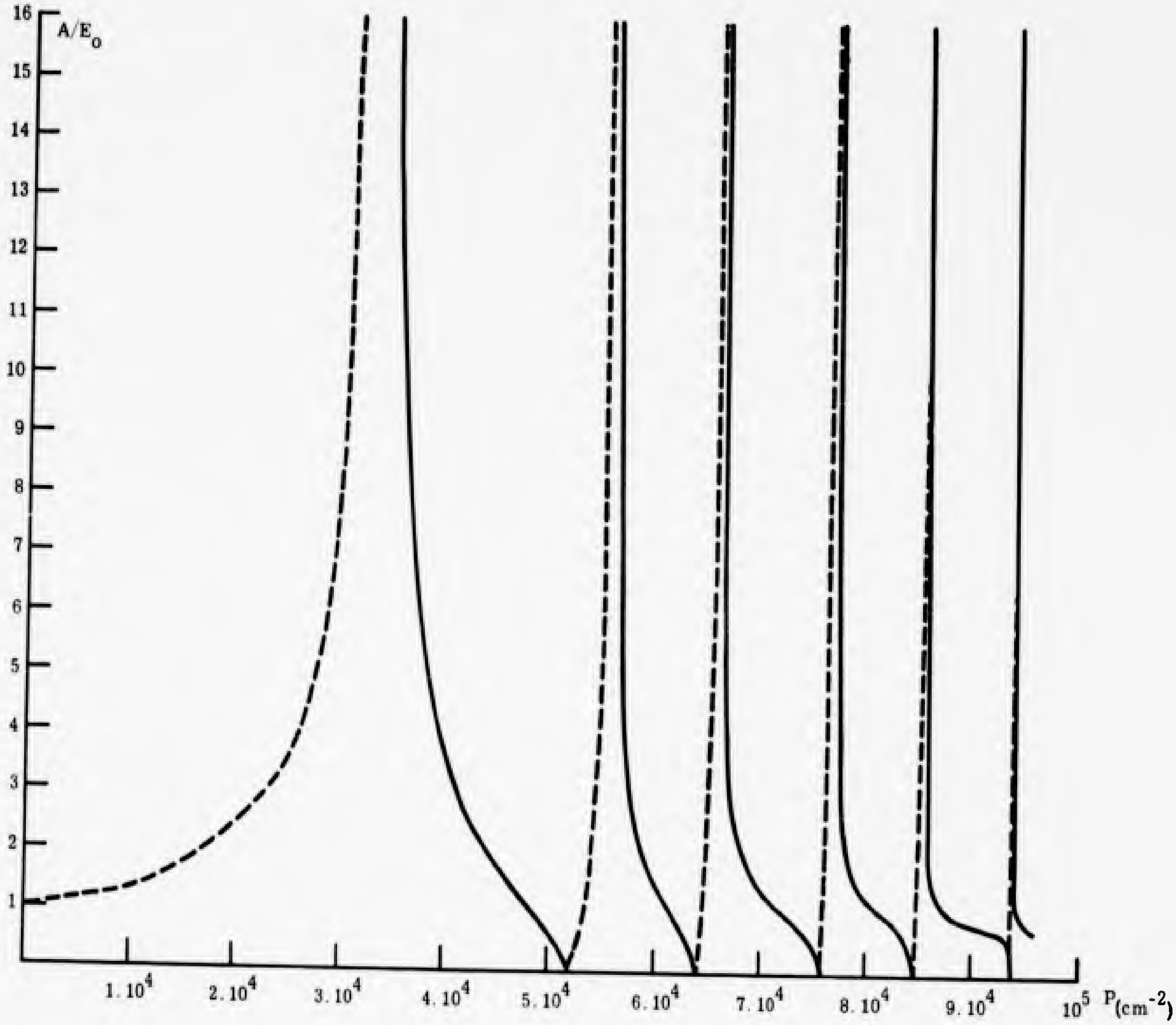

Figure 1a. The ratio A/E_o of the field of the scattered wave to the field of the incident wave, as a function of $P = (m/\gamma KT)\omega^2$, for $R_1 = 0.1$ cm, $R = 0.2$ cm, $S(R_1) = S(0) = 10^5$ cm^{-2}, $S(R) = 3.10^4$ cm^{-2}.

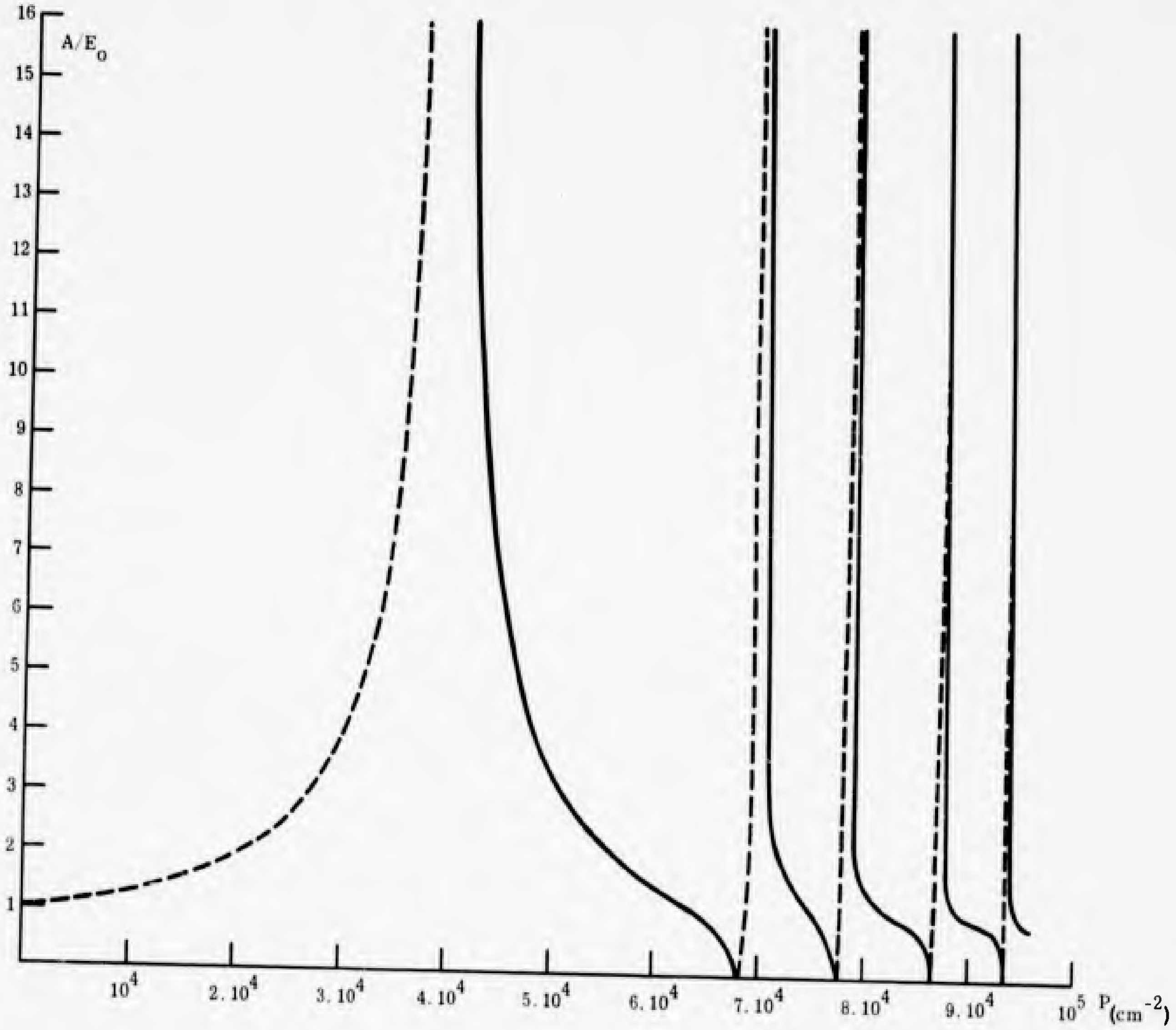

Figure 1b. The ratio A/E_o, as a function of P, for $R_1 = 0.1$ cm $R = 0.2$ cm $S(R_1) = S(0) = 10^5\ cm^{-2}$, $S(R) = 5.10^4\ cm^{-2}$.

The existence of electron acoustic waves, in the space between $r = R$ and the radial distance for which $P = S$, is clearly shown by Figure 2, where the computed potential φ and perturbation number density n (of electrons) are shown on an arbitrary scale for a constant value of θ, as functions of the radial distance r. Figure 2 has been computed for the same model as Figure 1a and for a frequency determined by $P = 0.8$. The radial distance, for which $P = S$, is then $r \cong 0.1286$ cm. The presence of standing electron acoustic waves in the space between $r \cong 0.1286$ and $r = 0.2$ is clearly shown by the plot of $n(r)$ in Figure 2.

Figure 2 also shows the absence of any form of singularity at $r = 0.1286$. It was pointed out in the introduction that a singularity would exist at that radial distance if the plasma were treated as a dielectric medium. It appears that the introduction of the pressure term removes this singularity, at least in the case of a hot plasma.

Conclusions: Resonant scattering has been shown to occur at several frequencies when an electromagnetic wave is incident on a cylindrically symmetrical distribution of ionization, in which the electron density decreases continuously from the center to the boundary with increasing radial distance and then drops abruptly from a still finite value to zero at the boundary.

In the calculation of the scattered waves the thermal motion of the electrons has been taken into account by an isotropic pressure term and collisions have been neglected. Qualitative agreement with previous experimental results has been obtained.

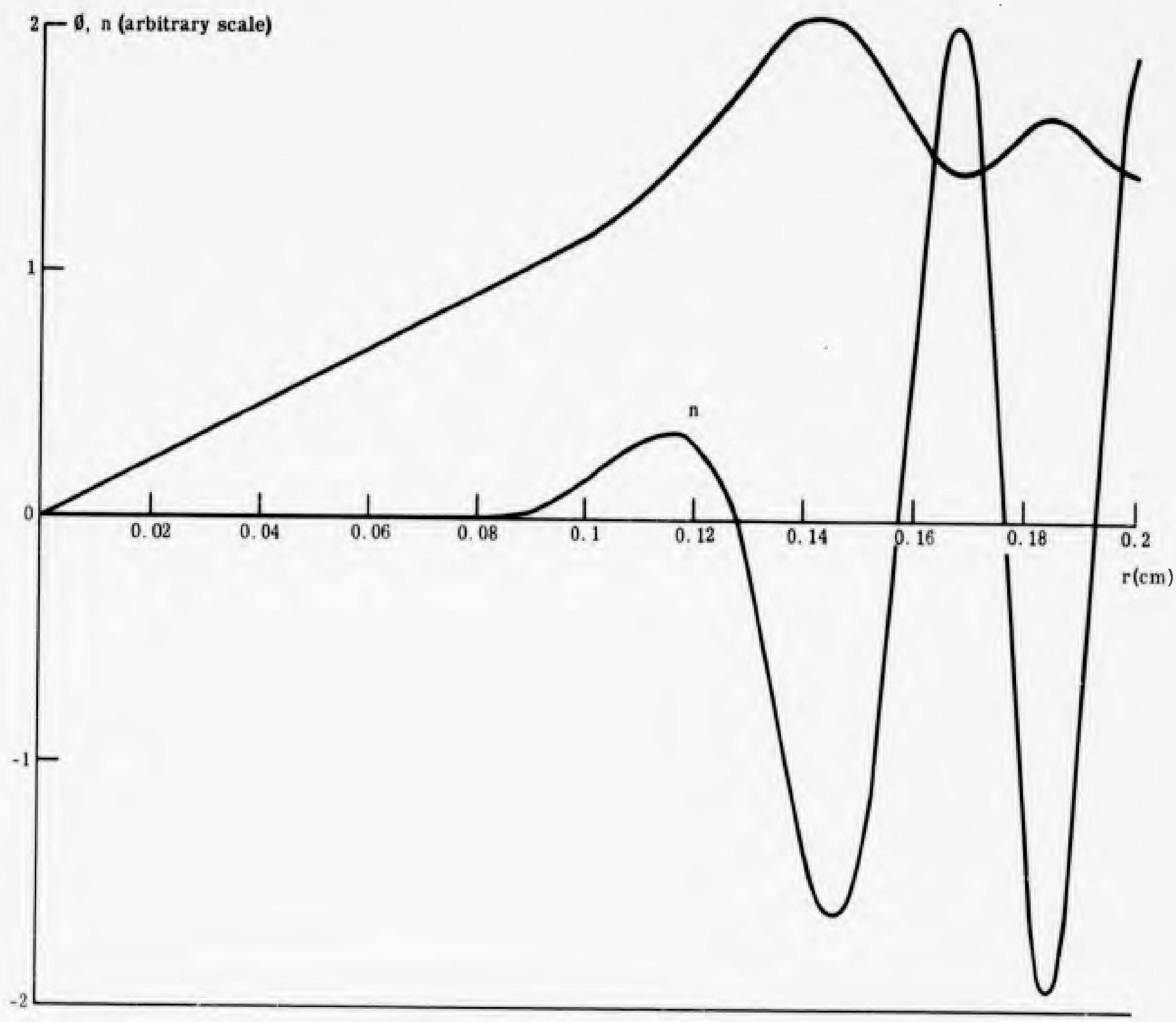

Figure 2. The perturbation potential φ and the perturbation electron number density n as functions of the radial distance r, for a constant value of the azimuth angle θ. Arbitrary scales are used for φ and n.

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