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GENERAL MOTORS CORPORATION

TECHNICAL REPORT
ON

THE INCIDENCE OF FOCUSED MICROWAVES UPON IONIZED DISTRIBUTIONS

PART 1. PLANE DISTRIBUTIONS

CONTRACT NO. DA-04-495-ORD-3567
HYPERVELOCITY RANGE RESEARCH PROGRAM

DEFENSE RESEARCH LABORATORIES
SANTA BARBARA, CALIFORNIA

AEROSPACE OPERATIONS DEPARTMENT

TR62-209N

DECEMBER 1962

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PREPARED BY F. H. NORTHOVER

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ABSTRACT

The effect of a plane slab of ionization located in the focal plane of a focused lens transmission system is investigated. It is found that if the effective dielectric constant is close to unity (no loss) then the transmission through the slab is the same as if it were illuminated with a uniform plane wave.*

*This research is part of Project DEFENDER, sponsored by the Advanced Research Projects Agency, Department of Defense.

INTRODUCTION

The properties of the ionized wake behind a hypervelocity projectile are currently of great interest. These properties may be determined through the use of microwave radiation — a powerful technique for probing the ionized region.

This problem is very difficult to analyze mathematically, but apparently some progress can be made in the case of plane and cylindrical distributions. The present work will, therefore, be restricted to consideration of such distributions, and a discussion of cylindrical distributions will be reserved for Part II.

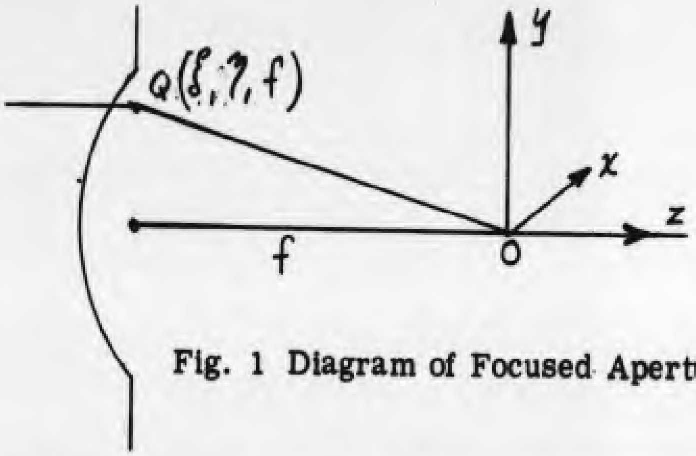

Fig. 1 Diagram of Focused Aperture

Figure 1 shows the section of a collimating system along its optic axis, with the aperture of the lens on the left. This represents the focused antenna. The origin O is taken at the geometrical focus, and Oz is along the optic axis away from the aperture. Let $Q(\xi,\eta,-f)$ be a point on the aperture and let

$$R = OQ$$
$$= (\xi^2 + \eta^2 + f^2)^{1/2}$$
$$r = PQ = \left\{(x-\xi)^2 + (y-\eta)^2 + (z+f)^2\right\}^{1/2}$$

Suppose that the incident waves on the circular aperture are excited by a source (such as a waveguide), which makes the field distribution thereon plane polarized. It has been shown by Matthew and Cullen (Ref. 1) that the resulting field in the image space, inside a sphere around O whose radius is small compared with f is also plane polarized and is given by

$$\underset{\sim}{E} = \left[\frac{1}{4\pi}\int_S \left\{\frac{\partial\phi}{\partial n}\left(\frac{e^{-ikr}}{r}\right) - \phi\frac{\partial}{\partial n}\left(\frac{e^{-ikr}}{r}\right)\right\}ds; 0; 0\right] \tag{2}$$

where $(\phi, 0, 0)$ is the value of E over the aperture, the integral is taken over the aperture plane, and the usual approximations of the Kirchoff solution for the scalar wave function E_x are made.

The effect of the collimating system is to direct a spherical wave, converging towards O through its aperture.

Accordingly,

$$\phi = e^{ikR}/R \tag{3}$$

At distances from the focus, O small compared with Eq. (2) leads to (Ref. 2)

$$E_{\chi} = \frac{i\,e^{-ikz}}{\lambda f^{2}} \int \exp\Big[ik\,\Big(\frac{x\xi + y\eta}{f} + \frac{\eta^{2} + \xi^{2}}{2f^{2}}\,z\Big]\,d\xi\,d\eta \qquad (4)$$

In Eq. (4) the effect of the incident field upon a plane slab of uniformly ionized gas must be examined.

Let this slab be located at $-\tau < z < \tau$ (i. e., have thickness 2τ) and be considered to be equivalent to the effect of a dielectric, of (possibly complex) dielectric constant K.

BOUNDARY CONDITIONS

Let the field inside the slab be $\underset{\sim}{E}'(E_1', 0, 0)$; $\underset{\sim}{H}'(H_1', H_2', 0)$

then $E_1' = E_1$, $H_1 = H$, $H_2' = H_2$

but $\underset{\sim}{H} = \frac{ic}{\omega}$ curl $\underset{\sim}{E}$

$$= \frac{ic}{\omega}\left\{0, \frac{\partial Ex}{\partial y}, -\frac{\partial Ex}{\partial y}\right\}$$

Hence, boundary conditions are

$$E_1' = E_1 \text{ , } \partial E_1'/\partial z = \partial E_1/\partial z$$

for all x, y, for then also

$$\partial E_1'/\partial y = \partial E_1/\partial y$$

From Eq. (4) the incident field on the slab is a superposition of elementary plane waves, the typical wave being

$$\frac{i}{\lambda f^2} \exp\left[ik\left\{\frac{x\xi + y\eta}{f} - \left(1 - \frac{\xi^2+\eta^2}{f^2}\right)^{1/2}\right\}\right] d\xi\, d\eta \qquad (5)$$

for Eq. (5) is equivalent to Eq. (4) as far as $O(\varpi^2/f^2)$, and the equivalent form (Eq. (5)) has been used because it makes*

$$\nabla^2 E_x + k^2 E_x = 0 \qquad (6)$$

and so Eq. (5) is a true plane wave.

*It is easy to verify that form (Eq. (4)) satisfies Eq. (6) as far and up to $O(\varpi^2/f^2)$, which is all that Eq. (4) is an approximation for.

The plane wave (Eq. (5)) sets up in the medium waves of the form

$$\frac{i}{\lambda f^2} \exp\left[ik_1\left\{\frac{x\xi_1 + y\eta_1}{f} \pm \left(1 - \frac{\xi^2 + \eta^2}{f^2}\right)^{1/2} z\right]d\xi d\eta$$

where $k_1 (= k\sqrt{K})$ is the propagation constant for the dielectric K and the numbers ξ_1, η_1, are different from ξ, η because $(\xi/f, \eta/f)$ $(\xi_1/f, \eta_1/f)$ are the x and y direction cosines of the inclination of the waves (Eqs. (5) and (7)) in their respective media, which are, of course, different.

The boundary conditions at $z = 0$ are, assuming also a reflected wave

$$\frac{iR}{\lambda f^2} \exp\left[ik\left\{\frac{x\xi + y\eta}{f} + \left(1 - \frac{\xi^2 + \eta^2}{f^2}\right)^{1/2} z\right\}\right]d\xi d\eta \tag{8}$$

that

$$(1 + R) \exp\left(ik\frac{x\xi + y\eta}{f}\right) = (A + B) \exp\left(ik_1\frac{x\xi_1 + y\eta_1}{f}\right) \tag{9}$$

where A and B are the constants in the linear combination of the types (Eq. (7)) for the internal forward and backward waves inside the slab.

Thus by Eq. (9) $k\xi = k_1\xi_1$, $k\eta = k_1\eta_1$

the wave types set up by $d\xi d\eta$ inside the slab in the form are taken as

$$\frac{i}{\lambda f^2} \exp\left[ik\left\{\frac{x\xi + y\eta}{f} \pm \left(K - \frac{\xi^2 + \eta^2}{f^2}\right) z\right]d\xi d\eta$$

In accordance with the above and assuming a reflected disturbance

$$\frac{Ri}{\lambda f^2} \exp\left[ik\left\{\frac{x\xi+y\eta}{f} + \left(1-\frac{\xi^2+\eta^2}{f^2}\right)^{1/2} z\right\}\right] \tag{12}$$

for the region $-f<z<-\tau$, and assuming

$$\frac{i}{\lambda f^2}\left\{A\exp\left[ik\left\{\frac{x\xi+y\eta}{f} - \left(K-\frac{\xi^2+\eta^2}{f^2}\right)^{1/2} z\right\}\right]\right.$$
$$\left. + B\exp\left[ik\left\{\frac{x\xi+y\eta}{f} + \left(K-\frac{\xi^2+\eta^2}{f^2}\right)^{1/2} z\right\}\right]\right\} \tag{13}$$

for the slab region $-\tau<z<\tau$, and, finally,

$$\frac{i}{\lambda f^2} R^1 \exp\left[ik\left\{\frac{x\xi+y\eta}{f} - \left(1-\frac{\xi^2+\eta^2}{f^2}\right)^{1/2} z\right\}\right] \tag{14}$$

for the transmitted wave for region $z>\tau$.

The boundary conditions now give on $z=-\tau$

$$\exp\left\{+ik\tau\phi\right\} + R\exp\left\{-ik\tau\phi\right\} =$$
$$\left[A\exp\left\{+ik\tau\phi_K\right\} + B\exp\left\{-ik\tau\phi_K\right\}\right] \tag{15}$$

where

$$\phi = \left(1-\frac{\xi^2+\eta^2}{f^2}\right)^{1/2}, \quad \phi_K = \left(K-\frac{\xi^2+\eta^2}{f^2}\right)^{1/2}$$

$$\exp\left\{ik\tau\phi\right\} - R\exp\left\{-ik\tau\phi\right\} =$$
$$\frac{\phi_K}{\phi}\left[A\exp\left\{ik\tau\phi_K\right\} + B\exp\left\{-ik\tau\phi_K\right\}\right] \tag{16}$$

On $z = \tau$

$$R' \exp\left\{-ik\tau\phi\right\} = \left[A \exp\left\{-ik\tau\phi_K\right\} - B \exp\left\{ik\tau\phi_K\right\}\right] \tag{17}$$

$$R' \exp\left\{-ik\tau\phi\right\} = \frac{\phi_K}{\phi}\left[A \exp\left\{-ik\tau\phi_K\right\} - B \exp\left\{ik\tau\phi_K\right\}\right] \tag{18}$$

The field inside the slab will be found by determing A and B and integrating the resulting expression (Eq. (13)) over the aperture (multiplying Eq. (13) by $d\xi d\eta$ and integrating over the aperture).

The field well beyond the slab will be found by the technique of this report, for the present expressions for π are only valid at distances from the geometrical focus O small compared with f. This technique will be to find π and $\partial\pi/\partial z$ at the back of the slab (i.e., on $z = \tau + 0$) and determine π at far points by a Kirchoff integral over this face, noting that points over the back face whose distance from O is large, or comparable with f, give negligible contribution to this Kirchoff integral. π and $\partial\pi/\partial z$ at $z = \tau$ may, therefore, be calculated (for this purpose) from the present theory. We have from the last two equations

$$R'\left[\frac{\phi_K}{\phi} + 1\right] = 2A\,\frac{\phi_K}{\phi}\,\exp\left\{-ik\tau\left[\phi_K - \phi\right]\right\} \tag{19}$$

$$A\left[\frac{\phi_K}{\phi} - 1\right]\exp\left\{-ik\tau\phi_K\right\} = B\left[\frac{\phi_K}{\phi} + 1\right]\exp\left\{ik\tau\phi_K\right\} \tag{20}$$

Also, by the first two equations

$$2=\left\{\frac{\phi_K}{\phi}+1\right\}A\exp\left\{ik\tau(\phi_K-\phi)\right\}-\left\{\frac{\phi_K}{\phi}\right\}\times B\exp\left\{-ik\tau(\phi_K+\phi)\right\} \tag{21}$$

Hence

$$2\exp(ik\tau\phi)=A\left[\left\{\frac{\phi_K}{\phi}+1\right\}\exp(ik\tau\phi_K)-\frac{\left\{\frac{\phi_K}{\phi}-1\right\}^2}{\left\{\frac{\phi_K}{\phi}+1\right\}}\times\exp(-3k\tau\phi_K\right] \tag{22}$$

Hence

$$A=\frac{2\left\{\frac{\phi_K}{\phi}+1\right\}\exp\left\{-ik\tau(\phi_K-\phi)\right\}}{\left\{\frac{\phi_K}{\phi}+1\right\}^{1/2}-\left\{\frac{\phi_K}{\phi}-1\right\}^{1/2}\exp(-4ik\tau\phi_K)} \tag{23}$$

and this gives

$$B=\frac{2\left\{\frac{\phi_K}{\phi}-1\right\}\exp\left\{-ik\tau(3\phi_K-\phi)\right\}}{\left\{\frac{\phi_K}{\phi}+1\right\}^2-\left\{\frac{\phi_K}{\phi}-1\right\}^2\exp\left\{-4ik\tau\phi_K\right\}} \tag{24}$$

$$R^{1}=\frac{4K\frac{\phi_K}{\phi}\exp\left\{-2ik\tau(\phi_K-\phi)\right\}}{\left\{\frac{\phi_K}{\phi}+1\right\}^2-\left\{\frac{\phi_K}{\phi}-1\right\}^2\exp\left\{-4ik\tau\phi_K\right\}} \tag{25}$$

It is obvious that R^1 reduces to unity, as it should, when $\tau\to 0$ or when $K\to 1$, and when $K\to 1$, $B\to 0$, $A\to 1$ as it should.

The Field in the Vicinity of the Focus Within the Slab

Using the above values of A and B, the expression (Eq. (13)) for the field due to $d\xi d\eta$ inside the slab is

$$F=\frac{\frac{2i}{\lambda f^2}\exp\left\{ik\frac{x\xi+y\eta}{f}+ik\tau\phi\right\}}{\left\{\frac{\phi_K}{\phi}+1\right\}^2-\left\{\frac{\phi_K}{\phi}-1\right\}^2\exp\left\{-4ik\tau\phi_K\right\}}\left[\left\{\frac{\phi_K}{\phi}+1\right\}\exp\left\{-ik(z+\tau)\phi_K\right\}\right.$$
$$\left.+\left\{\frac{\phi_K}{\phi}-1\right\}\exp\left\{ik(z-3\tau)\phi_K\right\}\right]$$

$$=\frac{\frac{2i}{\lambda f^2}(\phi)\exp\left\{ik\frac{x\xi+y\eta}{f}-ik\tau(\phi_K-\phi)\right\}}{\left[\phi_K+\phi\right]^2-\left[\phi_K-\phi\right]^2\exp\left\{-4ik\tau\phi_K\right\}}\times$$

$$\left[(\phi_K+\phi)\exp\left\{-ikz\phi_K\right\}+(\phi_K-\phi)\exp\left\{ik(z-2\tau)\phi_K\right\}\right] \tag{26}$$

Hence the field in the slab has, for its Hertzian vector (multiplying the above by $d\xi d\eta$ and integrating over the aperture)

$$\frac{4\pi i}{\lambda f^2}\int_0^a\frac{\phi(\bar{\omega})\exp\left\{-ik\tau\left(\phi_K(\bar{\omega})-\phi(\bar{\omega})\right)\right\}J_0\left(\frac{kr\bar{\omega}}{f}\right)}{\left(\phi_K(\bar{\omega})+\phi(\bar{\omega})\right)^2-\left(\phi_K(\bar{\omega})-\phi(\bar{\omega})\right)^2\exp\left\{-4ik\tau\phi_K(\bar{\omega})\right\}}\times \tag{27}$$

$$\left[\left\{\phi_K(\bar{\omega})+\phi(\bar{\omega})\right\}\exp\left\{-ikz\phi_K(\bar{\omega})\right\}+\left\{\phi_K(\bar{\omega})-\phi(\bar{\omega})\right\}\right.$$
$$\left.\exp\left\{-ik(z-2\tau)\phi_K(\bar{\omega})\right\}\right]\bar{\omega}\,d\bar{\omega}$$

where now $\phi(\bar{\omega})=\left(1-\frac{\bar{\omega}^2}{f^2}\right)^{1/2}$ $\qquad \bar{\omega}^2=\xi^2+\eta^2$

and $\phi_K(\bar{\omega})=\left(K\;\frac{\bar{\omega}^2}{f^2}\right)^{1/2}$

the integral being taken over the aperture.

VERIFICATION - SPECIAL CASE z = 0

Putting $z = 0$ gives the field over the focal plane inside the slab. If further, $K = 1$, the above reduces to

$$\frac{2\pi i}{\lambda f^2}\int_0^a J\left(\frac{kr\bar{\omega}}{f}\right)\bar{\omega}\,d\bar{\omega}$$

as it should.

Considering now the limiting case $\tau \to 0, K \neq 1$; the integral $(z = 0)$

$$\longrightarrow \frac{2\pi i}{\lambda f^2}\int_0^a J(kr\bar{\omega}/f)\,\bar{\omega}\,d\bar{\omega}$$

as it should

THE FIELD JUST OUTSIDE THE SLAB

Using the formula derived for R' and Eq. (11), the expression for the Hertzian vector due to $d\xi d\eta$ is

$$\frac{\dfrac{4ik}{\lambda f^2}\phi_K \phi \exp\left\{ik\left(\dfrac{x\xi+y\eta}{f}\right)-2ik\tau(\phi_K-\phi)-ikz\phi\right\}}{\{\phi_K+\phi\}-\{\phi_K-\phi\}\exp\{-4ik\tau\phi_K\}} \tag{28}$$

so, (multiplying by $d\xi d\eta$ and integrating) the required field is

$$\frac{8\pi ik}{\lambda f^2}\int_0^a \frac{\phi_K\phi \exp\{2ik\tau(\phi_K-\phi)-ikz\phi\} J_0\left(\dfrac{kr\bar{\omega}}{f}\right)\bar{\omega}\,d\bar{\omega}}{\{\phi_K-\phi\}^2-\{\phi_K-\phi\}^2\exp(-4ik\tau\phi_K)} \tag{29}$$

the integral being taken over the aperture.

This expression will, of course, be valid only at distances from the focus small compared with f. For distances where this condition is not satisfied, we would adopt the previous technique of expressing the field as a Kirchoff-type integral over the plane $z=\tau+0$. For calculating the integrand of this Kirchoff integral, Eq. (29) is, of course, useful.

A USEFUL CASE

The present theory is restricted to neglect of the squares of a^2/f^2. Thus, we may replace in the above

$$\phi = \left(1 - \frac{\bar{\omega}^2}{f^2}\right)^{1/2} \quad \text{by} \quad 1 - \frac{\bar{\omega}^2}{2f^2}$$

If K is not too small we may also replace

$$\phi_K = \left(K - \frac{\bar{\omega}^2}{f^2}\right)^{1/2} \quad \text{by} \quad \sqrt{K} - \frac{\bar{\omega}^2}{2f^2\sqrt{K}}$$

In doing this, $\bar{\omega}^4/8K^4f^4$ is neglected in comparison with 1; this allows K to become even as small as a/f

Thus

$$\left(K - \frac{\bar{\omega}^2}{f^2}\right)^{1/2} - \left(1 - \frac{\bar{\omega}^2}{f^2}\right)^{1/2}$$

$$\div \sqrt{K} - \frac{\bar{\omega}^2}{2f^2\sqrt{K}} - 1 + \frac{\bar{\omega}^2}{2f^2} \tag{30}$$

Further, if K is near 1, write

$$K = 1 + \varepsilon \tag{31}$$

then we have

$$\sqrt{K} - \frac{\bar{\omega}^2}{2f^2\sqrt{K}} - 1 + \frac{\bar{\omega}^2}{2f^2} \div + \frac{\varepsilon}{2} + \frac{1}{2}\frac{\bar{\omega}^2}{f^2}\varepsilon = \frac{\varepsilon}{2}\left(1 + \frac{\bar{\omega}^2}{f^2}\right) \tag{32}$$

Also

$$\left(K-\frac{\bar{w}^2}{f^2}\right)^{1/2}+\left(1-\frac{\bar{w}^2}{f^2}\right)^{1/2} \doteq 2+\frac{1}{2}\varepsilon \tag{33}$$

Hence, neglecting $\varepsilon^2/8$ in comparison with unity, the denominator of the integrand in Eq. (29) may be replaced by 1.

Approximating the numerator of the integrand, Eq. (29) is now equal to

$$\frac{2\pi i}{\lambda f^2}\int_0^a \left(1+\frac{\varepsilon}{2}-\frac{\bar{w}^2}{f^2}\right)\exp\left\{-i\varepsilon k\tau-ikz\left(1-\frac{\bar{w}^2}{2f^2}\right)\right\} J_0\left(\frac{kr\bar{w}}{f}\right)\bar{w}\,d\bar{w} \tag{34}$$

which is simply $\sqrt{K}\exp(ik\varepsilon\tau)$, the undisturbed focused field. It must be emphasized that this result has been established only on the hypothesis that squares of ε are neglected in comparison with unity.

PHYSICAL INTERPRETATION

The forms of the solutions in Eqs. (27) and (29) bear an interesting physical interpretation in terms of "reflected and transmitted waves".

Field Inside Slab

This is given by Eq. (27).

The result, in the form of an integral is the sum of elementary plane "cylindrical" waves of the general form

$$\exp\left\{\pm ikz\left(K-\frac{w^2}{f^2}\right)^{1/2}\right\} J_0\left(\frac{k\bar{w}}{f}r\right) \tag{35}$$

Note that $\left(\frac{k\bar{w}}{f}\right)^2+\left[ik\left(K-\frac{\bar{w}^2}{f^2}\right)^{1/2}\right]^2=-k^2K$

The term involving $\exp\left[-ikz\left(K-w^2/f^2\right)^{1/2}\right]$ represents the sum of the forward* propagated waves within the slab, and that involving $\exp\left[ikz\left(K-w^2/f^2\right)\right]$ represents the sum of the backward propagated waves within the slab.

Forward Propagated Waves

Placing the denominator of the integrand effectively equal to 4 (this

*"Forward" means away from the aperture.

means neglecting all but the first "transmitted" wave within the slab, i.e., neglecting secondary forward propagating waves caused by reflection from the 1st edge), the effective focal plane of this wave is given by putting the phase equal to zero, i.e., by

$$z\left(K-\frac{\bar{\omega}^2}{f^2}\right)^{1/2} = -\tau\left[\left(K-\frac{\bar{\omega}^2}{f^2}\right)^{1/2}-\left(1-\frac{\bar{\omega}^2}{f^2}\right)^{1/2}\right]$$

i.e., $z = -\frac{1}{2}(K-1)\tau$ (first approx)

This is shown at F_1 in the diagram.

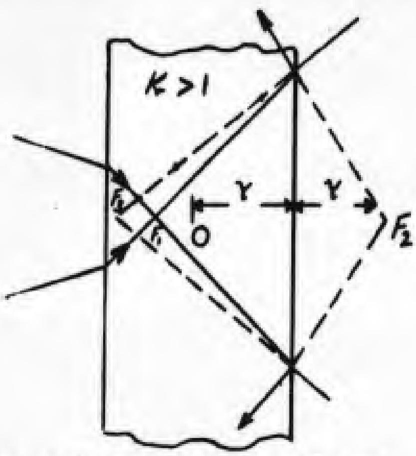

Fig. 2. Focusing in Plasma Slab

Backward Propagating Waves

Conducting the same approximation with the denominator (this means taking account of only the 1st internal reflection — see Fig. 2), the effective (virtual) focus will be given by

$$z - 2\tau = +\tau\left[\left(K-\frac{\bar{\omega}^2}{f^2}\right)^{1/2}-\left(1-\frac{\bar{\omega}^2}{f^2}\right)^{1/2}\right]$$

$$z = 2\tau + \frac{1}{2}(K-1)\tau \qquad \text{(first approx)} \qquad (36)$$

This is shown at F_2 in Fig. 2.

F_2 is the mirror image of F_1 in the 2nd face $(z = \tau)$ of the slab which is as it should be.

Field Outside Slab

By referring to Eq. (29), it can be seen that there is only one forward propagated wave.

With the same approximation as the above for the denominator of the integrand (this means that only the primary transmitted ray is taken — see Fig. 2) the effective (virtual) focus will be given by

$$z = -2\tau\left\{\left(K - \frac{\bar{w}^2}{f^2}\right)^{1/2} - \left(1 - \frac{\bar{w}^2}{f^2}\right)^{1/2}\right\}$$

$$z = -(K-1)\tau \qquad \text{(first approx)} \qquad (37)$$

This is shown at F_3 in Fig. 2.

Note that $OF_3 = 2OF_1$, which can be easily proved from Snell's law using ray theory and elementary geometry and remembering that all the angles with the optic axis are supposed small.

THE APPROXIMATIONS TO THE FIELD

The Field Inside the Slab

The denominator of the integrand will now be approximated.

Let $z = F_1$, $z = F_3$ be the positions of the real focus for the main transmitted wave (within slab) and the virtual focus for the main reflected wave (within slab).

Thus, Eq. (27) is

$$\frac{2\pi i}{\lambda f^2}\int_0^a \left\{\exp -ik(z-F_1) - \frac{K-1}{4}\exp ik(z-F_3)\right\} J_0\left(\frac{kr\bar{w}}{f}\right)\bar{w}\,d\bar{w} \qquad (38)$$

where

$$F_1 \doteq -\frac{1}{2}(K-1)\tau \;; \quad F_2 \doteq 2\tau + \frac{1}{2}(K-1)\tau$$

The Field Outside the Slab

With the above-mentioned approximation for the demonimator of the integrand, Eq. (29) becomes

$$\frac{4\pi i}{\lambda f^2}\int_0^a \left(K-\frac{\bar{w}^2}{f^2}\right)^{1/2}\left(1-\frac{\bar{w}^2}{f^2}\right)^{1/2} \exp -ik(z-F_3) \cdot J_0\left(\frac{kr\bar{w}}{f}\right)\bar{w}\,d\bar{w} \qquad (39)$$

where $F_3 \doteq -(K-1)\tau$

In the last formula, the effective distribution over the new focal plane (virtual focal plane) at $z = F_3 = -(K-1)\tau$ is $\left(K - \frac{\bar{w}^2}{f^2}\right)^{1/2} \left(1 - \frac{\bar{w}^2}{f^2}\right)^{1/2}$.

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APPENDIX

Far Field for Slab Distribution

With a slab of dielectric, of thickness 2τ, centered at the geometrical focus $(0, 0, 0)$ whose plane faces are parallel to xOy, it was found that, if K was near to unity, the field just outside the slab $(z > \tau)$ is $\exp\left\{-2ik\tau\left(\sqrt{K}-1\right)\right\}$ times the "free space" value.

Hence, the perturbation field anywhere to the right of the slab is

$$\exp\left\{-2ik\tau\left(\sqrt{K}-1\right)\right\}-1$$

times the "free space" field.

This agrees with a factor obtained for the perturbation field of a thick cylinder.

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