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A BROAD LOOK AT THE PERFORMANCE
OF INFRARED DETECTORS*

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I. INTRODUCTION

In this paper we approach the subject of infrared detectors in a somewhat different manner than has been done in the past. Instead of a detailed approach stressing, for example, the solid state physics or the ways in which it is possible to classify detec~~tors, we discuss~~ the capabilities of present day infrared detectors from a more or less absolute viewpoint, i.e., in terms of radiant power and time as measured. The intent ~~here~~ was ~~has~~ purposely ~~been~~ to suppress many of the fine points in order to attain a perspective of the subject. ~~Hopefully, we are trying to show the forest and not the trees. Those interested in more detail are referred to the literature which includes the extensive work of R. Clark Jones, a recent book by Smith, Jones, and Chasmar, a discussion of photoconductors by R. Petritz, and articles by several other authors.~~(1-5)

~~The~~ This method of approach starts with a derivation of the performance limits for ideal detectors, thus establishing a basis of comparison for real detectors. The derivations include both the limits set by background fluctuations and by signal fluctuations, ~~the latter subject being treated here in some detail.~~* Next some recent data on detector performance are reduced to a form allowing intercomparisons. Our method stresses the fundamental fact that the minimum detectable power is directly related to the time taken for the detection process or, more loosely, to the time constant of the detector. It then becomes possible to treat the subject in a very simple way. On a graph whose ordinate is minimum detectable power and whose

*The justification for dwelling upon signal fluctuations arises not only from the fact that the subject is important conceptually, but it also may have more practical relevance in the future. For example, very narrow band (high-Q) infrared detectors may be signal fluctuation limited against a target with a continuous spectrum. Also, in very low temperature background situations, e.g., the detection of space vehicles from a high altitude platform, signal fluctuations might easily become important.

abscissa is time constant, both the signal and background limits of ideal detectors and the performance of all types of real detectors can be shown. Since there are two methods of rating detectors, the results take the form of two graphs, one for monochromatic radiation and the other for 500°K black body radiation. The limitations to this approach are discussed: essentially an exact intercomparison between detectors of different noise spectral densities and different time constants is, to an appreciable extent, both involved and arbitrary. Our method may understate the abilities of some detectors (those with both a short time constant and also a 1/f noise spectrum) but the error is rarely more than a factor of 3. Compensating this is the fact that our method gives very quickly an approximate value for the minimum detectable power of practically any detector at any infrared wave length. An additional point is that in the future as detectors presumably tend closer to ideal behavior, the error involved becomes even less.

For completeness, the background fluctuation limits are extended through the long wave infrared region of the spectrum to the microwave region. Not only does this serve to tie the subject together but it points up the interesting fact that in the 'classical' region, quanta behave in a very strange manner.

The material presented here should be of assistance to anyone who is attempting to form an estimate of the reasonableness of claims for novel schemes of infrared detection. Regardless of the principle of operation, it should be possible to arrive at a performance rating for any infrared detector that can be plotted on Figs. 6 and 7 thus allowing a direct comparison with existing detectors.

II. THE IDEAL DETECTOR

As radiation consists ultimately of quanta, it might be considered reasonable to define an ideal detector simply as a device which can count or indicate the arrival of each incident quantum. This is equivalent to requiring that it have a detective quantum efficiency of unity for radiation of all wave lengths. Furthermore, this ideal detector would be perfectly noiseless, that is, it would generate no interfering signals nor would it give a false count when, in fact, there were no incident quanta. Needless to say, such an ideal detector is very far from physical realization at the present time. There are, however, some detectors with a detective quantum efficiency near unity for all wave lengths less than a certain value λ_o with a very rapid fall off in response beyond λ_o. It is therefore a useful concept to define an 'ideal quantum detector' as a device capable of noiselessly detecting all incident quanta with wave lengths less than the cutoff wave length λ_o, and having zero response beyond λ_o.

A detector need not respond to the number of incident quanta but alternatively can respond to the energy or heating effect of the incident radiation; absorption of radiation is indicated by a temperature rise of the detector. Such devices are known as thermal detectors and usually respond uniformly to radiation covering a very wide wave length region. The ultimate limits of detectability of a thermal detector are set by temperature fluctuations due to the statistical interchange of energy between the detector and its surroundings. Incident signal radiation which produces a detector temperature rise less than the r.m.s. value of this fluctuating temperature cannot be said to be 'detected.' The numerical value of this fluctuation will be given later; for the present an 'ideal' thermal detector will be defined as a device which completely absorbs all incident energy, is coupled to its surroundings by only one totally blackened surface through radiation alone (i.e., has no conductive or convective losses), and finally, is in thermal equilibrium with its surroundings.

III. BACKGROUND FLUCTUATIONS

In almost all real situations the target or the source of radiation to be measured is not the only source which is supplying radiation to the detector. In the daytime scattered sunlight is often incident upon the detector but, even if this can be avoided, there is still the unavoidable radiation from the background. This background radiation would be of small consequence if it were perfectly steady; however, the rate of arrival of background quanta at the detector is a statistically fluctuating quantity. The magnitude of this fluctuation sets a lower limit to the signal radiation that can be detected.* Quanta obey Bose-Einstein instead of classical statistics; however, for a background temperature of 300°K and wave lengths less than about 20 microns the difference between the two types of statistics is negligible. Under these conditions, the rate of arrival of background quanta N may then be considered as being completely random and the r.m.s. fluctuation per sec, $\sqrt{\overline{\Delta n^2}}$, is simply equal to $\sqrt{N}$. For an ideal quantum detector the signal photon current for a unity signal to noise ratio need only be equal to the r.m.s. fluctuation per sec $\sqrt{N}$.

Before numerical values of the background fluctuation can be computed it is necessary to know the effective temperature and spectral distribution of the background and the solid angle through which background radiation is incident upon the detector. It has been customary to consider the background as a 300°K black body supplying radiation over 2π steradians. This

*To make this point perfectly clear, our usage of the term 'background fluctuations' has nothing whatsoever to do with the fluctuations of the sky background which would be obtained, for instance, by scanning across sunlit clouds. The 'background fluctuations' we are referring to would exist if the detector were placed in a perfectly lightless environment such as the 'dark tunnel' used to test infrared equipment.

is reasonable since in the majority of detector applications background radiation of roughly 300°K and near unity emissivity comes through the optical system and from the local surroundings. Actually, an uncooled detector receives background radiation from 4π steradians; only a cooled detector without a cooled shield can be considered to be receiving background radiation from 2π steradians.

Of course one can imagine situations in which the background can be far lower than the 300°K, 2π value, e.g., the local surroundings of the detector can be cooled and the optical system pointed at a near zenith angle. A further reduction can be obtained with a cooled filter transmitting an atmospheric "window." In such cases it is usually quite straightforward to modify the 300°K, 2π value of background appropriately.*

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*When considering the improvement in system performance in a low background situation it is obviously necessary that the detector be able to sense background fluctuations. Until recently, the internally generated noise of almost all infrared detectors was so high that it masked the background noise by a large factor. It should also be noted that with background limited detectors, the system performance is independent of the f number of the optics. That is to say, if background radiation comes only through the optical system, it makes no difference whether the detector receives this radiation from 2π steradians (an f/0.5 system) or from some smaller solid angle.

A. Background Fluctuation Limits for an Ideal Quantum Detector

It will be assumed that the ideal quantum detector accepts background radiation through a 2π solid angle and does not reradiate. Further, the ideal detector will be assumed to have the capability of counting quanta for any desired time interval. In effect, the ideal detector has a completely adjustable 'time constant.'

The number of background quanta falling on a unit area per sec is then given by the integral of the Planck equation of photon spectral density $N_\lambda(T,\lambda)$ up to the detector cut off wave length λ_c:*

$$N_B = \int_0^{\lambda_c} N_\lambda (T,\lambda)\, d\lambda$$

The detector will require the least radiation power if all the signal quanta have wave lengths just short of the cut off wave length. For a unity signal to noise, unit area, and a one second counting time, the minimum amount of monochromatic radiation power to equal background fluctuations becomes:

$$P_{B \underset{\lambda_c}{\min}}(1) = \frac{hc}{\lambda_c} \sqrt{N_B}$$

For area A and counting time t:

$$P_{B \underset{\lambda_c}{\min}}(t) = \frac{hc}{\lambda_c} \sqrt{N_B \frac{A}{t}}$$

*Expressions for $N_\lambda(T,\lambda)d\lambda$ are given in Ref. 2. The Radiation Slide Rule of the Admiralty Research Laboratory is a very convenient means of obtaining numerical values for both spectral and integrated forms of the Planck equation.

$P_{B\,min_{\lambda_c}}(t)$ can thus be made as small as desired by decreasing A and increasing t.

Table 1 gives the values of the background photon current and $P_{B\,min_{\lambda_c}}(1)$ for a unit area detector and 300°K background radiation with λ_c varying between 1 and 10 microns. The interesting point here is the very rapid change in minimum detectable power between 1 and 3 microns, while beyond 5 or 6 microns it is relatively constant.

Table 1

MINIMUM DETECTABLE POWER FOR IDEAL QUANTUM DETECTOR, ONE SECOND COUNTING TIME, 1 CM^2 AREA, 300°K BACKGROUND, 2π SOLID ANGLE

λ_c Cut Off Wave Length (Microns)	N_B photons per sec on 1 cm^2 area	$P_{B\,min_{\lambda_c}}(1)$, Watts
1.0	6.6	(5×10^{-19})*
2.0	4.2×10^{10}	2.0×10^{-14}
3.0	5.8×10^{13}	5.0×10^{-13}
4.0	1.9×10^{15}	2.2×10^{-12}
5.0	1.3×10^{16}	4.5×10^{-12}
6.0	4.9×10^{16}	7.3×10^{-12}
8.0	2.2×10^{17}	1.2×10^{-11}
10.0	5.0×10^{17}	1.4×10^{-11}
∞ (Ideal Thermal Detector)	4.15×10^{18}	3.9×10^{-11}

*The expression for $P_{B\,min}$ breaks down for very small values of N_B. This number is included only to show the rapid variation of $P_{B\,min}$ between 1 and 2 microns. It will become evident later that at 1 micron and 1 sec counting time, the detector is actually signal fluctuation limited.

The value of $P_{B\ min}$ for an ideal thermal detector cannot be obtained from the above expressions for the ideal quantum detector. As $\lambda \rightarrow \infty$ the total mean square fluctuation and hence the required signal photon current approaches a finite value. The energy per signal quantum approaches zero as $\lambda \rightarrow \infty$, thus $P_{B\ min}$ would also approach zero, an unreasonable result. For wave lengths much greater than 1 mm, however, the assumption that signal quanta can be directed onto the detector surface and be completely absorbed becomes questionable. Without going into the exact derivation at this time, it is interesting to note that a very simple approximate argument gives a value for $P_{B\ min}$ for an ideal thermal detector within 22 per cent of the generally accepted value: the mean square fluctuation per sec of the background photon current is $N_B^{1/2}$. (This neglects any effects due to the departure of Bose statistics from classical statistics.) The energy represented by this fluctuation is: $h\bar{\nu}\ N_B^{1/2}$ where $h\bar{\nu}$ is the average energy per quantum of a 300°K background. The value of this fluctuation energy per sec is thus $\frac{\sigma T^4}{N_B} \cdot N_B^{1/2} = \sigma T^4\ N_B^{-1/2} = \frac{0.046}{\sqrt{4.15 \times 10^{18}}} = 2.25 \times 10^{-11}$ watts.

Reradiation by the detector doubles the mean square fluctuation, hence this approximate value of $P_{B\ min}$ is 3.19×10^{-11} watts for a 1 cm^2 detector subject to a 300°K background.*

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*The value given for example in Ref. 2 for $P_{B\ min} = 5.55 \times 10^{-11}$ watts refers to a unit bandwidth and hence, by the relation $2t\Delta f = 1$, to a $t = 1/2$ sec. For a 1 sec counting time $P_{B\ min} = 3.9 \times 10^{-11}$ watts.

B. Background Fluctuation Limits for a 500°K Black Body Source

The relative abilities of detectors to sense black body signal radiation is of considerable practical importance, hence, laboratory measurements generally include the response to a standardized black body source at 500° K. It is therefore necessary to compute the minimum amount of 500° K radiant power necessary to equal background fluctuations. For a given cut off wave length the required signal current is the same as calculated above. The average energy carried by the effective signal quanta is now:

$$h\bar{\nu} = \frac{\int_0^{\lambda_c} H_\lambda(500^\circ)\, d\lambda}{\int_0^{\lambda_c} N_\lambda(500^\circ)\, d\lambda}$$

where $H_\lambda(500^\circ)$ is the spectral radiant flux density in watts/cm^2 - cm $\Delta\lambda$ and $N_\lambda(500^\circ)$ is the spectral radiant photon density. However, the detector must be charged with all the radiant signal power, hence the average energy per quantum must be multiplied by

$$\frac{H(500^\circ)}{\int_0^{\lambda_c} H_\lambda(500^\circ)\, d\lambda} ,$$

where $H(500^\circ)$ is the total radiant power from a 500° K black body of unit area. The resulting expression for minimum detectable power becomes

$$P_{B\,min\,500^\circ}\,(1) = \frac{H(500^\circ)}{\int_0^{\lambda_c} N_\lambda(500^\circ)\, d\lambda} \sqrt{\int_0^{\lambda_c} N_\lambda(T_B)\, d\lambda}$$

Figure 1 shows $P_{B\ min\ 500^o}$ for background temperatures ranging from 200^{o}K to 350^{o}K. For a 300^{o}K background there is a wide range of λ_c for which $P_{B\ min\ 500^o}$ remains almost constant. In other words, for an ideal quantum detector looking at a 500^{o}K black body against a 300^{o}K background it makes essentially no difference what cut off wave length is chosen (beyond 1 μ): the increased response to background by raising λ_c is almost exactly compensated by the increased signal. For black body radiation it can be shown in general that $P_{B\ min}$ is insensitive to λ_c for a target temperature approximately twice the background temperature.

For background temperature less than 300^{o}K (and a 500^{o}K source) it is advantageous to use as short a cutoff wave length as possible, while the opposite conclusion holds for background temperatures higher than 300^{o}K.

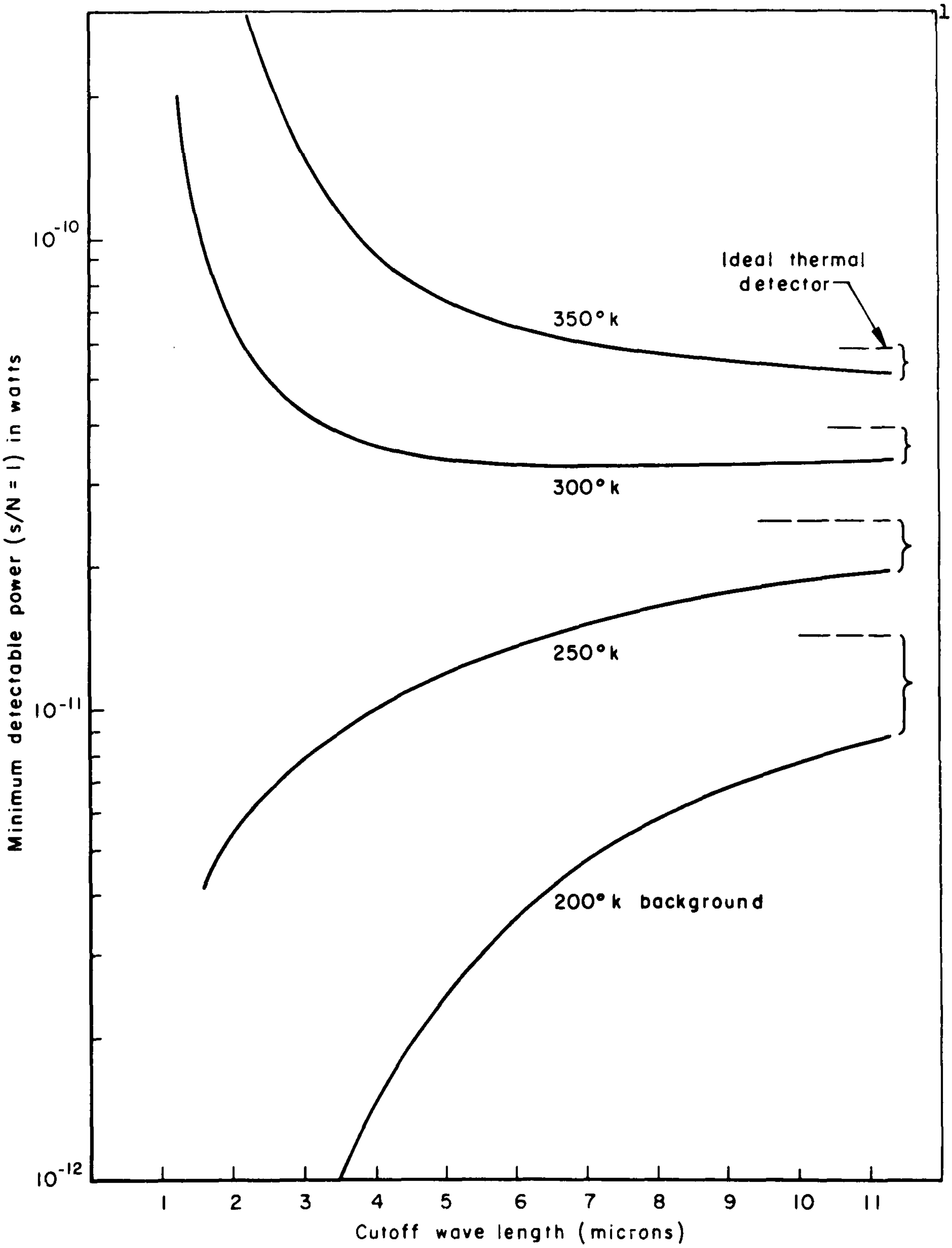

Fig. 1 — Ideal quantum detector performance (1. cm area, 500° k target, 1 sec counting time) for various background temperatures

IV. SIGNAL FLUCTUATIONS

It is possible to imagine situations in which the background supplies so few quanta to the detector that fluctuations or noise from this source becomes unimportant. Under these circumstances, fluctuations in the arrival of signal quanta will set a limit to the minimum detectable power. During the time that the detector is pointed at the target, it is obviously necessary that at least one quantum of the proper wave length arrive at the detector. Indeed it will be necessary to require that more than one quantum be received on the average to insure a low probability of missing a real target due to these fluctuations. If the arrival of quanta can be considered as a random process, the Poisson formula may be used to treat the fluctuations. Then, if m events occur on the average in a given time interval t, the probability $P_m(0)$ of no events occurring in t is e^{-m}. Thus if m = 1, the probability that a real target could be missed is 36 per cent. To reduce this probability below 1 per cent, m must be 4.6 or more. Using this rather arbitrary criterion the minimum detectable power of wavelength λ and for a counting time t becomes:

$$P_{S\,\underset{\lambda}{min}}(t) = 4.6\,\frac{hc}{\lambda t}$$

For t = 1 sec and λ = 3 microns, $P_{S\,\underset{3\mu}{min}}(1) = 3 \times 10^{-19}$ watts while, for comparison, $P_{B\,\underset{3\mu}{min}}(1)$ set by 300°K background fluctuations is 5×10^{-13} watts for a 1 cm^2 detector. Notice that $P_{S\,\underset{\lambda}{min}}(t)$ for signal fluctuations varies as t^{-1} and is independent of detector area while $P_{B\,\underset{\lambda}{min}}(t)$ for background fluctuations varies as $t^{-1/2}$ and as $A^{1/2}$. Figure 2 shows this dependence graphically for several wave lengths assuming a 1 cm^2 detector area. No infrared detector regardless of its principle of operation can do better than the signal fluctuation limits; no quantum detector of cutoff wavelength λ_c exposed to 300°K background radiation can do better than the appropriate background fluctuation limits.

For monochromatic 1 micron radiation the crossover point between the background and signal fluctuation limits occurs at a counting time t of 3 sec. A 1 cm^2 detector with a shorter time constant than 3 sec would be signal fluctuation limited; with a longer time constant it would be background fluctuation limited.

The signal fluctuation limits for monochromatic radiation are perhaps less interesting than those for a 500°K black body source. Using the above criterion, on the average 4.6 quanta are required per counting time interval to avoid missing a real target more than 1 per cent of the time. The average energy per effective quantum is:

$$\frac{\int_0^{\lambda_c} H_\lambda \, d\lambda}{\int_0^{\lambda_c} N_\lambda \, d\lambda}$$

using the same symbols that were defined earlier. As in the background fluctuation case, however, all of the incident signal radiation must be charged to the detector, hence the minimum power for a counting time t required to exceed the signal fluctuations from a 500°K source becomes:

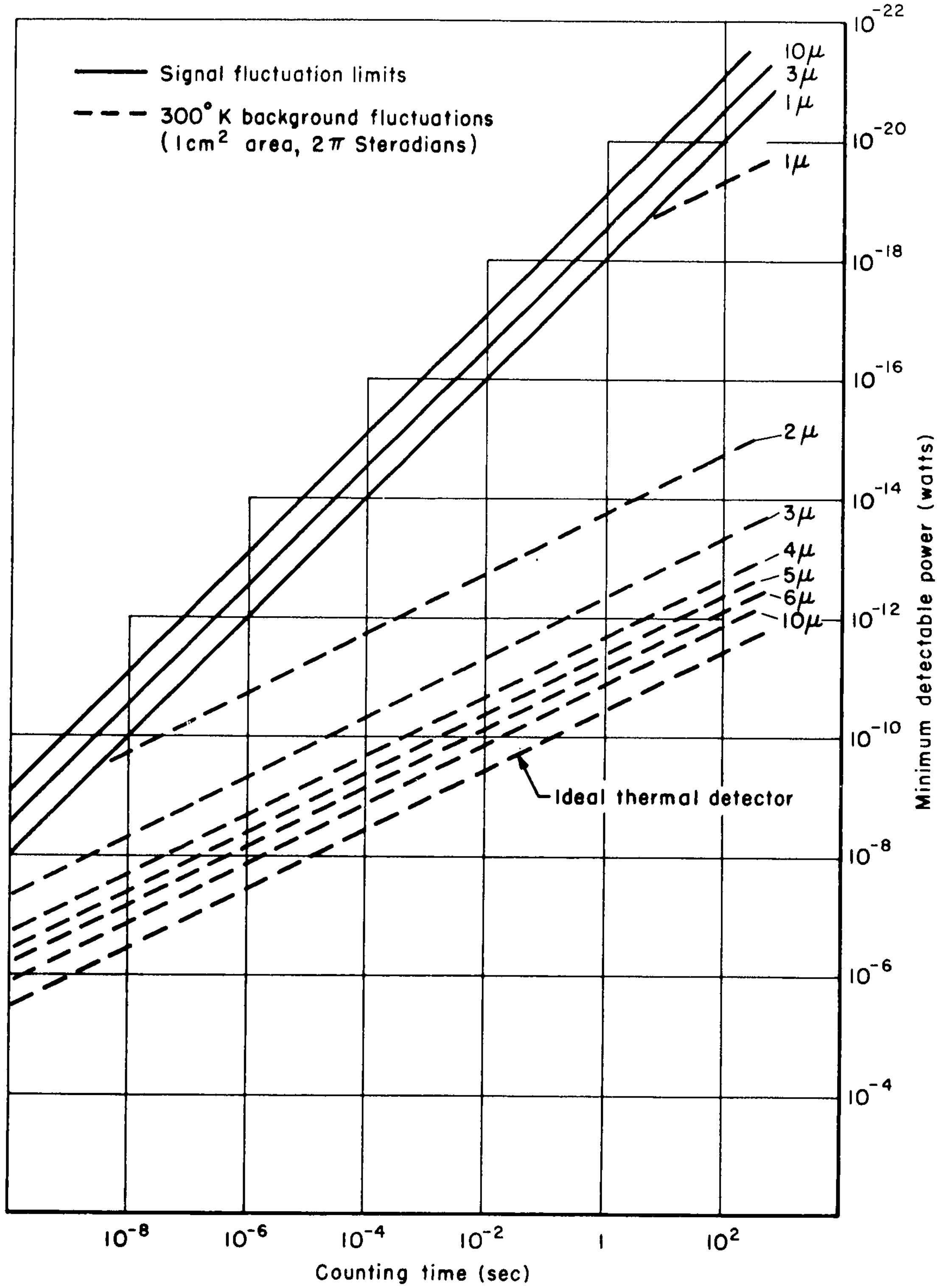

Fig. 2 — Signal and 300° K background fluctuation limits for monochromatic signal radiation

$$P_{S\,min}(t) = \frac{4.6\ H\ (500^\circ)}{t \int_0^{\lambda_c} N_\lambda\ (500^\circ)\ d\lambda}$$

Table 2 gives $P_{S\,min\ 500^\circ K}(1)$ for various cutoff wave lengths.

Table 2

Signal Fluctuation Limits for 500° K Black Body Radiation, 1 Sec Counting Time, Ideal Quantum Detector Cutoff Wave Length λ_c

λ_c	$P_{S\,min\ 500^\circ K}(1)$, (Watts)
1.0	5.2×10^{-10}
1.5	1.0×10^{-13}
2.0	1.4×10^{-15}
3.0	2.5×10^{-17}
4.0	3.9×10^{-18}
10.0	2.2×10^{-19}
∞	8.4×10^{-20}

The large values of $P_{S\,min\ 500^\circ K}$ at short wave lengths merely reflects the fact that a 500°K source emits only a small fraction of its total energy at short wave lengths.

Figure 3 is a plot of $P_{S\ min}(t)_{500^{o}K}$ and $P_{B\ min}(t)_{500^{o}K}$ against counting time t. The interesting point here is that the 300°K background limits are very insensitive to wave length while the signal fluctuation limits are quite sensitive to wave length, in considerable contrast to Fig. 2 showing the monochromatic limits.

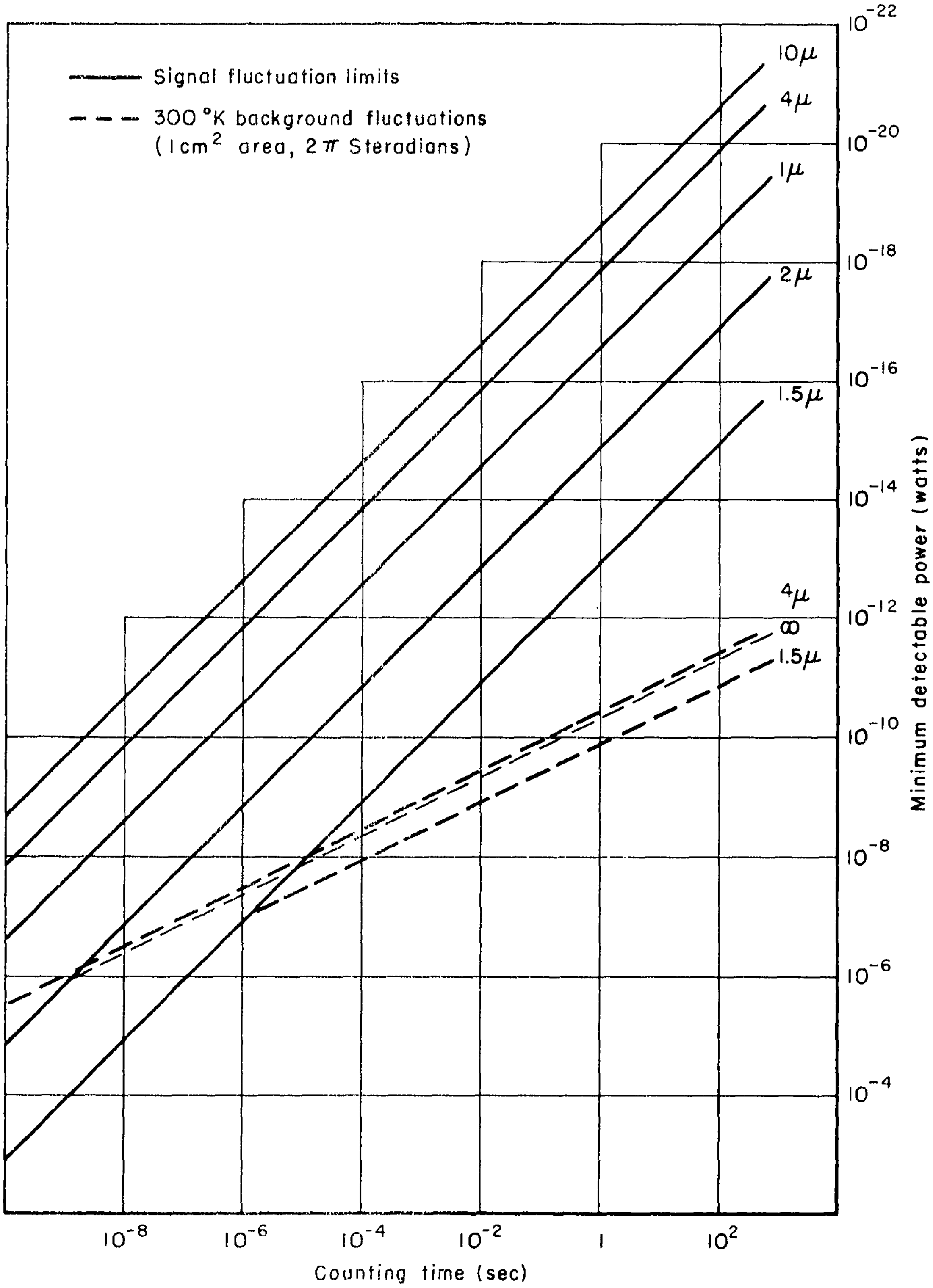

Fig 3 — Signal and 300° K background fluctuation limits for a 500° K black body source

V. REAL DETECTORS

Real detectors fall short of ideal performance in a number of respects, the most obvious way being that real detectors generate considerable internal noise. Except for a very few of the best detectors, the internally generated noise masks the $300^{\circ}K$ background fluctuations by a considerable factor. An alternative way of saying the same thing is that if the incident ambient radiation is lowered by cooling the surroundings, there are relatively few detectors in existence for which there is a reduction in output noise.

One way of characterizing a detector's performance would be to find the background temperature that gives rise to a noise equal to that generated internally by the detector. Unfortunately, the noise spectral densities of many real detectors differ markedly from the flat or "white" noise spectrum of background fluctuations. The equivalent noise temperature would thus be a function of the particular choice of bandwidth used in the measurement. Detectors with a more or less flat noise spectrum might, however, be usefully described in terms of an equivalent background temperature.

This brings up a second major difference between the real and the ideal detectors: except in very rare cases it is not possible to count the arrival of individual infrared quanta as was assumed possible for the ideal detector. Instead, the detector is usually connected to a linear amplifier of bandwidth Δf whose output voltage is therefore proportional to the average value of the incident photon current. Measurements of the detector's signal and noise output are often referred to one c.p.s. bandwidth. It is possible to express the background noise of an ideal detector in terms of electrical bandwidth merely by substituting for the counting time t the expression $(2\Delta f)^{-1}$.* In the counting time terminology the expression was obtained:

$$P_{B\,\underset{\lambda}{\min}}(t) = \frac{hc}{\lambda_c}\sqrt{N\,\frac{A}{t}}$$

In the bandwidth terminology this expression becomes:

$$P_{B\,\underset{\lambda}{\min}} = \frac{hc}{\lambda_c}\sqrt{2NA\Delta f}$$

Since both modes of description are completely equivalent, it is only a matter of taste or of convenience which is chosen for comparing the performance of real and ideal detectors.

Although in the past it has been conventional to use the bandwidth description, there are advantages for our purposes in using the counting time instead:

1. The results are given directly in terms of minimum detectable power for a given time constant, a quantity that should be useful in discussing detector applications.
2. The dependence of the minimum detectable power on counting time is made explicit and kept continuously in view. In the bandwidth description it is too easy to forget the fundamental fact that "minimum detectable power" for a given detector can be varied over a wide range of values merely by changing the effective counting time, or the bandwidth.

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*This assertion is proven in Ref. 2.

The problem now arises of how to reduce the existing experimental data to a single number for each detector that accurately characterizes its ultimate performance, both relative to detectors of the same type and to those of different types. This problem would be fairly simple if real detectors behaved more or less like an ideal detector, for example, if real detectors all displayed a white noise spectrum but had detective quantum efficiencies less than unity.* Unfortunately, real detectors display a variety of noise spectral densities and time constants. In the most extensive published data all detectors were measured under one set of conditions (90 c.p.s. chopping frequency and a 5 c.p.s. bandwidth). The result is that some detectors are measured near their optimum chopping frequency while the majority are not.

R. Clark Jones has made a study of the problem of rating detectors, the results being given in an article in <u>Advances in Electronics</u>.[1] Elsewhere Jones introduced a rating which others have called "Jones' S," or S_J, and defined as:

$$S_J = \frac{\text{NEP}}{\sqrt{A}} \left(\frac{f}{\Delta f}\right)^{1/2}$$

This rating removes the dependence on chopping frequency; however, it is only valid if the detector displays a 1/f noise spectral density. This holds true to a fair approximation for the film photoconductors PbS, PbTe, and PbSe, but even for these detectors at higher chopping frequencies, and especially for the newer crystal detectors, InSb and doped Ge, the trend is

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*The expression for the background fluctuation limits for a detector with uniform detective quantum efficiency ε out to λ_c (and zero beyond) is identical with those derived earlier multiplied by $\varepsilon^{-1/2}$. For signal fluctuations the factor is ε^{-1}.

more towards a flat noise spectrum. For this reason Jones has recommended that the S_J rating be dropped (at least for non-1/f detectors), but he has no simple alternative rating scheme to offer in its place.[↓](6)

In the Advances in Electronics article, Jones separates all detectors into two classes depending upon how detectivity is traded for speed of response. ("Detectivity" is the reciprocal of the minimum detectable power measured under certain conditions of chopping frequency and bandwidth.) Jones can then assign a merit rating M_1 or M_2 to any given detector depending upon its class. M_1 is essentially a comparison of a real Class I detector to an ideal thermal detector; M_2 a comparison of a Class II detector with "Haven's limit" (a semi-empirical expression representing the best attainable performance of real thermal detectors). In Ref. (6) Jones suggests that M_2 be used for all photoconductive and photovoltaic detectors not definitely proven to be Class I.

Jones' system of rating detectors is not without its drawbacks. First, and perhaps unavoidably, it is complex. The above discussion does not begin to show this complexity: for example, it turns out to be necessary to divide each of the main classes into two sub-classes. A "reference time constant" is introduced which equals detector time constant τ for class Ia and IIb but only one-fourth τ for class Ib and IIa. Secondly, Jones' M ratings are only able to compare detectors belonging to the same class. His "Detectivity in reference condition A" compares only detectors in the same sub-class (since Δf is by definition different for each sub class).[↓↓] This may again be

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[↓]Jones' D^* rating, although simple, does not do full justice to each detector regardless of its noise spectrum and time constant. The limitations of D^* are treated in this paper, for we use $(D^*)^{-1}$ as our measure of performance.

[↓↓]It is assumed that the comparison is between detectors with the same class and time constant, differing only in sub class.

logically correct but it can hardly be said to be desirable. Thirdly, there is a slight tendency for the M and D ratings to overstate the detector's real abilities. Essentially, the maximum signal-to-noise ratio (or $D_1(f_m)$) is assumed over the entire reference bandwidth instead of the average. (This objection does not apply to the D^* rating.) Fourthly, Jones' rating system is set up in a somewhat arbitrary manner. There is nothing inescapable or compelling about his particular choices and, in a sense, the final results do not convey much fundamental quantitative information to the reader. While one knows that a high merit rating means a good detector, it is not easy to answer the question, "How good?"

In view of the above considerations we adopt the following simple procedure for treating detector performance data: the "counting time" t for the ideal detector will be replaced by the physical time constant τ for the real detector. The measured noise equivalent power of the real detector is first normalized to unit area and unit bandwidth by dividing by $\sqrt{A\Delta f}$.[↓] Since $2\Delta f \cdot t = 1$, this normalized power will correspond to $P_{min}(1/2)$, the minimum detectable power for a 1/2 sec counting time. In order to plot power against τ correctly we must calculate the quantity $P_{min}(\tau)$, which is simply $\left(\sqrt{2\tau}\right)^{-1} P_{min}\left(\frac{1}{2}\right)$. Equivalently, we could translate the point $\left[\frac{1}{2}, P_{min}\left(\frac{1}{2}\right)\right]$ on a straight line of slope $\frac{1}{2}$ until it intersects the vertical line through τ (Fig. 4). The distance from the point $(\tau, P_{min}(\tau))$ to the line representing P_{min} for an ideal quantum detector is of course an indication of the quality of the particular detector.[↓↓]

[↓]The reciprocal of the normalized noise equivalent power is equal to $D^*(\lambda$, 90 c.p.s.) or D^*(500°K, 90 c.p.s.) depending on whether the NEP is a spectral or 500°K black body measurement.

[↓↓]According to Petritz the limits for an ideal photoconductor are poor-

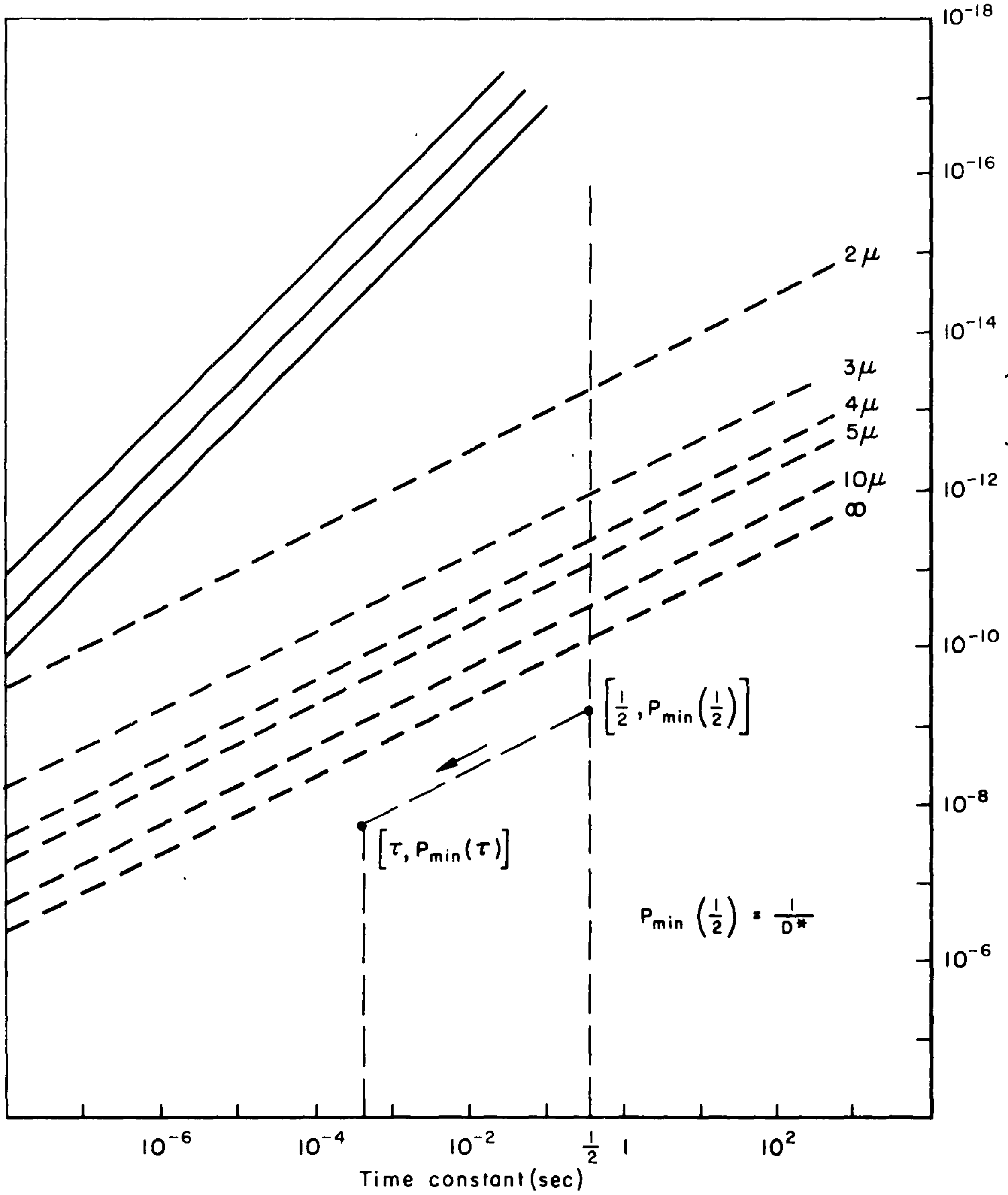

Fig. 4 — Method of plotting detector performance data

The point $(\tau, P_{min}(\tau))$ gives immediately the minimum detectable power of a 1 cm^2 detector with pnysical time constant τ when used in a circuit with electrical bandwidth $\Delta f = \frac{1}{2\tau}$, which is of the order of the bandwidth needed to reproduce a pulse of duration τ. If this bandwidth is larger than is required for the particular detector application, the point may be moved upwards to the right along a line of slope 1/2 until the desired bandwidth or overall circuit time constant is obtained. In some cases, the point may even be moved downward to the left to shorter time constants, e.g., if the detector's responsivity falls with increasing f at the same rate as the noise spectral density, then equalizing circuits can be used to boost both signal and noise. When the noise spectral density begins to " flatten out " to white noise, this technique can no longer be used except at the expense of a degradation in signal to noise.

To gain more insight into the approximations involved in this procedure, it is convenient to use the bandwidth viewpoint. Figure 5 is a plot of signal-to-noise ratio vs. chopping frequency for two detectors differing in noise spectral density but having the same time constant. Curve A would hold for a white noise spectrum, curve B for a 1/f noise spectrum. Customarily, the data establishes the ordinates at 90 c.p.s. for each detector. Evidently the 90 c.p.s. S/N value is somewhat unfair to detector B but is quite satisfactory as an indication of detector A's performance. The maximum slope of curve B is +1/2 below $f_m = (2\pi r)^{-1}$ and -1/2 above. Thus the 90 c.p.s. S/N

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er by the factor $\sqrt{2}$ than the background limits given here.[3] Essentially this is due to energy exchanges between the carriers and the lattice. Reflection at the surface of the detector (Fresnel losses) and less than complete absorption of the incident radiation are additional factors that degrade the performance of real detectors.

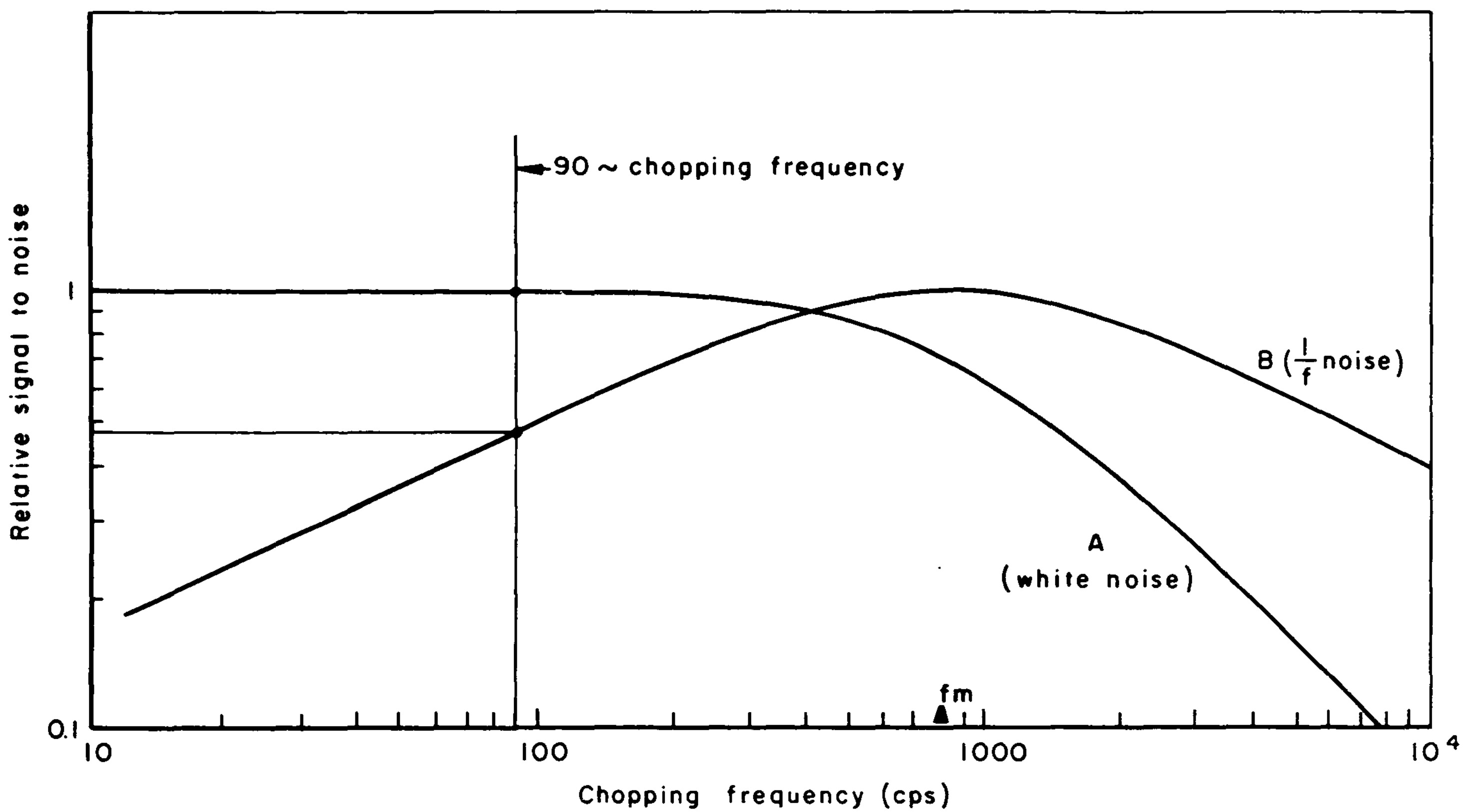

Fig 5 — Variation of signal to noise ratio with chopping frequency understates the peak of detector B by the approximate factor $\left(\frac{f_m}{2 \times 90}\right)^{1/2}$ for $f_m >> 90$ c.p.s. Stated another way, for the 90 c.p.s. S/N to be an order of magnitude low, $f_m = 18$ kc corresponding to a physical time constant of 9 microseconds. It is extremely unlikely that any detector exists with a true 1/f spectrum and a 9 μs time constant. More likely, a 9 μs detector might display a 1/f spectrum below a few kc but would have a relatively flat noise spectrum above, say, 5 kc. The conclusion is that the 90 c.p.s. measurement is likely to be unfair by no more than a factor of about 3 for the majority of PbS detectors and will be a somewhat better indication of performance for other photoconductors.

VI. RESULTS FOR PHOTOCONDUCTORS

Data on a large number of photoconductors, reduced to the $(\tau, P_{min}(\tau))$ form, are plotted in Figs. 6 and 7. The points corresponding to individual detectors are not plotted separately, but instead a line is drawn around all the points belonging to a given detector type and temperature of operation. There is a considerable amount of overlapping of these areas which is not surprising. The best samples of cooled PbS and PbTe are seen to be about an order of magnitude poorer than the 300° K background limits for an ideal quantum detector. The bulk of the samples are perhaps two orders of magnitude poorer while some are even three orders of magnitude from the appropriate limits.

The shape of the areas is interesting. Using a little imagination, one can say that the detectors of a given type cluster about a line whose slope is slightly greater than unity (Fig. 7). It has been pointed out earlier that the 90 c.p.s. measurement discriminates against the shorter time constant 1/f detectors. Had this factor been accounted for, the slope would have been closely equal to unity, in other words, minimum detectable power is traded directly with time constant. In essence, this is what Jones means by a Class II detector.

The scale was purposely chosen to be small to de-emphasize the details. There is nothing particularly fundamental about the precise extent of each area--the sample size was uncontrolled, the detectors were manufactured over a number of years from about 1950 to 1956 during which time many improvements in all infrared photodetectors were made. Undoubtedly, the best modern detectors of each type would fall closer to the background limits and one would also expect present production detectors to show less variability and better average performance than the detectors plotted here. The importance of Figs. 6 and 7 is that the domain of operation of present infrared detectors is shown with respect to absolute limits.

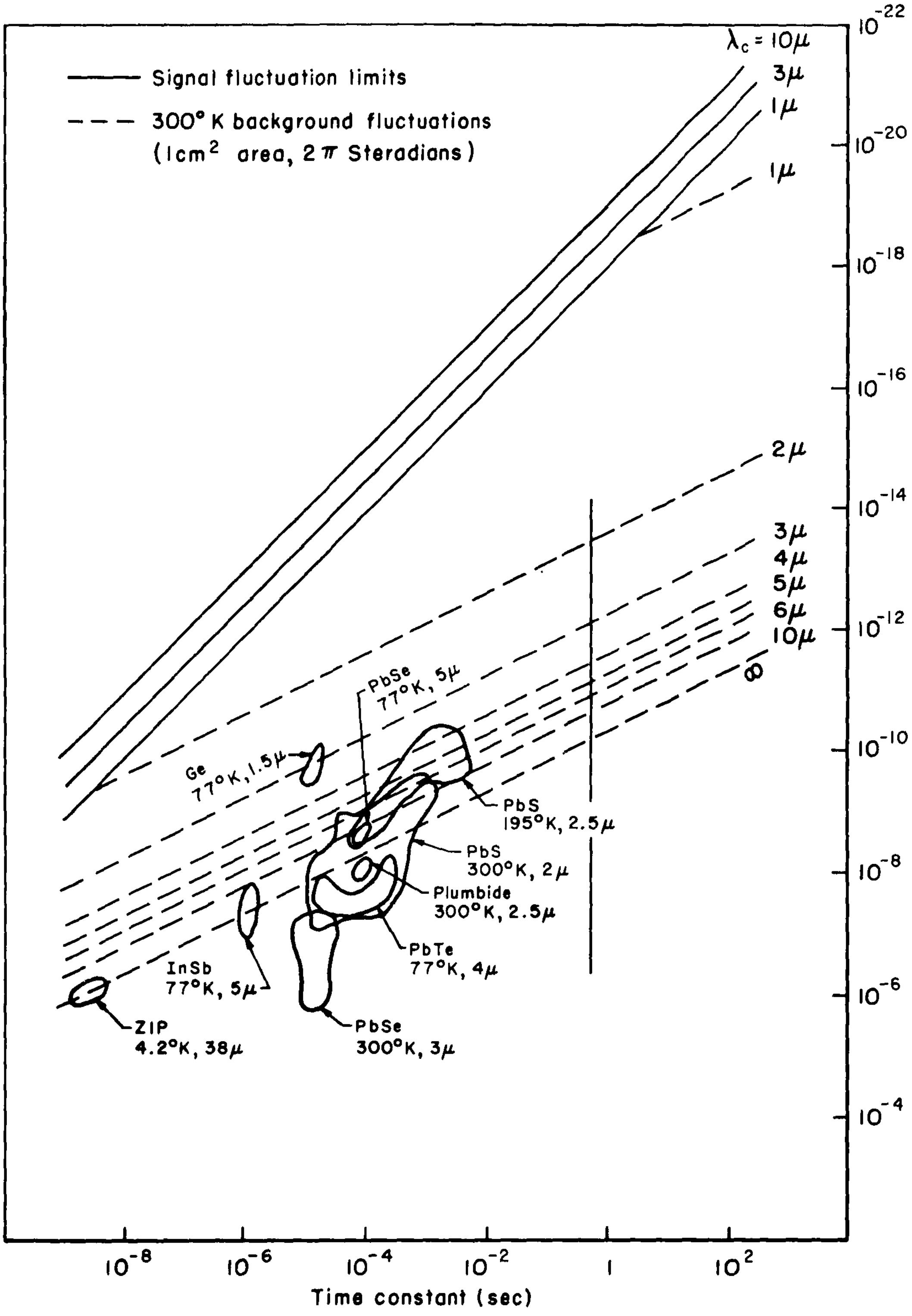

Fig. 6 — Performance of various photoconductors for monochromatic radiation

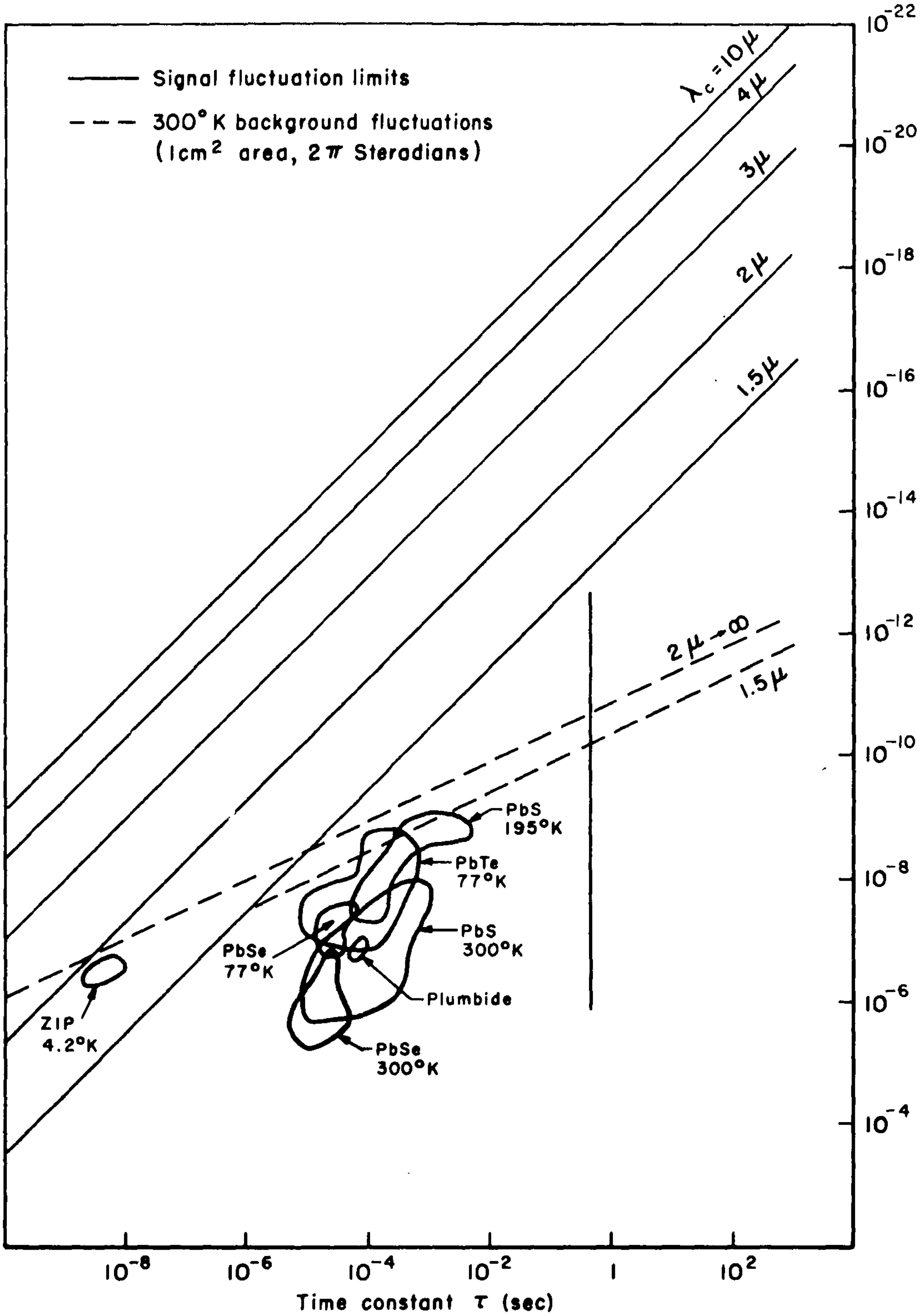

Fig. 7 — Performance of various photoconductors for a 500°K black body source

VII. EXTENSION TO VERY LONG WAVE LENGTHS

The preceding derivations for the performance limits of ideal detectors were based on the assumption of a random arrival of quanta. As mentioned earlier, photons obey Bose-Einstein statistics instead of classical statistics and there are situations when the fluctuations are considerably greater than in a random process. It can be shown that if n_ν quanta in the frequency interval $d\nu$ are incident upon a detector per sec the mean square fluctuation in n_ν (per sec) is given by:

$$\overline{\Delta n^2} = n_\nu \left(1 + \frac{1}{\exp \frac{h\nu}{kT} - 1}\right)$$

Thus deviations from randomness begin to occur when the parameter $x = \frac{h\nu}{kT}$ becomes of the order unity and become very significant for $x \ll 1$. Figure 8 (taken from Ref. 4) is a plot of both $\overline{\Delta n^2}$ and n_ν vs. x for black body radiation. Although n_ν approaches zero as x approaches zero, Δn^2 remains constant. This is a rather surprising result and is in marked contrast to the behavior that is ordinarily expected of quanta. As Fellgett has pointed out, this increased fluctuation in principle allows temperature to be obtained by a measurement of $\overline{\Delta n^2}$. Kittel has an apt description of the situation at small values of x when he says that "photons like to travel in packs."[7]

From the standpoint of the Nyquist theorem, the constancy of $\overline{\Delta n^2}$ is not particularly remarkable since the noise power per unit bandwidth is also independent of ν and equals kT.*

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*Actually, Nyquist recognized that deviations from a uniform noise spectrum would exist at very high frequencies. See Ref. 8.

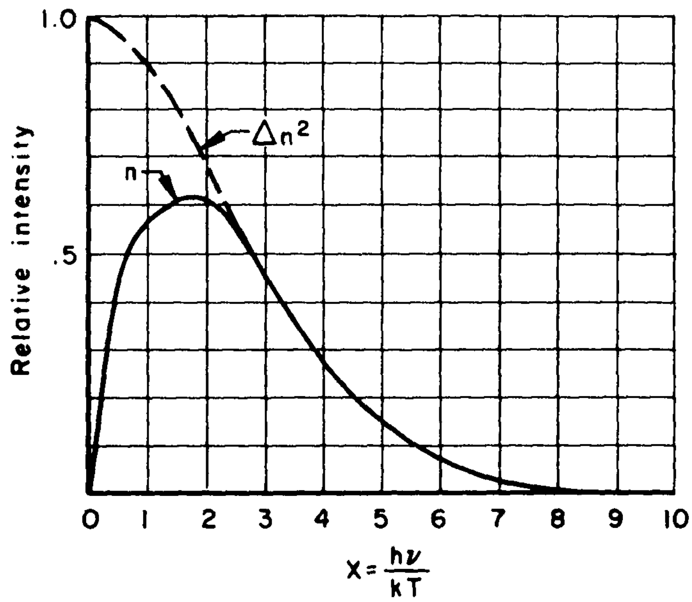

Fig. 8 — Photon spectral density and mean square fluctuation vs X (Taken from Fellgett Ref. 4)

The fluctuations in background radiation can be related to the electrical fluctuations in a resistance by means of an antenna. At first look, it might be supposed that a paradox exists since the constancy of $\overline{\Delta n^2}$ implies that $\overline{\Delta E^2}$, the mean square fluctuation in energy per second of the photon flux, varies as ν^2 (i.e., $\overline{\Delta E^2} = h^2\nu^2\Delta n^2$) rather than being independent of ν. The paradox is resolved when it is noted that the cross section of an antenna varies as λ^2 or $\frac{1}{\nu^2}$ which exactly compensates for the fall off in energy of the photon background at low ν's.

An interesting fact emerges from a study of radiation fluctuations: at microwave frequencies the fluctuation in the background energy is of the same order as the background energy itself (as seen by an antenna); in the quantum domain the fluctuation in energy is far less than the background. To prove this it is convenient to start with the black body photon spectral density:

$$n_\nu = \frac{2\pi}{c^2} \frac{\nu^2 d\nu}{\exp \frac{h\nu}{kT} - 1}$$

For x very small:

$$n_\nu = \frac{2\pi}{c^2} \frac{kT}{h} \nu d\nu$$

The average background energy per sec falling on a collecting area A becomes

$$\bar{E} = \frac{2\pi}{c^2} A\nu^2 kTd\nu = \frac{2\pi}{\lambda^2} A\, kTd\nu$$

It can be shown that the cross section of any antenna for background radiation (averaging over all angles of incidence and considering the fact that an antenna accepts only one polarization) is $\frac{\lambda^2}{2\pi}$.[4] This means that the electrical power delivered to a resistor matched to the antenna is given by the incident radiation power density multiplied by $\frac{\lambda^2}{2\pi}$. The average electrical power collected by the antenna due to the background radiation is then $\overline{W}_N = kTd\nu$, which is also the result of applying Nyquist's theorem.

For x very small the mean square fluctuation per sec of the background radiation energy $\overline{\Delta E^2} = h^2\nu^2\overline{\Delta n^2}$ becomes:

$$\overline{\Delta E^2} = \frac{2\pi\nu^2}{c^2} k^2T^2 d\nu \qquad \text{(per unit area) per sec}$$

The antenna transforms energy fluctuations in the radiation to energy fluctuations in the antenna circuit. Thus

$$\overline{\Delta W^2} = k^2T^2 d\nu \qquad \text{(per sec)}$$

To convert a fluctuation from a time to a bandwidth viewpoint, we multiply $\overline{\Delta W^2}$ by $2d\nu$ hence:

$$\overline{\Delta W^2} = 2\,k^2T^2(d\nu)^2$$

$\overline{\Delta W^2}$ is the mean square deviation of the noise energy from the average value $\overline{W}_N$. Fellgett shows (Ref. 4) that $\overline{\Delta W^2} = 2\,\overline{W}_N$ hence:

$$\overline{W}_N = kTd\nu$$

(W_N is the instantaneous noise energy and equals $\frac{V^2}{R}$, where V is a voltage in the antenna circuit.) Thus at microwave frequencies (and at ordinary temperatures*) the fluctuations in the background energy are seen to be $\sqrt{2}$ times the average background.

In the quantum domain we have random behavior, hence $\overline{\Delta n^2} = n_\nu$ and

$$\overline{\Delta E^2} = h^2\nu^2\overline{\Delta n^2} = h^2\nu^2 n_\nu = h\nu\bar{E}$$

where $\bar{E}$ is the average background energy. Since $\sqrt{\overline{\Delta E^2}} \ll \bar{E}$ it is possible to use "space filtering," i.e., chopping techniques that allow relatively weak targets of small angular extent to be detected against a large steady background.

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*At very low temperatures even the microwave frequencies might be in the "quantum domain." The important parameter is of course $x = \frac{h\nu}{kT}$ which must be very much less than unity for the above to hold.

VIII. THE IDEAL THERMAL DETECTOR

Using the above arguments for energy fluctuations, we can now indicate the method used in Ref. 2 to derive the performance limits of an ideal thermal detector. The energy fluctuation in $d\nu$ is:

$$(\overline{\Delta E^2})_\nu = h^2\nu^2\,\overline{\Delta n^2} = h^2\nu^2 n_\nu \left(1 + \frac{1}{\exp\frac{h\nu}{kT} - 1}\right) = \frac{2\pi}{c^2}\,\frac{h^2\nu^4 \exp\frac{h\nu}{kT}\,d\nu}{\left(\exp\frac{h\nu}{kT} - 1\right)^2}$$

Since fluctuations at each frequency are all independent the total mean square energy fluctuation is the integral of the above expression over all ν.

$$\overline{\Delta E^2} = \frac{2\pi}{c^2}\,\frac{k^5T^5}{h^3}\int_0^\infty \frac{x^4 e^x dx}{(e^x - 1)^2}$$

Evaluating the integral, substituting for σ its equivalent $\frac{2\pi^5 k^4}{15c^2h^3}$, and then expressing the fluctuation on a bandwidth basis one obtains:

$$\overline{\Delta E^2} = 8\,k\,T^5\,\sigma\,\Delta f$$

for radiation falling on a detector of unit area and emissivity. As the detector itself reradiates randomly the fluctuation is doubled and the final expression becomes

$$\overline{\Delta E^2} = 16\,A\,\sigma\,k\,T^5\,\Delta f\ .$$

IX. REAL THERMAL DETECTORS

To complete our survey of detector performance we have plotted in Figs. 9 and 10 the $(\tau, P_{min}(\tau))$ limits for various types of thermal detectors. The data used were taken from various sources[1,9-12] but no attempt was made to plot only the best samples of each detector type. Good thermocouples are less than an order of magnitude from the ideal thermal detector limit; the best Golay cells are within a factor of 3 from this limit. Thermistors with time constants less than 10^{-2} secs are about two orders of magnitude poorer in performance than an ideal detector. The superconducting bolometer is seen to combine a close approach to the ideal with a short ($\sim$1 millisec) time constant.

When comparing detectors on their ability to detect monochromatic radiation, Fig. 9 shows that photoconductors are considerably superior to thermal detectors, especially at the shorter wave lengths. For detection of 500°K black body radiation the situation is changed: a good thermal detector is seen to compare rather favorably with a good photoconductor (Fig. 10).

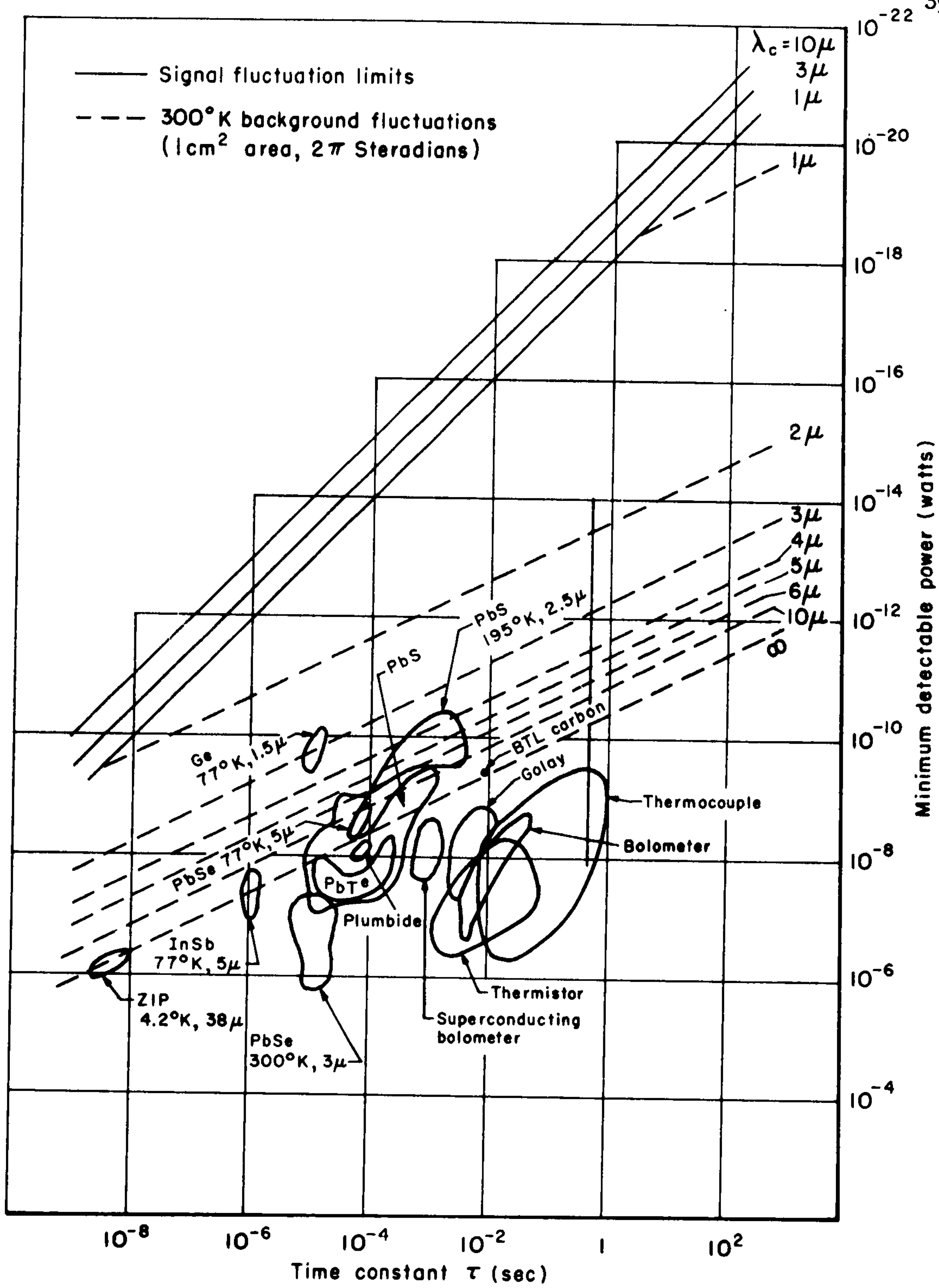

Fig. 9 — Performance of various photoconductors and thermal detectors for monochromatic radiation

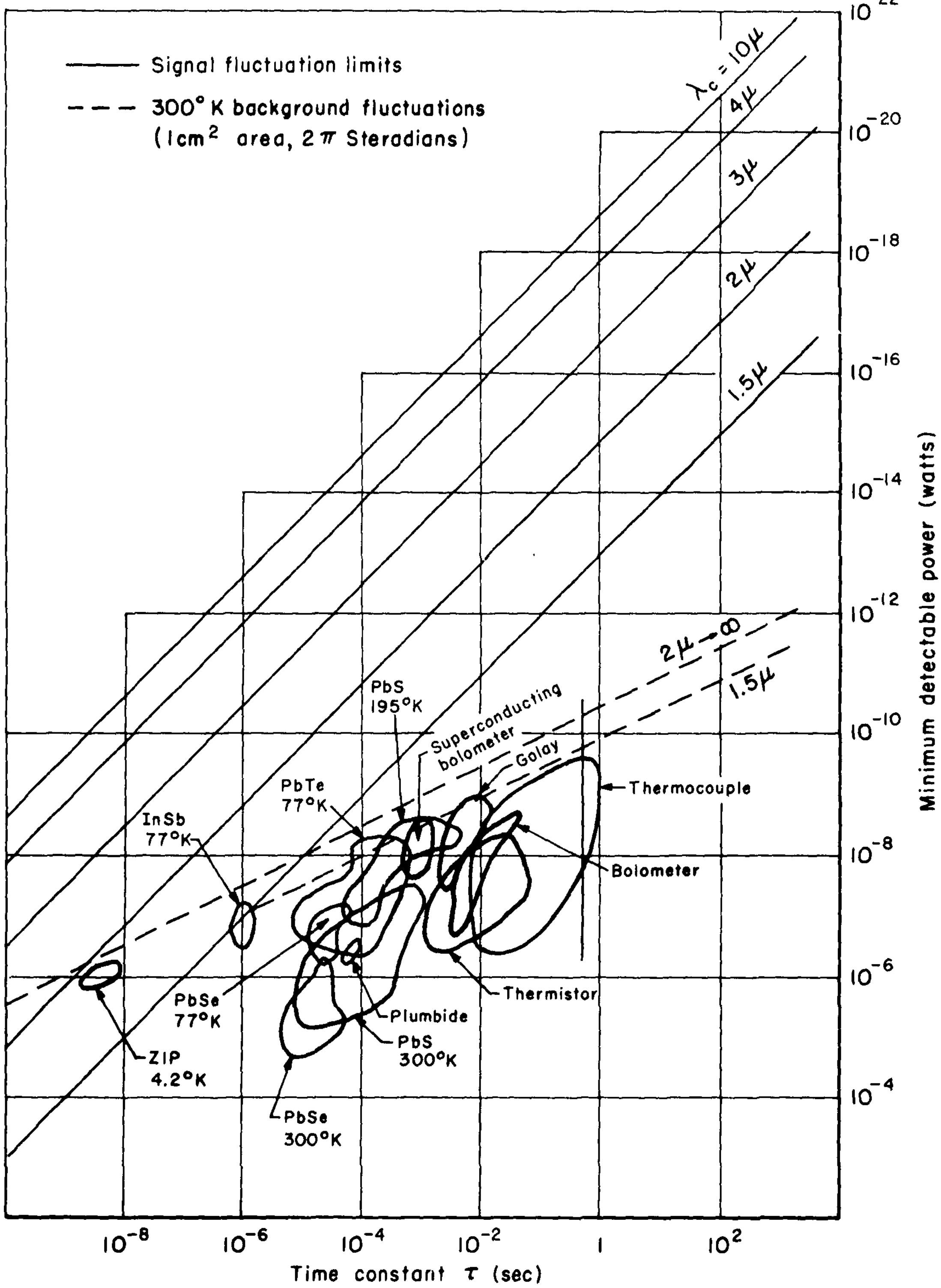

Fig. 10 — Performance of various photoconductors and thermal detectors for a 500°K black body source

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