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**THE APPARENT TEMPERATURE OF ISOLATED OBJECTS**

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## NOTICES

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# ABSTRACT

The emission and absorption cross-sections of an arbitrary object are defined in terms of its scattering cross-section, and the principle of detailed balance is then used to derive Kirchhoff's radiation law for the object, taking into account both the polarization of the emitted and absorbed radiation, and the orientation of the object. The result is used to calculate the apparent brightness temperature of an object in an arbitrary radiation field. Finally a simple formula for the change in antenna temperature caused by introducing the object into the beam of a high-gain microwave radiometer antenna is found.

TABLE OF CONTENTS

# THE APPARENT TEMPERATURE OF ISOLATED OBJECTS

## 1. INTRODUCTION

Radiometer systems (of which a radio telescope is a common example) are sometimes used at microwave frequencies to measure the apparent temperature of isolated objects. For example, it was suggested in Reference 1 that a sensitive radiometer might be used to determine the presence of a heated object at high altitudes. Conversely, the presence of a large number of objects in the beam of a radio telescope might interfere with its performance; Lilley (Reference 2) has recently estimated the changes in antenna temperature that might be expected from the "West-Ford" dipole belt.

If the objects under consideration are large in terms of the operating wavelength of the radiometers, as is usually true in the infrared region, the apparent temperature can usually be related to a surface emissivity and the surface area of the object. For small objects, however, the concept of surface emissivity breaks down, and it is necessary to use some parameter which takes account of the radiating properties of the body as a whole. Since the scattering properties of such bodies are often expressed in terms of a scattering or absorption cross section, it seems natural to introduce a corresponding emission cross section for the thermal energy radiated by the body (see, for example, Reference 3, p. 452).

In this report, the emission cross section for isolated objects will be defined, and the proper form for Kirchhoff's law, i.e., the relation between the emission and the absorption cross sections, will be derived. As a by-product of the derivation, it will be shown that the emission cross section can be written in terms of the scattering cross sections, which are usually known, or can easily be measured, at microwave frequencies. Finally, an expression will be found for the apparent temperature of the radiation coming from an isolated object illuminated by an arbitrary radiation field. From this apparent temperature, the change in antenna temperature of an operating radiometer system can easily be found.

The relations between the reciprocity principle, the principle of detailed balance, and the proper form of Kirchhoff's law have been previously discussed in great detail by S. M. Rytov,[4] and some of the results given here have previously been derived by Rytov, Levin,[5] and others. However, we have given an alternative derivation of Kirchhoff's law in some detail because of the insight it gives into the problem of calculating apparent temperatures under nonequilibrium conditions. The main difference between the approach taken in this report and that of References 4 and 5 is that here the entire calculation is made in terms of the scattering coefficients of the body, rather than by introducing expansion coefficients or Green's functions. This has the practical advantage of allowing temperature calculations to be made in terms of easily measured properties of the object, at the cost of a more restricted range of validity (the body must be at a uniform temperature, and distances must be large enough for far-field approximations to hold).

## 2. THE SCATTERING PROPERTIES OF AN ISOLATED OBJECT

The first step in the program of determining the thermal radiation from an isolated object is to define the scattering properties of the object for monochromatic radiation, and to express the forward-scattering theorem in terms of them. For this purpose, we shall follow the procedure of Bolljahn and Lucke (Reference 6). Consider a monochromatic plane wave, with polarization state denoted by the vector $\vec{P}_{c1}$, incident on an isolated body from the direction $\theta_c$, $\phi_c$ (see Fig. 1). In Reference 6, linear polarization is used, i.e., the incoming wave may be in one of two states, $\vec{P}_{c1} = \vec{\theta}_c$ or $\vec{P}_{c2} = \vec{\phi}_c$, where $\vec{\theta}_c$, $\vec{\phi}_c$ are unit vectors. In general, however, any pair of orthogonal polarization states may be used; for example,

$$\vec{P}_{c1} = (\vec{\theta}_o + i\,\vec{\phi}_o)/\sqrt{2} \quad \text{and} \quad \vec{P}_{c2} = (\vec{\theta}_o - i\,\vec{\phi}_o)/\sqrt{2}$$

represent circularly polarized waves. The incident plane wave may now be written

$$\vec{E}^i = \vec{P}_{c1}\, E_o\, e^{i\,\vec{k}_c \cdot \vec{r}} \tag{1}$$

where $E_o$ is the amplitude and $\vec{k}_c$ is the vector of magnitude $k = 2\pi/\lambda$ in the direction of propagation. We have assumed that the incident wave is in polarization state (1).

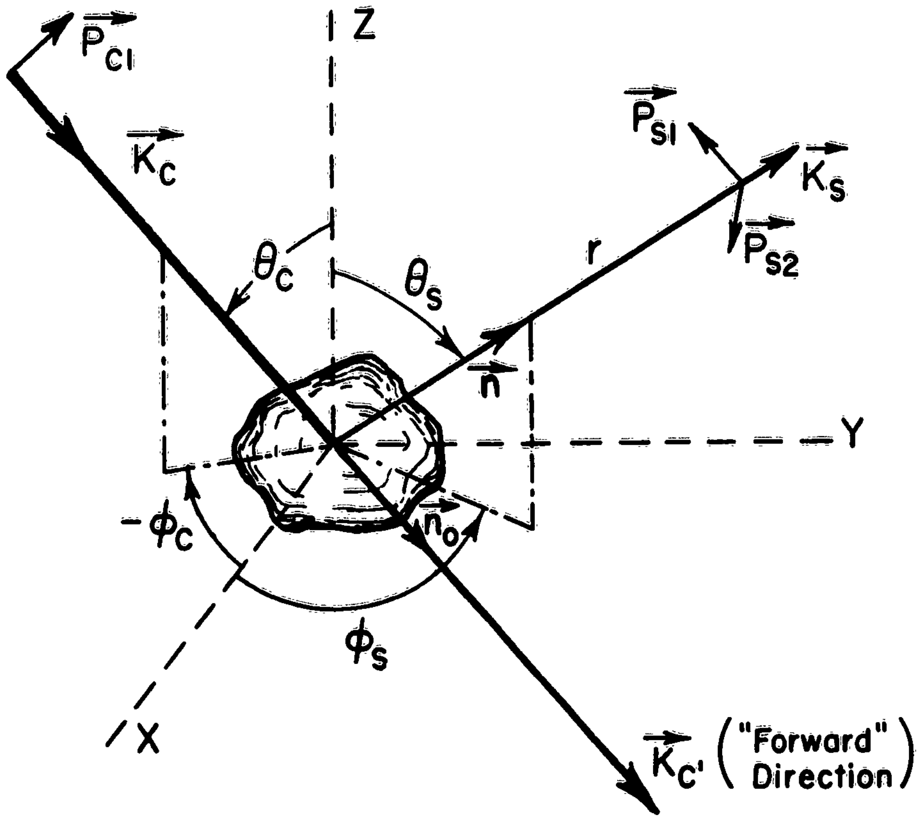



Fig. 1. Geometry of the scattering process.

The scattered wave in the direction ($\theta_s$, $\phi_s$) will also have a definite polarization state which can be resolved into two orthogonal components represented by unit vectors $\vec{P}_{s1}$ and $\vec{P}_{s2}$, so that the scattered wave itself can be written

$$\vec{E}^s = \vec{E}_1^s + \vec{E}_2^s \tag{2}$$

$$= E_o \left[ \frac{f_{11}(c, s)}{kr} \vec{P}_{s1} + \frac{f_{12}(c, s)}{kr} \vec{P}_{s2} \right] e^{i \vec{k}_s \cdot \vec{r}} .$$

Here the $f_{ij}(c, s)$ are called the scattering amplitudes: the subscripts i, j refer to the polarization states of the incident and scattered radiation respectively; the variables (c, s) refer to the directions $\theta_c \phi_c$ and $\theta_s \phi_s$ of the incident and scattered radiation. It is a consequence of the Lorentz reciprocity condition that

$$f_{ij}(c, s) = f_{ji}(s, c) \,. \qquad \begin{array}{l} i = 1, 2 \\ j = 1, 2 \end{array} \tag{3}$$

The total field at any point is then the sum of the incident and scattered fields, or $\vec{E}^t = \vec{E}^i + \vec{E}^s$. The several cross sections of interest may now be defined, following Reference 6.

i. <u>The Absorption Cross Section</u>

$a_i(c)$, for an incident plane-wave from the direction (c) with polarization state (i) is defined as the net flow of energy into a large sphere surrounding the target, divided by the energy density in the incident wave:

$$a_i(c) \equiv - \frac{\mathrm{Re} \int (\vec{E}^t \times \vec{H}^{t\,*}) \cdot \vec{n} \, dS}{\mathrm{Re}\, (\vec{E}^i \times \vec{H}^{i\,*}) \cdot n_o} \,. \tag{4}$$

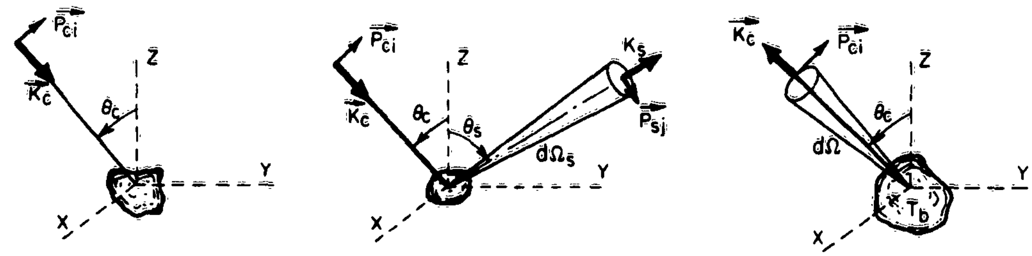
Here $\vec{H}^i$ and $\vec{H}^t$ are the magnetic fields associated with $\vec{E}^i$ and $\vec{E}^t$; $\vec{n}$ and $\vec{n}_o$ are unit vectors in the radial direction and the propagation direction of the incoming wave, respectively (see Fig. 2a).

ii. <u>The Differential Scattering Cross Section</u>

$\partial\sigma_{ij}(c, s)/\partial\Omega$, for a plane wave incident from the $c$ direction with polarization $i$, scattered into the $s$ direction with polarization $j$, is defined as the ratio of the scattered power of the appropriate polarization flowing outward through a unit solid angle, , divided by the energy density of the incident wave:

$$\frac{\partial\sigma_{ij}(c, s)}{\partial\Omega} \equiv \frac{\mathrm{Re}\, (\vec{E}^s_j \times \vec{H}^{s\,*}_j) \cdot \vec{n} \; r^2}{\mathrm{Re}\, (\vec{E}^i \times \vec{H}^{i\,*}) \cdot \vec{n}_o} \,. \tag{5}$$

(It is implied that the incident field $\vec{E}^i$ has polarization state $i$; see Fig. 2b.)




(a) Absorption Cross-Section. Power Absorbed by (Dissipated Within) Body Is $I_0 a_i(c)$ Watts

(b) Differential Scattering Cross-Section. Power Scattered Into $d\Omega'_s$ With Polarization State (j) Is $I_0 \frac{\partial \sigma_{ij}}{\partial \Omega}(c,s)\, d\Omega$ Watts

(c) Emisson Cross-Section. Power Emitted Into $d\Omega$ With Polarization State (i) Is $K T_b \lambda^{-2} d\Omega e_i(c) \Delta f$ Watts

Fig. 2. Power relations for the several cross sections. It is assumed that the incident wave has intensity $I_0$ watts/meter$^2$ and polarization state (i) from the ($\theta_c$) direction. The emitted wave in (2c) goes into the ($\theta_c$) direction.

iii. <u>The Total Scattering Cross Section</u>

$\sigma_i(c)$, for a plane wave incident from the (c) direction with polarization (i) is defined as the total power scattered into all directions, divided by the energy density in the incident wave,

(6a) $$\sigma_i(c) \equiv \frac{\int \mathrm{Re}[\vec{E}^s \times \vec{H}^{s\,*}] \cdot \vec{n}\, r^2 \, d\Omega_s}{\mathrm{Re}[\vec{E}^i \times \vec{H}^{i\,*}] \cdot \vec{n}_o} \quad ;$$

and this is also equal to the integral of the differential scattering cross section,

(6b) $$\sigma_i(c) \equiv \int \left( \frac{\partial \sigma_{ij}}{\partial \Omega} + \frac{\partial \sigma_{ii}}{\partial \Omega} \right) d\Omega_s \quad .$$

It includes contributions from both polarizations of the scattered power.

iv. <u>The Total or Extinction Cross Section</u>

$t_i(c)$, is defined as the sum of the absorption and scattering cross sections,

(7) $$t_i(c) \equiv a_i(c) + \sigma_i(c) \quad .$$

v. <u>Relation Between Cross Sections and Scattering Amplitudes</u>

All of the cross sections just defined can be directly related to the scattering amplitudes $f_{ij}$. For example, it is easy to show (see, e.g., Reference 6) that the proper form of the well-known forward scattering theorem is

(8) $$t_i(c) = -4\pi\, k^{-2} \mathrm{Im}\,[f_{ii}(c, c')]$$

where c' is the "forward" direction; i.e., $\theta_{c'} = \pi - \theta_c$ ; $\phi_{c'} = \pi + \phi_c$, (see Fig. 1). From the definitions of the amplitudes and the differential scattering cross section, one has

$$(9) \qquad \frac{\partial \sigma_{ij}(c,s)}{\partial \Omega} = k^{-2} \; f_{ij}(c,s) \; f_{ij}^{*}(c,s)$$

whence, by integration,

$$(10) \qquad \sigma_i(c) = k^{-2} \int \left[ |f_{ij}(c,s)|^2 + |f_{ii}(c,s)|^2 \right] d\Omega_s \; .$$

Clearly, from Eqs. (8) and (10) the absorption cross section $a_i(c)$ can also be found from the scattering amplitudes.

## 3. KIRCHHOFF'S LAW

In order to specify the thermal power emitted by a body maintained at constant temperature, one further cross section, the emission cross section $e_i(c)$, must be introduced. This may be defined as the area of a black body surface, at the same temperature as the object, required to produce the same brightness in the direction (c), with polarization (i). See Fig. 2. It is assumed that the black body test surface radiates like an "ideal" black body even though $e_i << \lambda^2$.

Having defined the emission cross section, we now seek a relation between it and the absorption cross section; not unexpectedly, the two are equal,

$$(11) \qquad a_i(c) = e_i(c) \qquad .$$

Note that the polar angles $(\theta_c, \phi_c)$ which define direction (c) refer to the line from object to observer for $e_i(c)$, and from source to object for $a_i(c)$. Thus the propagation vectors $\vec{k}_c$ in Figs. 2a and 3 are shown in opposite directions. Similarly the polarization state i for the radiated power is that which would be absorbed by an antenna which produces polarization state i in the incident radiation.

Equation (11) is the formal expression of Kirchhoff's law. It may be seen that, because of Eqs. (7), (8), and (10), the emission cross section of a body can be calculated from the scattering amplitudes.

Although Kirchhoff's law is well known, at least in the form in which polarization effects are ignored, we wish to give a formal derivation of it, since the details of the proof offer considerable insight into the more general problem of calculating the apparent temperature

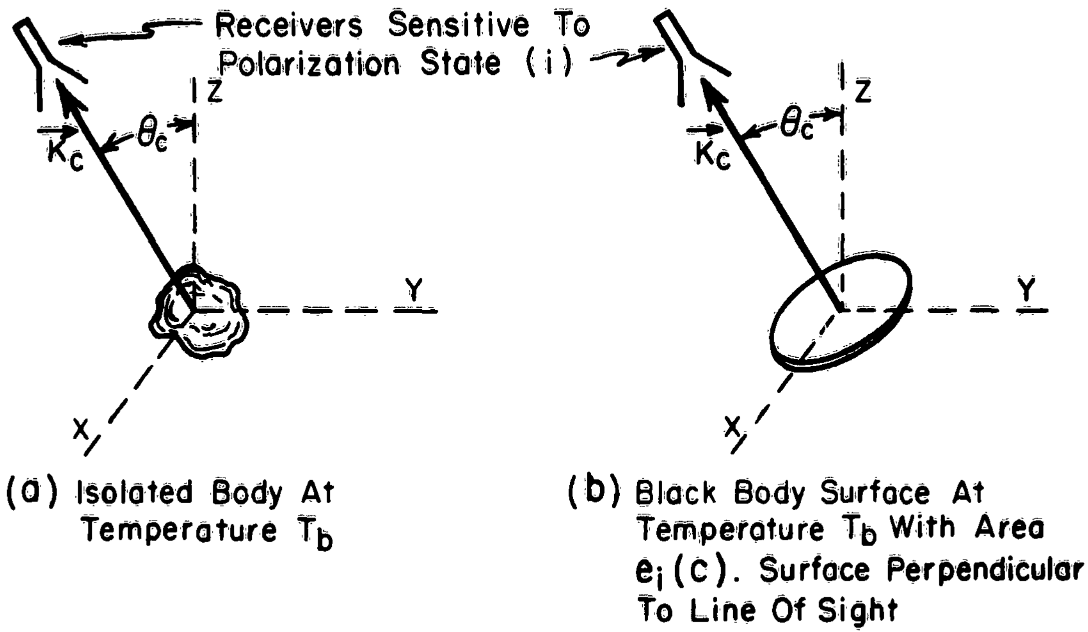



Fig. 3. The emission cross section of the body is $e_i(c)$ if the power in the two idential receivers is equal.

of a body under certain kinds of nonequilibrium conditions. The proof given here depends, as usual,[7] on the principle of detailed balance and the reciprocity theorem for the scattering coefficients.

Consider first a body in a very large isothermal enclosure, and consider the thermal radiation with propagation vectors lying within a solid angle $d\Omega_c$ in the direction (c) (see Fig. 4). If the spectral brightness of the black body radiation in the enclosure is $2B_o$ watts-meter $^{-2}$-Steradian$^{-1}$- cycle$^{-1}$ , i.e., if there is $B_o$ in each of two orthogonal polarization states, then the power density flowing towards the object from the range of angles $d\Omega_c$ is

(12) $$d\, I_{in} = B_o\, d\Omega_c\, \Delta f \text{ watts-meter}^{-2}$$

in each polarization, in the frequency interval $\Delta f$. This must be balanced[7] by the power flowing away from the object, within the same $d\Omega_c$. The outward-flowing power is the sum of three components.

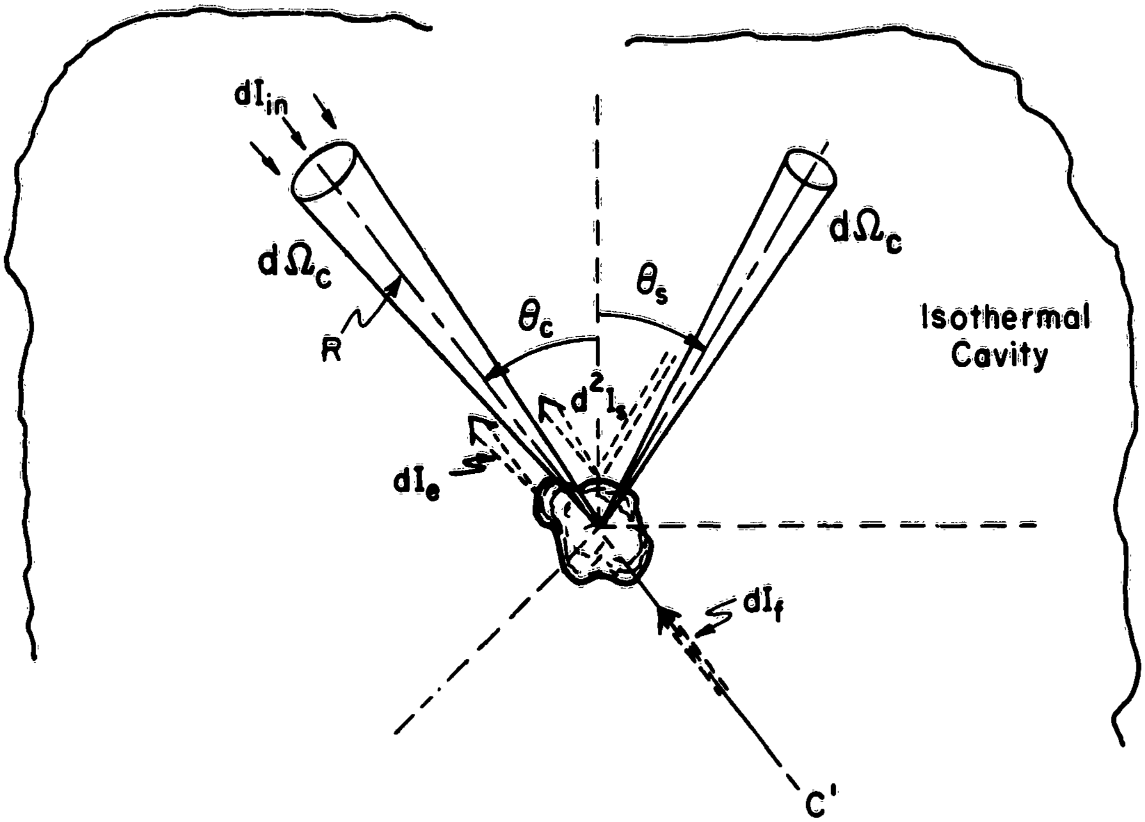



Fig. 4. Geometry for Kirchhoff's law.

i. The emission: the body emits $e_i(c)B_o\Delta f$ watts/steradian in the direction (c) with polarization (i). Thus

$$d\,I_e = \frac{e_i(c)B_o\,d\Omega_c}{R^2}\quad \Delta f \text{ watts-meter}^{-2} \tag{13}$$

flows out through the solid angle $d\Omega_c$.

ii. The "scattered" power: thermal radiation incident on the body from the range of solid angles $d\Omega_s$ is scattered into $d\Omega_c$. The power density at distance R from the object is

$$d^2 I_s = B_o\left[\frac{\partial\sigma_{ii}(s,c)}{\partial\Omega} + \frac{\partial\sigma_{ji}(s,c)}{\partial\Omega}\right]\frac{[d\Omega_s\,d\Omega_c\,\Delta f]}{R^2}\,. \tag{14}$$

Thus the total power scattered into $d\Omega_c$ is just the integral of $d^2 I_s$ over all $d\Omega_s$. Making use of the reciprocity theorem (Eq. (3)), and the definition of the total scattering cross section (Eq. (6b)), one finds that

$$dI_s = B_o \frac{d\Omega_c}{R^2} \sigma_i(c, s) \Delta f \text{ watts/meter}^2 \tag{15}$$

is the power scattered into $d\Omega_c$ with polarization (i), caused by radiation of both polarizations falling on the object.

iii. The "forward"-scattered energy: the evaluation and interpretation of the third contribution is less straightforward. It is caused formally by the "scattered" energy originating from the range of solid angles around the direction c' opposite $d\Omega_c$ (see Fig. 4). It will turn out that this contribution is effectively a negative one; that is, the body blocks out sufficient radiation from the (c') direction to account for its extinction cross section. To evaluate this contribution, consider a plane wave incident on the body from (c') with propagation vector $\vec{k}$ nearly parallel to the "blocked" direction $k_{c'}$ (see Fig. 5). By "blocked" direction we refer here to the direction from which the radiation would have to come to place the receiver P in the forward scattering zone. The direction $\vec{k}$ can now be specified by the point Q at which it crosses a plane $(\xi, \eta)$ perpendicular to $\vec{k}_{c'}$ at some large distance $R'$ from the object. For convenience, define two angles $\psi = \tan^{-1} \xi/R'$ and $\chi = \tan^{-1} \eta/R'$. Then for $\psi << 1$ and $\chi << 1$, the vector $\vec{k}$ can be expanded as

$$\vec{k} = [\vec{k}_{c'} + k(\vec{\xi}_o\psi + \vec{\eta}_o\chi)]/[1 + \psi^2 + \chi^2]^{1/2} \tag{16}$$

where $\vec{\xi}_o$ and $\vec{\eta}_o$ are unit vectors, and $|\vec{k}| = |\vec{k}_{c'}| = k$. Now an incident plane wave $\vec{P}_i E_o \exp(i\vec{k} \cdot \vec{R})$ with amplitude $E_o$ and polarization state $\vec{P}_i$ gives rise to a field in the vicinity of P that may be written as

$$\vec{E}(P) = \vec{P}_i E_o \left[ e^{i\vec{k} \cdot \vec{R}} + \frac{f_{ii}(c', c)}{kR} e^{i\vec{k}_{c'} \cdot \vec{R}} \right] + \vec{P}_j E_o \left[ \frac{f_{ij}(c', c)}{kR} e^{ik_{c'} \cdot R} \right] \tag{17}$$

since $\vec{k}_{c'}$ is the direction from the object to P. Thus the power density at P is

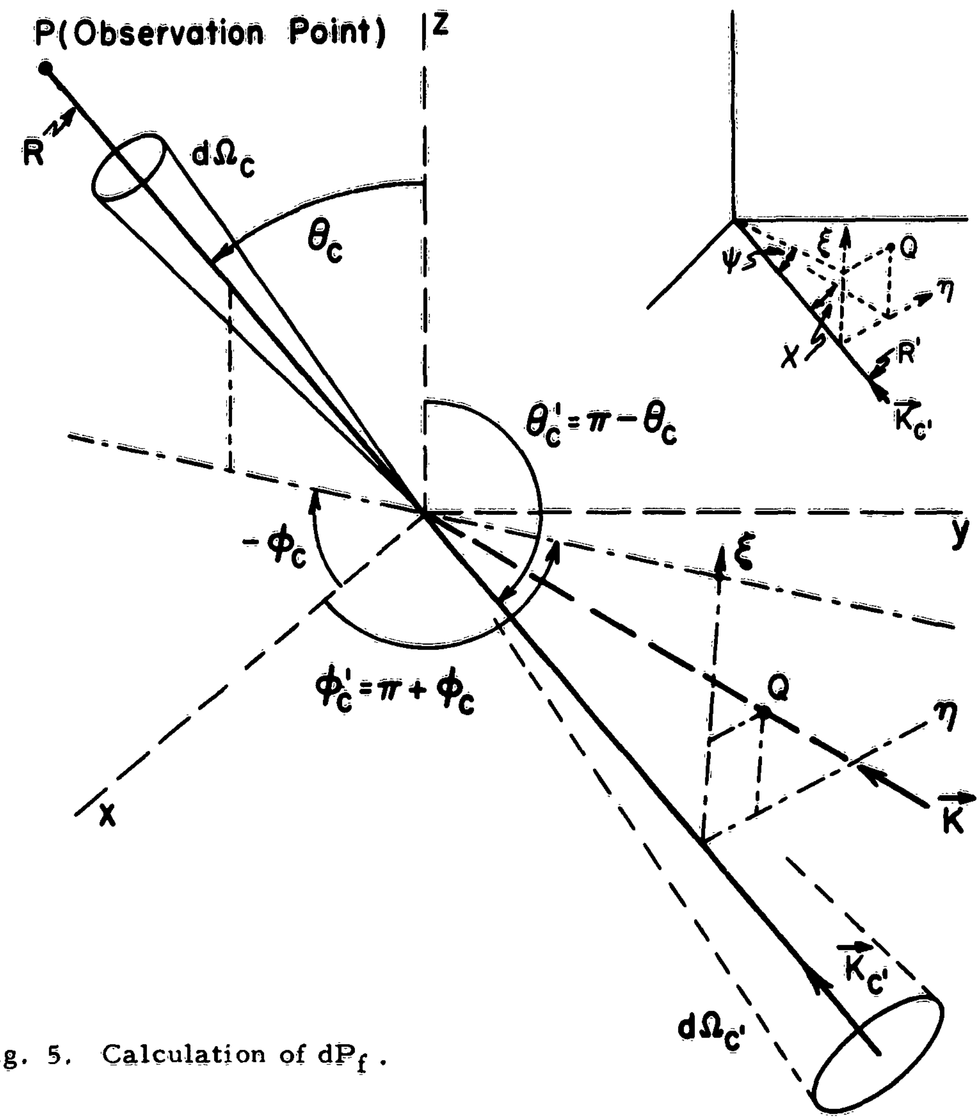



Fig. 5. Calculation of $dP_f$.

(18)
$$d^2 I_f = \overline{\vec{E}(P) \times \vec{H}(P)^*}$$
$$= Y_o |E_o|^2 \left[ 1 + 2\,\mathrm{Re}\, \frac{f_{ii}(c', c)}{k R} e^{i(\vec{k}_{c'} - \vec{k}) \cdot R} + \frac{|f_{ii}|^2 + |f_{ij}|^2}{k^2 R^2} \right]$$

where $Y_o$ is the admittance of free space. Since this is the power density at P caused by a single plane wave from direction $\vec{k}$, the total power density at P caused by black body radiation from the cone $d\Omega_c$ can be found by integrating over $\psi$ and $\chi$. This is simplified by observing, from Eq. (16), that

$$(19)\qquad (\vec{k}_{c'} - \vec{k}) \cdot \vec{R} \cong \frac{kR}{2} (\psi^2 + \chi^2).$$

Now if Eq. (18) is integrated over $d\Omega_{c'}$, assuming that the contributing plane waves all have the same intensity $Y|E_o|^2$, and are uniformly distributed in direction, then the first term is $Y|E_o|^2 \int d\chi\, d\psi = Y|E_o|^2\, d\Omega_{c'}$. This corresponds to the thermal radiation that would be falling on P from $d\Omega_c$ if the scatterer were not there. Thus $E_o$ must be chosen so that $Y|E_o|^2\, d\Omega_{c'} = B_o\, d\Omega_{c'}/\Delta f$. Consequently the total power density at P becomes

$$(20)\qquad d\, I_f = \int d^2\, I_f\, d\chi\, d\psi$$

$$= B_o \Delta f\, d\Omega_{c'} + B_o \Delta f \left(\frac{2}{kR}\right) \iint_{d\Omega_{c'}} \mathrm{Re}\, f_{ii}(c', c)\, e^{\frac{ikR}{2}(\psi^2 + \chi^2)}\, d\psi\, d\chi .$$

Now the limits on $\psi$ and $\chi$ may be replaced by $\pm\infty$, if $kR$ is sufficiently large (see, for example, Reference 3, p. 30); thus

$$(21)\qquad dI_f = \left\{ B_o\, d\Omega_{c'} + B_o \frac{4\pi}{k^2 R^2} \mathrm{Re}[i\, f_{ii}(c', c)] \right\} \Delta f$$

or, using the forward-scattering theorem,

$$(22)\qquad dI_f = \left( d\Omega_{c'} - \frac{t_i(c)}{R^2} \right) B_o\, \Delta f \qquad .$$

In getting Eqs. (20) and (21) from Eq. (18), the secord-order term in $(f^2 / kR)^2$ have been neglected.

Thus the thermal radiation reaching P from the directions in the vicinity of $\vec{k}_f$ is diminished by an amount sufficient to account for the extinction cross section of the body. We may now use the principle of detailed balance in the form

$$dI_{in} = dI_f + dI_s + dI_e$$

or

$$(23) \qquad B_o \Delta f \, d\Omega_c{}' = B_o \left[ d\Omega_c{}' + \frac{e_i + \sigma_i - t_i}{R^2} \right] \Delta f \quad .$$

Substituting from Eq. (7) the definition for $t_i(c)$, we find that there is a power balance only if $e_i(c) = a_i(c)$, which is Kirchhoff's law.

## 4. THE APPARENT BRIGHTNESS TEMPERATURE OF AN ISOLATED BODY

We are now in a position to calculate the apparent brightness temperature (defined by Eq. (24)) of an isolated body which is maintained at a constant temperature when it is placed in an arbitrary thermal noise field. That is, the body is at a constant, uniform temperature, but it is not in thermodynamic equilibrium with its surroundings. It will be assumed that the temperatures and wavelength regions of interest are those in which the Rayleigh-Jeans law is valid. In that case, $B_o = kT\lambda^{-2}$, so that the total brightness temperature of the radiation coming from several sources is simply the sum of the brightness temperatures of the individual sources.

The situation we wish to consider is that of an observer in a field of thermal radiation who is to measure the brightness temperature of the radiation coming from a given direction before and after an object, at a large distance R, is introduced into the line of sight (see Fig. 6a). Operationally, the measurement could be made by a radiometer attached to a high-gain antenna with effective aperture A.

That is, the quantity to be measured is the power $dP_i(o)$ passing through an area A (perpendicular to the line of sight) coming from the range of solid angles subtended by the antenna beam $d\Omega$. The antenna is assumed to be sensitive only to radiation of polarization state (i). If the incoming radiation has brightness temperature $T_i(o)$, then $dP_i(o) = kT_i(o) \, A \, \Delta f \, d\Omega / \lambda^2$ watts. Conversely, the temperature is given by

$$(24) \qquad T_i(o) = \left( \frac{\lambda^2}{k \, A \, \Delta f} \right) \frac{dP_i(o)}{d\Omega} \quad .$$

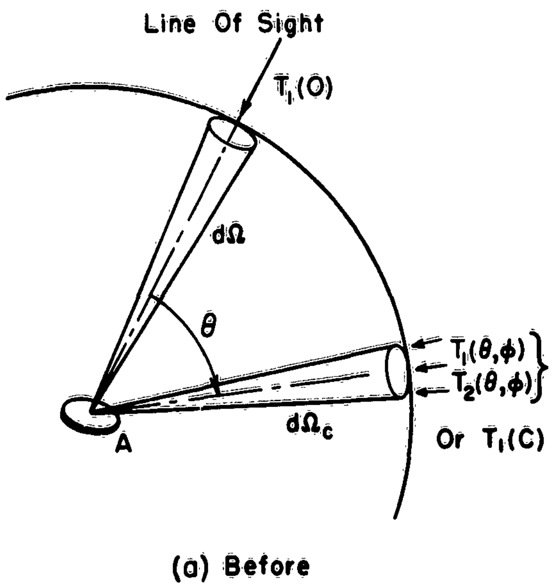



Fig. 6. The apparent temperature of an object.

For black body radiation, the true brightness temperature may be defined as the limit of Eq. (24) as $d\Omega$ approaches zero. When a finite object is introduced into a finite beam, then we say that Eq. (24) gives the apparent temperature of the radiation coming from within $d\Omega$ .

Now suppose that an object is placed in the beam at a distance R along the line of sight (see Fig. 6b). The object is illuminated by black body radiation with temperature distributions $T_i'(c')$ for the two orthogonal polarizations states (i.e., $i = 1, 2$; $c'$ represents the co-ordinates $\theta_c'$, $\phi_c'$ in a system with origin at the object; $T'$ represents a brightness temperature observed from the object).

After the object is introduced into the "beam" (see Fig. 6b), the power crossing A in polarization state (1) will be changed. (To avoid some confusion in interpreting subscripts, we have chosen to calculate the temperature in a specific state, labelled (1), in the remainder of this section.) Again, there are three contributions to the change:

(i) From the definition of the emission cross section, the body will radiate power

$$dP_e = \frac{k\, T_b}{\lambda^2} \; \epsilon_1(o) \; \frac{A\, \Delta f}{R^2} \tag{25}$$

across the surface A. Here $T_b$ is the physical temperature of the body, assumed to be at a uniform temperature throughout.

(ii) The radiation fields $T_i'(c')$ will be reflected by the body. The reflected power crossing A is

$$dP_s = \frac{k\, \Delta f}{\lambda^2 R^2} \int \left[ \frac{\partial \sigma_{12}(c', c)}{\partial \Omega} \, T_2'(c') + \frac{\partial \sigma_{11}(c', o)}{\partial \Omega} \, T_1'(c') \right] d\Omega_{c'} \tag{26}$$

which may be written

$$dP_s = \frac{k\, \Delta f}{\lambda^2 R^2} \; \langle \sigma\, T' \rangle_1 \tag{27}$$

where

$$\langle \sigma\, T' \rangle_1 = \sum_{k=1}^{2} \int \frac{\partial\, \sigma_{k1}(c', o)}{\partial \Omega} \, T_k'(c') \, d\Omega_{c'}$$

represents the average product of the bistatic scattering cross section and the temperature of the radiation illuminating object. It has been assumed, in deriving Eq. (26), that the two orthogonal components $T_1'(c')$ and $T_2'(c')$ are uncorrelated. If this is not true, a more complete treatment in terms of the Stokes parameters or the density matrix (see, for example, Reference 8) is necessary to find $dP_s$.

(iii) The radiation from the "blocking" direction is modified by the presence of the scatterer. Following the procedure leading to Eq. (22), it is clear that this contribution to the power crossing A is

$$\text{(28)} \qquad dP_f = \frac{k\, T_i'(\sigma)}{\lambda^2}\, A\, \Delta f \left[ d\Omega - \frac{t_i(o)}{R^2} \right] \Delta f \quad .$$

Thus the change in <u>apparent</u> brightness temperature, as measured by a system with an angular resolution of $d\Omega$, from the definition, Eq. (24), is

$$\text{(29)} \qquad \Delta T_i(o) = \left[ \frac{e_i(o) T_b + \langle \sigma T' \rangle_i - t_i(o) T_i'(\sigma')}{R^2} \right] d\Omega^{-1} \quad .$$

The limitations on the magnitude of $d\Omega$ may now be considered. In the first place, the target should be in the far-field of the receiving aperture A, and, conversely, the receiver should be in the far-field of the target. If we take the "aperture" of the target equal to $t_i$, the largest of its cross sections, then the following should hold:

$$\text{(30a)} \qquad R > t_i/\lambda$$

$$\text{(30b)} \qquad R > A/\lambda \quad .$$

Also, the target should not be resolvable by the aperture A, if the apparent brightness is to have significance, so that

$$d\Omega \cong \frac{\lambda^2}{A} > t_i/R^2$$

or

$$\text{(30c)} \qquad \lambda^2 R^2 > A t_i$$

which is actually already implied by conditions (30a) and (30b).

## 5. ANTENNA TEMPERATURES BEFORE AND AFTER AN OBJECT IS PLACED IN THE BEAM

In practice, the antenna of a radiometer system does not have the ideal pattern assumed in Fig. 6. The actual antenna pattern may be specified by its power gain function for radiation of each polarization, viz., $g_1(\theta, \phi)$ and $g_2(\theta, \phi)$. (More elegant methods for treating the response of an antenna to polarized radiation are available, and have recently been discussed by Ko[8] ; the straightforward method is used here because the gain functions correspond to the standard antenna pattern measurements.) If the axis of a polar coordinate system is taken along the main beam direction, as in Fig. 6a, and the antenna is designed to receive polarization state (1) in the main beam direction, then $g_1(0, \phi) = g_m$ ; $g_2(0, \phi) = 0$, where $g_m$ is the maximum gain of the antenna. The antenna temperature $T_a$ can now be written in terms of the pattern functions as

$$T_a = \frac{\sum_1^2 \int g_k(\theta,\phi)\, T_k(\theta,\phi)\, d\Omega_c}{\sum_1^2 \int g_k(\theta,\phi)\, d\Omega_c} \qquad (31)$$

The use of this expression implies the assumption that the two components of the incoming radiation $T_1(c)$ and $T_2(c)$ are uncorrelated; if there is correlation between them, then again the more complete treatment[8] in terms of the statistical matrix or the Stokes parameters is required.

We can now calculate the change in antenna temperature, $\Delta T_a$, due to introduction of an isolated object into the main beam. From Eq. (29), we see that, if it can be assumed that the gain of the antenna is constant over the effective solid angle $t_1/R^2$ subtended by the target, then

$$\Delta T_a = g_m \frac{\Delta\, T_1(o)\, d\Omega}{\sum \int g_k\, d\Omega} \qquad (32)$$

or

$$\Delta T_a = \frac{g_m}{\sum_{1}^{2} \int g_k \, d\Omega} \left[ \frac{e_1 T_b + < \sigma T' >_1 - t_1 T_1'(o')}{R^2} \right] \tag{33}$$

Because of the assumption just mentioned, Eq. (33) for $\Delta T_a$ is correct as long as $<\sigma T'>_1$ is correct, i.e., as long as the two polarizations of radiation incident on the target are uncorrelated.

Notice also that in calculating $T_a$ and $\Delta T_a$, the incident radiation $T_i'(c')$ on the target need not be the same as the incident radiation $T_i(c)$ on the antenna. In practice such a situation might arise if a target at a high altitude were observed with a ground-based radiometer, and there were appreciable atmospheric absorption between object and receiver. In such a situation, of course, it would be necessary to take the absorption into account. For example, if the absorption coefficient for the path R were $\alpha$, the observed change in antenna temperature would be $\Delta T_a(obs) = (1-\alpha)\Delta T_a + \alpha T_{atm}$ where $T_{atm}$ is the temperature of the intervening medium.

A convenient rearrangement of Eq. (33) may be obtained by relating the scattering cross sections to some reference cross-sectional area $A_p$, which is usually taken as the projected area of the object. That is, let

$$\begin{aligned} e_i(o) &= f_{ei}(o) A_p \\ t_i(o) &= f_{ti}(o) A_p \\ <\sigma T'>_i &= <f_s T'>_i A_p \end{aligned} \tag{34}$$

where the f's are dimensionless numbers, often referred to[3] as the efficiency factors for emission or absorption ($f_{ei}$), extinction ($f_{ti}$), etc.

Also, if the radiometer antenna were used as a transmitter, the expression

$$\gamma = \frac{g_m}{\sum \int g_i \, d\Omega} \; \frac{A_p}{R^2} \tag{35}$$

would be just the fraction of total transmitter power intercepted by $A_p$, the geometrical area of the target. Thus one can write

$$(36) \qquad \Delta T_a = \gamma [f_e T_b + < f_s T' > - f_t T_i'(o) ]$$

with the understanding that the f's refer to radiation of the same polarization state as that for which the antenna is designed.

That is, the change of antenna temperature is proportional to the fraction of power intercepted by the reference target, multiplied by the sum of the appropriate temperatures weighted according to their efficiency factors.

## 6. CONCLUSIONS

The reciprocity theorem and the principle of detailed balancing can be used to calculate the emission cross section of an isolated target in terms of its scattering cross sections. The apparent temperature of such an object when it lies within a radiometer beam (or the change in antenna temperature caused by its introduction into the beam) can then also be found from its scattering properties, and the temperature of the thermal radiation incident on it.

## 7. REFERENCES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