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INVESTIGATING THE STABILITY OF A RIBBED CYLINDRICAL
SHELL DURING LONGITUDINAL COMPRESSION

By

I. Ya. Amiro



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UNEDITED ROUGH DRAFT TRANSLATION

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BY: I. Ya. Amiro

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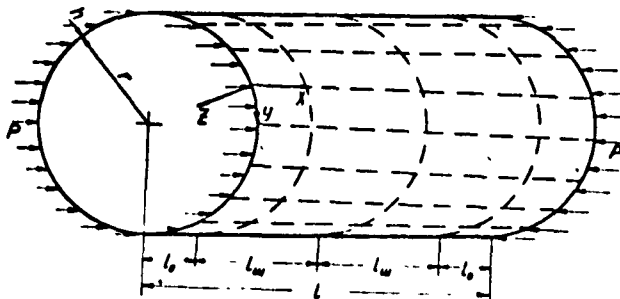
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Investigating the Stability of a Ribbed Cylindrical Shell
during Longitudinal Compression ¹

by

I.Ya.Amiro (Kiev)

In contrast to a majority of known investigations in this report the critical stresses for a ribbed cylindrical shell are designated with consideration of discrete arrangement of reinforcing ribs. This offers the possibility of examining also such types of stability losses, at which the ribs only bend or are only twisted and which cannot be investigated when — bringing the problem down to the calculation of a structurally-orthotropic shell.



Drawing

The problem of designation the critical longitudinal stresses is solved approximately by an energy method. It is considered here, that for cylindrical shells, reinforced by ribs, the critical stresses of total loss in stability can be designated from the viewpoint of small deformations.

1. Let us discuss a ribbed closed cylindrical shell, which has along the butt-ends rigid diaphragms. We will designate the value of uniform distribution of longitudinal compressive stresses p , at which the shell under question may lose its stability.

A ribbed shell (drawing) consists of shell proper (walls) and reinforcing ribs

¹. Report read on January 28, 1960 at the All Union Conference on Theoretical and Applied Mechanics.

(stringers and bulkheads). To examine the wall we utilize the technical theory of thin shells[1]. The problem of investigating the performance of a cylindrical shell can be reduced to finding two functions: buckling w and function of stresses φ . In conformity with the number of unknown components the method leads to two differential equations which bind these functions together. We will confine ourselves to the being satisfied with only one of these equations, with the equations of deformation compatibility

$$\frac{\partial^4 \varphi}{\partial x^4} + 2 \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + \frac{\partial^4 \varphi}{\partial y^4} = - \frac{E}{r} \frac{\partial^4 w}{\partial x^4}, \quad (1)$$

and instead of the equilibrium equation we utilize the minimum conditions of the potential energy of the system under consideration.

2. The potential energy of the system is designated as a function, which is carried out by inner and outer forces when transforming the system from the deformed state at the beginning, to nondeformed state.

The potential energy of internal forces consists of potential energy of the very shell U_0 , potential energy of stringers U_s and potential energy of bulkheads U_b . Assuming that the jointing of ribs with shell assures uniformity of rib deformation corresponding to the deformations of the shell, we find:

$$U_0 = \frac{D}{2} \int_0^{2\pi r} \int_0^l \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dx dy +$$

$$+ \frac{h}{2E} \int_0^{2\pi r} \int_0^l \left\{ \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \right)^2 - 2(1+\nu) \left[\frac{\partial^2 \varphi}{\partial x^2} \frac{\partial^2 \varphi}{\partial y^2} - \left(\frac{\partial^2 \varphi}{\partial x \partial y} \right)^2 \right] \right\} dx dy; \quad (2)$$

$$U_s = \sum_{i=1}^k \int_0^l \left[\frac{B}{2} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{K}{2} \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 + \frac{F}{2E} \frac{\partial^2 \varphi}{\partial y^2} - \nu \frac{\partial^2 \varphi}{\partial x^2} \right]_{y=y_i} dx; \quad (3)$$

$$U_b = \sum_{j=1}^{k_1} \int_0^{2\pi r} \left[\frac{B_1}{2} \left(\frac{\partial^2 w}{\partial y^2} + \frac{w}{r} \right)^2 + \frac{K_1}{2} \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 + \frac{F_1}{2E} \left(\frac{\partial^2 \varphi}{\partial x^2} - \nu \frac{\partial^2 \varphi}{\partial y^2} \right) \right]_{x=x_j} dy. \quad (4)$$

In formulas (2) - (4) are designated: D -cylindrical rigidity on the shell, B, K , and F -rigidity of the stringer during bending and torsion and the area of its later

al cross section; k - number of stringers; B_1 , F_1 and k_1 - corresponding values for bulkheads.

The potential energy of external forces consists of the work of longitudinal forces, which act against the shell, and the work of longitudinal forces which affect the stringers:

$$T = \rho h \int_0^l \int_0^{2\pi} \left[\frac{1}{E} \left(\frac{\partial^2 \varphi}{\partial y^2} - \nu \frac{\partial^2 \varphi}{\partial x^2} \right) - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] dx dy + \\ + \sum_{i=1}^I \rho F \int_0^l \left[\frac{1}{E} \left(\frac{\partial^2 \varphi}{\partial y^2} - \nu \frac{\partial^2 \varphi}{\partial x^2} \right) - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right]_{y=y_i} dx. \quad (5)$$

3. The expression for the bending of the shell, which lost stability, is accepted in form of:

$$w = C_1 \sin \frac{m\pi x}{r} \cos \frac{ny}{r} + C_2 \sin \frac{m\pi x}{l} \sin \frac{ny}{r}. \quad (6)$$

The first item corresponds to symmetrical deformation, and the second one - inversely symmetrical with respect to that diametral symmetrical area of the shell where the origin of the coordinates is situated. Principally these deformations can be considered as independent from each other.

Substituting the term for the bending of the shell (6) in the equation of deformation compatibility (1) and untie in the remaining, we find with consideration of the conditions on the butt-ends a function of stresses in such a form:

$$\varphi = -\frac{\rho y^2}{2} + Er \frac{m^2}{(m^2 + n^2)^2} \sin \frac{mx}{r} \left(C_1 \cos \frac{ny}{r} + C_2 \sin \frac{ny}{r} \right), \quad (7)$$

where

$$m = \frac{m \pi r}{l}$$

Replacing w and φ respectively by (6) and (7), for complete potential energy ($3 = U_c + U_s + U_b + T$) we find

See page 32 for Eq. 8.

$$\begin{aligned}
\mathcal{J} = & \left\{ \frac{\pi l D}{4r^3} (m^2 + n^2)^3 + \frac{\pi h l E}{4r} \frac{m^4}{(m^2 + n^2)^3} + \right. \\
& + \sum_{j=1}^{k_1} \left[\frac{\pi B_1}{2r^3} (n^2 - 1)^3 \sin^2 \frac{m x_l}{r} + \frac{\pi K_1}{2r^3} m^2 n^2 \cos^2 \frac{m x_l}{r} + \right. \\
& + \left. \frac{\pi E F_1 m^4 (m^4 - v n^2)^3}{2r} \frac{(m^2 + n^2)^3}{(m^2 + n^2)^4} \sin^2 \frac{m x_l}{r} \right] - \frac{\pi h l m^3}{4r} \rho \left(C_1^2 + C_2^2 \right) + \\
& + \sum_{l=1}^k \left[\frac{B l}{4r^4} m^4 + \frac{l E F}{4r^3} \frac{m^4 (n^2 - v m^2)^3}{(m^2 + n^2)^4} - \frac{l F m^3}{4r^3} \rho \right] \times \\
& \times \left(C_1 \cos \frac{n y_l}{r} + C_2 \sin \frac{n y_l}{r} \right)^2 + \frac{l K}{4r^4} m^2 n^2 \left(C_1 \sin \frac{n y_l}{r} - C_2 \cos \frac{n y_l}{r} \right)^2 \Bigg\} - \\
& - \left(\frac{l F k}{2E} + \frac{\pi h l r}{E} - \frac{v^2 \pi r F_1 k_1}{E} \right) \rho^2.
\end{aligned}$$

4. To designate the critical value of compressive stresses we utilize the condition of minimum of the potential energy of the system. Bringing the ^{first} derivatives of the potential energy (8) down to zero, by the parameters C_1 and C_2 we will obtain two equations:

$$\begin{cases} (A + A_1)C_1 + HC_2 = 0 \\ HC_1 + (A + A_2)C_2 = 0 \end{cases},$$

where

$$A = \frac{\pi l D}{4r^3} (m^2 + n^2)^2 + \frac{\pi h l E}{4r} \frac{m^4}{(m^2 + n^2)^2} +$$

$$+ \sum_{i=1}^k \left[\frac{\pi B_i}{2r^3} (n^2 - 1)^2 \sin^2 \frac{mx_i}{r} + \frac{\pi K_i}{2r^3} m^2 n^2 \cos^2 \frac{mx_i}{r} + \right.$$

$$\left. + \frac{\pi E F_i}{2r} \frac{m^4 (n^2 - \nu n^2)^2}{(m^2 + n^2)^4} \sin^2 \frac{mx_i}{r} \right] - \frac{\pi h l}{4r} m^2 p; \quad (10)$$

$$A_1 = \sum_{i=1}^k \left\{ \left[\frac{l B_i}{4r^4} m^4 + \frac{l E F_i}{4r^3} \frac{m^4 (n^2 - \nu m^2)^2}{(m^2 + n^2)^4} - \frac{l F_i m^2}{4r^3} p \right] \cos^2 \frac{ny_i}{r} + \right.$$

$$\left. + \frac{l K_i}{4r^4} m^2 n^2 \sin^2 \frac{ny_i}{r} \right\}; \quad (11)$$

$$A_2 = \sum_{i=1}^k \left\{ \left[\frac{l B_i}{4r^4} m^4 + \frac{l E F_i}{2r^3} \frac{m^4 (n^2 - \nu m^2)^2}{(m^2 + n^2)^4} - \frac{l F_i m^2}{4r^3} p \right] \sin^2 \frac{ny_i}{r} + \right.$$

$$\left. + \frac{l K_i}{4r^4} m^2 n^2 \cos^2 \frac{ny_i}{r} \right\}; \quad (11a)$$

$$H = \left(\frac{l B}{4r^4} m^4 + \frac{l E F}{2r^3} \frac{m^4 (n^2 - \nu m^2)^2}{(m^2 + n^2)^4} - \frac{l F m^2}{4r^3} p - \frac{l K}{4r^4} m^2 n^2 \right) \sum_{i=1}^k \sin \frac{ny_i}{r} \cos \frac{ny_i}{r}. \quad (12)$$

Since the distances between the stringers are identical, then the ordinate of the i -stringer equals $y_i = \frac{2\Omega x}{k} (i-1)$. Under such a condition it is easy to show that

$$\sum_{i=1}^k \sin \frac{ny_i}{r} \cos \frac{ny_i}{r} = 0 \quad (13)$$

coefficient $H = 0$. The system of equations (9) splits into two independent equations:

$$(A + A_1)C_1 = 0 \quad (14); \quad (A + A_2)C_2 = 0. \quad (15)$$

Equations (14) correspond to the symmetrical, and equations (15) - inversely symmetrical form of deformation (with respect to this area of shell symmetry, is situated the origin of the coordinates).

Losses in shell stability correspond to the change from zero value C_1 and C_2 . In this way, the equations to designate the critical stresses of condition $A + A_1 = 0$ and $A + A_2 = 0$.

5. The general case of shell deformation during loss in stability resulting in deformation, at which the ribs reinforcing the shell become simultaneously bent and distorted. The corresponding form of deformation is described by equations (6), because the number of half-waves along the ring $2n$ will not be a multiple to the number of stringers ($2m \neq n_1 k$), and the number of half waves along the shell m will not be a multiple: a) at $l_0 = l_b$ number of bulkheads, increased by the unit ($m \neq n_1 k_1 + 1$); b) at $l_0 = \frac{l_b}{2}$ number of bulkheads ($m \neq n_1 k_1$). Under such conditions it can be shown, that the sums, which are included in terms (10)-(12), pertain to:

$$\sum_{i=1}^k \cos^2 \frac{n y_i}{r} = \sum_{i=1}^k \sin^2 \frac{n y_i}{r} = \frac{k}{2}; \quad (16)$$

$$a) \text{ при } l_0 = l_b \quad \sum_{i=1}^{k_1} \cos^2 \frac{m x_i}{r} = \frac{k_1 - 1}{2}, \quad \sum_{i=1}^{k_1} \sin^2 \frac{m x_i}{r} = \frac{k_1 + 1}{2}; \quad (17)$$

$$b) \text{ при } l_0 = \frac{l_b}{2} \quad \sum_{i=1}^{k_1} \cos^2 \frac{m x_i}{r} = \sum_{i=1}^{k_1} \sin^2 \frac{m x_i}{r} = \frac{k_1}{2}. \quad (18)$$

Expressions for A , A_1 and A_2 with inclusion of (16)-(18) is equalized:

$$A = \frac{\pi l D}{4 r^3} (m^2 + n^2)^2 + \frac{\pi h l E}{4 r} \frac{m^4}{(m^2 + n^2)^3} + \frac{\pi B_1}{4 r^3} (n^2 - 1)^2 k_2 + \\ + \frac{\pi K_1}{4 r^3} m^2 n^2 (2k_1 - k_2) + \frac{\pi E F_1}{4 r} \frac{m^4 (m^2 - \nu n^2)^2}{(m^2 + n^2)^4} k_2 - \frac{\pi h l}{4 r} m^2 p; \quad (19) \\ A_1 = A_2 = \left[\frac{l B}{8 r^4} m^4 + \frac{l K}{8 r^4} m^2 n^2 + \frac{l E F}{4 r^3} \frac{m^4 (n^2 - \nu m^2)^2}{(m^2 + n^2)^4} - \frac{l F m^2}{4 r^3} p \right] k. \quad (20)$$

Тут і дані $k_2 = k_1 + 1$ при $l_0 = l_b$ та $k_2 = k_1$ при $l_0 = \frac{l_b}{2}$.

Here $k_2 = k_1 + 1$ at $l_0 = l_b$ and $k_2 = k_1$ at $l_0 = \frac{l_b}{2}$

Since $A_1 = A_2$, then in the general case of deformation of the shell the losses in stability from (14) and (15) we obtain one equation for the designation of critical stresses: $A + A_1 = 0$. Substituting instead of A and A_1 the expressions (19) and (20) and designating from the obtained equation the critical longitudinal stresses, we

find:

$$\rho_{xp} = \frac{Et}{1 + \gamma k} \left\{ \frac{t}{12(1 - \nu^2)} \left[\Phi + (\alpha m^2 + \beta n^2) k + \alpha_1 k_1 \frac{(n^2 - 1)^2}{m^2} + \beta_1 (2k_1 - k_2) n^2 \right] + \frac{1}{t\Phi} (1 + \gamma k \Phi_1 + \gamma_1 k_1 \Phi_2) \right\}. \quad (21)$$

here
Tyr

$$\left. \begin{aligned} t &= \frac{h}{r}; \quad \alpha = \frac{B}{2\pi r D}; \quad \beta = \frac{K}{2\pi r D}; \quad \gamma = \frac{F}{2\pi r h} \\ \alpha_1 &= \frac{B_1}{l D}; \quad \beta_1 = \frac{K_1}{l D}; \quad \gamma_1 = \frac{F_1}{l h} \\ \Phi &= \frac{(m^2 + n^2)^2}{m^2}; \quad \Phi_1 = \frac{(n^2 - \nu m^2)^2}{(m^2 + n^2)^2}; \quad \Phi_2 = \frac{(m^2 - \nu n^2)^2}{(m^2 + n^2)^2} \end{aligned} \right\}. \quad (22)$$

In each concrete case for m and n it is necessary to select such numbers (integrals) which correspond to the least value (21).

For a shell without ribs $\alpha = \beta = \gamma = \alpha_1 = \beta_1 = \gamma_1 = 0$ and the formula for critical stresses has the form of

$$\rho_{xp} = Et \left[\frac{t\Phi}{12(1 - \nu^2)} + \frac{1}{t\Phi} \right]. \quad (23)$$

If R_{or} is to be considered as a continuous function of ϕ , then the smallest value in expression (23) will be derived, when

$$\Phi = \frac{2}{t} \sqrt{3(1 - \nu^2)}. \quad (24)$$

In this case for critical stresses we have the known formula (for [2], p.403)

$$\rho_{xp} = \frac{Et}{\sqrt{3(1 - \nu^2)}}. \quad (25)$$

6. In addition to the general case of shell deformation, when the stringers (ribs) reinforcing the shell bend and distort simultaneously it is necessary to take into consideration other instances of shell deformation during loss in stability. In these individual cases the stringers or bulkheads or the first and the second together only bend or become distorted. Such forms of stability losses are described by formula (6), if for m and n are taken numbers which satisfy certain ratios. . Accepting, for example, for the half-waves along the rings numbers multiple to the number of stringers ($2n = n_1 k$), we obtain a form of deformation, at which the stringers either bend (at $C_2=0$) or only twist (at $C_1=0$). Such individual cases of deformation of shells may be eight (8) in toto (table 1.)

TABLE 1.

1. Stringers are only bent
2. Stringers only twisted
3. Bulkheads only bent*
4. Bulkheads only twisted
5. Stringers only bent
Bulkheads only twisted
6. Stringers and bulkheads
only bent*
7. Stringers and bulkheads
only bent*
8. Stringers and bulkheads only twisted
9. General
10. Instances of deformation
11. Conditions for numbers of half-waves

*This type of deformation is possible only in case b), when $l_0 = \frac{l_b}{2}$.

10. Випадки деформації	11. умови для чисел півхвиль	n	s	s_1	s_2	
9. Загальний	a) $m \neq m_1(k_1+1)$ б) $m \neq m_1 k_1$	$2n \neq n_1 k$	$1 + \gamma k$	$(\alpha m^2 + \beta n^2)k$	$\alpha_1 k_2 \frac{(n^2-1)^2}{m^2} + \beta_1 (2k_1 - k_2)n^2$	$\gamma k \Phi_2 + \gamma_1 k_2 \Phi_2$
1. Стрингери тільки згинаються	a) $m \neq m_1(k_1+1)$ б) $m \neq m_1 k_1$	$2n = n_1 k$	$1 + 2\gamma k$	$2\alpha k m^2$	$\alpha_1 k_2 \frac{(n^2-1)^2}{m^2} + \beta_1 (2k_1 - k_2)n^2$	$2\gamma k \Phi_1 + \gamma_1 k_2 \Phi_2$
2. Стрингери тільки закручуються	a) $m \neq m_1(k_1+1)$ б) $m \neq m_1 k_1$	$2n = n_1 k$	1	$2\beta k n^2$	$\alpha_1 k_2 \frac{(n^2-1)^2}{m^2} + \beta_1 (2k_1 - k_2)n^2$	$\gamma_1 k_2 \Phi_2$
3. Шпангоути тільки згинаються *	a) — б) $m = (2m_1 - 1)k_1$	$2n \neq n_1 k$	$1 + \gamma k$	$(\alpha m^2 + \beta n^2)k$	$2\alpha_1 k_1 \frac{(n^2-1)^2}{m^2}$	$\gamma k \Phi_1 + 2\gamma_1 k_1 \Phi_2$
4. Шпангоути тільки закручуються	a) $m = m_1(k_1+1)$ б) $m = 2m_1 k_1$	$2n \neq n_1 k$	$1 + \gamma k$	$(\alpha m^2 + \beta n^2)k$	$2\beta_1 k_1 n^2$	$\gamma k \Phi_1$
5. Стрингери тільки згинаються, шпангоути тільки закручуються	a) $m = m_1(k_1+1)$ б) $m = 2m_1 k_1$	$2n = n_1 k$	$1 + 2\gamma k$	$2\alpha k m^2$	$2\beta_1 k_1 n^2$	$2\gamma k \Phi_1$
6. Стрингери тільки закручуються, шпангоути тільки згинаються *	a) — б) $m = (2m_1 - 1)k_1$	$2n = n_1 k$	1	$2\beta k n^2$	$2\alpha_1 k_1 \frac{(n^2-1)^2}{m^2}$	$2\gamma_1 k_1 \Phi_2$
7. Стрингери і шпангоути тільки згинаються *	a) — б) $m = (2m_1 - 1)k_1$	$2n = n_1 k$	$1 + 2\gamma k$	$2\alpha k m^2$	$2\alpha_1 k_1 \frac{(n^2-1)^2}{m^2}$	$2(\gamma k \Phi_1 + \gamma_1 k_1 \Phi_2)$
8. Стрингери і шпангоути тільки закручуються	a) $m = m_1(k_1+1)$ б) $m = 2m_1 k_1$	$2n = n_1 k$	1	$2\beta k n^2$	$2\beta_1 k_1 n^2$	0

Offering no accurate conclusions for the individual cases, conclusions which are perfectly similar to the one given above in the general case, we will write the formula for critical stresses in form of

$$p_{kp} = \eta E t, \quad (26)$$

where the coefficient η , which for a ribless shell, as is evident, equals $\eta = \sqrt{\frac{1}{3(1-\nu^2)}}$ for the discussed finned (ribbed) shells are designated by formula

$$\eta = \frac{1}{\kappa} \left[\frac{t}{12(1-\nu^2)} (\Phi + S + S_1) + \frac{1}{t\Phi} (1 + S_2) \right]. \quad (27)$$

The coefficient κ , S , S_1 and S_2 of formula (27) are designated in conformity with with the case of deformation from table 1.

The numbers of half-waves m and $2m$, which satisfy for each case of deformation certain ratios (table 1), are selected so, that η would have the smallest values. From 9 possible cases of deformation (one general and eight individual) during loss in stability we find this case to which the lowest value of critical stresses corresponds.

7. We will investigate the stability of closed cylindrical shells on concrete examples. We will consider at first a cylindrical shell ($r = 100$ cm, $h = 0.4$ cm), which is reinforced only by longitudinal ribs ($B = 33.3$ E; $K = 50$ G; $F = 6.98$ cm²). Assuming that the Poisson coefficient equals $\nu = 0.1$ we will designate by formulas (22) the ~~the~~ dimensionless parameters of the shell and stringers: $t = 0.004$; $\alpha = 9.06$; $\beta = 5.23$; $\gamma = 0.0278$; $\alpha_1 = \beta_1 = \gamma_1 = 0$. For such a shell, in addition to the general case of deformation, it is necessary to take under consideration two individual ones (the first and second in table 1).

For shells of three lengths at different number of reinforcing ribs in table 2 are given the lowest values of the coefficient η , which corresponds to three discussed cases of deformation of a shell during the loss in stability. The lowest of the three value η for each number of ribs is designated in thick letters. For each η are given corresponding numbers of half-waves m along the shell, and the number of

waves n along the ring. In parentheses for the purpose of comparison are given the values.....calculated without consideration of the members, which depend upon the energy along the deformation of ribs, i.e. members proportional to the coefficient S_2 in formula (27). On the basis of analysing data, given in table 2, it is possible to make the following conclusions.

The critical stresses (26) for a shell, reinforced only by longitudinal ribs, depend upon the length of the shell with an increase in length the coefficient η , as a rule, decreases.

In the general case of deformation with a rise in the number of longitudinal ribs there is also a rise in the critical stresses. Not at any given number of ribs we do have the the genarl form of stability loss: at small twin number of longitudinal ribs the critical stresses, which correspond to the individual form of deformation, at which the shell reinforcing ribs only bend, appear to be smaller, than in the general case of deformation; in this case the transformation from unpaired to greater by a unit of a paired number of ribs is accompanied by a considerable reduction in critical stresses. For a great number of longitudinal ribs (this number depends upon the length of the shell) dyping loss in stability we have the general form of deformation, because these forms correspond to least critical stresses.

Table 2.

1. Cases of deformation

2. General	3. individual	
	4. First	5. Second

SEE PAGE 9a FOR TABLE 2.

TABLE 2.

	1. Випадки деформації								
	2. Загальний			3. Окремі					
				4. перший			5. другий		
	γ	т	л	γ	т	л	γ	т	л
15	1,554 (1,415)	1	6	1,658 (1,632)	2	15	13,54	8	15
16	1,591 (1,445)	1	6	0,650 (0,564)	1	8	4,56	1	8
23	1,807 (1,628)	1	6	3,865 (3,852)	3	23	47,5	12	23
24	1,836 (1,647)	1	6	1,178 (1,152)	1	12	13,85	3	12
31	2,003 (1,784)	1	6	6,645 (6,440)	3	31	113,8	15	31
32	2,101 (1,866)	1	6	1,882 (1,852)	2	16	32,0	8	16
15	1,074 (0,977)	1	5	1,658 (1,632)	4	15	13,54	15	15
16	1,098 (0,995)	1	5	0,650 (0,564)	2	8	4,56	2	8
23	1,223 (1,095)	1	5	3,699 (3,659)	5	23	47,5	23	23
24	1,240 (1,107)	1	5	1,111 (1,064)	3	12	13,85	6	12
31	1,339 (1,184)	1	5	6,445 (6,440)	6	31	113,8	31	31
32	1,351 (1,194)	1	5	1,747 (1,728)	3	16	32,0	16	16
15	0,840 (0,773)	1	4	1,635 (1,613)	7	15	13,54	30	15
16	0,851 (0,780)	1	4	0,628 (0,573)	3	8	4,56	4	8
23	0,910 (0,822)	1	4	3,669 (3,659)	10	23	47,5	46	23
24	0,918 (0,827)	1	4	1,059 (1,023)	5	12	13,85	13	12
31	0,964 (0,859)	1	4	6,409 (6,403)	13	31	113,8	63	31
32	0,970 (0,863)	1	4	1,747 (1,729)	6	16	32,0	32	16

Values of the coefficient η , calculated without consideration of members, which depend upon the energy of longitudinal rib deformation in most unfavorable cases differ by 10-13%. Since the energy method gives a higher value of critical stresses, then it is permissible for practical calculations to utilize simplified formulas, which have a small flexure in direction of reduction.

8. Let us examine a cylindrical closed shell ($r = 100$ cm; $l = 628$ cm; $h = 0.4$ cm), reinforced by 24 longitudinal ribs and a different number of annular ribs ($k_1 = 1, 2, 3$). Assuming that stringers and bulkheads give lateral cross section so, as stringers in the foregoing shell, according to formulas (22) we designate the non-dimensional parameters of the bulkheads: $\alpha_1 = 9.06$;

$B_1 = 5.23$; $\gamma_1 = 0.0278$. Since the distance of the end bulkheads from the frontal diaphragms is equal to the distance between the bulkheads ($l_0 = l_b$), then, in addition to the general case of deformation, it is necessary to take into consideration also five individual ones.

In table 3 are listed the lowest values of the coefficient η , which correspond to six investigated instances of shell deformation during loss in stability. For the purpose of comparison are given values η , which correspond to upper critical stresses, designated by formulas for structural-orthotropic shells (see [3], page 476). For each η is given the corresponding number of half waves m along the shell and numbers of waves n along the circle.

On the basis of analyzing data, given in table 3, we arrive at a conclusion, that critical stresses, calculated by formulas for structural-orthotropic shells, are quite close * to each, which are designated for the general case of shell deformation. But the lowest critical stresses may correspond not to the general, but to one of the individual cases of shell deformation during loss in stability. These

* Critical stresses, calculated by formulas for structural-orthotropic shell, appear to be somewhat lower than the ones, whose formulas do not consider the resistance of ribs to twisting.

lowest critical stresses are quite different from the stresses of the general case (for the shell under question by 40%).

Table 3. (SEE PAGE 12 FOR TABLE 3.)

1. Cases of deformation

2. General

3. Individual

4. ORTHOTROPIC SHELL

In this way, examination of the structurally-orthotropic shell, which made it possible to investigate only the general case of shell deformation, at which the ribs reinforcing the shell bend and twist simultaneously, may be insufficient for general analysis of a ribbed shell stability. The formulas suggested above allow, in addition to the general, to examine also individual instances of deformation and determine forms of deformation, to which least critical stresses do correspond.

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TABLE 3.

1. Випадки деформації		$k_1=1$			$k_1=2$			$k_1=3$		
		η	m	n	η	m	n	η	m	n
2. Загальний		2,05	1	3	2,56	1	3	2,89	3	1
3. Окремі	1	4,27	11	12	5,32	11	12	6,12	13	12
	2	14,0	59	12	14,4	62	12	14,7	66	12
	4	1,29	2	5	1,67	3	5	2,06	4	6
	5	1,33	6	12	1,56	6	12	1,85	4	12
	8	14,4	12	12	14,9	12	12	15,5	12	12
4. Ортотропна оболонка		1,91	1	3	2,41	1	3	2,60	3	1

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